

Matrix Theory (EE5609) Assignment 1

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Abstract—This document contains the solution to find a unit vector perpendicular to two vectors

Download all python codes from

[https://github.com/utkarshsurwade/
Matrix_Theory_EE5609/tree/master/codes](https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes)

and latex-tikz codes from

[https://github.com/utkarshsurwade/
Matrix_Theory_EE5609/tree/master/
Assignment1](https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/Assignment1)

Using row reduction to find an expression for n.

$$\begin{array}{l} R_1 \leftarrow \frac{R_1}{4} \\ R_2 \leftarrow R_2 - 2R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 \end{array} \right) \quad (2.0.6)$$

$$\begin{array}{l} R_2 \leftarrow \frac{R_2}{-2} \\ R_1 \leftarrow R_1 - R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array} \right) \quad (2.0.7)$$

From above equations we get,

$$n_1 + 2(n_3) = 0 \quad (2.0.8)$$

$$n_2 - 2(n_3) = 0 \quad (2.0.9)$$

$$\therefore n_1 = -2(n_3) \quad (2.0.10)$$

$$\therefore n_2 = 2(n_3) \quad (2.0.11)$$

$$\therefore \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} -2n_3 \\ 2n_3 \\ n_3 \end{pmatrix} = n_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (2.0.12)$$

Let us consider n_3 to be 1 which gives us:

$$\therefore \mathbf{n} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \quad (2.0.13)$$

2 SOLUTION

Let $\mathbf{A} = \mathbf{a} + \mathbf{b}$ and $\mathbf{B} = \mathbf{a} - \mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \quad (2.0.2)$$

Let n be a vector Perpendicular to A and B both

$$\mathbf{A}^T \mathbf{n} = 0 \quad (2.0.3)$$

$$\mathbf{B}^T \mathbf{n} = 0 \quad (2.0.4)$$

The augmented matrix can be represented as follows:

$$\left(\begin{array}{ccc|c} 4 & 4 & 0 & 0 \\ 2 & 0 & 4 & 0 \end{array} \right) \quad (2.0.5)$$

$$\|\mathbf{n}\| = \sqrt{(-2)^2 + 2^2 + 1^2} = 3 \quad (2.0.14)$$

Let \mathbf{u} be the unit vector of C which can be found as follows:

$$\mathbf{u} = \frac{\mathbf{n}}{\|\mathbf{n}\|} \quad (2.0.15)$$

Solving the above equation gives the unit vector \mathbf{u} which is perpendicular to vectors \mathbf{A} and \mathbf{B}

$$\therefore \mathbf{u} = \frac{1}{3} \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \quad (2.0.16)$$