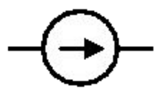
BASIC ELECTRICAL AND ELECTRONICS ENGINEERING
Unit-1

Dr.M.Arun Noyal Doss
Associate Professor

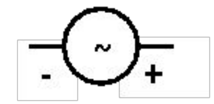
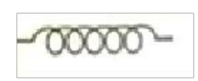
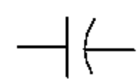
ELECTRIC CIRCUITS

Electric circuits are broadly classified as Direct Current (D.C.) circuits and Alternating Current (A.C.) circuits. The following are the various elements that form electric circuits.

D.C. Circuits

Elements	**Representation**
Voltage source	
Current source	
Resistor	

A.C. Circuits

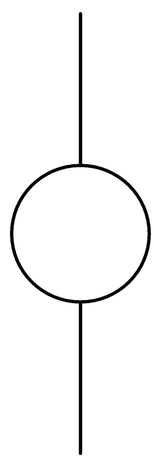
Elements	**Representation**
Voltage source	
Current source	
Resistor	
Inductor	
Capacitor	

We also will classified sources as **Independent** and **Dependent** sources

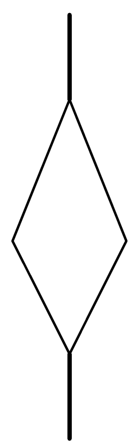
Independent source establishes a voltage or a current in a circuit without relying on a voltage or current elsewhere in the circuit

Dependent sources establishes a voltage or a current in a circuit whose value depends on the value of a voltage or a current elsewhere in the circuit

We will use circle to represent **Independent source** and diamond shape to represent **Dependent sources**



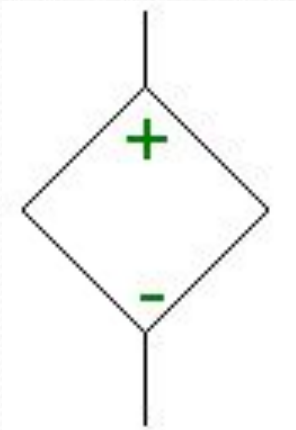
Independent source



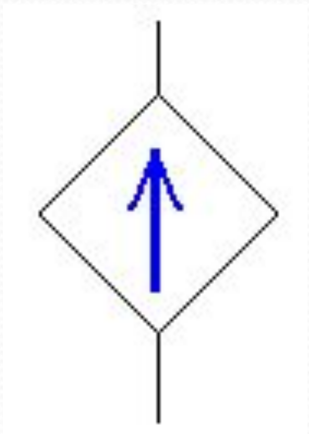
Dependent sources

Dependent Power Sources

- Voltage controlled voltage source
  - (VCVS)
- Current controlled voltage source
  - (CCVS)



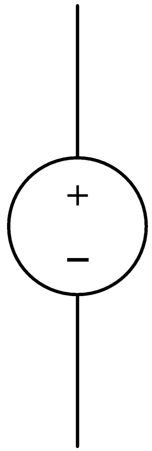
- Voltage controlled current source
  - (VCCS)
- Current controlled current source
  - (CCCS)



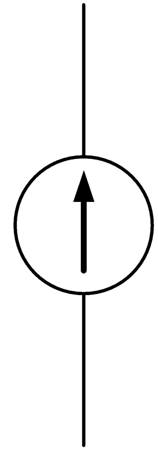
Summary

- Dependent sources are voltage or current sources whose output is a function of another parameter in the circuit.
  - Voltage controlled voltage source (VCVS)
  - Current controlled current source (CCCS)
  - Voltage controlled current source (VCCS)
  - Current controlled voltage source (CCVS)
- Dependent sources only produce a voltage or current when an independent voltage or current source is in the circuit.
- Dependent sources are treated like independent sources when using nodal or mesh analysis, but <u>**not**</u> with superposition.

Independent and dependent voltage and current sources can be represented as



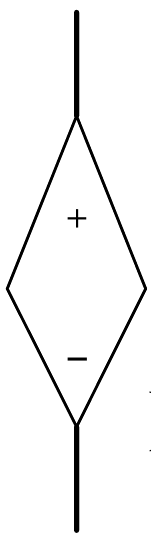
V 5



A 3

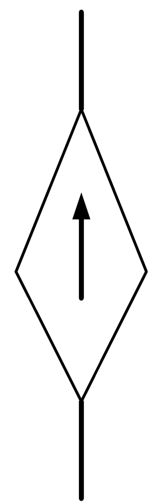
Independent voltage source

Independent current source



$\boldsymbol{i_x}$ $\boldsymbol{V}$ **4**

were i_x is some current through an element



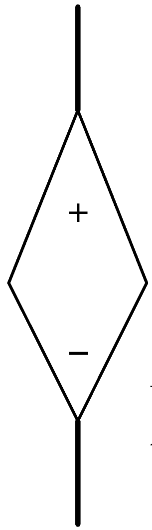
$\boldsymbol{v_x}$ $\boldsymbol{A}$ **4**

were v_x is some voltage across an element

Dedependent voltage source
Voltage depend on current

Dedependent current source
Current depend on voltage

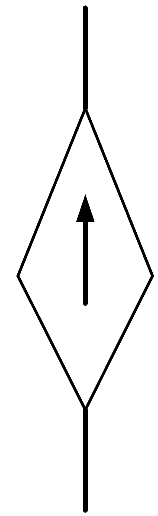
The dependent sources can be also as



$\boldsymbol{v_x}$ ***V*** **4**

were v_x is some current through an element

Dedependent voltage source
Voltage depend on voltage



$\boldsymbol{i_x}$ ***A*** **7**

were i_x is some voltage across an element

Dedependent current source
Current depend on current

First we shall discuss about the analysis of DC circuit. The voltage across an element is denoted as E or V. The current through the element is I.

Conductor is used to carry current. When a voltage is applied across a conductor, current flows through the conductor. If the applied voltage is increased, the current also increases. The voltage current relationship is shown in Fig. 1.

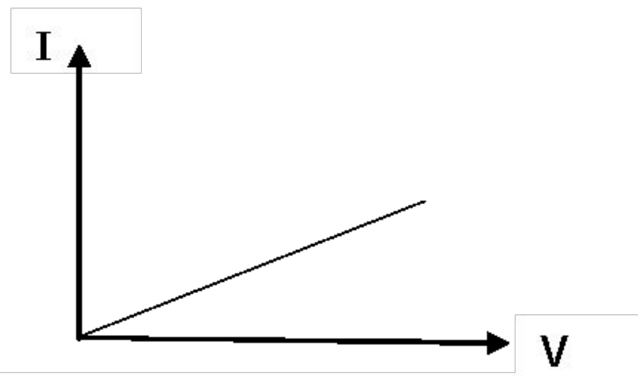


Fig. 1 Voltage – current relationship

It is seen that $I \infty$ V. Thus we can write

$$I = G\,V \tag{1}$$

where G is called the conductance of the conductor.

Very often we are more interested on RESISTANCE, R of the conductor, than the conductance of the conductor. Resistance is the opposing property of the conductor and it is the reciprocal of the conductance, Thus

$$R = \frac{1}{G} \quad \text{or} \quad G = \frac{1}{R} \tag{2}$$

Therefore

$$I = \frac{V}{R} \tag{3}$$

The above relationship is known as OHM's law. Thus Ohm law can be stated as the current flows through a conductor is the ratio of the voltage across the conductor and its resistance. Ohm's law can also be written as

$$V = R\,I \tag{4}$$

$$R = \frac{V}{I} \tag{5}$$

The resistance of a conductor is directly proportional to its length, inversely proportional to its area of cross section. It also depends on the material of the conductor. Thus

$$R = \rho \, \frac{\ell}{A} \tag{6}$$

where ρ is called the specific resistance of the material by which the conductor is made of. The unit of the resistance is Ohm and is represented as Ω. Resistance of a conductor depends on the temperature also. The power consumed by the resistor is given by

$$P = V\,I \tag{7}$$

When the voltage is in volt and the current is in ampere, power will be in watt. Alternate expression for power consumed by the resistors are given below.

$$P = R\,I \times I = I^2 R \tag{8}$$

$$P = V \times \frac{V}{R} = \frac{V^2}{R} \tag{9}$$

KIRCHHOFF’s LAWS

There are two Kirchhoff’s laws. The first one is called Kirchhoff’s current law, KCL and the second one is Kirchhoff’s voltage law, KVL. Kirchhoff’s current law deals with the element currents meeting at a junction, which is a meeting point of two are more elements. Kirchhoff’s voltage law deals with element voltages in a closed loop also called as closed circuit.

Kirchhoff's current law

Kirchhoff's currents law states that the algebraic sum of element current meeting at a junction is zero.

Consider a junction P wherein four elements, carrying currents I_1, I_2, I_3 and I_4, are meeting as shown in Fig. 2.

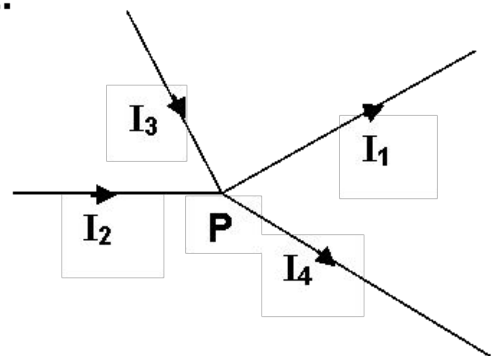


Fig. 2 Currents meeting at a junction

Note that currents I_1 and I_4 are flowing out from the junction while the currents I_2 and I_3 are flowing into the junction. According to KCL,

$$I_1 - I_2 - I_3 + I_4 = 0 \quad (10)$$

The above equation can be rearranged as

$$I_1 + I_4 = I_2 + I_3 \quad (11)$$

From equation (11), KCL can also stated as at a junction, the sum of element currents that flows out is equal to the sum of element currents that flows in.

Kirchhoff's voltage law

Kirchhoff's voltage law states that the algebraic sum of element voltages around a closed loop is zero.

Consider a closed loop in a circuit wherein four elements with voltages V_1, V_2, V_3 and V_4, are present as shown in Fig. 3.

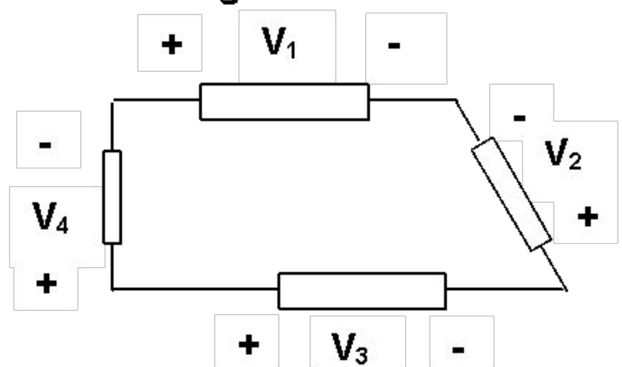


Fig. 3 Voltages in a closed loop

Assigning positive sign for voltage drop and negative sign for voltage rise, when the loop is traced in clockwise direction, according to KVL

$$V_1 - V_2 - V_3 + V_4 = 0 \tag{12}$$

The above equation can be rearranged as

$$V_1 + V_4 = V_2 + V_3 \tag{13}$$

From equation (13), KVL can also stated as, in a closed loop, the sum of voltage drops is equal to the sum of voltage rises in that loop.

Resistors connected in series

Two resistors are said to be connected in series when there is only one common point between them and no other element is connected in that common point. Resistors connected in series carry same current. Consider three resisters R_1, R_2 and R_3 connected in series as shown in Fig. 4. With the supply voltage of E, voltages across the three resistors are V_1, V_2 and V_3.

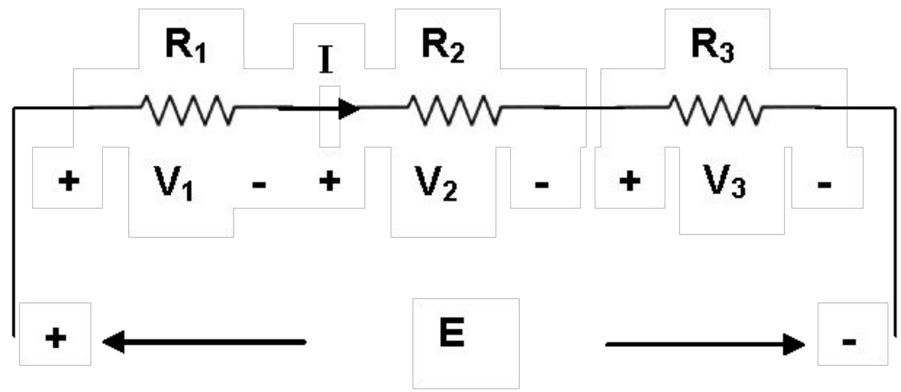


Fig. 4 Resistors connected in series

As per Ohm's law

$$\left.\begin{aligned} V_1 &= R_1 I \\ V_2 &= R_2 I \\ V_3 &= R_3 I \end{aligned}\right\} \qquad (14)$$

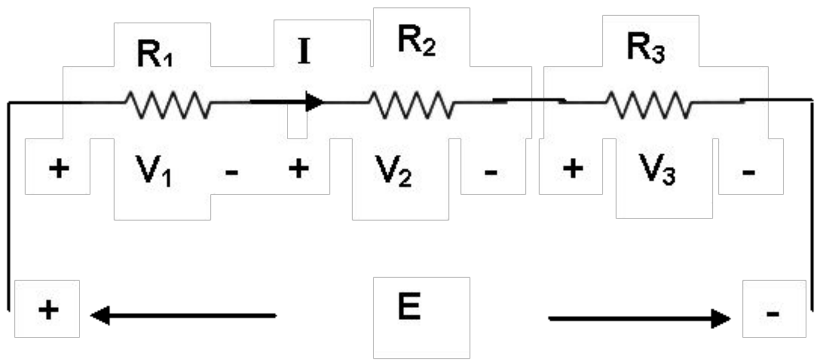


Fig. 4 Resistors connected in series

As per Ohm's law

$V_1 = R_1 \mathbf{I}$

$V_2 = R_2 \mathbf{I}$

$V_3 = R_3 \mathbf{I}$

Applying KVL,

$$E = V_1 + V_2 + V_3 \tag{15}$$

$$= (R_1 + R_2 + R_3)\, \mathbf{I} = R_{e\,q}\, \mathbf{I} \tag{16}$$

Thus for the circuit shown in Fig. 4,

$$E = R_{e\,q}\, \mathbf{I} \tag{17}$$

where E is the circuit voltage, $\mathbf{I}$ is the circuit current and $R_{e\,q}$ is the equivalent resistance. Here

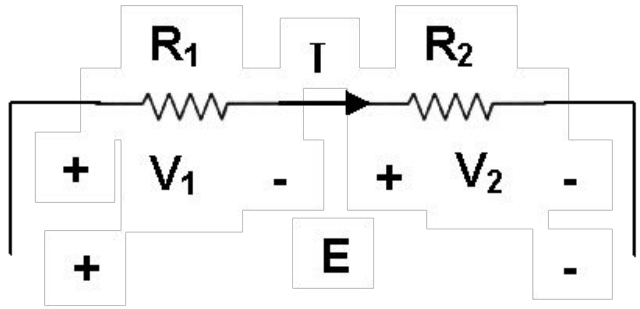
$$R_{e\,q} = R_1 + R_2 + R_3 \tag{18}$$

This is true when two are more resistors are connected in series. When n numbers of resistors are connected in series, the equivalent resistor is given by

$$R_{e\,q} = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n \tag{19}$$

Voltage division rule

Consider two resistors connected in series. Then



$V_1 = R_1\ I$

$V_2 = R_2\ I$

$E = (R_1 + R_2)\ I$ and hence $I = E\ /\ (R_1 + R_2)$

Total voltage of E is dropped in two resistors. Voltage across the resistors are given by

$$V_1 = \frac{R_1}{R_1 + R_2}\ E \quad \text{and} \tag{20}$$

$$V_2 = \frac{R_2}{R_1 + R_2}\ E \tag{21}$$

Resistors connected in parallel

Two resistors are said to be connected in parallel when both are connected across same pair of nodes. Voltages across resistors connected in parallel will be equal.

Consider two resistors R_1 and R_2 connected in parallel as shown in Fig. 5.

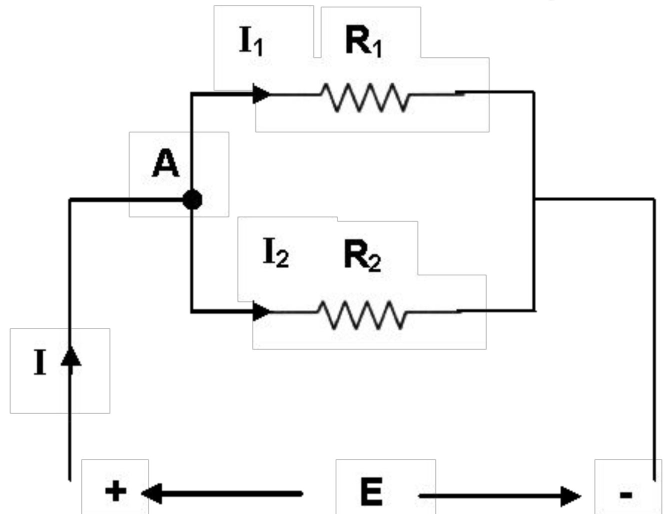
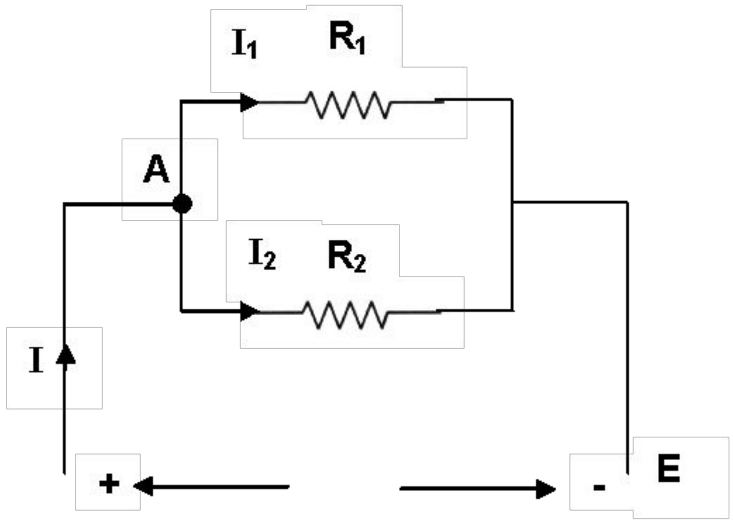


Fig. 5 Resistors connected in parallel

As per Ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \qquad (22)$$



As per Ohm's law

$$I_1 = \frac{E}{R_1}$$

$$I_2 = \frac{E}{R_2}$$

Applying KCL at node A

$$I = I_1 + I_2 = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_{eq}} \tag{23}$$

Thus for the circuit shown in Fig. 5

$$I = \frac{E}{R_{eq}} \tag{24}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (25)$$

From the above $\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$

Thus $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ **(26)**

When n numbers of resistors are connected in parallel, generalizing eq. (25), R_{eq} can be obtained from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots + \frac{1}{R_n} \qquad (27)$$

Current division rule

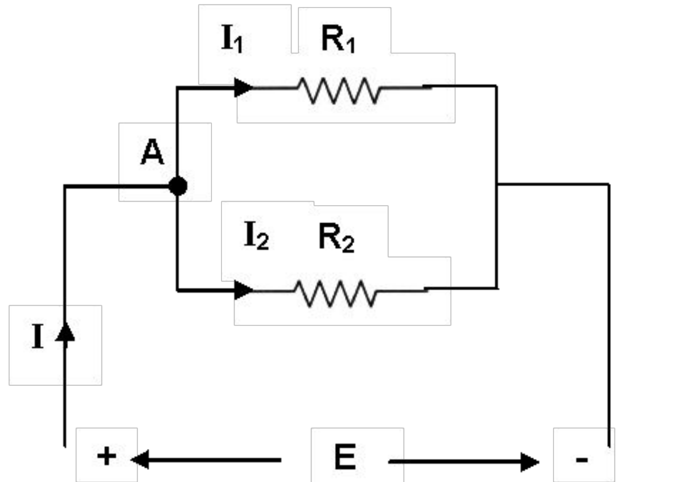


Fig. 5 Resistors connected in parallel

Referring to Fig. 5, it is noticed the total current gets divided as I_1 and I_2. The branch currents are obtained as follows.

From eq. (23)

$$E = \frac{R_1 R_2}{R_1 + R_2} I \tag{29}$$

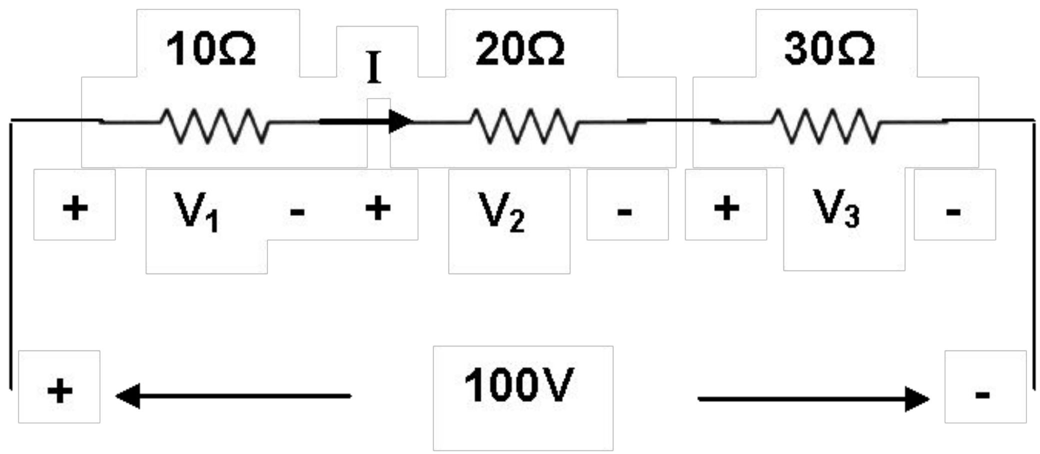
Substituting the above in eq. (22)

$$\left.\begin{aligned} I_1 &= \frac{R_2}{R_1 + R_2} I \\ I_2 &= \frac{R_1}{R_1 + R_2} I \end{aligned}\right\} \tag{30}$$

Example 1

Three resistors 10Ω, 20Ω and 30Ω are connected in series across 100 V supply. Find the voltage across each resistor.

Solution



Current $I = 100 / (10 + 20 + 30) = 1.6667$ A

Voltage across 10Ω = 10 x 1.6667 = 16.67 V

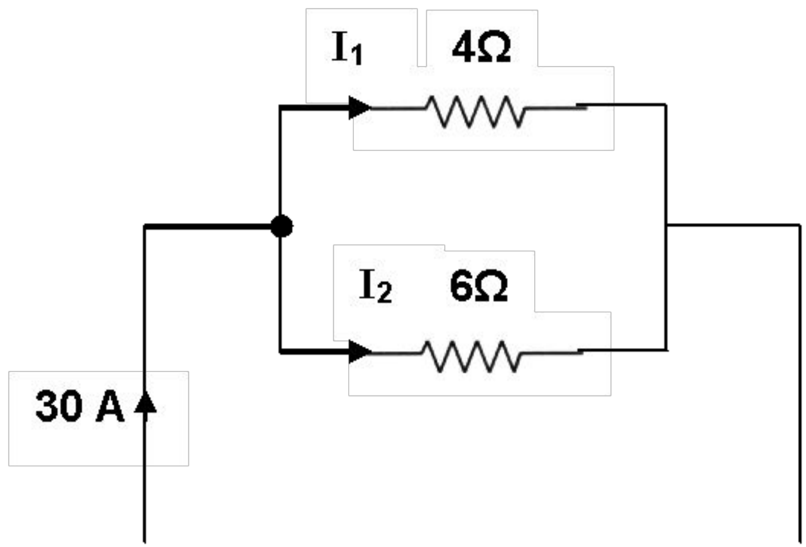
Voltage across 20Ω = 20 x 1.6667 = 33.33 V

Voltage across 30Ω = 30 x 1.6667 = 50 V

Example 2

Two resistors of 4Ω and 6Ω are connected in parallel. If the supply current is 30 A, find the current in each resistor.

Solution



Using the current division rule

$$\text{Current through } 4\Omega = \frac{6}{4+6} \times 30 = 18\,\text{A}$$

$$\text{Current through } 6\Omega = \frac{4}{4+6} \times 30 = 12\,\text{A}$$

Example 3

Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that the total power of 100 W is absorbed?

