BEEE-UNIT 5

Digital Systems
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NUMBER SYSTEM

Definition: In digital system, the number system is used for representing the information. The number system has different bases and the most common of them are the decimal, binary, octal, and hexadecimal.

BASE OR RADIX (r)

The base or radix of the number system is the total number of the digit used in the number system.

Sl No	Name of the number system	Symbol Used	Base or radix
1	Decimal	0,1,2,3,4,5,6,7,8,9	10
2	Binary	0,1	2
3	Octal	0,1,2,3,4,5,6,7	8
4	Hexadecimal	0,1,2,3,4,5,6,7,8,9, A,B,C,D,E,F	16

I. Decimal to any number system

1. Decimal to Binary

Problem:

Convert 53.62_{10} to binary

Sol :

2 | 53
2 | 26-1
2 | 13-0
2 | 6-1
2 | 3-0
1-1

110101_2

$0.62 \times 2 = 1.24$
$0.24 \times 2 = 0.48$
$0.48 \times 2 = 0.96$
$0.96 \times 2 = 1.92$
$0.92 \times 2 = 1.84$
$0.84 \times 2 = 1.68$

100111_2

5 bit after decimal point enough

$53 = 110101_2$

$0.62 = 100\,111_2$

$53.62 = 110101.100\,111_2$

2. Decimal to octal

Problem:

Convert 444.456_{10} to octal

Sol :

$$\begin{array}{r|l} 8 & 444 \\ \hline 8 & 55-4 \\ \hline 8 & 6-7 \\ \hline \end{array} \qquad 674_8$$

0.456x8=3.648-3
0.648x8=5.184-5
0.184x8=1.472-1
0.472x8=3.776-3
0.776x8=6.208-6

0.35136_8

5 bit after decimal point enough

$444.456_{10} = 674.35136_8$

3. Decimal to hexadecimal

Problem:

Convert 444.456_{10} to hexadecimal

Sol :

A-10 B-11 C-12 D-13 E-14 F-15

$$\begin{array}{r|l} 16 & 444 \\ \hline 16 & 27-12 \\ \hline & 1-11 \end{array} \qquad 1BC$$

0.456X16=7.296-7
0.296X16=4.736-4
0.736X16=11.776-11

0.74B

444.456 = 1BC.74B

II. Any number system to Decimal [Use formula]

Decimal = $a_n \times r^n + a_{n-1} \times r^{n-1} + \ldots\ldots a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2} \ldots a_{-m} \times r^{m-1}$

Where: r- base or radix of given number system

a_n – n^{th} bit/digit value

a_0 – 0^{th} bit/digit value (Left immediate bit/digit at decimal point)

a_{-1} – 1^{st} bit/digit value (Right immediate bit/digit after decimal point)

4. Binary to Decimal

Problem:

Convert 101111.0101_2 to decimal

Sol :

Decimal= $a_5 \times 2^5 + a_4 \times 2^4 + a_3 \times 2^3 + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0 + a_{-1} \times 2^{-1} + a_{-2} \times 2^{-2} + a_{-3} \times 2^{-3} + a_{-4} \times 2^{-4}$

= $1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$ [Use calci to solve this]

= 32 +0+8+4+2+1+0_0.25+0+0.0625

= 47.3125_{10}

5. Octal to Decimal

Problem:

Convert 573. 26_8 to decimal

Sol :

Decimal= $a_2 \times 8^2 + a_1 \times 8^1 + a_0 \times 8^0 + a_{-1} \times 8^{-1} + a_{-2} \times 8^{-2}$

$= 5 \times 8^2 + 7 \times 8^1 + 3 \times 8^0 + 2 \times 8^{-1} + 6 \times 8^{-2}$

$= 320+56+3+0.25+0.09375$

$= 379.34375_{10}$

6. Hexadecimal to Decimal

A-10
B-11
C-12
D-13
E-14
F-15

Problem:

Convert $D2A.5C_{16}$ to decimal

Sol :

Decimal= $a_2 \times 16^2 + a_1 \times 16^1 + a_0 \times 16^0 + a_{-1} \times 16^{-1} + a_{-2} \times 16^{-2}$

$= 13 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 + 5 \times 16^{-1} + 12 \times 16^{-2}$

$= 3370.359375_{10}$

III. Other than Decimal conversion

7. Octal to Binary

Oct-Bin
0-000
1-001
2-010
3-011
4-100
5-101
6-110
7-111

Problem:

Convert 365.71_8 to binary

Sol : For each octal digit, write 3 bit binary

3	6	5 .	7	1
011	110	101	111	001

$= 011110101.111001_2$

8. Binary to Octal

Problem:

Convert 11001111.1101_2 to Octal

Sol : Choose 3 bit binary before decimal point and after decimal point.

Note: You can add 0, in prefix/suffix if needed

011 001 111.110 100

3 1 7 6 4

$= 317.64_8$

9. Hexadecimal to Binary

Note

0-0000	8- 1000
1-0001	9- 1001
2-0010	A- 1010
3-0011	B- 1011
4-0100	C- 1100
5-0101	D- 1101
6-0110	E- 1110
7-0111	F-1111

Problem:

Convert $AB85.2F_{16}$ to binary

Sol : For each hexadecimal digit, write 4 bit binary

A	B	8	5.	2	F
1010	1011	1000	0101	0010	1111

= $1010101110000101.00101111_2$

10. Binary to Hexadecimal

Problem:

Convert 110111101.1010111_2 to Hexadecimal

Sol : Choose 4 bit binary before decimal point and after decimal point

Note: You can add 0 in prefix/suffix, if needed

0001	1011	1101.	1010	1110
1	B	D	A	E

= $1BD.AE_{16}$

11. Octal to Hexadecimal [Via Binary]

Problem:

Convert 7320.12_8 to Hexadecimal

Sol : First convert Octal to Binary, then convert Binary to hexa

$$7320.12 = 111\ 011\ 010\ 000.001\ 010_2$$

1110 1101 0000. 0010 1000

E D 0 2 8

$$7320.12_8 = ED0.28_{16}$$

12. Hexadecimal to Octal [Via Binary]

Problem:

Convert $EEE.ECE_{16}$ to Octal

Sol : First convert hexa to Binary, then convert Binary to octal

$$EEE.ECE = 111011101110.111011001110_2$$

111 011 101 110.111 011 001 110

7 3 5 6 7 3 1 6

$$7356.7316_8$$

Why Binary used in Digital system

- Digital system use binary - the bits 0 and 1 - to store/process data. The circuits in Digital system are made up of billions of transistors. A transistor is a tiny switch that is activated by the electronic signals it receives.
- **The digits 1 and 0 used in binary reflect the on and off states of a transistor.**

SIGNED BINARY NUMBERS

To represent negative numbers, signed number system is used. The four best-known methods of extending the binary numeral system to represent signed numbers are: signed magnitude, ***ones' complement, two's complement***, and offset binary.

1. Signed magnitude / Sign-and-magnitude

In this approach, a number's sign is represented with a sign bit: setting that bit (often the **most significant bit**) to 0 for a positive number and setting it to 1 for a negative number. The remaining bits in the number indicate the magnitude (or absolute value). For example, in an eight-bit byte, only seven bits represent the magnitude, which can range from 0000000 (0) to 1111111 (127). Thus numbers ranging from -127_{10} to $+127_{10}$ can be represented once the sign bit (the eighth bit) is added. For example, -43_{10} encoded in an eight-bit byte is 10101011 while 43_{10} is 00101011.

2. One's Complement

The one's complement form of a negative binary number is the **bitwise NOT** applied to it, i.e. the "complement" of its positive counterpart**.**

43_{10} = 00101011
-43_{10} = 11010100

3. Two's Complement

In two's complement, negative numbers are represented by the bit pattern which is one greater than the ones' complement of the positive value. ie, one's complement it and add 1.

