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Assignment 4

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Abstract—This document solves a question based on triangle.

All the codes for the figure in this document can be found at

https://github.com/utkarshsurwade/ Matrix_Theory_EE5609/tree/master/ Assignment4

1 Problem

E and F are respectively the mid-points of equal sides AB and AC of $\triangle ABC$. Show that BF = CE.

2 Solution

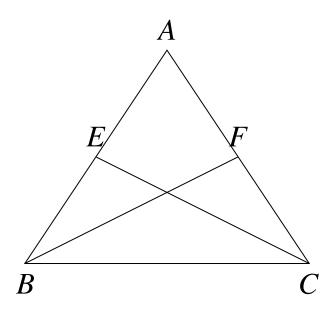


Fig. 1: Isosceles Triangle with mid-points E and F on equal sides

According to figure:

$$(\mathbf{A} - \mathbf{B}) + (\mathbf{B} - \mathbf{F}) = (\mathbf{A} - \mathbf{F}) \qquad (2.0.1)$$

$$\therefore (\mathbf{B} - \mathbf{F}) = (\mathbf{A} - \mathbf{F}) - (\mathbf{A} - \mathbf{B}) \qquad (2.0.2)$$

$$\therefore (\mathbf{B} - \mathbf{F}) = \frac{1}{2}(\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B}) \qquad (2.0.3)$$

similarly,

$$(A - C) + (C - E) = (A - E)$$
 (2.0.4)

$$\therefore (\mathbf{C} - \mathbf{E}) = (\mathbf{A} - \mathbf{E}) - (\mathbf{A} - \mathbf{C}) \qquad (2.0.5)$$

:
$$(\mathbf{C} - \mathbf{E}) = \frac{1}{2}(\mathbf{A} - \mathbf{B}) - (\mathbf{A} - \mathbf{C})$$
 (2.0.6)

Since AB = AC

$$\therefore (AB)^2 = (AC)^2$$

$$\|(\mathbf{A} - \mathbf{B})\|^2 + \frac{1}{4} \|(\mathbf{A} - \mathbf{C})\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) =$$

$$\|(\mathbf{A} - \mathbf{C})\|^2 + \frac{1}{4} \|(\mathbf{A} - \mathbf{B})\|^2 - (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C})$$
(2.0.8)

$$(\frac{1}{2}(\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B}))^{T}(\frac{1}{2}(\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B})) =$$

$$(\frac{1}{2}(\mathbf{A} - \mathbf{B}) - (\mathbf{A} - \mathbf{C}))^{T}(\frac{1}{2}(\mathbf{A} - \mathbf{B}) - (\mathbf{A} - \mathbf{C}))$$
(2.0.9)

$$\|(\mathbf{B} - \mathbf{F})\|^2 = \|(\mathbf{C} - \mathbf{E})\|^2$$
 (2.0.10)

$$\therefore ||\mathbf{B} - \mathbf{F}|| = ||\mathbf{C} - \mathbf{E}|| \tag{2.0.11}$$

Hence, BF is equal to CE