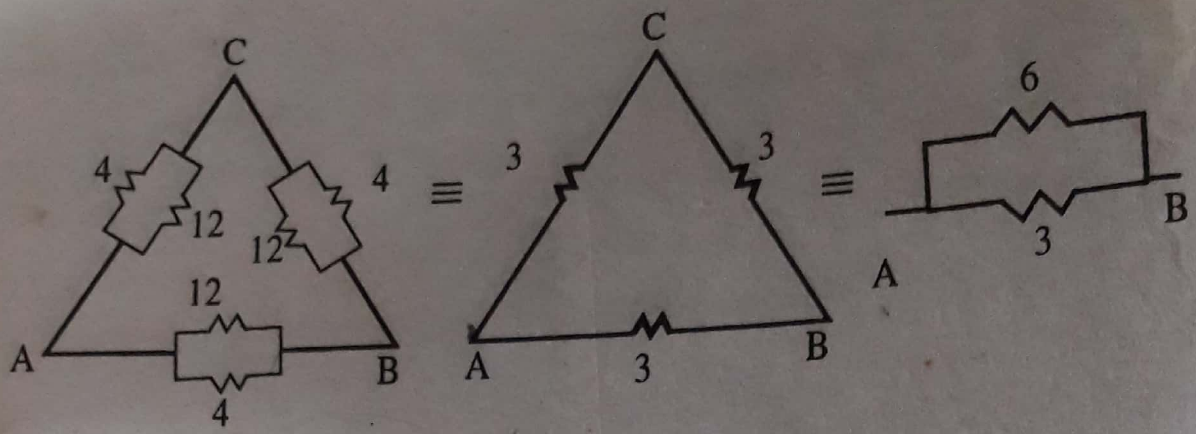
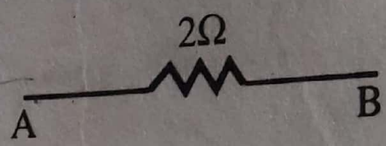


Fig 4.45

$$R_{AB} = 3 \parallel 6 = \frac{3 \times 6}{3 + 6}$$

$$= 2\,\Omega$$



4.3 Network Theorems

4.3.1 Super Position Theorem

With the help of this theorem, we can find the current through or the voltage across a given element in a linear circuit consisting of two or more sources the statement is as follows :

"In a linear circuit containing more than one source, the current that flows at any point or the voltage that exists between any two points is the algebraic sum of the currents or the voltages that would have been produced by each source taken separately with all other sources removed."

Note :

1. Removal of ideal voltage source means short circuiting
2. Removal of ideal voltage source means replacing by open circuit.
3. Removal of practical voltage source means replacing by internal resistance. Example : Consider the following simple circuit

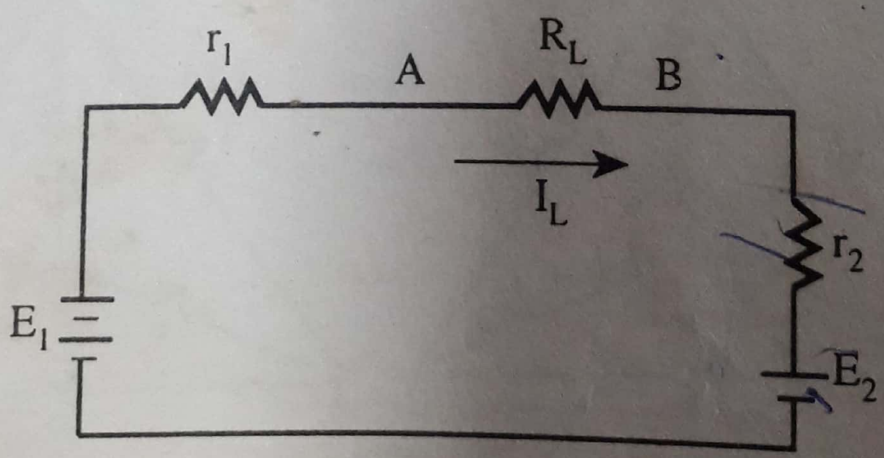


Fig 4.46

Let E_1 be greater than E_2, I_L is the load current ie., current through the load resistance. The net voltage circuit equals to $E_1 - E_2$.

Total resistance = $r_1 + r_2 + R_L$

$$\text{Therefore, } I_L = \frac{E_1 - E_2}{r_1 + r_2 + R_L}$$

$$= \left(\frac{E_1}{r_1 + r_2 + R_L}\right) + \left(\frac{-E_2}{r_1 + r_2 + R_L}\right)$$

$$I_L = I'_L + (-I''_L)$$

I'_L is the current flowing through R_L because of E_1 alone, flowing from A to B.

I''_L is the current flowing through R_L because of E_2 alone, flowing from B to A.

I_L is the current flowing through R_L when both sources are acting.

Note :

Superposition theorem can be applied for finding the current or voltage across a particular element in a linear circuit containing more than two sources. But, this theorem can not be used for the calculation of the power.

WORKED EXAMPLES

SUPERPOSITION THEOREM

Example 1 : Compute the current in 23 ohm resistor of the figure below using superposition theorem

[MU. Nov 94]

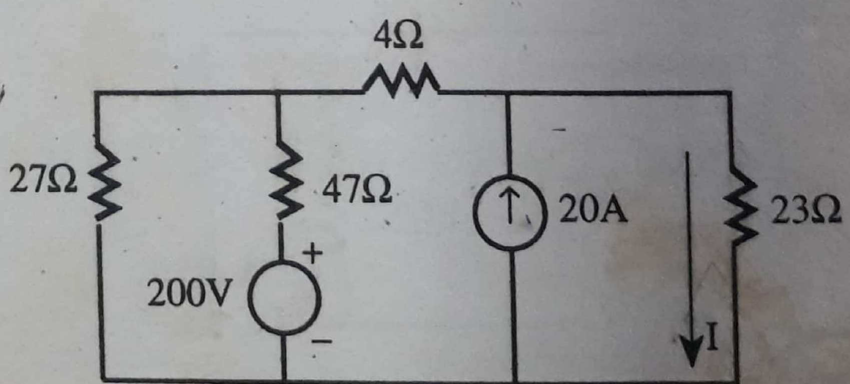


Fig 4.47

***Solution** : Step 1 :* Allow only the voltage source to act. The correspondent circuit is as below :

47

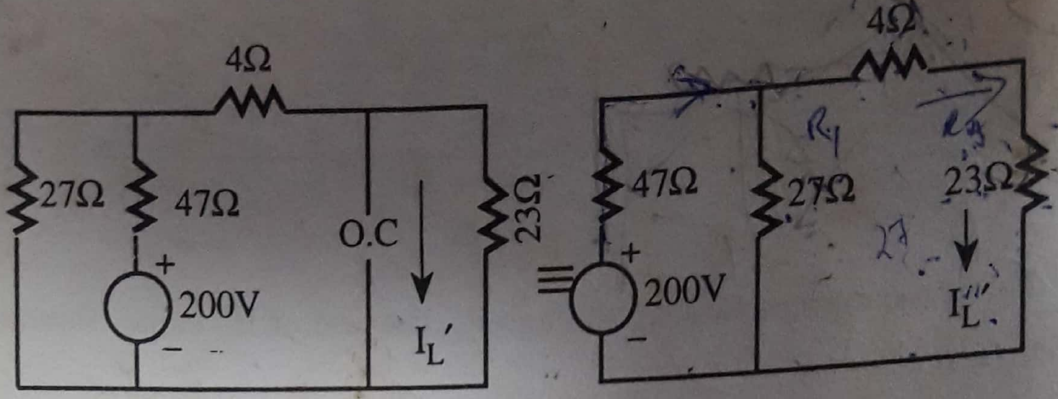


Fig 4.48

Total resistance connected to the source = $47+\dfrac{27 \times 27}{27+27} = 60.5$ ohms.

Total current from the source = $\dfrac{200}{60.5} = 3.306$ A

The current through 23 ohms $= I'_L = \dfrac{3.306 \times 27}{27+27} \doteq 1.653$ A

Step 2: Allow only the current source to act. The circuit becomes as

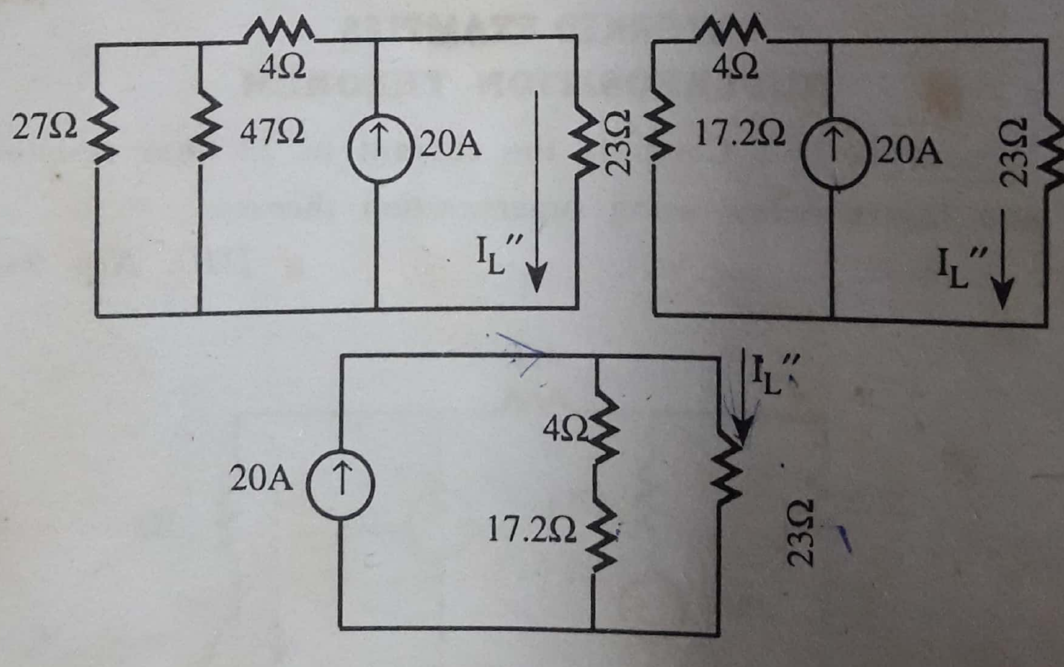


Fig 4.49

$$I''_L = \frac{20 \times 21.2}{21.2+23} = 9.6 \text{ A}$$

By superposition theorem, $I_L = I'_L + I''_L = 1.653 + 9.6 = 11.253$ A

Example 2 : **Using superposition theorem, find the current in resistance R_3 in the circuit shown below.** *[MU]*

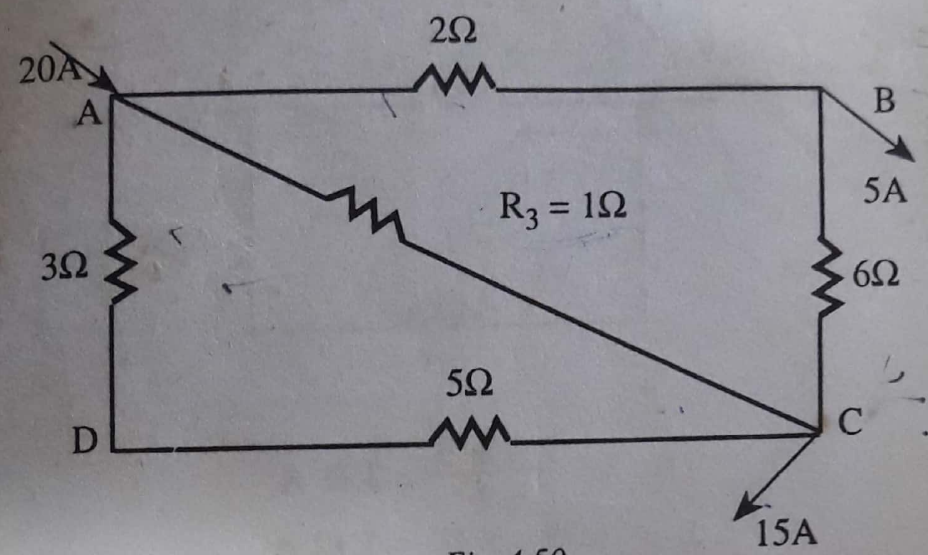
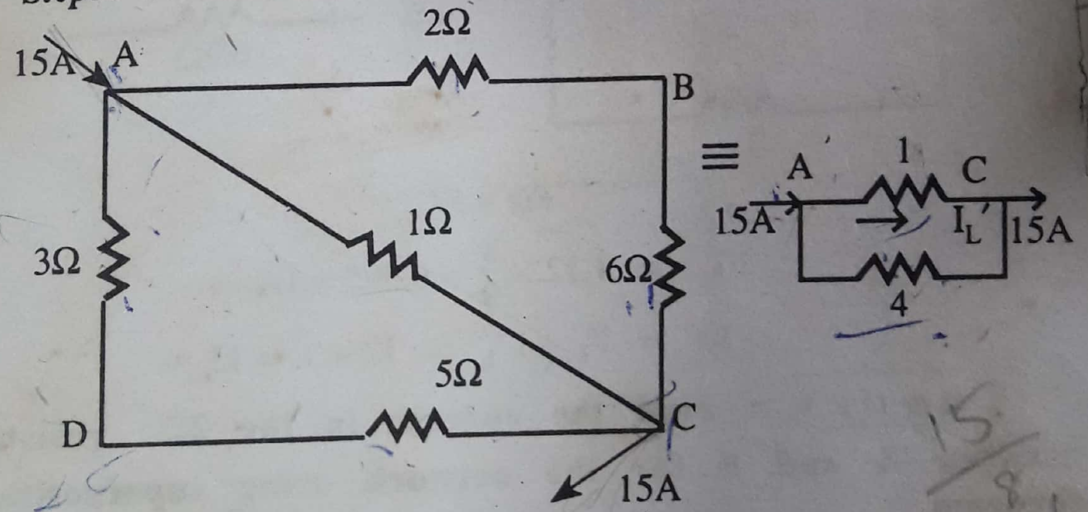


Fig 4.50

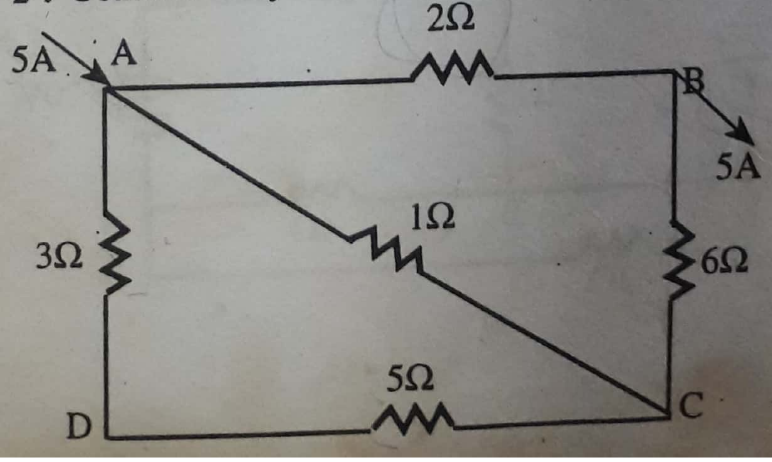
Solution : The above circuit diagram may be assumed to be consisting of two currents. One current of 15 A entering at A and leaving at C. Another current of 5A, entering at A and leaving at B.

