

Course - CVL867
Assignment 2 Solutions
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2022SRZ8573

February 15, 2023

Code:

0.1 Description of code:

- a) `utils/rng.py` — functions to generate random number
- b) `utils/random_walk.py` — Implements random walk in 1d and higher dimensions. Functions for finding when collision will happen and mean square displacement of the random walk.
- c) `utils/plot.py` — utility functions to plot
- d) `main.py` — i) Function returning list of average displacement for multiple simulations of different jump lengths. ii) Implements all the questions at one place, inside main function.

0.2 How to run the code:

`python main.py 1` : return solution for Q1
`python main.py 2` : return solution for Q2
... so on till Q6

- 1 **Implement an algorithm to run a 1D random walk with 1000 jumps. Take all constants equal to 1. Plot x and x^2 with respect to the step number.**

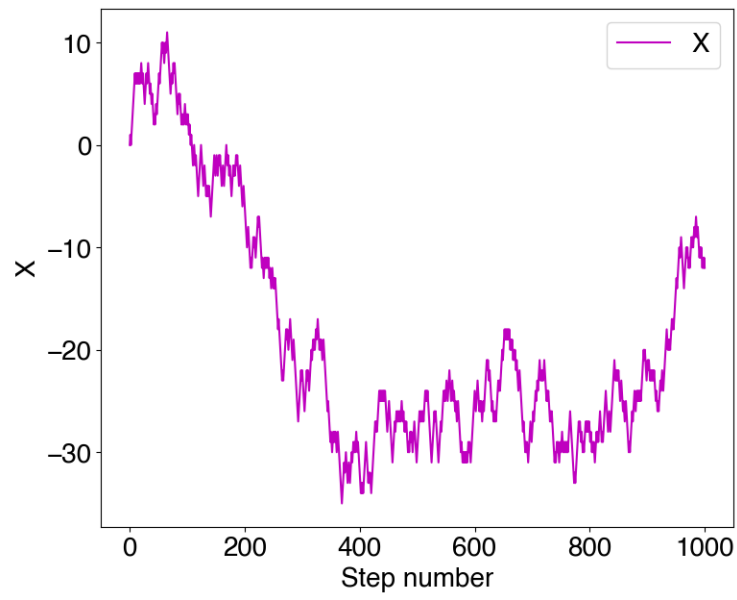


Figure 1: X v/s Step

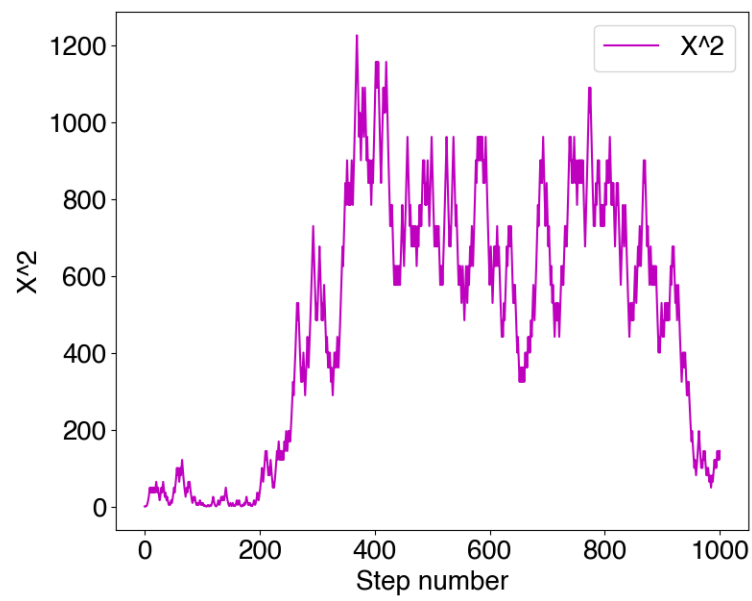


Figure 2: X^2 v/s step

2 By averaging multiple simulations, check that the mean-square displacement tends to a linear function of the number of steps, with a slope of 1.

MSD(N): mean square displacement after step N

a(i) : random ith step: either -1 or 1

$$\begin{aligned}
 \text{MSD}(N) &= \langle (a(1) + a(2) + \dots + a(N))^2 \rangle \\
 &= \left\langle \left(\sum_{i=1}^N a_i \right)^2 \right\rangle \\
 &= \left\langle \sum_{i=1}^N a_i^2 + 2 \sum_{i < j} a_i a_j \right\rangle \\
 &= \sum_{i=1}^N \langle a_i^2 \rangle + 2 \sum_{i < j} \langle a_i a_j \rangle \\
 &= N \langle a^2 \rangle + 0 \\
 &= N(0.5 * (-1)^2 + 0.5 * (+1)^2) \\
 &= N
 \end{aligned}$$

