# ASDS 6304 Project 3
# Due Oct. 25, 2024

Submission requirements:

(a) One pdf or word with all the answers. List your answers with question orders clearly.
(b) The coding file needs to submit.
(c) Write down both of group members name and group number.
(d) One day late submission will have 10% penalty.

1. Compute the gradient $\nabla f(x)$ and Hessian $\nabla^2 f(x)$ of the Rosenbrock function
$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$
   (a) Show that $x^* = (1,1)^T$ is the only local minimizer of this function, and that the Hessian matrix at this point is positive definite
   (b) Is the function convex or not? Justify your answer.
   (c) Plot the contour of this function for the interval chosen by yourself.
2. Show that the function $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$ has only one stationary point and that it is neither a maximum nor minimum, but a saddle point. Sketch the contour lines of $f$.
3. Let $a$ be a given n-vector, and $A$ be a given $n \times n$ symmetric matrix. Compute the gradient and Hessian of $f_1(a) = a^T x$ and $f_2(x) = x^T Ax$. Are both function convex or not? Justify your answer. If one is not convex, can you add some conditions so that the one becomes convex?
4. Program the steepest descent algorithms using the backtracking line search, Algorithm 3.1 in the slide. Use them to minimize the Rosenbrock function in (1). Set the initial step length $\alpha_0 = 1$ and print the step length used by each method at each iteration. First try the initial point $x_0 = (1.2,1.2)^T$, and then the more difficult starting point $x_0 = (-1.2,1)^T$. Compare the result you get by using trust region method from the package SciPy. Minimize in python. Make your conclusions.
5. Load the dataset California Housing dataset from sklearn. datasets. You need to figure out which is the target variable, which are independent variable, and run preprocessing on the dataset before you start build your model.
   (a) Build linear regression model. Write down the objective function in the form of matrix and vectors and explain your model. Find the gradient and Hessian of your objective function.
   (b) Build lasso regression model. Write down the objective function in the form of matrix and vectors and explain your model. Find the gradient and Hessian of your objective function.
   (c) Build ridge regression model. Write down the objective function in the form of matrix and vectors and explain your model. Find the gradient and Hessian of your objective function.
   (d) Which objective functions are convex, and which are not? Justify your answers.
   (e) Solve the above three models by the built-in functions in python. Solve the above three models the trust region method in the scipy.minimize. If you need to choose some parameters, please state that why you choose those specific values.

(f) Compare the computational results you get from (e) and make your conclusion. Which model is the best for this dataset? Justify your answer.

Some information about the California Housing Data:
**MedInc**: Median income of households in the block group (measured in tens of thousands of dollars). This is a continuous variable.
Example: A value of 3.2 means the median income for households in that block is $32,000.
**HouseAge**: Median age of the houses in the block group. This is a continuous variable.
Example: A value of 25 means that the median age of the houses in that block is 25 years.
**AveRooms**: Average number of rooms per household in the block group. This is a continuous variable.
Example: A value of 6 means that, on average, there are 6 rooms per household in that block.
**AveBedrms**: Average number of bedrooms per household in the block group. This is a continuous variable.
Example: A value of 2 means that, on average, there are 2 bedrooms per household in that block.
**Population**: Total population of the block group. This is a continuous variable.
Example: A value of 1500 means that there are 1,500 people living in that block.
**AveOccup**: Average number of occupants per household in the block group. This is a continuous variable.
Example: A value of 3 means that, on average, each household in that block has 3 occupants.
**Latitude**: Geographical latitude of the block group. This is a continuous variable.
Example: A value of 34.19 means that the block group is located at 34.19 degrees latitude.
**Longitude**: Geographical longitude of the block group. This is a continuous variable.
Example: A value of -118.39 means that the block group is located at -118.39 degrees longitude.
**MedHouseVal**: Median house value in the block group (target variable), measured in hundreds of thousands of dollars. This is a continuous variable.
Example: A value of 2.0 means that the median house value for households in that block is $200,000.