Solution

Let R_T be the total equivalent resistor. Then

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$$

Resistance $R_T = \frac{120}{154} = 0.7792\,\Omega$

Let E be the supply voltage. Then total current taken = E / 0.7792 A

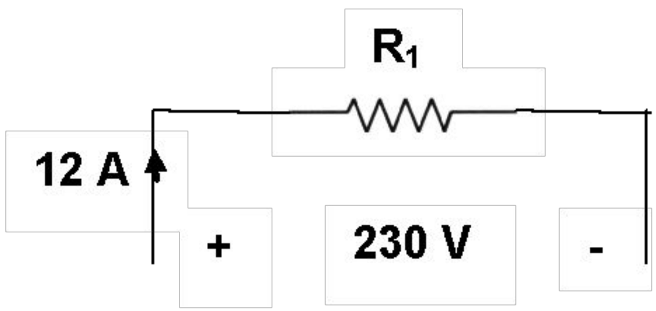
Thus $(\frac{E}{0.7792})^2$ x $0.7792 = 100$ and hence E^2 = 100 x 0.7792 = 77.92

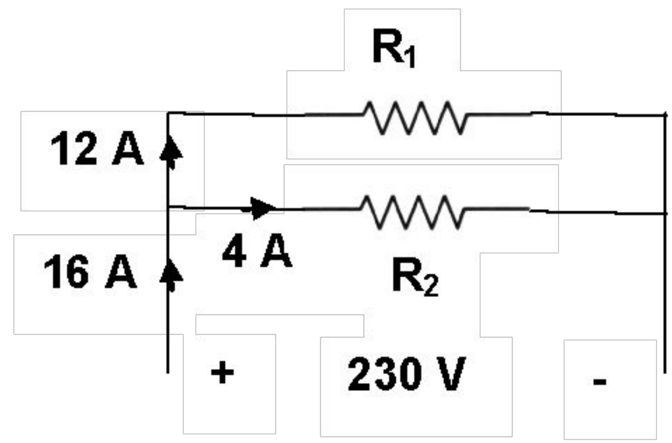
Required voltage = $\sqrt{77.92} = 8.8272$ V

Example 4

When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel, to increase the load to 16 A

Solution





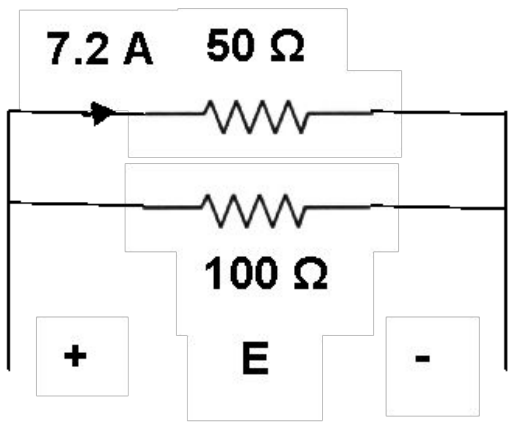
To make the load current 16 A, current through the second resistor = 16 –12 = 4 A

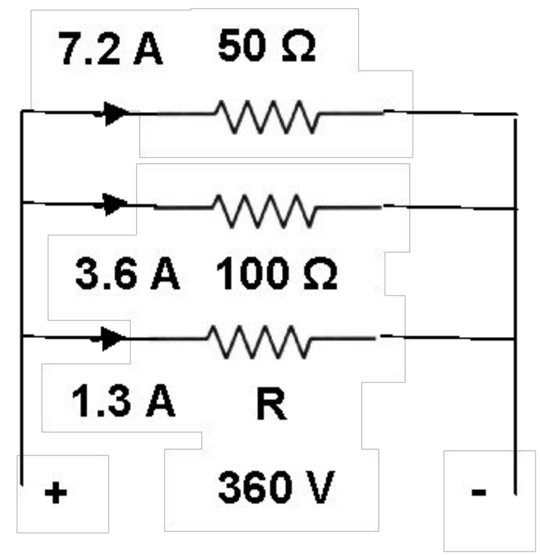
Value of second resistor R_2 = 230/4 = 57.5 Ω

Example 5

A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistor to be added in parallel to make the line current as 12.1A?

Solution





Supply voltage E = 50 x 7.2 = 360 V

Current through 100 Ω = 360/100 = 3.6 A

When the line current is 12.1 A, current through third resistor = 12.1 – (7.2 + 3.6)

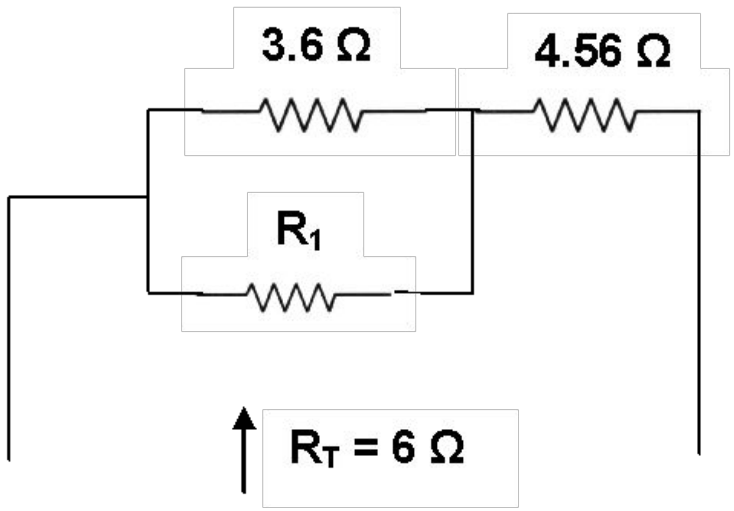
= 1.3 A

Value of third resistor = 360/1.3 = 276.9230 Ω

Example 6

A resistor of 3.6 ohms is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms, so that the total resistance of the circuit shall be 6 ohms?

Solution



$3.6 \parallel R_1 = 6 - 4.56 = 1.44\ \Omega$

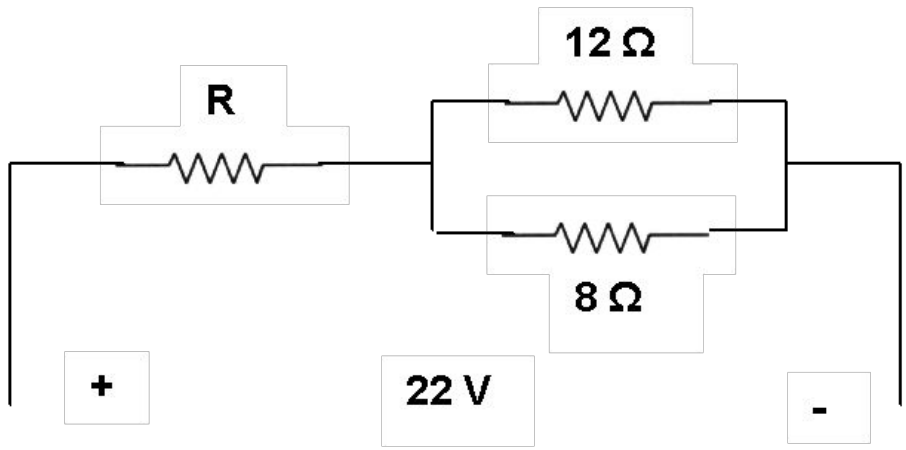
Thus $\dfrac{3.6 \times R_1}{3.6 + R_1} = 0.4$; Therefore $\dfrac{3.6 + R_1}{R_1} = \dfrac{1}{0.4} = 2.5$; $\dfrac{3.6}{R_1} = 1.5$

Required resistance $R_1 = 3.6/1.5 = 2.4\ \Omega$

Example 7

A resistance R is connected in series with a parallel circuit comprising two resistors 12 Ω and 8 Ω respectively. Total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Calculate the value of the resistor R.

Solution



Total current taken = 70 / 22 = 3.1818 A

Equivalent of 12 Ω ‖ 8 Ω = 96/20 = 4.8 Ω

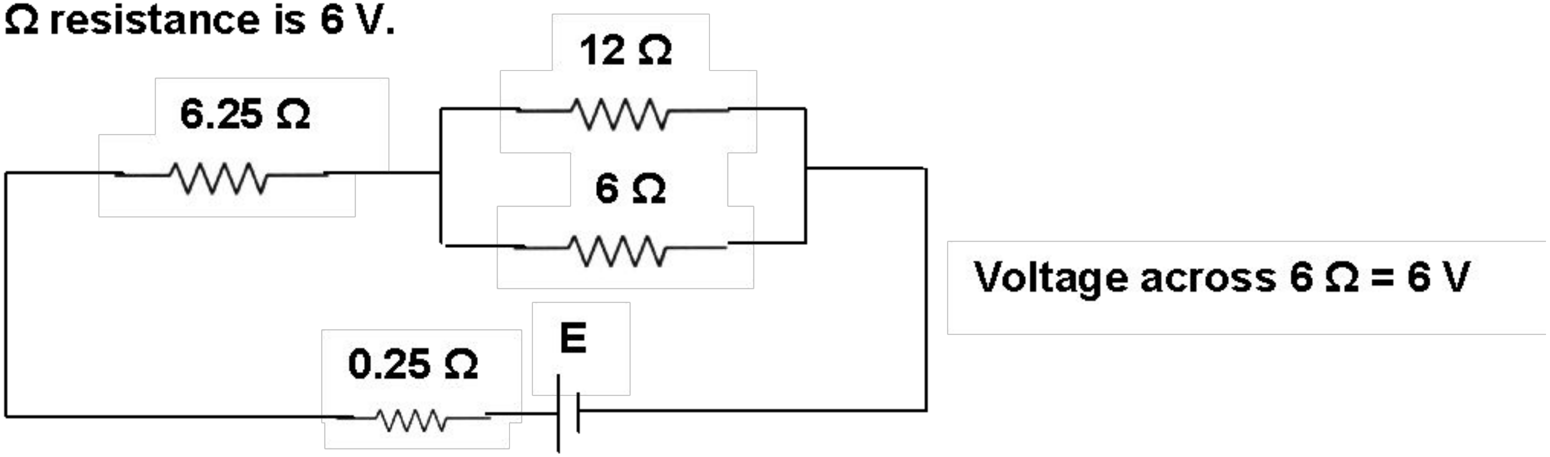
Voltage across parallel combination = 4.8 x 3.1818 = 15.2726 V

Voltage across resistor R = 22 – 15.2726 = 6.7274 V

Value of resistor R = 6.7274/3.1818 = 2.1143 Ω

Example 8

The resistors 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance and a battery which has an internal resistance of 0.25 Ω. Determine the emf of the battery if the potential difference across 6 Ω resistance is 6 V.



Solution

Current in 6 Ω = 6/6 = 1 A

Current in 12 Ω = 6/12 = 0.5 A

Therefore current in 25 Ω = 1.0 + 0.5 = 1.5 A

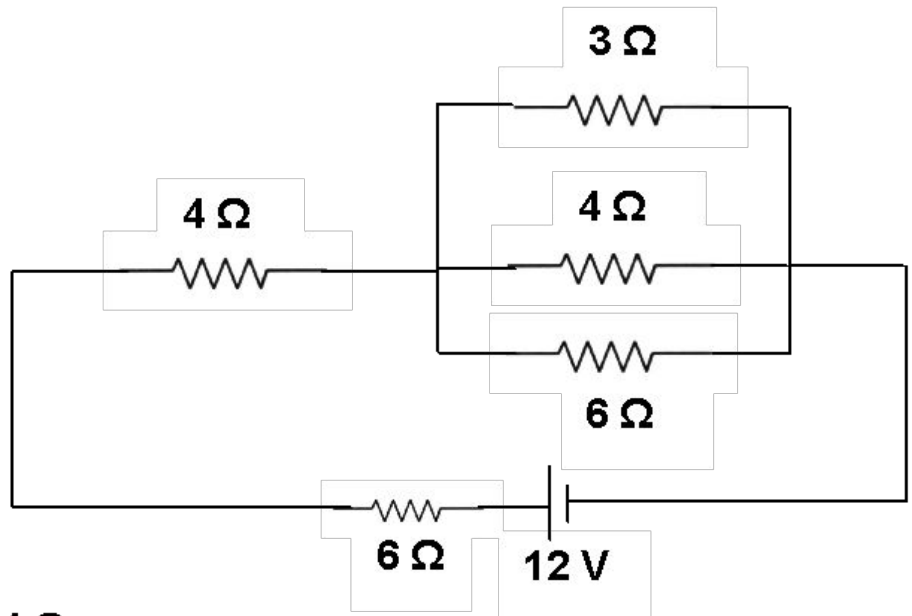
Using KVL   E = (0.25 x 1.5) + (6.25 x 1.5) + 6 = 15.75 V

Therefore battery emf E = 15.75 V

Example 9

A circuit consist of three resistors 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of 12 V and an internal resistance of 6 Ω is connected across the circuit. Find the total current in the circuit and the terminal voltage across the battery.

Solution



$4\ \Omega \parallel 6\ \Omega = 24/10 = 2.4\ \Omega$

$1.4\ \Omega \parallel 3\ \Omega = 7.2/5.4 = 1.3333\ \Omega$

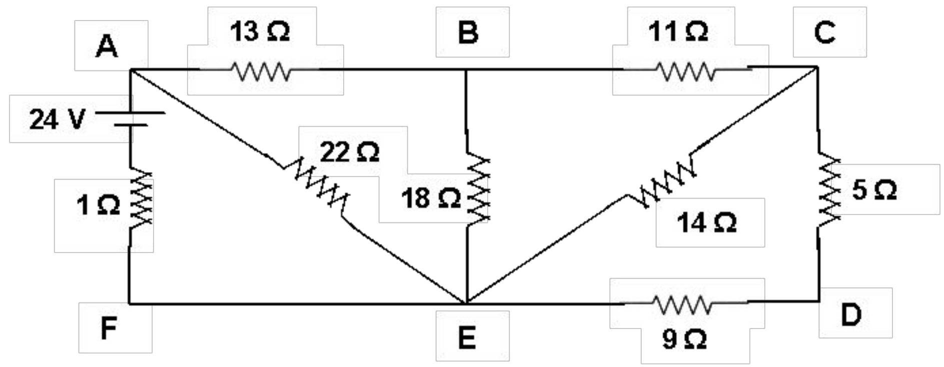
Total circuit resistance = 4 + 6 + 1.3333 = 11.3333 Ω

Circuit current = 12/11.3333 = 1.0588 A

Terminal voltage across the battery = 12 – (6 x 1.0588) = 5.6472 V

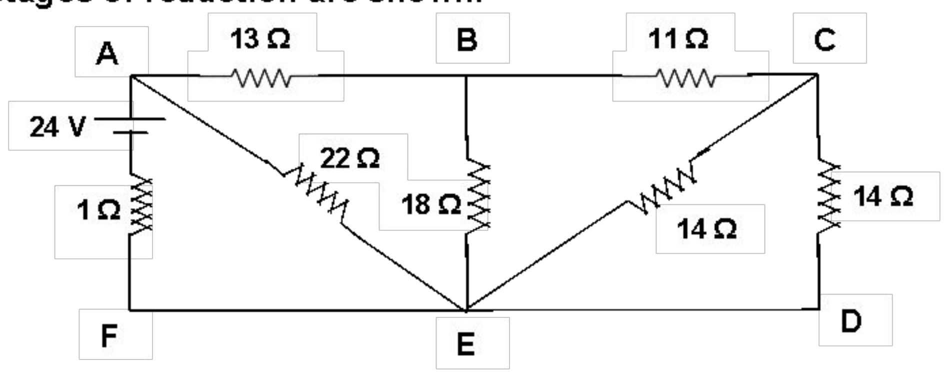
Example 10

An electrical network is arranged as shown. Find (i) the current in branch AF (ii) the power absorbed in branch BE and (iii) potential difference across the branch CD.



Solution

Various stages of reduction are shown.



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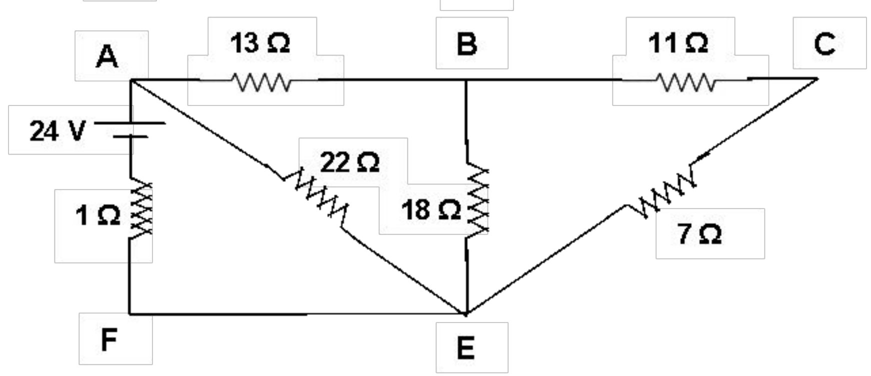

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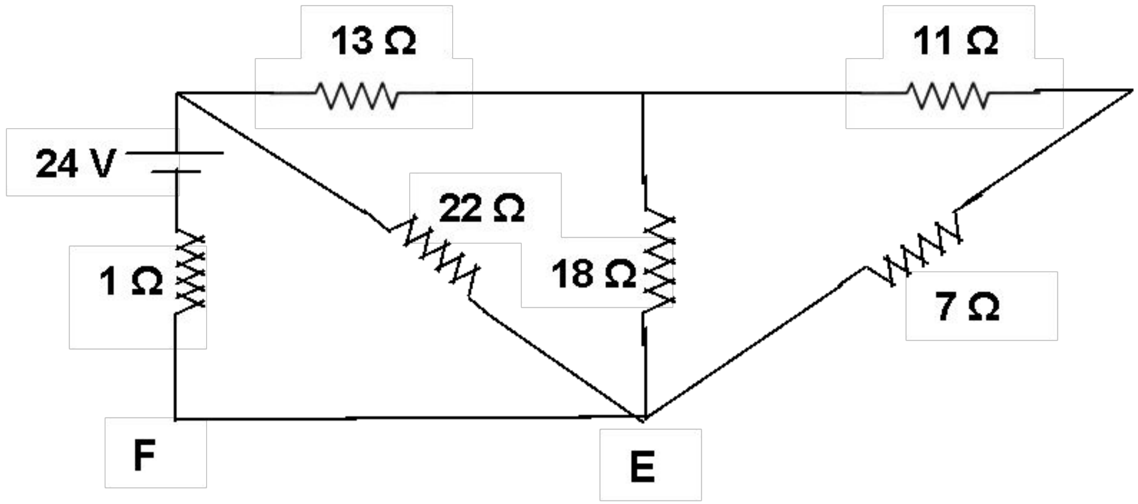

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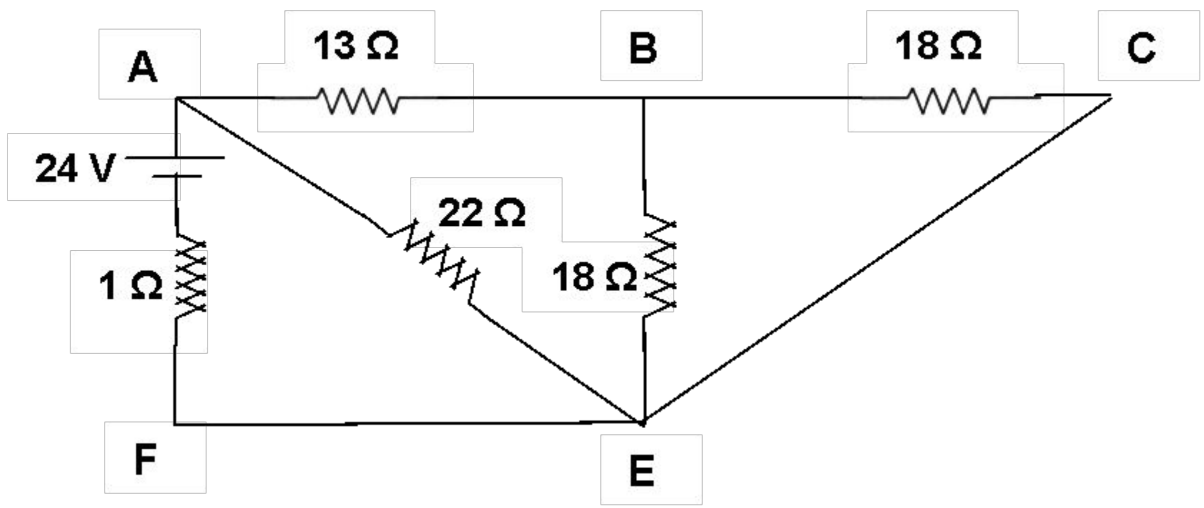
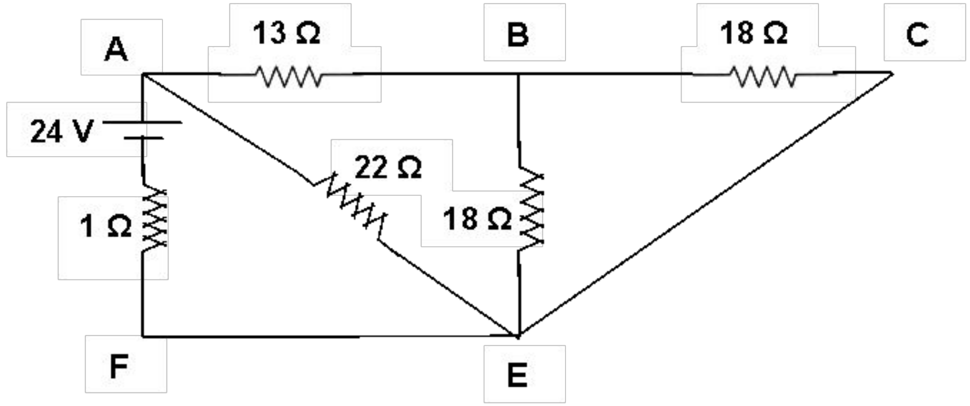
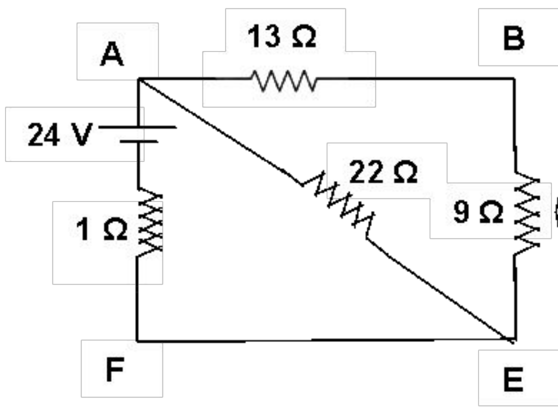


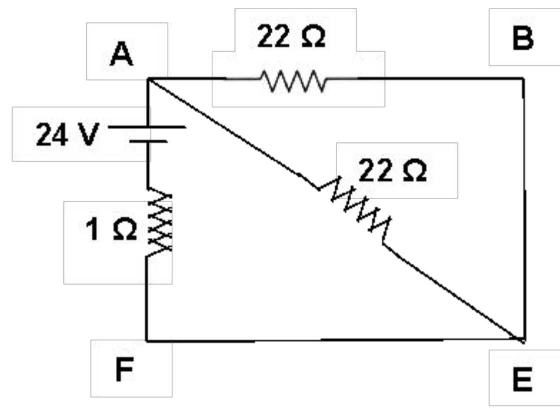
3


A
13 Ω
B
18 Ω
C
24 V
22 Ω
18 Ω
1 Ω
F
E

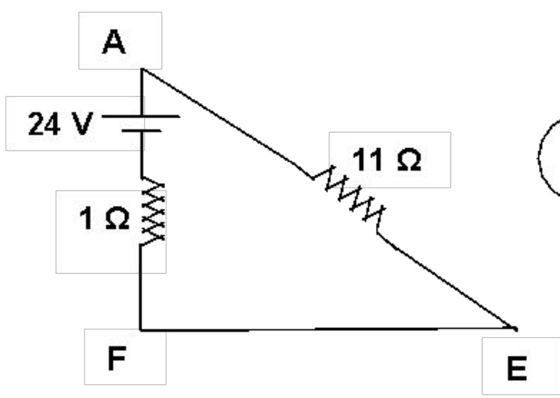
3


A
13 Ω
B
24 V
22 Ω
9 Ω
1 Ω
F
E

4


A
22 Ω
B
24 V
22 Ω
1 Ω
F
E

5


A
24 V
11 Ω
1 Ω
F
E

6

Current in branch AF = 24/12 = 2 A from F to A

Using current division rule current in 13 Ω in Fig. 4= 1 A

Referring Fig. 3, current in branch BE = 0.5 A

Power absorbed in branch BE = 0.5^2 x 18 = 4.5 W

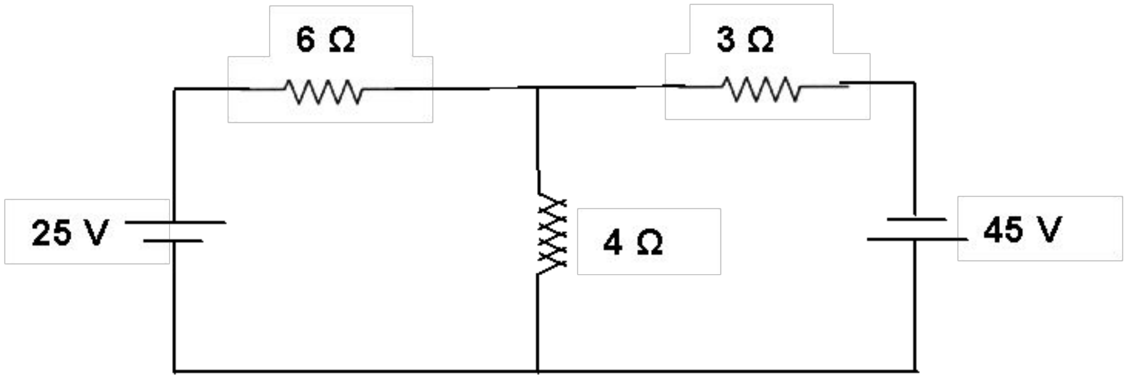
Voltage across BE = 0.5 x 18 = 9 V

Voltage across CE in Fig. 1 = $\frac{7}{18} \times 9 = 3.5$ V

Referring Fig. given in the problem, using voltage division rule, voltage across in branch CD = $\frac{5}{14} \times 3.5 = 1.25$ V

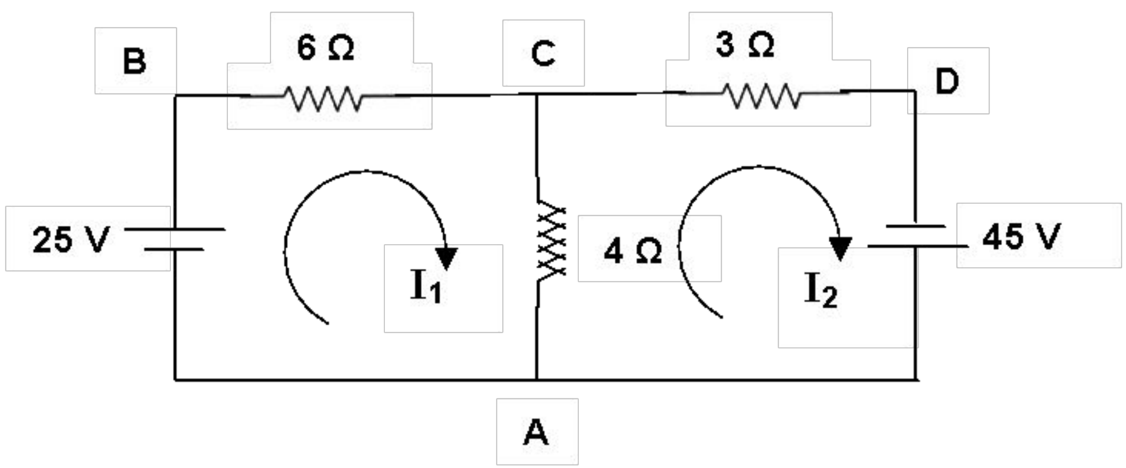
Example 11

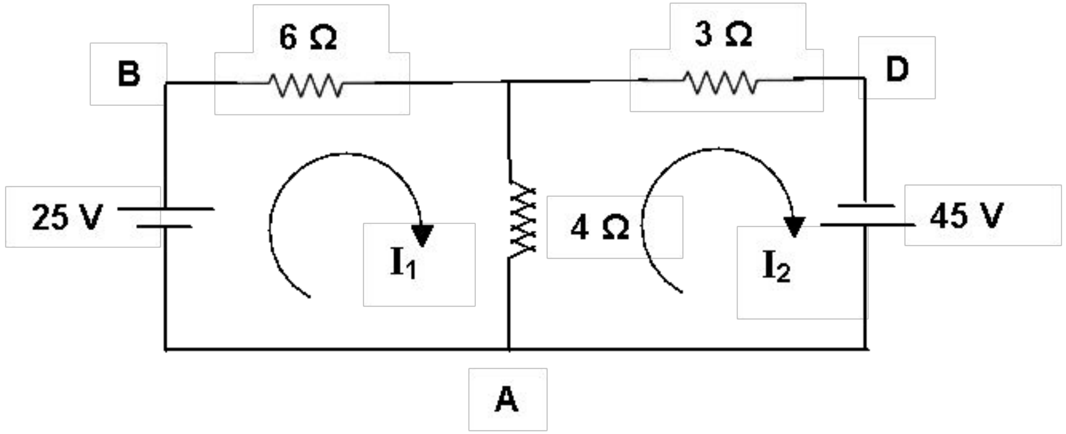
Using Kirchhoff's laws, find the current in various resistors in the circuit shown.



