



DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB103J –Semiconduuctor Physics

Concepts of optical transition in bulk semiconductor

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- For the optical properties of semiconductors, the photons should interact with charge carriers.
- In the process of interaction three process occurs
 - Absorption
 - Recombination
 - Emission

i.e the photons are absorbed and emitted , these processes are important in photonic devices using semiconductors

There are several type of transition possibilities are occur

1. Band to band transition (Inter band transition)
2. Impurity level to band transition
3. Free carrier transition (Intra band transition)

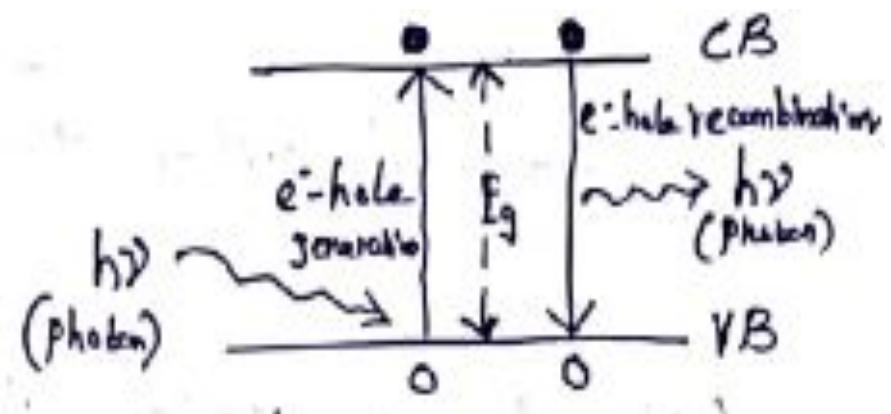


In semiconductors electrons can make transitions between two energy states and create or destroy photons in the process.

1. Band to band transition (Inter band transition)

An absorbed photon can result in an electron in the valence band making an upward transition to conduction band. This results electron-hole pair generation, followed by this electron-hole recombination takes place by the emission of Photon

Eg: Band to Band transition in GaAs can results absorption and emission of photons with wavelength of 0.087 μm or ($E_g = 1.42 \text{ eV}$)



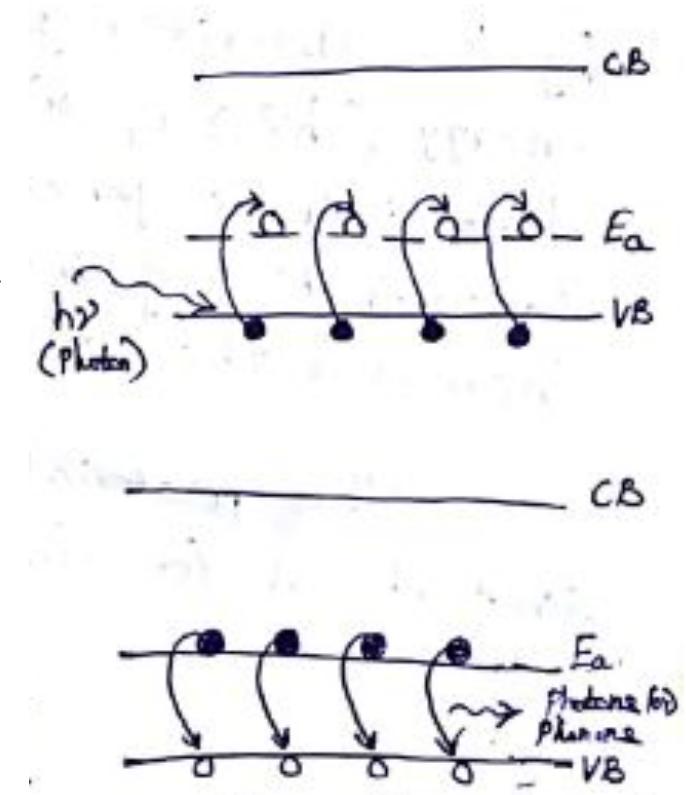
2. Impurity level to band transition:

An absorbed photon results in a charge carriers transition between a donor (or) acceptor level to a band in semiconductor, mostly observed in doped semiconductors.

For example if a p-type material is considered, the low energy photon absorbed by p-type semiconductor material leads transition of electron from valence band to acceptor level where its trapped by acceptor atom. Thus hole is created in Valence band and acceptor atom is ionized.

Similarly a hole may be trapped by an ionized acceptor atom. The result may be the electron decay from its acceptor level to recombine with hole. The energy may be released radiatively (photons) or non-radiatively (phonons)

Eg: In Hg doped Ge the wavelength of absorption and emission between valence and conduction band is $14 \mu\text{m}$





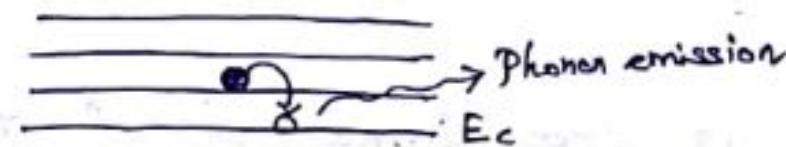
3. Free carrier transition (Intra band transition):

An absorbed photon can impart its energy to an electron in a given band, causing it to move higher level in that band. If a lower level conduction band is considered, by absorbing photon energy the electron moves to next higher energy level in the same conduction band.

Similarly due to thermalization, electron relaxes down to the bottom of the conduction band while releasing its energy in the form of phonons.



Photon - (Absorption process)



Phonon - (Recombination process)

Optical absorption process :

Absorption is the process in which the photons are absorbed by the semiconductor materials causes transition of electron from valence band to conduction band.

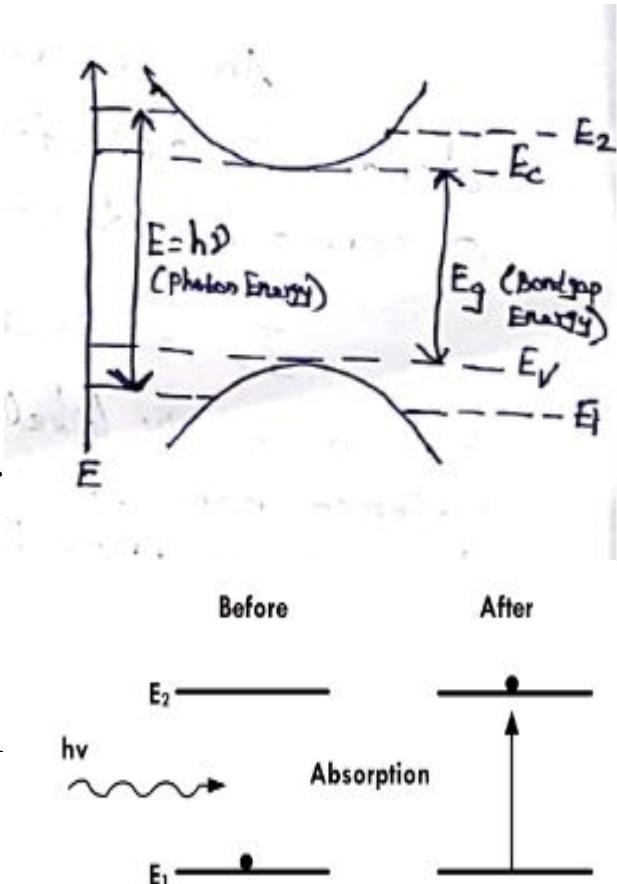
Consider direct band to band absorption

The condition for absorption to happen, the incident photon energy must be equal Or grater than the bandgap i.e $h\nu \geq E_g$, corresponding wavelength $\lambda = hc/E_g$.

The electron hole pairs are generated due to absorption of photon having energy grater Or equal to bandgap.

This leads to the increase of concentration of mobile charge carriers and increase the Conductivity of the material.

So the material behaves as a photoconductor with a conductivity proptaional to photon Flux, the effect is used to detect light.



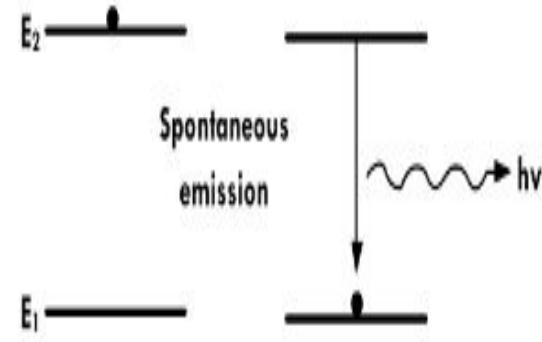


Emission process:

Generally the emission process are takes place in two types in optical devices

1. Spontaneous emission
2. Stimulated emission

Spontaneous emission: spontaneous emission, this process requires a conduction band energy state occupied by an electron and an empty valence band energy state. The electron Itself transit from conduction band to valence band spontaneously by releasing a photon.



This photon has a random direction and phase.

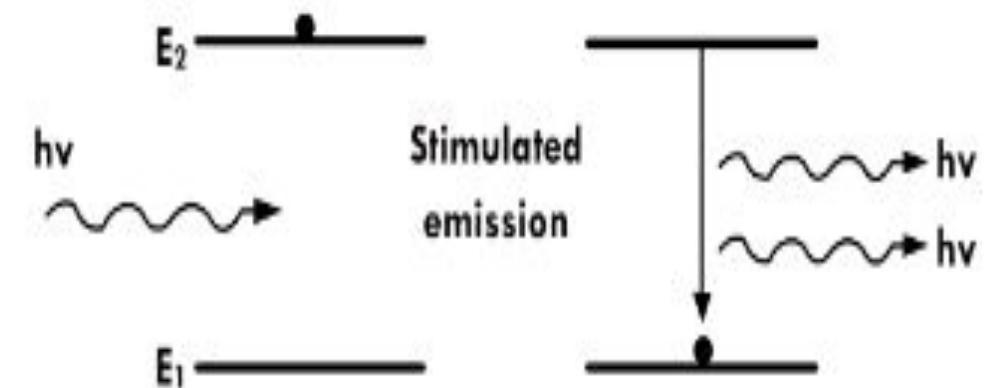
This is the opposite of the common situation in equilibrium, but at a finite temperature there will be a small number of full states in the conduction band and empty states in the valence band. Also, electrons and holes can be created via optical absorption and other pumping mechanisms.



Stimulated emission:

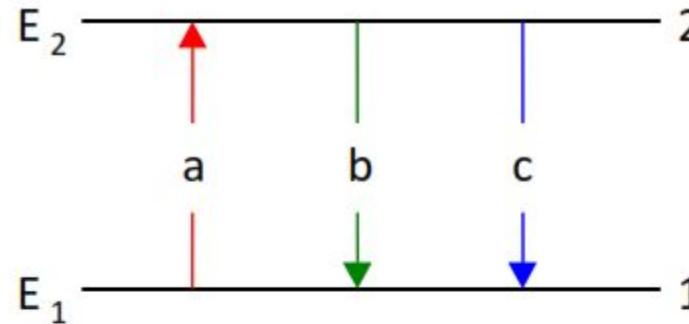
An incident photon causes an upper level atom to decay, emitting a “stimulated” photon whose properties are identical to those of the incident photon.

The term “stimulated” underlines the fact that this kind of radiation only occurs if an incident photon is present





Over all picture of Absorption and emission process



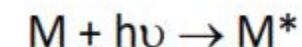
a absorption

b spontaneous emission

c stimulated emission

Absorption

Molecule absorbs a quantum of radiation (a photon) and is excited from 1 to 2.

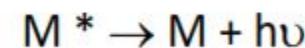


(state 1)

(state 2)

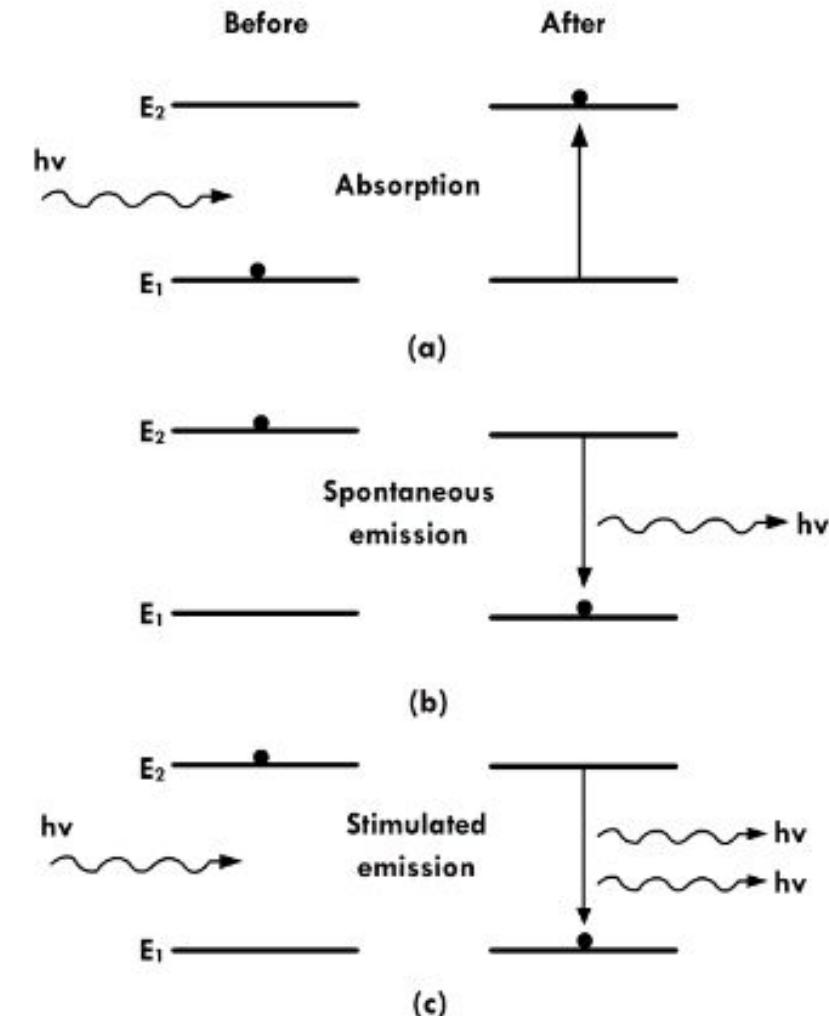
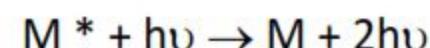
Spontaneous emission

M^* (in state 2) spontaneously emits a photon of radiation.



