

# Assignment 14

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Download latex-tikz codes from

[https://github.com/utkarshsurwade/Matrix\\_Theory\\_EE5609/tree/master/codes](https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes)

## 1 PROBLEM

Let  $\mathbf{A}$  be an  $n \times m$  matrix with each entry equal to  $+1, -1$  or  $0$  such that every column has exactly one  $+1$  and exactly one  $-1$ . We can conclude that

$$1. \text{Rank } \mathbf{A} \leq n - 1 \quad (1.0.1)$$

$$2. \text{Rank } \mathbf{A} = m \quad (1.0.2)$$

$$3. n \leq m \quad (1.0.3)$$

$$4. n - 1 \leq m \quad (1.0.4)$$

## 2 EXPLANATION

option	Solution
1.	<p>Let us Consider <math>\mathbf{A}</math> as follows, where <math>n=4</math> and <math>m=3</math></p> $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix} \quad (2.0.1)$ <p>Calculating Row Reduced Echelon Form of <math>\mathbf{A}</math> as follows:</p> $\begin{array}{l} \xleftrightarrow[R_4 \leftarrow R_2 + R_4]{R_4 \leftarrow R_1 + R_4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \end{array} \quad (2.0.2)$ $\xleftrightarrow{R_4 \leftarrow R_3 + R_4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.3)$ <p>Since the Rank <math>\mathbf{A}=3</math> and <math>n=4</math>, Therefore the Rank <math>\mathbf{A} \leq n - 1</math> statement is true.</p>

2.	<p>Let us Consider <math>\mathbf{A}</math> as follows,where <math>n=2</math> and <math>m=2</math></p> $\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \quad (2.0.4)$ <p>Applying elementary transformations on <math>\mathbf{A}</math> as follows:</p> $\xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \quad (2.0.5)$ <p>Since the Rank <math>\mathbf{A}=1</math> and <math>m=2</math>, Therefore the Rank <math>\mathbf{A} \neq m</math>, Hence the statement is false.</p>
3.	<p>Let us Consider <math>\mathbf{A}</math> as follows,where <math>n=3</math> and <math>m=2</math></p> $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 0 \end{pmatrix} \quad (2.0.6)$ <p>Since there exists a matrix <math>\mathbf{A}</math> when <math>n&gt;m</math>, Therefore the statement is false.</p>
4	<p>Let us Consider <math>\mathbf{A}</math> as follows,where <math>n=4</math> and <math>m=2</math></p> $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.7)$ <p>Since there exists a matrix <math>\mathbf{A}</math> when <math>n-1&gt;m</math>, Therefore the statement is false.</p>

TABLE 1: Solution summary