Assignment 14

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Download latex-tikz codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

1 Problem

Let **A** be an n × m matrix with each entry equal to +1,-1 or 0 such that every column has exactly one +1 and exactly one -1. We can conclude that

$$1.\ \text{Rank } \mathbf{A} \leq n-1 \tag{1.0.1}$$
$$2.\ \text{Rank } \mathbf{A} = m \tag{1.0.2}$$
$$3.\ n \leq m \tag{1.0.3}$$
$$4.\ n-1 \leq m \tag{1.0.4}$$

2 Explanation

option	Solution
1.	Let us consider **A** as follows and let s be the summation of all column entries: $$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}$$ $$\left\|\mathbf{A} - \lambda\mathbf{I}\right\| = \begin{pmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22}-\lambda & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm}-\lambda \end{pmatrix} = 0$$ $$= \begin{pmatrix} a_{11}+a_{21}+\dots+an1-\lambda & a_{11}+a_{21}+\dots+an1-\lambda & \dots & a_{11}+a_{21}+\dots+an1-\lambda \\ a_{21} & a_{22}-\lambda & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm}-\lambda \end{pmatrix}$$ $$\implies (s-\lambda)\begin{pmatrix} 1 & 1 & \dots & 1 \\ a_{21} & a_{22}-\lambda & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm}-\lambda \end{pmatrix} = 0$$

<table>
<tr><td>Example</td><td>
Since s=0 according to question,
Therefore $\lambda = 0$ is an eigen value of $\mathbf{A}$.
Since $\lambda = 0$, Hence $\mathbf{A}$ is singular.
Which means at least two rows are linearly dependent.
Therefore,

$$\text{Rank}(\mathbf{A}) < n$$
$$\text{Rank}(\mathbf{A}) \leq n - 1$$

Let us Consider $\mathbf{A}$ as follows,where n=4 and m=3

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

Calculating Row Reduced Echelon Form of $\mathbf{A}$ as follows:

$$\xleftrightarrow[R_4 \leftarrow R_2 + R_4]{R_4 \leftarrow R_1 + R_4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\xleftrightarrow{R_4 \leftarrow R_3 + R_4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
</td></tr>
<tr><td>Conclusion</td><td>
Since the Rank $\mathbf{A}$=3 and n=4,
Therefore the Rank $\mathbf{A} \leq n - 1$ statement is true.
</td></tr>
<tr><td>2.</td><td>
Let us Consider $\mathbf{A}$ as follows,where n=2 and m=2

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

Applying elementary transformations on $\mathbf{A}$ as follows:

$$\xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$$
</td></tr>
<tr><td>Conclusion</td><td>
Since the Rank $\mathbf{A}$=1 and m=2,
Therefore the Rank $\mathbf{A} \neq m$, Hence the statement is false.
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3.	Let us Consider **A** as follows,where n=3 and m=2 $$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 0 \end{pmatrix} \quad (2.0.1)$$
Conclusion	Since there exists a matrix **A** when n>m, Therefore the statement is false.
4	Let us Consider **A** as follows,where n=4 and m=2 $$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.2)$$
Conclusion	Since there exists a matrix **A** when n-1>m, Therefore the statement is false.

TABLE 1: Solution summary