Matrix Theory (EE5609) Assignment 5

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***Abstract*—This document contains the solution to find points on a curve at which the tangent is parallel to the x-axis**

Download all python codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

and latex-tikz codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/Assignment5

1 Problem

Find the points on the curve

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} - 3 = 0$$

at which the tangents are parallel to the x-axis

2 Solution

Equation of circle is:

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

The centre and the radius is as follows:

$$f = -3 \tag{2.0.2}$$

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.3}$$

$$r = \sqrt{\|\mathbf{u}\|^2 - f} = 2 \tag{2.0.4}$$

Since the tangents is parallel to the x-axis,their direction and normal vectors, $\mathbf{m}$ and $\mathbf{n}$ are as follows:

$$\mathbf{m} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$

$$\mathbf{n} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.6}$$

For a circle, given the normal vector $\mathbf{n}$, the tangent points of contact to circle is given by equation as follows:

$$\mathbf{q_i} = (\kappa_i\mathbf{n} - \mathbf{u}), i = 1, 2 \tag{2.0.7}$$

where

$$\kappa_i = \pm\sqrt{\frac{\mathbf{u^T u} - f}{\mathbf{n^T n}}} \tag{2.0.8}$$

$$\kappa = \pm\sqrt{\frac{\begin{pmatrix}-1 & 0\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix} - (-3)}{\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}} \tag{2.0.9}$$

$$\therefore \kappa = \pm\sqrt{\frac{4}{1}} \tag{2.0.10}$$

$$\therefore \kappa = \pm 2 \tag{2.0.11}$$

Therefore from (2.0.7) , the point of contact $\mathbf{q_i}$ are as follows:

$$\mathbf{q_1} = 2\begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.13}$$

$$\mathbf{q_2} = -2\begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix}1\\-2\end{pmatrix} \tag{2.0.15}$$

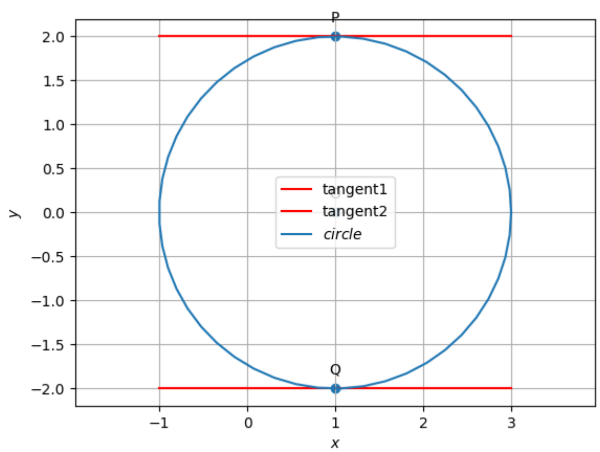

Fig. 0: Figure depicting tangents of circle parallel to x-axis