R .V. COLLEGE OF ENGINEERING, BENGALURU- 560 059
(Autonomous Institution Affiliated to VTU, Belagavi)

TITLE: EULER'S POLYGON DIVISION PROBLEM

Seminar/Assignment Report

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CERTIFICATE

Certified that the Assignment topic **"EULER'S POLYGON DIVISION PROBLEM"** is carried out by **UTKARSH SINGH (1RV18CS181)** who are bonafide students of **R V College of Engineering, Bengaluru** in partial fulfilment of the award of seminar/assignment marks for the **3rd semester academic year 2019 - 20 in Discrete Mathematical Structures Course.** It is certified that all corrections/ suggestions indicated for the internal assessment have been incorporated in the report, and a soft copy is deposited in the department library. The seminar/assignment report has been approved as it satisfies the academic requirement in respect of the work prescribed by the institution for the said course.

Marks awarded:

Cos	CO1	CO2	CO3	CO4	Total
Max. Marks	03	02	01	04	10
Max. obtd.					

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INTRODUCTION

Whenever mathematicians hit on an invariant feature, a property that is true for a whole class of objects, they know that they're onto something good. They use it to investigate what properties an individual object can have and to identify properties that all of them must have. Euler's formula can tell us, for example, that there is no simple polyhedron with exactly seven edges. You don't have to sit down with cardboard, scissors and glue to find this out — the formula is all you need. The argument showing that there is no seven-edged polyhedron is quite simple, so have a look at it if you're interested.

Using Euler's formula in a similar way we can discover that there is no simple polyhedron with ten faces and seventeen vertices.

WHAT IS A POLYHEDRON?

- A polyhedron is a solid object whose surface is made up of a number of flat *faces* which themselves are bordered by straight lines. Each face is in fact a *polygon*, a closed shape in the flat 2-dimensional plane made up of points joined by straight lines.

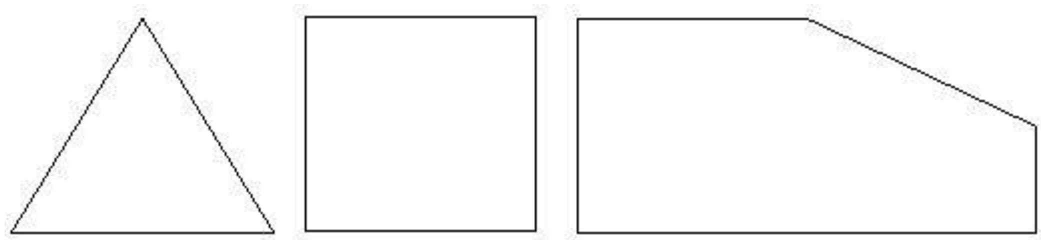


FIG: The familiar triangle and square are both polygons, but polygons can also have more irregular shapes like the one shown on the right.

- Polygons are not allowed to have holes in them, as the figure below illustrates: the left-hand shape here is a polygon, while the right-hand shape is not.

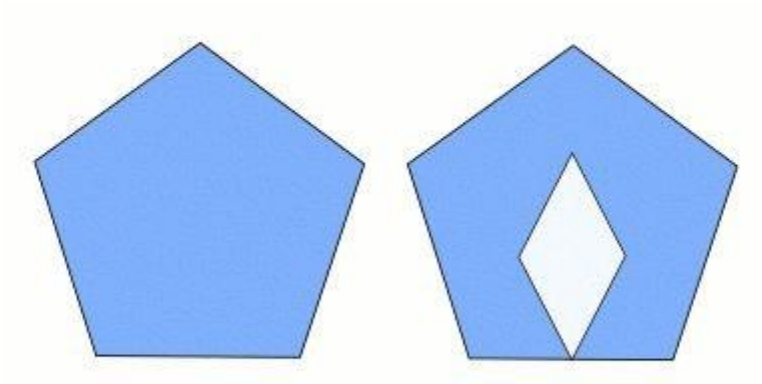


FIG: The shape on the left is a polygon, but the one on the right is not, because it has a 'hole'.

- A polygon is called regular if all of its sides are the same length, and all the angles between them are the same; the triangle and square in figure 1 and the pentagon in figure 2 are regular.
- A polyhedron is what you get when you move one dimension up. It is a closed, solid object whose surface is made up of a number of polygonal faces. We call the sides of these faces edges — two faces meet along each one of these edges. We call the corners of the faces vertices, so that any vertex lies on at least three different faces. To illustrate this, here are two examples of well-known polyhedra.

