

# Matrix Theory (EE5609) Assignment 1

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**Abstract**—This document contains the solution to find a unit vector perpendicular to two vectors

Download all python codes from

[https://github.com/utkarshsurwade/  
Matrix\\_Theory\\_EE5609/tree/master/codes](https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes)

and latex-tikz codes from

[https://github.com/utkarshsurwade/  
Matrix\\_Theory\\_EE5609/tree/master/  
Assignment1](https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/Assignment1)

Using row reduction to find an expression for n.

$$\begin{array}{l} \xrightarrow{R_1 \leftarrow \frac{R_1}{4}} \\ \xleftarrow{R_2 \leftarrow R_2 - 2R_1} \end{array} \left( \begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 \end{array} \right) \quad (2.0.6)$$

$$\begin{array}{l} \xrightarrow{R_2 \leftarrow \frac{R_2}{-2}} \\ \xleftarrow{R_1 \leftarrow R_1 - R_2} \end{array} \left( \begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array} \right) \quad (2.0.7)$$

$$\therefore \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (2.0.8)$$

From above equations we get,

$$n_1 + 2(n_3) = 0 \quad (2.0.9)$$

$$n_2 - 2(n_3) = 0 \quad (2.0.10)$$

$$\therefore n_1 = -2(n_3) \quad (2.0.11)$$

$$\therefore n_2 = 2(n_3) \quad (2.0.12)$$

$$\therefore \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = n_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (2.0.13)$$

Let us consider  $n_3$  to be -8 which gives us:

$$\therefore \mathbf{n} = \begin{pmatrix} 16 \\ -16 \\ -8 \end{pmatrix} \quad (2.0.14)$$

$$\|\mathbf{n}\| = \sqrt{16^2 + (-16)^2 - 8^2} = 24 \quad (2.0.15)$$

Let  $\mathbf{u}$  be the unit vector of C which can be found as follows:

$$\mathbf{u} = \frac{\mathbf{n}}{\|\mathbf{n}\|} \quad (2.0.16)$$

Solving the above equation gives the unit vector  $\mathbf{u}$  which is perpendicular to vectors  $\mathbf{A}$  and  $\mathbf{B}$

$$\therefore \mathbf{u} = \frac{1}{3} \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \quad (2.0.17)$$

## 1 PROBLEM

Find a unit vector perpendicular to each of the vectors  $\mathbf{a} + \mathbf{b}$  and  $\mathbf{a} - \mathbf{b}$ , where

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \quad (1.0.1)$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \quad (1.0.2)$$

## 2 SOLUTION

Let  $\mathbf{A} = \mathbf{a} + \mathbf{b}$  and  $\mathbf{B} = \mathbf{a} - \mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \quad (2.0.2)$$

Let  $\mathbf{n}$  be a vector Perpendicular to  $\mathbf{A}$  and  $\mathbf{B}$  both

$$\mathbf{A}^T \mathbf{n} = 0 \quad (2.0.3)$$

$$\mathbf{B}^T \mathbf{n} = 0 \quad (2.0.4)$$

The augmented matrix can be represented as follows:

$$\left( \begin{array}{ccc|c} 4 & 4 & 0 & 0 \\ 2 & 0 & 4 & 0 \end{array} \right) \quad (2.0.5)$$