Stimulated emission

A quantum of radiation is required to stimulate M^* to go from 2 to 1.





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18PYB103J –Semiconductor Physics

Lecture 2 and 3

Concepts of Optical Recombination and emission process

- When a semiconductor is illuminated with light an electron in the valence band making an upward transition to conduction band.
- This results electron-hole pair generated, the reverse process of electron-hole annihilation is called recombination.
- Recombination process may be radiative or non- radiative
 1. If electron annihilated with hole energy is released equal to $E \geq E_g$ called radiative recombination, in this process a photon of energy $E = h\nu$ is released
 2. If electron annihilated with hole energy is released equal to $E < E_g$ called non radiative recombination, in this process phonons are released

Recombination electron – hole pairs observed in different optical process

- Luminescence: Process where electron hole pairs created and recombined radiatively
- Photoluminescence: electron- hole pairs are generated due to absorption of light and recombination occurs radiatively
- Cathodoluminescence: electron – hole pairs are generated by the electron bombardment, and radiative recombination occurs
- Electroluminescence: process of radiative recombination following injection with pn-junction or similar device.



When a semiconductor is under equilibrium without any incident photon (or) injection of electron the carrier density can be calculated from an equilibrium Fermi level using Fermi Dirac statistics

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

But when light is illuminated non equilibrium carrier concentration is created and above relation is not valid, hence Fermi Dirac distribution for electrons and holes in non equilibrium condition are

$$f(E)_n \propto \exp\left(\frac{E_{Fn} - E_c}{kT}\right) \text{ for electrons}$$

$$f(E)_p \propto \exp\left(\frac{E_v - E_{fp}}{kT}\right) \text{ for holes}$$

Further carrier concentration is calculated as

$$n = N_c \exp\left[\frac{E_{Fn} - E_c}{kT}\right] \text{ for electrons}$$

$$p = N_v \exp\left[\frac{E_v - E_{fp}}{kT}\right] \text{ for holes}$$

The excess carriers generated in semiconductor at non equilibrium condition must eventually recombine

$$\text{Generation rate (G)} = \text{recombination rate (R)}$$

The generation recombination process involves transition of charge carriers across the energy bandgap and is different for direct & indirect bandgap semiconductor materials.

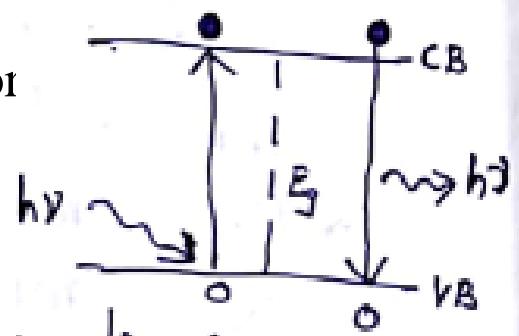
The probability of radiative recombination is very high and direct bandgap semiconductor due to momentum & energy conservation

Recombintaion rate of charge carriers is depends upon the lifetime charge carriers

In general both radiative and noradiative recombinations are considered, the total life time is given as

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

where $R = \frac{\Delta n}{\tau}$





τ_r is radiative life time & τ_{nr} is non-radiative life time of charge carriers. Also total Recombination rate is given by

$$R = R_r + R_{nr}$$

Internal quantum efficiency due to recombination process is

$$\eta_r = \frac{1}{1 + [\tau_r / \tau_{nr}]}$$

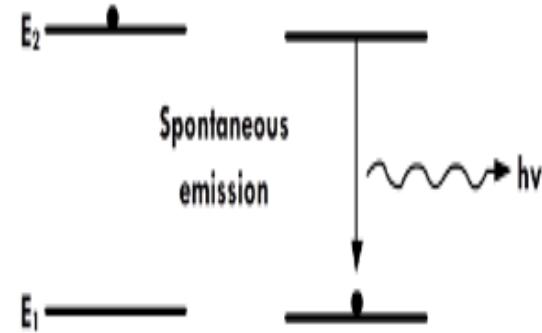
If τ_r / τ_{nr} is small in which τ_{nr} is large as possible, η_r increases leads to high radiative recombination in Semiconductor

Emission process:

Generally the emission process are takes place in two types in optical devices

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This photon has a random direction and phase.

This is the opposite of the common situation in equilibrium, but at a finite temperature there will be a small number of full states in the conduction band and empty states in the valence band. Also, electrons and holes can be created via optical absorption and other pumping mechanisms.



Absorption:

Let us consider two energy levels in semiconductor E_1 & E_2

where E_1 corresponds to ground state

E_2 corresponds to exited state

At room temperature most of the electrons are in ground state

When photons of energy grater or equal to bandgap incident on

Semiconductor electron hole pairs are generated, this process is

Called absorption

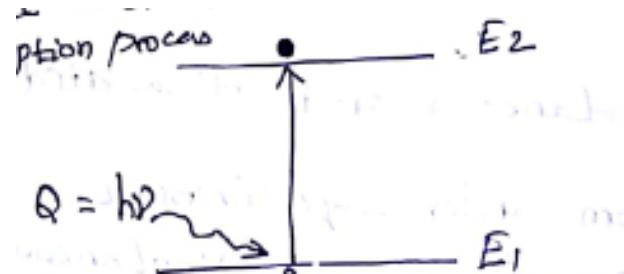
$$N_{ab} \propto Q N_1$$

N_{ab} → number of atoms undergoing absorption process/Vt

N_1 → number of atoms in E_1 / V

Q → energy density of incident radiation

B_{12} → proportionality constant



Spontaneous emission:

When electron hole pairs are generated due to the absorption of incident radiation.

After a short time without any external stimulus the electron come back from

Unstable exited to state (E_2) to ground state (E_1) by emitting a photon of energy

$$h\nu \geq E_2 - E_1$$

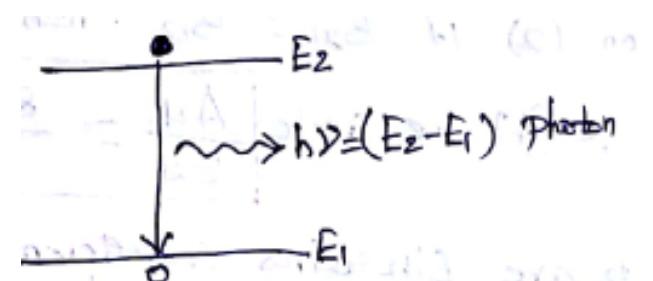
This process is called spontaneous emission

$$\text{Then } N_{sp} \propto N_2 \Rightarrow N_{sp} = A_{21} N_2$$

N_{sp} → is the number of atoms undergoing spontaneous emission process/vt

N_2 → number of atoms in E_2 / V

A_{21} → proportionality constant

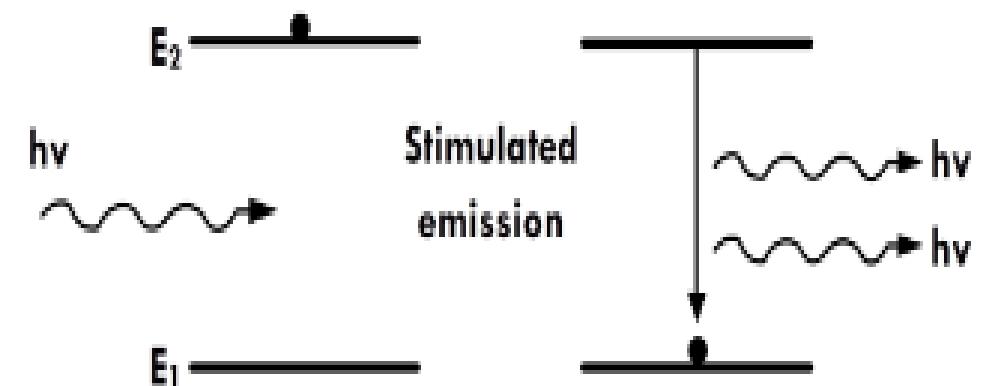




Stimulated emission:

An incident photon causes an upper level atom to decay, emitting a “stimulated” photon whose properties are identical to those of the incident photon.

The term “stimulated” underlines the fact that this kind of radiation only occurs if an incident photon is present



Stimulated emission:

If a photon of energy ($h\nu$) impinges on the electron which is presented in Exited state (E_2).

The electron stimulated back to the ground state by releasing the energy $h\nu \geq E_2 - E_1$ which is phase with the incident radiation this process is called stimulated emission.

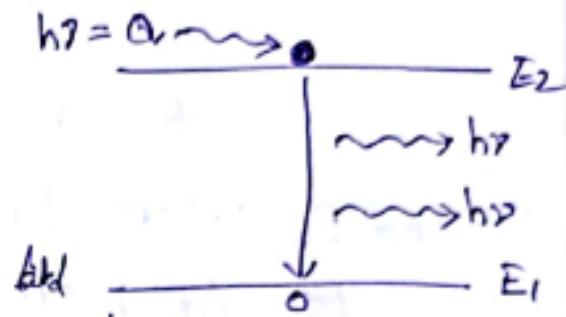
$$N_{st} = B_{21} Q N_2$$

N_{st} → number of atoms undergoing stimulated process/Vt

N_2 → number of atoms in E_2 / V

Q → energy density of incident radiation

B_{21} → proportionality constant



Einstein's theory of spontaneous and stimulated emission

At thermal equilibrium, the number of upward transition should be equal to the number of downward transitions per unit volume per unit time.

$$B_{12}N_1Q = A_{21}N_2 + B_{21}N_2Q \quad (4)$$

(or)

$$Q = \frac{A_{21}}{\left(\frac{N_1}{N_2} \right) B_{12} - B_{21}} \quad (5)$$

From Boltzmann's distribution law, at a given temperature T , the ratio of the population of two levels is given by

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT} \quad (6)$$

(or)

$$\frac{N_1}{N_2} = e^{h\nu / kT} \quad (7)$$

where k is Boltzmann constant. Substituting the value of N_1/N_2 in this Eqn

$$Q = \frac{\mathbf{A}_{21}}{\left(\frac{N_1}{N_2}\right)\mathbf{B}_{12} - \mathbf{B}_{21}} \quad \text{we get,}$$

$$Q = \frac{A_{21}}{B_{12}e^{h\nu / kT} - B_{21}} \quad (8)$$

According to Planck's black body radiation theory, we have

$$Q = \frac{8\pi hc}{\lambda^5} \frac{1}{(e^{h\nu/kT} - 1)} \quad (9)$$

Here c is the velocity of light.

If $B_{12} = B_{21} = B$, Eqn (8) can be expressed as

$$Q = \frac{A_{21}}{B_{21}(e^{h\nu/kT} - 1)} \quad (10)$$

Comparing the above Eqns we get

$$\frac{\mathbf{A}_{21}}{\mathbf{B}_{21}} = \frac{8\pi hc}{\lambda^5} \quad (11)$$

This eqn gives the ratio between spontaneous and stimulated coefficients. A and B are called Einstein's coefficients.

Find the relative population of the two states in Nd:YAG laser that produces a light beam of wavelength $1.06\mu\text{m}$ at 300K.

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1) / kT} = e^{-hv / kT}$$
$$= \mathbf{2.39 \times 10^{20}}$$

What fraction of sodium atom is in the first excited state in a sodium vapor lamp at a temperature of 250°C.

$$T = 250 + 273 = 523 \text{ K}$$

$$K = 1.38 \times 10^{-23} \text{ J/K}$$

$$\lambda = 5900 \times 10^{-10} \text{ m}$$

$$N_2/N_1 = e^{-(E_2 - E_1)/kT} = e^{-hv/kT}$$

$$v = C/\lambda$$

$$N_2/N_1 = 5.364 \times 10^{-21}$$

A He-Ne laser emits light at a wavelength of 632.8nm and has an output power of 3 mw. How many photons are emitted in each minute by this laser when operating?

$$\lambda = 6328 \times 10^{-10} \text{m} \quad P = 3 \text{mw} = 3 \times 10^{-3} \text{W}$$

$$v = c/\lambda = 4.74 \times 10^{14} \text{Hz}$$

$$E = hv = 3.14 \times 10^{-19} \text{J}$$

$$\text{Photons /minute} = n \times 60$$

$$5.7324 \times 10^{10} \text{ photons / minute.}$$



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Joint density of states

(Conservation of energy and momenta of
electron
with photon interacts)



Optical Joint Density of States

How many states are possible for photon interaction of energy $h\nu$ in valence and conduction band is given by optical joint density of states. To determine the density of state ρ_ν with which a photon of energy $h\nu$ interacts under a condition of energy and momentum conservation in a direct band gap semiconductor.