43_{10} = 00101011

-43_{10} = 11010100 (one's complement)

1

11010101 (Two's complement)

511_{10} = 0111111111

-511_{10} = 1000010000 (one's complement)

1 (Add 1)

1000000001 (Two's complement)

Note: In our calculator [Casio fx 991 MS], we have 10 bits. So maximum we can use 9 bit only. 10^{th} bit is used (MSB) to represent 0 for a positive number. [We can represent only 511 to -512 in our calculator]

Binary arithmetic [2's complement arithmetic]

Perform the following using 2's complement arithmetic. [Use 10 bit]

(i) Y=85-42

(ii) Z=42-85

Sol (i) Y = 85-42

= 85+(-42)

85 in 10 Bit : 0001010101

42 in 10 Bit : 0000101010

-42 in 1's complement : 1111010101

(+) 1

-42 in 2's complement : 1111010110

85 = 0001010101

(+)

-42 = 1111010110

~~1~~0000101011

Discard the carry.

Y=0000101011

$\boxed{Y=43}$

Sol (ii) Z = 42-85

= 42+(-85)

42 in 10 Bit : 0000101010

85 in 10 Bit : 0001010101

-85 in 1's complement : 1110101010

(+) 1

-85 in 2's complement : 1110101011

-85 = 1110101011

(+)

42 = 0000101010

1111010101

Since MSB is 1, it's a negative number.

To know its value, 2's complement it.

0000101010

(+) 1

0000101011

$\boxed{Z= -43}$

BCD CODES

Human can understand and use decimal number easily. But Digital system can understand only binary numbers. So, to interface among human and Digital system, BCD codes were developed. There are many types of BCD codes that were introduced by Scientist. But 8421 code is very much popular and widely used.

8421 code is a 4 bit BCD code having codes for 0 to 9 (Decimal Numbers). Each decimal digit represents 4 bit binary. It obeys positional weightage principle. It's easy to understand and remember because it's a normal 4 bit binary number from 0 to 9.

BINARY CODES FOR DECIMAL-BCD

Binary Codes for Decimal digits

The following table shows the various binary codes for decimal digits 0 to 9.

Decimal Digit	8421 Code BCD Code	2421 Code	84-2-1 Code	Excess 3 Code
0	0000	0000	0000	0011
1	0001	0001	0111	0100
2	0010	0010	0110	0101
3	0011	0011	0101	0110
4	0100	0100	0100	0111
5	0101	1011	1011	1000
6	0110	1100	1010	1001
7	0111	1101	1001	1010
8	1000	1110	1000	1011
9	1001	1111	1111	1100

BCD CODES

Express 7915_{10} in BCD and Excess-3

7915_{10} in BCD = 0111 1001 0001 0101

7915_{10} in Excess-3= 1010 1100 0100 1000

BCD Addition

BCD is a numerical code, and many applications require that arithmetic operations be performed. Addition is the most important operation because the other three operations like subtraction, multiplication and division can be accomplished using addition. The rule for adding two BCD numbers is given below.

1. Add the two numbers, using the rules for binary addition.
2. If a four-bit sum is equal to or less than 9, it is a valid BCD number.
3. If a four-bit sum is greater than 9, or if a carry-out of the group is generated, it is an invalid result. Add 6 (0110_2) to the four-bit sum in order to skip the six invalid states and return the code to 8421. If a carry results when 6 is added, simply add the carry to the next four-bit group.

Perform the following using BCD Arithmetic

459 + 278

Sol:

	1	111	1
459 =	0100	0101	1001
278 =	0010	0111	1000
	0111	1101	0001
		0110	0110
	0111	0011	0111
	7	3	7

Legal Code but contains Carry, so add 0110

Illegal Code, So add 0110

Boolean Algebra

- Invented by George Boole in 1854
- An algebraic structure defined by a set $B = \{0, 1\}$, together with two binary operators (+ and ·) and a unary operator ($\overline{}$)

OR Laws	AND Laws
1. $A + 0 = A$	5. $A \cdot 0 = 0$
2. $A + 1 = 1$	6. $A \cdot 1 = A$
3. $A + A = A$	7. $A \cdot A = A$
4. $A + \overline{A} = 1$	8. $A \cdot \overline{A} = 0$

9. $\overline{\overline{A}} = A$		Involution
10. $A + B = B + A$	11. $AB = BA$	Commutative
12. $(A + B) + C = A + (B + C)$	13. $(AB)\,C = A(BC)$	Associative
14. $A(B + C) = AB + AC$	15. $A + BC = (A + B)(A + C)$	Distributive
16. $\overline{A + B} = \overline{A} \cdot \overline{B}$	17. $\overline{A \cdot B} = \overline{A} + \overline{B}$	DeMorgan's
18. A+AB = A	19. A(A+B) = A	Absorption

Prove A+A'B = A+B

Similarly we can prove A'+AB = A'+B

A	B	A'	A'B	A+A'B	A+B
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

State and prove DeMorgan's Theorem

1. $\overline{A+B} = \overline{A} \cdot \overline{B}$
2. $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	A+B	(A+B)'	A'	B'	A'.B'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0
			LHS			RHS

A	B	A.B	(A.B)'	A'	B'	A'+B'
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0
			LHS			RHS

Simplification of Boolean expressions using Boolean algebra

1. Simplify the following expressions using Boolean algebra

$Y = A + AB + AB'C$

Sol: $Y = A(1+B+B'C)$

$= A.1$

$Y = A$

2. Simplify the following expressions using Boolean algebra

$Y = (A' + B)C + ABC$

Sol: $Y = A'C + BC + ABC$

$= A'C + BC(1+A)$

$= A'C + BC$

3. Simplify the following expressions using Boolean algebra

$Y = AB'C(BD + CDE) + AC'$

$= AB'CBD + AB'CCDE + AC'$ | $B'B=0$; $CC=C$

$= 0 + AB'CDE + AC'$

$= A(C' + B'CDE)$ | $C'+CX = C'+X$

$= A(C' + B'DE)$

4. Simplify the following expressions using Boolean algebra

$$Y = AB + \overline{AC} + A\overline{B}C\,(AB+C)$$

Sol: $Y = AB + A' + C' + AB'CAB + AB'CC$ — $B'B=0;\ CC=C$

$= AB + A' + C' + 0 + AB'C$

$= AB + A' + C' + AB'C$ — $C'+CX=C'+X$

$= AB + A' + C' + AB'$

$= A[B+B'] + A' + C'$ — $B+B'=1$

$= A + A' + C'$ — $A+A'=1$; $1+\text{ANY}=1$

$= 1$

5. Simplify the following expressions using Boolean algebra

$Y(A,B,C) = A'B+BC'+BC+AB'C'$

Sol: $Y = A'B+B[C'+C]+AB'C'$

$= A'B+B+AB'C'$

$= B[A'+1]+AB'C$

$= B+AB'C'$ — $B+B'X = B+X$, Where $X=AC'$

$Y = B+AC'$

6 Simplify the following expressions using Boolean algebra

$$Y=\{\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{\overline{F}}})}$$

$$Y=\{\overline{\overline{A + B\overline{C}}}\}\cdot\{\overline{D(\overline{E + \overline{\overline{F}}})}\}$$

$$Y=\{A + B\overline{C}\}\cdot\{\overline{D(\overline{E + \overline{\overline{F}}})}\}$$

$$Y=\{A + B\overline{C}\}\cdot\{\overline{D(\overline{E + \overline{\overline{F}}})}\}$$

$$Y=\{A + B\overline{C}\}\cdot\{\overline{D(\overline{E} \cdot \overline{\overline{F}})}\}$$

$$Y=\{A + B\overline{C}\}\cdot\{\overline{D(\overline{E}\,F)}\}$$

$$Y=\{A + B\overline{C}\}\{\overline{D} + (\overline{\overline{E}\,F})\}$$

$$Y=\{A + B\overline{C}\}\{\overline{D} + (\overline{\overline{E}\,F})\}$$

$$Y=\{A + B\overline{C}\}\{\overline{D} + (\overline{\overline{E}} + \overline{F})\}$$

$$Y=\{A + B\overline{C}\}\{\overline{D} + E + \overline{F}\}$$

Y = AD'+AE+AF'+BC'D'+BC'E+BC'F'

