Matrix Theory (EE5609) Assignment 1

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Abstract—**This document contains the solution to find a unit vector perpendicular to two vectors**

Download all python codes from

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/codes
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and latex-tikz codes from

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https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment1
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1 Problem

Find a unit vector perpendicular to each of the vectors $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$, where

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \tag{1.0.2}$$

2 Solution

Let $\mathbf{A} = \mathbf{a} + \mathbf{b}$ *and* $\mathbf{B} = \mathbf{a} - \mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \tag{2.0.2}$$

Let $\mathbf{n}$ be a vector Perpendicular to $\mathbf{A}$ and $\mathbf{B}$ both

$$\mathbf{A}^T\mathbf{n} = 0 \tag{2.0.3}$$

$$\mathbf{B}^T\mathbf{n} = 0 \tag{2.0.4}$$

The augmented matrix can be represented as follows:

$$\left(\begin{array}{ccc|c} 4 & 4 & 0 & 0 \\ 2 & 0 & 4 & 0 \end{array}\right) \tag{2.0.5}$$

Using row reduction to find an expression for n.

$$\xleftrightarrow[R_2 \leftarrow R_2 - 2R_1]{R_1 \leftarrow \frac{R_1}{4}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 \end{array}\right) \tag{2.0.6}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - R_2]{R_2 \leftarrow \frac{R_2}{-2}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \tag{2.0.7}$$

From above equations we get,

$$n_1 + 2(n_3) = 0 \tag{2.0.8}$$

$$n_2 - 2(n_3) = 0 \tag{2.0.9}$$

$$\therefore n_1 = -2(n_3) \tag{2.0.10}$$

$$\therefore n_2 = 2(n_3) \tag{2.0.11}$$

$$\therefore \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} -2n_3 \\ 2n_3 \\ n_3 \end{pmatrix} = n_3 \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \tag{2.0.12}$$

Let us consider n_3 to be 1 which gives us:

$$\therefore \mathbf{n} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \tag{2.0.13}$$

$$||\mathbf{n}|| = \sqrt{(-2)^2 + 2^2 + 1^2} = 3 \tag{2.0.14}$$

Let $\mathbf{u}$ be the unit vector of $\mathbf{n}$ which can be found as follows:

$$\mathbf{u} = \frac{\mathbf{n}}{||\mathbf{n}||} \tag{2.0.15}$$

Solving the above equation gives the unit vector $\mathbf{u}$ which is perpendicular to vectors $\mathbf{A}$ *and* $\mathbf{B}$

$$\boxed{\therefore \mathbf{u} = \frac{1}{3}\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}} \tag{2.0.16}$$