To approximate this relation for a direct band-gap semiconductor by two parabolas,

$$E_2 = E_c + \frac{\hbar^2 K^2}{2m_c}$$

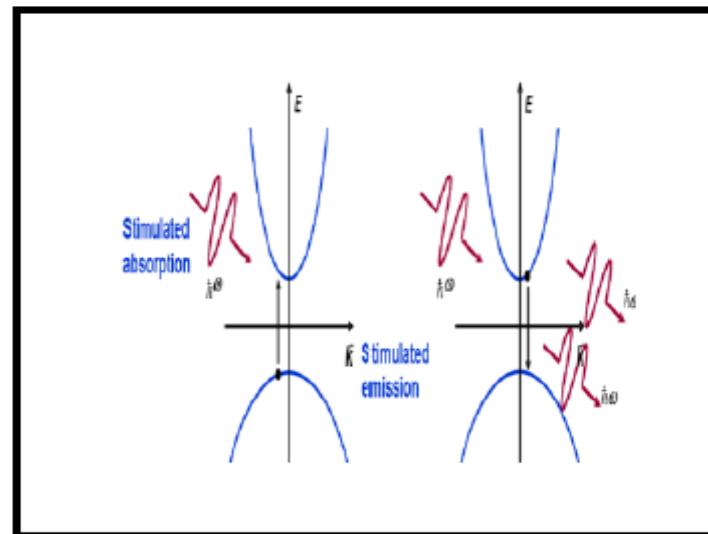
$$E_1 = E_v - \frac{\hbar^2 K^2}{2m_v}$$

$$h\nu = E_2 - E_1$$

$$h\nu = E_g + \frac{\hbar^2 K^2}{2m_g}$$

$$K^2 = \frac{2m_g}{\hbar^2} (h\nu - E_g)$$

$$\rho_\nu = (m_c - m_v)$$





Here, substitute the value of K^2 in eq (1) & eq (2)

$$E_2 = E_c + \frac{m\gamma}{m_c} (h\gamma - E_g)$$

Similarly,

$$E_1 = E_v - \frac{m\gamma}{m_c} (h\gamma - E_g)$$

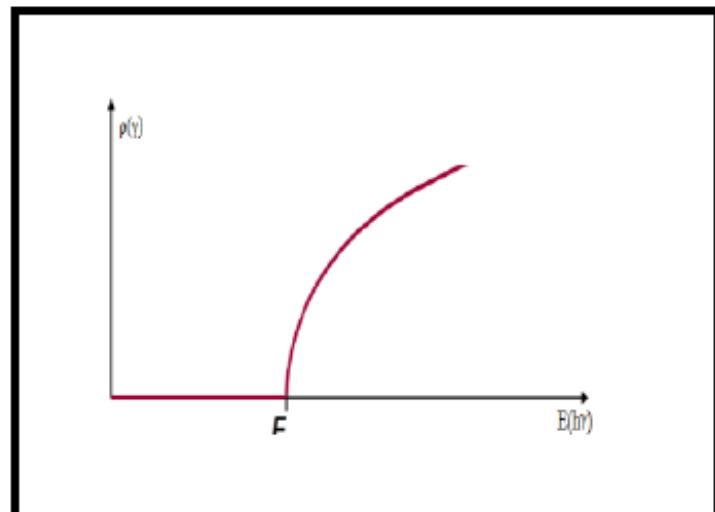
The one-to-one correspondence between E_2 and γ permits us to readily relate $\rho(\gamma)$ to the density of states $\rho_c(E_2)$ in conduction band by use of the incremental relation

$$\rho_c(E_2)dE = \rho(\gamma) d\gamma$$

Here $\rho_c(E_2)dE$ is no of states between E_2 and dE_2 and $\rho(\gamma) d\gamma$ is the number of states per unit volume of energy between $h\gamma$ and $h(\gamma+d\gamma)$ to interact.

Therefore,

$$\rho(\gamma) = \rho_c(E_2) \frac{dE}{d\gamma}$$





$$\rho(\gamma) = \frac{(2m\gamma)^{3/2}}{\pi\hbar^2} (h\gamma - E_g)^{1/2} \text{ for } h\gamma \geq E_g$$

The density of states which a photon of energy $h\gamma$ interact increases with $h\gamma \geq E_g$ in accordance with a square root law. Similarly One-to-One correspondence between E_1 and $\rho(\gamma)$ in equation, together with $\rho(\gamma) E_1$, results in an expression for $\rho(\gamma)$ identical.



Density of States for Photons

To define the density of states for photons we assume that the photon is enclosed in a large cube of side length L, such that volume is $V = L^3$. The wave function of photon is a plane wave $e^{ik\cdot r}$. We use the periodic boundary conditions that the wave function should be periodic in the x,y and z directions with a period L.

Because of the wave function has to be zero at boundaries. We have Quantization of wave number

$$L \cdot K = n2\pi$$

$$K_x = \frac{2\pi}{L}; K_y = \frac{2\pi}{m}; K_z = \frac{2\pi}{n}$$

The volume of state in K space is $(\frac{2\pi}{L})^3$



Density of States for Photons

Let us look at the integral using the number of states with a differential volume in the K-space.

$$\frac{d^3K}{(\frac{2\pi}{L})^3} = \frac{K^2 dk d\Omega}{(\frac{2\pi}{L})^3}$$

Where $d\Omega$ is the differential solid angle.

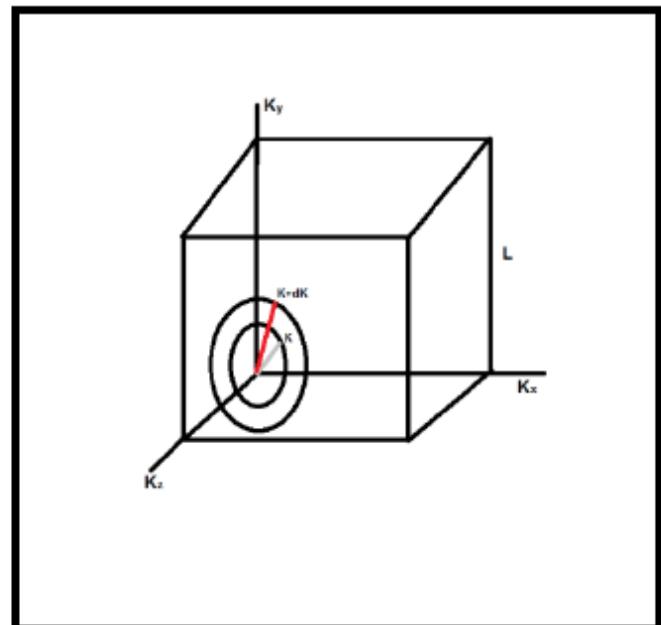
Therefore

$$N(E_{21}) = \frac{2}{V} \sum_K \delta(E_2 - E_1 - E_k)$$

$$N(E_{21}) = 2 \int \frac{K^2 dk d\Omega}{(2\pi)^3} \delta(E_2 - E_1 - E_k)$$

$$E_k = \hbar\omega_k = \frac{\hbar KC}{n_r}$$

Where, C/n_r is the speed of light in medium with refractive index of n_r . Here integration over solid angle is 4π .





Density of States for Photons

$$K = \frac{E_k n_r}{\hbar C}$$

$$dK = \frac{n_r 2\pi}{hC} dE_k$$

$$N(E_{21}) = 2 \int \frac{K^2 dk d\Omega}{(2\pi)^3} \delta(E_{21} - E_k)$$

$$N(E_{21}) = 2 \int \frac{K^2}{(2\pi)^3} \frac{n_r 2\pi}{hC} dE_k (4\pi) \delta(E_{21} - E_k)$$

$$N(E_{21}) = 2 \int \frac{1}{(2\pi)^3} \left(\frac{E_k n_r 2\pi}{hC} \right)^2 \frac{n_r 2\pi}{hC} dE_k (4\pi) \delta(E_{21} - E_k)$$

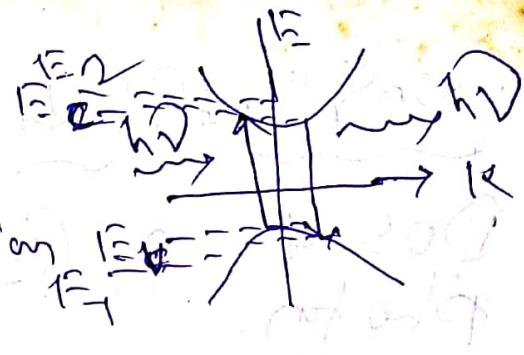
$$N(E_{21}) = \frac{2 \times 4\pi \times (2\pi)^3 (n_r)^3}{(2\pi)^3 (hC)^3} \int (E_k)^3 dE_k \delta(E_{21} - E_k)$$

$$N(E_{21}) = \frac{8\pi (n_r)^3}{(hC)^3} E_{21}^2 \quad [\hbar = \frac{h}{2\pi}; h = \hbar 2\pi]$$

$$N(E_{21}) = \frac{8\pi E_{21}^2 (n_r)^3}{8\pi^3 \hbar^3 C^3} = \frac{E_{21}^2 (n_r)^3}{\pi^2 \hbar^3 C^3}$$

Which is the number of states with photon energy E_{21} per unit volume per energy interval.

Optical Joint Density of State:



No. of emission (or) Absorption per unit volume is given by

$$n = \int n(E) dE = \int P(E) f(E) dE \rightarrow ①$$

$$P = \int P(E) dE = \int P(E) [1 - f(E)] dE \rightarrow ②$$

No. of Pairs of States in Conducting and Valence Band

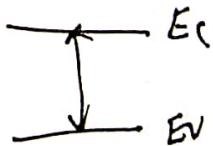
$$E_2 = E_e + \frac{\hbar^2 k_e^2}{2m_e} \rightarrow ③$$

$$E_1 = E_v - \frac{\hbar^2 k_v^2}{2m_v} \rightarrow ④$$

$$E_2 - E_1 = E_e - E_v + \frac{\hbar^2 k_e^2}{2} \left[\frac{1}{m_e} + \frac{1}{m_v} \right]$$

$$E_2 - E_{pl} = E_g + \frac{\hbar^2 k_e^2}{2m_v} \rightarrow ⑤$$

$$\therefore h\nu = E_g + \frac{\hbar^2 k_e^2}{2m_v}$$



$$(h\nu - E_g) = \frac{\hbar^2 k_e^2}{2m_v}$$

$$E_{pl} = \frac{2m_v}{\hbar^2} (h\nu - E_g) \rightarrow ⑥$$

using ex ⑥ in eq ③ we get

$$E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g) \rightarrow ④$$

$$\sqrt{(E_2 - E_c)^2} = \frac{m_r}{m_c} (h\nu - E_g) \rightarrow ⑤$$

optical Joint density of States

$$p(\beta) d\beta = p(E_2) dE_2 \rightarrow ⑥$$

$$\begin{aligned} \therefore p(\beta) &= \frac{p_c(E_2) dE_2}{\frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E_2 - E_c)^{1/2}} \\ &= \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \frac{m_r}{m_c} (h\nu - E_g)^{1/2} \end{aligned}$$

$$p(\beta) = \frac{1}{\pi h^2} (2m_r)^{3/2} (h\nu - E_g)^{1/2} \rightarrow ⑦$$

Eqn ⑦ is optical Joint Dos

Density of State of Photons

DOS of photon $D(E) = \frac{dn(E)}{dE} \rightarrow ①$

$n(E)$ is the no. of photons in phase space
 $dn(E)$ is the no. of photons in infinitesimal space
 $dn(E) = \frac{d^3r d^3p}{h^3} \rightarrow ②$

$$\begin{aligned}
 D(E) &= \frac{1}{h^3} \int d^3r d^3p \xrightarrow{\text{Energy}} \\
 &= \frac{1}{h^3} V \int p^2 dp \xrightarrow{\substack{\text{solid angle} \\ \text{spherical} \\ \text{polar} \\ \text{coordinates}}} \sin\theta d\theta d\phi \\
 &= \frac{V}{h^3} \int p^2 dp (4\pi) \\
 &= \frac{4\pi V}{h^3} \int p^2 dp \\
 &= \frac{4\pi V}{h^3} \int \frac{E^2}{c^2} \cdot \frac{dE}{c} \quad \boxed{\frac{E}{c} = \frac{pc}{c} = p} \\
 &\quad \boxed{\frac{dE}{c} = dp}
 \end{aligned}$$

$$n(E) = \frac{4\pi N}{h^3 c^3} \int_{E_0}^{E_0 + \Delta E} \frac{E^2}{E^2 - E_0^2} dE \quad \text{approx}$$

$$\therefore n(E) = \frac{4\pi N}{h^3 c^3} \left[\frac{E^3}{3} \right]_{E_0}^{E_0 + \Delta E} + \dots \quad \text{approx}$$

$$n(E) = \frac{4\pi N E^3}{3 h^3 c^3} \rightarrow \textcircled{3}$$

Photons have two polarizabian state

$$\therefore n(E) = 2 \cdot \frac{4\pi N E^3}{3 h^3 c^3} \Rightarrow \frac{8\pi N E^3}{3 h^3 c^3} \rightarrow \textcircled{4}$$

$$D(E) = \frac{dn(E)}{dE} \Rightarrow \frac{8\pi N}{3 h^3 c^3} \cdot E^2$$

$$D(E) = \frac{8\pi N E^2}{h^3 c^3} \rightarrow \textcircled{5}$$

In terms of frequency

$$E = h\nu \quad \boxed{E = h\nu}$$

$$D(\nu) = \frac{8\pi N \nu^2}{h c^3} \rightarrow \textcircled{6}$$

Density of states of photons.



DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB103J –Semiconduuctor Physics

Optical Transitions Using Fermi's Golden Rule

Optical Transitions Using Fermi's Golden Rule

Introduction

Fermi's golden rule is a simple expression for the transition probabilities between states of a quantum system, which are subjected to a perturbation. It is used for a large variety of physical systems covering, e.g., nuclear reactions, optical transitions, or scattering of electrons in solids.