Simplify the given Boolean expression using Boolean laws and theorems SRM-MAY-2019 (3+3)

(1) $Y = ABC + A\overline{B}C + A\overline{B}C + AB\overline{C}$. **(2)** $Y = AB + A\overline{B}(\overline{\overline{A}}\,\overline{\overline{B}})$

Sol-1: $Y = ABC + A\overline{B}C + A\overline{B}C + AB\overline{C}'$

$$Y = ABC + A\overline{B}C + AB\overline{C}$$

$$Y = AC(1 + \overline{B}) + AB\overline{C}$$

$$Y = AC + AB\overline{C} = A(C + B\overline{C}) = A(C + B) = AC + AB$$

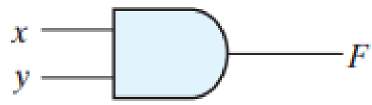
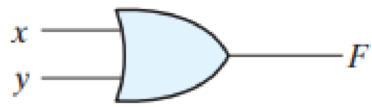
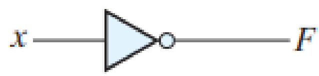
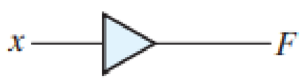
Sol-1: $Y = AB + A\overline{B}(\overline{\overline{A}}\,\overline{\overline{B}})$

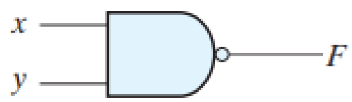
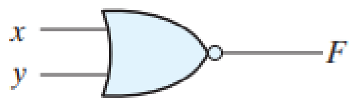
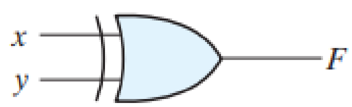
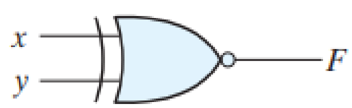
$$Y = AB + A\overline{B}(\overline{\overline{A}} + \overline{\overline{B}})$$

$$Y = AB + A\overline{B}(A + B)$$

$$Y = AB + A\overline{B}A + A\overline{B}B = AB + A\overline{B} = A$$

LOGIC GATES

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	x y F 0 0 0 0 1 0 1 0 0 1 1 1
OR		$F = x + y$	x y F 0 0 0 0 1 1 1 0 1 1 1 1
Inverter		$F = x'$	x F 0 1 1 0
Buffer		$F = x$	x F 0 0 1 1

			x	y	F
NAND		$F = (xy)'$	0	0	1
			0	1	1
			1	0	1
			1	1	0
			x	y	F
NOR		$F = (x + y)'$	0	0	1
			0	1	0
			1	0	0
			1	1	0
			x	y	F
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	0	0	0
			0	1	1
			1	0	1
			1	1	0
			x	y	F
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	0	0	1
			0	1	0
			1	0	0
			1	1	1

SUM OF PRODUCTS [SOP] AND PRODUCT OF SUMS [POS]

Logical functions (Boolean expression) are generally expressed in terms of logical variables (inputs) in following forms. (Each input variable can have the value, either 0 or 1 only)

- **SUM OF PRODUCTS [SOP] Ex: AB'+ BC+C'D**
- **PRODUCT OF SUMS [POS] Ex: (A'+B') (B'+C) (C'+D)**

MINTERMS

A product term containing all the inputs of the functions in either complemented or uncomplemented form is called ***MINTERMS.***

Let us consider 3 variable (input) function. It has 2^3 all possible combinations. [A 'n' variable (input) function has 2^n all possible combinations]. Let the inputs are A, B, C and output is Y.

TRUTH TABLE

	Inputs				Output
	A	B	C	MINTERMS	Y
0	0	0	0	A’B’C’	0
1	0	0	1	A’B’C	0
2	0	1	0	A’BC’	1
3	0	1	1	A’BC	1
4	1	0	0	AB’C’	1
5	1	0	1	AB’C	0
6	1	1	0	ABC’	1
7	1	1	1	ABC	0

- In minterms, 0 are assigned with bar letter and 1 are assigned with unbar letter.
- Within the row, all are multiplied (Product)
- Choose only the output 1.
- Add the minterms which having 1 output.
- In this example, we get Y= A'BC' + A'BC+ AB'C'+ ABC'. This expression is called canonical SOP form. [Standard SOP form]
- Each input is assigned with it equivalent decimal value. In the truth table, only the output Y= 1 is chosen, it corresponding input's decimal values are stated as below.

$$Y = \sum m\ (2,3,4,6)$$

MAXTERMS

A sum term containing all the inputs of the functions in either complemented or uncomplemented form is called ***MAXTERMS.*** Let us consider the same truth table.

	Inputs				Output
	A	B	C	MAXTERMS	Y
0	0	0	0	(A+B+C)	0
1	0	0	1	(A+B+C’)	0
2	0	1	0	(A+B’+C)	1
3	0	1	1	(A+B’+C’)	1
4	1	0	0	(A’+B+C)	1
5	1	0	1	(A’+B+C’)	0
6	1	1	0	(A’+B’+C)	1
7	1	1	1	(A’+B’+C’)	0

- In maxterms, 1 are assigned with bar letter and 0 are assigned with unbar letter.
- Within the row, all are summed (Added)
- Choose only the output 0.
- Product the maxterms which having 0 output.
- In this example, we get Y= (A+B+C) (A+B+C’) (A’+B+C’) (A’+B’+C’). This expression is called canonical POS form. [Standard POS form]
- Each input is assigned with it equivalent decimal value. In the truth table, only the output Y= 0 is chosen, it corresponding input’s decimal values are stated as below.

$$Y = \prod M\,(0,1,5,7)$$

1. For the Boolean function given below, obtain the (i) canonical SOP form (ii) canonical POS form.

$Y(A,B,C) = A + B'C$

$= AXX + XB'C$

$= AB'C' + AB'C + ABC' + ABC + A'B'C + AB'C$

[Remove the common term; Since A+A=A]

$Y = AB'C' + AB'C + ABC' + ABC + A'B'C$ [Canonical SOP form]

100 101 110 111 001

(m_4 m_5 m_6 m_7 m_1)

$Y = \sum m\,(1,4,5,6,7)$

$Y = \prod M\,(0,2,3)$ **[** Minterms and Maxterms are complement with each other**]**

M_0 M_2 M_3

000 010 011

$Y = (A+B+C)\,(A+B'+C)\,(A+B'+C')$ [Canonical POS form]

Karnaugh maps/ K-map

If the number of input variables is more than 2, its very difficult to minimize the Boolean function by Boolean algebra. Karnaugh maps/ K map overcomes this difficulty.

