Matrix Theory (EE5609) Assignment 7

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***Abstract*—This document contains the solution to QR decomposition problem**

Download all python codes from

```
https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/codes
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and latex-tikz codes from

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment7
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1 Problem

Find QR decomposition of $\begin{pmatrix} 3 & 1 \\ -4 & 1 \end{pmatrix}$

2 Solution

Let α and β be transpose of column vectors of the given matrix.

$$\alpha = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.2}$$

We can express these as

$$\alpha = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\beta = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

where

$$k_1 = \|\alpha\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\alpha}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T \beta \tag{2.0.9}$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

From above we can see that $\mathbf{R}$ is an upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now by using equations (2.0.5) to (2.0.9)

$$k_1 = 5 \tag{2.0.13}$$

$$\mathbf{u}_1 = \frac{1}{5} \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \tag{2.0.14}$$

$$r_1 = \frac{-1}{5} \tag{2.0.15}$$

$$\mathbf{u}_2 = \frac{5}{7} \begin{pmatrix} \frac{28}{25} \\ \frac{21}{25} \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \frac{7}{5} \tag{2.0.17}$$

Thus obtained QR decomposition is

$$\begin{pmatrix} 3 & 1 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 5 & -\frac{1}{5} \\ 0 & \frac{7}{5} \end{pmatrix} \tag{2.0.18}$$