Matrix Theory (EE5609) Assignment 11

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Abstract

This document provides solutions to ugc problems

Download all python codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/codes

and latex-tikz codes from

https://github.com/utkarshsurwade/Matrix_Theory_EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{A}$ be a real symmetric matrix. Then we can conclude that

1. $\mathbf{A}$ does not have 0 as an eigenvalue
2. All eigenvalues of $\mathbf{A}$ are real
3. If $\mathbf{A}^{-1}$ exists, then $\mathbf{A}^{-1}$ is real and symmetric
4. $\mathbf{A}$ has at least one positive eigenvalue

2 Solution

Options	Solutions
1.	Let us consider $\mathbf{A}$ as follows: $$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$ Eigenvalues corresponding to $\mathbf{A}$ are, $$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0$$ $$\implies \begin{pmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{pmatrix} = 0$$ $$\implies \lambda(\lambda - 2) = 0$$ $$\therefore \lambda_1 = 0, \lambda_2 = 2$$ Therefore symmetric matrix $\mathbf{A}$ can have 0 as an eighenvalue.
2.	Let $\mathbf{v}$ be an eigenvector with respect to eigenvalue λ for matrix $\mathbf{A}$ where $\mathbf{v} \neq 0$ and $$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$ $$\implies \mathbf{v}^H\mathbf{A}\mathbf{v} = \mathbf{v}^H(\lambda\mathbf{v}) = \lambda\mathbf{v}^H\mathbf{v}$$ where H denotes Hermitian conjugation.Taking Hermitian transpose, we get $$(\lambda\mathbf{v}^H\mathbf{v})^H = (\mathbf{v}^H\mathbf{A}\mathbf{v})^H$$ $$\therefore \overline{\lambda}\mathbf{v}^H\mathbf{v} = \mathbf{v}^H\mathbf{A}^H\mathbf{v} = \mathbf{v}^H\mathbf{A}\mathbf{v}$$ $$\therefore (\lambda - \overline{\lambda})\mathbf{v}^H\mathbf{v} = 0$$ $$\text{since } \mathbf{v} \neq 0,\ \therefore \lambda - \overline{\lambda} = 0$$ $$\therefore \lambda = \overline{\lambda} \text{ is real}$$ Hence eigenvalues of a Hermitian matrix(in particular real symmetric matrix) are real.
3.	$$\text{Since } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$ $$\therefore (\mathbf{A}^{-1}\mathbf{A})^T = \mathbf{I}^T$$ $$\implies (\mathbf{A}^T)(\mathbf{A}^{-1})^T = \mathbf{I}$$ $$\text{Since } \mathbf{A}^T = \mathbf{A}$$ $$\therefore \mathbf{A}(\mathbf{A}^{-1})^T = \mathbf{I}$$ $$\therefore \mathbf{A}^{-1} = (\mathbf{A}^{-1})^T$$ Since $\mathbf{A}$ is real and the inverse of a matrix is unique,therefore $\mathbf{A}^{-1}$ is real and symmetric.
4.	Let us consider $\mathbf{A}$ as follows: $$\mathbf{A} = \begin{pmatrix} -6 & 2 \\ 2 & -9 \end{pmatrix}$$ Eigenvalues corresponding to $\mathbf{A}$ are, $$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0$$ $$\implies \begin{pmatrix} -6-\lambda & 2 \\ 2 & -9-\lambda \end{pmatrix} = 0$$ $$\implies (\lambda + 5)(\lambda + 10) = 0$$ $$\therefore \lambda_1 = -5, \lambda_2 = -10$$ Therefore symmetric matric $\mathbf{A}$ can have all negative eigenvalues.