Consider a semiconductor illuminated by electromagnetic radiations (light). The interaction between photons and the electrons in the semiconductor can be described by the Hamiltonian operator.

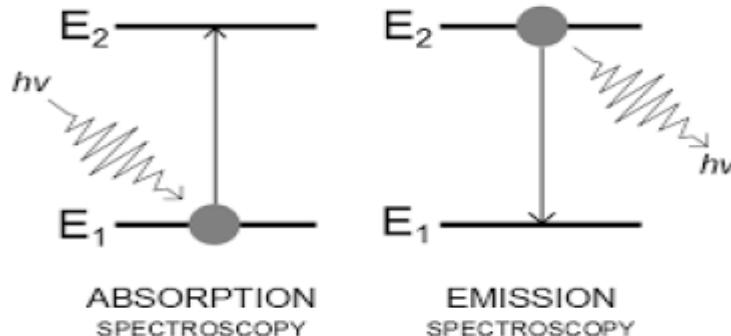
$$\vec{H} = \frac{1}{2m_0} (\vec{p} - e\vec{A})^2 + \vec{V}(r)$$

Where ,

m_0 is the free electron mass, \vec{A} is the vector potential accounting part of electromagnetic field.

$\vec{V}(r)$ is the periodic potential and $e = -|e|$

Optical Transitions Using Fermi's Golden Rule



Using the time dependent perturbation theory, the transition rate for the absorption of a photon can be derived, assuming an electron is initially at state E_1 is given by Fermi's Golden rule

$$W_{abs} = \frac{2\pi}{\hbar} | \langle b | H' (r) | a \rangle |^2 \delta (E_b - E_a - \hbar\omega)$$

Where $E_b > E_a$ is assumed.

The total upward transition rate per unit volume

Where $E_b > E_a$ has been assumed. The total upward transition rate per unit volume ($S^{-1}, \text{ cm}^{-3}$) in the crystal taking into account the probability that state a is occupied and state b is empty is

$$R_{a-b} = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} | H'_{ba} |^2 \delta (E_b - E_a - \hbar\omega) f_a (1 - f_b)$$

Optical Transitions Using Fermi's Golden Rule

Where we sum over the initial and final states and assume that the Fermi-Dirac distribution f_a is the probability that the state a is occupied. A similar expression holds for f_b with E_a replaced by E_b , and $(1 - f_b)$ is probability that the state b is empty. The prefactor 2 takes into account the sum over spins, and the matrix element H'_{ba} is given by

$$H'_{ba} = | \langle b | H'(r) | a \rangle |^2 = \int \psi^*(r) H'(r) \psi_a(r) d^3r$$

Similarly, The transition rate for the emission of a photon (fig.2) if an electron is initially at state b is.

$$W_{\text{ems}} = \frac{2\pi}{\hbar} | \langle a | H'^+(r) | b \rangle |^2 \delta(E_a - E_b + \hbar\omega)$$

The downward transition rate per unit volume ($\text{S}^{-1} \text{ cm}^{-3}$) is

$$R_{b-a} = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} | H'^+_{ab} |^2 \delta(E_a - E_b + \hbar\omega) f_b (1 - f_a)$$



Optical Transitions Using Fermi's Golden Rule

Using the even property of the delta function, $\delta(-x) = \delta(x)$ and $|H'_{ba}| = |H^+_{ab}|$.

The net upward transition rate per unit volume can be written as,

$$R = R_{a \rightarrow b} - R_{b \rightarrow a}$$

$$R = \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a - \hbar\omega)(f_a - f_b)$$

An Optical absorption coefficient

The absorption coefficient $\alpha_0 (\frac{1}{cm})$ in the crystal is the fraction of photons absorbed per unit distance

$$\alpha_0 = \frac{\text{Number of Photons absorbed per second per unit volume}}{\text{Number of injected photons per second per unit area}}$$

Optical Gain in Semiconductor



Optical gain in the material is attained when we injected a carrier density beyond E₂ such that the Quasi-Fermi level are separated by an energy greater (E_{fa}-E_{fb}). The process of stimulated downward transition is called optical gain and the process of upward transition is called optical loss. The simple formula for optical gain is

$$g \equiv \frac{1}{\phi} \frac{d\phi}{dz}$$

Where ϕ is photon flux (number of photons per cross section area unit in the unit time) and Z is the direction of electromagnetic field propagation is equal to

$$R = R_{a \rightarrow b} - R_{b \rightarrow a}$$



Optical Gain in Semiconductor

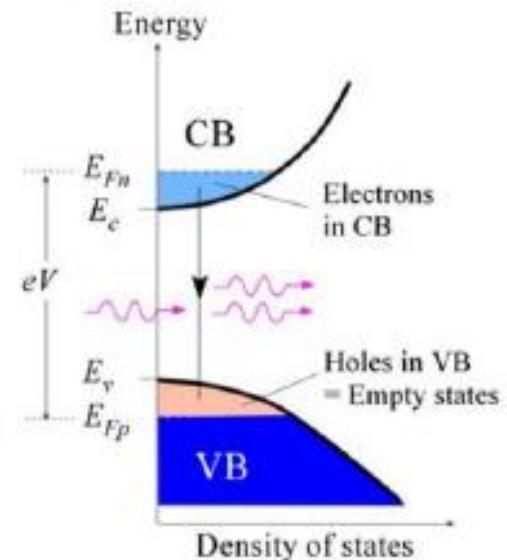
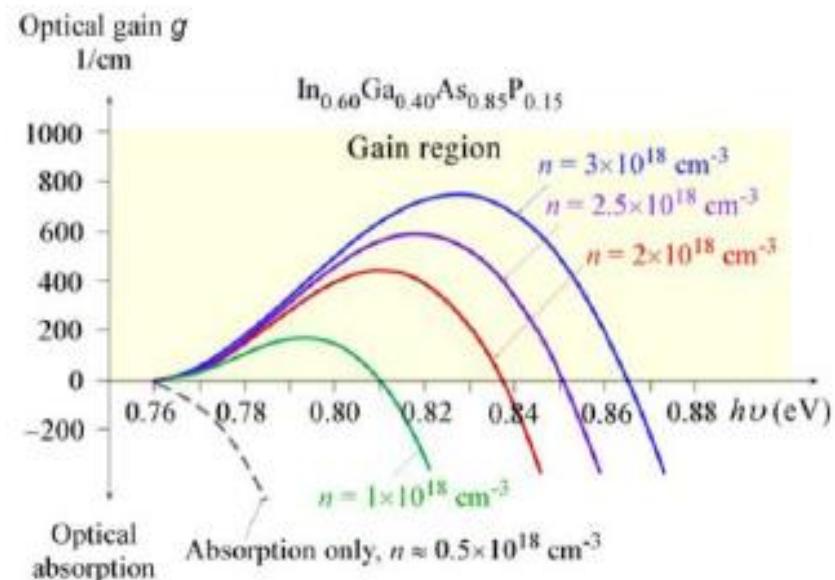


So the resultant gain we explained as

$$g = \frac{1}{\varphi} \cdot \frac{2}{V} \sum_{K_a} \sum_{K_b} \frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_a - E_{cb} + \hbar\omega)(f_b - f_a)$$

The gain and absorption (Loss) profiles as a function of energy is shown in Fig.

Optical Gain Curve





Thank you

Fermi Golden rule - used to know transition rates in semiconductors
 obtained - Applying time dependent perturbation theory to a system undergoing transition per unit time from Initial state (E_1) to final state (E_2) that is part of Continuum states.

The transition rate per unit time for absorption of photon \downarrow
 (Assuming e^- in initial state (E_1)), we write
 $f(E) = N(E) / N(E) \Rightarrow N(E) = f(E) \times N(E)$
 $W_{\text{abs}} = \frac{2\pi}{\hbar} | \langle b | H'_{\text{ab}} | a \rangle |^2 \delta(E_b - E_a - \hbar\omega)$ \uparrow absorption R

Total upward transition per unit volume in unit time

$$R_{a \rightarrow b} = \frac{2}{V} \sum_k \sum_k \frac{2\pi}{\hbar} | H'_{\text{ab}} |^2 \delta(E_b - E_a - \hbar\omega) f_a (1 - f_b)$$

2 - Sum over spin

$$| H'_{\text{ab}} |^2 = \langle b | H'_{\text{ab}} | a \rangle = \int \psi_b^*(r) H'_{\text{ab}} \psi_a(r) d^3r$$

Up transition rate for emission of Photon (Assuming e^- is initially at state E_b is)

$$W_{\text{em}} = \frac{2\pi}{\hbar} | \langle a | H'^+(r) | b \rangle |^2 \delta(E_a - E_b + \hbar\omega)$$

Total downward transition per unit volume per unit time

$$R_{b \rightarrow a} = \frac{2}{V} \sum_k \sum_k \frac{2\pi}{\hbar} | H'^+_{ba} |^2 \delta(E_a - E_b + \hbar\omega) f_b (1 - f_a)$$

Using even property of δ $\Rightarrow \delta(-x) = \delta(x) \Rightarrow | H'_{ba} | = | H'^+_{ba} |$

Then net upward transition rate per unit volume

$$R = R_{a \rightarrow b} - R_{b \rightarrow a}$$

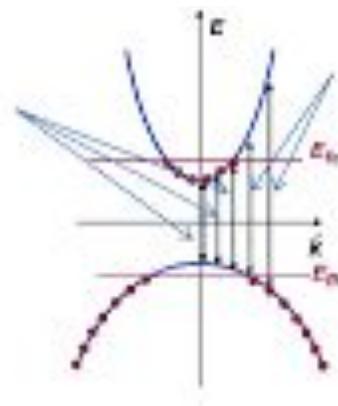
$$R = \frac{2}{V} \sum_k \sum_k \frac{2\pi}{\hbar} | H'_{\text{ab}} |^2 \delta(E_b - E_a - \hbar\omega) (f_a - f_b)$$

Transition rate is effectively independent of time & Proportional to Strength of Coupling bet. Initial & final state as well as density of states of Photon.



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18PYB103J – Physics: Semiconductor Physics Module – III (Lecture S9 – SLO 1 & 2)

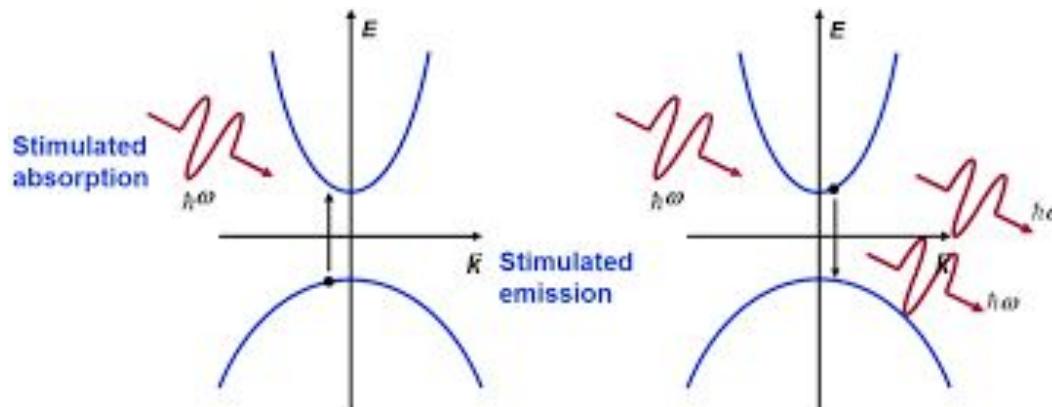


Optical Gain and Optical Loss



Optical Gain and Loss in semiconductor

Optical Gain in semiconductor defines the stimulated emission associated with light emission created by recombination of electrons and holes.



Optical Loss in semiconductor defines the stimulated absorption associated with light absorption created by generation of electrons and holes.



In a semiconductor crystal, consider an electron initially occupies a single state and makes a transition to one of a large number of final states due to photon interaction.

The electron-photon interactions in the crystal is characterized by Fermi's Golden Rule and gives the transition rate for a single pair of conduction and valence band states.

Each downward transition generates one photon and upward transition absorbs one photon.



$W_{c \rightarrow v}$ is the downward transition rate and $W_{v \rightarrow c}$ is the upward transition rate which can be find using Fermi Golden rule

$$W_{c \rightarrow v} = \frac{2\pi}{\hbar} \left| H'_{eh} \right|^2 \rho_{red} f_c (1 - f_v)$$

$$W_{v \rightarrow c} = \frac{2\pi}{\hbar} \left| H'_{eh} \right|^2 \rho_{red} f_v (1 - f_c)$$

Where H' is time dependent perturbation to the original Hamiltonian, It is to induce electronic transition between conduction and valence band. f_v and f_c the Fermi distribution and ρ_{red} is reduced density of state.



Explanation for Optical Gain

Optical gain in semiconductor is caused by photon-induced transitions of electrons from the conduction band to the valence band

Optical gain in the material is attained when we inject a carrier density beyond N_{tr} such that the quasi-Fermi levels are separated by an energy greater than the band gap.

If the number of downward transition per seconds exceeds the number of upward transition, there will be a net generation of photons, and optical gain can be achieved.