Step1: Consider only 15A. The relevant circuit diagram is as below:



$$I'L = \frac{15 \times 4}{4 + 1} = 12 \text{ A}$$

Step 2 : Consider only 5A. The corresponding circuit is :



1Ω and 8Ω are in parallel their equivalent is $\frac{8}{9} = 0.9\Omega$. The combination being equal to $6.9\ \Omega$

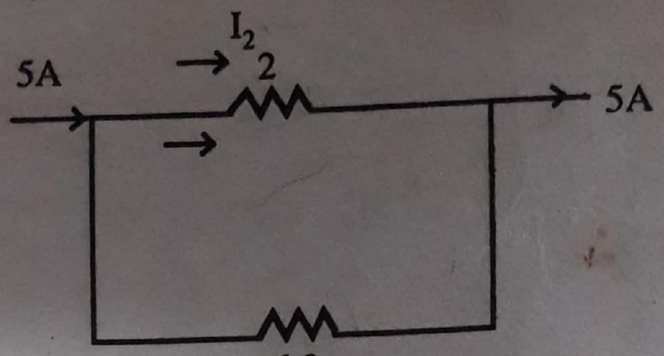


Fig 4.53

$$I_2 = \frac{5 \times 6.9}{8.9} = 3.88\ A$$

$$I_{6.9} = 5 - 3.88 = 1.12\ A$$

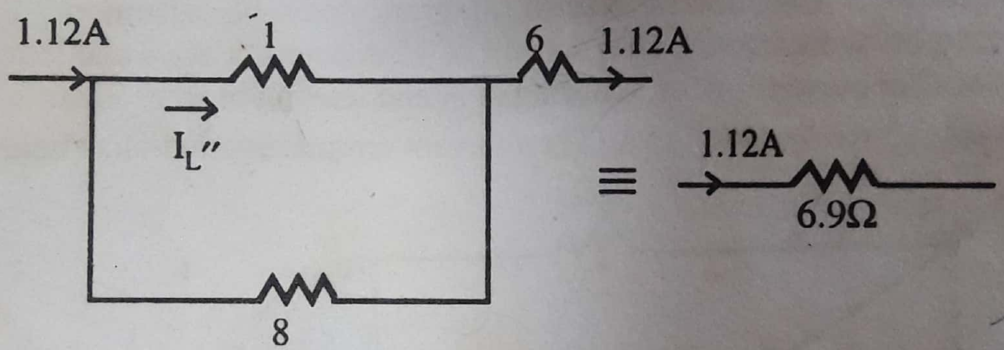


Fig 4.54

$$\therefore\ I''_L = 1.12 \times \frac{8}{9} = 0.995 = 1A$$

$$\therefore\ I_L = I'_L + I''_L = 12 + 1 = 13\ A$$

Example 3 : **Find the current in the 2Ω resistor between A and B for the network using superposition theorem.** *[MU. April 94]*

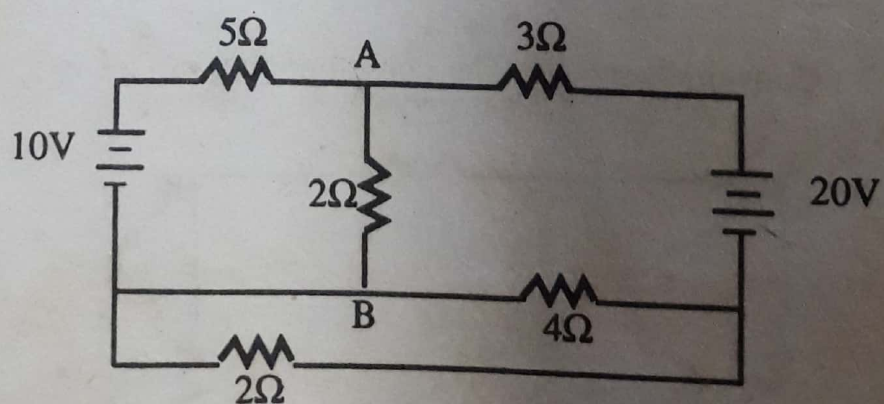


Fig 4.55

Solution : Step 1 : When only 10 V source is acting. Let the current through 2Ω Be I'_L. The circuit diagram with 10 V source acting, and 20V source killed is as below :

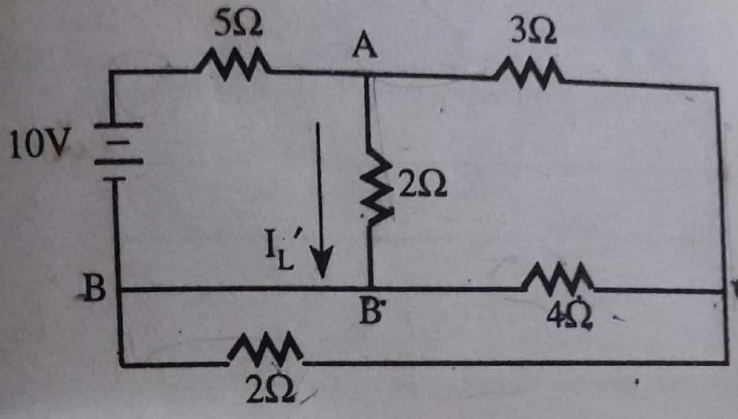


Fig 4.56

Between B and C, 2Ω and 4Ω are in parallel. Their equivalent $= \frac{2 \times 4}{2 + 4} = 1.33\ \Omega$. Now, the circuit diagram can be re–drawn as below :

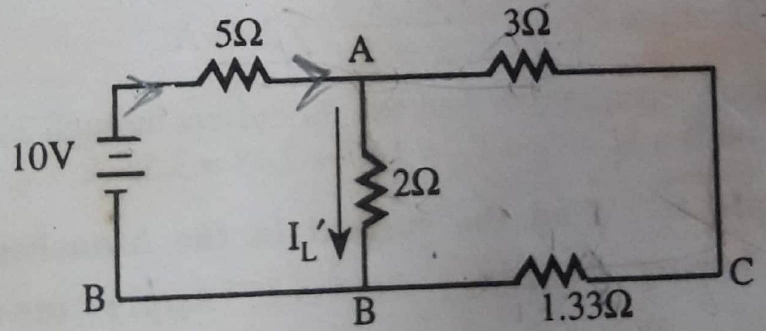


Fig 4.57

For the above circuit, the total resistance connected to the 10V source

$$= 5 + \frac{2 \times 4.33}{2 + 4.33} = 6.37\ \Omega$$

$$\text{Current from the source} \Rightarrow \frac{10}{6.37} = 1.57\ \text{A}$$

$$\text{Therefore } I'_L = \frac{1.57 \times 4.33}{2 + 4.33}$$

$$= 1.07\text{A}$$

Step 2 : Allow only 20V source to act, killing the other source. The circuit becomes as :

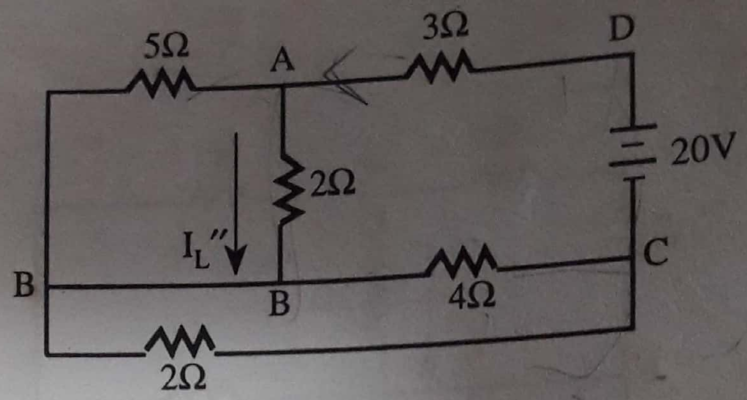


Fig 4.58

By inspecting the above figure, we can say that across the voltage source 20V there is series parallel combination of resistances.

Between A and B 2Ω and 5Ω are in parallel. Between B and C 4Ω and 2Ω are in parallel so, the equivalent resistance between C and D

$$= 3 + \frac{5 \times 2}{5 + 2} + \frac{4 \times 2}{4 + 2} = 3 + 1.43 + 1.33 = 5.76$$

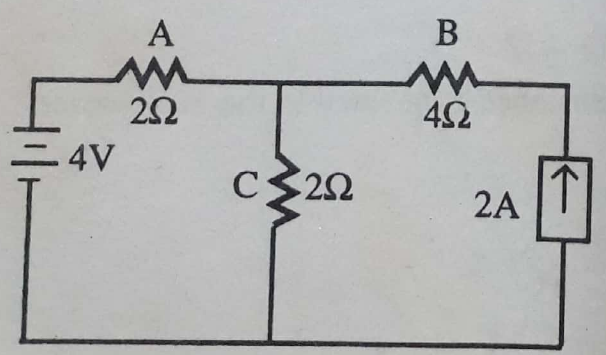
The current from the source $= \dfrac{20}{5.76} = 3.47$ A

This current is divided into two parallel paths between A and B.

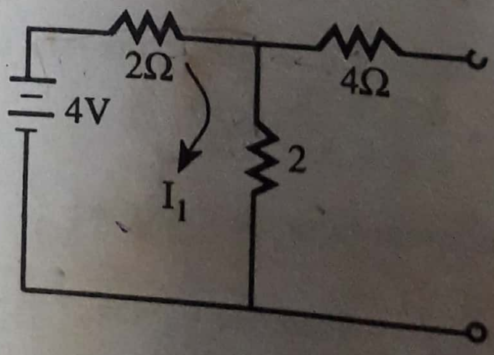
Therefore $I''_L = \dfrac{3.47 \times 5}{5 + 2} = 2.48$ A

By applying superposition theorem, the current through 2Ω connected between A and B $= I_L = I'_L + I'_L = 1.07 + 2.48 = 3.55$ A

Example 4 : Find the current in the branches A, B, C of the following 2 source network. Apply superposition principle



(*a*) Network



(*b*) With V source only

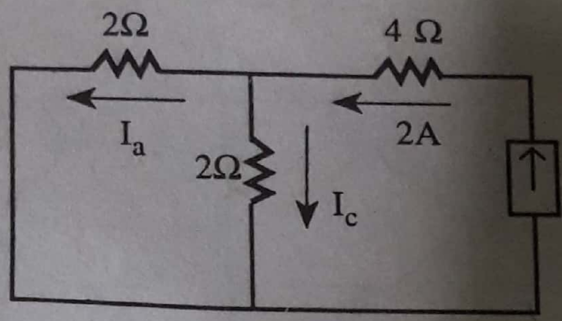


Fig 4.59 (*c*) with I source only

Solution : Step 1 : When only V source is acting, I source becomes open circuit. Refer fig *(b)*. The current from V source flows only through 2 Ω and 2Ω. No current flows through 4Ω as it is open circuit

$$I_1 = \frac{4}{2+2} = 1A.$$

Step 2 : Allow only the I source to act, then V source becomes short circuited. For circuit refer fig *(c)*. The source current of 2A flows through 4Ω and is divided into two paths. The current shown in the figure are as $I_a = \frac{2 \times 2}{2+2} = 1A$ and $I_c = 2 - 1 = 1A$

By applying superposition principle,

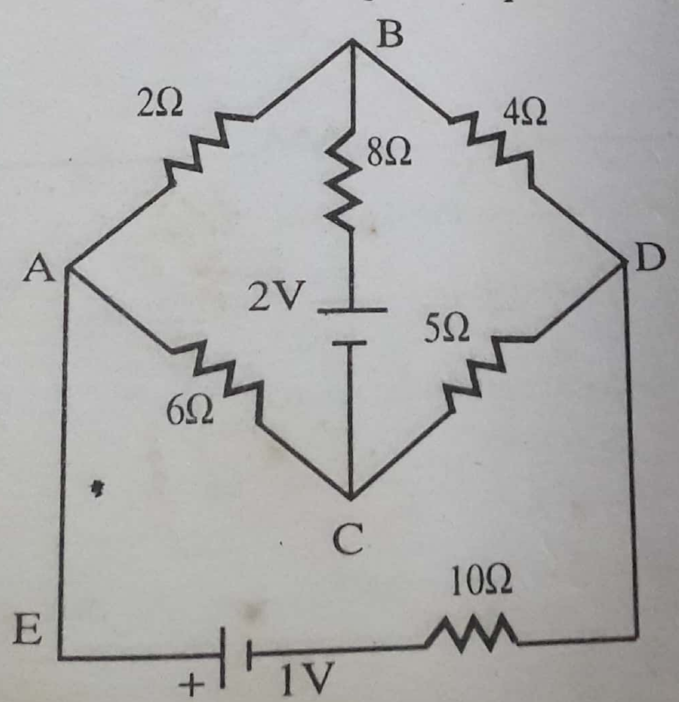
Current through branch A = $I_1 - I_a = 1 - 1 = 0$

Current through branch C = $I_1 + I_c = 1 + 1 = 2A$

Current through branch B = 2A.

Example 5 :Using superposition theorem or otherwise, obtain the current in EA in figure below :

[MU. April 96 BE, ICE Branch]



Step 1 : First allow only 1V source to act. Let the current in EA be I'. The relevant circuit diagram is as below :

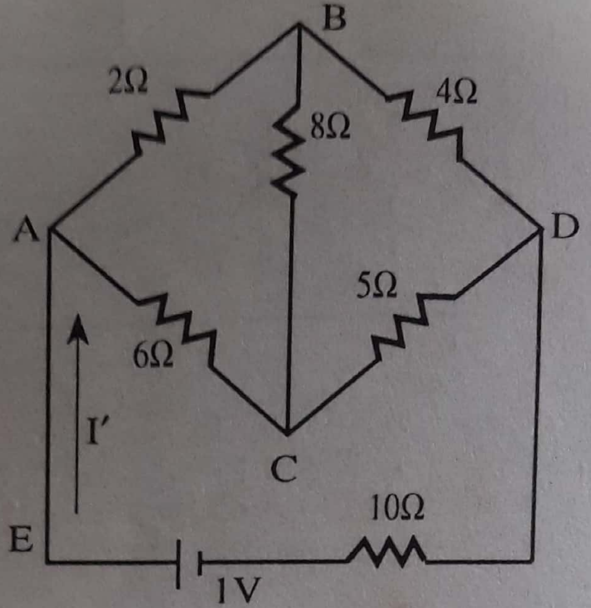


Fig 4.61

I' can be found by applying mesh current method or converting Delta to star and then applying ohm's law. The above circuit becomes as drawn below after converting Delta connection between A, B and C into its equivalent star.

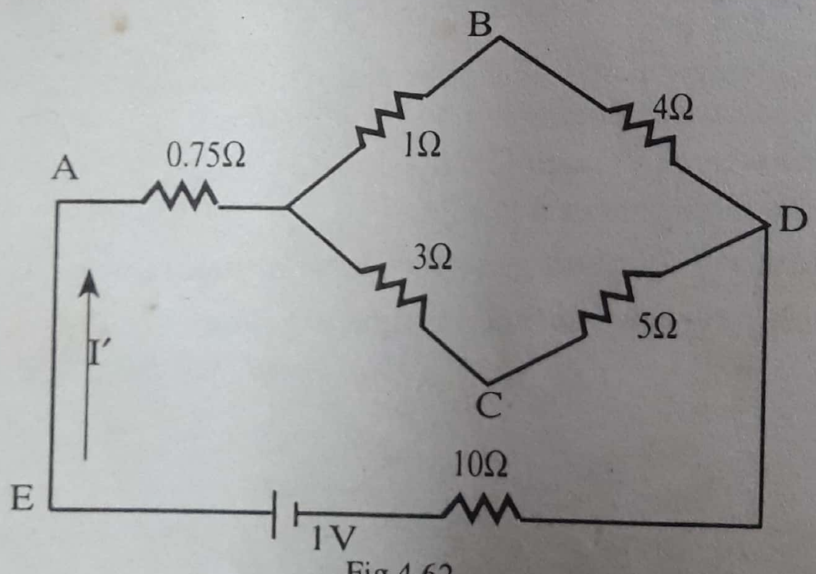


Fig 4.62

$R_A = \frac{2 \times 6}{2 + 8 + 6} = 0.75\Omega \quad R_B = \frac{8 \times 2}{16} = 1\Omega \quad R_C = \frac{6 \times 8}{16} = 3\Omega$

Total resistance connected across the source

$$= 0.75 + \frac{5 \times 8}{5 + 8} + 10 = 13.83\ \Omega$$

$$\therefore I' = \frac{\text{Voltage}}{\text{resistance}} = \frac{1}{13.83} = 0.0723\ A = 72.3\ mA$$

It flows form E to A.