Solution

Let the loop current be I_1 and I_2





Considering the loop ABCA, KVL yields

$6\ I_1 + 4\ (I_1 - I_2) - 25 = 0$

For the loop CDAC, KVL yields

$3\ I_2 - 45 + 4\ (I_2 - I_1) = 0$

Thus $10\ I_1 - 4\ I_2 = 25$

$-4\ I_1 + 7\ I_2 = 45$

On solving the above $I_1 = 6.574$ A; $I_2 = 10.1852$ A

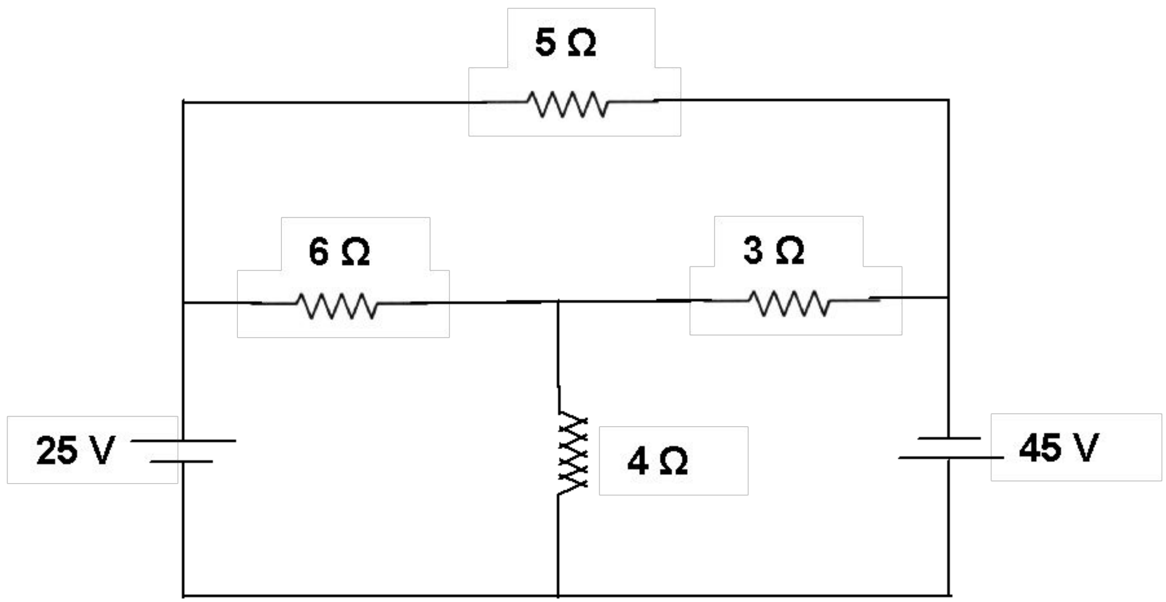
Current in 4Ω resistor = $I_1 - I_2 = 6.574 - 10.1852 = -3.6112$ A

Thus the current in 4Ω resistor is 3.6112 A from A to C

Current in 6 Ω resistor = 6.574 A; Current in 3 Ω resistor = 10.1852 A

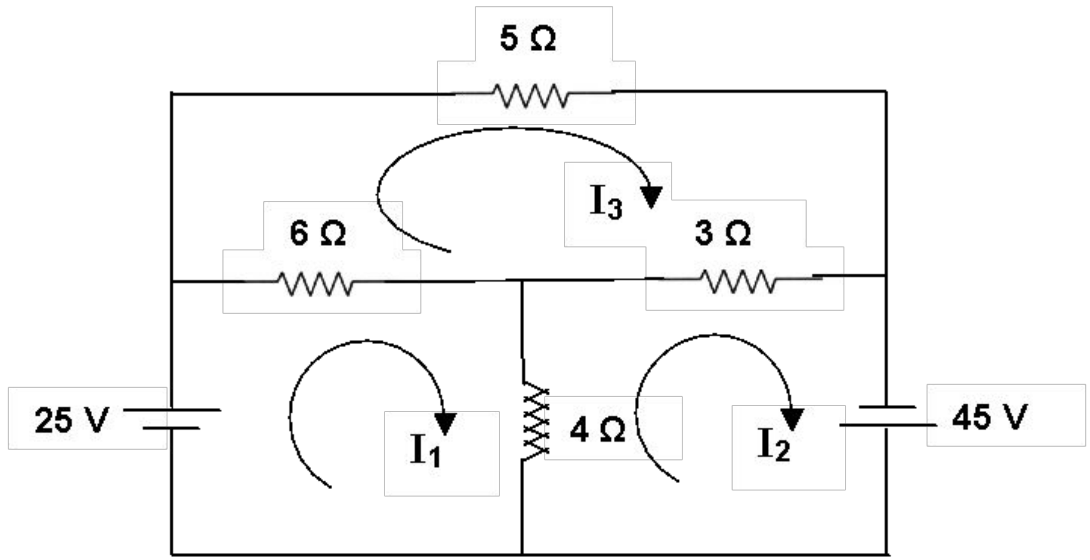
Example 12

Find the current in 5 Ω resistor in the circuit shown.



Solution

Let the loop current be I_1, I_2 and I_3.



Three loops equations are:

$6\,(I_1 - I_3) + 4\,(I_1 - I_2) - 25 = 0$

$4\,(I_2 - I_1) + 3\,(I_2 - I_3) - 45 = 0$

$5\,I_3 + 3\,(I_3 - I_2) + 6\,(I_3 - I_1) = 0$

On solving

Current in 5 Ω resistor, $I_3 = 14$ A

CIRCUIT THEOREMS

THEVENIN'S THEOREM

In many practical applications, we may not be interested in getting the complete analysis of the circuit, namely finding the current through all the elements and voltages across all the elements. We may be interested to know the details of a portion of the circuit; as a special case **it may be a single element such as load impedance.** In such a situation it is very convenient to use Thevenin's theorem to get the solution.

Fig. illustrates the Thevenin's equivalent of sub-circuit A.

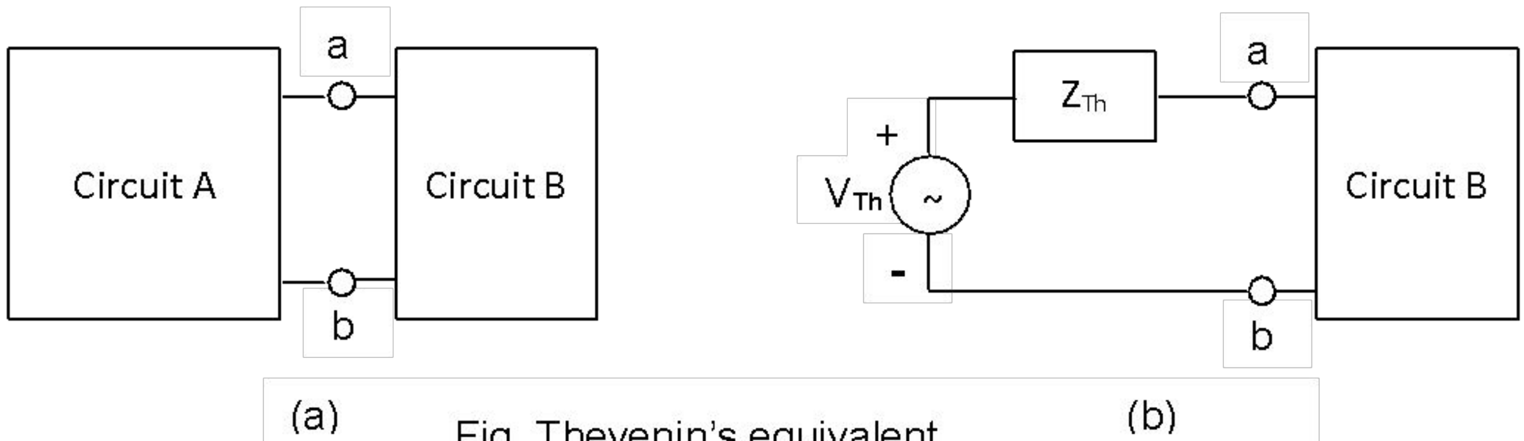
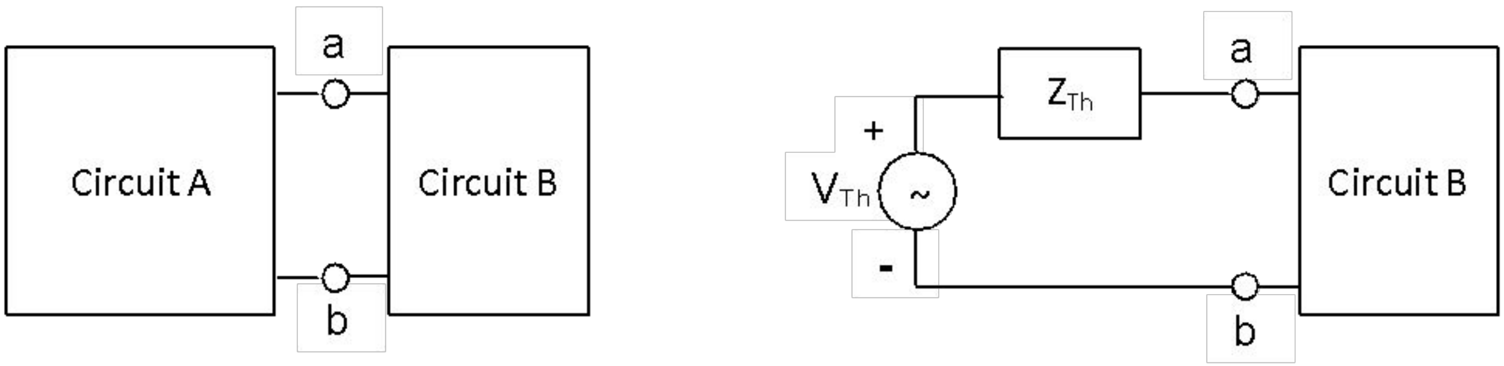


Fig. Thevenin's equivalent.



(a) Fig. Thevenin's equivalent. (b)

In Fig. (a) a circuit partitioned into two parts, namely circuit A and circuit B, is shown. They are connected by a single pair of terminals. In Fig.(b) circuit A is replaced by Thevenin's equivalent circuit, which consists of a voltage source V_{Th} in series with an impedance Z_{Th}.

To obtain the Thevenin's equivalent circuit, we need to find Thevenin's voltage V_{th} and Thevenin's impedance Z_{Th}. Unique procedure is available to find the Thevenin's voltage V_{Th}. When we need the Thevenin's voltage of circuit A, measure or calculate the **OPEN CIRCUIT VOLTAGE of circuit A.** This will be the Thevenin's voltage.

Thevenin's impedance can be calculated in three different ways **depending on the nature of voltage and current sources** in the circuit of our interest.

The circuit for which Thevenin's impedance is to be calculated consists of impedances and **one or more independent sources.** That is, **the circuit does not contain any dependent source**. To determine Thevenin's impedance, circuit shown in Fig. (b) is to be used.

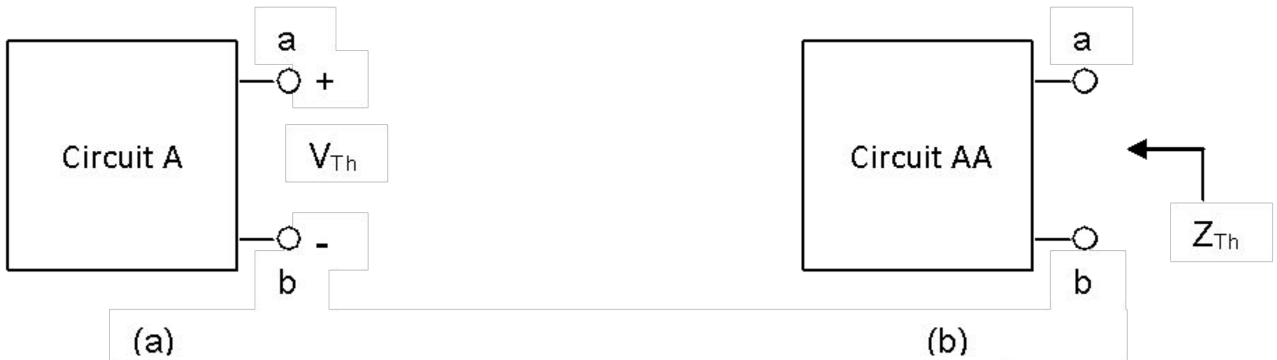


Fig. Determining Thevenin's equivalents.

The circuit AA in Fig. (b) is obtained from circuit A by replacing all the independent voltage sources by short circuits and replacing all independent current sources by open circuits. Thus in circuit AA, all the independent sources are set to zero. Then, Thevenin's impedance is the equivalent circuit impedance of circuit AA which can be obtained using reduction techniques.

The methods of finding the Thevenin's impedance depend on the nature of the circuit for which the Thevenin's equivalent is sought for. These methods are summarized below:

Circuit with independent sources only - ANY ONE OF THE FOLLOWING

1. **Make independent sources zeros and use reduction techniques to find Z_{Th}.**

2. Short circuit terminals a and b and find the short circuit current I_{sc} flowing from *a* to *b*. Then $Z_{Th} = V_{Th} / I_{sc}$

3. Set all independent sources to zero. Apply 1 V across the open circuited terminals *a*-*b* and determine the source current I_s entering the circuit through *a.* Then $Z_{Th} = 1 / I_s$. Alternatively introduce a current source of 1 A from *b* to *a* and determine the voltage V_{ab}. Then, Thevenin's impedance $Z_{Th} = V_{ab}$.

Example 1

Find the Thevenin's voltage with respect to the load resistor R_L in circuit shown in Fig.

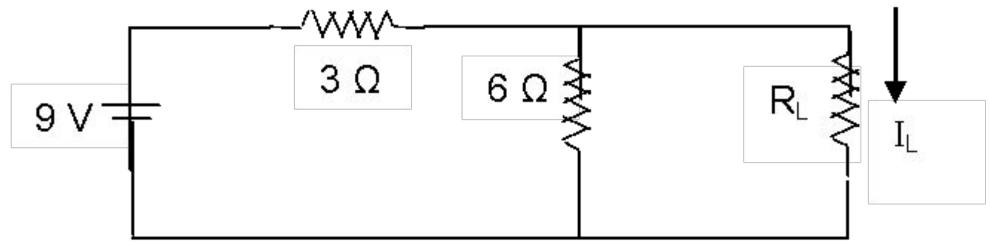
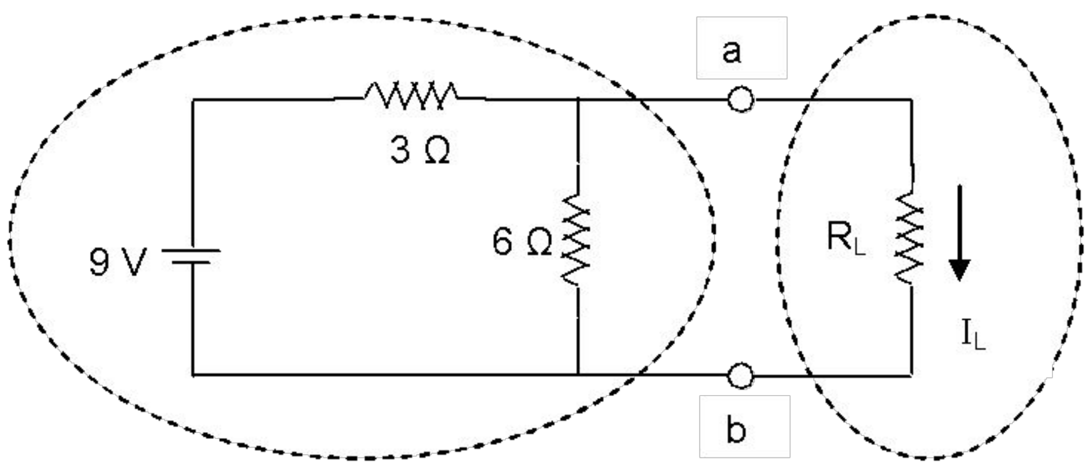


Fig. Circuit for Example1

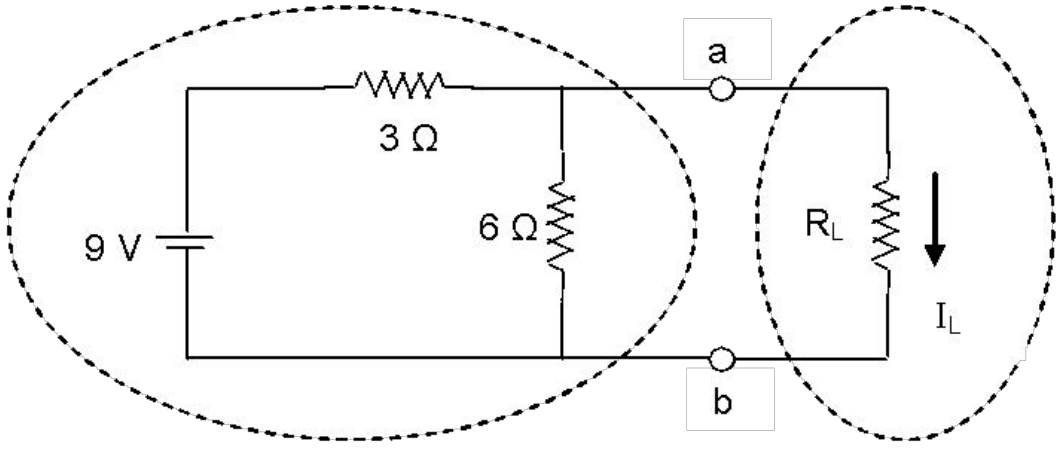
Solution

The given circuit can be divided into two circuits as shown in Fig.



Circuit A

Circuit B



Thevenin's voltage of circuit A can be obtained from the circuit shown in Fig.

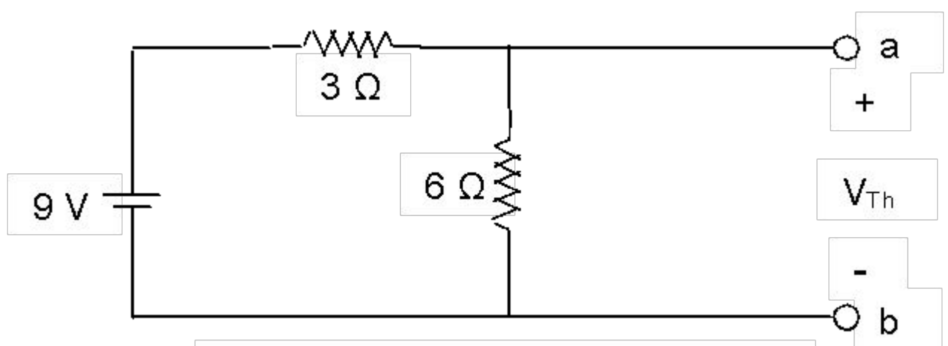


Fig. Circuit.

Using voltage division rule $V_{Th} = V_{6\Omega} = \frac{6}{9} \times 9 = 6\,V$

Example 2

Obtain the Thevenin's equivalent for the circuit shown in Fig.

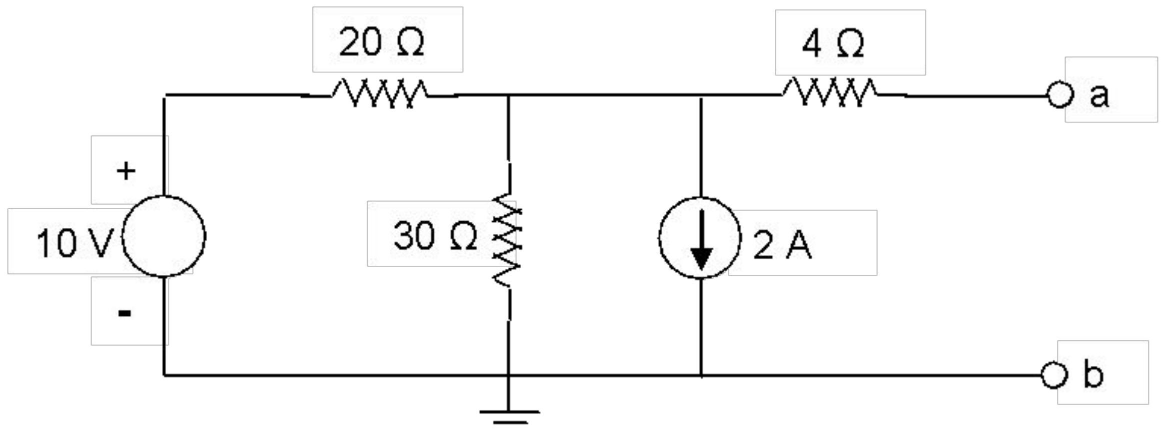


Fig. Circuit for Example 2.

Solution:

Open circuit voltage V_{ab} is the Thevenin's voltage V_{Th}.

To find Thevenin's voltage:

Note that there is no current flow in resistor of 4 Ω. Therefore, voltage V_{Th} is same as the voltage across 30 Ω resistor. Then, the node voltage equation is

$$\frac{V_{Th} - 10}{20} + \frac{V_{Th}}{30} + 2 = 0$$

On solving this, we get V_{Th} = - 18 V

To find Thevenin's impedance: Since the circuit has only independent sources, it falls under case 1

Reducing the sources to zero, the resulting circuit is shown in Fig.

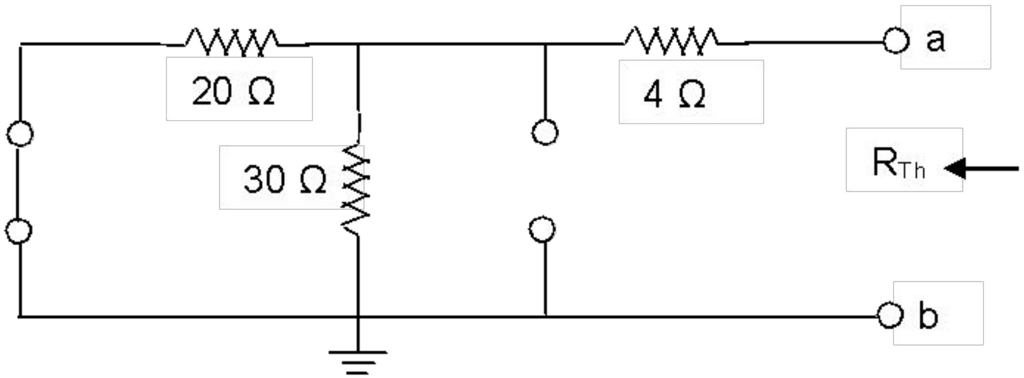


Fig. Circuit - Example 2.

Thus $R_{Th} = 4 + 20 || 30 = 16\ \Omega$ Thevenin's equivalent circuit is shown in Fig.

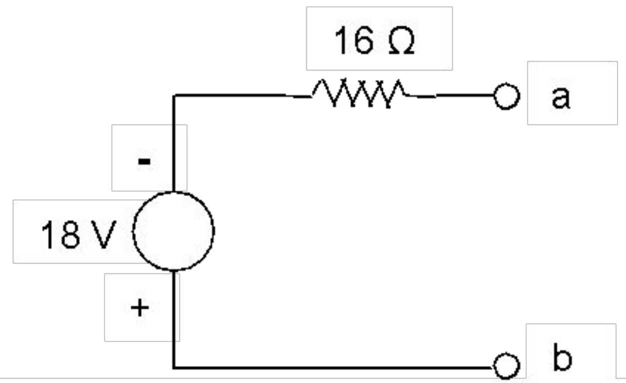
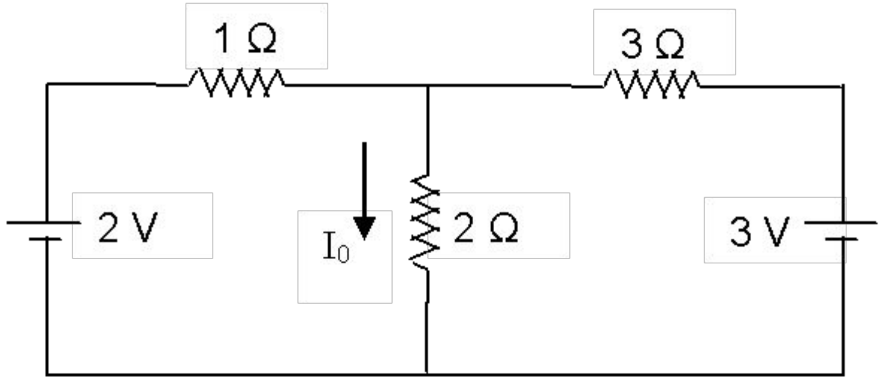


Fig. Thevenin's equivalent circuit – Example 2.

R_{Th} can be obtained by two other methods also

Example 3 Using Thevenin's equivalent circuit, calculate the current I_0 through the 2 Ω resistor in the circuit shown below.



Solution: Circuit by which V_{Th} and R_{Th} can be calculated are shown in Fig.