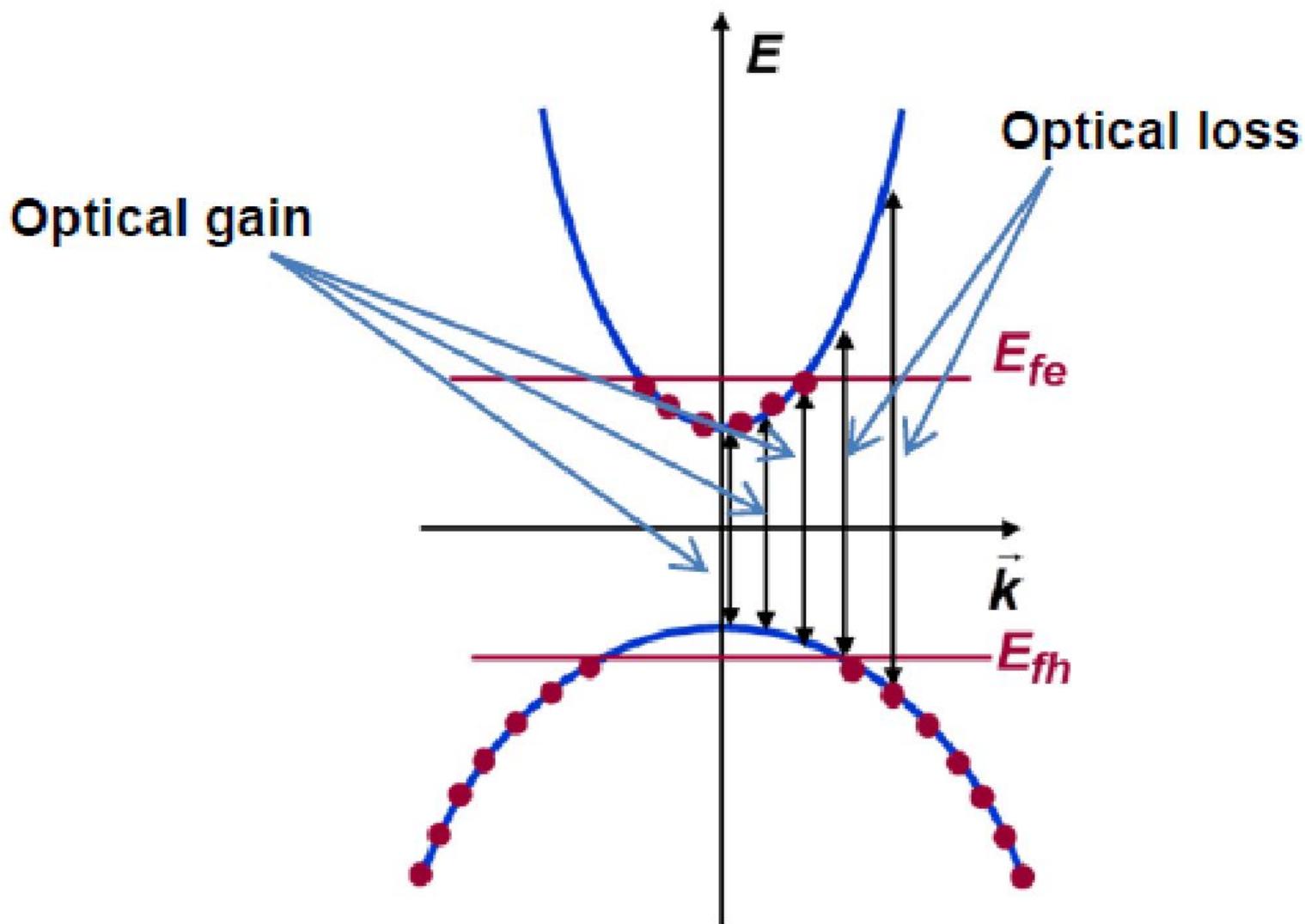
The optical gain is given as $(g) = \frac{1}{\phi} \left(\frac{d\Phi}{dz} \right)$

Where,

ϕ is the photon flux (the number of photons per cross section area unit in the unit of time)

z is the direction of the electromagnetic field propagation,

$$\frac{d\Phi}{dz} = W_{C \rightarrow V} - W_{V \rightarrow C}$$





The optical gain experienced by an incoming photon is very much dependent on the photon's energy.

From the figure, Given a value for the Fermi level splitting, optical frequencies for which $Eg < \hbar\omega < E_{fe} - E_{fh}$ experience optical gain

The condition $Eg < \hbar\omega < E_{fe} - E_{fh}$ is the condition for population inversion and can be realized if, for example, electrons are removed from the valence band and placed in the conduction band.

The light loss coefficient $\alpha(\omega)$ for optical gain is less than zero or negative

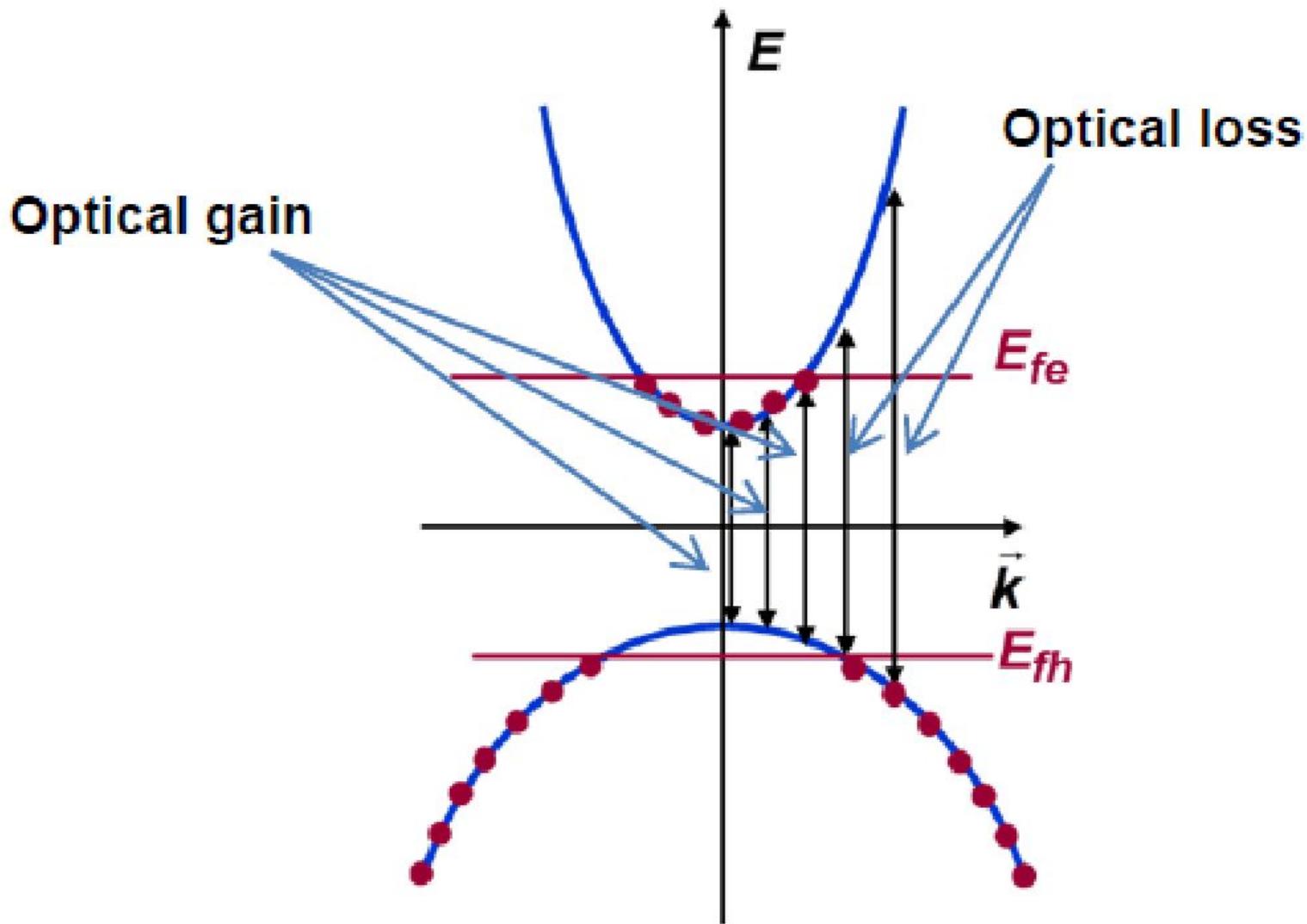


Explanation for Optical Loss

Optical gain in semiconductor is caused by photon-induced transitions of electrons from the valence band to the conduction band

Optical loss in the material is attained when we inject a carrier density beyond N_{tr} such that the quasi-Fermi levels are separated by an energy greater than the band gap.

If the number of upward transition per seconds exceeds the number of downward transition, there will be a net absorption of photons, and optical loss can be achieved.





From the figure, Given a value for the Fermi level splitting, optical frequencies for which $Eg > \hbar\omega$, E_{fe} - E_{fh} experience optical loss

The condition $Eg < \hbar\omega$, E_{fe} - E_{fh} is the condition for stimulated absorption and can be realized if, for example, electrons are removed from the conduction band and placed in the valence band.

The light loss coefficient $\alpha(\omega)$ for optical loss is greater than zero or positive

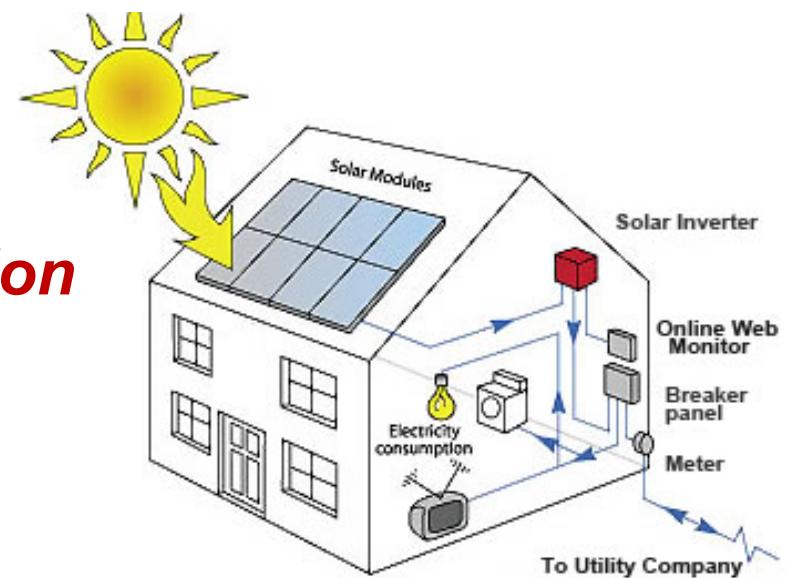


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**18PYB103J – Physics: Semiconductor Physics
Module - III (Lecture S13 – SLO 1 & 2)**

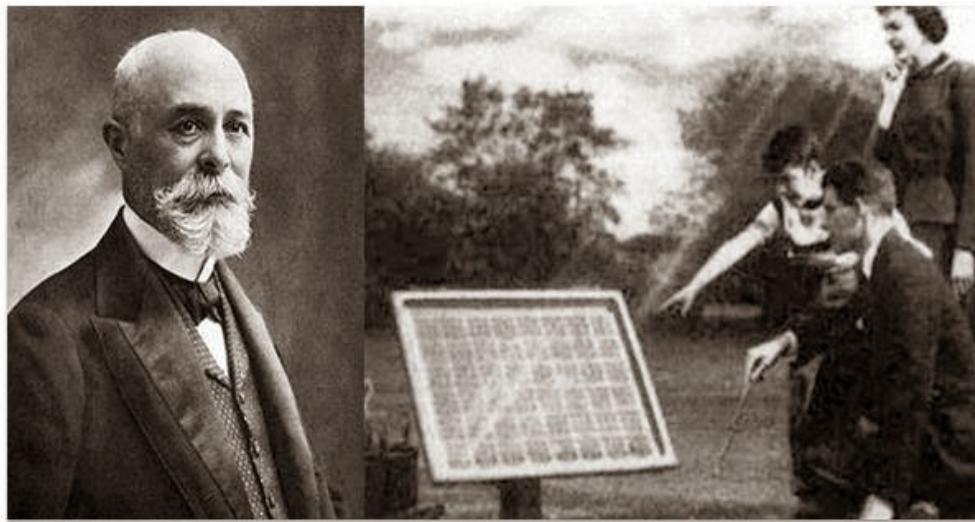
Photovoltaic Effect

***Photovoltaic effect in PN Junction
under illumination***





Photovoltaic effect: It is a process that generates voltage or electric current in semiconductor device when it is exposed to sunlight.



The photovoltaic effect was discovered in the year 1839 by French physicist, **Edmond Becquerel**. It find's application in solar or photovoltaic cell to generate electricity.



History of Photovoltaic's

- **In 1839** – French physicist, **Edmond Becquerel** observed photovoltaic effect while experimenting with two silver-coated platinum electrodes immersed in a dilute acid and observed two electrodes altered their electric power under illumination.
- **In 1876** – **William G. Adams and Richard E. Day** who showed that electric power is directly produced under illumination.
- **In 1904** – **Albert Einstein** described the photoelectric effect on which photovoltaic technology is based.



- **1918** – A Polish scientist **Jan Czochralski** discovered a method for monocrystalline silicon production.
- **1941** – The first **photovoltaic monocrystalline** photovoltaic cell was constructed based on Czochralski method.
- **1951** – The first **germanium photovoltaic cells** was made in Bell's Laboratories with 4.5% efficiency.
- **1957** – **Hoffman Electronics** introduced photovoltaic cell with 8% efficiency and a year later, the efficiency of increased to 9% .



Introduction to solar radiation

Solar radiation is a radiant energy emitted by the sun as a result of its nuclear fusion reactions.

