ASDS 6304 Project 1 Due Sep. 10th, 2024

Requirement of the report:

1. All the reports need to be submitted in PDF.
2. In the report, the contribution of each person needs to be listed clearly.
3. For all the parameters or variables used in your model, please write down their definitions and meanings clearly.
4. For each question, you need to summarize your steps, models or the method you use. Put the corresponded coding and graphs after your solutions for each question.
5. If your report is not well-organized, you will not get any credits.

Q1: Given $f(x) = 2x^4 - 10x^2 + x - 5$, draw the curve and find $\min f(x)$ by the package SciPy in python. Interpret your result.

Q2: Given $f(x, y) = 2x^4 - 10y^2 + x - y - 25$,.

(a) draw the graph of $f(x, y)$. Hint: when drawing the graph, you need to import the function Axes3D from mpl_toolkits.mplot3d.

(b) $\min f(x, y)$ and $\max f(x, y)$. If one of them can't be solved, please add some constraints so that you can solve it.

(c) When min f(x,y), show which method you choose. Try some other methods and check if there is difference. Interpret your result.

Q3: Given the problem

$$\min(x-2)^2 + (y-1)^2$$
$$s.t.\, y - (x+1)^2 \geq 0, x + 1 \leq y$$

(a) Draw the feasible region

(b) Solve the problem by using the function minimize from scipy.optimize

(c) Interpret your result.

Q4: Given linear program problem:

$\min 2x + 3y,\ s.t.\, x + y \leq 1, y - x \geq 0, x \geq 0$

(a) Draw the feasible region

(b) Solve it by using the function linprog from Scipy.optimize

(c) Interpret your result

Q5: A small company has developed two versions of a new product. Each version of the product is made from the same raw material that costs $10 per gram and requires two different types of specialized labor. U is the higher-priced version of the product. U sells for $270 per unit and requires 10g of raw material, 1 hour of labor type A, and 2 hours of labor type B. Due to the higher price, the market demand for U is limited to 40 units per week. V is the lower-priced version of the product that sells for $210 per unit with unlimited demand and requires 9g of raw material, 1 hour of labor type A, and 1 hour of labor type B. These data are summarized in Table 1.1.

Product version	Raw material required	Labor A required	Labor B required	Market demand	Price
U	10 g	1 hour	2 hours	≤40 units	$270
V	9 g	1 hour	1 hour	unlimited	$210

Table 1.1 *Summary of the problem data and parameters.*

The availability of labor and the raw materials inventory limits weekly production in the company. The raw material must be ordered in advance and has a short shelf life. Any raw material left over at the end of the week is discarded. Table 1.2 details the cost and availability of raw materials and labor.

Resource	Amount available	Cost
Raw Material	No limits	$10 / g
Labor A	80 hours	$50 / hour
Labor B	100 hours	$40 / hour

Table 1.2 *Summary of the cost and availability of raw material and labor.*

The company wants to maximize its gross profit. How much raw material should be ordered in advance for each week? How many units of U and V should the company produce each week?

(a) build an optimization problem and solve it. When you build your math model, you should list the unknown variables with symbols, description, and any lower or upper bounds known from the problem data. You need to state the steps that you build your objective functions and give the math expression of the objective function and list all the possible constraints you know from this problem.

(b) solve your optimization problem by the module linprog in scipy.optimize. Please also show the feasible region as well in your work. Interpret your results.

Bonus Question:
Given a vector-valued function:

$$f(x,y) = \begin{pmatrix} (x-1)^2 + (y+2)^2) \\ x^2 \end{pmatrix}$$

Minimize $f(x,y)$with the constraints that $x + y \geq 1$ and $x^2 + y^2 \leq 4$.