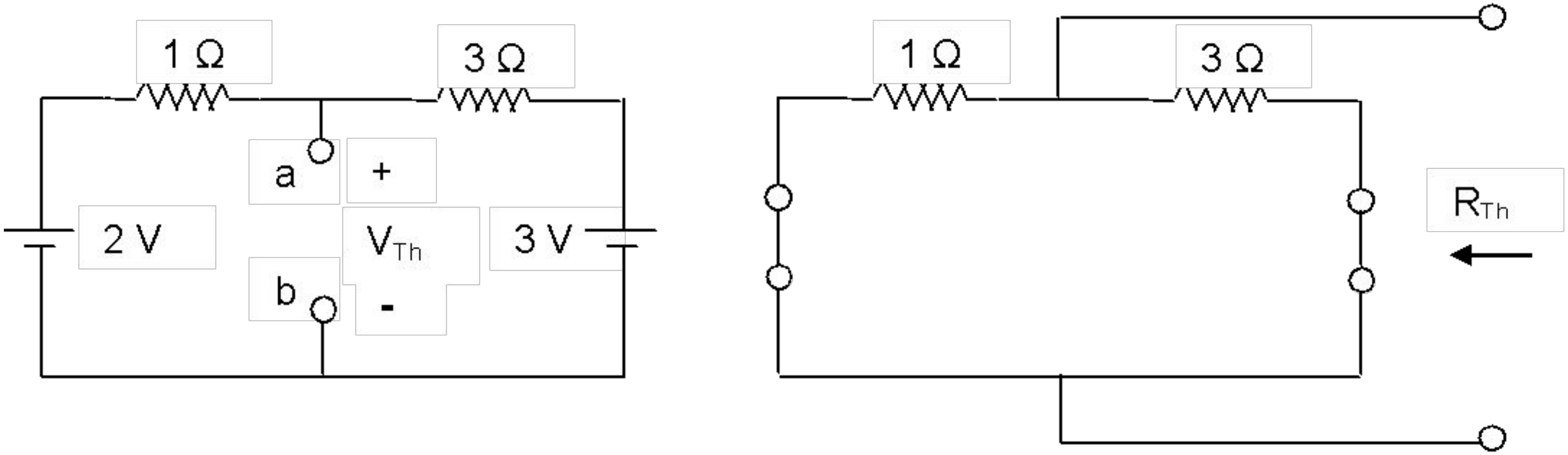
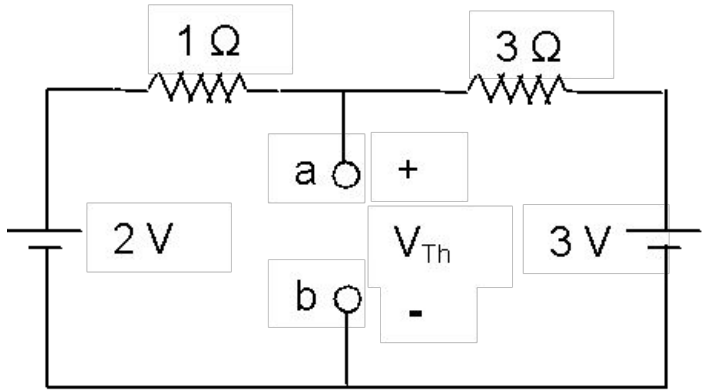
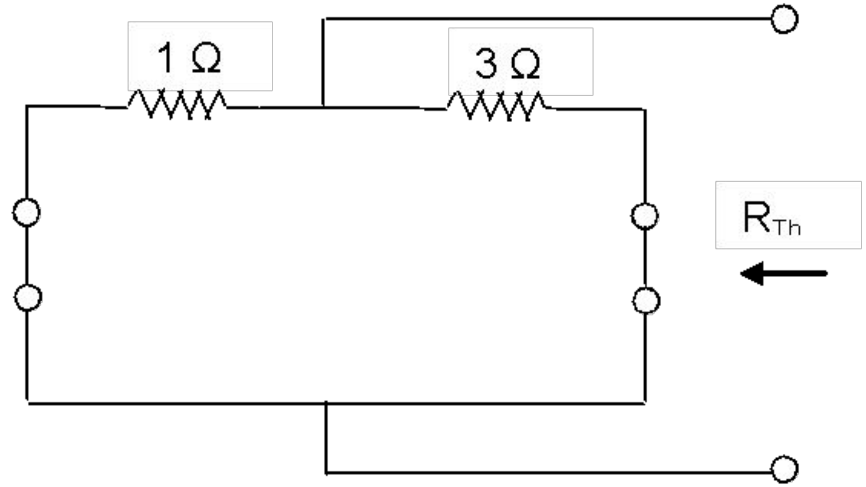


Fig. Circuits for V_{Th} and R_{Th} - Example 3.

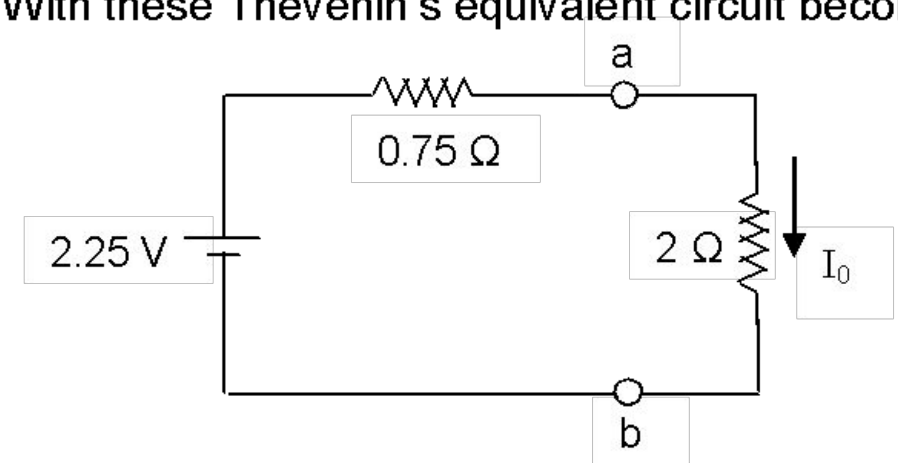




Knowing the anticlockwise current as 0.25 A

$- 2 - (1 \times 0.25) + V_{Th} = 0$. i.e. $V_{Th} = 2.25$ V; Also $R_{Th} = 1 || 3 = 0.75\ \Omega$

With these Thevenin's equivalent circuit becomes



Current $I_0 = 2.25 / 2.75 = 0.8182$ A

NORTON'S THEOREM

Much similar to Thevenin's theorem, Norton's theorem is also used to obtain the equivalent of two terminal sub-circuit.

Fig. illustrates the Norton's equivalent of sub-circuit A.

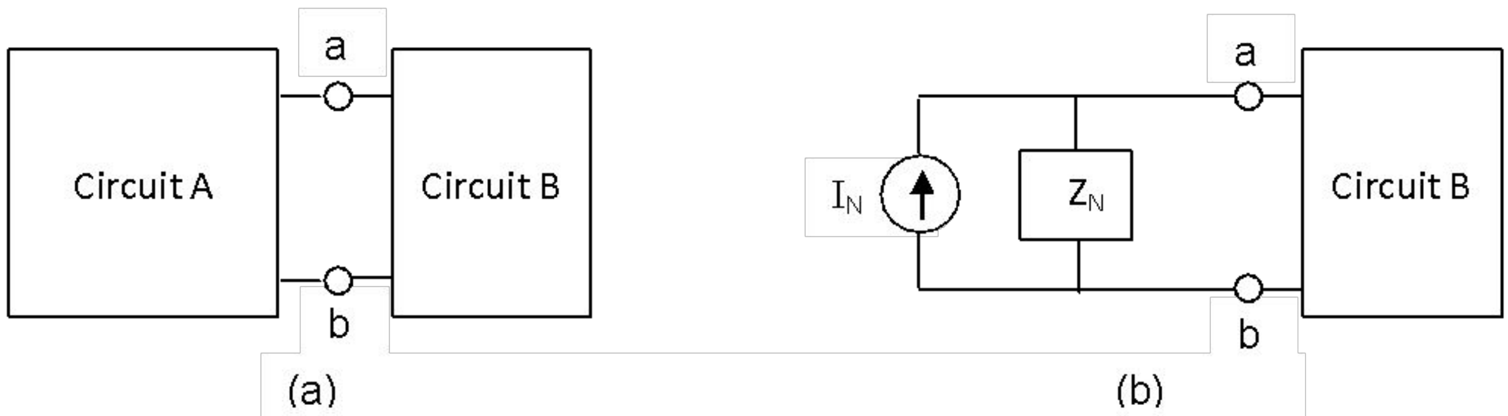


Fig. Norton's equivalent.

In Fig. (a) a circuit partitioned into two parts, namely circuit A and circuit B, is shown. They are connected by a single pair of terminals. In Fig. (b), circuit A is replaced by Norton's equivalent circuit, which consists of a current source I_N in parallel with an impedance Z_N.

Looking at the Thevenin's and Norton's equivalents shown in Fig. (a) and (b), it is clear that one can be obtained from the other through source transformation.

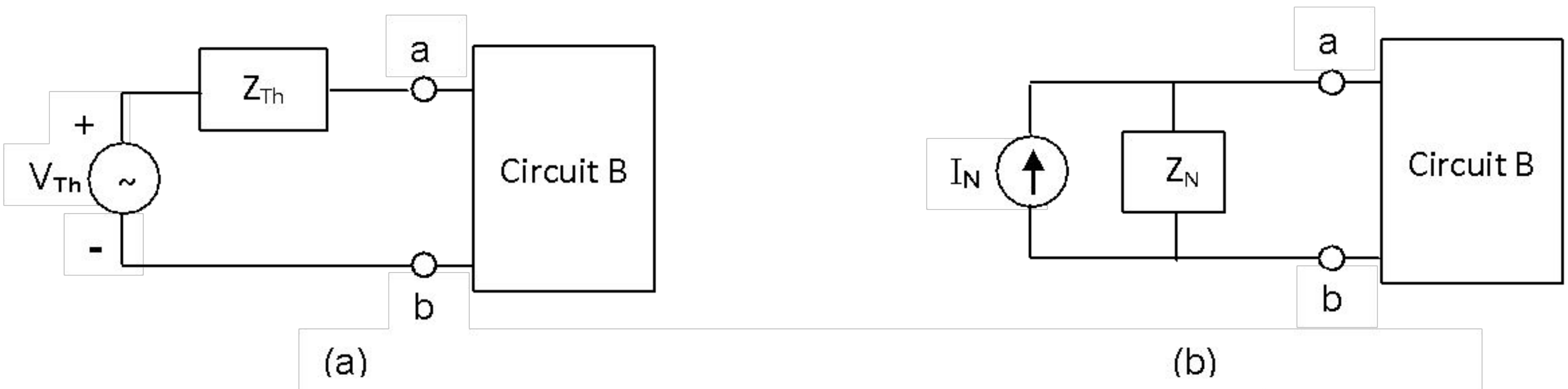


Fig. Thevenin's and Norton's equivalents.

It is to be noted that

$$Z_N = Z_{Th}$$

$$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{V_{Th}}{Z_N}$$

To obtain Norton's equivalent circuit, we need to find current I_N and the impedance Z_N. They can be obtained from Thevenin's voltage and impedance.

Otherwise Norton's current can be obtained by finding the short circuit current as indicated in Fig.

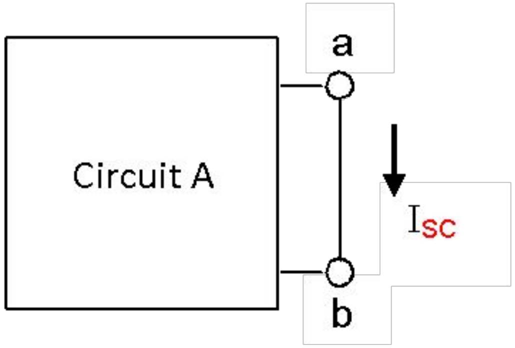


Fig. Getting short circuit current.

It is to be noted that the short circuit current is from terminal *a* to terminal *b* while Norton's current is from terminal *b* to terminal *a*.

The impedance Z_N can be got exactly same way we got Z_{Th} as discussed in previous section except that the method indicated under Case 2 is not applicable as it requires the value of V_{Th}.

Example 1

Using Norton's theorem, determine the current through the resistor R_L when $R_L = 0.7$, 1.2 and 1.6 Ω in the circuit shown in Fig.

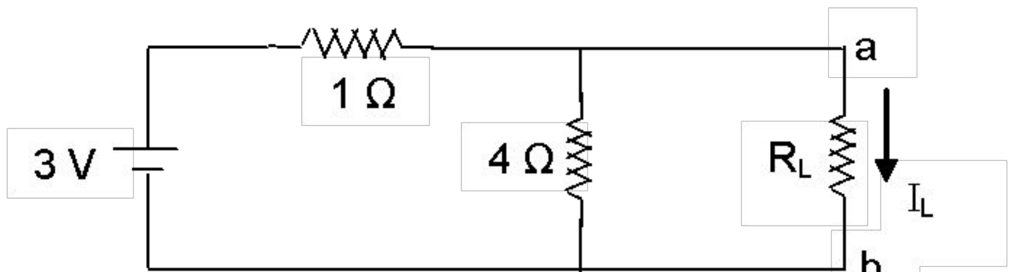


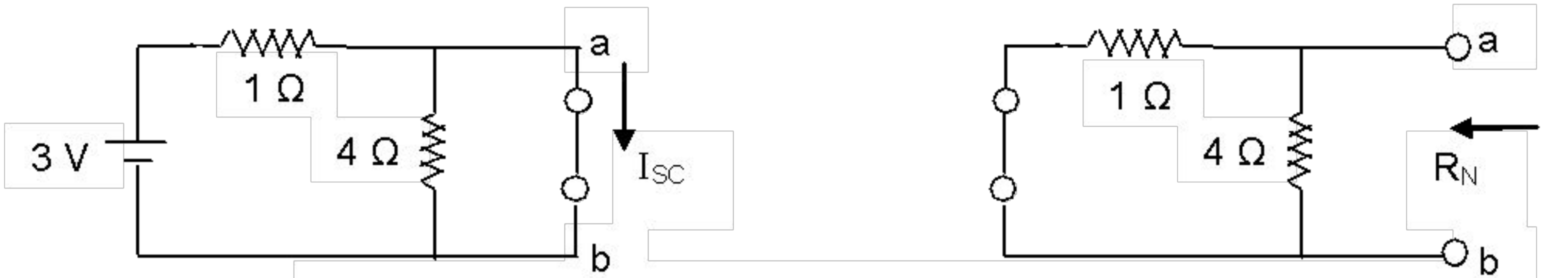
Fig. Circuit for Example 1.

Solution:

Circuits to determine I_{SC} and R_N are shown in Fig. (a) and (b).

(a) (b)

Fig. Short circuit current and Norton's resistance.



(a) (b)

It is to be noted that since there is a short circuit parallel to 4 Ω no current flows in it.

Norton's current $I_N = 3$ A; Norton's resistance $R_N = 1||4 = 0.8\ \Omega$

Norton's equivalent circuit is shown in Fig.

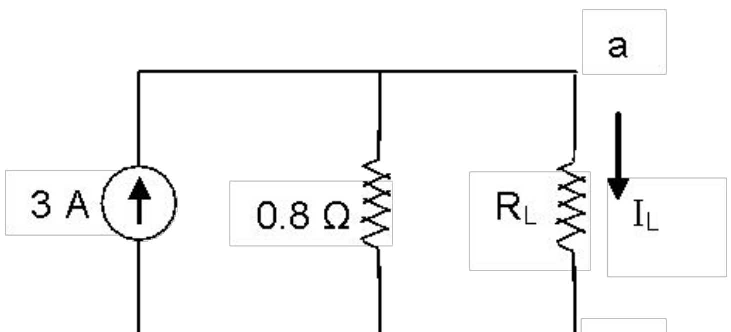


Fig. Norton's equivalent.

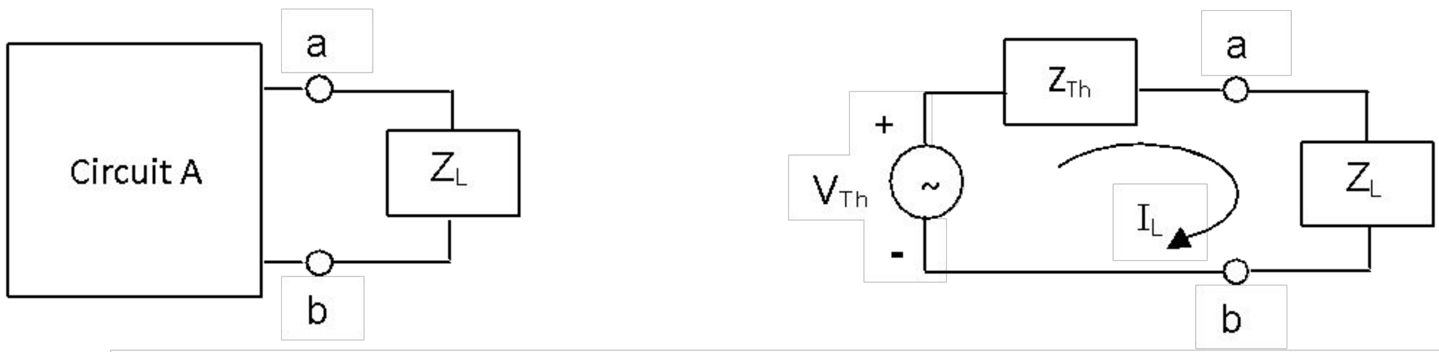
R_N can be obtained by another method also.

When $R_L = 0.7\ \Omega$, $I_L = (0.8 / 1.5) \times 3 = 1.6$ A; When $R_L = 1.2\ \Omega$, $I_L = (0.8 / 2) \times 3 = 1.2$ A

When $R_L = 1.6\ \Omega$, $I_L = (0.8 / 2.4) \times 3 = 1.0$ A

MAXIMUM POWER TRANSFER THEOREM

There are some applications wherein maximum power needs to be transferred to the load connected. Consider a linear ac circuit *A*, connected to a load of impedance Z_L as shown in Fig. (a). It is required to transfer maximum real power to the load. The circuit A can be replaced by its Thevenin's equivalent as shown in Fig. (b).



(a) (b)

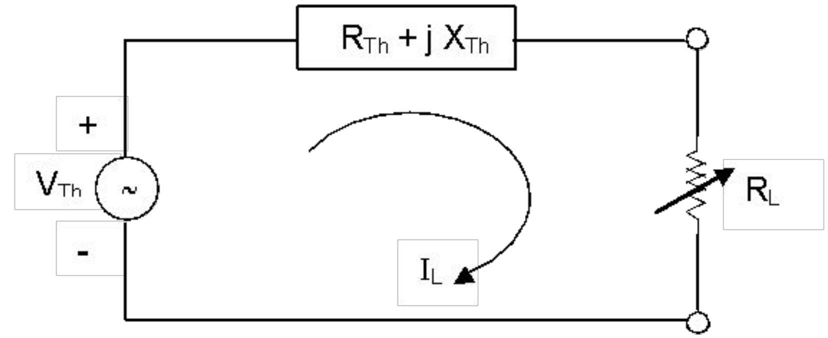
Fig. Maximum power transfer theorem - Illustration.

Let $Z_{Th} = (R_{Th} + j X_{Th})$ and $Z_L = (R_L + j X_L)$

The following maximum power transfer theorems determine the values of load impedance Z_L for which maximum real power is transferred to the load impedance.

Case 1:

Load is a variable resistance R_L



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + jX_{Th}}$ This gives $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + X_{Th}^2}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + X_{Th}^2}$

This can be written as $P_L = \dfrac{|V_{Th}|^2}{\dfrac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^2}{R_L}}$

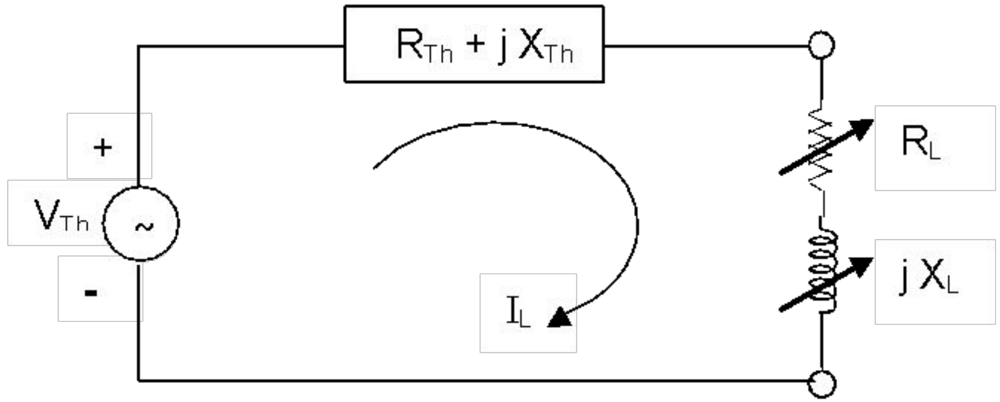
For power P_L to be maximum, $\dfrac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^2}{R_L}$ must be minimum. Thus power P_L will be maximum when

$\dfrac{d}{dR_L}\left(\dfrac{R_{Th}^2}{R_L} + 2R_{Th} + R_L + \dfrac{X_{Th}^2}{R_L}\right) = 0$ i.e. when $-\dfrac{R_{Th}^2}{R_L^2} + 1 - \dfrac{X_{Th}^2}{R_L^2} = 0$

i.e. when $R_L^2 = R_{Th}^2 + X_{Th}^2$ i.e. when $R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$

Using this value of R_L, the current I_L and hence maximum power can be computed.

Case 2 In the load impedance, R_L and X_L are varied independently as shown.



Load current $I_L = \dfrac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + jX_L)}$ Thus $|I_L| = \dfrac{|V_{Th}|}{\sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}}$

Real power delivered to the load $P_L = |I_L|^2 R_L = \dfrac{|V_{Th}|^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$

If R_L in Eq. is held fixed, the value of P will be maximum when $(X_{Th} + X_L)^2$ is minimum. This will occur when $X_{Th} + X_L = 0$ i.e. when

$X_L = - X_{Th}$

Keeping $X_L = - X_{Th}$ Eq. becomes

$$P_L = \frac{|V_{Th}|^2 \; R_L}{(R_{Th} + R_L)^2} = \frac{|V_{Th}|^2}{\dfrac{R_{Th}^{\;2}}{R_L} + 2R_{Th} + R_L}$$

For P_L given by Eq. to become maximum, $\dfrac{R_{Th}^{\;2}}{R_L} + 2R_{Th} + R_L$ must be minimum. This will occur when $\dfrac{d}{dR_L}\left(\dfrac{R_{Th}^{\;2}}{R_L} + 2R_{Th} + R_L\right) = 0$ i.e. when $-\dfrac{R_{Th}^{\;2}}{R_L^{\;2}} + 1 = 0$ i.e. when

$R_L = R_{Th}$

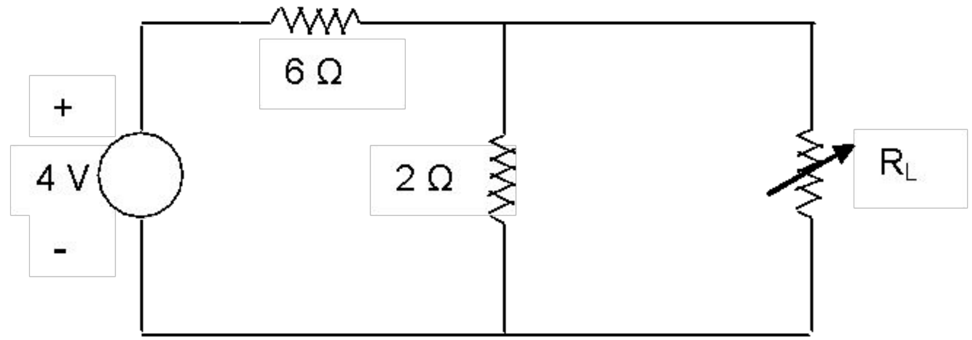
Combining Eqs. and, we can state that real power transferred to the load will be maximum when

$Z_L = R_{Th} - j\,X_{Th} = Z_{Th}^{\;*}$

Setting $R_L = R_{Th}$ and $X_L = -\,X_{Th}$ in Eq., maximum real power can be obtained as

$$P_{max} = \frac{|V_{Th}|^2}{4\,R_{Th}}$$

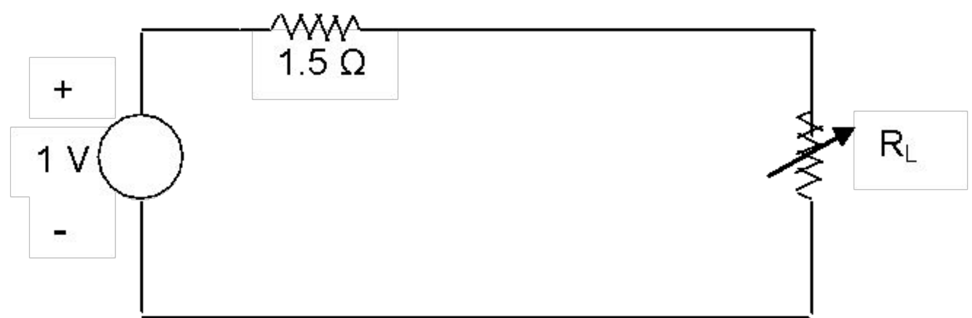
Example 1 Consider the circuit shown below. Determine the value of R_L when it is dissipating maximum power. Also find the value of maximum power dissipated.



Solution:

As a first step, Thevenin's equivalent across the load resistor is obtained.

$V_{Th} = \frac{2}{2+6} \times 4 = 1\,V$; $R_{Th} = 6||2 = 1.5\,\Omega$ Resulting circuit is shown.



For P_L to be maximum, $R_L = 1.5\,\Omega$; Then circuit current = 1/3 = 0.3333 A

Maximum power dissipated $P_{max} = 0.3333^2 \times 1.5 = 0.16667$ W

SUPERPOSITION THEOREM

The idea of superposition rests on the linearity property. Superposition theorem is applicable to linear circuits having two or more independent sources.

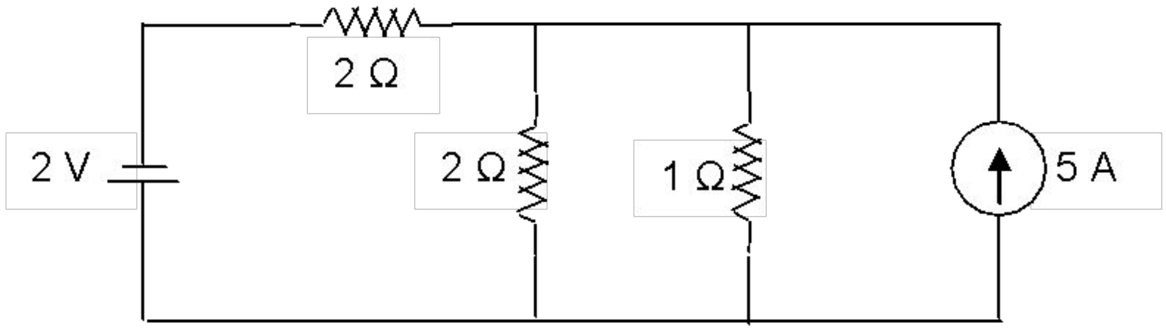
In a linear circuit having two or more independent sources, total response in an element (voltage across the element or current through the element) is equal to the algebraic sum of responses in that element due to each source applied separately while the other sources are reduced to zero.