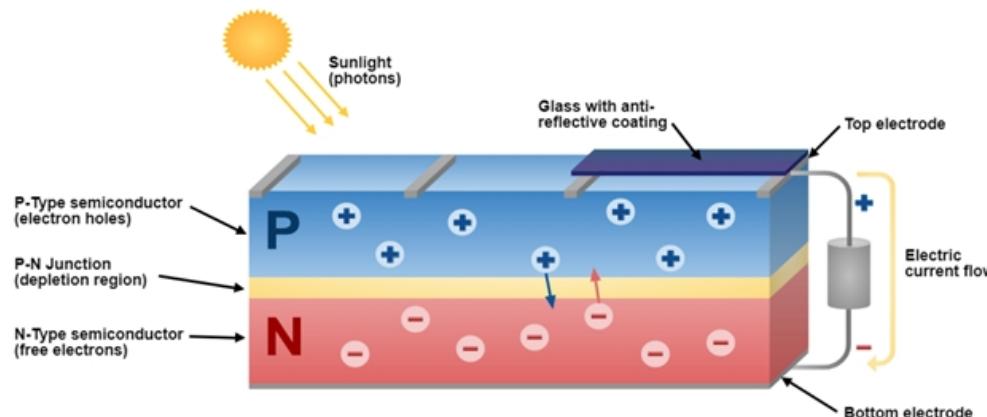
Over 99% of the energy flux from the sun is in the spectral region of $0.15 - 4\mu\text{m}$, with approximately 50% in the visible light region of $0.4 - 0.7\mu\text{m}$.

Sunlight is composed of photons and the photons contain different amounts of energy that correspond to the different wavelengths.



Photovoltaic in Semiconductor

The Solar or Photovoltaic cells are composed of two different types of semiconductor (a P-type and an N-type) that are joined together to create a PN Junction.





When photons strike a photovoltaic cell made by semiconductor PN junction, they may be reflected, absorbed or transmitted.

If photons is absorbed by the semiconductor then result is generation of electron-hole pair across PN Junction.

This electron-hole pair when separated from each other across the PN junction , results generation of voltage across the junction.

This voltage can drive a current in an external circuit called photocurrent

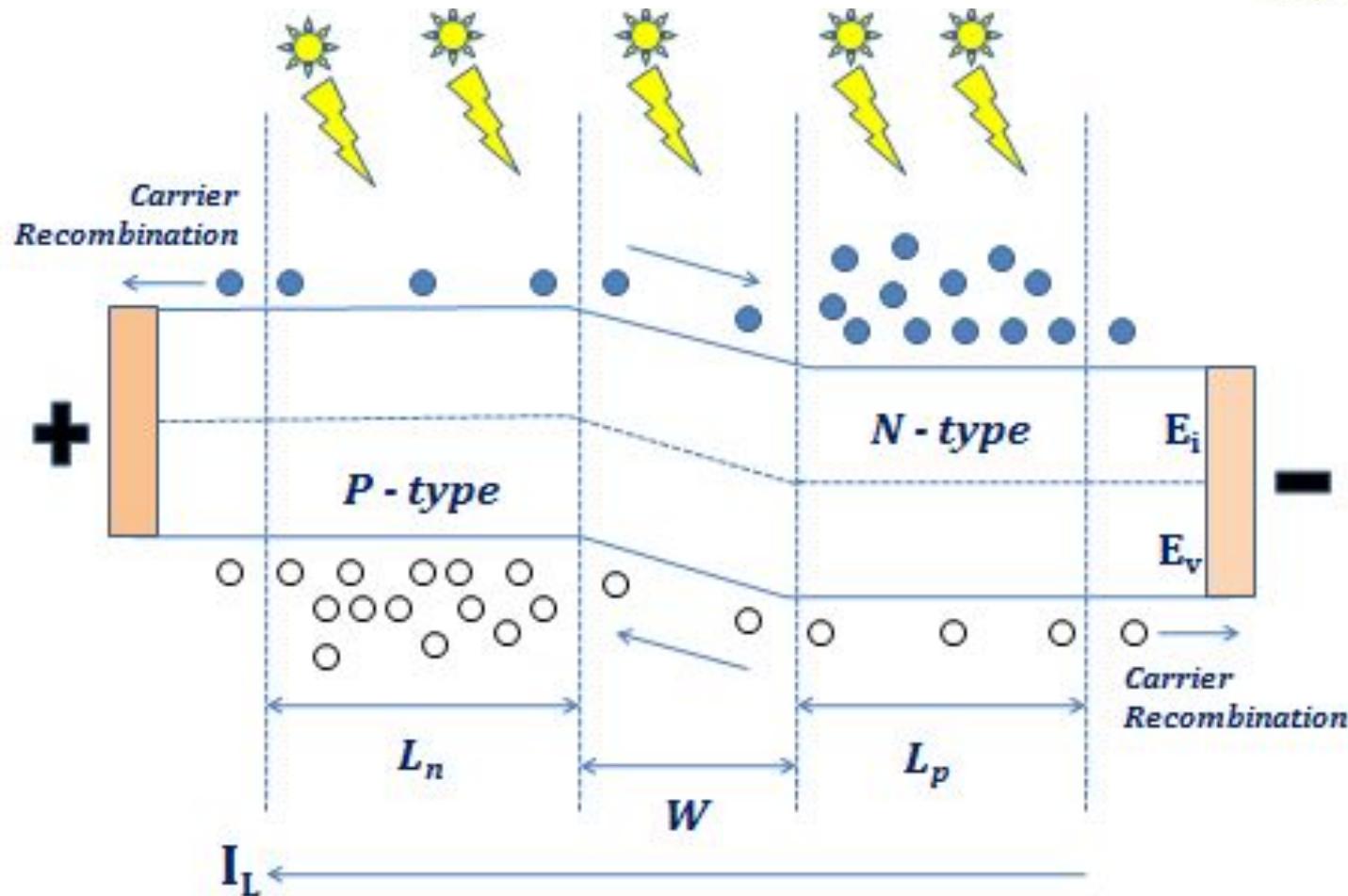


Photovoltaic in PN Junction under Illumination

When there is no light falling on the PN Junction, no electron-hole pair is generated. But when light is illuminated, it absorbs solar radiation and electron-hole pairs are generated.

Under the uniform illumination condition, generation of carrier will occur in the space-charge region as well as quasi-neutral region.

The carrier that are generated in the space charge region will be immediately swept away due to the electric field (electron towards N-region and holes towards P-region).



L_n and L_p - Diffusion length of electron and hole in Quasi neutral region, W - Width of depletion region or Space charge region, E_v - Energy of valence band and E_i - Fermi energy



Due to the electric field, chances of recombination of these electron pairs are quite less

The electron-hole pairs which are generated in the quasi neutral region will move around in a random manner.

In their random motion, some of the generated minority carriers will come near to the space charge region edge, where they will experience a force due to electric field and will be pulled at the other side

Only the minority charge carriers will cross the junction



Minority electrons from P-side will come to N-side leaving behind their positive charge called hole.

Similarly minority hole from N-side moves to P-side leaving behind their negative charge called electrons.

The drift of charge carriers creates a net increase in the positive charges at P-side and a net increase in negative charges at N-side.

This buildup of a positive and negative charge causes a potential difference to appear across the PN Junction under illumination. The potential difference created across light illuminated PN junction is called photo voltage.



The contribution to the photo voltage is coming only from the carriers that are generated within the semiconductor width ($L_n + W + L_p$)

Light generated Current

In a semiconductor PN Junction, four current components are present in equilibrium condition: drift current due to electron and hole, diffusion current due to electron and hole

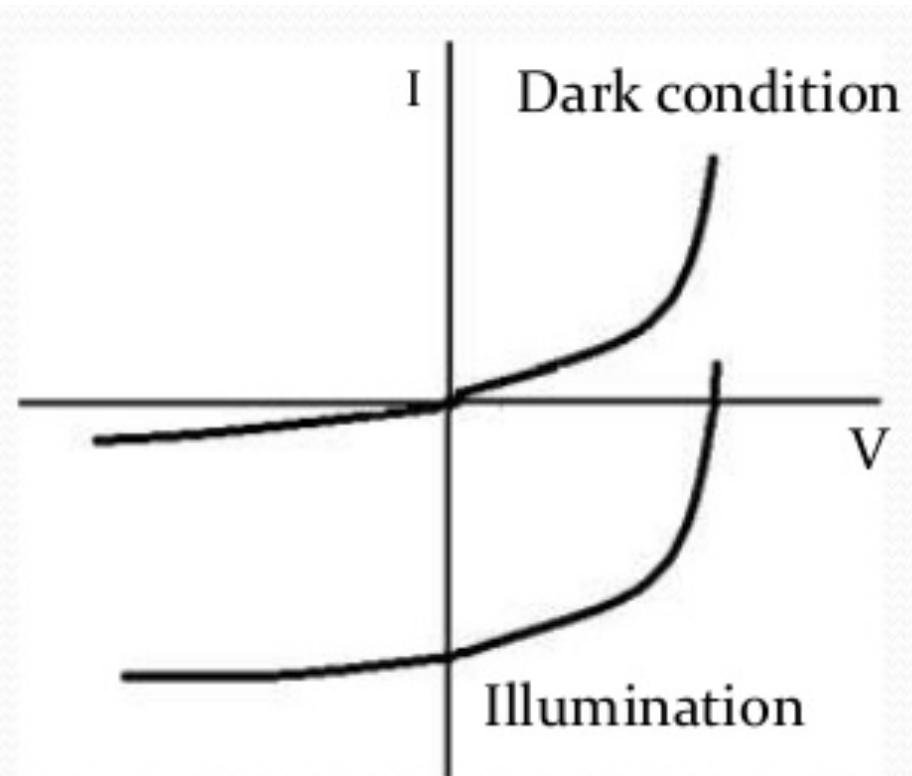
In equilibrium condition, net current is zero which requires the drift and diffusion currents of carriers to be equal and opposite.



When semiconductor PN Junction is illuminated, there is large drift current is observed due to minority electron and holes carrier.

Therefore in the I-V curve under dark, $I \propto V$ is observed as normally like PN diode.

But on illuminating the PN junction, large negative current caused by the light known as light generated current I_L is observed



**Dark I-V curve and I-V curve
under illuminated P-N Junction**



The large negative current in illuminated PN junction is due to voltage which is generated due to light biases the PN Junction in a forward bias mode, the diffusion current flows opposite to the direction of light generated current

The light generated current is given by the following equation

$$I_{\text{total}} = I_0 (e^{qV/kT} - 1) - I_L$$



**DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
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**18PY103J – Physics: Semiconductor Physics
Module-III, Lecture-14**

Solar Cell



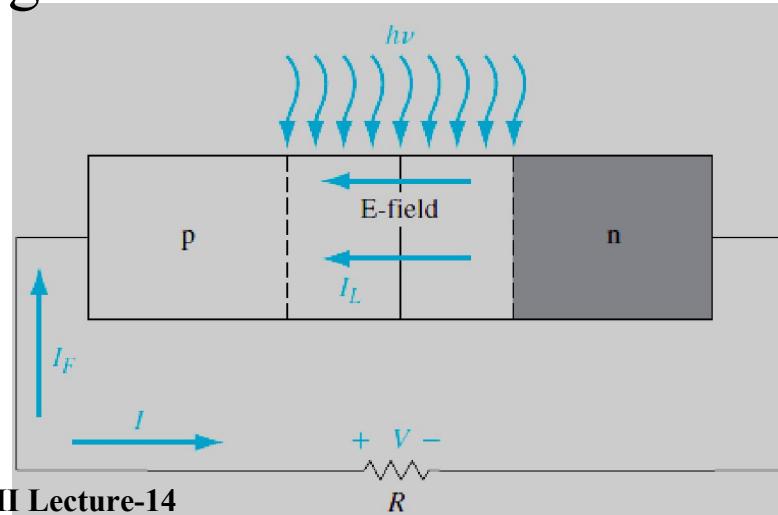
Solar Cell

A solar cell is a pn junction device with no voltage directly applied across the junction. The solar cell converts photon power into electrical power and delivers this power to a load.

The pn Junction Solar Cell

Consider the pn junction shown in Fig. 1, with a resistive load. Even with zero bias applied to the junction, an electric field exists in the space charge region as shown in the Fig. 1.

Figure 1 | A pn junction solar cell with resistive load.



Incident photon illumination can create electron-hole pairs in the space charge region that will be swept out producing the photocurrent I_L in the reverse-biased direction as shown in Fig. 1.

The photocurrent I_L produces a voltage drop across the resistive load which forward biases the pn junction. The forward-bias voltage produces a forward-bias current I_F as indicated in the Fig. 1.

The net pn junction current, in the reverse-biased direction, is

$$I = I_L - I_F = I_L - I_S \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] \quad (1)$$

here the ideal diode equation has been used.

As the diode becomes forward biased, the magnitude of the electric field in the space charge region decreases, but does not go to zero or change direction. The photocurrent is always in the reverse-biased direction and the net solar cell current is also always in the reverse-biased direction.

There are ***two limiting cases of interest.***

First limiting case is the ***short-circuit condition*** occurs when $R = 0$ so that $V = 0$. The current in this case is referred to as the ***short-circuit current***,

Or

$$I = I_{sc} = I_L \quad (2)$$



The *second limiting case* is the *open-circuit condition* and occurs when $R \rightarrow \infty$.