Step 2 : When only 2V source is acting and the other source short circuited, the circuit becomes as shown below : Now Let the current in EA be I''

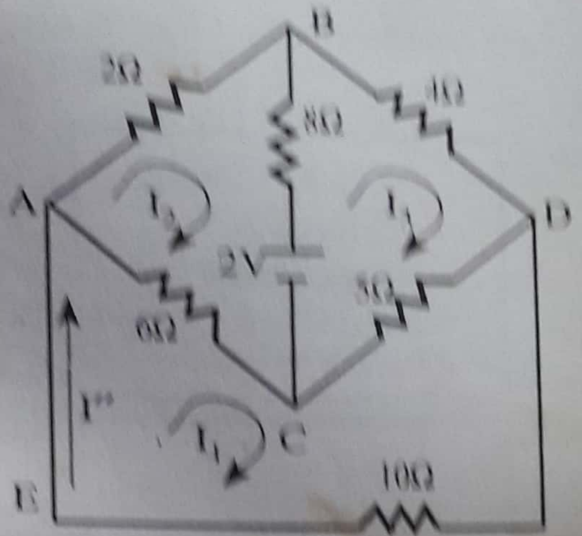


Fig 4.63

For the calculation of I'', loop current method is applied. let the loop currents be I_1, I_2 and I_3 as shown in the figure above. $I'' = I_1$

$$\text{By inspection, } \begin{bmatrix} 21 & -6 & -5 \\ -6 & 16 & -8 \\ -5 & -8 & +17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 21 & -6 & -5 \\ -6 & 16 & -8 \\ -5 & -8 & +17 \end{vmatrix} = 21(+272-64)+6(-102-40)-5(48+80)$$

$$= 4368 - 852 - 640 = 2876$$

$$\Delta_1 = \begin{vmatrix} 0 & -6 & -5 \\ -2 & 16 & -8 \\ 2 & -8 & 17 \end{vmatrix} = 6(-34+16)-5(16-32) = -108+80$$

$$= -28$$

$$\therefore I_1 = I'' = \frac{\Delta_1}{\Delta}$$

$$= \frac{-28}{2876} = -9.736 \, mA$$

Step 3 : By applying superposition theorem, the current in EA = I = I' + I'' = 72.3 − 9.736 = 62.564 *mA*

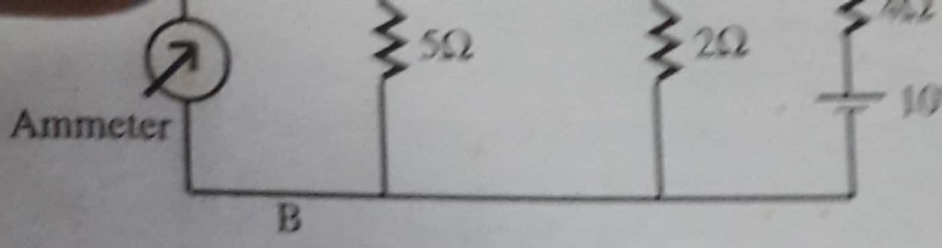


Fig 4.74

In this figure, there is a short circuit (through ammeter) between the parallel combination of 5Ω and 2Ω. Hence, no current will flow through 5Ω and 2Ω. The current from the voltage source flows through only 4Ω and the same thing is read by the ammeter. By applying ohm's law, reading of ammeter $= \frac{10}{4} = 2.5A$

Thus, reciprocity theorem is verified for the given circuit.

[*Note* : After connecting the voltage source in 4Ω resistor, the current through branches of resistances 2Ω and 5Ω becomes 0.]

4.3.3 Thevenin's Theorem

Consider the following active circuit

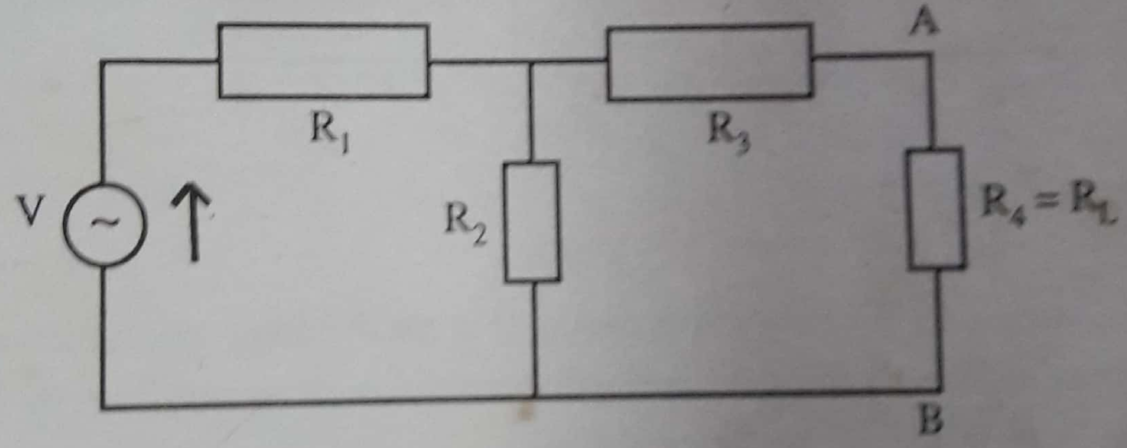


Fig 4.75

Suppose that the current through the load resistance R_L is required. It can be calculated by the following methods.

a) series parallel simplification

b) branch current method

c) Loop current method.

If the circuit consist of one more loop, in addition to the existing two loops, the first method will become more tedious. If R_L takes various values, whatever method you are applying it will be some what much laborious. For example, if you apply loop current method, we must form the matrix and solve for I_L. This procedure is to be repeated as many times as number of values of R_L. The Thevenin's theorem helps us to avoid the repeated procedure by this theorem we can replace a given active circuit between two terminals by a constant voltage source.

Then, by applying ohm's law, we can compute the value of I_L.

Statement of Thevenin's theorem :

Any linear active network with output terminals A, B as shown in fig (a) can be replaced by a single voltage source ($V_{Th} = V_{0c}$) in series with a single impedance ($Z_{Th} = Z_i$)

V_{Th} is the Thevenin's voltage. It is the voltage between the terminals A and B on open circuit condition, Hence it is called open circuit voltage denoted by V_{0c}

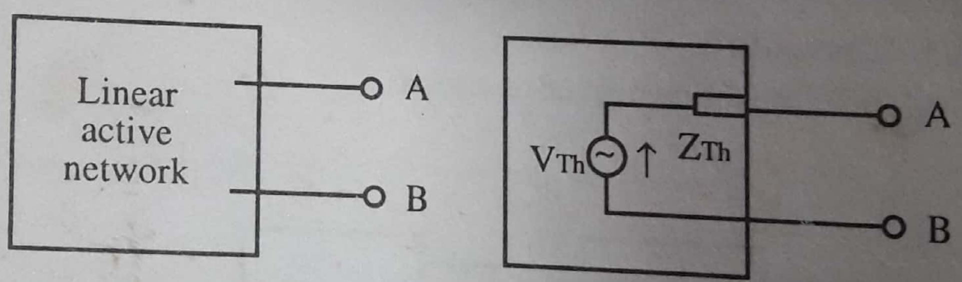


Fig 4.76

Z_{Th} is called Thevenin's impedance. It is the driving point impedance at the terminals A, B when all internal sources are set to zero too. In case of D.C, Z_{Th} is replaced by R_{Th}

If a load impedance Z_L is connected across AB, we can find the current through it by the following formula $I_L = \dfrac{V_{Th}}{Z_{Th} + Z_L}$.

Note : 1. This Theorem can be applied for both D.C and A.C circuits. 2. In the Thevenin's equivalent circuit (Voltage source), if AB terminals are short circuited, the current flowing through AB is obtained by ohm's law.

$$I_{SC} = \frac{V_{Th}}{Z_{Th}} \Rightarrow Z_{Th} = \frac{V_{Th}}{I_{SC}}$$

Steps to be followed in applying Thevenin's Theorem :

The following steps are necessary in applying Thevenin's theorem for the given network :

1. Let the Load resistance be R_L through which the current I is required. Mark the terminals A and B (for convenience) across R_L is connected *i.e.*, R_L is supposed to be connected between the terminals marked as A and B.
2. Blindly, draw the Thevenin's equivalent circuit between A and B terminals. It is a constant voltage source with voltage V_{Th} and resistance R_{Th}.
3. In the given circuit disconnect R_L and redraw the fig after removing R_L. Find the voltage between A and B. It is V_{Th}.
4. From the circuit in the above step, kill all the energy sources properly and obtain the equivalent resistance between A and B when looked back. it is R_{Th}.

R_{Th} can also be calculated in the following way :

a) In a given circuit, remove R_L and replace by short-circuit and find the current through this. It is I_{SC}. So, $R_{Th} = = \frac{V_{Th}}{I_{SC}}$.

Open and Short Circuit Method for finding Internal Z ($Z_i = Z_{Th}$) of a network.

Every network according to Thevenin's theorem can be replaced by an internal source V_{Th} in series with an internal resistance R_{Th}. For a practical network, the values of V_{Th} and R_{Th} may be found by two experiments :

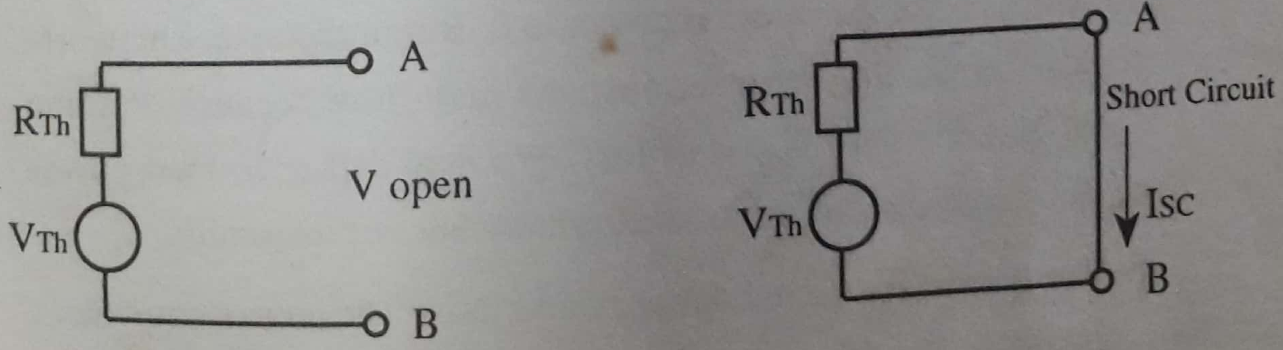


Fig (a) *Fig 4.77* *Fig (b)*

1. Keep the terminals open as shown in fig(a). The voltage between the open terminals A and B is V_{Th}. It can be measured with a high resistance voltmeter, connected to the terminals.

2. Short circuit the terminals and connect an ammeter in series with the short-circuit refer fig (*b*). The ammeter measures the shortcircuit current I_{SC}. As the resistance of the meter is negligible, the ammeter does not affect the conditions of a short-circuit.

From fig (*b*), $I_{SC} = \frac{V_{Th}}{R_{Th}}$. (by ohm's law)

Therefore $$R_{Th} = \frac{V_{Th}}{I_{SC}}$$

$$= \frac{\text{Open circuit Voltage}}{\text{short circuit current}}$$

These two tests are the ones which are generally employed to determine the network equivalent of electric machines such as transformer, alternater, Induction motor and so on.

Power Calculation using Thevenin's Theorem

We have seen that this theorem replaces an active network by a voltage V_{Th} and a series resistance R_{Th}, as far as any external load connected to the terminals of the network is concerned. For instance, if the load current is I_L, then the power dissipated in R_L will be $I^2_L R_L$. As I_L is actual current, the power calculated will also be correct. That means, the theorem can be used to find the power in external circuit.

In the Thevenin's equivalent there is no current flow when the terminals are open. So, the internal power is 0. But in the actual network, even when the terminals are open, currents may flow because of some closed loops. Hence, there is power loss. We may call it no load power loss. The power calculations are true externally but not internally.

The power loss in the actual network and in its Thevenin's equivalent are not equal. But, the difference between the two losses is constant. It does not vary with change in the external load. The constant difference is the no–load power loss.

WORKED EXAMPLES

THEVENIN'S THEOREM

Example 1 : **Determine the current I in the network by using Thevenin's theorem.** *[Bharathidasan Uni. Nov 95]*

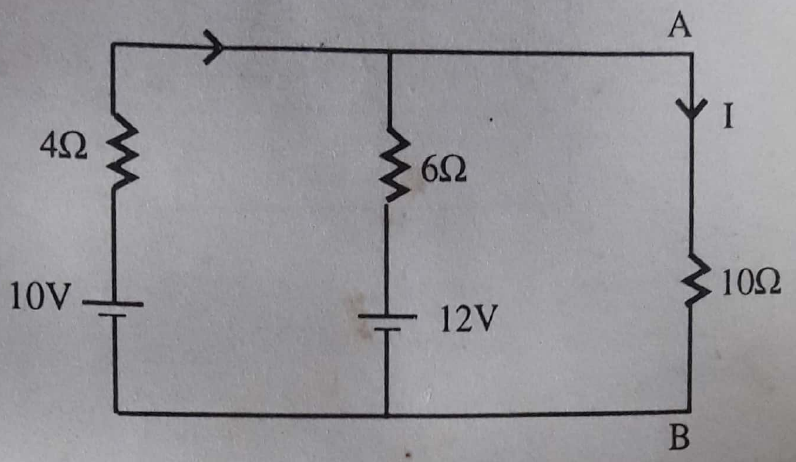


Fig 4.78

Solution : Step 1 : The Thevenin's equivalent circuit is

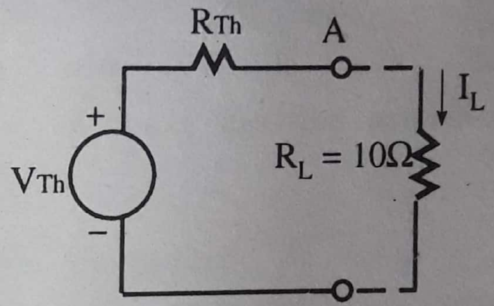


Fig 4.79 B

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Step 2 : To find V_{Th}: From the given circuit disconnect $R_L = 10\Omega$.