To make a current source to zero, it must be open circuited. Similarly, **if any voltage source is to be made zero, it must be short circuited.** When this theorem is used in circuit with initial conditions, they are to be treated as sources. Further, dependent sources if any are left intact because they are controlled by circuit variables.

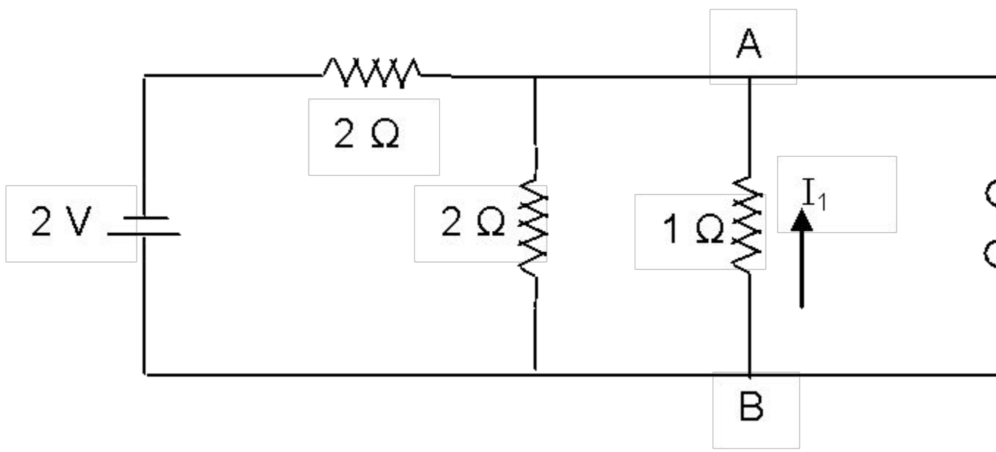
One disadvantage of analyzing a circuit using Superposition theorem is that it involves more calculations. If the circuit has three independent sources, we need to solve three simpler circuits each having only one independent source. However, when the circuit has only one independent source, several short-cut techniques can be readily applied to get the solution.

Major advantage of Superposition theorem is that it can be used to solve ac circuit having more than one source with **different frequencies.** In such case, solution in time frame is obtained corresponding to each source and added up to get the total solution.

Example 1 Calculate the current through the 1 Ω resistor in the circuit shown below.

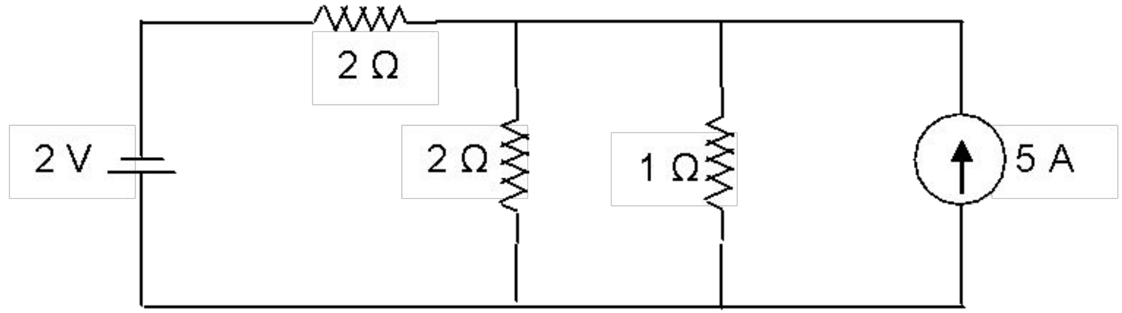


Solution: First calculate current I_1 due to voltage source alone. The current source is open circuited. The resulting circuit is shown below.



Total circuit resistance $R_T = 2.6667\ \Omega$. Circuit current $I_T = \dfrac{2}{2.6667} = 0.75$ A

$$\text{Current } I_1 = \frac{2}{3} \times 0.75 = 0.5 \text{ A from B to A}$$



Now calculate current I_2 due to current source alone. The voltage source is short circuited as shown in Fig.

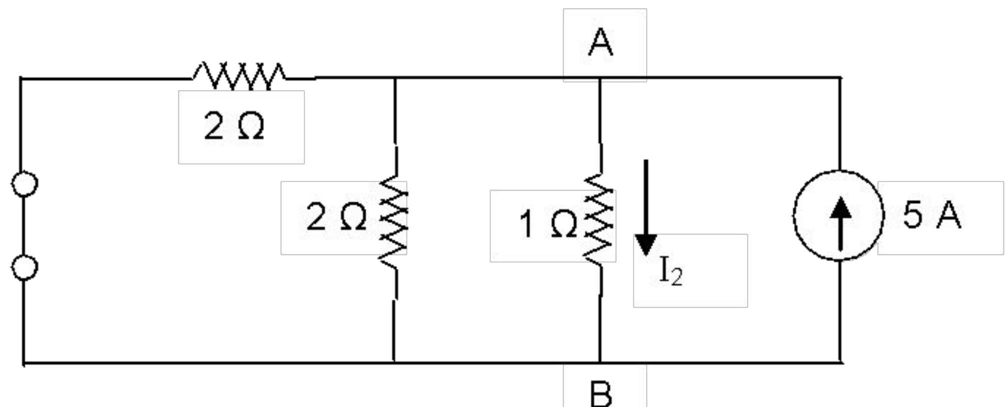


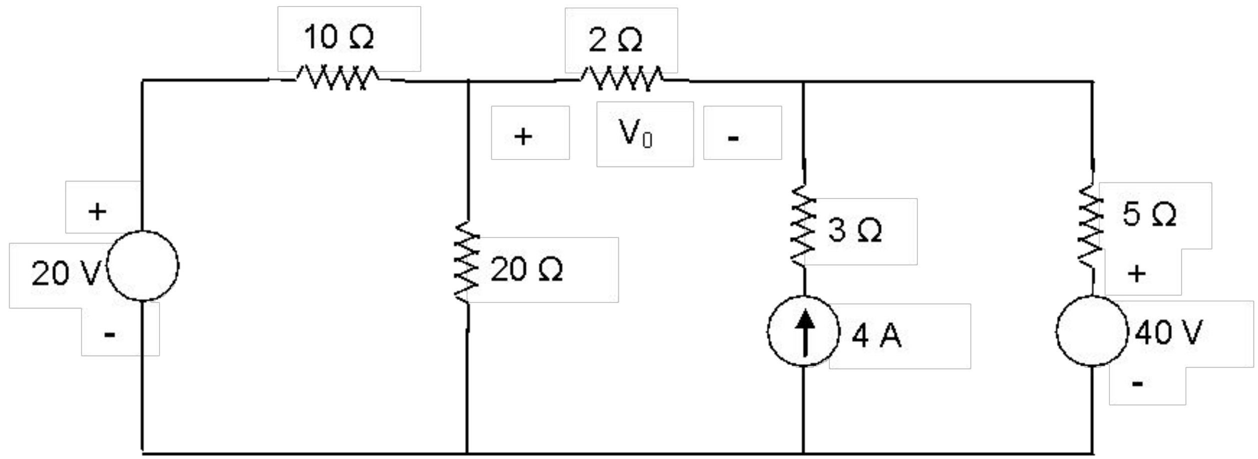
Fig. Circuit - Example 1

Noting that two 2 Ω resistors are in parallel, current I_2 = 2.5 A from A to B.

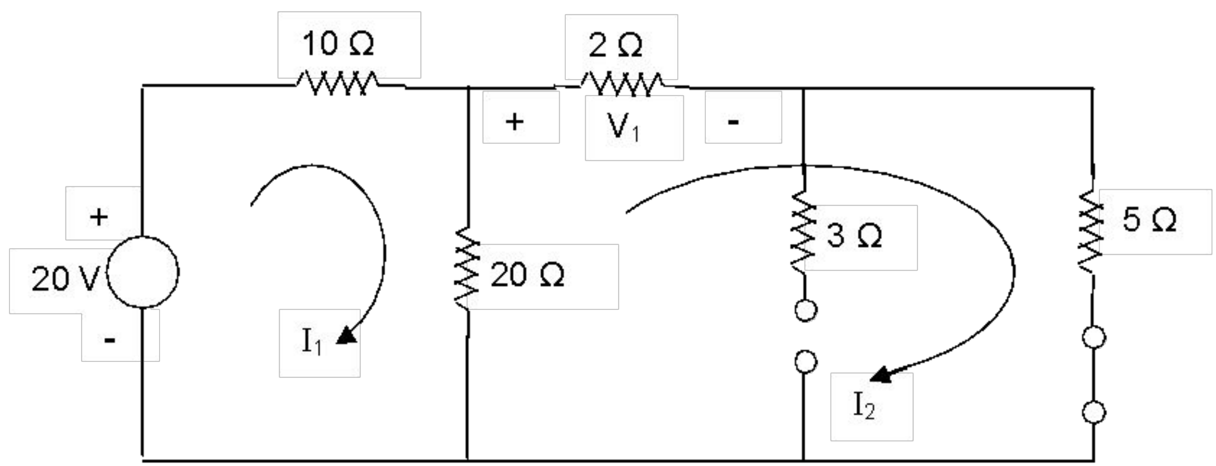
When both the sources are simultaneously present:

Current through 1 Ω resistor = 2.5 - 0.5 = 2 A from A to B.

Example 2 In the circuit shown, find the voltage drop, V_0 across the 2 Ω resistor using Superposition theorem.



Solution: 20 V source alone present: The circuit will be as shown below.

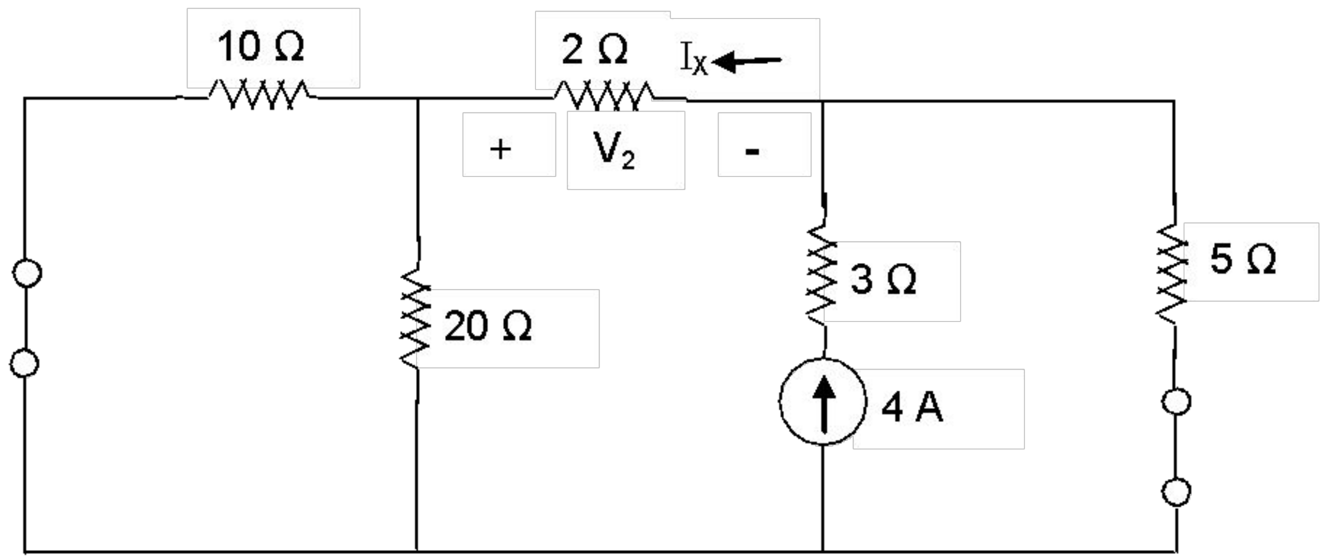


Mesh current equations : $\begin{bmatrix} 30 & -20 \\ -20 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$ On solving, $I_2 = 0.9756$ A

Thus voltage $V_1 = 2 \times 0.9756 = 1.9512$ V

4 A source alone present:

The circuit will be as shown below.



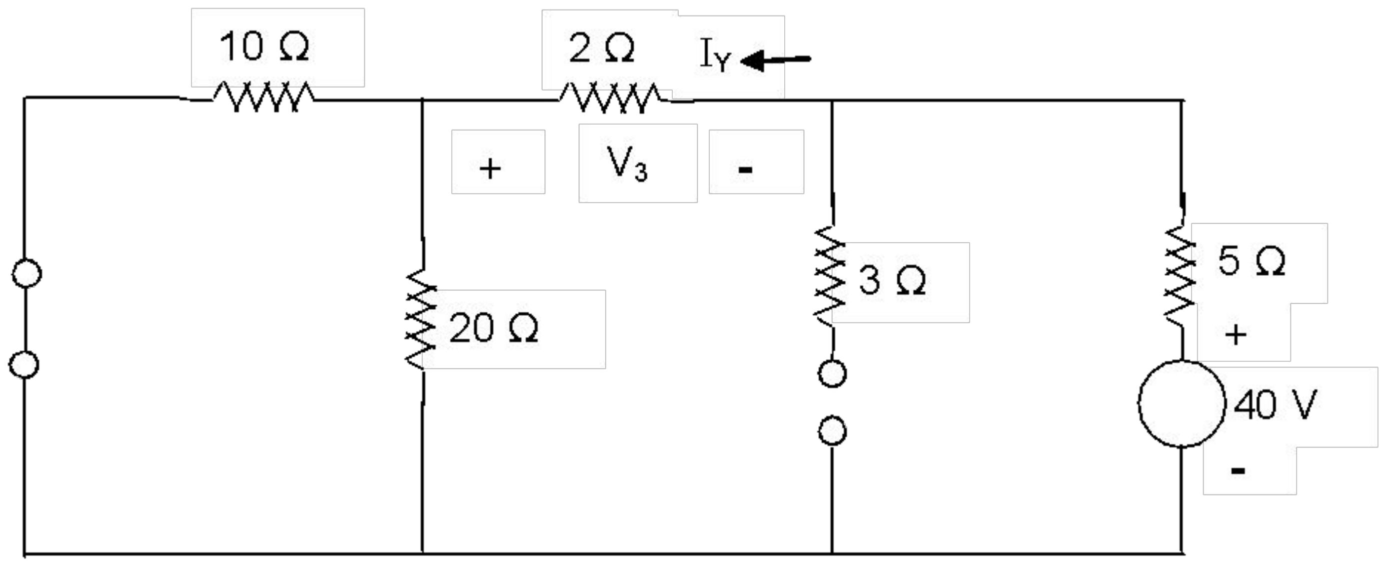
$$2 + 10 \,||\, 20 = 8.6667\ \Omega$$

Therefore current $I_X = \dfrac{5}{13.6667} \times 4 = 1.4634$ A

Thus voltage $V_2 = -\,2 \times 1.4634 = -\,2.9268$ V

40 V source alone present:

Resulting circuit is shown below.



Circuit resistance $R_T = 5 + 2 + (10||20) = 13.6667\ \Omega$

Current $I_Y = 40 / 13.6667 = 2.9268$ A; Thus voltage $V_3 = -2 \times 2.9268 = -5.8537$ V

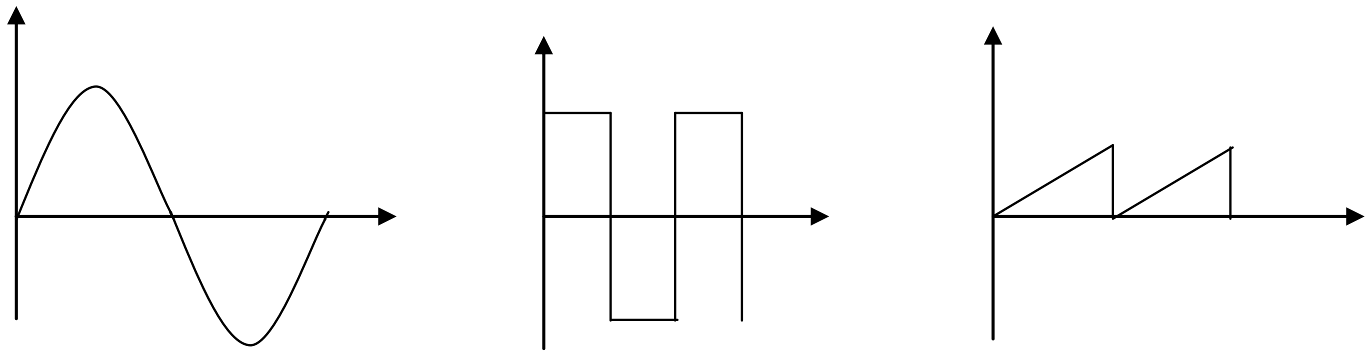
When all the three sources are simultaneously present,

voltage across 2 Ω, i.e. $V_0 = V_1 + V_2 + V_3 = 1.9512 - 2.9268 - 5.8537 = -6.8293$ V

FUNDAMENTALS OF AC

Electrical appliances such as lights, fans, air conditioners, TV, refrigerators, mixy, washing machines and industrial motors are more efficient when they operate with AC supply. The required AC voltage is generated by AC generator also called as alternator.

A <u>waveform</u> is a graph in which the instantaneous values of any quantity are plotted against time. A periodic waveform is the one which repeats itself at regular intervals. A waveform may be sinusoidal or non sinusoidal. Examples of a few periodic waveforms are shown in Fig.1.



(a) Sinusoidal waveform **(b) Rectangular waveform** **(c) Sawtooth waveform**

Fig. 1

Alternating waveform is a waveform which reverses its direction at regular intervals. Sinusoidal and rectangular waveforms shown above are alternating waveforms. Let us see more details about sinusoidal waveform.

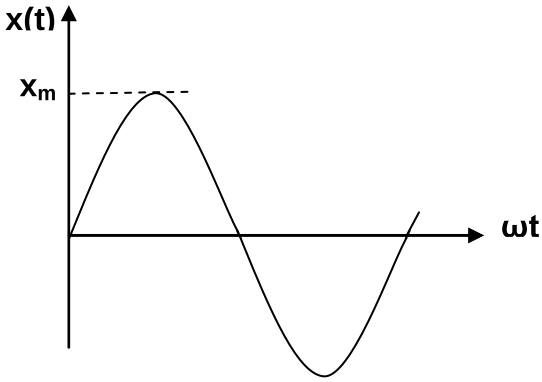


Fig. 2

Fig. 2 shows a sinusoidal waveform, which can be called as a sinusoid. It can represent a voltage or current. Its equation can be written as

$$x(t) = x_m \sin(\omega t + \varphi) \qquad (1)$$

Thus a sinusoid is described in terms of

i) its maximum value

ii) its angular frequency, ω and

iii) its phase angle φ

It is evident that sinusoid repeats in a cyclic manner. The number of cycles it makes in one second is called the frequency (f). Thus the unit for frequency is cycles per second which is also commonly known as hertz (Hz). Electric supply has a frequency of 50 or 60 Hz. In communication circuit, the frequency will be in the order of Mega Hz.

The time taken by the sinusoid to complete one cycle is called the period (T) of the sinusoid. When the supply frequency is 50 Hz, the sinusoid makes 50 cycles in one second. Thus the period is 1/50 = 0.02 second. The frequency and the period are related as

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T} \qquad (2)$$

The angular frequency of sinusoid is represented by ω and its unit is radians per second. In one cycle the angle covered is 2π radians. When the frequency is f cycles per second, the angle covered in one second will be 2πf radians. Thus

$$\omega = 2\,\pi\,f \qquad (3)$$

While drawing a sinusoid, instead of ωt, time t can be taken in the x-axis.

Example 1

Consider the voltage sinusoid

$$v(t) = 70 \sin(60t + 20^0)\ V$$

Find the amplitude, phase, angular frequency, frequency, period and the value of voltage at time t = 0.25 s.

Solution

Amplitude $v_m = 70\ V$

Phase $\varphi = 20^0$

Angular frequency $\omega = 60\ rad/s$

Frequency $f = \frac{\omega}{2\pi} = \frac{60}{2\pi} = 9.5511\ Hz$

Period $T = \frac{1}{f} = \frac{1}{9.5511} = 0.1047\ s$

Voltage value at t = 0.25 s is

$$v(0.25) = 70 \sin(60 \times 0.25 \times \frac{180}{\pi} + 20^0) = 24.59\ V$$

The two sinusoids shown in Fig. 3 are $x(t) = x_m \sin \omega t$ and $x(t) = x_m \sin(\omega t + \varphi)$

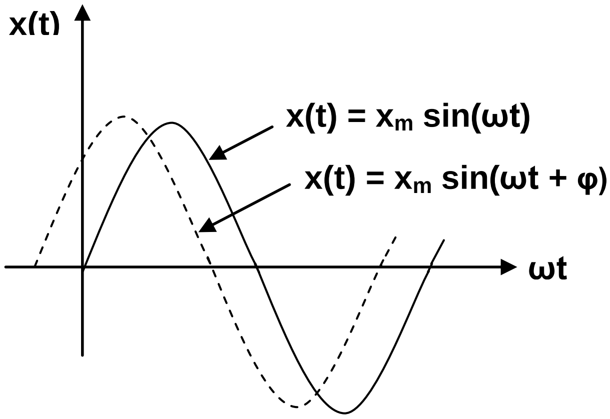


Fig. 3

The sinusoid $x(t) = x_m \sin(\omega t + \varphi)$ leads the sinusoid $x(t) = x_m \sin \omega t$ by an angle of φ. The sinusoids can also be written as

$$x(t) = x_m \sin(\theta + \varphi) \tag{4}$$

The average value of the periodic waveform can be obtained as:

$$\text{Average value} = \frac{\text{Area under one complete cycle}}{\text{Period}} \tag{5}$$

Average value is also called as mean value.

The Root Mean Square (RMS) value of periodic waveform is:

$$\textbf{RMS value} = \sqrt{\frac{\textbf{Area under squared curve for one cycle}}{\textbf{Period}}} \qquad \textbf{(6)}$$

Form Factor is defined as

$$\textbf{Form Factor} = \frac{\textbf{RMS Value}}{\textbf{Average value}} \qquad \textbf{(7)}$$

Peak Factor is defined as

$$\textbf{Peak Factor} = \frac{\textbf{Peak Value}}{\textbf{RMS value}} \qquad \textbf{(8)}$$

Consider a current waveform described by

$$i(t) = I_m \sin\theta \tag{9}$$

Its positive half cycle and negative half cycle of such sinusoids are negative of each other. Hence the area in one cycle is zero. For such sinusoidal wave form the average value is the average value over half cycle.

Thus

$$\text{Area of the curve} = \int_0^{\pi} I_m \sin\theta \, d\theta = I_m(-\cos\theta)\Big|_0^{\pi} = I_m(1+1) = 2\, I_m$$

$$I_{av} = \frac{2\, I_m}{\pi} = 0.6366\, I_m \tag{10}$$

When we square the waveform $i(t) = I_m \sin\theta$, the first and the second half of the cycle will be same. Therefore while computing the R M S value of $i(t) = I_m \sin\theta$ it is enough to consider only one half cycle.

Area of square curve = $\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$

$$= \frac{I_m^2}{2}\int_0^{\pi} (1 - \cos 2\theta)\, d\theta = \frac{I_m^2}{2}\left(\theta - \frac{\sin 2\vartheta}{2}\right)\Big|_0^{\pi} = \frac{I_m^2}{2}[\pi - 0] = \frac{\pi}{2} I_m^2$$

Mean square value = $\frac{I_m^2}{2}$

RMS value $= \frac{I_m}{\sqrt{2}}$ (11)

$= 0.7071\, I_m$ (12)

Form factor = $\frac{\text{RMS Value}}{\text{Average value}} = \frac{0.7071\, I_m}{0.6366\, I_m} = 1.11$ (13)

Peak factor = $\frac{\text{Peak Value}}{\text{RMS value}} = \frac{I_m}{0.7071\, I_m} = 1.414$ (14)

We may be calculating average and RMS values of waveforms in which inclined straight line variations are present. Consider the waveform shown in Fig. 4. Its square curve is shown in Fig. 5. Area A_1 of the square curve can be calculated as follows.

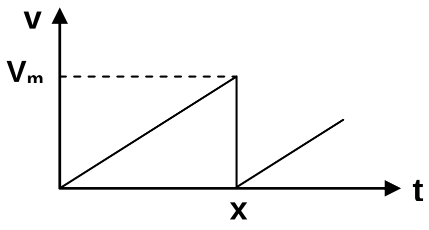


Fig. 4

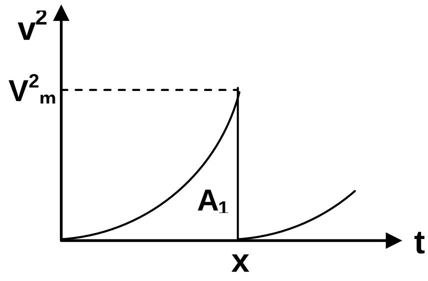


Fig. 5

Equation of the straight line is: $v = \frac{V_m}{x} t$; Then $v^2 = \frac{V_m^2}{x^2} t^2$

$$\text{Area } A_1 = \int_0^x \frac{V_m^2}{x^2} t^2 \, dt = \frac{V_m^2}{x^2} \left. \frac{t^3}{3} \right|_0^x = \frac{1}{3} V_m^2 \, x$$

It can be verified that the above result is true for the waveform shown in Fig. 6 also.