The net current is zero and the voltage produced is the *open-circuit voltage* V_{oc} . The photocurrent is just balanced by the forward-biased junction current, so we have

$$I = 0 = I_L - I_S \left[\exp\left(\frac{eV_\infty}{kT}\right) - 1 \right] \quad (3)$$

Then can find the open circuit voltage V_{oc} as

$$V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_S}\right) \quad (4)$$

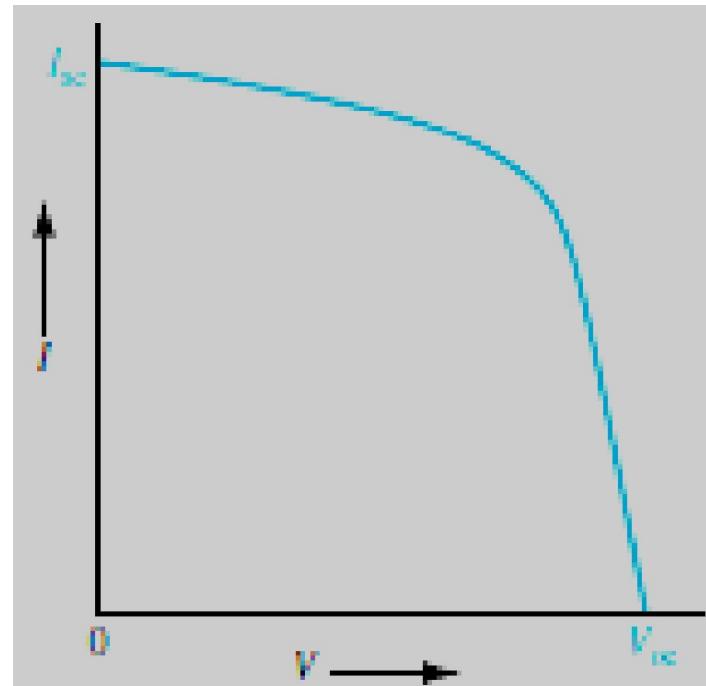
where $V_t = nkT/q$



Solar Cell

A plot of the diode current I as a function of the diode voltage V from Eqn (4) is shown in Fig. 2. We may note the short-circuit current and open circuit voltage points on the figure.

Figure 2 | I - V characteristics of a pn junction solar cell.



The power delivered to the load is

$$P = V * I = I_L * V - I_S \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] * V \quad (5)$$



Solar Cell



We may find the current and voltage which will deliver the maximum power to the load by setting the derivative equal to zero, or $dP/dV = 0$.

Using Eqn. (5), we find

$$\frac{dP}{dV} = 0 = I_L - I_S \left[\exp\left(\frac{eV_m}{kT}\right) - 1 \right] - I_s V_m \left(\frac{eV}{kT} \right) \exp\left(\frac{eV_m}{kT}\right) \quad (6)$$

where V_m is the voltage that produces the maximum power.

We may rewrite Eqn. (6) in the form

$$\left(1 + \frac{V_m}{V_t} \right) \exp\left(\frac{eV_m}{kT}\right) = 1 + \frac{I_L}{I_S} \quad (7)$$



Solar Cell



The value of V_m may be determined by trial and error. Figure 3 shows the maximum power rectangle where I_m is the current when $V = V_m$.

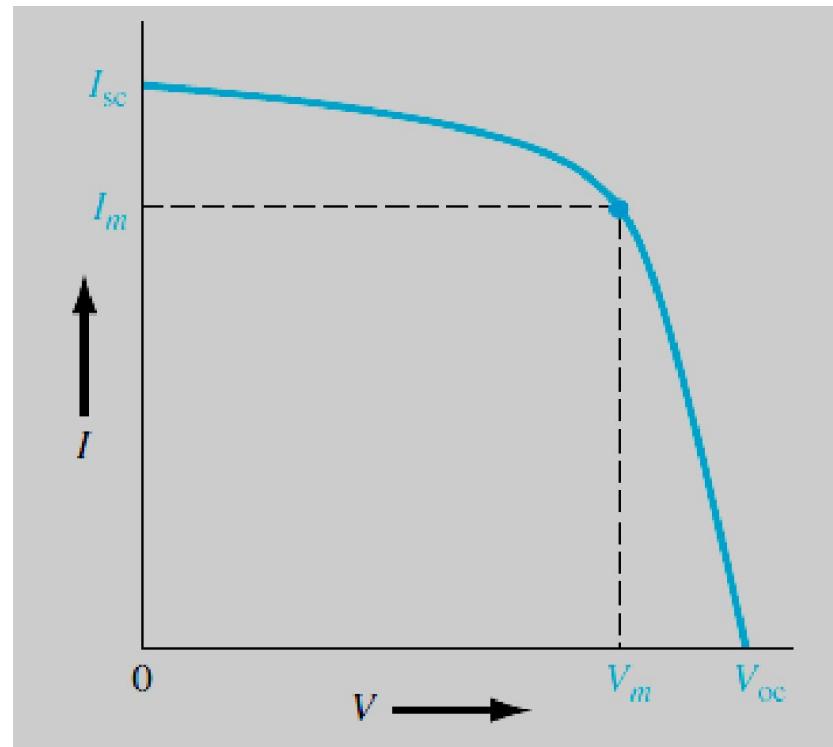


Figure 3 | Maximum power rectangle of the solar cell I - V characteristics.



Efficiency of a Solar Cell:

To determine the efficiency of the solar cell following parameters are to be described

□ Short-Circuit Current I_{sc}

The short-circuit current is the current through the solar cell when the voltage across the solar cell is zero (i.e., when the solar cell is short circuited).

$$I_{sc} = -I_L \quad (8)$$

Here Short-Circuit Current is nothing but light generated current.



□ Open-Circuit Voltage V_{OC}

The open-circuit voltage, V_{OC} , is the maximum voltage available from a solar cell, and this occurs at zero current. The open-circuit voltage corresponds to the amount of forward bias on the solar cell due to the bias of the solar cell junction with the light-generated current.

$$V_{OC} = \frac{n k T}{q} \ln\left(1 + \frac{I_L}{I_S}\right) \quad (9)$$

□ Fill Factor FF

Fill Factor FF, is a measure of the realizable power from a solar cell. Typically, the fill factor is between 0.7 and 0.8. The FF is defined as the ratio of the maximum power from the solar cell to the product of V_{oc} and I_{sc} so that

$$FF = \frac{P_m}{I_{sc} V_{oc}} = \frac{I_m V_m}{I_{sc} V_{oc}} \quad (10)$$



□ Conversion Efficiency η

The *conversion efficiency* η of a solar cell is defined as the ratio of output electrical power to incident optical power. For the maximum power output, we can write

$$\eta = \frac{P_m}{P_{in}} \times 100\% = \frac{I_m V_m}{P_{in}} \times 100\% = \frac{I_{sc} V_{oc} FF}{P_{in}} \times 100\% \quad (11)$$

The maximum possible current density and the maximum possible voltage in the solar cell are I_{sc} and V_{oc} , respectively.

Solar Cell



The conventional pn junction solar cell has a single semiconductor bandgap energy. When the cell is exposed to the solar spectrum, a photon with energy less than E_g will have no effect on the electrical output power of the solar cell.

A photon with energy greater than E_g will contribute to the solar cell output power, but the fraction of photon energy that is greater than E_g will eventually only be dissipated as heat.

The maximum efficiency of a silicon pn junction solar cell is approximately 28 percent. Non ideal factors, such as series resistance and reflection from the semiconductor surface, will lower the conversion efficiency typically to the range of 10 to 15 percent.



Solar Cell

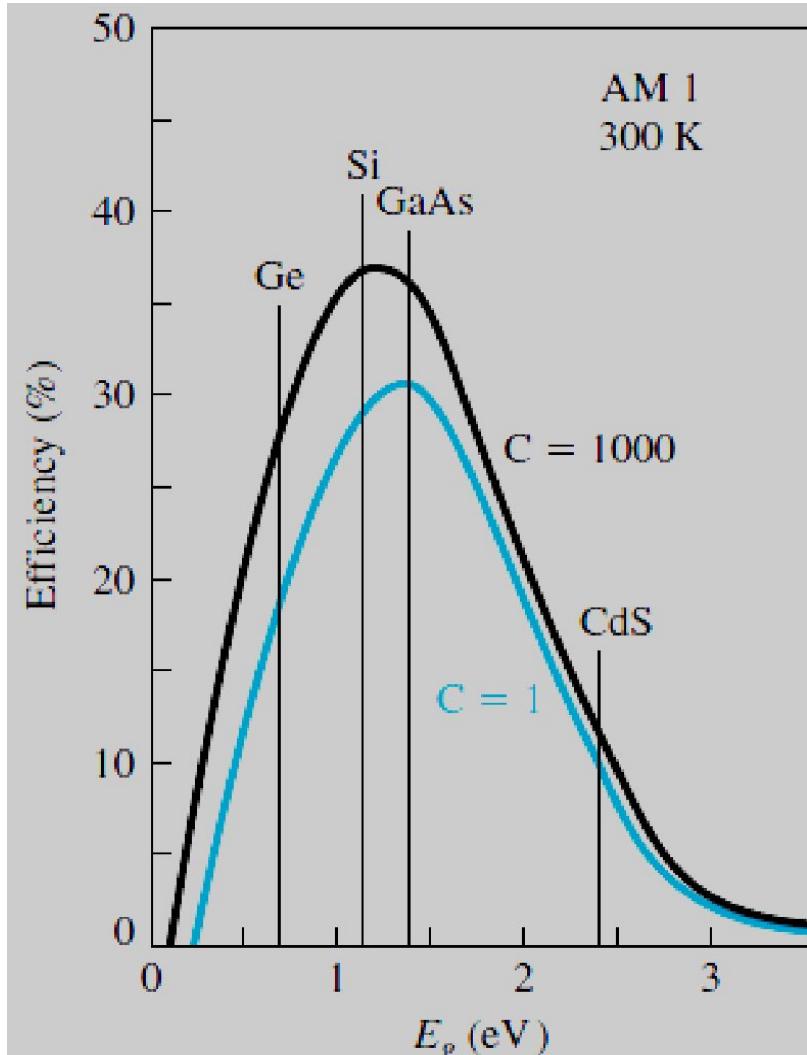


Figure 4 | Ideal solar cell efficiency at $T = 300$ K for $C = 1$ sun and for a $C = 1000$ sun concentrations as a function of bandgap energy.



A large optical lens can be used to concentrate sunlight onto a solar cell so that the light intensity can be increased up to several hundred times. The short-circuit current increases linearly with light concentration while the open-circuit voltage increases only slightly with concentration.

We can see that the conversion efficiency increases only slightly with optical concentration through Fig. 4. The primary advantage of using concentration techniques is to reduce the overall system cost since an optical lens is less expensive than an equivalent area of solar cells.



Loss in Solar Cell : Factors which affect Efficiency of the Solar Cell

1. Loss due to Low Energy Photons ($h\nu < E_g$)
2. Loss due to High Energy Photons ($h\nu > E_g$)
3. Voltage Loss
4. Fill Factor Loss
5. Loss due to Reflection
6. Loss due to less Absorption
7. Loss due to covering of Metal Contact
8. Recombination Loss



**DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
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**18PY103J – Physics: Semiconductor Physics
Module-III, Lecture-15**

The Drude Model of electrical conduction

Due to the quantum mechanical nature of electrons, a problem of electron movement in a solid (i.e. conduction) would require consideration of not only all the positive ion cores interacting with each electron, *but also each electron with every other electron.*

Even with advanced models, this rapidly becomes far too complicated to model adequately for a material of macroscopic scale.

The *Drude model simplifies* things considerably by using classical mechanics approach to *describe the conductivity in metals*. This model makes several key assumptions (some of which are better approximations than others)



Drude Theory

In Drude model, when atoms of a metallic element are brought together to form a metal, the valence electrons from each atom become detached and wander freely through the metal, while the metallic ions remain intact and play the role of the immobile positive particles.

Electrons in a metal behave much like particles in an ideal gas (no Coulombic interaction and no collisions between particles).



In a single isolated atom of the metallic element has a nucleus of charge $e Z_a$ as shown in Fig. 1 below.

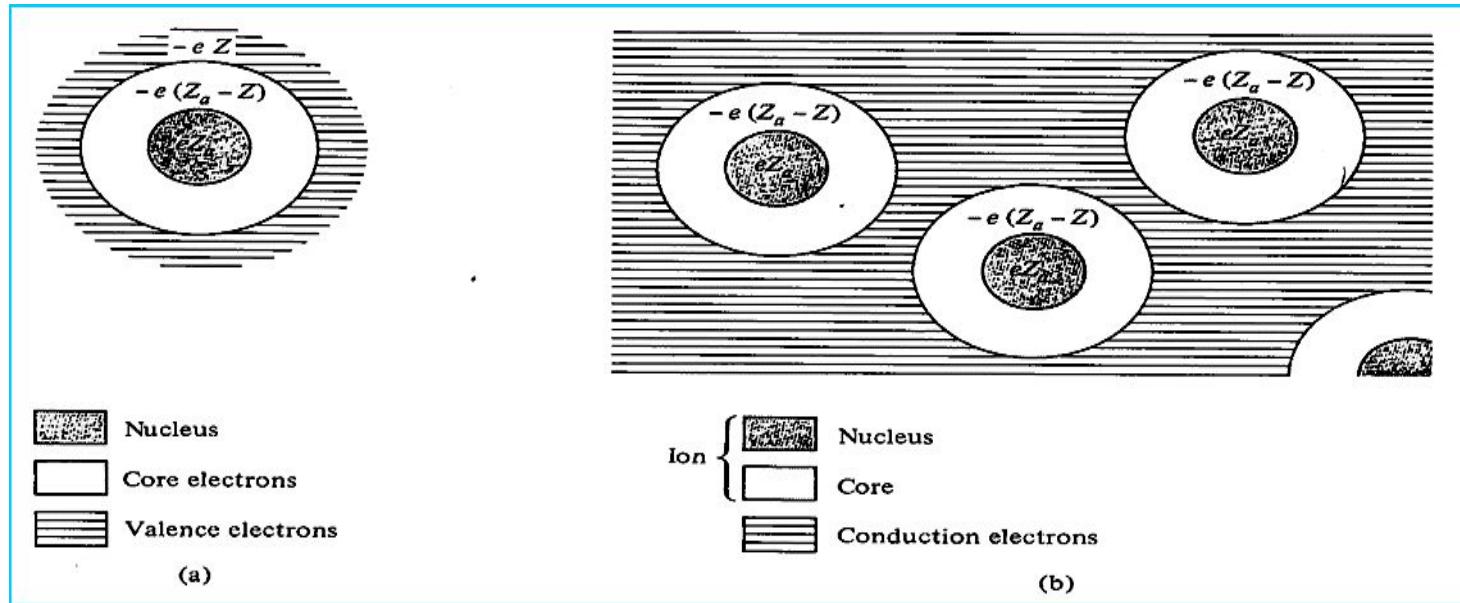


Figure 1 | Arrangement of atoms in a metal

where Z_a - is the atomic number and e - is the magnitude of the electronic charge [$e = 1.6 \times 10^{-19}$ coulomb] surrounding the nucleus, there are Z_a electrons of the total charge $-e Z_a$.