Karnaugh maps/ K map

- A visual way to simplify logic expressions
- It gives the most simplified form of the expression
- K-Maps are a graphical technique used to simplify a logic equation.
- K-Maps can be used for any number of input variables, BUT are only practical for **two, three, and four variables**

General Structure of K-Map

Two variable (Inputs- A,B)

A \ B	0	1
0	0	1
1	2	3

Three variable (Inputs- A,B,C)

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Four variable (Inputs- A,B,C,D)

AB \ CD	00	1	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Procedure to minimize Boolean expression by K-map:

1. We have to cheek, number of variables (Inputs).

(i)If the maximum number in the Boolean expression is ≤ 3, it is 2 variable function.

(ii)If the maximum number in the Boolean expression is ≤ 7, it is 3 variable function.

(iii)If the maximum number in the Boolean expression is ≤ 15, it is 4 variable function.

Note: Some times, in the question itself, inputs will be given. Ex: $Y(A,B,C)=\sum(0,4,5,7)$

2. Check the given question is Minterms or Maxterms. If $\sum$ is given, it is Minterms. In K-map, for the given decimal location, we have to enter 1. In remaining location, we have to enter 0.

If $\prod$ is given, it is Maxterms. In K-map, for the given decimal location, we have to enter 0. In remaining location, we have to enter 0.

3. Draw the K-map and fill it. (use step 1 & 2)

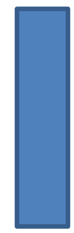
4 (a) Solution Procedure for SOP method

(i) We have box ALL the 1.

(ii) Larger the box, smaller the equation. Since all are minimization problem, we have chose larger box.

(iii) The number of 1's inside the box must be 2^n. [ie we have to try boxing 16 , if not possible we have to try boxing 8, if not possible we have to try boxing 4, if not possible we have to try boxing 2, if not possible we have to box 1]

(iv) The shape of the box must be square or rectangular. ie



(v) For each box, we have to find unchanged input. For that, we have see K-map from right to left, then bottom to top. The unchanged input within the box should be **product**. The product of one box should be sum with next box.[In input, 0 are assigned with bar letter and 1 are assigned with unbar letter]

(vi) Overlapping is allowed to make larger box.

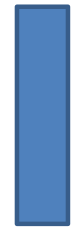
4 (b) Solution Procedure for POS method

(i) We have box ALL the 0.

(ii) Larger the box, smaller the equation. Since all are minimization problem, we have chose larger box.

(iii) The number of 0's inside the box must be 2^n. [ie we have to try boxing 16 , if not possible we have to try boxing 8, if not possible we have to try boxing 4, if not possible we have to try boxing 2, if not possible we have to box 1]

(iv) The shape of the box must be square or rectangular. ie



(v) For each box, we have to find unchanged inputs. For that, we have see K-map from right to left, then bottom to top. The unchanged input within the box should be **summed**. The sum of one box should be product with next box.[In input, 1 are assigned with bar letter and 0 are assigned with unbar letter]

(vi) Overlapping is allowed to make larger box.

K-MAP-SOP METHOD

1 Simply the following Boolean expression

$Y(A,B,C) = \sum m(1,2,3,6,7)$

Method 1: Boolean Algebra

Binary of minterms: 001, 010, 011, 110, 111

$$Y = A'B'C + A'BC' + A'BC + ABC' + ABC$$

$$Y = A'B'C + A'B[C' + C] + AB[C' + C]$$

$$Y = A'B'C + A'B + AB$$

$$Y = A'B'C + B[A + A']$$

$$Y = A'B'C + B$$

B+B'X=B+X

$$\mathbf{Y = A'C + B}$$

1 Simply the following Boolean expression

$Y(A,B,C) = \sum m(1,2,3,6,7)$

Method 2: K-MAP

Three variable (Inputs- A,B,C)

A \ BC	B'C' 00	B'C 01	BC 11	BC' 10
A' 0	$0\ _{0}$	$1\ _{1}$	$1\ _{3}$	$1\ _{2}$
A 1	$0\ _{4}$	$0\ _{5}$	$1\ _{7}$	$1\ _{6}$

Y=B+A'C

1

2

2 Simply the following Boolean expression

$Y\ (A,B,C) = \sum m\ (0,1,2,3,6)$

A \ BC	B'C' 00	B'C 01	BC 11	BC' 10
A' 0	1 $_0$	1 $_1$	1 $_3$	1 $_2$
A 1	$_4$	$_5$	$_7$	1 $_6$

$Y = A' + BC'$

Three-Variable K-Map : Examples

$f = \sum(0,4) = \bar{B}\bar{C}$

A \ BC	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$f = \sum(4,5) = A\bar{B}$

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	0	0

$f = \sum(0,1,4,5) = \bar{B}$

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$f = \sum(0,1,2,3) = \bar{A}$

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$f = \sum(0,4) = \bar{A}C$

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$f = \sum(4,6) = A\bar{C}$

A \ BC	00	01	11	10
0	0	0	0	0
1	1	0	0	1

$f = \sum(0,2) = \bar{A}\bar{C}$

A \ BC	00	01	11	10
0	1	0	0	1
1	0	0	0	0

$f = \sum(0,2,4,6) = \bar{C}$

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

3. Simply the following Boolean expression and implement it using logic gates

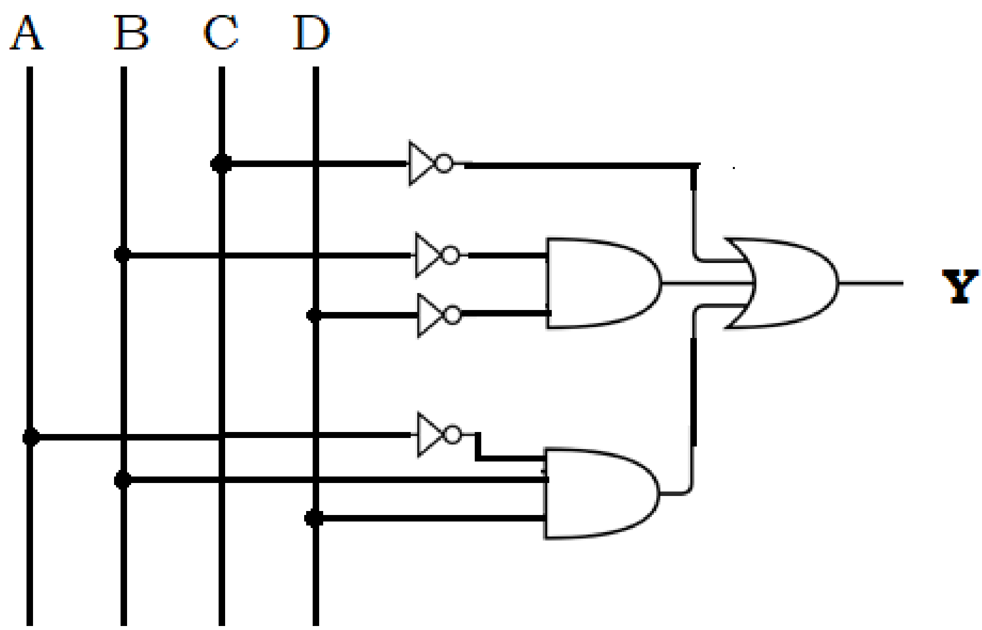
Y (A,B,C,D)= $\sum$m (0,1,2,4,5,7,8,9,10,12,13)

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1 (0)	1 (1)	0 (3)	1 (2)
A'B 01	1 (4)	1 (5)	1 (7)	0 (6)
AB 11	1 (12)	1 (13)	0 (15)	0 (14)
AB' 10	1 (8)	1 (9)	0 (11)	1 (10)