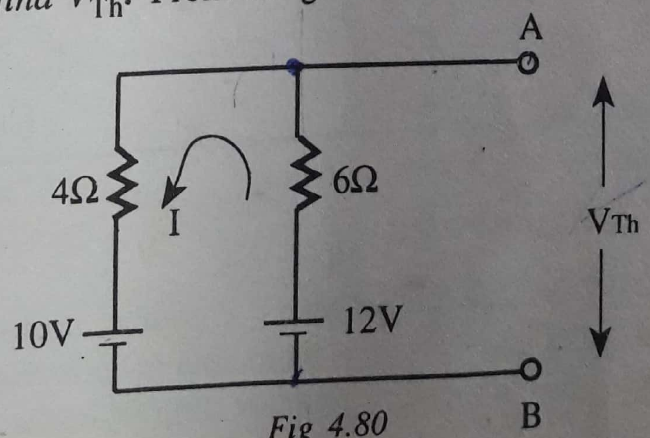


Fig 4.80

$$I = \frac{12 - 10}{4 + 6} = 0.2 \text{ A}$$

$\therefore V_{Th} = V_{AB} = 6I - 12 = 6 \times 0.2 - 12 = -10.8$ volts

[B is –ve polarily and A is +ve polarity]

Step 3 : To calculate R_{Th} : From the above circuit, kill the sources. The resultant CKt is

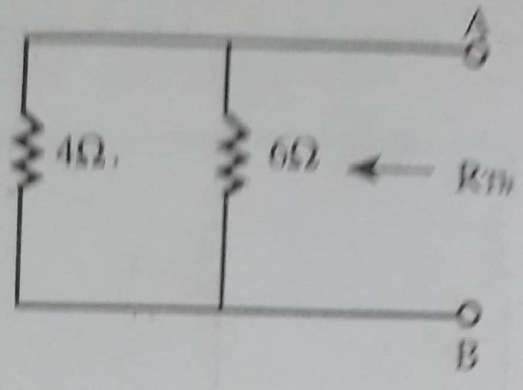


Fig 4.81

CKt to find R_{Th}

$$R_{Th} = R_{AB} = 4 \parallel 6 = \frac{4 \times 6}{4+6} = 2.4\ \Omega$$

Step 4 : $$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{10.8}{2.4 + 10} = 0.871\ A$$

Example 2 : **Find the Thevenin's equivalent for the network of the figure between *a* and *b*.** ***[MU. Oct. 1995]***

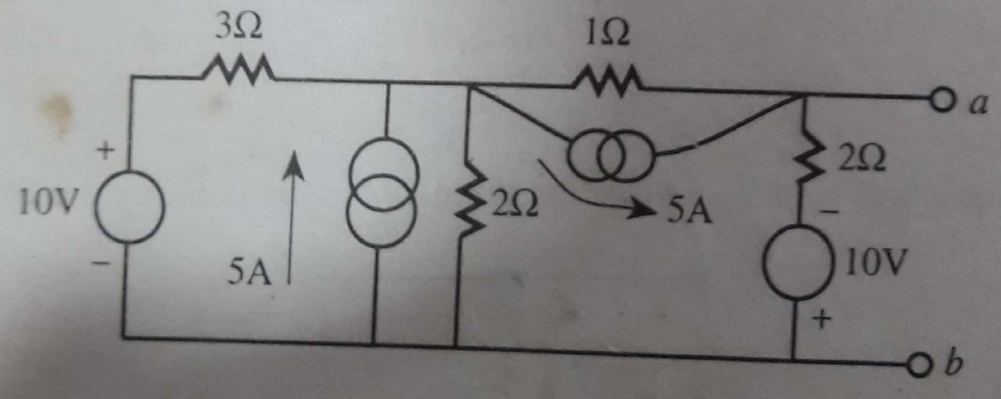


Fig 4.82

Solution : The network given is a combination of voltage and current sources. By converting voltage source into current source and vice–versa, wherever necessary and the simplifying we can obtain the required network.

Step 1 : Converting or transforming the voltage source of 10V in series with resistance 3Ω into equivalent current source, the following circuit is obtained.

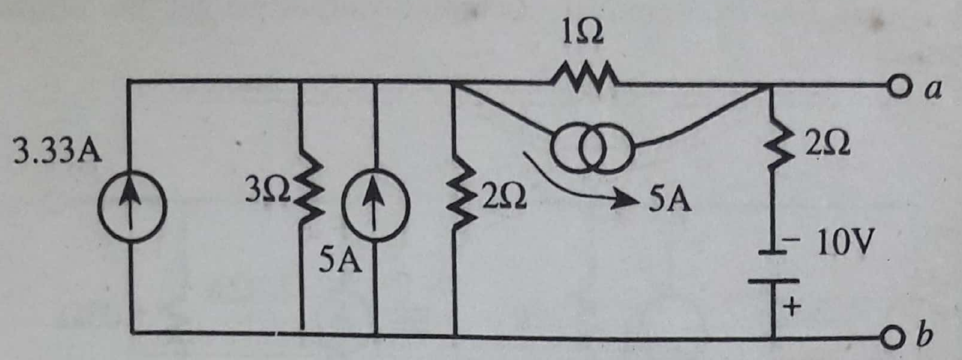


Fig 4.83

Step 2 : Replacing the 2 current sources in parallel by its equivalent current sources, we get the following circuit

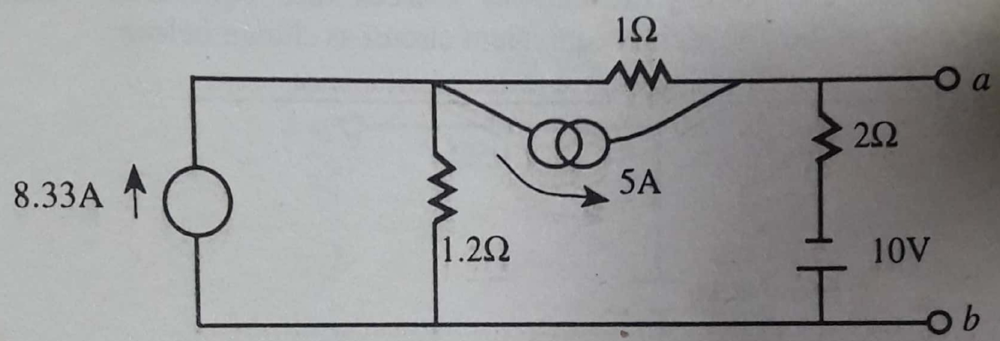


Fig 4.84

Step 3 : Transforming the 2 current sources in series by their equivalent voltage sources we get the following network:

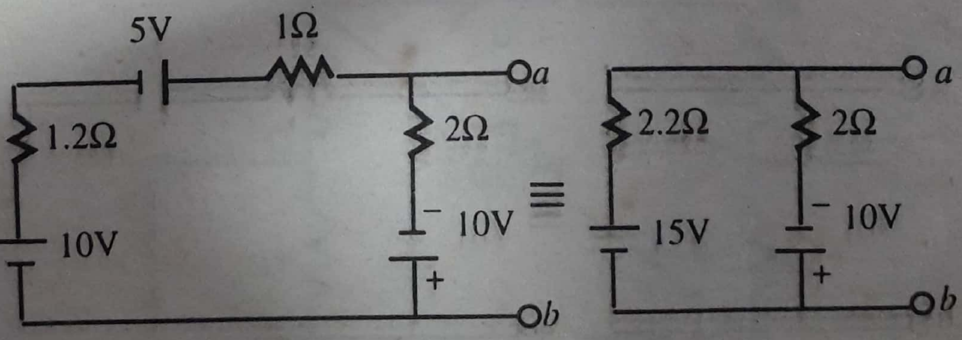


Fig 4.85

Step 4 : Transforming the voltage sources which are parallel in the above circuit, into their equivalent current sources we get the following network :

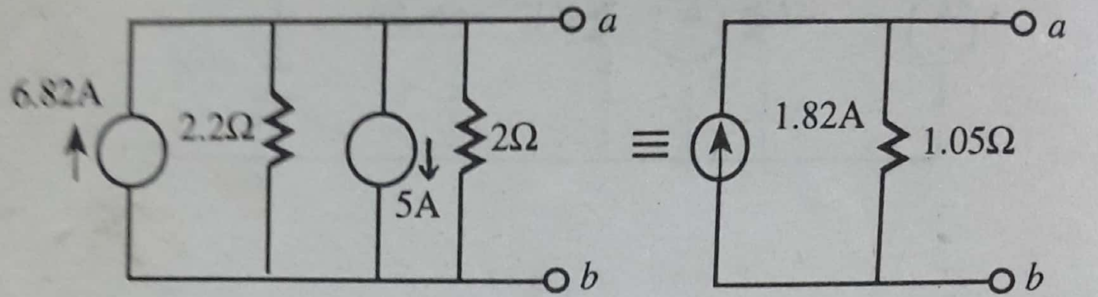


Fig 4.86

Step 5 : Converting the current sources into equivalent voltage source we get the Thevenin's equivalent circuit as shown below :

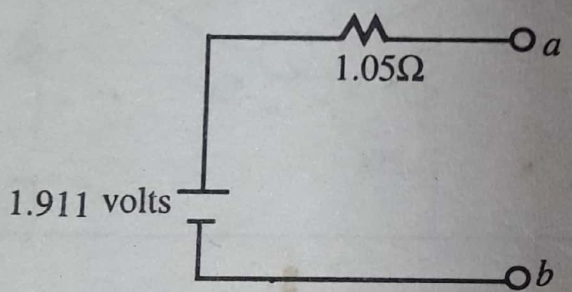


Fig 4.87

Note : From step 3 we can proceed to find V_{Th} and R_{Th} without going through step 4.

Example 3 : **Calculate, using Thevenin's Theorem the current through the branch FC** *[MU. Nov. 1984]*

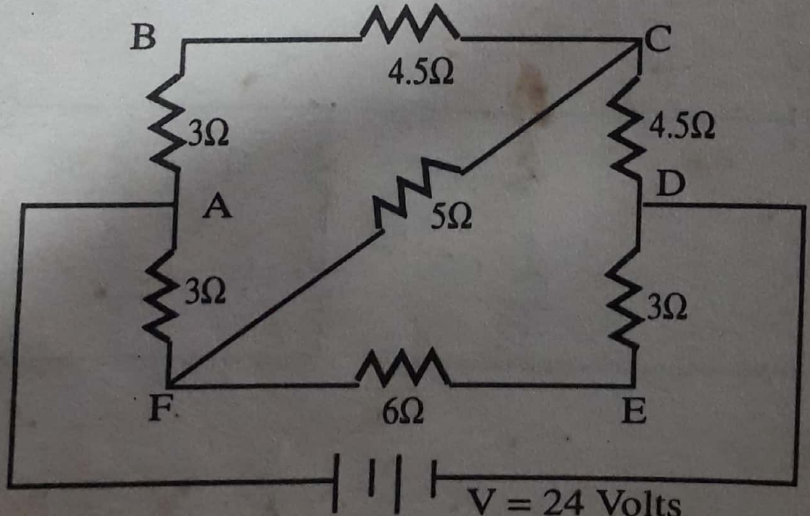


Fig 4.88

Solution : Step 1 : The Thevenin's equivalent circuit is

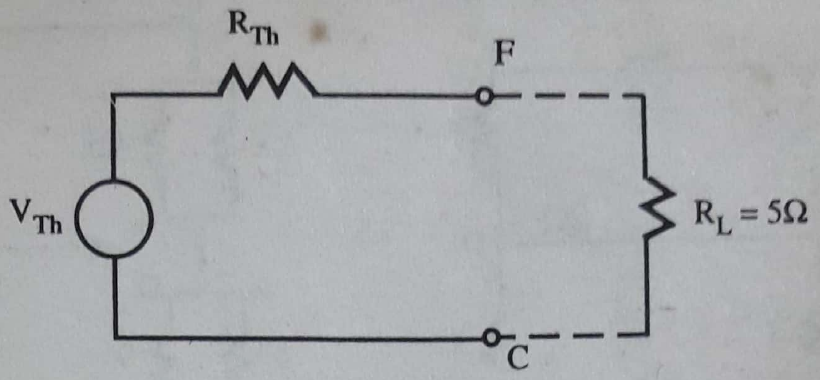


Fig 4.89

Step 2 : To calculate V_{Th}. Disconnect $R_L = 5\Omega$, between F and C terminals.

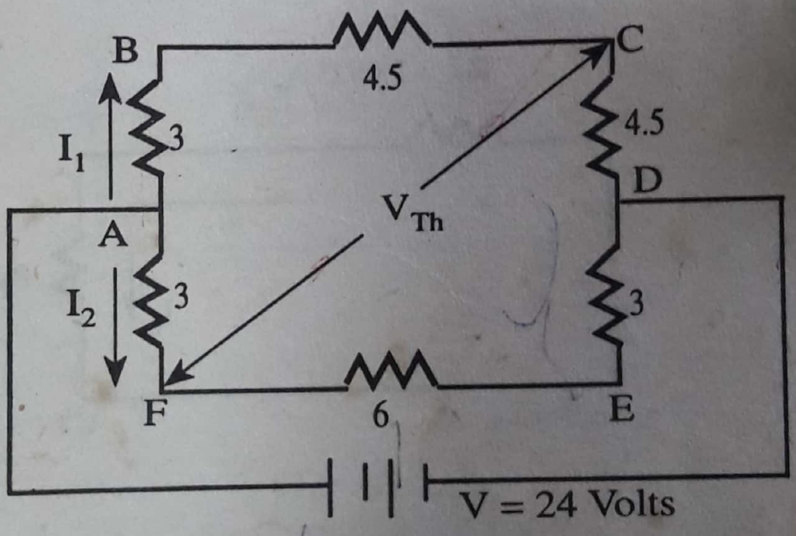


Fig 4.90

$$I_1 = \frac{24}{3 + 4.5 + 4.5} = 2A$$

$$I_2 = \frac{24}{3 + 6 + 3} = 2A$$

$$\begin{aligned} V_{Th} &= V_{FC} = -6 I_2 - 3I_2 + 4.5 I_1 \\ &= -9I_2 + 4.5 I_1 \\ &= -9 \times 2 + 4.5 \times 2 \\ &= -9 \text{ Volts} \end{aligned}$$

[C is –ve and F is +ve]

Step 3 : To calculate R_{Th} : Re–draw the above circuit, after killing the voltage source

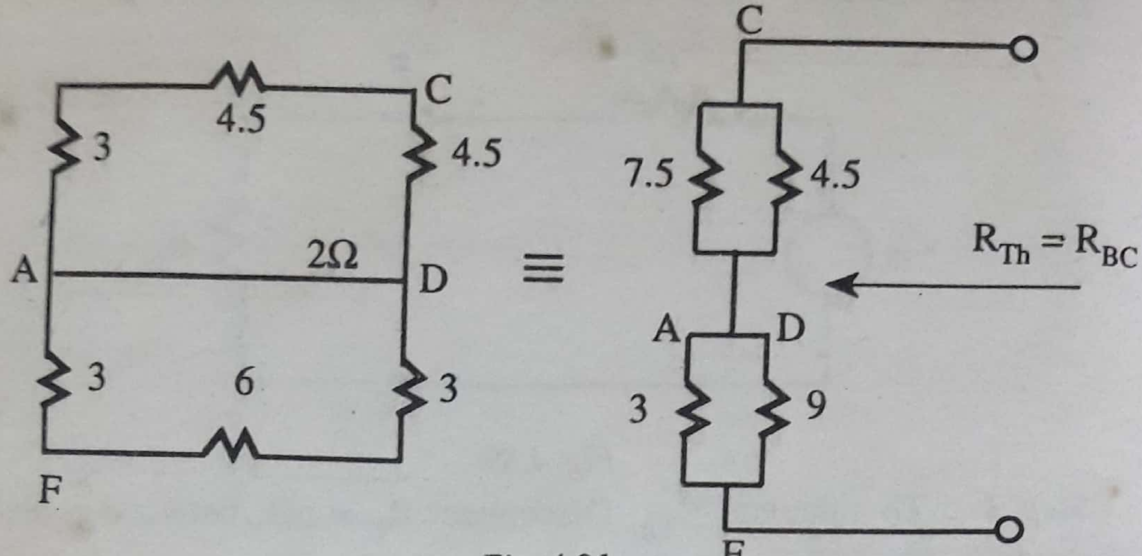


Fig 4.91

$$R_{Th} = R_{FC} = \frac{7.5 \times 4.5}{7.5 + 4.5} + \frac{3 \times 9}{3 + 9}$$

$$= 2.8125 + 2.25 = 5.0625\ \Omega$$

Step 4 :

