Assignment 4

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment4
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1 Problem

E and F are respectively the mid-points of equal sides AB and AC of $\triangle ABC$.Show that BF = CE.

2 Solution

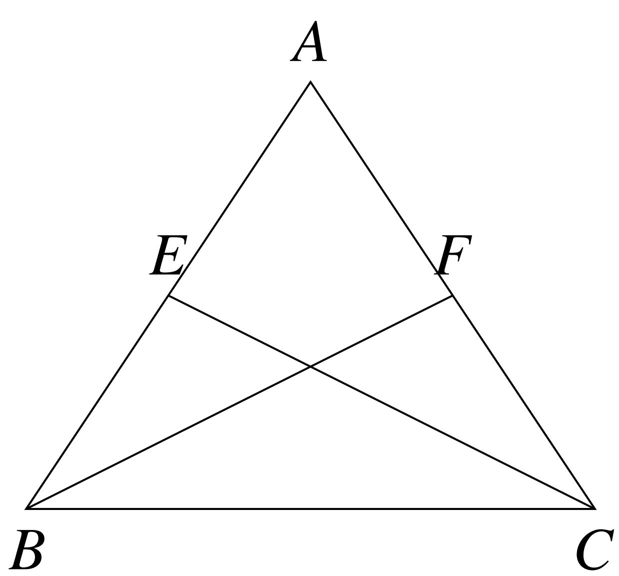

Fig. 1: Isosceles Triangle with mid-points E and F on equal sides

According to figure:

$$(\mathbf{A}-\mathbf{B})+(\mathbf{B}-\mathbf{F})=(\mathbf{A}-\mathbf{F}) \tag{2.0.1}$$

$$\therefore (\mathbf{A}-\mathbf{B})=(\mathbf{A}-\mathbf{F})-(\mathbf{B}-\mathbf{F}) \tag{2.0.2}$$

$$\therefore (\mathbf{A}-\mathbf{B})=\frac{1}{2}(\mathbf{A}-\mathbf{C})-(\mathbf{B}-\mathbf{F}) \tag{2.0.3}$$

similarly,

$$(\mathbf{A}-\mathbf{C})+(\mathbf{C}-\mathbf{E})=(\mathbf{A}-\mathbf{E}) \tag{2.0.4}$$

$$\therefore (\mathbf{A}-\mathbf{C})=(\mathbf{A}-\mathbf{E})-(\mathbf{C}-\mathbf{E}) \tag{2.0.5}$$

$$\therefore (\mathbf{A}-\mathbf{C})=\frac{1}{2}(\mathbf{A}-\mathbf{B})-(\mathbf{C}-\mathbf{E}) \tag{2.0.6}$$

Since AB = AC
$\therefore (AB)^2 = (AC)^2$
Solving LHS:

$$\therefore \|(\mathbf{A}-\mathbf{B})\|^2 \tag{2.0.7}$$

$$\left\|\frac{1}{2}(\mathbf{A}-\mathbf{C})-(\mathbf{B}-\mathbf{F})\right\|^2 \tag{2.0.8}$$

$$\frac{1}{4}\|(\mathbf{A}-\mathbf{C})\|^2+\|(\mathbf{B}-\mathbf{F})\|^2-(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{F}) \tag{2.0.9}$$

Solving RHS:

$$\therefore \|(\mathbf{A}-\mathbf{C})\|^2 \tag{2.0.10}$$

$$\left\|\frac{1}{2}(\mathbf{A}-\mathbf{B})-(\mathbf{C}-\mathbf{E})\right\|^2 \tag{2.0.11}$$

$$\frac{1}{4}\|(\mathbf{A}-\mathbf{B})\|^2+\|(\mathbf{C}-\mathbf{E})\|^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{E}) \tag{2.0.12}$$

Comparing LHS AND RHS
Since AB=AC

$$\frac{1}{4}\|(\mathbf{A}-\mathbf{C})\|^2=\frac{1}{4}\|(\mathbf{A}-\mathbf{B})\|^2 \tag{2.0.13}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{F})=(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{E})=0 \tag{2.0.14}$$

$$\|\mathbf{B}-\mathbf{F}\|^2=\|\mathbf{C}-\mathbf{E}\|^2 \tag{2.0.15}$$

$$\therefore \|\mathbf{B}-\mathbf{F}\|=\|\mathbf{C}-\mathbf{E}\| \tag{2.0.16}$$

Hence, BF is equal to CE