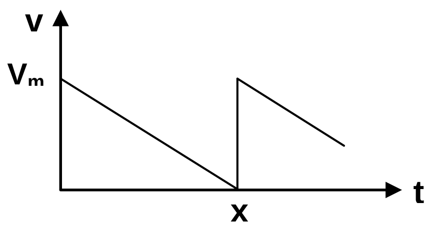


Fig. 6

Example 2

Find the average and RMS values of the waveform shown in Fig. 7

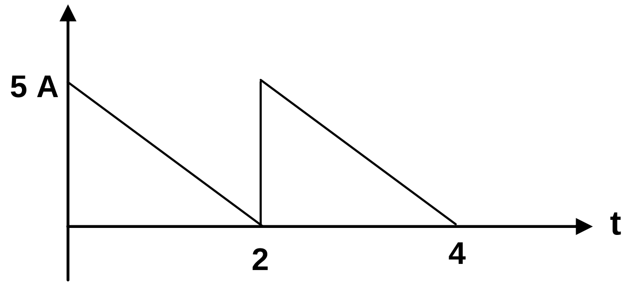


Fig. 7

Solution

$I_{av} = \frac{1}{2}$ x area of the triangle $= \frac{1}{2} \text{x} \frac{1}{2} \text{x} 5 \text{x} 2 = 2.5\text{ A}$

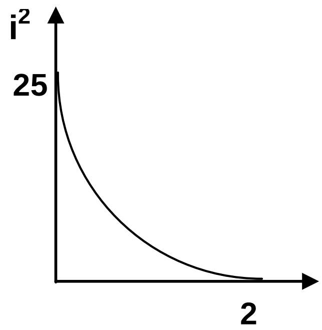


Fig. 8

The square curve is shown in Fig. 8.

Area of square curve = $\frac{1}{3}$ x 25 x 2 $= 16.6663$

Mean square value = $\frac{16.6663}{2} = 8.3332$

RMS value = $\sqrt{8.3332} = 2.8867\text{ A}$

Example 3

Find the average and RMS value of the waveform shown in Fig. 9.

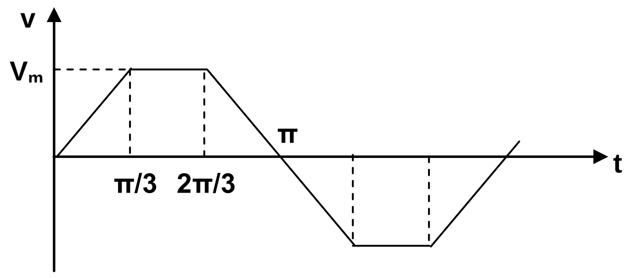


Fig. 9

Solution

Area of positive half cycle = $\frac{1}{2}\frac{\pi}{3}V_m + \frac{\pi}{3}V_m + \frac{1}{2}\frac{\pi}{3}V_m = \frac{2\pi}{3}V_m$

Average value = $\frac{2}{3}V_m = 0.6667\ V_m$

The square curve is shown in Fig. 10.

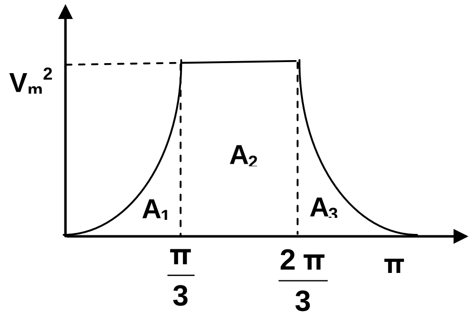


Fig. 10

Area of square curve = $\frac{\pi}{9}V_m^2 + \frac{\pi}{3}V_m^2 + \frac{\pi}{9}V_m^2 = \frac{5}{9}\pi\ V_m^2$

Mean Square value = $\frac{5}{9}V_m^2$; **RMS value** = $0.7454\ V_m$

Example 4

Find the average and RMS values of the half wave rectified sine wave shown in Fig. 11.

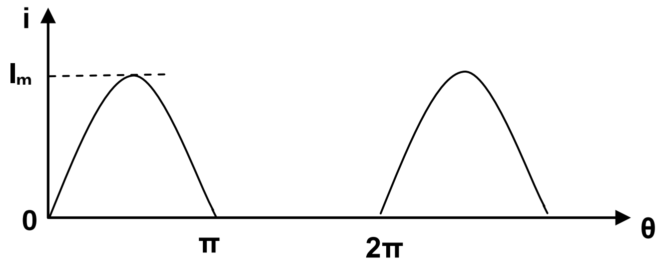


Fig. 11

Solution

As seen earlier, area of half sine wave = $2\ I_m$

Total area = $2\ I_m + 0 = 2\ I_m$

Average value $I_{av} = \dfrac{2\ I_m}{2\ \pi} = 0.3183\ I_m$

As seen earlier, area of square of half sine wave = $\dfrac{\pi}{2}\ I_m^2$

Total area of square curve = $\dfrac{\pi}{2}\ I_m^2$

Mean of square curve = $\dfrac{1}{2\pi}\ \dfrac{\pi}{2}\ I_m^2 = \dfrac{1}{4}\ I_m^2 = 0.25\ I_m^2$

RMS value $I_{RMS} = 0.5\ I_m$

Example 5

Find the average and RMS values of the full wave rectified sine wave shown in Fig. 12.

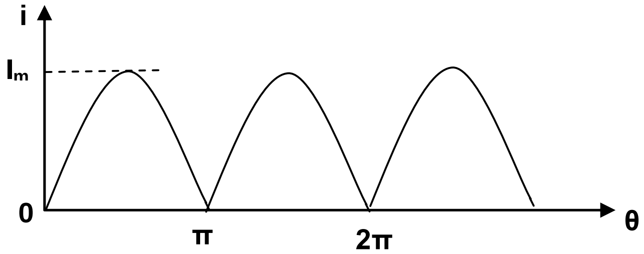


Fig. 12

Solution

As seen earlier, area of half sine wave = 2 I_m

Total area = 2 I_m + 2 I_m = 4 I_m

Average value $I_{av} = \dfrac{4\,I_m}{2\,\pi} = \dfrac{2}{\pi}\,I_m = 0.6366\,I_m$

As seen earlier, area of square of half sine wave = $\dfrac{\pi}{2}\,I_m^2$

Total area of square curve = $\pi\,I_m^2$

Mean of square curve = $\dfrac{1}{2\,\pi}\,\pi\,I_m^2 = \dfrac{1}{2}\,I_m^2$

RMS value $I_{RMS} = \dfrac{I_m}{\sqrt{2}} = 0.7071\,I_m$

If the waveform is the sum of several waveforms, its RMS values can be obtained as follows.

Let

$W = W_1 + W_2 + W_3$ and their RMS values be $W_{1\,RMS}$, $W_{2\,RMS}$ and $W_{3\,RMS}$ respectively. Then

$$W_{RMS} = \sqrt{W_{1RMS}^2 + W_{2RMS}^2 + W_{3RMS}^2}$$

<u>Example 6</u>

A conductor carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 10 A. Find the RMS value of the conductor current.

<u>Solution</u>

Conductor current $i(t) = (10 + 10 \sin \omega t)$ A

Here $W_1 = 10$ A and $W_2 = 10 \sin \omega t$ A

Therefore $W_{1\,RMS} = 10$ A; $W_{2\,RMS} = 7.071$ A

RMS value of conductor current = $\sqrt{10^2 + 7.071^2} = 12.2474$A

SINGLE PHASE AC CIRCUITS

PHASORS

Consider a linear ac circuit having one or more sinusoidal inputs having same frequency as shown in Fig. 1. The amplitudes and phase angles of the inputs may be different while their frequency should be same.

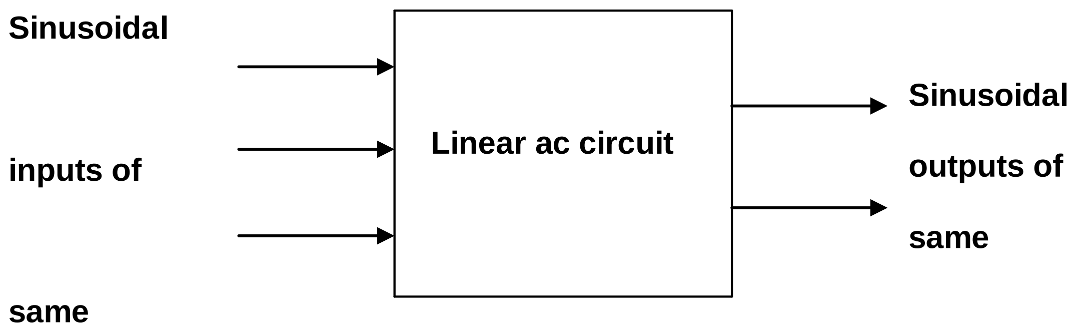


Fig. 1

The output what we may be interested may be voltage across an element or current through an element. The output waveform will be sinusoidal with the same frequency as the input signals. This could be easily verified experimentally. Since we are dealing with linear ac circuits with sinusoidal inputs, it follows that, the steady-state response in any part of the circuit is also sinusoidal. The steady-state analysis of such circuits can be carried out easily using phasors.

A sinusoid is fully described when its maximum value, angular frequency and phase are specified. A question may arise whether we should always deal with such sinusoidal time function to represent voltage and current in ac circuits. When all the inputs are sinusoidal time function with the same angular frequency ω, the voltage or the current in any part of the circuit will also be of sinusoidal time function with the SAME ANGULAR FREQUENCY ω. Hence it is redundant to carry information of ω, while representing voltages and currents in ac circuits. This idea gives birth to the concept of PHASORS.

The phasor corresponding to sinusoid $x(t) = x_m \cos(\omega t + \varphi)$ is $X = \frac{x_m}{\sqrt{2}} \angle \varphi$ (1)

In case $x(t)$ is expressed as $x(t) = x_m \sin(\omega t + \varphi)$, it can be written as

$x(t) = x_m \sin(\omega t + \varphi) = x_m \cos(\omega t + \varphi - \frac{\pi}{2})$ and the corresponding phasor is

$$X = \frac{x_m}{\sqrt{2}} \angle \varphi - \frac{\pi}{2} \qquad (2)$$

In a similar way we can state:

If $x(t) = - x_m \cos(\omega t + \varphi)$ its phasor is $X = \frac{x_m}{\sqrt{2}} \angle \varphi - \pi$ (3)

If $x(t) = - x_m \sin(\omega t + \varphi)$ its phasor is $X = \frac{x_m}{\sqrt{2}} \angle \varphi + \frac{\pi}{2}$ (4)

Eqs. (1) to (4) are useful to find the phasor for a given sinusoid.

Fig. 1 is useful to locate the quadrant in which the phasor lies.

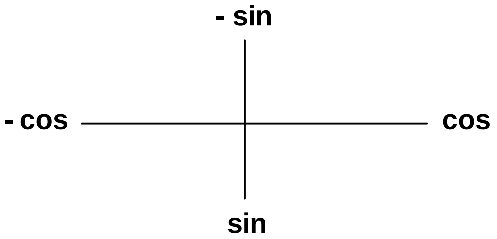


Fig. 1 Quadrants for Phasor

A few sinusoids and the corresponding phasors are;

$x_1(t) = \sqrt{2}\ 150 \cos(\omega t + 15^0)$ $\qquad X_1 = 150 \angle 15^0$

$x_2(t) = \sqrt{2}\ 150 \cos(\omega t - 75^0)$ $\qquad X_2 = 150 \angle -75^0$

$x_3(t) = \sqrt{2}\ 100 \sin \omega t$ $\qquad X_3 = 100 \angle -90^0$

$x_4(t) = \sqrt{2}\ 100 \sin(\omega t + 30^0)$ $\qquad X_4 = 100 \angle -60^0$

$x_5(t) = \sqrt{2}\ 100 \sin(\omega t - 150^0)$ $\qquad X_5 = 100 \angle -240^0$

$x_6(t) = -\sqrt{2}\ 80 \cos(\omega t + 30^0)$ $\qquad X_6 = 80 \angle 210^0$

$x_7(t) = -\sqrt{2}\ 80 \sin(\omega t - 30^0)$ $\qquad X_7 = 80 \angle 60^0$

The above phasors are shown in Fig. 2

.

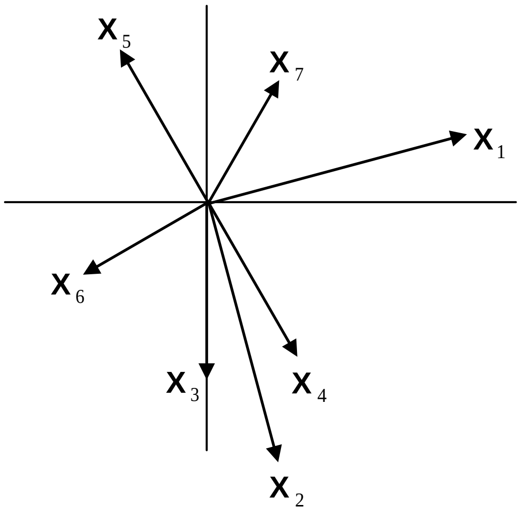


Fig. 2 Phasors of given sinusoids

The important motivation for the use of phasors is the ease with which two or more sinusoids at the same frequency can be added or subtracted. In the sinusoidal steady state, all the currents and voltages are of same frequency. Hence phasors can be used to combine currents or voltages. KCL and KVL can be easily interpreted in terms of phasor quantities.

A phasor is a transformed version of a sinusoidal voltage or current waveform and it contains the amplitude and phase angle information of the sinusoid. Phasors are complex numbers and can be depicted in a complex plane. The relationship of phasors on a complex plane is called a phasor diagram.

Example 1

Using phasor concept, find the sum of 4 voltages given by :

$v_1 = \sqrt{2}\ 50\ \sin \omega t$ $\qquad$ $v_2 = \sqrt{2}\ 40\ \sin(\omega t + \pi / 3)$

$v_3 = \sqrt{2}\ 20\ \sin(\omega t - \pi / 6)$ $\qquad$ $v_4 = \sqrt{2}\ 30\ \sin(\omega t + 3\pi / 4)$

Solution

In phasors corresponding to the sinusoids are:

$$V_1 = 50 \angle -90° = 0.0 - j\,50.0$$

$$V_2 = 40 \angle -30° = 34.6410 - j\,20.0$$

$$V_3 = 20 \angle -120° = -10.0 - j\,17.3205$$

$$V_4 = 30 \angle 45° = \underline{21.2132 + j21.2132}$$

$$V_1 + V_2 + V_3 + V_4 = \underline{45.8542 - j66.1073}$$

$$= 80.4536 \angle -55.25^0$$

Corresponding sinusoid is obtained as

$$v_T = \sqrt{2}\ 80.4536\ \cos(\omega t - 55.25^0) = 113.7786 \cos(\omega t - 55.25^0)$$

$$= 113.7786\ \sin(\omega t + 34.75^0)$$

SINGLE ELEMENT IN STEADY STATE

Voltage-current relationship of resistor, inductor and capacitor can be obtained in phasor form. Such phasor representations are useful in solving ac circuits.

RESISTOR

Let the voltage v(t) across the resistor terminals be

$$v(t) = V_m \cos \omega t \tag{5}$$

The current through it is given by

$$i(t) = \frac{v(t)}{R} = \frac{V_m}{R} \cos \omega t \tag{6}$$

Expressing the equations (5) and (6) in phasor form we get

$$V = \frac{V_m}{\sqrt{2}} \angle 0^0 \tag{7}$$

$$I = \frac{V_m}{\sqrt{2}\,R} \angle 0^0 \tag{8}$$

<u>The impedance of an element is defined as the ratio of the phasor voltage across it to the phasor current through it</u>. Thus

$$Z = \frac{V}{I} \tag{9}$$

For a resistor $$Z = \frac{V}{I} = R \angle 0^0 \tag{10}$$

Thus in the case of a resistor, voltage-current relationship is

$$V = R\, I \tag{11}$$

Representation of resistor in time frame and its phasor form are shown in Fig. 3.

Fig. 3 Representation of a resistor

It is to be noted that as seen by the Eqns. (7) and (8),both the voltage V and the current I have the same phase angle of 0^0. The phasor diagram showing the voltage and current in a resistor is shown in Fig. 4.

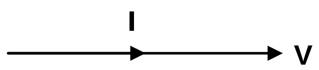


Fig. 4 Phasor diagram - resistor

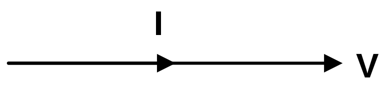


Fig. 4 Phasor diagram - resistor

In the phasor diagram shown in Fig. 4, importance must be given to the phase angles of the voltage V and the current I. The lengths of the phasors depend on their magnitude and the scale chosen. In no occasion length of a voltage phasor and the length of a current phasor can be compared since they have different units. The scale for current phasors will be like

1 cm = x Volts while the scale for the voltage phasors will be like

1 cm = y Ampere.

The impedance of the resistor is R $\angle 0°$. In a general network where R is embedded, the phasor corresponding to the voltage across R and the phasor corresponding to the current through R are always in phase.

INDUCTOR

For the inductor, the voltage-current relationship is

$$v(t) = L\frac{di(t)}{dt} \tag{12}$$

In steady state, let the current through it be

$$i(t) = I_m \cos \omega t \tag{13}$$

Then $$v(t) = L\frac{di(t)}{dt} = -\omega L\ I_m \sin \omega t \tag{14}$$

Expressing the above two equations in phasor form we have

$$I = \frac{I_m}{\sqrt{2}}\angle 0^0 \tag{15}$$

and $$V = \omega L\frac{I_m}{\sqrt{2}}\angle 90^0 \tag{16}$$

The impedance of the inductor is given by

$$Z = \frac{V}{I} = \omega L \angle 90^0 = j\,\omega L = j\,X_L \qquad (17)$$

where $X_L = \omega L$ (18)

Thus, the terminal relationship of an inductor in phasor form is

$$V = j\,X_L\,I \qquad (19)$$

Representation of inductor in time frame and its phasor form are shown in Fig. 5.

Fig. 5 Representation of a inductor

It is to be noted that as seen by the Eqns. (15) and (16), the voltage V leads the current I by a phase angle of 90^0. The phasor diagram showing the voltage and current in an inductor is shown in Fig. 6.

Fig. 6 Phasor diagram - inductor

Fig. 6 Phasor diagram - inductor

It is to be noted that the voltage V leads the current I by 90° or we can also state that the current I lags the voltage V by 90°. The steady state impedance corresponding to the inductance L is jX_L where $X_L = \omega L$. The quantity X_L is known as the INDUCTIVE REACTANCE.

CAPACITOR

For the capacitor, the voltage-current relationship is

$$i(t) = C\frac{dv(t)}{dt} \tag{20}$$

In steady state, let the voltage across it be

$$v\,(t) = V_m \cos \omega t \tag{21}$$

Then $i(t) = C\frac{dv(t)}{dt} = -\ \omega C\ V_m \sin \omega t$ **(22)**

Expressing the above two equations in phasor form we have

$$V = \frac{V_m}{\sqrt{2}}\angle 0^0 \tag{23}$$

and $I = \omega C\frac{V_m}{\sqrt{2}}\angle 90^0$ **(24)**

The impedance of the capacitor is given by

$$Z = \frac{V}{I}$$

$$= \frac{1}{\omega C} \angle -90^0 = -\frac{j}{\omega C} = -j X_C \qquad (25)$$

where $X_C = \frac{1}{\omega C}$ (26)

Thus, the terminal relationship of a capacitor in phasor form is

$$V = -jX_C I \qquad (27)$$

Representation of capacitor in time frame and its phasor form are shown in Fig. 7.

Fig. 7 Representation of a capacitor

It is to be noted that as seen by the Eqns. (23) and (24), the current I leads the voltage V by a phase angle of 90^0. The phasor diagram showing the voltage and current in a capacitor is shown in Fig. 8.

Fig. 8 Phasor diagram of - capacitor

It is to be noted that the current I leads the voltage V by 90° or we can also state that the voltage V lags the current I by 90°. The steady state impedance corresponding to the capacitance C is $-jX_C$ where $X_C = \frac{1}{\omega C}$. The quantity X_C is known as the CAPACITIVE REACTANCE.

It is conventional to say how the current phasor is relative to voltage phasor. Thus for the resistor, the current phasor is in phase with the voltage phasor. In an inductor, the current phasor lags the voltage phasor by 90^0. In the case of a capacitor, the current phasor leads the voltage phasor by 90^0.

Example 2

The voltage of $v = \sqrt{2}\ 80 \cos(100t - 55^0)$ V is applied across a resistor of 25Ω. Find the steady state current through the resistor.

Solution

Here $V = 80 \angle -55^0$ and $R = 25\ \Omega$

Thus, current $I = \dfrac{V}{R} = \dfrac{80\angle -55^0}{25} = 3.2\angle -55^0$ A

Current $i(t) = 4.5255 \cos(100t - 55^0)$ A

Example 3

The voltage of $v = \sqrt{2}\ 20 \sin(50\,t - 25^0)$ V is applied across an inductor of 0.1 H. Find the steady state current through the inductor.

Solution

Phasor voltage $V = 20 \angle -25^0 - 90^0 = 20 \angle -115^0$ **V**

Impedance $Z = j\,\omega\,L = j\,50 \times 0.1 = 5 \angle 90^0\ \Omega$

$$\text{Current } I = \frac{V}{Z} = \frac{20\angle -115^0}{5\angle 90^0} = 4\angle -205^0 \text{ A}$$

Converting this to the time domain

$$\text{Current } i(t) = 5.6569 \cos(50\,t - 205^0) \text{ A}$$

$$= -\ 5.6569 \cos(50\,t - 25^0) \text{ A}$$

Example 4

The voltage of v = $\sqrt{2}$ 12 cos (100 t - 25^0) V is applied across a capacitor of 50 μF. Find the steady state current through the capacitor.

Solution

Phasor voltage V = 12 $\angle$- 25^0 V

Impedance $Z = -j\,\dfrac{1}{\omega C} = -j\,\dfrac{1}{100 \times 50 \times 10^{-6}} = -j\,200\,\Omega$

Current $I = \dfrac{V}{Z} = \dfrac{12\angle -25^0}{200\angle -90^0}\ \text{A} = 0.06\angle 65^0\ \text{A} = 60\angle 65^0\ \text{mA}$

Converting this to the time domain

Current i(t) = 84.8528 cos (100 t + 65^0) mA

ANALYSIS OF RLC CIRCUITS

An ac circuit generally consists of resistors, inductors and capacitors connected in series, parallel and series-parallel combinations. Often we need to simplify the circuit by finding the equivalents. Further to this, we have to make use of KVL, KCL, source transformation, voltage division and current division what we discussed in previous chapter, by replacing resistors by impedances and dc voltages and currents by voltage phasors and current phasors.

A coil used in ac circuit will have its own resistance in addition to the inductive reactance due to its inductance. One such coil is shown in Fig. 9.

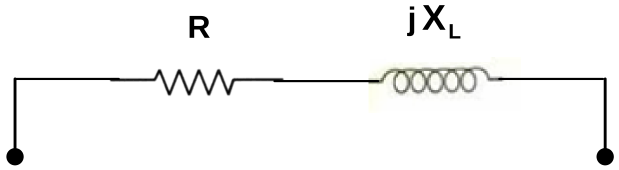


Fig. 9 A coil in an ac circuit

It is clear that the resistance R and inductive reactance jX_L are connected in series. The impedance of this coil is

$$Z = R + jX_L \tag{28}$$

Now consider a case where a resistance R and a capacitance having a capacitive reactance $-jX_C$ are connected in series as shown in Fig. 10.

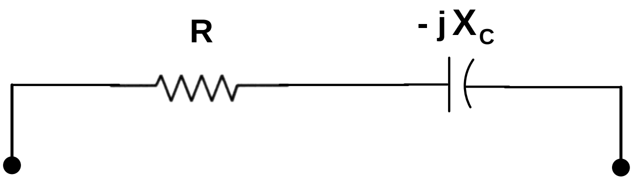


Fig. 10 A resistance and a capacitance in series

The impedance of the circuit is

$$Z = R - jX_C \tag{29}$$

IMPEDANCE AND ADMITTANCE

The steady state impedance (a complex quantity) can be written in two forms, namely Rectangular form and Polar form as

Rectangular form: $Z = R + jX$

Polar form: $Z = |Z| \angle \varphi$

RL CIRCUIT

Having studied how to combine the series and parallel impedances we shall now see how the RL, RC and RLC circuits can be analyzed.

Let us consider a simple circuit in which a resistor and an inductor are connected in series as shown in Fig. 13.

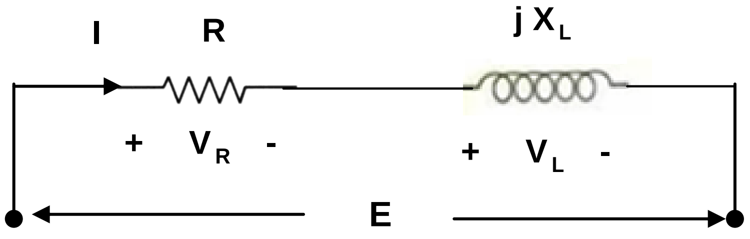


Fig. 13 RL circuit

Taking the supply voltage as reference

$$\mathbf{E} = |\mathbf{E}| \angle 0^0 \quad (41)$$

Circuit impedance $\mathbf{Z} = \mathbf{R} + j\mathbf{X_L} = |\mathbf{Z}|\angle\boldsymbol{\theta}$ (42)

Circuit current $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}} = \dfrac{|\mathbf{E}|\angle 0^0}{|\mathbf{Z}|\angle\boldsymbol{\theta}} = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}\angle -\boldsymbol{\theta}$ (43)

$= |\mathbf{I}|\angle -\boldsymbol{\theta}$ (44)

where $|\mathbf{I}| = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}$ (45)

Further $\mathbf{V_R} = \mathbf{RI} = \mathbf{R}\,|\mathbf{I}|\angle -\boldsymbol{\theta}$ (46)

$\mathbf{V_L} = j\mathbf{X_L}\,\mathbf{I} = \mathbf{X_L}|\mathbf{I}|\angle -\boldsymbol{\theta} + 90^0$ (47)

Using KVL, we get $\mathbf{V_R} + \mathbf{V_L} = \mathbf{E}$ (48)

The phasor diagram for this RL circuit can be got by drawing the phasors V_R, V_L, E and I as shown in Fig. 14.

$$E = |E| \angle 0^0 \, V$$

$$= |Z||I| \angle 0^0$$

$$I = |I| \angle -\theta$$

$$V_R = R|I| \angle -\theta$$

$$V_L = X_L |I| \angle -\theta + 90^0$$

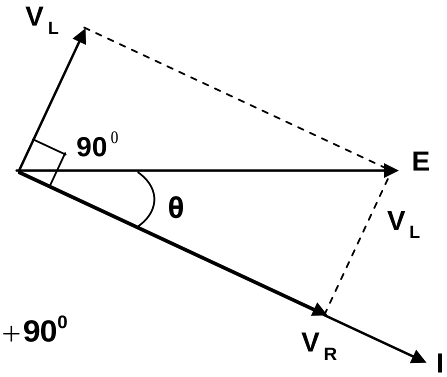


Fig.14 Phasor diagram of RL circuit

Consider the triangle formed by the phasors V_R, V_L and E. Recognizing that $|V_R| = R|I|$, $|V_L| = X_L|I|$ and $|E| = |Z||I|$ if each side of the triangle is divided by $|I|$ then R, X_L and $|Z|$ will form a triangle as shown in Fig, 15. This triangle is known as the IMPEDANCE TRIANGLE.

