Matrix Theory (EE5609) Assignment 10

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Abstract—**This document uses properties of vector spaces and subspaces.**

Download all python codes from

```
https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/codes
```

and latex-tikz codes from

```
https://github.com/utkarshsurwade/
    Matrix_Theory_EE5609/tree/master/
    Assignment10
```

1 Problem

Let **V** be the set of all 2 X 2 matrices **A** with complex entries which satisfy $\mathbf{A_{11}} + \mathbf{A_{22}} = 0$ and Show that **V** is a vector space over the field of real numbers, with the usual operations of matrix addition and multiplication of a matrix by a scalar.

2 Theorem 1.

A non-empty subset **W** of **V** is a subspace of **V** if and only if for each pair of vectors **a**, **b** in **W** and each scalar c in **F** the vector c**a** + **b** is again in **W**

3 Solution

Let **M** be the vector space of all 2 x 2 matrices over $\mathbb{C}$ i.e set of complex numbers and let $\mathbb{R}$ be set of real numbers.
Consider **A**,**B** ϵ **V** and c ϵ $\mathbb{C}$

$$\text{Let } \mathbf{A} = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \tag{3.0.1}$$

$$\text{and } \mathbf{B} = \begin{pmatrix} x' & y' \\ z' & w' \end{pmatrix} \tag{3.0.2}$$

$$x + w = x' + w' = 0 \tag{3.0.3}$$

$$c\mathbf{A} + \mathbf{B} = \begin{pmatrix} cx + x' & cy + y' \\ cz + z' & cw + w' \end{pmatrix} \tag{3.0.4}$$

$$\therefore (cx + x') + (cw + w') = c(x + w) + (x' + w') = 0 \tag{3.0.5}$$

Since **V** is a subset of **M**.
Therefore using above Theorem,

$$c\mathbf{A} + \mathbf{B} \ \epsilon \ \mathbf{V} \tag{3.0.6}$$

Hence **V** is a subspace of **M** as a vector space over $\mathbb{C}$.Hence **V** is a vector space over $\mathbb{R}$.