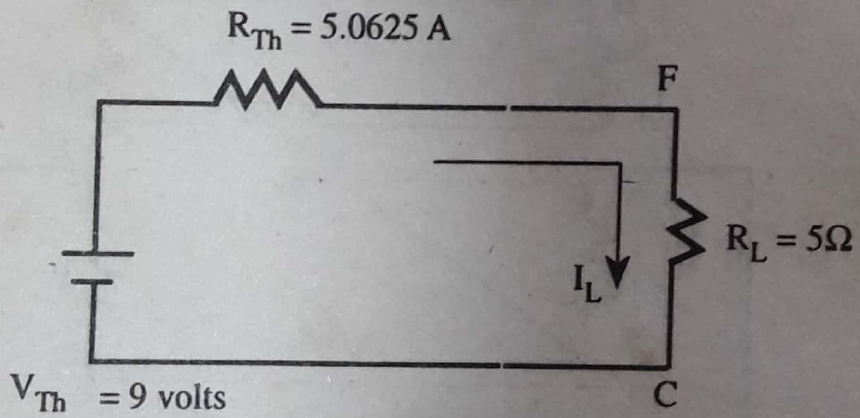


Fig. 4.92

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{9}{5.0625 + 5} = 0.894 \text{ A}$$

Example 4 : **It is required to find current through the 0.1 Ω resistor in the figure, using Thevenin's method.**
[MK University, April 1995]

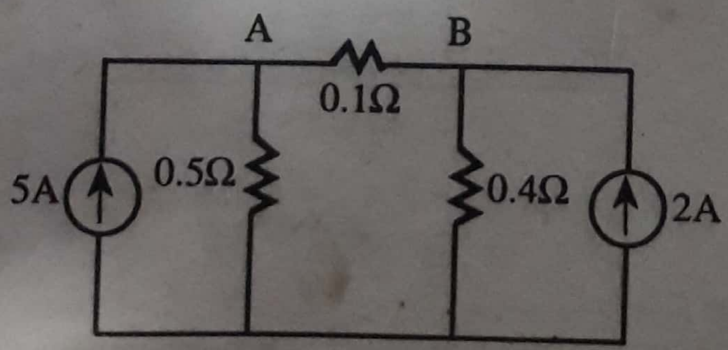
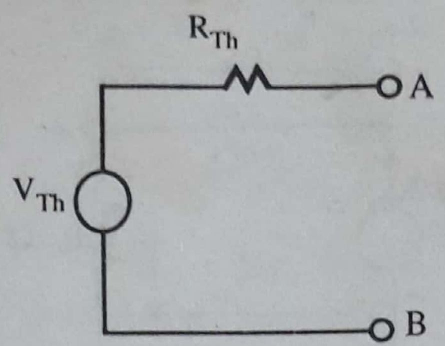


Fig. 4.93

Solution : Step 1 : The Thevenin equation circuit



Step 2 : To calculate V_{Th} ; From the given network disconnect (remove) $R_L = 0.1\Omega$

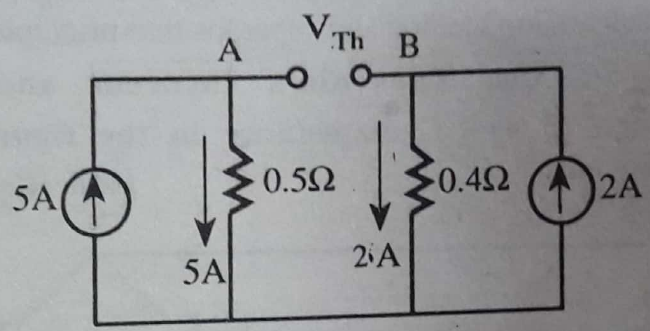


Fig. 4.95

CKt to Calculate V_{Th}

$$V_{Th} = V_{AB}$$

$$= -5 \times 0.5 + 2 \times 0.4 = -1.7\text{ V [B – ve and A + ve]}$$

Step 2 : To calculate R_{Th} : Kill the current in the above circuit by open circuit (O.C)

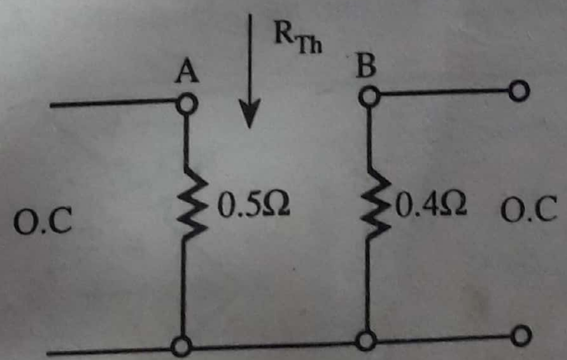


Fig. 4.96

CKt to calculate R_{Th}

$$R_{Th} = R_{AB} = 0.5 + 0.4 = 0.9\Omega$$

Step 3 : $I_L = \dfrac{V_{Th}}{R_{Th} + R_L} = \dfrac{1.7}{0.9 + 0.1} = 1.7A$

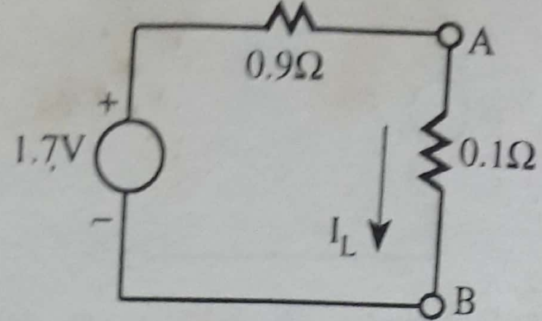


Fig 4.97

Fig. 4.97

[*Note :* The above problem can also be solved by other methods such as source conversion method and superposition principle]

Example 5 : Use Thevenin's theorem and find the current through (5 +*j*4) Ω impedance in the figure.

[MK Uni. Apr 95]

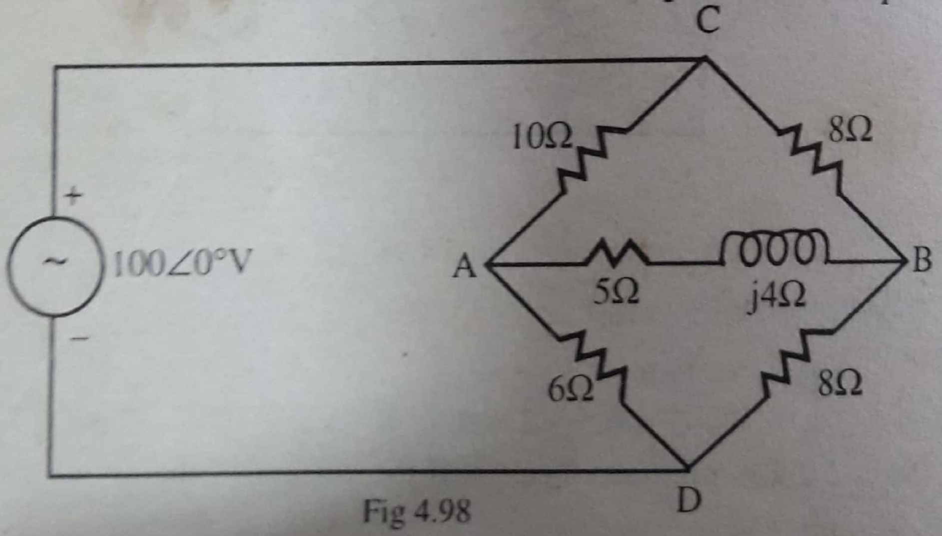


Fig 4.98

Solution : ***Step 1 :*** To find V_{Th}. Disconnect $Z_L = (5 + j4)\Omega$

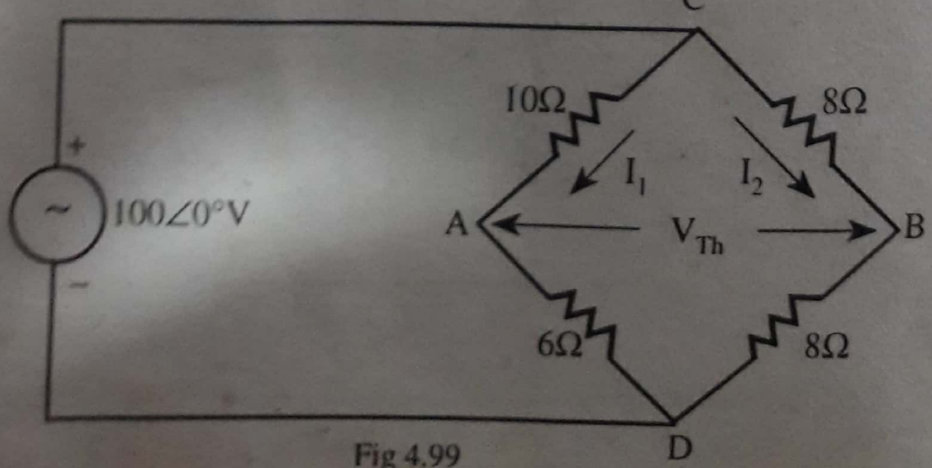


Fig 4.99

$$I_1 = \frac{100\angle 0}{10+6} = 6.67\ \angle 0^\circ \text{ A}$$

$$I_2 = \frac{100\angle 0}{8+8} = 6.67\ \angle 0^\circ \text{ A}$$

$$\therefore\ V_{Th} = V_{AB} = -6I_1 + 8I_2 = -6 \times 6.67\angle 0^\circ + 8 \times 6.67\ \angle 0^\circ$$

$$= 13.33\angle 0^\circ = \frac{40}{3}\angle 0^\circ \text{ volts}$$

Step 2 : To calculate $Z_{Th} = R_{Th}$. In the above circuit, kill the voltage source, by shorting C and D terminals.

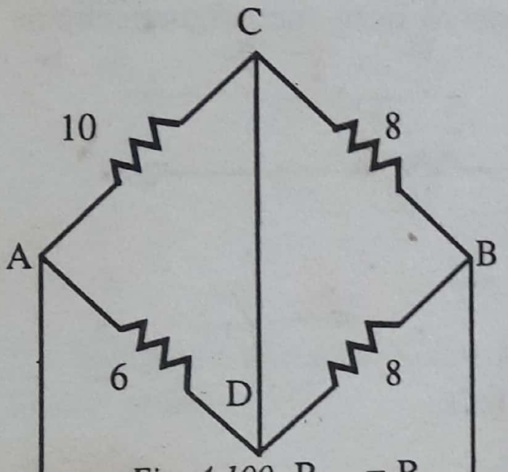


Fig. 4.100 $R_{Th} = R_{AB}$

$$R_{Th} = \frac{10 \times 6}{10+6} + \frac{8 \times 8}{8+8}$$

$$= 7.75\ \Omega$$

Step 3 : Thevenin equivalent CK*t*

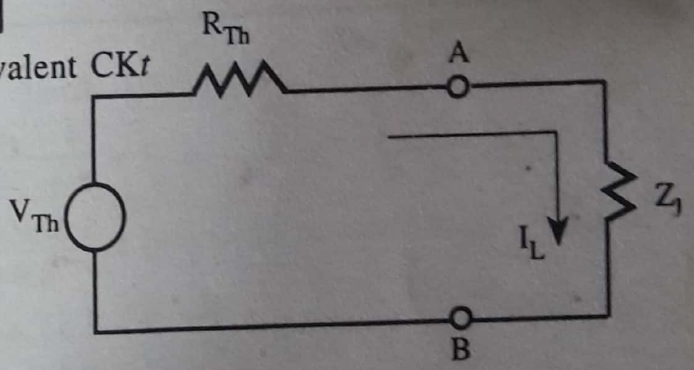


Fig.4.101

$$\therefore\ I_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{13.33\angle 0}{(7.75 + 5 + j4)}$$

$$= \frac{13.33\angle 0}{13.363\ \angle 17.42^\circ}$$

$$= 0.998\ \angle -17.42^\circ \text{ A}$$

Example 6 **: Find the Thevenin's equivalent circuit for the network given in the figure**

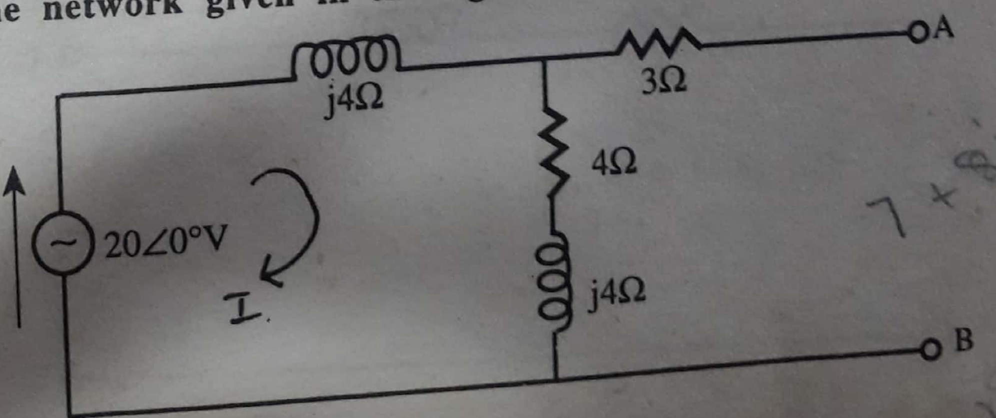


Fig. 4.102

Example 7 : **Determine Thevenin's equivalent across the terminals A, B.**

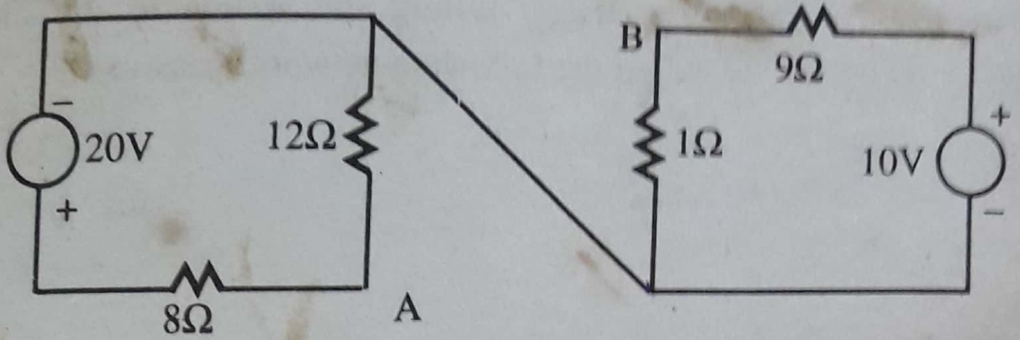


Fig. 4.105

Solution **:** For clear understanding, Let us re-draw the given circuit as shown below :

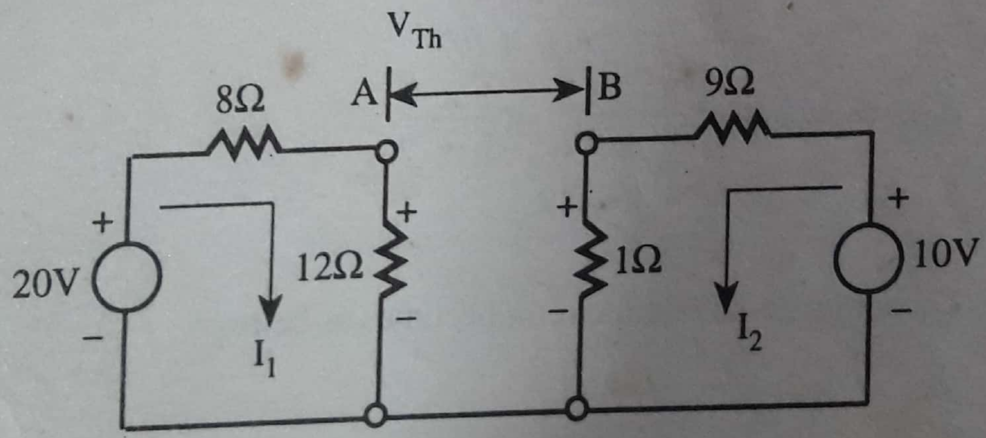


Fig. 4.106

Step 1 : To find V_{Th} For each loop, there are two resistors in series. By distribution of voltage formula

$$V_{12} = \frac{20}{8+12} \times 12$$

$$= 12 \text{ volts}$$

$$V_1 = \frac{10}{1+9} \times 1 = 1 \text{ volts}$$

$$\therefore V_{Th} = V_{AB} = -V_{12} + V_1$$
$$= -12 + 1$$
$$= -11 \text{ volts}$$

