Assignment 4

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment4
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1 Problem

E and F are respectively the mid-points of equal sides AB and AC of $\triangle ABC$.Show that BF = CE.

2 Solution

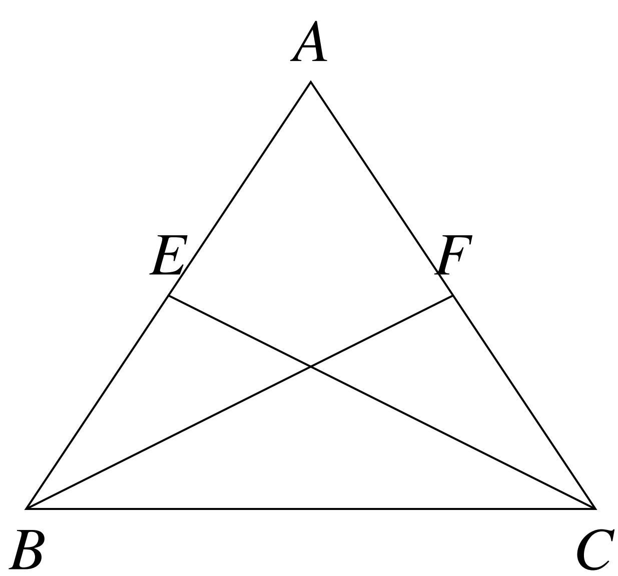

Fig. 1: Isosceles Triangle with mid-points E and F on equal sides

According to figure:

$$(\mathbf{A}-\mathbf{B})+(\mathbf{B}-\mathbf{F})=(\mathbf{A}-\mathbf{F}) \tag{2.0.1}$$
$$\therefore (\mathbf{B}-\mathbf{F})=(\mathbf{A}-\mathbf{F})-(\mathbf{A}-\mathbf{B}) \tag{2.0.2}$$
$$\therefore (\mathbf{B}-\mathbf{F})=\frac{1}{2}(\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B}) \tag{2.0.3}$$

similarly,

$$(\mathbf{A}-\mathbf{C})+(\mathbf{C}-\mathbf{E})=(\mathbf{A}-\mathbf{E}) \tag{2.0.4}$$
$$\therefore (\mathbf{C}-\mathbf{E})=(\mathbf{A}-\mathbf{E})-(\mathbf{A}-\mathbf{C}) \tag{2.0.5}$$
$$\therefore (\mathbf{C}-\mathbf{E})=\frac{1}{2}(\mathbf{A}-\mathbf{B})-(\mathbf{A}-\mathbf{C}) \tag{2.0.6}$$

Since AB = AC
$\therefore (AB)^2 = (AC)^2$

$$\therefore \|(\mathbf{A}-\mathbf{B})\|^2 = \|(\mathbf{A}-\mathbf{C})\|^2 \tag{2.0.7}$$

$$\|(\mathbf{A}-\mathbf{B})\|^2+\frac{1}{4}\|(\mathbf{A}-\mathbf{C})\|^2-(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) = \|(\mathbf{A}-\mathbf{C})\|^2+\frac{1}{4}\|(\mathbf{A}-\mathbf{B})\|^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{2.0.8}$$

$$\left(\frac{1}{2}(\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B})\right)^T\left(\frac{1}{2}(\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B})\right) = \left(\frac{1}{2}(\mathbf{A}-\mathbf{B})-(\mathbf{A}-\mathbf{C})\right)^T\left(\frac{1}{2}(\mathbf{A}-\mathbf{B})-(\mathbf{A}-\mathbf{C})\right) \tag{2.0.9}$$

$$\|(\mathbf{B}-\mathbf{F})\|^2 = \|(\mathbf{C}-\mathbf{E})\|^2 \tag{2.0.10}$$
$$\therefore \|\mathbf{B}-\mathbf{F}\| = \|\mathbf{C}-\mathbf{E}\| \tag{2.0.11}$$

Hence, BF is equal to CE