Some of these electrons ' Z ', are the relatively weakly bound valence electrons. The remaining ($Z_a - Z$) electrons are relatively tightly bound to the nucleus and are known as the *core electrons*.

These isolated atoms condense to form the metallic ion, and the valence electrons are allowed to wander far away from their parent atoms. They are called '*conduction electron gas*' or '*conduction electron cloud*'.

Due to kinetic theory of gas Drude assumed, conduction electrons of mass ' m ' move against a background of heavy immobile ions.



The density of the electron gas is calculated as follows. A metallic element contains 6.023×10^{23} atoms per mole (Avogadro's number) and ρ_m/A moles per m³

Here ρ_m is the mass density (in kg per cubic metre) and 'A' is the atomic mass of the element.

Each atom contributes 'Z' electrons, the number of electrons per cubic meter.

The conduction electron densities are of the order of 10^{28} conduction electrons for cubic meter, varying from 0.91×10^{28} for cesium upto 24.7×10^{28} for beryllium.



These densities are typically a thousand times greater than those of a classical gas at normal temperature and pressures.

Due to strong electron-electron and electron-ion electromagnetic interactions, the Drude model boldly treats the dense metallic electron gas by the methods of the kinetic theory of a neutral dilute gas.

In the absence of an externally applied electromagnetic fields, each electron is taken to move freely here and there and it collides with other free electrons or positive ion cores. This collision is known as elastic collision.

The neglect of electron–electron interaction between collisions is known as the “*independent electron approximation*”.



In the presence of externally applied electromagnetic fields, the electrons acquire some amount of energy from the field and are directed to move towards higher potential. As a result, the electrons acquire a constant velocity known as *Drift velocity* V_d .

In Drude model, due to kinetic theory of collision, that abruptly alter the velocity of an electron. Drude attributed the electrons bouncing off the impenetrable ion cores.

Let us assume an electron experiences a collision with a probability per unit time $1/\tau$. That means the probability of an electron undergoing collision in any infinitesimal time interval of length ds is just ds/τ .



The time ‘ τ ’ is known as the relaxation time and it is defined as the time taken by an electron between two successive collisions. That relaxation time is also called *mean free time* [or] *collision time*.

Electrons are assumed to achieve thermal equilibrium with their surroundings only through collision. These collisions are assumed to maintain local thermodynamic equilibrium in a particularly simple way.

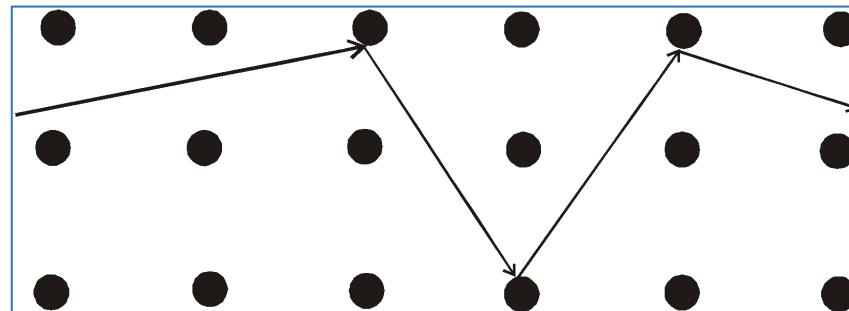


Figure 2 | Trajectory of a conduction electron

Determination of Electrical Conductivity of Semiconductor

To determine electrical conductivity by applying Drude theory, we consider a rectangular bar of intrinsic semiconductor connected to a battery as shown in Fig. 1.

If the direction of electric field is along x -direction then the free electrons will accelerate along negative x -axis and holes along x -direction.

So, the velocity of electrons along negative x -direction increases and attains some constant resultant velocity. This constant velocity is called drift velocity, represented as V_d .



Determination of Electrical Conductivity

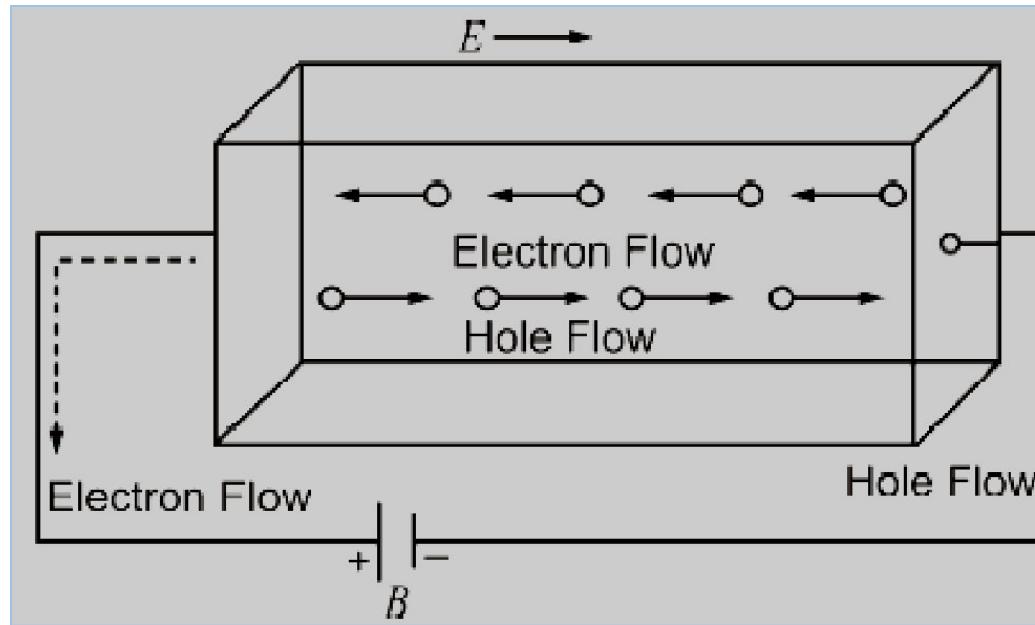


Figure 3 | A rectangular bar of semiconductor under the influence of Electric field E .

The total electrical current through the semiconductor is the sum of electron current I_e and hole current I_h .

To find the expression for electrical conductivity, consider a rectangular bar of length L and cross-section area A .

Let ‘ n ’ be the number of electrons per unit volume of the semiconductor *i.e.* its electron density, and E be the applied electric field.

Due to this applied electric field, let V_d be the average or drift velocity of the electrons. Assume a plane perpendicular to cross sectional area A .

Then the number of electrons crossing the imaginary plane in 1sec is

$$= nAV_d \quad (1)$$

Then the current flowing across the plane is

$$I_e = neAV_d \quad (2)$$

Then the electron current density is,

$$J_e = \frac{I_e}{A} = neV_d \quad (3)$$

From Ohm's law, the current density J_e due to electrons is given as

$$\begin{aligned} V &= IR = \frac{IL\rho}{A} \\ \frac{I}{A} &= J_e = \frac{V}{L\rho} = E\sigma_n \end{aligned} \quad (4)$$

Here electrical conductivity of electrons.

From eqn. 3 and eqn. 4, we have

$$J_e = neV_d = E\sigma_n \quad (5)$$

The drift velocity produced per unit applied electric field is called the mobility of electrons represented as

Or $\mu_e = \frac{V_d}{E}$

$$V_d = \mu_e E \quad (6)$$

Substituting eqn. 6 in eqn. 5 gives

$$\sigma_n = ne\mu_e \quad (7)$$

Eqn. 7 represents electrical conductivity due to electrons.

Similarly, the electrical conductivity of holes can be obtained.

Let p be the number of holes per unit volume of the material, μ_h is the mobility of holes and the charge of a hole is ‘ e ’, then

$$\sigma_p = pe\mu_h \quad (8)$$

The total conductivity of a intrinsic semiconductor is given by the sum of eqn. 7 and eqn. 8.

$$\sigma = \sigma_n + \sigma_p = ne\mu_e + pe\mu_h$$

$$\sigma = ne[\mu_e + \mu_h] \quad (9)$$



**DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY**

**18PY103J – Physics: Semiconductor Physics
Module-III, Lecture-16**

*Numerical based on Solar Cell
& Electrical Conductivity*

Solved Numerical:

1. Determine the Open-Circuit Voltage V_{oc} of the solar cell, if Saturation Current (I_s) = 1×10^{-10} A, Light Generated Current(I_L) = 0.5 A, Ideality Factor (n) = 1, and Temperature (T) = 300 K

Solution:

Open-Circuit Voltage $V_{OC} = \frac{nkT}{q} \ln\left(1 + \frac{I_L}{I_S}\right)$

$$= \frac{1 \times 1.38 \times 10^{-23} \times 300}{1.61 \times 10^{-19}} \ln\left(1 + \frac{0.5}{1 \times 10^{-10}}\right)$$
$$= 0.57V$$

2. Determine the Fill Factor FF of the solar cell, if Short-Circuit Current (I_{sc}) = 2.75 A, Open-Circuit Voltage (V_{oc}) = 0.6V, Current at Maximum Power (I_m) = 2 A and Voltage at Maximum Power (V_m) = 0.5V

Solution:

Fill Factor

$$FF = \frac{I_m V_m}{I_{sc} V_{oc}} = \frac{2 \times 0.5}{2.75 \times 0.6}$$
$$= 0.606$$



3. Determine the Conversion Efficiency η of the solar cell, if Short-Circuit Current (I_{sc}) = 3.5A, Open-Circuit Voltage (V_{oc}) = 0.6V, Fill Factor (FF) = 0.7 and Input Power (P_{in}) = 10W

Solution:

Efficiency

$$\eta = \frac{I_{sc} V_{oc} FF}{P_{in}} \times 100\%$$
$$= \frac{3.5 \times 0.6 \times 0.7}{10} \times 100\%$$
$$= 14.7\%$$



4. Determine the Conductivity (σ) of the Intrinsic Semiconductor. The given parameters are: $\mu_e = 0.145 \text{ m}^2/\text{V-s}$; $\mu_h = 0.055 \text{ m}^2 / \text{V-s}$; $n_i = 1.5625 \times 10^{16} / \text{m}^3$; $q = 1.602 \times 10^{-19} \text{ C}$.

Solution:

Conductivity
$$\begin{aligned}\sigma &= ne[\mu_e + \mu_h] \\ &= 1.5625 \times 10^{16} \times 1.602 \times 10^{-19} [0.145 + 0.055] \\ &= 5.0 \times 10^{-4} \text{ mhos/m}\end{aligned}$$



5. The resistivity of intrinsic germanium at 300 K is 0.47 Ωm. If the electron and hole mobilities are $0.38 \text{ m}^2/\text{V}\cdot\text{s}$ and $0.18 \text{ m}^2/\text{V}\cdot\text{s}$, then calculate the Intrinsic carrier density (n) at 300 K.

Solution:

Conductivity

$$\sigma = ne[\mu_e + \mu_h]$$

$$n = \frac{\sigma}{e[\mu_e + \mu_h]}$$

$$= \frac{1/0.47}{1.602 \times 10^{-19} [0.38 + 0.18]}$$

$$= 2.3 \times 10^{19} \text{ m}^{-3}$$



Unsolved Problems:

1. Determine the Open-Circuit Voltage V_{oc} of the solar cell, if Saturation Current (I_s) = 0.75×10^{-10} A, Light Generated Current(I_L) = 0.65 A, Ideality Factor (n) = 0.9, and Temperature (T) = 310K. [Ans: $V_{oc} = 0.55V$]
2. Determine the Fill Factor FF of the solar cell, if Short-Circuit Current (I_{sc}) = 2.25 A, Open-Circuit Voltage (V_{oc}) = 0.75V, Current at Maximum Power (I_m) = 1.5 A and Voltage at Maximum Power (V_m) = 0.85V. [Ans: FF = 0.7556]
3. Determine the Conversion Efficiency η of the solar cell, if Short-Circuit Current (I_{sc}) = 2.8A, Open-Circuit Voltage (V_{oc}) = 0.55V, Fill Factor (FF) = 0.8 and Input Power (P_{in}) = 10W. [Ans: $\eta = 12\%$]
4. The following data are given for an intrinsic semiconductor at 27°C; $n_i = 2.4 \times 10^{19} m^{-3}$, $\mu_e = 0.39 m^2/V-s$ and $\mu_h = 0.19 m^2/V-s$. Calculate the conductivity of the intrinsic semiconductor. [Ans: $\sigma = 2.22 \Omega^{-1}m^{-1}$]