[B is –ve and A is +ve]

***Step 2 :* To calculate R_{Th}** : killing the source in the circuit shown in the fig above, we get the following network (passive)

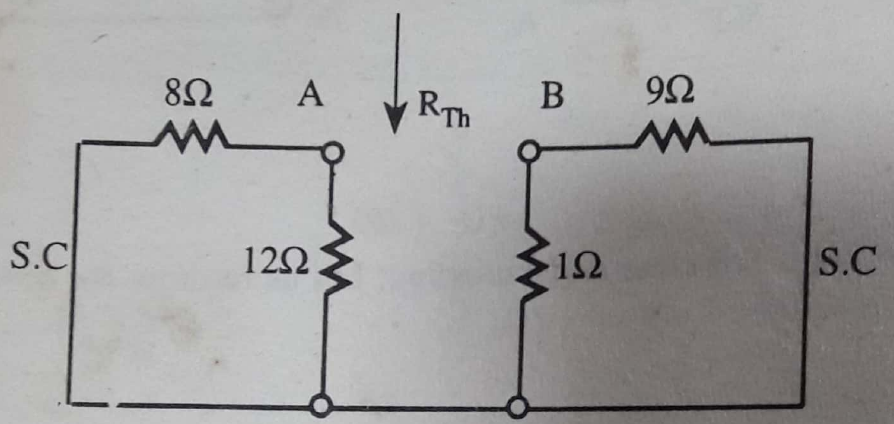


Fig. 4.107

$$R_{Th} = R_{AB} = \frac{8 \times 12}{8 + 12} + \frac{1 \times 9}{1 + 9}$$
$$= 4.8 + 0.9$$
$$= 5.7\ \Omega$$

The Thevenin's equivalent circuit becomes as below :

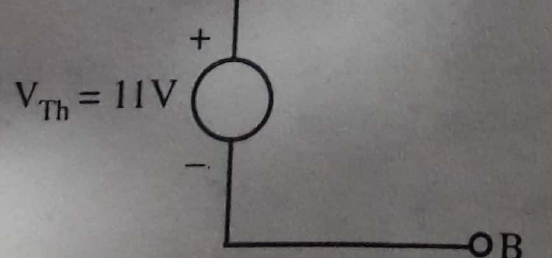


Fig. 4.108

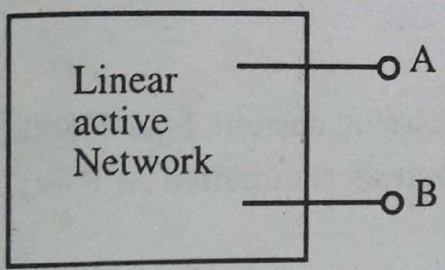
4.3.4. Norton's Theorem

Norton's theorem is the dual of the Thevenin's Theorem :

Statement : "Any linear active network with output terminals A, B as shown in the figure can be replaced by a single current source. I_{SC} (I_N) in parallel with a single impedance Z_{Th} (Z_n) = R_{Th} (R_N)"

I_{SC} is the current through the terminals AB of the active network when shorted. Z_{Th} is the Thevenin's impedance.

The current through an impedance connected to the terminals of the Norton's equivalent circuit must have the same direction as the current through the same impedance connected to the original active network.



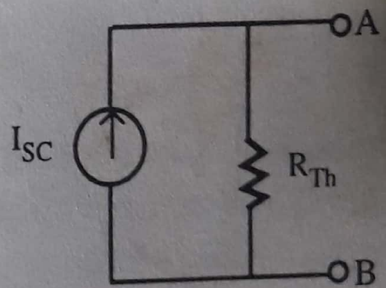


Fig. 4.109

Fig (a) Original Network

Fig (b) Norton's equivalent circuit or constant current source

After evaluating the values of I_{SC} and R_{Th}, the current through R_L connected across A and B can be connected by applying division of current formula which is expressed below :

$$\text{Current through } R_L = I_L = \frac{I_{SC} \times R_{Th}}{R_{Th} + R_L}$$

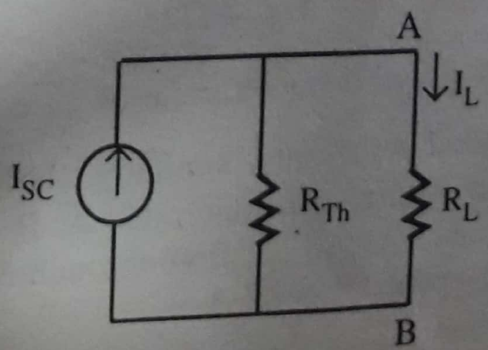


Fig. 4.110

WORKED EXAMPLES

Example 1 : **Determine the voltage across 200 Ω resistor in circuit by Norton's theorem** *[MU, April 96]*

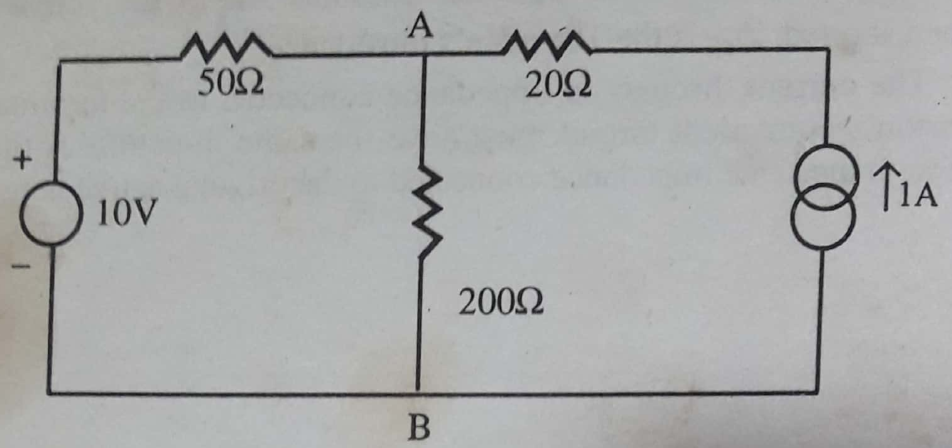


Fig. 4.111

Solution : Step 1 : To find the short circuit current I_{SC}, Replace the 200 Ω by short-circuit. The current through short circuited A, B is I_{SC} Refer the following figure.

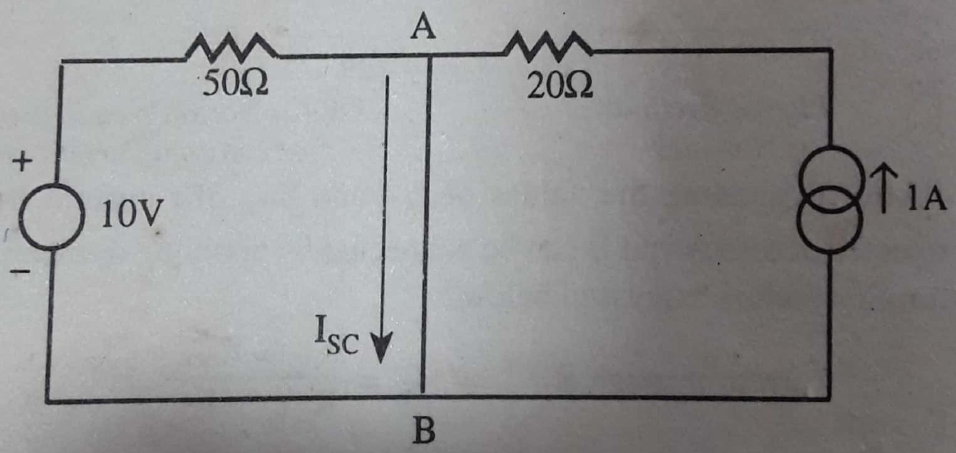


Fig. 4.112

The voltage source will drive a current of $\frac{10}{50}$ = 0.2 A, which flows through short circuited A, B only. similarly, the current of 1A flows through 20 and then through short-circuited A, B only. 0.2 A will not flow through 20Ω and 1A will not flow through 50Ω. It is because of short-circuit. So, I_{SC} = 0.2 + 1 = 1.2 A

Step 2 : To find R_{Th}. From the given circuit, disconnect R_L = 200 Ω between A and B and also kill the sources. The resultant circuit becomes as below :

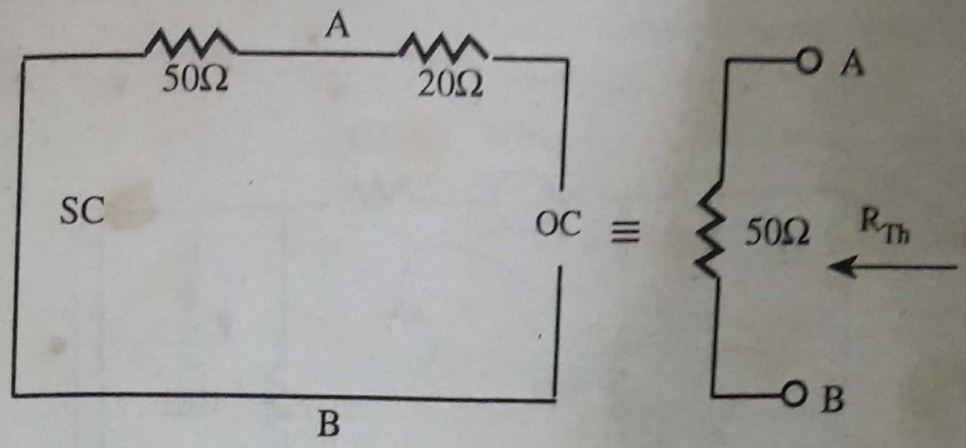
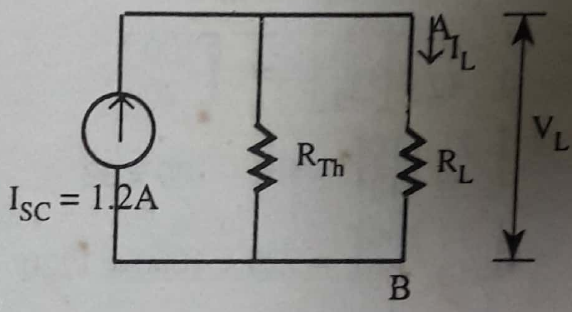


Fig. 4.113

$R_{Th} = 50\Omega.$

Step 3 : Drawing Norton's equivalent circuit and showing the load resistance R_L we get the following circuit :

57
26



Absent

Fig. 4.114

$$I_L = \frac{I_{SC}\ R_{Th}}{R_{Th} + R_L} = \frac{1.2 \times 50}{50 + 200} = 0.24 \text{ A}$$

$$\therefore V_L = I_L\ R_L = 0.24 \times 200 = 48 \text{ volts}$$

Example 2 : **Obtain the Norton's equivalent circuit at the terminals A, B for the network shown.** ***[BHN.U.Apr96]***

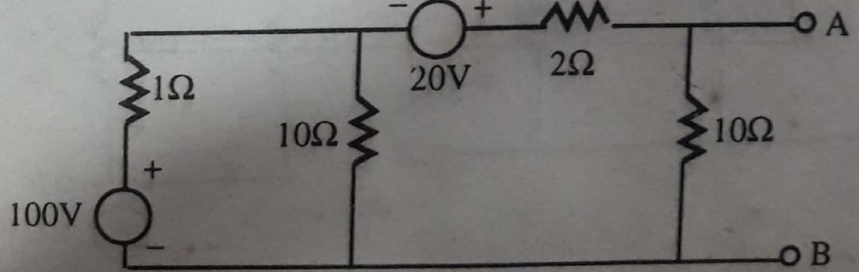


Fig. 4.115

Solution : Step 1 : To calculate I_{SC} : Short circuit A and B terminals to get the following circuit :

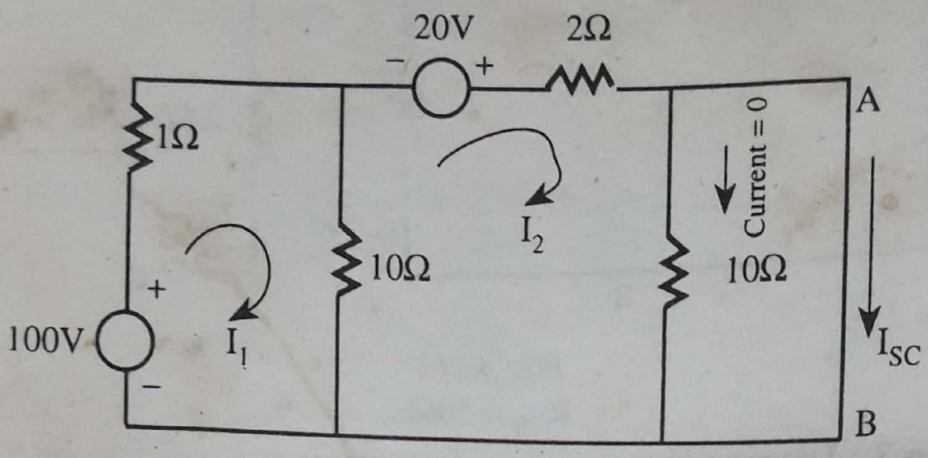


Fig. 4.116 CKt to find I_{SC}

As 10Ω is shorted, no current flows through it. Hence $I_{SC} = I_2$. Neglect presence of 10Ω. By inspection.

$$\begin{bmatrix} 11 & -10 \\ -10 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -10 \\ -10 & 12 \end{vmatrix} = 132 - 100 = 32$$

$$\Delta_2 = \begin{vmatrix} 11 & 100 \\ -10 & 20 \end{vmatrix} = 220 + 1000 = 1220$$

$$\therefore I_2 = \frac{\Delta_2}{\Delta} = \frac{1220}{32} = 38.125 \text{ A} \quad \therefore I_{SC} = I_2 = 38.125 \text{ A}$$

Step 2 : To find R_{Th} : From the original circuit kill the sources. Remember that 10Ω must be taken and it is effective.

