# Matrix Theory (EE5609) Assignment 6

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*Abstract*—**This document contains the solution to finding the point at which the respective line is tangent to the curve**

Download all python codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

and latex-tikz codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/Assignment6

## 1 Problem

Find the point at which the line $\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$

## 2 Solution

Comparing $y^2 = 4x$ to standard equation,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

$\therefore$ a = b = e = 0, d =-2, c = 1, f = 0.

$$\therefore \mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\therefore \mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\text{Now, } \left|V\right| = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \tag{2.0.4}$$

$\implies$ That the curve is a parabola.

$$\text{Since } \mathbf{V}\mathbf{p_1} = 0 \tag{2.0.5}$$

$$\therefore \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

Since the slope of the line is 1 The direction vector $\mathbf{m}$ is as follows:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

$$\text{Since } \mathbf{m}^T\mathbf{n} = 0 \tag{2.0.8}$$

$$\therefore \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.9}$$

Now, the equation for the point of contact for the parabola is given as,

$$\begin{pmatrix} \mathbf{u}^T + \kappa\mathbf{n}^T \\ \mathbf{V} \end{pmatrix}\mathbf{q} = \begin{pmatrix} -f \\ \kappa\mathbf{n} - \mathbf{u} \end{pmatrix} \tag{2.0.10}$$

$$\text{where, } \kappa = \frac{\mathbf{p_1}^T\mathbf{u}}{\mathbf{p_1}^T\mathbf{n}} = -2 \tag{2.0.11}$$

By substituting the values ,we get:

$$\begin{pmatrix} -4 & 2 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \tag{2.0.12}$$

Solving for $\mathbf{q}$ by removing the zero row and representing (2.0.12) as augmented matrix and then converting the matrix to echelon form,

$$\implies \begin{pmatrix} -4 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \left(\frac{-R_1}{4}\right)} \begin{pmatrix} 1 & \frac{-1}{2} & 0 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{2.0.14}$$

Threrefore the point at which the line $\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} = 1$ is a tangent to the curve $y^2 = 4x$ is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

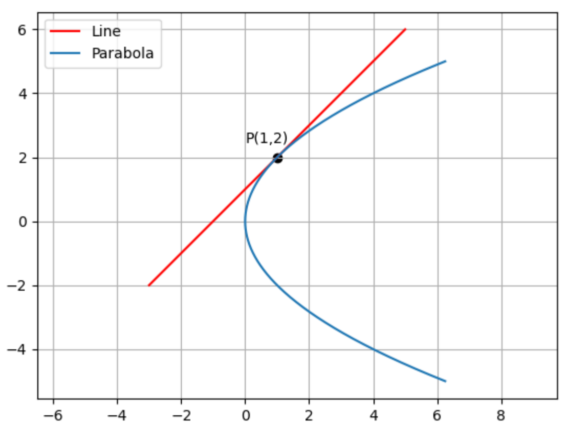


Fig. 0: Figure depicting the point at which the line is tangent to the parabola