Y=C'+B'D'+A'BD

Implementation

$Y = C' + B'D' + A'BD$



Four-Variable K-Maps Examples

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	0	0

$$f = \sum(0,8) = \overline{B} \bullet \overline{C} \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	0	0
10	0	0	0	0

$$f = \sum(5,13) = B \bullet \overline{C} \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	0	0	0

$$f = \sum(13,15) = A \bullet B \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,6) = \overline{A} \bullet B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(2,3,6,7) = \overline{A} \bullet C$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$$f = \sum(4,6,12,14) = B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

$$f = \sum(2,3,10,11) = \overline{B} \bullet C$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \sum(0,2,8,10) = \overline{B} \bullet \overline{D}$$

Four-Variable K-Maps Examples

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$f = \sum(4,5,6,7) = \overline{A} \bullet B$

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$f = \sum(3,7,11,15) = C \bullet D$

AB \ CD	00	01	11	10
00	0	0	0	1
01	0	0	0	1
11	0	0	0	1
10	0	0	0	1

$f = \sum(2,6,10,14)$

$f = C\,\overline{D}$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

$f = \sum(12,13,14,15) = AB$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$f = \sum(1,3,5,7,9,11,13,15)$

$f = D$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$f = \sum(0,2,4,6,8,10,12,14)$

$f = \overline{D}$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$f = \sum(4,5,6,7,12,13,14,15)$

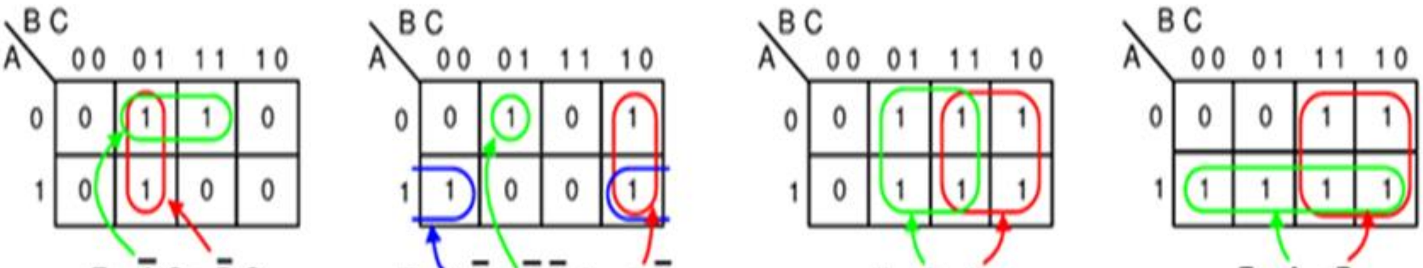
$f = B$

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

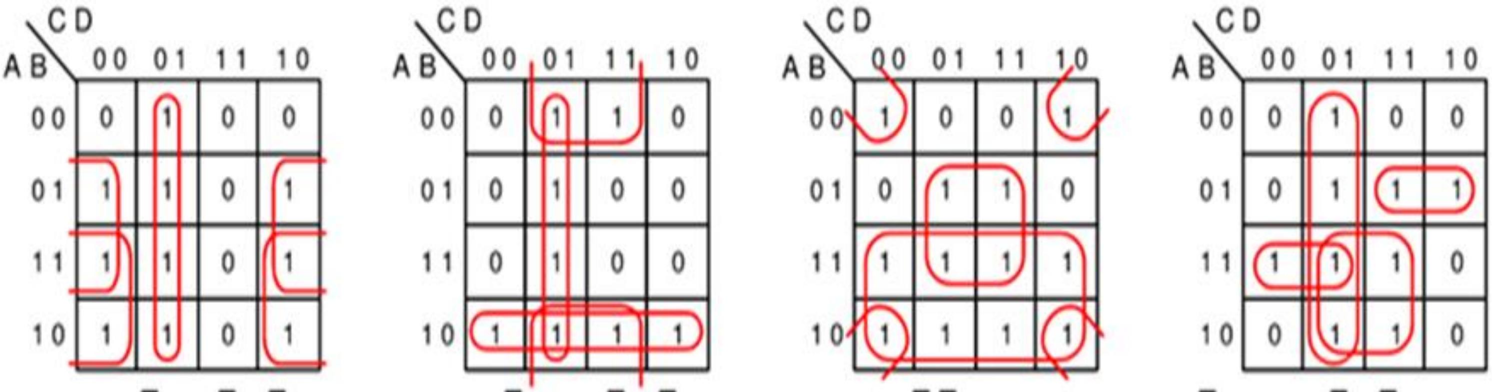
$f = \sum(0,1,2,3,8,9,10,11)$

$f = \overline{B}$

3,4-Variable K-Maps Examples



$F = \overline{A}\cdot C + \overline{B}\cdot C$ $\quad$ $F = A\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + B\cdot\overline{C}$ $\quad$ $F = B + C$ $\quad$ $F = A + B$



$F = B\cdot\overline{D} + A\cdot\overline{D} + \overline{C}\cdot D$ $\quad$ $F = \overline{B}\cdot D + A\cdot\overline{B} + \overline{C}\cdot D$ $\quad$ $F = \overline{B}\cdot\overline{D} + B\cdot D + A$ $\quad$ $F = \overline{C}\cdot D + A\cdot B\cdot\overline{C} + \overline{A}\cdot B\cdot C + A\cdot D$

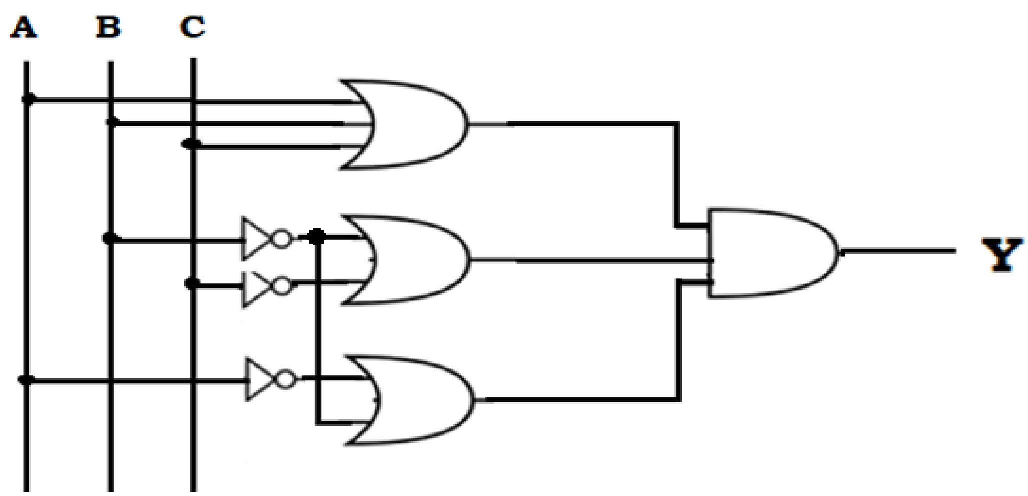
K-MAP-POS METHOD

1. Simply the following Boolean expression by POS method and implement it using logic gates

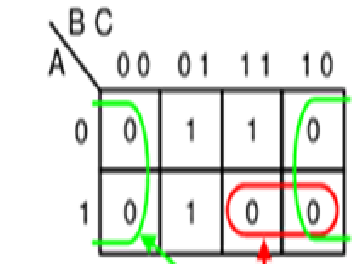
$$Y\ (A,B,C) = \prod m\ (0,3,6,7)$$

A \ BC	BC 00	BC' 01	B'C' 11	B'C 10
A 0	0 (0)	1 (1)	0 (3)	1 (2)
A' 1	1 (4)	1 (5)	0 (7)	0 (6)