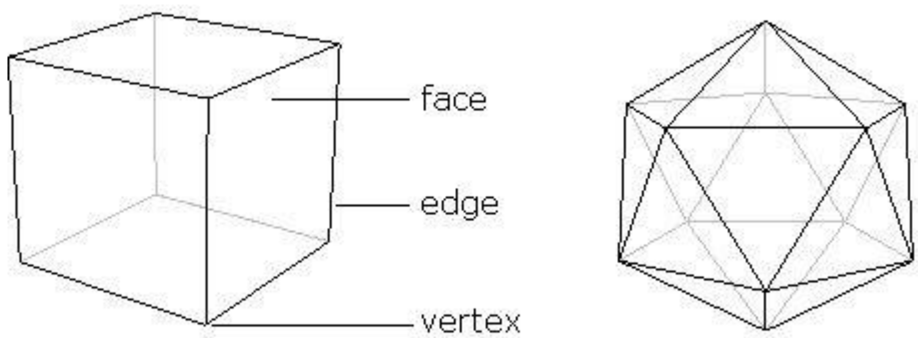


FIG: The familiar cube on the left and the icosahedron on the right. A polyhedron consists of polygonal faces, their sides are known as edges, and the corners as vertices.

- A polyhedron consists of just one piece. It cannot, for example, be made up of two (or more) basically separate parts joined by only an edge or a vertex. This means that neither of the following objects is a true polyhedron.

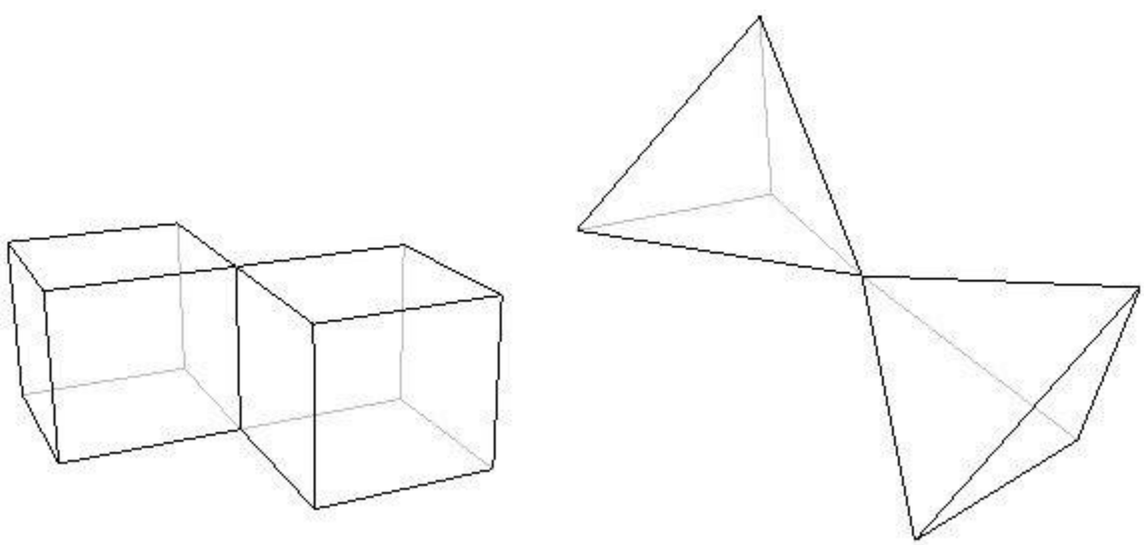


FIG: These objects are not polyhedra because they are made up of two separate parts meeting only in an edge (on the left) or a vertex (on the right).

PROBLEM FORMULATION AND DESCRIPTION

Look at a polyhedron, for example the cube or the icosahedron above, count the number of vertices it has, and call this number *V*. The cube, for example, has 8 vertices, so V = 8. Next, count the number of edges the polyhedron has, and call this number E. The cube has 12 edges, so in the case of the cube E = 12. Finally, count the number of faces and call it F. In the case of the cube, F = 6. Now Euler's formula tells us that.

$V - E + F = 2$;

or, in words: the number of vertices, minus the number of edges, plus the number of faces, is equal to two.

In the case of the cube, we've already seen that V = 8, E = 12 and F = 6. So,

$V - E + F = 8 - 12 + 6 = 14 - 12 = 2$

which is what Euler's formula tells us it should be. If we now look at the icosahedron, we find that V = 12, E = 30 and F = 20. Now,

$V - E + F = 12 - 30 + 20 = 32 - 30 = 2$,

as we expected.

Euler's formula is true for the cube and the icosahedron. It turns out, rather beautifully, that it is true for pretty much every polyhedron. The only polyhedra for which it doesn't work are those that have holes running through them like the one shown in the figure below.

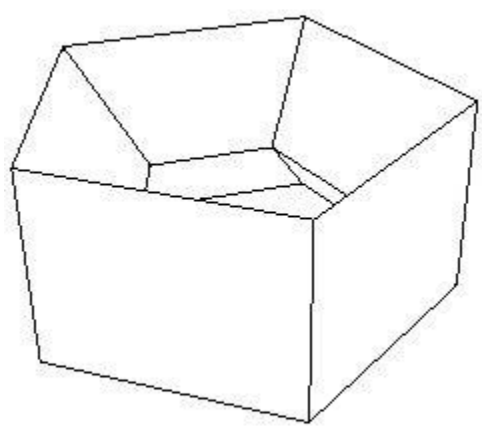


FIG: This polyhedron has a hole running through it. Euler's formula does not hold in this case.

