1

Assignment 4

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All the codes for the figure in this document can be found at

https://github.com/utkarshsurwade/ Matrix_Theory_EE5609/tree/master/ Assignment4

1 Problem

E and F are respectively the mid-points of equal sides AB and AC of $\triangle ABC$. Show that BF = CE.

2 Solution

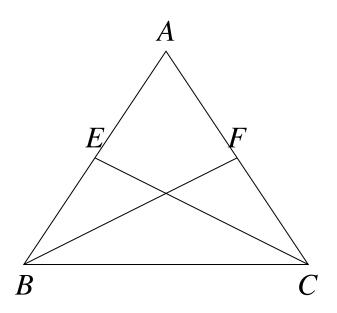


Fig. 1: Isosceles Triangle with mid-points E and F on equal sides

According to figure:

$$(\mathbf{A} - \mathbf{B}) + (\mathbf{B} - \mathbf{F}) = (\mathbf{A} - \mathbf{F}) \qquad (2.0.1)$$

$$\therefore (\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{F}) - (\mathbf{B} - \mathbf{F}) \qquad (2.0.2)$$

$$\therefore (\mathbf{A} - \mathbf{B}) = \frac{1}{2}(\mathbf{A} - \mathbf{C}) - (\mathbf{B} - \mathbf{F}) \qquad (2.0.3)$$

similarly,

$$(A - C) + (C - E) = (A - E)$$
 (2.0.4)

$$\therefore (\mathbf{A} - \mathbf{C}) = (\mathbf{A} - \mathbf{E}) - (\mathbf{C} - \mathbf{E}) \qquad (2.0.5)$$

$$\therefore (\mathbf{A} - \mathbf{C}) = \frac{1}{2}(\mathbf{A} - \mathbf{B}) - (\mathbf{C} - \mathbf{E}) \quad (2.0.6)$$

Since AB = AC $\therefore (AB)^2 = (AC)^2$

Solving LHS:

$$(\mathbf{A} - \mathbf{B})^2$$
 (2.0.7)

$$(\frac{1}{2}(\mathbf{A} - \mathbf{C}) - (\mathbf{B} - \mathbf{F}))^2$$
 (2.0.8)

$$\frac{1}{4}(\mathbf{A} - \mathbf{C})^2 + (\mathbf{B} - \mathbf{F})^2 - (\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{F}) \quad (2.0.9)$$

Solving RHS:

$$(\mathbf{A} - \mathbf{C})^2$$
 (2.0.10)

$$(\frac{1}{2}(\mathbf{A} - \mathbf{B}) - (\mathbf{C} - \mathbf{E}))^2$$
 (2.0.11)

$$\frac{1}{4}(\mathbf{A} - \mathbf{B})^2 + (\mathbf{C} - \mathbf{E})^2 - (\mathbf{A} - \mathbf{B})^T(\mathbf{C} - \mathbf{E}) (2.0.12)$$

Comparing LHS AND RHS

$$||\mathbf{B} - \mathbf{F}|| = ||\mathbf{C} - \mathbf{E}||$$

Hence, BF is equal to CE