Assignment 4

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment4
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1 Problem

E and F are respectively the mid-points of equal sides AB and AC of $\triangle ABC$.Show that BF = CE.

2 Solution

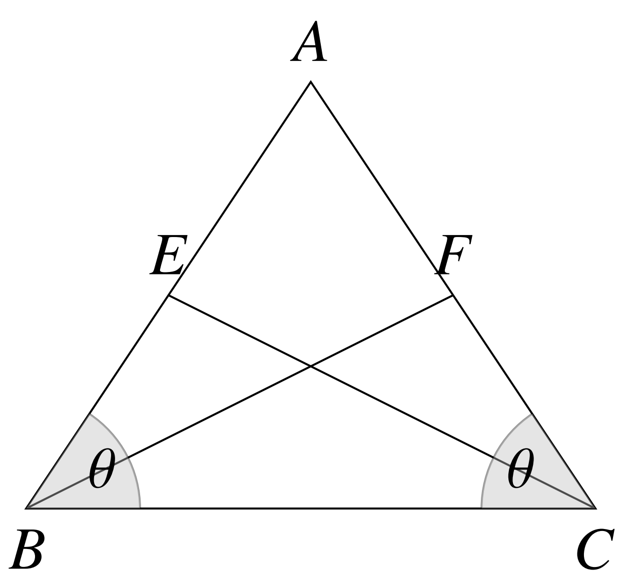

Fig. 1: Isosceles Triangle with mid-points E and F on equal sides

Let θ be the angle opposite to equal sides.

$$\cos(\theta) = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.1}$$

$$\cos(\theta) = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.2}$$

From above equations:

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.3}$$

Solving LHS:

$$(\mathbf{A}-\mathbf{E}+\mathbf{E}-\mathbf{C}+\mathbf{C}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.4}$$

$$(\mathbf{A}-\mathbf{E})^T(\mathbf{B}-\mathbf{C}) + (\mathbf{E}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) + \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.5}$$

Solving RHS:

$$(\mathbf{A}-\mathbf{F}+\mathbf{F}-\mathbf{B}+\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.6}$$

$$(\mathbf{A}-\mathbf{F})^T(\mathbf{B}-\mathbf{C}) + (\mathbf{F}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) + \|\mathbf{B}-\mathbf{C}\|^2 \tag{2.0.7}$$

From LHS and RHS

$$(\mathbf{E}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = (\mathbf{F}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.8}$$

$$(\mathbf{E}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}+\mathbf{E}-\mathbf{C}) = (\mathbf{F}-\mathbf{B})^T(\mathbf{B}-\mathbf{F}+\mathbf{F}-\mathbf{C}) \tag{2.0.9}$$

$$(\mathbf{E}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) + \|\mathbf{E}-\mathbf{C}\|^2 = (\mathbf{F}-\mathbf{B})^T(\mathbf{F}-\mathbf{C}) + \|\mathbf{B}-\mathbf{F}\|^2 \tag{2.0.10}$$

Since AB = AC

$$(\mathbf{E}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) = (\mathbf{F}-\mathbf{B})^T(\mathbf{F}-\mathbf{C}) = 0 \tag{2.0.11}$$

$$\therefore \|\mathbf{E}-\mathbf{C}\|^2 = \|\mathbf{B}-\mathbf{F}\|^2 \tag{2.0.12}$$

$$\therefore \|\mathbf{E}-\mathbf{C}\| = \|\mathbf{B}-\mathbf{F}\| \tag{2.0.13}$$

Hence, BF is equal to CE