- These polyhedra are called non-simple, in contrast to the ones that don't have holes, which are called simple. Non-simple polyhedra might not be the first to spring to mind, but there are many of them out there, and we can't get away from the fact that Euler's Formula doesn't work for any of them. However, even this awkward fact has become part of a whole new theory about space and shape.

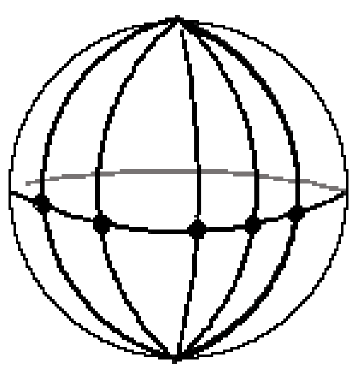
EULER’S FORMULA FOR SPHERE

A triangulation of a surface is a network on the surface all of whose faces are triangular (that is, they are bounded by three edges). In fact, surveyors map the countryside by triangulating it with `trig points' (usually on mountain tops) and measuring angles and distances between these trig points. In this case the trig points are the vertices of the triangulation.

The simplest example of a triangulation on the sphere (which we think of as the surface of the earth) is found by drawing the equator and, say,

n lines of longitude. In this example, there are $2n$ triangular faces, $n+2$ vertices (n on the equator, and one at each pole), and $3n$ edges, so that

(number of faces)−(number of edges)+(number of vertices)$=2n-3n+(n+2)=2$.



Thus, for any triangulation of the sphere with, say, T triangles, E edges and V vertices, Euler's formula for the sphere is that

$T-E+V=2.$

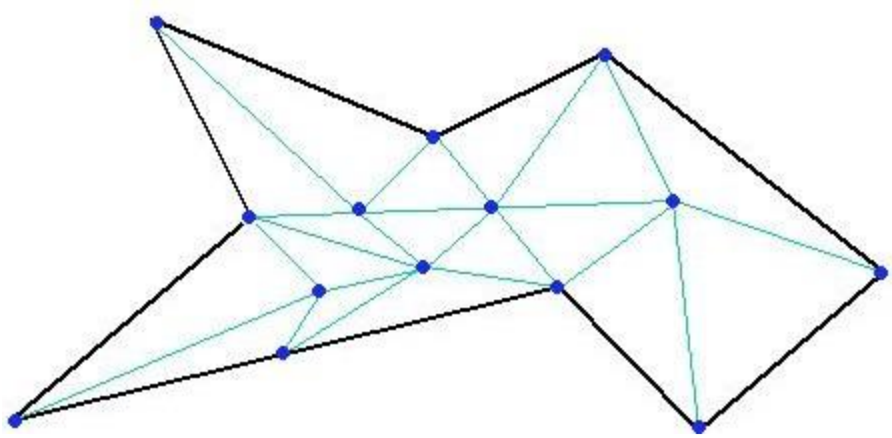
The important thing to realise is that this formula is a topological invariant : this means that if we deform the triangulation and the sphere continuously then the numbers T , E and V will not change and the formula will still be true. For example, as we can deform the sphere to a cube, the formula will still hold for a cube, so let us check this with one particular triangulation of the cube. We divide each face of the cube into two triangles by drawing a diagonal across each face. Then there are six faces of the cube and each gives two triangles; thus

$T=12$. Clearly $V=8$ (there are no new vertices introduced when we draw the diagonals), and $E=18$ (twelve from the original cube and the six added diagonals). Thus , $T-E+V=12-18+8=2$.

EULER'S FORMULA FOR A SIMPLE CLOSED POLYGON

Given a polygon that does not cross itself, we can triangulate the inside of the polygon into non-overlapping triangles such that any two triangles meet (if at all) either along a common edge, or at a common vertex. Suppose that there are T triangles, E edges and V vertices; then Euler's formula for a polygon is

$T-E+V=1.$



Proof:–

The terms T, E and V are unchanged when we apply a continuous change (or deformation) to the picture, we can imagine the picture to be drawn on a flexible rubber sheet. We cut the polygon out of the sheet, and then manipulate it until it fits exactly onto the lower half of a sphere sitting on the table. We can now regard it as a triangulation of the lower hemisphere of this sphere, and we shall use geographical terms, like `equator' and `north pole' to describe the points on the sphere. There will be a certain number of vertices, say N, on the `equator' of the sphere (exactly the same number as were on the boundary of the original polygon) and, as the vertices and edges alternate around the equator, there will also be exactly $N$ edges on the equator. Now draw arcs of circles from the `North pole' of the sphere to each of these N vertices (see Figure 4). This will now give us a triangulation of the sphere with$N$ new triangular faces, N new edges (all from the North pole) and one new vertex (at the North pole). Thus, from Euler's formula for the sphere,

$$(T+N)-(E+N)+(V+1)=2,$$

and this gives Euler's formula for the polygon, namely

$$T-E+V=1.$$