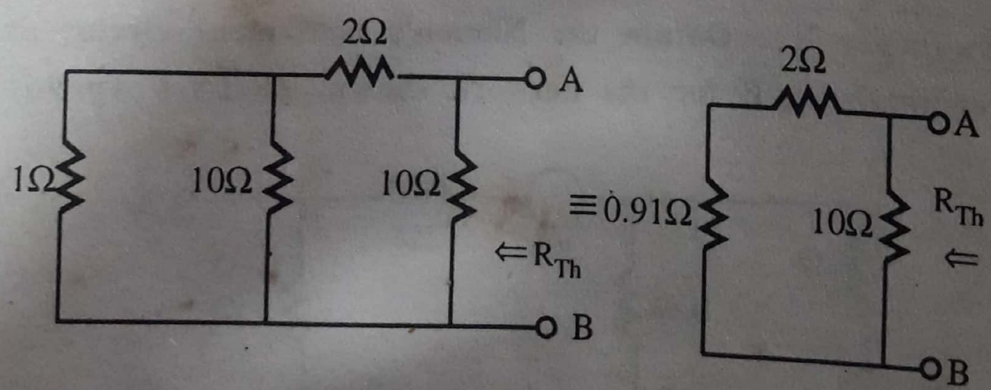


Fig. 4.117

$$\therefore R_{Th} = \frac{2.91 \times 10}{2.91 + 10} = 2.254 \ \Omega$$

Step 3 : Norton's equivalent circuit is

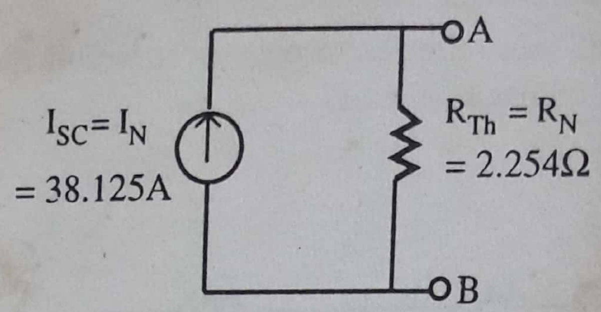


Fig. 4.118

[*Note :* The student is advised to find Thevenin's equivalent CKt for the above problem. Ans : V_{Th} = 85.94 volts and R_{Th} = 2.254 Ω]

Example 3 : Obtain the Norton's equivalent circuit for the following figure. ***[Bharathidasan University. Nov 1994]***

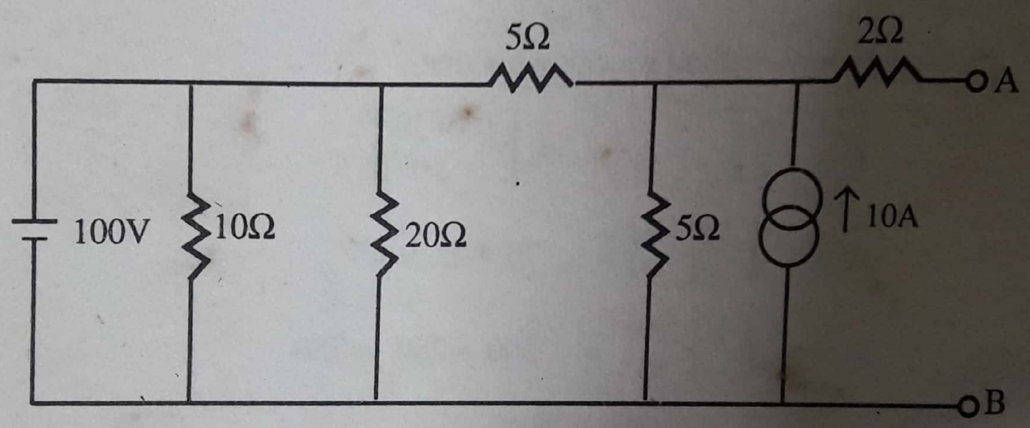


Fig. 4.119

Solution : 10Ω and 20Ω are connected across the ideal voltage source. Hence these resistors can be removed without altering the performances of the network. Also converting the current source in to voltage source we get the following circuit :

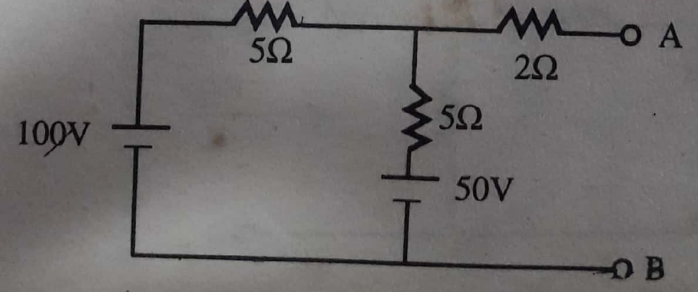


Fig. 4.120

Norton's equivalent can be found for the above circuit converting the Thevenin's equivalent source into current source. or, we can directly find the Norton's equivalent which is done as below :

Step 1 : To find short-circuit current, short circuit the terminals A and B to obtain the following circuit :

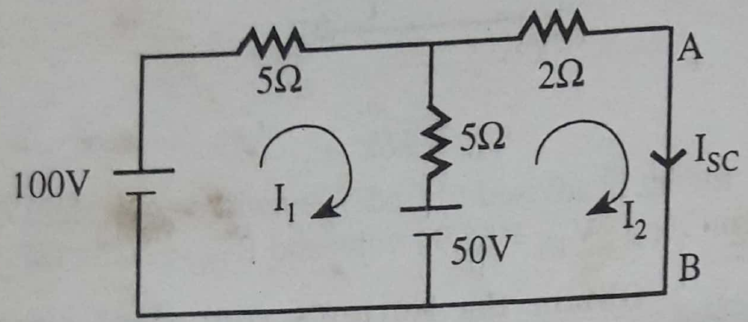


Fig. 4.121

By loop current method we can write that

$$\begin{bmatrix} 10 & -5 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100-50 \\ 50 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 10 & -5 \\ -5 & 7 \end{vmatrix} = 70 - 25 = 45$$

$$\Delta_2 = \begin{vmatrix} 10 & 50 \\ -5 & 50 \end{vmatrix} = 500 + 250 = 750$$

$$\therefore I_2 = I_{SC} = \frac{750}{45} = \frac{50}{3} \text{ A}$$

$$= 16.67 \text{ A}$$

Step 2: To find R_{Th} : Killing the sources and keeping A and B terminals open, we get

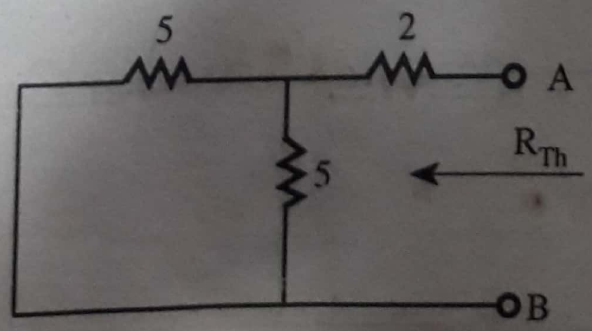


Fig. 4.122

$$R_{Th} = R_{AB} = 2 + \frac{5 \times 5}{10} = 4.5\ \Omega$$

∴ Norton's equivalent is

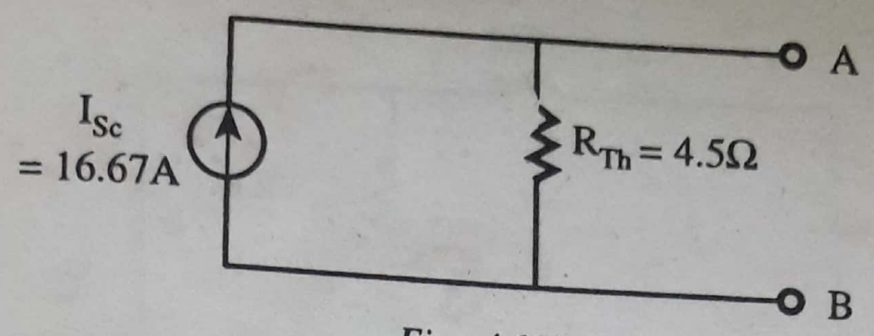


Fig. 4.123

[*Note* : The student is advised to solve the above problem without removing the resistance of 10Ω and 20Ω. He can convert the current source and voltage source, for convenience.]

Example 4 **Find the voltage across the 15Ω, resistor using Norton's theorem for the circuit given below :**

[MU. April 95]

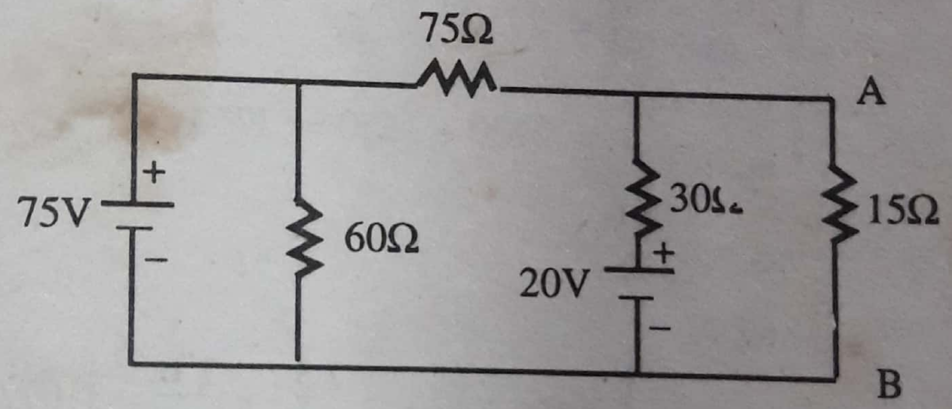
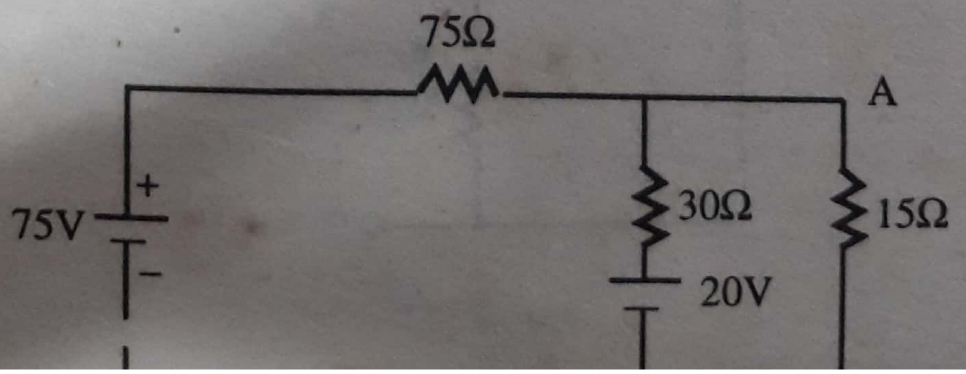


Fig. 4.124

Solution : Here also since 75V is the ideal voltage source, 60 Ω resistor can be removed without altering the performance of the circuit. The resultant figure is re–drawn as below :



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To find Norton's current I_{SC}, short circuit terminals A and B, as a result of which the circuit becomes as below :

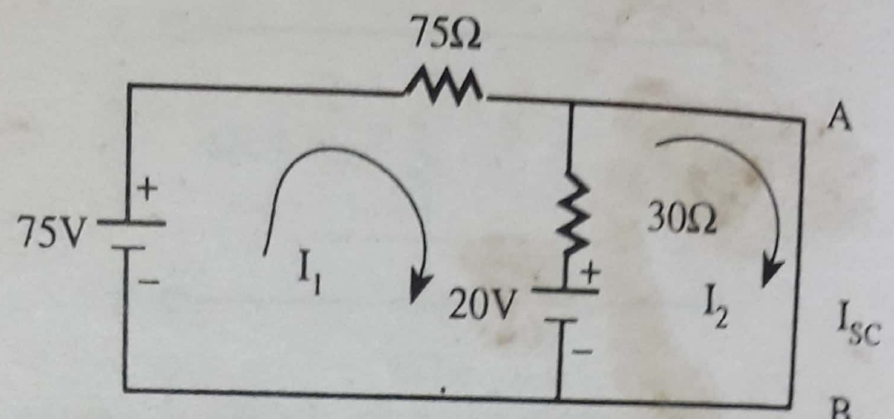


Fig. 4.126

By Inspection

$$\begin{bmatrix} 105 & -30 \\ -30 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 55 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 105 & -30 \\ -30 & 30 \end{vmatrix} = 2250$$

$$\Delta_2 = \begin{vmatrix} 105 & 55 \\ -30 & 20 \end{vmatrix} = 2100 + 1650 = 3750$$

$$\therefore I_2 = I_{SC} = \frac{3750}{2250}$$

$$= 1.67 \text{ A}$$

$$\left[\textit{Note} : I_{SC} = \frac{75}{75} + \frac{20}{30} = 1.67\text{A} \right]$$

Step 2 : To find R_{Th}

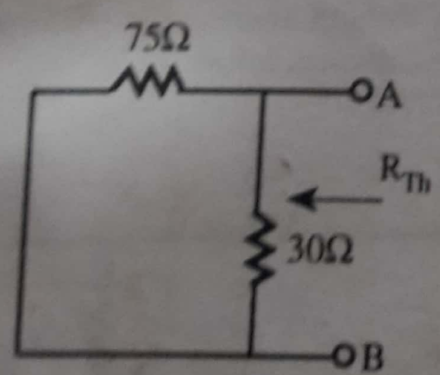


Fig. 4.127

Ckt to calculate R_{Th}

$$R_{Th} = R_{AB} = \frac{75 \times 30}{75 + 30} = 21.43\ \Omega$$

Step 3 : In the original CKt, replacing the left part of AB by Norton's equivalent we get,

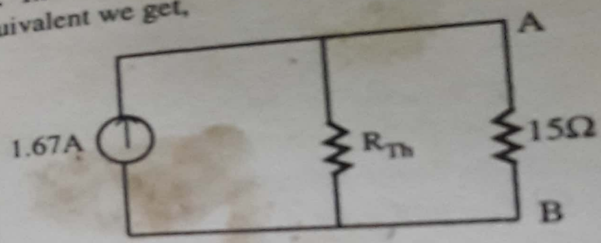


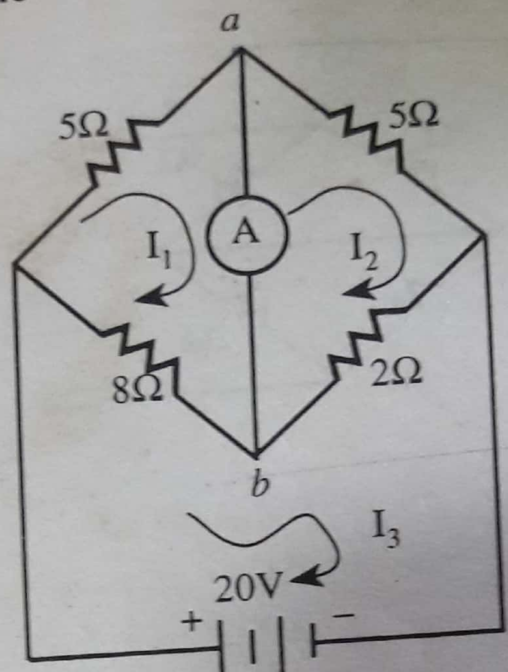
Fig. 4.128

$$I_{15} = 1.61 \times \frac{21.43}{21.43 + 15} = 0.982 \text{ A}$$

$$\therefore V_{15} = 0.982 \times 15 = 14.75 \text{ Volts}$$

Example 5 : Obtain Norton's equivalent across

Example 6 : In the circuit of the figure, compute the current through the O resistance ammeter. Use Norton's theorem.



Solution : Step 1 : To find I_{SC}. For this we have to short circuit the terminals A and B. As the resistance of the ammeter is $O(R_L = 0)$, we can make use of the given circuit itself to find I_{SC}.

$$I_{SC} = I_1 \sim I_2$$

By inspection

$$\begin{bmatrix} 13 & 0 & -8 \\ 0 & 7 & -2 \\ -8 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 13 & 0 & -8 \\ 0 & 7 & -2 \\ -8 & -2 & 10 \end{vmatrix} = 13\,(70-4) - 8\,(56) = 410$$

$$\Delta_1 = \begin{vmatrix} 0 & 0 & -8 \\ 0 & 7 & -2 \\ 20 & -2 & 10 \end{vmatrix} = 20\,(56) = 1120$$

$$\Delta_2 = \begin{vmatrix} 13 & 0 & -8 \\ 0 & 0 & -2 \\ -8 & 20 & 10 \end{vmatrix} = 13\,(40) - 8(0) = 520$$

$$\therefore I_1 = \frac{1120}{410}\ \text{A} \quad ; \quad I_2 = \frac{520}{410}$$

$$\therefore I_{SC} = I_1 - I_2 = \frac{1120}{410} - \frac{520}{410}$$

$$= \frac{6}{4.1}\ \text{A} = 1.4634\ \text{A}$$

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Step 2 : To find R_{Th} : The current through $a\,b$ is required. Keep a and b terminals open and kill the source.