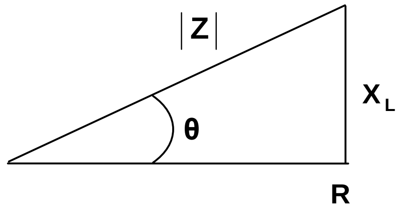


Fig. 15 Impedance diagram of RL circuit

RC CIRCUIT

Let us now consider the circuit in which a resistor and a capacitor are connected in series as shown in Fig. 16.

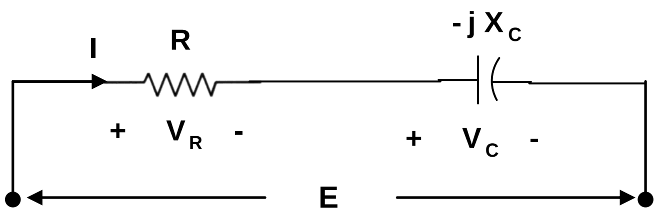


Fig. 16 RC circuit

Taking the supply voltage as reference

$$\mathbf{E} = |\mathbf{E}| \angle 0^0 \tag{49}$$

Circuit impedance $\mathbf{Z} = \mathbf{R} - \mathbf{j}\mathbf{X_C} = |\mathbf{Z}| \angle -\boldsymbol{\theta}$ **(50)**

Circuit current $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}} = \dfrac{|\mathbf{E}|\angle 0^0}{|\mathbf{Z}|\angle -\boldsymbol{\theta}} = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}\angle \boldsymbol{\theta}$ **(51)**

$$= |\mathbf{I}| \angle \boldsymbol{\theta} \tag{52}$$

where $|\mathbf{I}| = \dfrac{|\mathbf{E}|}{|\mathbf{Z}|}$ **(53)**

Further $\mathbf{V_R} = R\mathbf{I} = R|\mathbf{I}|\angle\theta$ (54)

$$\mathbf{V_C} = -jX_C\,\mathbf{I} = X_C|\mathbf{I}|\angle\theta - 90^0 \qquad (55)$$

Using KVL, we get $\mathbf{V_R} + \mathbf{V_C} = \mathbf{E}$ (56)

The phasor diagram for this RC circuit can be got by drawing the phasors $\mathbf{V_R}$, $\mathbf{V_C}$, $\mathbf{E}$ and $\mathbf{I}$ as shown in Fig. 17.

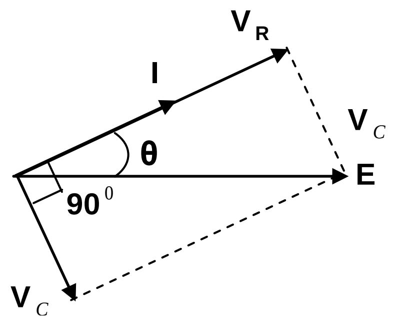


Fig. 17 Phasor diagram of RC circuit

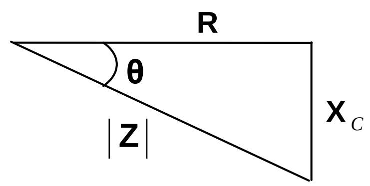


Fig. 18 Impedance triangle of RC circuit

Consider the triangle formed by the phasors $\mathbf{V_R}$, $\mathbf{V_C}$ and $\mathbf{E}$. Recognizing that $|\mathbf{V_R}| = R|\mathbf{I}|$, $|\mathbf{V_C}| = X_C|\mathbf{I}|$ and $|\mathbf{E}| = |Z||\mathbf{I}|$ if each side of the triangle is divided by $|\mathbf{I}|$ then R, X_C and $|Z|$ will form a triangle as shown in Fig, 18. This triangle is known as the IMPEDANCE TRIANGLE.

RLC CIRCUITS

Analysis of RLC circuits is the series, parallel and series-parallel combination of RL and RC circuits. Equivalent of RLC circuit will be R, or RL or RC circuit as illustrated in the examples to be discussed.

POWER AND POWER FACTOR

Let $|E| \angle 0^0$ be the supply voltage in an AC circuit. The supply current may lag or lead the supply voltage. Let the supply current be $|I| \angle -\theta$. The supply current can be resolved into two components (i) A component I_p in phase with the voltage and (ii) A component I_q at right angle to the voltage as shown in Fig. 19.

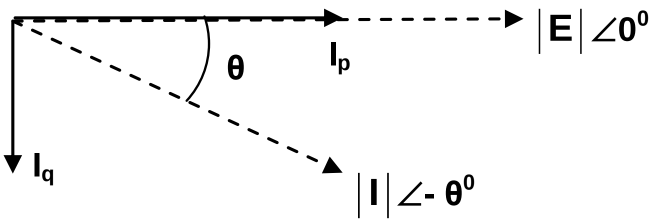


Fig. 19 Power and Power factor

Current I_p is called the active or in phase component while I_q is known as reactive or quadrature component. As seen from Fig. 19

$I_p = |I| \cos \theta$ and (57)

$I_q = |I| \sin \theta$ (58)

It is to be noted that

$|I| \cos \theta$, $|I| \sin \theta$ and $|I|$ form three sides of a right angle triangle as in Fig. 20.

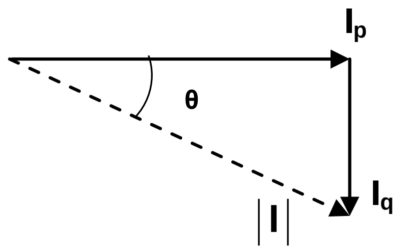


Fig. 20 Components of current

Active Power (P)

Active power is the real power consumed by the circuit. This is due to the in phase component.

Active or real power $P = |E| \, I_p$

$$= |E| \, |I| \cos\theta \text{ Watts} \qquad (59)$$

Reactive Power (Q)

The power associated with the reactive component of current I_q is known as reactive power. Its unit is Volt Ampere Reactive (VAR).

Reactive power $Q = |E| \, I_q$

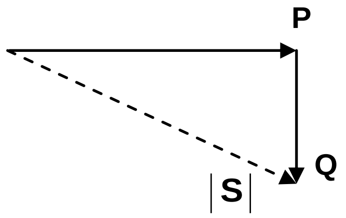
$$= |E| \, |I| \sin\theta \text{ VAR} \qquad (60)$$

Apparent Power and Power Factor

The product of voltage and current, $|E|\ |I|$ is called as Apparent Power, $|S|$. Its unit is Volt Ampere (VA).

Apparent power $|S| = |E|\ |I|$ VA **(61)**

Similar to Fig. 20, real power P, reactive power Q and apparent power $|S|$ form three sides of a right angle triangle as shown in Fig. 21.



$$P = |E|\,|I|\ \cos\theta$$

$$Q = |E|\,|I|\ \sin\theta$$

$$S = |E|\,|I|$$

Fig. 20 Components of power

Power Factor (pf) is the ratio of real power to apparent power.

$$\text{Thus power factor} = \frac{|E|\,|I|\ \cos\theta}{|E|\,|I|}$$

$$= \cos\theta \qquad (62)$$

By the above definition, it is not possible to distinguish whether the load is inductive or capacitive. If the load is inductive, the current is lagging the voltage and the nature of the power factor is LAGGING. On the other hand if the load is capacitive, the current is leading the voltage and hence the nature of the power factor is LEADING.

Whenever power factor is furnished, it must be clearly stated whether it is lagging or leading. For inductive load, the power factor is cos θ lagging; for capacitive load, the power factor is cos θ leading; for resistive load since the voltage and current are in-phase, power factor angle θ is zero and the power factor is said to be UNITY.

Power associated with R, L and C can be obtained as follows.

<u>In the case of resistor</u>, p.f. angle is zero and hence

$$P = |E|\ |I| \cos\theta = |E|\ |I| = |Z|\,|I|\,|I| = |I|^2\ R \qquad (63)$$

$$Q = |E|\ |I| \sin\theta = 0 \qquad (64)$$

<u>In the case of pure inductor and pure capacitor</u>, p.f. angle = 90^0 and hence

$$P = |E|\ |I| \cos\theta = 0 \qquad (65)$$

$$Q = |E|\ |I| \sin\theta = |E|\ |I| = |Z|\,|I|\,|I| = |I|^2\ X \qquad (66)$$

Example 5

In a series circuit containing pure resistance and pure inductance, the current and voltage are: i(t) = 5 sin (314 + $\frac{2\pi}{3}$) and v(t) = 20 sin (314 + $\frac{5\pi}{6}$). (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the power drawn by the circuit?

Solution

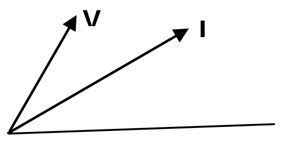
Current I = $\frac{5}{\sqrt{2}}\angle 120\text{-}90 = \frac{5}{\sqrt{2}}\angle 30^0$; Voltage V = $\frac{20}{\sqrt{2}}\angle 150\text{-}90 = \frac{20}{\sqrt{2}}\angle 60^0$

Impedance Z = $\frac{V}{I} = \frac{20\angle 60}{5\angle 30} = 4\ \angle 30^0\ \Omega = (3.4641 + j2)\Omega$

Resistance R = 3.4641 Ω

X_L = 2 Ω; 314 L = 2; L = $\frac{2}{314}$ H ; Inductance L = 6.3694 mH

Angle between voltage and current = 30^0



p.f. = cos 30^0 = 0.866 lagging

Power P = $|V||I|\cos\theta = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3$ W

Example 6

An inductive coil takes 10 A and dissipates 1000 W when connected to a supply of 250 V, 25 Hz. Calculate the impedance, resistance, reactance, inductance and the power factor.

Solution

$P = |I|^2 R$; Resistance $R = \dfrac{1000}{100} = 10\ \Omega$

$|Z| = \dfrac{250}{10} = 25\,\Omega$; From impedance triangle $X = \sqrt{25^2 - 10^2} = 22.9128\,\Omega$

Thus impedance $Z = (10 + j\,22.9128)\ \Omega = 25 \angle 66.42^0\ \Omega$

Resistance $R = 10\ \Omega$ Reactance $X = 22.9128\ \Omega$

Inductance $L = \dfrac{X}{2\pi f} = \dfrac{22.9128}{2\pi \times 25} = 0.1459$ H

From impedance triangle, power factor = $\dfrac{R}{|Z|} = \dfrac{10}{25} = 0.4$ lagging

Example 7

A resistance is connected in series with a coil. With a supply of 250 V, 50 Hz, the circuit takes a current of 5 A. If the voltages across the resistance and the coil are 125 V and 200 V respectively, calculate (i) impedance, resistance and reactance of the coil (ii) power absorbed by the coil and the total power. Draw the phasor diagram.

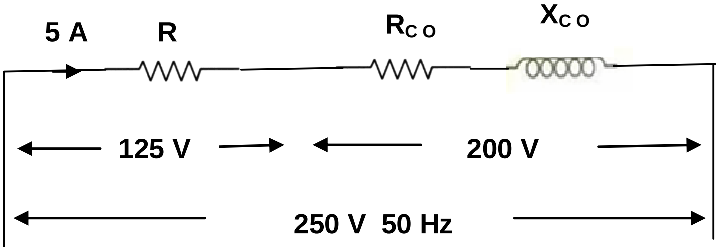


Fig. 21 Example 7

Resistance R = $\dfrac{125}{5} = 25\ \Omega$

$\left|Z_{CO}\right| = \dfrac{200}{5} = 40\ \Omega$; $\left|Z_T\right| = \dfrac{250}{5} = 50\ \Omega$

$\left|Z_{CO}\right| = \dfrac{200}{5} = 40\ \Omega$; **Therefore** $\left|R_{CO} + jX_{CO}\right| = 40$; $R_{CO}^2 + X_{CO}^2 = 1600$

$\left|Z_T\right| = \dfrac{250}{5} = 50\ \Omega$; $\left|(25 + R_{CO}) + jX_{CO}\right| = 50$

$625 + 50\ R_{CO} + R_{CO}^2 + X_{CO}^2 = 2500$ **i.e.** $50\ R_{CO} = 2500 - 625 - 1600 = 275$

Resistance of the coil $R_{CO} = 5.5\ \Omega$ **Also** $X_{CO}^2 = 1600 - 5.5^2 = 1569.75$

<u>Reactance of the coil</u> X_C = 39.62 Ω

<u>Impedance of the coil</u> Z_{CO} = (5.5 + j 39.62) = 40 $\angle 82.1^0$ Ω

<u>Power absorbed by the coil</u> $P_{CO} = 5^2 \times 5.5 = 137.5$ W

<u>Total power</u> $P_T = (5^2 \times 25) + 137.5 = 762.5$ W

Total impedance Z_T = (30.5 + j 39.62) = 50 $\angle 52.41^0$Ω

$|I|\,R_{CO} = 5 \times 5.5 = 27.5$ V ; $|I|\,X_{CO} = 5 \times 39.62 = 198.1$ V

Phasor diagram is shown in Fig. 22.

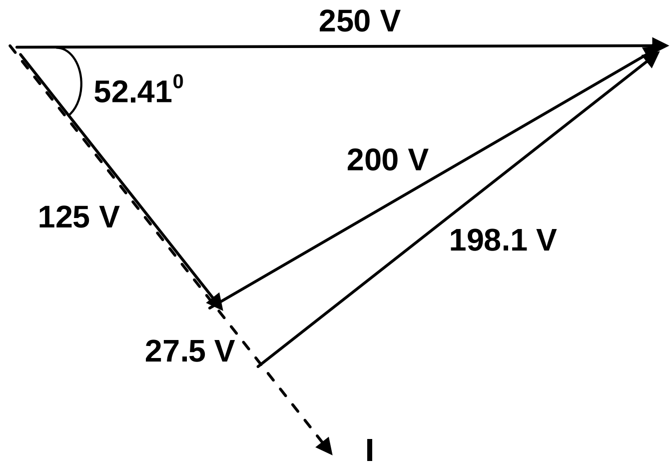


Fig. 22 Phasor diagram-Example 7

Example 8

When a resistor and a seriesly connected inductor coil, are supplied with 240 V, a current of 3 A flows lagging behind the supply voltage by 37^0. The voltage across the coil is 171 V. Find the value of the resistor, resistance and reactance of the inductor coil.

Solution

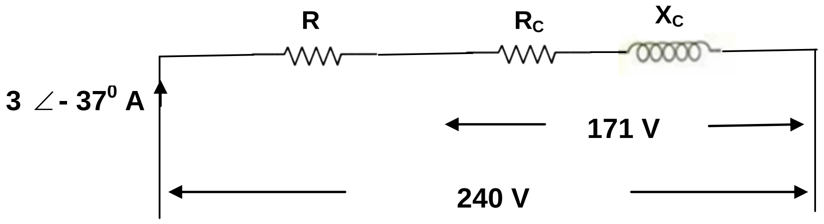


Fig. 23 Example 8

Supply voltage E = 240 $\angle 0^0$ V; Supply current I = 3 $\angle$ - 37^0 A

Circuit impedance Z = $\frac{E}{I}$ = 80 $\angle$ 37^0 Ω = (63.8908 + j 48.1452) Ω

Thus R + R_C = 63.8908 Ω and X_C = 48.1452 Ω

For the coil, $|Z_C| = \frac{171}{3} = 57$ Ω ; From impedance triangle of the coil

R_C = $\sqrt{57^2 - 48.1452^2} = 30.5129$ Ω

Value of resistor R = 63.8908 – 30.5129 = 33.3779 Ω

Resistance of inductor coil R_C = 30.5129 Ω

Reactance of inductor coil X_C = 48.1452 Ω

<u>Example 9</u>

When a voltage of 100 V at 50 Hz is applied to choking coil 1, the current taken is 8 A and the power is 120 W. When the same supply is applied to choking coil 2, the current is 10 A and the power is 500 W. Find the current and power when the supply is applied to two coils connected in series.

<u>Solution</u>

Resistance $R_1 = \dfrac{120}{8^2} = 1.875\ \Omega$

Impedance $|Z_1| = \dfrac{100}{8} = 12.5\ \Omega$;   Therefore $X_1 = \sqrt{12.5^2 - 1.875^2} = 12.3586\ \Omega$

Resistance $R_2 = \dfrac{500}{10^2} = 5\ \Omega$

Impedance $|Z_2| = \dfrac{100}{10} = 10\ \Omega$;   Therefore $X_2 = \sqrt{10^2 - 5^2} = 8.6603\ \Omega$

Total resistance $R_T = 6.875\ \Omega$;   Total reactance $X_T = 21.0189\ \Omega$

Total impedance $Z_T = (6.875 + j\,21.0189) = 22.1147\ \angle 71.89^0\ \Omega$

<u>Total current</u> $|I_T| = \dfrac{100}{22.1147} = 4.5219$ A   <u>Power</u> $P_T = 4.5219^2 \times 6.875 = 140.5771$ W

Example 10

A resistance of 100 ohm is connected in series with a 50 μF capacitor. When the supply voltage is 200 V, 50 Hz, find the (i) impedance, current and power factor (ii) the voltage across resistor and across capacitor. Draw the phasor diagram.

Solution

Resistor R = 100 Ω; Reactance of the capacitor $X_C = \dfrac{10^6}{2\pi \times 50 \times 50} = 63.662\ \Omega$

Impedance $Z = (100 - j\,63.662) = 118.5447\ \angle -32.48^0$

Taking the supply voltage as reference, $E = 200\ \angle 0^0$ V

Current $I = \dfrac{E}{Z} = \dfrac{200\angle 0^0}{118.5447\angle -32.48^0} = 1.6871\angle 32.48^0$ A

Power factor = $\cos 32.48^0 = 0.8436$ leading

Voltage across resistor $V_R = 100 \times 1.6871\ \angle 32.48^0 = 168.71\ \angle 32.48^0$ V

Voltage across capacitor $V_C = -j\,63.662 \times 1.6871\ \angle 32.48^0 = 107.4042\ \angle -57.52^0$ V

Phasor diagram is shown in Fig. 24.

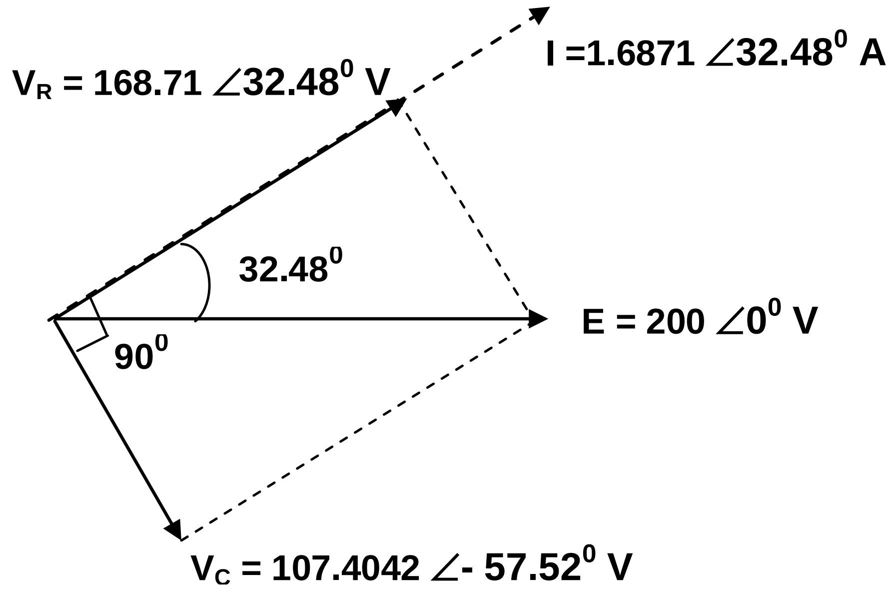


Fig. 24 Phasor diagram - Example 10

Example 11

In a circuit, the applied voltage of 150 V lags the current of 8 A by 40^0. (i) Find the power factor (ii) Is the circuit inductive or capacitive? (iii) Find the active and reactive power.

Solution

Power Factor = 0.766 leading

Circuit is capacitive.

Active Power P = 150 x 8 x 0.766 = 919.2 W

Reactive Power Q = 150 x 8 x 0.6428 = 771.36 VAR

Example 12

Find the circuit constants of a two elements series circuit which consumes 700 W with 0.707 leading power factor. The applied voltage is V = 141.4 sin 314 t volts.

Solution

$$\left|V\right| = \frac{141.4}{\sqrt{2}} = 99.9849\ V; \quad \text{Since Power } P = \left|V\right|\left|I\right| \cos\theta$$

$$\left|I\right| = \frac{700}{99.9849 \times 0.707} = 9.9025\ A \quad \text{and} \quad \left|Z\right| = \frac{99.9849}{9.9025} = 10.0969\ \Omega$$

From the impedance triangle

Resistance $R = \left|Z\right| \cos\theta = 10.0969 \times 0.707 = 7.1385\ \Omega$

Reactance $X_C = \left|Z\right| \sin\theta = 10.0969 \times 0.707 = 7.1385\ \Omega$

Capacitance $C = \dfrac{1}{314 \times 7.1385} = 446.132\ \mu F$

Example 13

A series R-C circuit consumes a power of 7000 W when connected to 200 V, 50 Hz supply. The voltage across the resistor is 130 V. Calculate (i) the resistance, impedance, capacitance, current and p.f. (ii) Write the equation for the voltage and current.

Solution

Data are shown in Fig. 25.

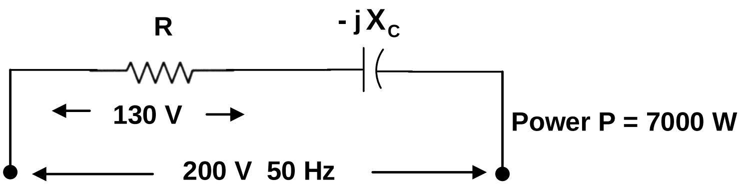


Fig. 25 Circuit – Example 13

Resistance R = $\frac{130^2}{7000} = 2.4143\ \Omega$;   Current $|I| = \frac{130}{2.4143} = 53.8458$ A

Since 200 x 53.8458 x cos θ = 7000, p.f. = 0.65 leading; $\theta = 49.46^0$

From impedance triangle, reactance X_C = R tan θ = 2.4143 x 1.1691 = 2.8226 Ω

Impedance Z = (2.4143 – j 2.8226) Ω = $3.7143\ \angle -49.46^0\ \Omega$

Capacitance C = $\frac{1}{2\ \pi \times 50 \times 2.8226} = 1127.72\ \mu F$

Current I = $53.8458\ \angle 49.46^0$ A;   **Power Factor** = 0.65 leading

Taking supply voltage as reference

$v(t) = \sqrt{2} \times 200 \cos(2\pi \times 50\,t) = 282.8427 \cos 314.16\,t$

$i(t) = \sqrt{2} \times 53.8458 \cos(2\pi \times 50\,t + 49.46^0) = 76.15 \cos(314.16\,t + 49.46^0)$ A

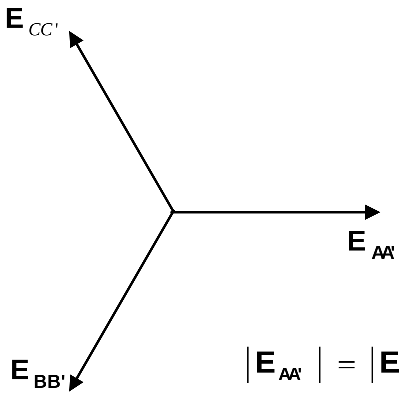
THREE PHASE SYSTEM

In general, generation, transmission and utilization of electric power is more economical in three phase system compared to single phase system.

The windings of three phase alternators are designated as AA', BB' and CC'. The voltages generated in these windings are

$$\left.\begin{aligned} e_{AA'} &= E_m \cos\omega t \\ e_{BB'} &= E_m \cos(\omega t - 120^0) \\ e_{CC'} &= E_m \cos(\omega t - 240^0) \end{aligned}\right\} \tag{77}$$

The phasor descriptions of three voltages are shown in Fig. 39. Here $E_{AA'}$ is taken as reference. Each voltage phasor is lagging the previous one by 120^0.



$$E_{AA'} = E\angle 0^0$$
$$E_{BB'} = E\angle -120^0$$
$$E_{CC'} = E\angle -240^0$$

$$|E_{AA'}| = |E_{BB'}| = |E_{CC'}| = E = \frac{E_m}{\sqrt{2}}$$

Fig. 39 Phasor representation of 3 phase voltages

Generally E $_{AA'}$ is written as E $_{A}$. Other phasors are represented likewise. Thus

$$\left.\begin{aligned} E_A &= E\angle 0^0 \\ E_B &= E\angle -120^0 \\ E_C &= E\angle -240^0 \end{aligned}\right\} \tag{78}$$

The three generator windings are connected either in STAR (wye) or in DELTA.

STAR CONNECTED GENERATOR

Fig. 40 shows the winding connections of star connected generator. The generator is connected to a 3 phase load.

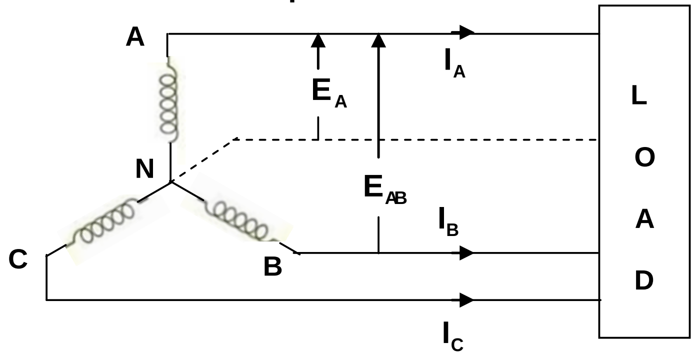


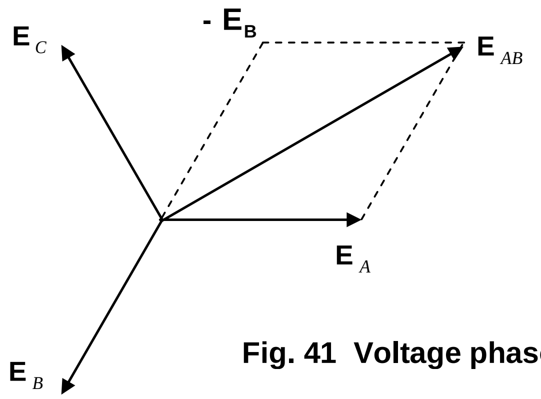
Fig. 40 Star connected generator

E_A, E_B and E_C are called the PHASE VOLTAGES. E_{AB}, E_{BC} and E_{CA} are called the LINE VOLTAGES or line to line voltages. The current flowing in each phase is called PHASE CURRENT and the current flowing in each line called LINE CURRENT.