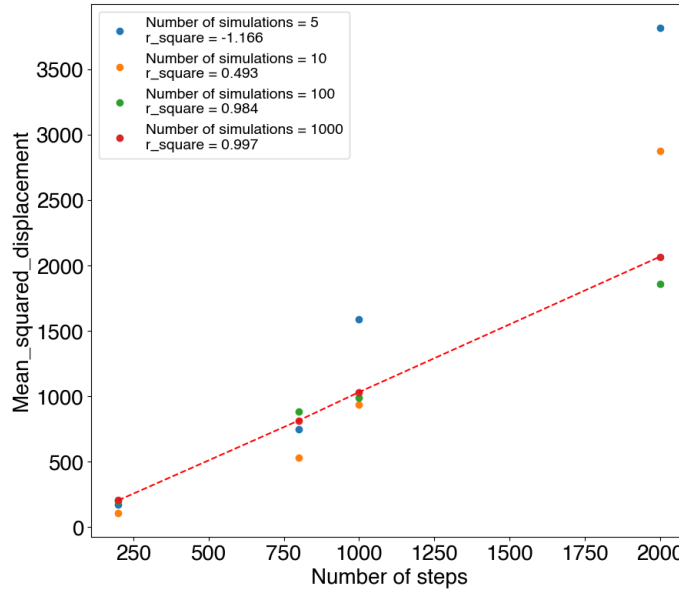


Figure 3: Plot showing Rsquare tending to 1 as number of simulations increasing.

- 3 Check the effect of a “biased” random walk, that is, when the probability to jump to the right is not 50% (try values between 0 and 100%). How does it affect the shape of the average mean-square displacement with respect to the step number?

Note: when $p = 0.5 \rightarrow$ unbiased case, above $\text{MSD}(t)$ is linear in t .

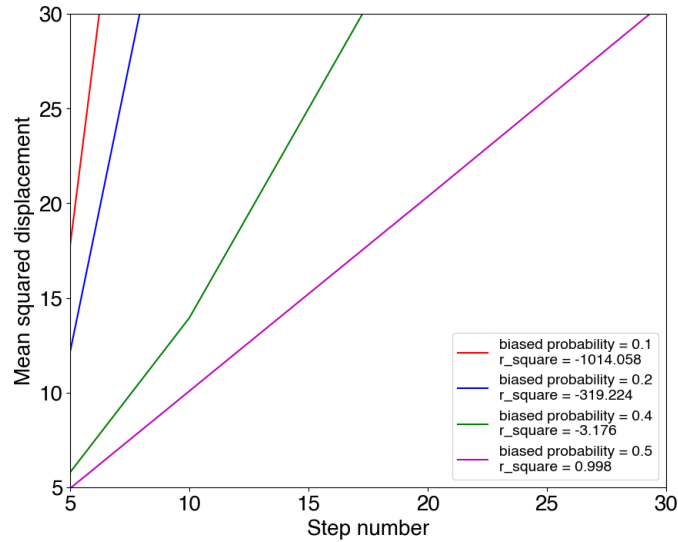


Figure 4: Mean squared displacement v/s step number averaged over multiple simulations. Also see the associated r_square value in plot. Biased probability = 0.5 implies unbiased random walk, thus is linear.

- 4 Implement a 2D random walk with 1000 jumps. Show an example of particle trajectory path

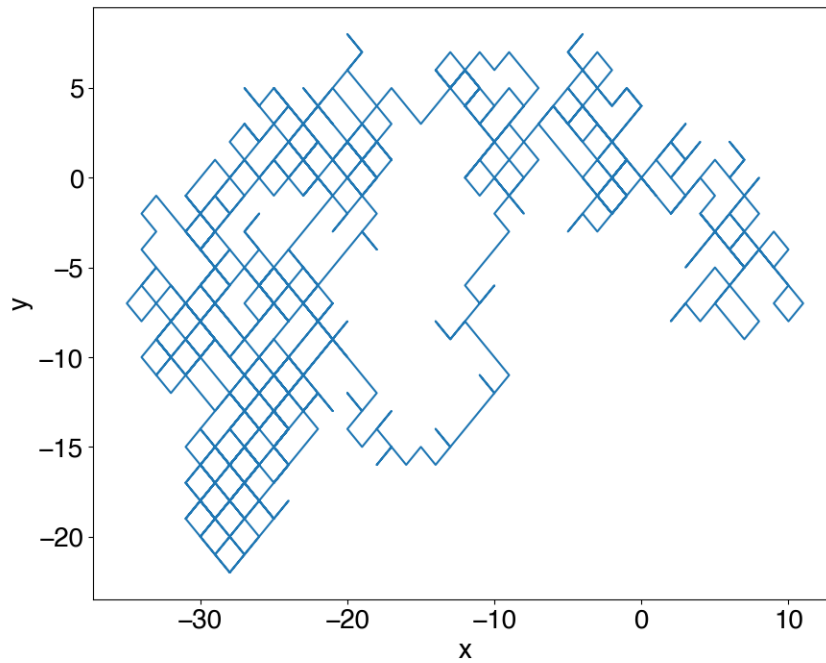


Figure 5: Trajectory of 2D random walk with 1000 jumps. Plot: y v/s x .