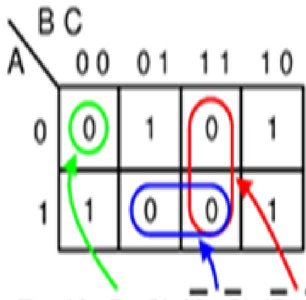
$Y = (A+B+C).\ (B'+C').\ (A'+B')$



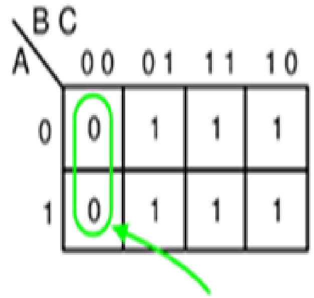
3,4-Variable K-Maps Examples [POS]



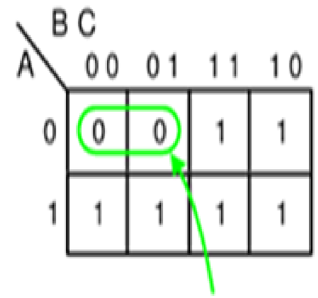
F = C · ($\bar{A}$+$\bar{B}$)



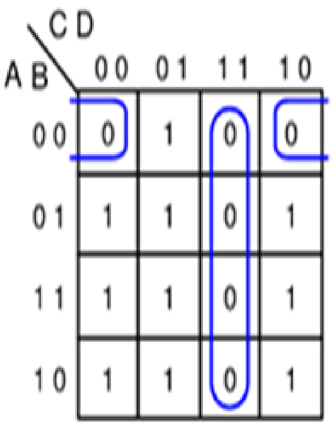
F = (A+B+C)·($\bar{A}$+$\bar{C}$)·($\bar{B}$+$\bar{C}$)



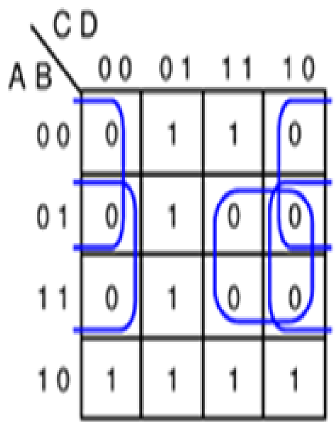
F = (B+C)



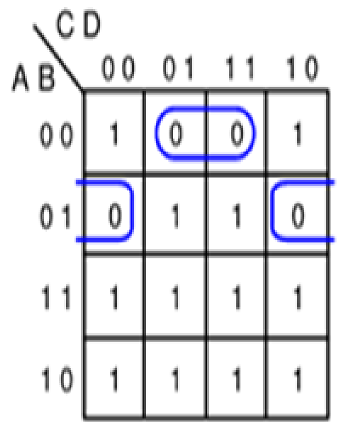
F = (A+B)



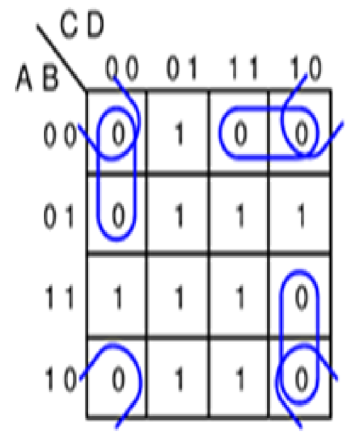
F = (A+B+D)·($\bar{C}$+$\bar{D}$)



F = (A+D)·($\bar{B}$+D)·($\bar{B}$+$\bar{C}$)



F = (A+B+$\bar{D}$)·(A+$\bar{B}$+D)



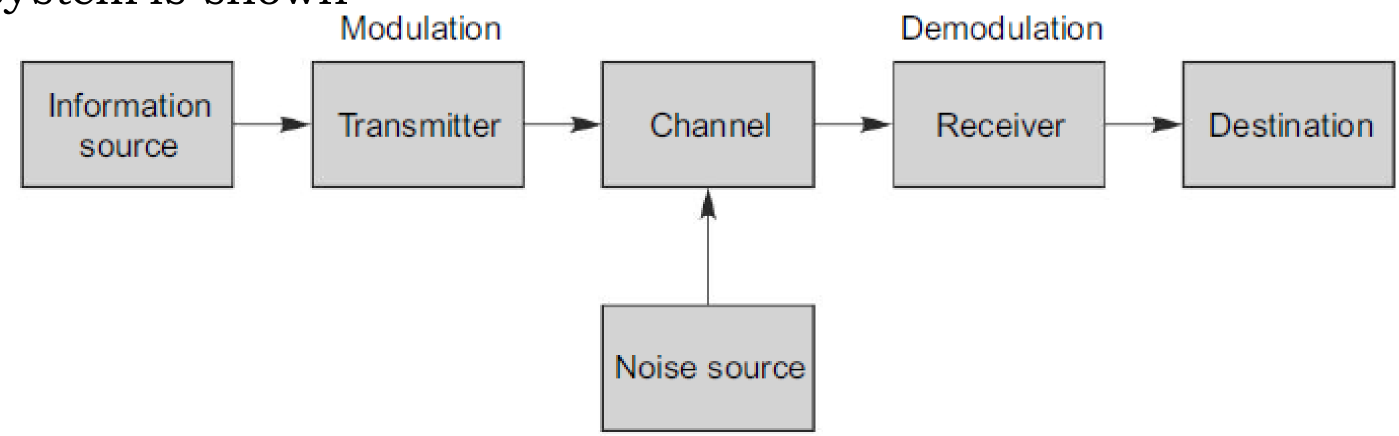
F = (A+C+D)·(A+B+$\bar{C}$)·
($\bar{A}$+$\bar{C}$+D)·(B+D)

COMMUNICATION SYSTEMS

Communication is a process of transfer of information bearing signals from one place to another. The equipment that transmits the information is the transmitter and the equipment that receives the information is the receiver. Channel is the medium through which the signal travels from the transmitter to the receiver. Telegraphy, telephony, facsimile, radio broadcast, TV transmission and computer communication are a few examples of communication services.

Block Diagram of Communication System

The basic function of a communication system is to communicate a message. The block diagram of a communication system is shown



The information to be transmitted is given by the information source. In most of the cases, the information will be non-electrical in nature. For example, audio signals in speech transmission and picture signals in television transmission. This information in the original form is converted into a corresponding electrical variation known as the message signal by using a transmitter. This message signal cannot be directly transmitted due to various reasons. Hence this message signal is superimposed on a high frequency carrier signal before transmission. This process is referred to as modulation.

After modulation, the modulated carrier is amplified by using power amplifiers in the transmitter and fed to the transmitting antenna. Channel is a medium through which the signal travels from the transmitter to the receiver. There are various types of channels, such as the atmosphere for radio broadcasting, wires for line telegraphy and telephony and optical fibers for optical communication.

As the signal gets propagated through the channel, it is attenuated by various mechanisms and affected by noise from the external source. Noise is an unwanted signal that interferes with the reception, of wanted signal. Noise is usually of random nature and in the design of communication system, careful attention should be paid to minimize the effect of noise on the reception of wanted signals.

