

Assignment 15

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https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

1 PROBLEM

Let \mathbf{A} be a real matrix with characteristic polynomial $(X - 1)^3$. Pick the correct statements from below:

1. \mathbf{A} is necessarily diagonalizable.
2. If the minimal polynomial of \mathbf{A} is $(x - 1)^3$, then \mathbf{A} is diagonalizable.
3. Characteristic polynomial of \mathbf{A}^2 is $(x - 1)^3$.
4. If \mathbf{A} has exactly two Jordan blocks, then $(\mathbf{A} - \mathbf{I})^2$ is diagonalizable.

2 DEFINITIONS

Characteristic Polynomial	For an $n \times n$ matrix \mathbf{A} , characteristic polynomial is defined by, $p(x) = x\mathbf{I} - \mathbf{A} $
Cayley-Hamilton Theorem	If $p(x)$ is the characteristic polynomial of an $n \times n$ matrix \mathbf{A} , then, $p(\mathbf{A}) = \mathbf{0}$
Minimal Polynomial	Minimal polynomial $m(x)$ is the smallest factor of characteristic polynomial $p(x)$ such that, $m(\mathbf{A}) = \mathbf{0}$ Every root of characteristic polynomial should be the root of minimal polynomial

TABLE 1: Definitions

3 EXPLANATION

Statement	Solution
1.	<p>Given that, $p(x) = (x - 1)^3$ Let us consider $m(x) = (x - 1)^2$</p> $\therefore \mathbf{J} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ <p>A matrix is diagonalizable iff its jordan form is a diagonal matrix. Since \mathbf{J} is not diagonalizable therefore \mathbf{A} is not diagonalizable.</p>
Conclusion	Therefore the statement is false.
2.	<p>Given that, $p(x) = (x - 1)^3$ and $m(x) = (x - 1)^3$</p> $\therefore \mathbf{J} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ <p>Since \mathbf{J} is not diagonalizable therefore \mathbf{A} is not diagonalizable.</p>
Conclusion	Therefore the statement is false.
3.	<p>Given that, $p(x) = (x - 1)^3$ Hence the eigen values of $\mathbf{A}=1,1,1$ Hence the eigen values of $\mathbf{A}^2 = 1^2, 1^2, 1^2$ or $1,1,1$ Therefore the characteristic polynomial of $\mathbf{A}^2 = (x - 1)^3$</p>
Conclusion	Therefore the statement is True.

4.	<p>We know that jordan form of a matrix is similar to the original matrix Let \mathbf{J} be the jordan form of the matrix \mathbf{A} then,</p> $\mathbf{A} = \mathbf{PJP}^{-1}$ $\mathbf{A} - \mathbf{I} = \mathbf{PJP}^{-1} - \mathbf{I}$ $\mathbf{A} - \mathbf{I} = \mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1}$ $(\mathbf{A} - \mathbf{I})^2 = \mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1}\mathbf{P}(\mathbf{J} - \mathbf{I})\mathbf{P}^{-1}$ $(\mathbf{A} - \mathbf{I})^2 = \mathbf{P}(\mathbf{J} - \mathbf{I})^2\mathbf{P}^{-1}$ <p>Therefore $(\mathbf{A} - \mathbf{I})^2$ is similar to $(\mathbf{J} - \mathbf{I})^2$ Since \mathbf{A} has exactly two jordan blocks and order of \mathbf{A} is 3.</p> $\therefore \mathbf{J} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\mathbf{J} - \mathbf{I} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(\mathbf{J} - \mathbf{I})^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ <p>Since $(\mathbf{J} - \mathbf{I})^2$ is diagonal matrix. Therefore $(\mathbf{A} - \mathbf{I})^2$ is diagonalizable.</p>
Conclusion	Therefore the statement is True.

TABLE 2: Solution summary