- 5 Assume a 10 by 10 square grid with periodic conditions. One drunk sailor is initially placed in (0,0), and a second one in (5,5). By performing multiple simulations, determine the average number of steps after which the two sailors bump into each other.

Algorithm 1 simulate_collision2d

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1: function SIMULATE_COLLISION2D(num_jumps, seed, grid)
2:   Initialize parameters:
3:   - sailor1_traj  $\leftarrow [0, 0]$ 
4:   - sailor2_traj  $\leftarrow [5, 5]$ 
5:   for step  $\leftarrow 0$  to num_jumps - 1 do
6:     sailor1_traj  $\leftarrow$  (sailor1_traj + [random(-1, 1), random(-1, 1)]) mod grid
7:     sailor2_traj  $\leftarrow$  (sailor2_traj + [random(-1, 1), random(-1, 1)]) mod grid
8:     if sailor1_traj = sailor2_traj then
9:       return step + 1
10:    end if
11:  end for
12:  return num_jumps
13: end function

```

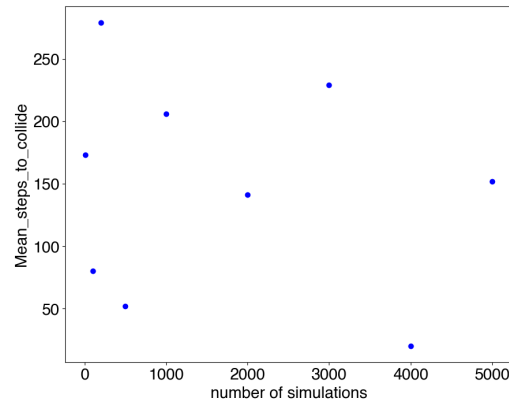


Figure 6: Showing number of mean steps to collide when total jumps are 1000. Value's on Y-axis represent mean across all the simulations.

6 Use Park–Miller random number generator to generate a series of 200 random number with initial seed=71, $a=18$, $m=167$. Find the period of the obtained series. Find the minimum value of ‘ a ’ for which the period is maximum.

6.1 Period

Period of obtained series: 83 (see red dashed vertical line in figure 7)

Period of a random number generator is the number of steps after which the random numbers start to repeat. I solved it by checking if the new random number generated is present in the set of random number that have been generated till now.

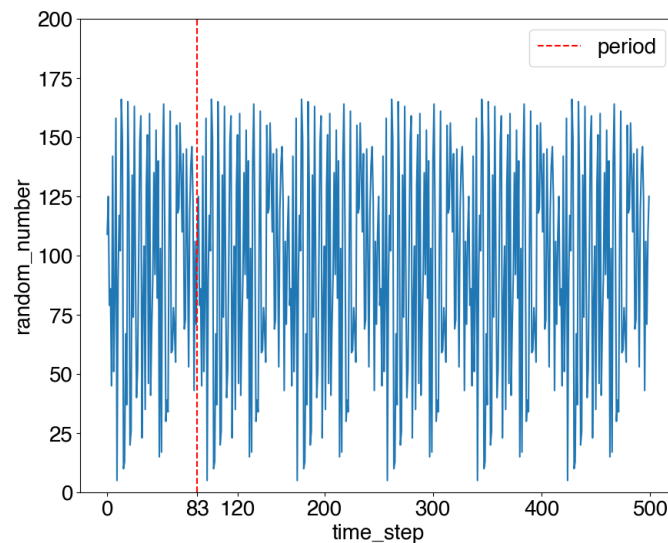


Figure 7: Identifying the period of random number generator

6.2 Minimum “ a ” for max period

Minimum value of “ a ” for which period is maximum: 71 (see red dashed vertical line in figure 8)

I repeated the Q6 part “1” with different values of “ a ” and then plotted it to find the solution.

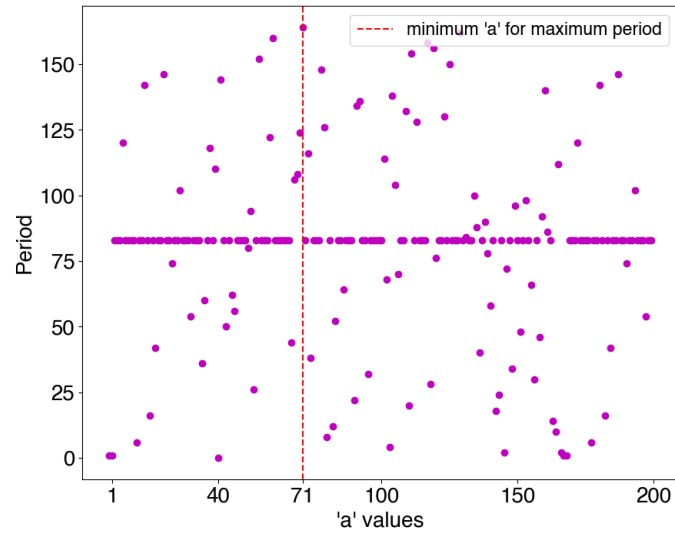


Figure 8: Finding out minimum "a" for which period is maximum