Assignment 15

Utkarsh Surwade
AI20MTECH11004

Download latex-tikz codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

1 Problem

Let $\mathbf{A}$ be a real matrix with characteristic polynomial $(x-1)^3$.Pick the correct statements from below:
1. $\mathbf{A}$ is necessarily diagonalizable.
2. If the minimal polynomial of $\mathbf{A}$ is $(x-1)^3$, then $\mathbf{A}$ is diagonalizale.
3. Characteristic polynomial of $\mathbf{A}^2$ is $(x-1)^3$.
4. If $\mathbf{A}$ has exactly two Jordan blocks, then $(\mathbf{A}-\mathbf{I})^2$ is diagonalizable.

2 Definitions

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $$p(x) = \left\lvert x\mathbf{I} - \mathbf{A} \right\rvert$$
Cayley-Hamilton Theorem	If $p(x)$ is the characteristic polynomial of an $n \times n$ matrix $\mathbf{A}$, then, $$p(\mathbf{A}) = \mathbf{0}$$
Minimal Polynomial	Minimal polynomial $m(x)$ is the smallest factor of characteristic polynomial $p(x)$ such that, $$m(\mathbf{A}) = 0$$ Every root of characteristic polynomial should be the root of minimal polynomial

TABLE 1: Definitions

3 Explanation

Statement	Solution
1.	Given that, $p(x) = (x-1)^3$ Let us consider $m(x) = (x-1)^2$ $\therefore \mathbf{J} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ A matrix is diagonalizable iff its jordan form is a diagonal matrix. Since $\mathbf{J}$ is not diagonizable therefore $\mathbf{A}$ is not diagonizable.
Conclusion	Therefore the statement is false.
2.	Given that, $p(x) = (x-1)^3$ and $m(x) = (x-1)^3$ $\therefore \mathbf{J} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Since $\mathbf{J}$ is not diagonizable therefore $\mathbf{A}$ is not diagonizable.
Conclusion	Therefore the statement is false.
3.	Given that, $p(x) = (x-1)^3$ Hence the eigen values of $\mathbf{A}$=1,1,1 Hence the eigen values of $\mathbf{A}^2 = 1^2, 1^2, 1^2$ or 1,1,1 Therefore the characteristic polynomial of $\mathbf{A}^2 = (x-1)^3$
Conclusion	Therefore the statement is True.

<table>
<tr><td>4.</td><td>
We know that jordan form of a matrix is similar to the original matrix Let $\mathbf{J}$ be the jordan form of the matrix $\mathbf{A}$ then,

$$\begin{aligned} \mathbf{A} &= \mathbf{PJP}^{-1} \\ \mathbf{A} - \mathbf{I} &= \mathbf{PJP}^{-1} - \mathbf{I} \\ \mathbf{A} - \mathbf{I} &= \mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1} \\ (\mathbf{A} - \mathbf{I})^2 &= \mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1}\mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1} \\ (\mathbf{A} - \mathbf{I})^2 &= \mathbf{P}(\mathbf{J} - \mathbf{I})^2\mathbf{P}^{-1} \end{aligned}$$

Therefore $(\mathbf{A} - \mathbf{I})^2$ is similar to $(\mathbf{J} - \mathbf{I})^2$
Since $\mathbf{A}$ has exactly two jordan blocks and order of $\mathbf{A}$ is 3.

$$\begin{aligned} \therefore \mathbf{J} &= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \mathbf{J} - \mathbf{I} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ (\mathbf{J} - \mathbf{I})^2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Since $(\mathbf{J} - \mathbf{I})^2$ is diagonal matrix.
Therefore $(\mathbf{A} - \mathbf{I})^2$ is diagonalizable.
</td></tr>
<tr><td>Conclusion</td><td>Therefore the statement is True.</td></tr>
</table>

TABLE 2: Solution summary