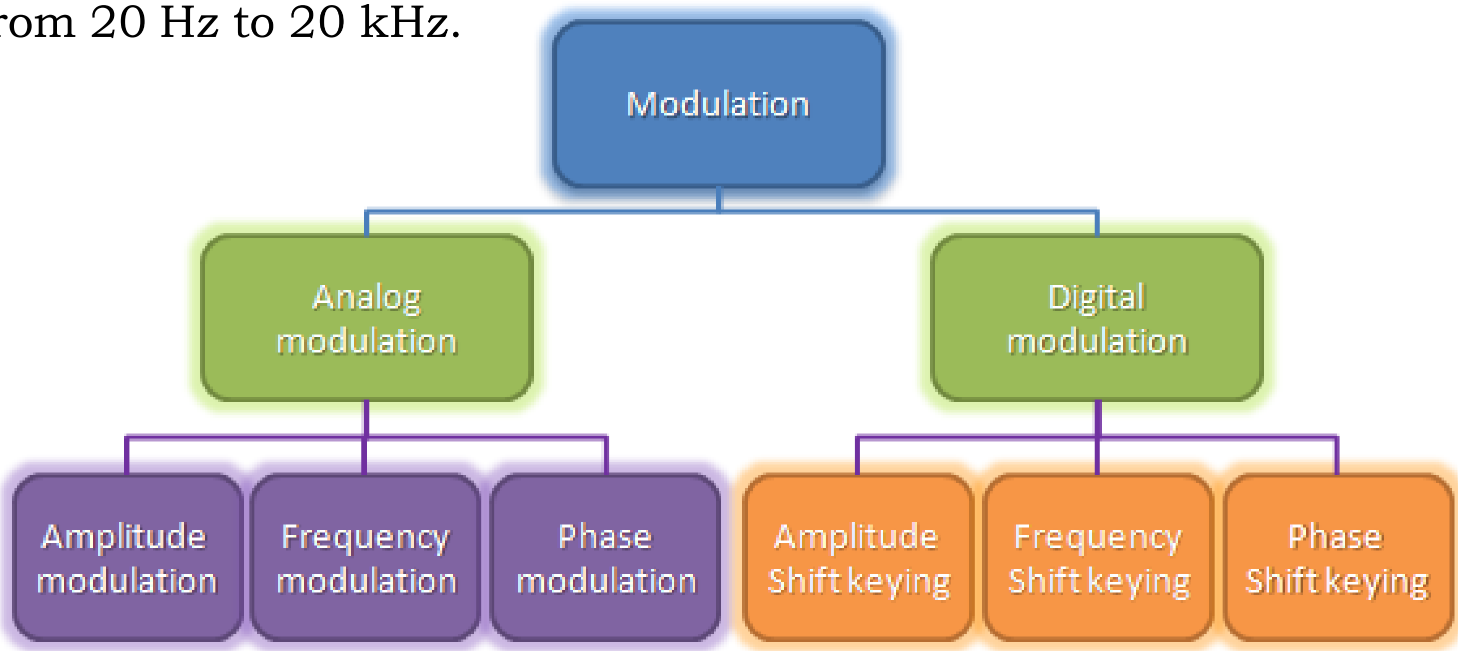
At the receiving end, a weak modulated carrier that is transmitted from the transmitter is received. As the received signal power will be very small, it is first of all amplified to increase the power level and the process of demodulation is done to recover the original message signal from the modulated carrier. The recovered message signal is further amplified to drive the output transducer such as a loud speaker or a TV receiver.

Electromagnetic Spectrum for Various Communication Services

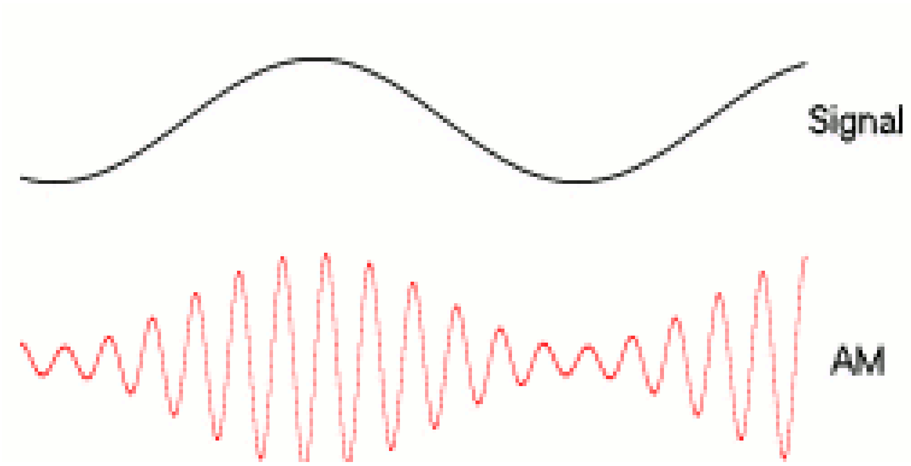
Classification	*Frequency Range*	*Wavelength*	*Uses*
Very Low Frequencies (V.L.F.)	10–30 kHz	30–10 km	Long distance point to point communications
Low Frequencies (L.F.)	30–300 kHz	10–1 km	Long distance point to point communication, Marine, Navigation, Power Line Carrier communication and Broadcast
Medium Frequencies (M.F.)	300–3000 kHz	1000–100 metres	Power Line Carrier communication, Broadcast, Marine communications, Navigation and harbour telephone
High Frequencies (H.F.)	3–30 MHz	100–10 metres	Moderate and long range communication of all types, Broadcast
Very High Frequencies (V.H.F.)	30–300 MHz	10–1 metres	Short distance communications, TV and FM broadcast, Data communication, Mobile and Navigation systems
Ultra High Frequencies (U.H.F.)	300–3000 MHz	1–0.1 metres	Short distance communications, TV broadcast, Radar, Mobile, Navigation and Microwave relay systems
Super High Frequencies (S.H.F.)	3–30 GHz	10–1 cms	Radar, Microwave relay and navigation systems
Extremely High Frequencies (E.H.F.)	30–300 GHz	1–0.1 cm	Radar, Satellite, Mobile, Microwave relay and navigations

BASIC PRINCIPLES OF MODULATION

Modulation is the process of changing some parameter of a high frequency carrier signal in accordance with the instantaneous variations of the message signal. The carrier signal has a constant amplitude and frequency. The function of a carrier signal is to carry the message signal and hence the name. The message or modulating signals are low frequency audio signals which contain the information to be transmitted. Generally message signal ranges from 20 Hz to 20 kHz.



Amplitude modulation (**AM**) is a modulation technique used in electronic communication, most commonly for transmitting messages with a radio carrier wave. In amplitude modulation, the amplitude (signal strength) of the carrier wave is varied in proportion to that of the message signal, such as an audio signal.



In amplitude modulation, the amplitude or *strength* of the carrier oscillations is varied. For example, in AM radio communication, a continuous wave radio-frequency signal (a sinusoidal carrier wave) has its amplitude modulated by an audio waveform before transmission. The audio waveform modifies the amplitude of the carrier wave and determines the *envelope* of the waveform. In the frequency domain, amplitude modulation produces a signal with power concentrated at the carrier frequency and two adjacent sidebands. Each sideband is equal in bandwidth to that of the modulating signal, and is a mirror image of the other. Standard AM is thus sometimes called "double-sideband amplitude modulation" (DSBAM). Single-sideband amplitude modulation.

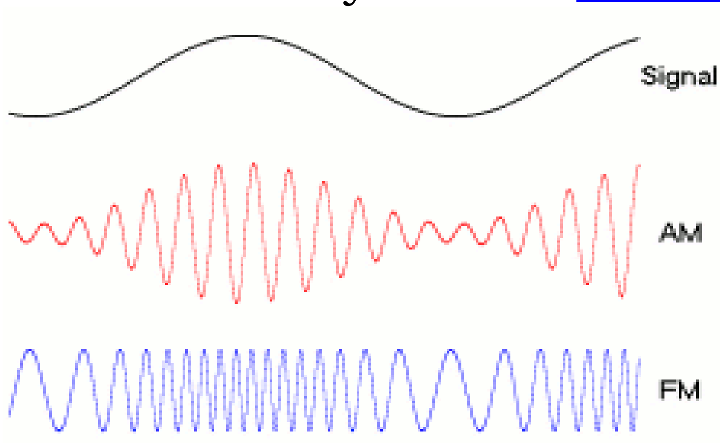
A disadvantage of all amplitude modulation techniques, not only standard AM, is that the receiver amplifies and detects noise and electromagnetic interference in equal proportion to the signal. AM broadcast is not favored for music and high fidelity broadcasting, but rather for voice communications and broadcasts (sports, news, talk radio etc.).