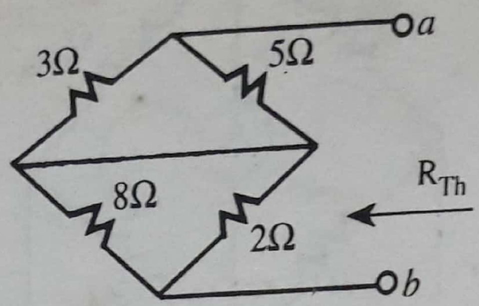

Fig. 4.134

$$R_{Th} = R_{AB}$$
$$= \frac{8 \times 2}{10} + \frac{5 \times 5}{10}$$
$$= 1.6 + 2.5 = 4.1\ \Omega$$

Step 3 : Norton's equivalent circuit.

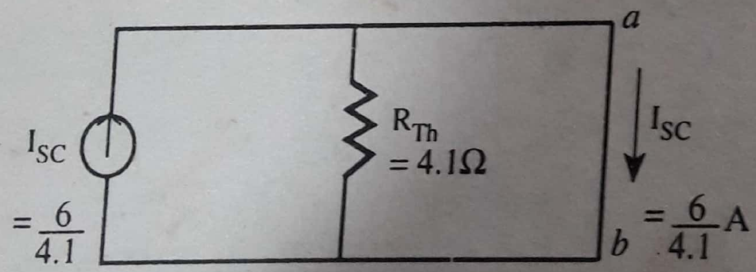

Fig. 4.135

$$I_{SC} = \frac{6}{4.1}\ \text{A}$$

[*Note :* The current through the zero resistance ammeter is I_{SC} which was found by loop current method. There is no need of R_{Th}. But to let the student to know as how to apply Norton's theorem through a short circuited path, the problem is solved by this method]

4.3.5 Maximum Power Transfer Theorem.

The statement of this theorem depends upon the conditions of the circuit. We shall consider three cases one by one as below :

Case I : Purely resistor circuit and the load resistance is variable

Statement :

"Maximum power will be delivered from a voltage source to a load, when the load resistance is equal to the internal resistance of the source".

For example, consider a voltage source of generated voltage V_g and internal resistance R_g, connected to a local resistance R_L as shown in the figure.

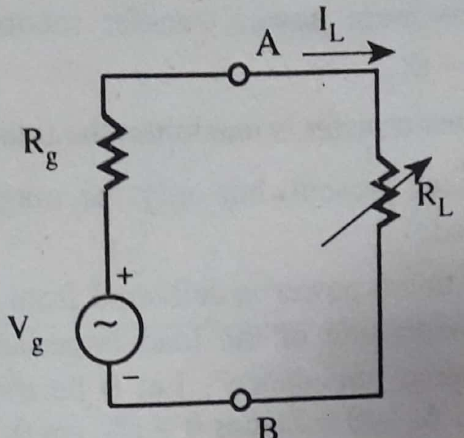

Fig 4.136

when the power transfer to the load is maximum $R_L = R_g$. At this condition the total resistance $= R_g + R_L = 2R_L$.

Therefore, the load current $= I_L = \dfrac{V_g}{2R_L}$

Power delivered to $R_L = I_L^2 R_L = \left(\dfrac{V_g}{2R_L}\right)^2 R_L = \dfrac{V_g^2}{4R_L}$

This is maximum power transferred to R_L

$$i.e.,\ P_{max} = \frac{V_g^2}{4R_L}$$

Under this condition the efficiency is only 50%, since one–half of total power generated is lost in dissipation within the source.

Note :

i) The Reader is to note that the maximum current and the maximum power transfer will not take place for the same value of R_L.

ii) Current is maximum when R_L is O and is given by $\dfrac{V_g}{R_g}$

iii) Power transfer is maximum when $R_L = R_g$.

MAXIMUM POWER TRANSFER

Example 1 :**The circuit shown in the figure R, absorbs maximum power. Compute the value of R and maximum power** *[Bharathiyar University. April 96]*

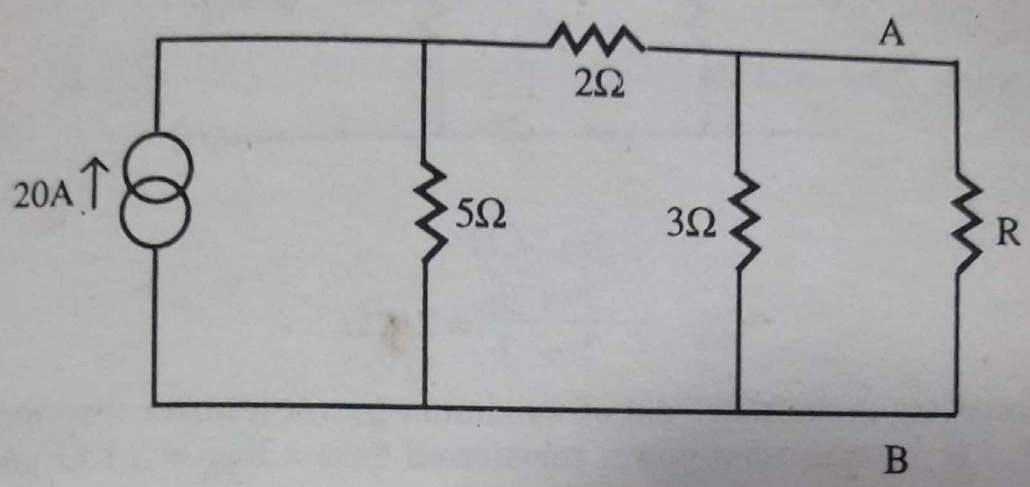

Fig. 4.137 - Original Circuit

Solution : Thevenin's equivalent circuit is drawn as below :

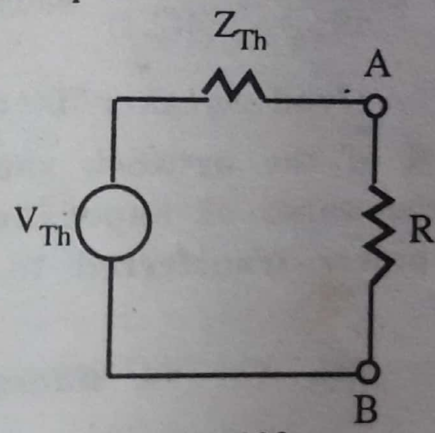

Fig. 4.138

To find V_{Th} : Disconnect R from the original circuit

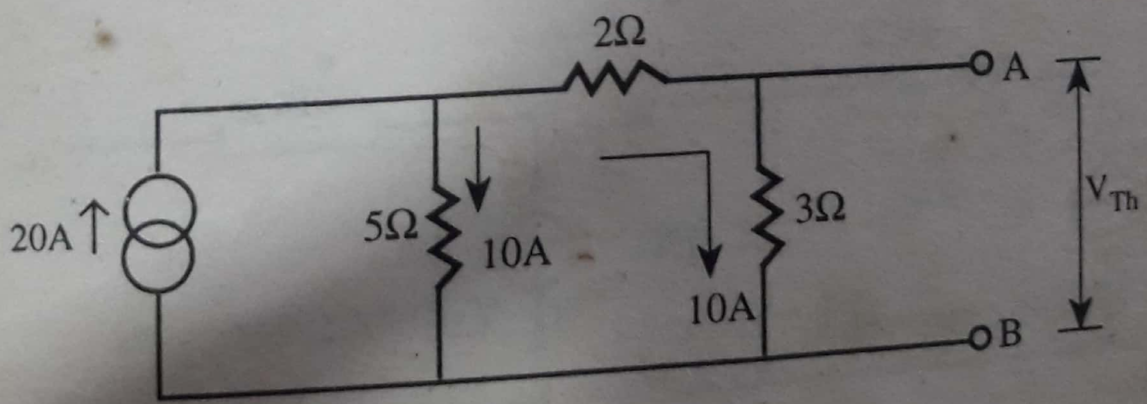

Fig. 4.139

$$V_{Th} = V_{AB} = -10 \times 3 = -30 \text{ Volts}$$

$= 30$ Volts [with B negative and A positive]

To find R_{Th} : From the above circuit, open circuit the current source.

Thus

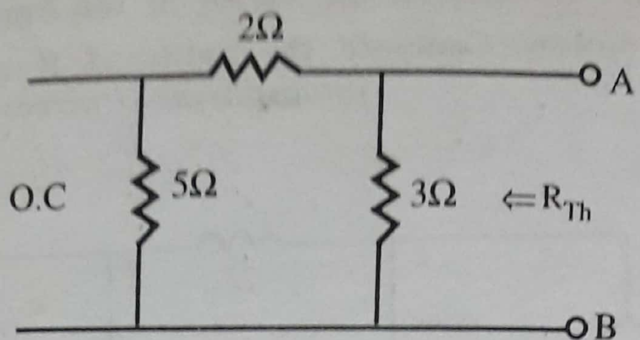

Fig. 4.140

$$R_{Th} = \frac{7 \times 3}{7 + 3} = 2.1\ \Omega$$

According to the statement of maximum power transfer theorem, the value of R for maximum power transferred to it = R_{Th} = 2.1 Ω and the maximum power transferred

$$= \frac{V_{Th}^2}{4R_L} = \frac{V_{Th}^2}{4R_{Th}} = \frac{30^2}{4(2.1)} \text{ watts} = 107 \text{ watts}$$

Example 2 : **A loud speaker is connected across the terminals A and B of the network shown in figure below. What should be the value of impedance of the speaker to obtain maximum power transferred to it and what is the maximum power ?**

[Bharathidasan Uni. Nov 94, Bharathiyar Uni. Nov 95]

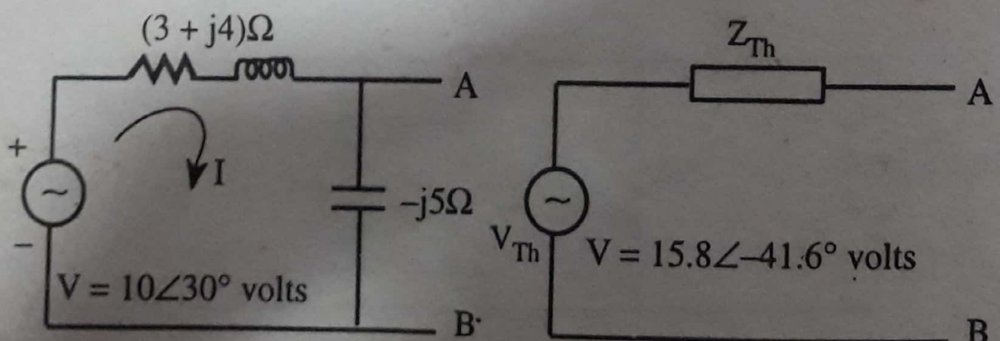

Fig. 4.141

Original Ckt *Thevenin's eq CKt*

Example 3 : **For the circuit of the figure, find the value of R_L for maximum power delivered to it. Calculate also the maximum load power.** *[M.U April 95]*

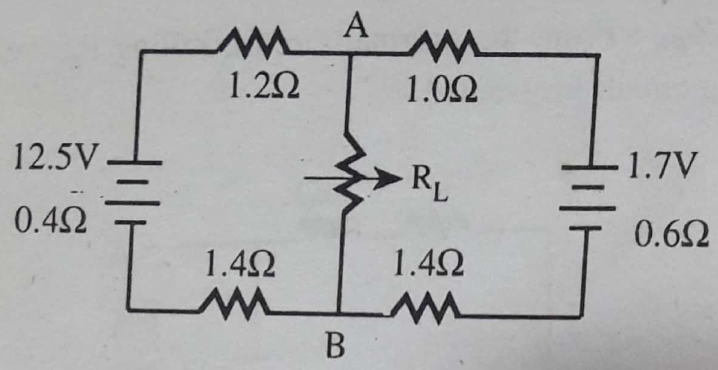

Fig. 4.144

Solution : Step 1 : To find V_{Th} : Disconnect R_L between A and B terminals

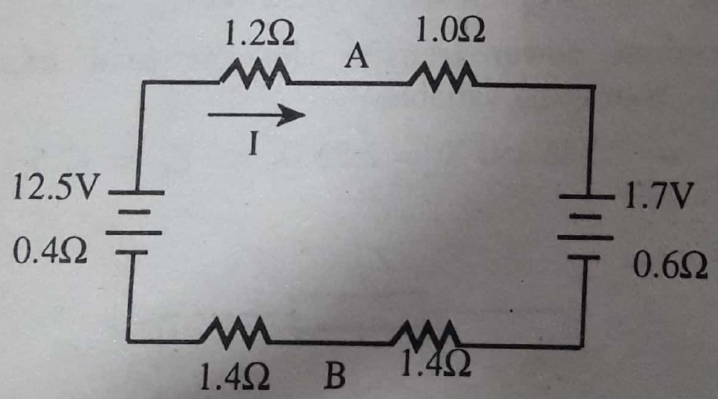

Fig. 4.145

$$I = \frac{12.5 - 1.7}{0.4 + 1.2 + 1.4 + 1 + 1.4 + 0.6}$$

$$= 1.8\text{A}$$

$$\therefore V_{Th} = V_{AB} = -1.7 - 3(1.8)$$

$$= -7.1 \text{ volts}$$

[B is negative polarity and A positive polarity]

Step 2 : To find R_{Th} : Kill the sources from the circuit in the step 1 to get the following circuit

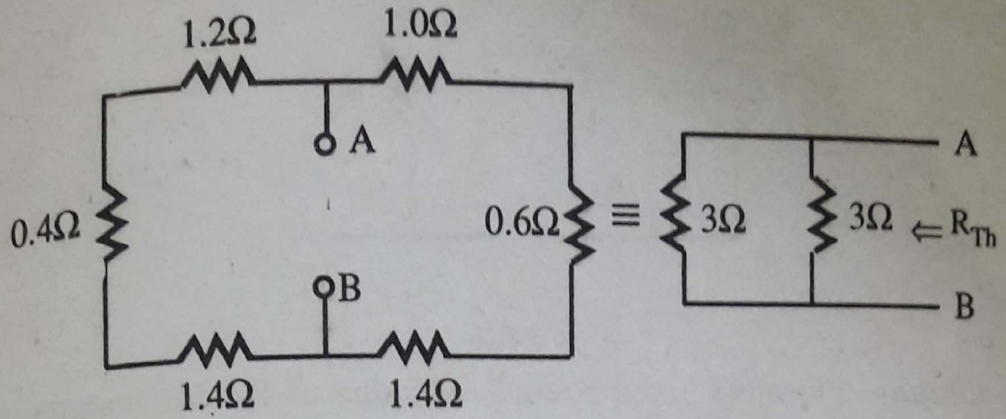

Fig. 4.146

$$R_{Th} = R_{AB} = \frac{3 \times 3}{3 + 3} = 1.5\ \Omega$$

Step 3 : Thevenin's equivalent circuit is drawn as below:

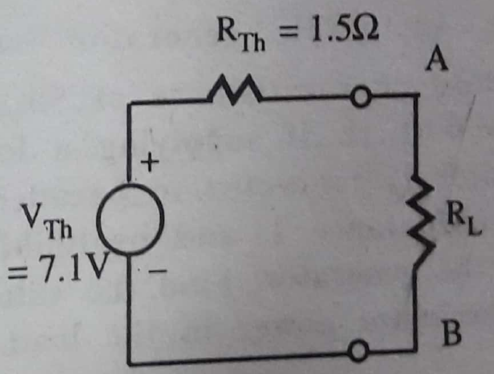

Fig. 4.147

From this, we can say that the values of R_L for maximum power transferred to it = $R_L = R_{Th} = 1.5\ \Omega$

$$\text{and } P_{max} = \frac{V^2_{Th}}{4R_L}$$

$$= \frac{(7.1)^2}{4\ (1.5)} = 8.4 \text{ watts}$$

Example 4 : In the circuit of fig below find the value of R_L which results in maximum power and calculate the value of maximum power. *[BHR Uni.Nov 94,MU.Apri 92]*