AM is also inefficient in power usage; at least two-thirds of the power is concentrated in the carrier signal.

Frequency modulation (**FM**) is the encoding of information in a carrier wave by varying the instantaneous frequency of the wave. The technology is used in telecommunications, radio broadcasting, signal processing, and computing.

In frequency modulation, such as radio broadcasting, of an audio signal representing voice or music, the instantaneous frequency deviation, i.e. the difference between the frequency of the carrier and its center frequency, has a functional relation to the modulating signal amplitude.

Frequency modulation is widely used for FM radio broadcasting.



It is also used in telemetry, radar, seismic prospecting, and monitoring newborns for seizures via EEG, two-way radio systems, sound synthesis, magnetic tape-recording systems and some video-transmission systems. In radio transmission, an advantage of frequency modulation is that it has a larger signal-to-noise ratio and therefore rejects radio frequency interference better than an equal power amplitude modulation (AM) signal. For this reason, most music is broadcast over FM radio.

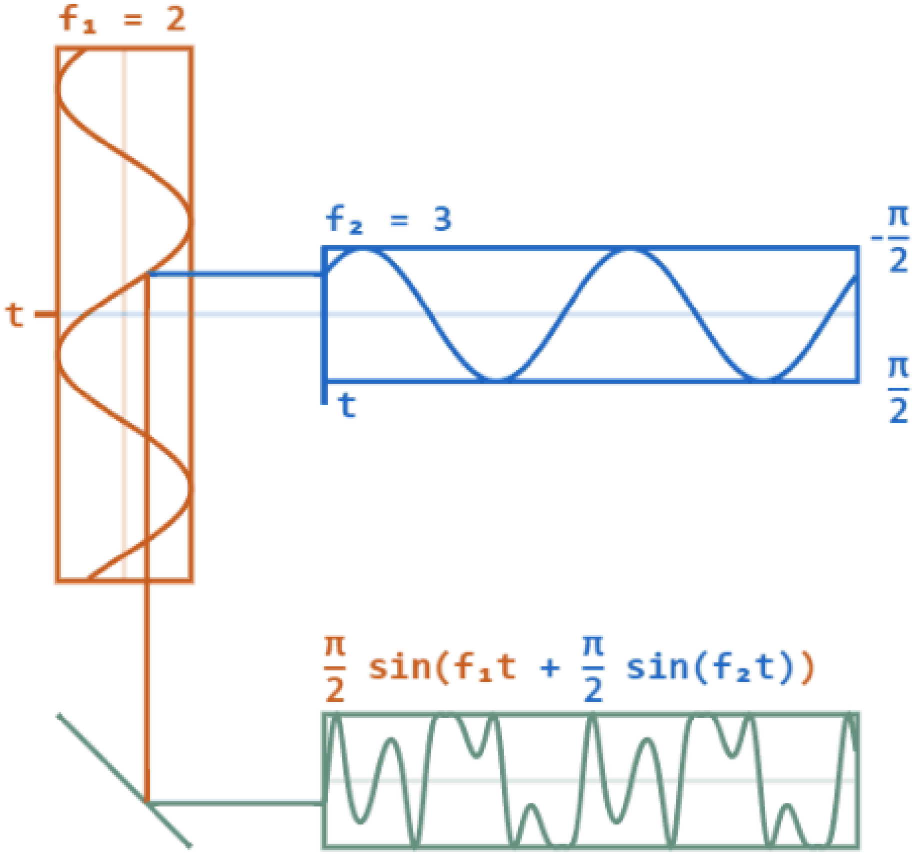
Frequency modulation and phase modulation are the two complementary principal methods of angle modulation; phase modulation is often used as an intermediate step to achieve frequency modulation. These methods contrast with amplitude modulation, in which the amplitude of the carrier wave varies, while the frequency and phase remain constant.

Phase modulation (**PM**) is a modulation pattern for conditioning communication signals for transmission. It encodes a message signal as variations in the instantaneous phase of a carrier wave. Phase modulation is one of the two principal forms of angle modulation, together with frequency modulation.

The phase of a carrier signal is modulated to follow the changing signal level (amplitude) of the message signal. The peak amplitude and the frequency of the carrier signal are maintained constant, but as the amplitude of the message signal changes, the phase of the carrier changes correspondingly.

Phase modulation is widely used for transmitting radio waves and is an integral part of many digital transmission coding schemes that underlie a wide range of technologies like Wi-Fi, GSM and satellite television.

PM is used for signal and waveform generation in digital synthesizers, such as the Yamaha DX7, to implement FM synthesis. A related type of sound synthesis called phase distortion is used in the Casio CZ synthesizers.



The modulating wave (blue) is modulating the carrier wave (red), resulting the PM signal (green). g(t) = π/2 * sin(2*2πt+ π/2*sin(3*2πt))

Demodulation is extracting the original information-bearing signal from a carrier wave. A **demodulator** is an electronic circuit that is used to recover the information content from the modulated carrier wave. There are many types of modulation so there are many types of demodulators. The signal output from a demodulator may represent, images or binary data.

There are several ways of demodulation depending on how parameters of the base-band signal such as amplitude, frequency or phase are transmitted in the carrier signal. For example, for a signal modulated with a linear modulation like AM , we can use a synchronous detector. On the other hand, for a signal modulated with an angular modulation, we must use an FM demodulator or a PM demodulator. Different kinds of circuits perform these functions.

Many techniques such as carrier recovery, clock recovery, bit slip, frame synchronization, rake receiver, pulse compression, Received Signal Strength Indication, error detection and correction, etc., are only performed by demodulators, although any specific demodulator may perform only some or none of these techniques.

AM demodulators:

An AM signal encodes the information into the carrier wave by varying its amplitude in direct sympathy with the analogue signal to be sent. There are two methods used to demodulate AM signals:

1.The **envelope detector** is a very simple method of demodulation that does not require a coherent demodulator. It consists of an envelope detector that can be a rectifier (anything that will pass current in one direction only) or other non-linear component that enhances one half of the received signal over the other and a low-pass filter. The rectifier may be in the form of a single diode or may be more complex. Many natural substances exhibit this rectification behavior, which is why it was the earliest modulation and demodulation technique used in radio. The filter is usually an RC low-pass type but the filter function can sometimes be achieved by relying on the limited frequency response of the circuitry following the rectifier. The crystal set exploits the simplicity of AM modulation to produce a receiver with very few parts, using the crystal as the rectifier and the limited frequency response of the headphones as the filter.

2.The **product detector** multiplies the incoming signal by the signal of a local oscillator with the same frequency and phase as the carrier of the incoming signal. After filtering, the original audio signal will result.

FM demodulators:

•The quadrature detector, which phase shifts the signal by 90 degrees and multiplies it with the unshifted version. One of the terms that drops out from this operation is the original information signal, which is selected and amplified.

•The signal is fed into a PLL and the error signal is used as the demodulated signal.

•The most common is a Foster-Seeley discriminator. This is composed of an electronic filter which decreases the amplitude of some frequencies relative to others, followed by an AM demodulator. If the filter response changes linearly with frequency, the final analog output will be proportional to the input frequency, as desired.

•A variant of the Foster-Seeley discriminator called the ratio detector

•Another method uses two AM demodulators, one tuned to the high end of the band and the other to the low end, and feed the outputs into a difference amplifier.

Using a digital signal processor, as used in software-defined radio.