Let I_l and I_{ph} be the magnitude of line current and phase current and E_l and E_{ph} be the magnitude of line voltage and phase voltage. In case of star connected system

Line current = Phase current i.e. $I_l = I_{ph}$ (79)

Taking E_A as the reference, the voltage phasors are shown in Fig. 41.



$$E_A = E_{ph}\angle 0^0$$
$$E_B = E_{ph}\angle -120^0$$
$$E_C = E_{ph}\angle -240^0$$

Fig. 41 Voltage phasors - Star system

The relationship between line voltage and phase voltage can be obtained as follows.

$$E_{AB} = E_A - E_B = E_{ph} - E_{ph}(-0.5 - j0.866)$$
$$= E_{ph}(1.5 + j0.866)$$
$$= \sqrt{3}\,E_{ph}\angle 30^0$$

The above result can be seen from the Fig. 41.

Similar expression can be obtained for E_{BC} and E_{CA} also. Collectively, we have

$$\left.\begin{aligned} E_{AB} &= \sqrt{3}\, E_{ph} \angle 30^0 \\ E_{BC} &= \sqrt{3}\, E_{ph} \angle -90^0 \\ E_{CA} &= \sqrt{3}\, E_{ph} \angle 150^0 \end{aligned}\right\} \qquad (80)$$

Thus $E_l = \left|E_{AB}\right| = \left|E_{BC}\right| = \left|E_{CA}\right| = \sqrt{3} E_{ph}$

Therefore for star connected system

$$\left.\begin{aligned} E_l &= \sqrt{3} E_{ph} \\ I_l &= I_{ph} \end{aligned}\right\} \qquad (81)$$

$$\left.\begin{array}{l}\textbf{Power supplied by the} \\ \textbf{three phase alternator}\end{array}\right\} \begin{aligned} &= 3 \text{ x phasepower} \\ &= 3\, E_{ph}\, I_{ph} \cos\theta \qquad (82) \\ &= \sqrt{3}\, E_l\, I_l \cos\theta \qquad (83) \end{aligned}$$

DELTA CONNECTED GENERATOR

Delta connected generator is shown in Fig. 42.

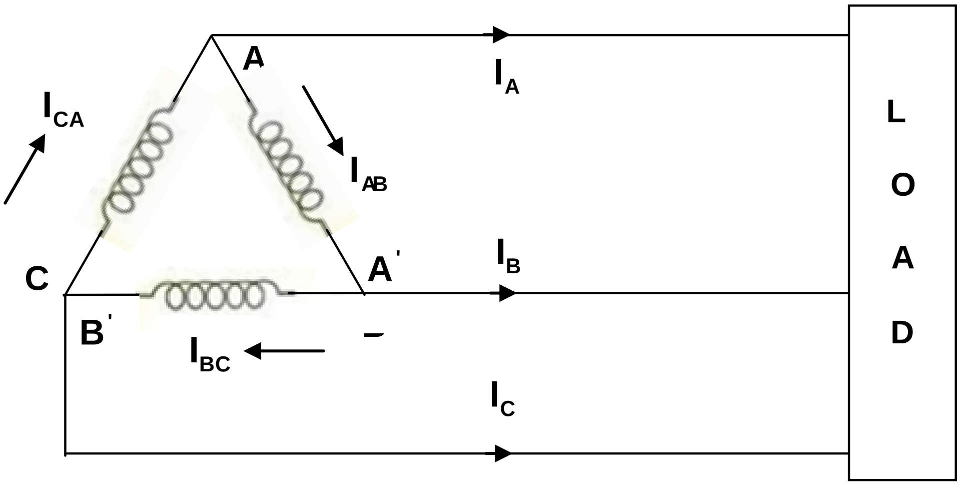


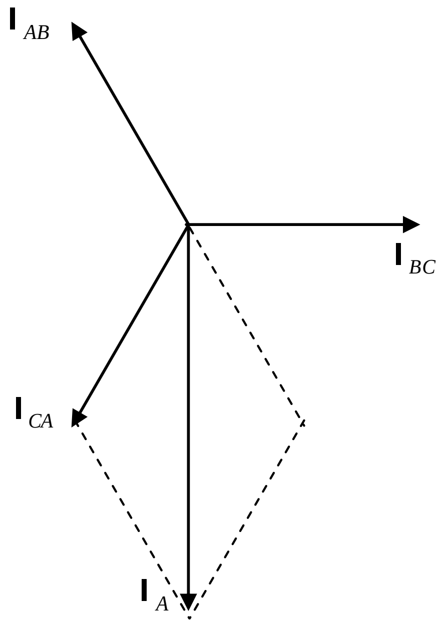
Fig. 42 Delta connected generator

I_A, I_B and I_C are called LINE CURRENTS. I_{AB}, I_{BC} and I_{CA} are called PHASE CURRENTS. The voltage across each phase is called PHASE VOLTAGE and voltage across two lines is called LINE VOLTAGE or line to line to line voltage.

In case of delta connected system, line voltage is equal to phase voltage. i.e.

$$E_l = E_{ph} \tag{84}$$

Taking $\mathbf{I}_{BC}$ as reference, current phasors are shown in Fig. 43.



$$\mathbf{I}_{BC} = \mathbf{I}_{ph} \angle 0^0$$
$$\mathbf{I}_{CA} = \mathbf{I}_{ph} \angle -120^0$$
$$\mathbf{I}_{AB} = \mathbf{I}_{ph} \angle -240^0$$

Fig. 43 Current phasors – Delta connected system

Considering the junction point formed by A and C

$$\mathbf{I}_A = \mathbf{I}_{CA} - \mathbf{I}_{AB} = \mathbf{I}_{ph}\,(-0.5 - j\,0.866) - \mathbf{I}_{ph}\,(-0.5 + j\,0.866) = -j\,\sqrt{3}\,\mathbf{I}_{ph}$$

The above result can be seen from Fig. 43. Similar expression can be obtained for $\mathbf{I}_B$ and $\mathbf{I}_C$. Collectively, we have

$$\left.\begin{aligned} I_A &= \sqrt{3}\, I_{ph} \angle -90^0 \\ I_B &= \sqrt{3}\, I_{ph} \angle -210^0 \\ I_C &= \sqrt{3}\, I_{ph} \angle 30^0 \end{aligned}\right\} \qquad (85)$$

Therefore $I_l = |I_A| = |I_B| = |I_C| = \sqrt{3}\, I_{ph}$

Thus for delta connected system

$$\left.\begin{aligned} E_l &= E_{ph} \\ I_l &= \sqrt{3}\, I_{ph} \end{aligned}\right\} \qquad (86)$$

$$\left.\begin{array}{l} \text{Power supplied by the} \\ \text{three phase alternator} \end{array}\right\} \begin{aligned} &= 3 \text{ x phase power} \\ &= 3\, E_{ph}\, I_{ph} \cos\theta && (87) \\ &= \sqrt{3}\, E_l\, I_l \cos\theta && (88) \end{aligned}$$

As seen from eq. (83) and eq.(88), the power supplied by the alternator is $\sqrt{3}\, E_l\, I_l \cos\theta$ whether it is connected in star or delta.

Unit-1

BALANCED THREE PHASE CIRCUITS

1.1 Introduction:

There are two types of systems available in electrical circuits, single phase and three phase. In single phase circuits, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, new invention has been done polyphase system, that more than one phase can be used for generating, transmitting and for load system. Three phase circuit is the polyphase system where three phases are send together from generator to the load. Each phase are having a phase difference of 120^0, i.e 120^0 angle electrically. So from the total of 360^0, three phase are equally divided into 120^0 each. The power in three phase system is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below .

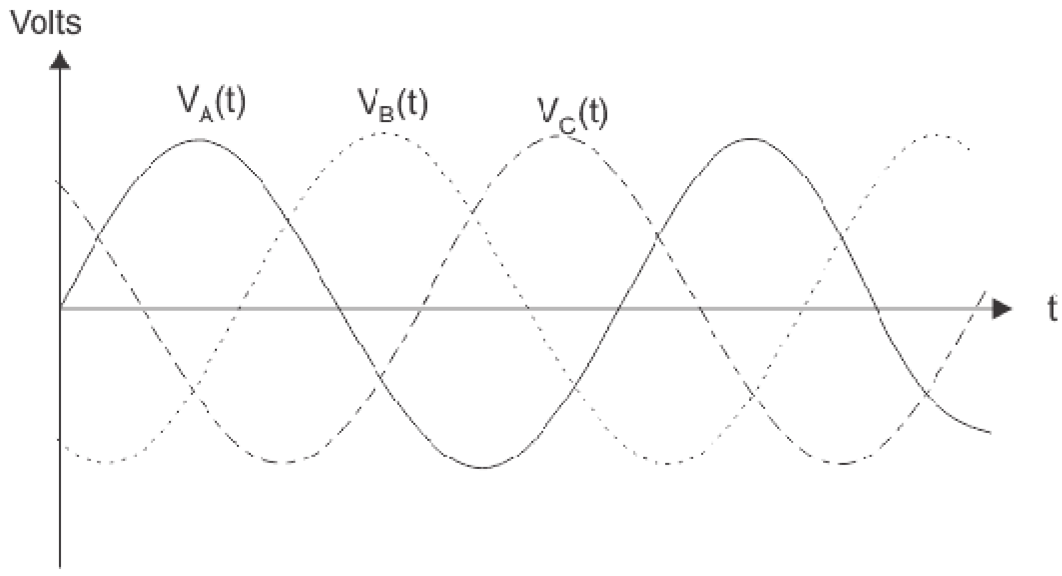


Fig.1.1

The three phase can be used as single phase each. So if the load is single phase, then one phase can be taken from the three phase circuit and the neutral can be used as ground to complete the circuit.

1.1.1 Why three phase is preferred over single phase?

There are various reasons for this question because there are numbers of advantages over single phase circuit. The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120^0 phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit.

And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load.

Till now we can say that there are three voltage sources connected together to form a three phase circuit and actually it is inside generator. The generator is having three voltage sources which are acting together in 120^0 phase difference. If we can arrange three single phase circuit with 120^0 phase difference, then it will become a three phase circuit. So 120^0 phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three phase devices is not having much difference. Now of we consider the transformer, it will be almost same size fro both single phase and three phase because transformer will make only the linkage of flux. So the three phase system will have higher efficiency compared to single phase for the same or little difference in mass of transformer, three phase line will be out whereas in single phase will be only one. And losses will be minimum in three phase circuit. So overall in conclusion the three phase system will have better and higher efficiency compared to the single phase system.

A balanced polyphase system is one in which there are two or more equal voltages of the same frequency displaced equally in time phase, which supply power to loads connected to the lines. In general, in a *n*-phase balanced polyphase system, there are *n*-equal voltages displaced in time phase by $\frac{360^0}{n}$ or $\frac{2\pi}{n}$ (except in the case of a 2-phase system, in which there are two equal voltages differing in phase by 90^0). Systems of six or more phases are used in polyphase rectifiers to obtain rectified voltage with low ripple. But three phase system is most commonly used polyphase system for generation and transmission of power. Hence we study in detail the 3-phase voltage generation and analysis of 3-phase circuit in this unit.

A 3-phase system has the following advantages over single phase system. For a given frame size of a machine a 3-phase machine will have large capacity than a single phase machine. The torque produced in a 3-phase motor will be more uniform where as in a 1-phase motor it is pulsating. The amount of copper required in a certain amount of power over a particular distance, is less compared to a single phase system.

1.1.2 Phase sequence:

It is the order in which the phase voltages will attain their maximum values. From the fig it is seen that the voltage in A phase will attain maximum value first and followed by B and C phases. Hence three phase sequence is ABC. This is also evident from phasor diagram in which the phasors with its +ve direction of anti-clockwise rotation passes a fixed point is the order ABC, ABC and so on. The phase sequence depends on the direction of rotation of the coils in the magnetic field. If the coils rotate in the opposite direction then the phase voltages attains maximum value in the order ACB. The phase sequence gets reversed with direction of rotation. Then the voltage for this sequence can be represented as

$$e_a = E_m \sin \omega t$$
$$e_c = E_m \sin(\omega t - 120^0)$$
$$e_b = E_m \sin(\omega t - 240^0)$$

The RMS values of voltage can be expressed as

$$E_A = E\angle 0^0$$
$$E_C = E\angle -120^0$$
$$E_B = E\angle -240^0$$

1.1.3 Star and Delta connection

The three phase windings have six terminals i.e., A,B,C are starting end of the windings and A’,B’ and C’ are finishing ends of windings. For 3 phase systems two types of common interconnections are employed.

1.1.3(a) Star connection: the finishing ends or starting ends of the three phase windings are connected to a common point as shown in. A’, B’, C’ are connected to a common point called neutral point. The other ends A, B, C are called line terminals and the common terminal neutral are brought outside. Then it is called a 3 phase 4 wire star connected systems. If neutral point is not available, then it is called 3 phase, 3 wire star connection.

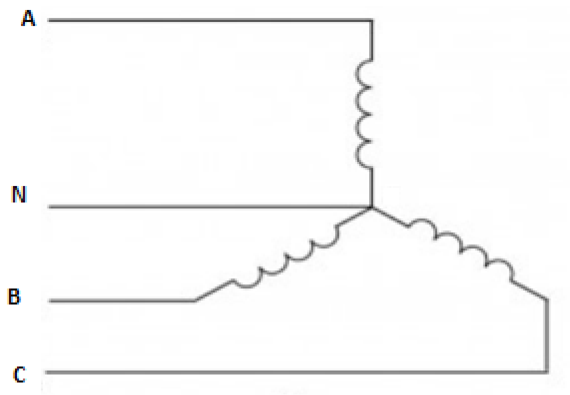


Fig.1.2

1.1.3(b) Delta connection: in this form of interconnection the dissimilar ends of the three coils i.e A and B’, B and C’, and C and A’ are connected to form a closed Δ circuit (starting end of one phase is connected to finishing end of the next phase). The three junction are brought outside as line terminal A, B, C. the three phase windings are connected in series and form a closed path. The sum of the voltages in the closed path for balanced system of voltages at any instant will be zero fig.

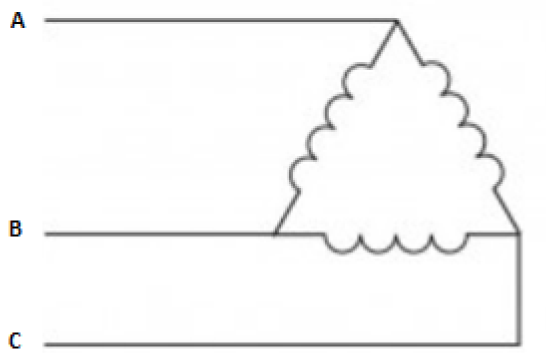


Fig.1.3

The main advantage of star connection is that we can have two different 3-phase voltages. The voltage that was the line terminals between A & B, B&C, and C & A are called line voltages and form a balanced three phase voltage. Another voltage is between the terminals A & N, B& N, and C &N are called phase voltage and form another balanced three phase voltage (line to neutral voltage or wye voltage).

1.2 Relation between line and phase voltage and currents in balanced systems:

In this section we will derive the relation between line and phase values of voltages and currents of 3-phase star connected and delta connected systems.

1.2.1 Star connection:

We will employ double subscript notation to represent voltages and currents. The terminal corresponding to first subscript is assumed to be at a higher potential with respect to the terminal corresponding to second subscript.

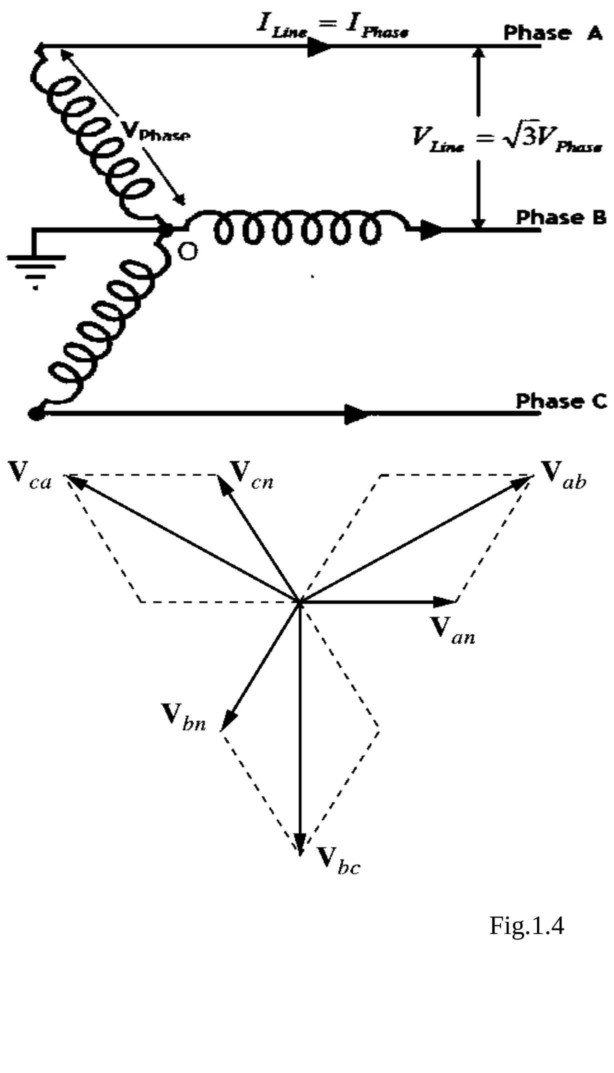


Fig.1.4

The voltage across each coil, i.e., the voltage between A & A',B& B', and C & C' are called phase voltages(acting from finishing end to starting end).

V_{AA}, V_{BB}, V_{CC}, or V_{AN}, V_{BN}, V_{CN} represent phase voltages.

The voltages across line terminals A & B, B & C, C & D are called line voltages. The connection diagram and the corresponding phasor diagram of voltages is shown in fig. From the star connected 3 phase system, it is clearly observed that whatever currents flow through the lines A, B, C also flow through the respective phase windings. Hence in star connected system, the phase currents and line currents are identical.

$$\text{Phase current } (I_{ph}) = \text{Line currents } (I_L)$$

$$I_{ph} = I_{Line}$$

The voltage V_{AB} between lines A and B is obtained by adding V_{AN} and V_{NB} respectively.

$$V_{AB} = V_{AN}+V_{NB} = V_{AN} - V_{BN}$$

Similarly

$$V_{BC} = V_{BN}+V_{NC} = V_{BN} – V_{CN}$$

$$V_{CA} = V_{CN}+V_{NA} = V_{CN} – V_{AN}$$

The line voltage V_{AB} is obtained by adding V_{AN} with reversed vector of V_{BN}.V_{AB} bisects the angle between V_{AN} and $-V_{BN}$

$$V_{AB}^2 = V_L^2 = V_{ph}^2 + V_{ph}^2 + 2\, V_{ph}V_{ph}\cos 60^0$$

$$= 3V_{ph}^2$$

$$V_{AB} = \sqrt{3}\, V_{ph}$$

$$\text{Line voltage} = \sqrt{3} \text{ phase voltage}$$

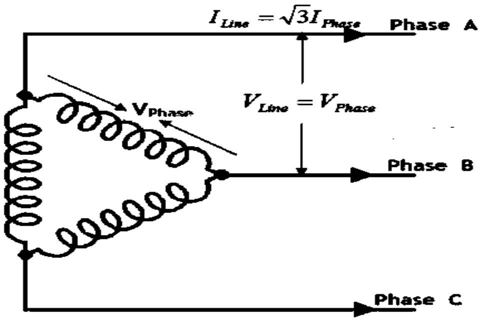
The line voltages V_{AB}, V_{BC}, V_{CA} are equal in magnitude and differ in phase by 120^0. Hence they form a balanced 3-phase voltage of magnitude $\sqrt{3}\, V_{ph}$. The two voltages differ in phase by 30^0. When the system is balanced, the three phase currents I_A, I_B, I_C are balanced. The magnitude and phase angle of current is determined by circuit parameters.

I_A, I_B, I_C are line or phase currents. The current in the neutral wire is I_N and is by applying kirchoff's current law at star point, we get

$$I_N = -(I_A + I_B + I_C)$$

If the currents are balanced, then the neutral current is zero.

1.2.2 Delta connection or MESH connection:



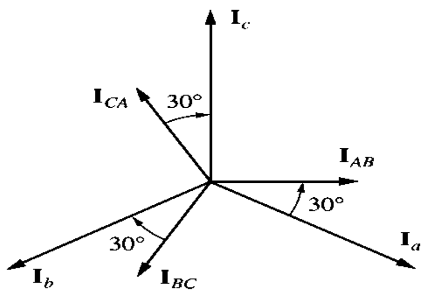


Fig.1.5.

The currents flowing through the phase windings I_{AA}', I_{BB}', and I_{CC}' or I_{AB}, I_{BC}, and I_{CA} are called phase currents and are balanced as shown in phase diagram Fig.1.5.

By applying KCL at node A

$$I_A + I_{CA} = I_{AB} \text{ , } I_A = I_{AB} - I_{CA}$$

Similarly by applying KCL at nodes B and C

$$I_B = I_{BC} - I_{AB} I_C = I_{CA} - I_{BC}$$

The line current I_A is obtained by adding I_{AB} and $-I_{CA}$ vectorially. I_A bisects the angle between I_{AB} and $-I_{CA}$

$$I_A^2 = I_{Line}^2 = I_{ph}^2 + I_{ph}^2 + 2\, I_{ph} I_{ph} \cos 60^0$$

$$= 3 I_{ph}^2$$

$$I_L = \sqrt{3}\, I_{ph}$$

Line current(I_L) = √3 phase voltage(I_{ph})

The line current I_A, I_B, I_C and also equal and differ in phase by 120^0. They form a balanced system of currents. The line and phase currents differ in phase by 30^0.

1.3 Analysis of balanced three phase circuits

A set of three impedances interconnected in the form of a star or delta form a 3-phase star or delta connected load. If the three impedances are identical and equal then it is a balanced 3-phase load, otherwise it is an unbalanced 3-phase load.

The analysis of balanced 3-phase circuits is illustrated as follows

1.3.1 Balanced delta connected load:

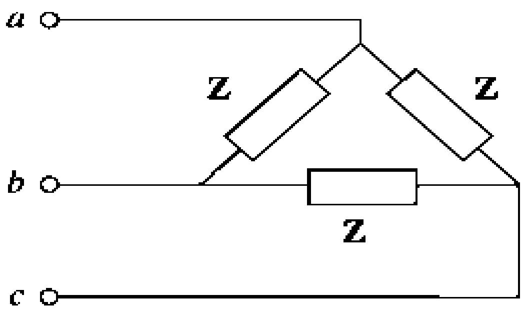


Fig.1.6

Let us consider a balanced 3-phase delta connected load

Determination of phase voltages:

$$V_{AB} = V\angle 0^0,\ V_{BC} = V\angle -120^0,\ V_{CA} = V\angle -240^0 = V\angle 120^0$$

Determination of phase currents:

Phase current = Phase voltage/ Load impedance

$$I_{AB}=\frac{V_{AB}}{Z}\ ;\ I_{BC}=\frac{V_{BC}}{Z}\ ;\ I_{CA}=\frac{V_{CA}}{Z}$$

Determination of line currents:

Line currents are calculated by applying KCL at nodes A,B,C

$$I_A = I_{AB} - I_{CA}\ ;\ I_B = I_{BC} - I_{AB}\ ;\ I_C = I_{CA} - I_{BC}$$

Note: Line currents are also balanced and equal to $\sqrt{3}$phase current.

1.3.2 Balanced star connected load:

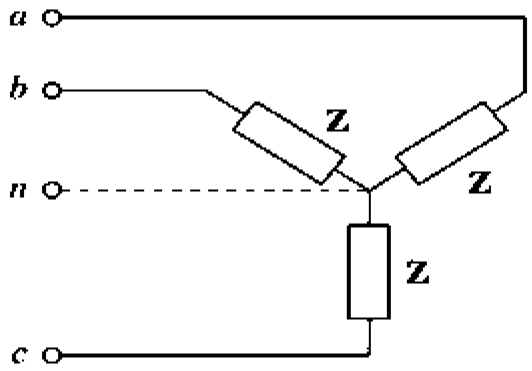


Fig.1.7

Let us consider a balanced 3-phase star connected load.

For star connection, phase voltage= Line voltage/(√3)

For ABC sequence, the phase voltage is polar form are taken as

$$V_{AN} = V_{ph}\angle -90^0 \; ; V_{CN} = V_{ph}\angle 150^0 \; ; V_{BN} = V_{ph}\angle 30^0$$

For star connection line currents and phase currents are equal

$$I_A = \frac{V_{AN}}{Z} \; ; I_B = \frac{V_{BN}}{Z} \; ; I_C = \frac{V_{CN}}{Z} \; ;$$

To determine the current in the neutral wire apply KVL at star point

$$I_N + I_A + I_B + I_C = 0$$

$$I_N = -(I_A + I_B + I_C) \quad \text{(since they are balanced)}$$

In a balanced system the neutral current is zero. Hence if the load is balanced, the current and voltage will be same whether neutral wire is connected or not. Hence for a balanced 3-phase star connected load, whether the supply is 3-phase 3 wire or 3-phase 4 wire, it is immaterial. In case of unbalanced load, there will be neutral current.

Module 2

DC Circuit

Lesson 6

Wye (Y) - Delta (Δ) OR Delta (Δ)-Wye (Y) Transformations

Objectives

- A part of a larger circuit that is configured with three terminal network Y (or Δ) to convert into an equivalent Δ (or Y) through transformations.
- Application of these transformations will be studied by solving resistive circuits.

L.6.1 Introduction

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration. A circuit configuration shown below

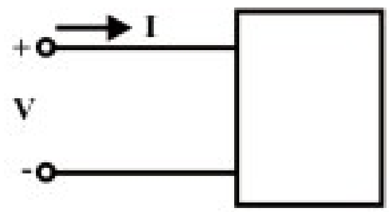


Fig. 6.1(a) One port network

is a general **one-port circuit.** When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a **one-port.** On the other hand, a **two-port** is a circuit having two pairs of terminals. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

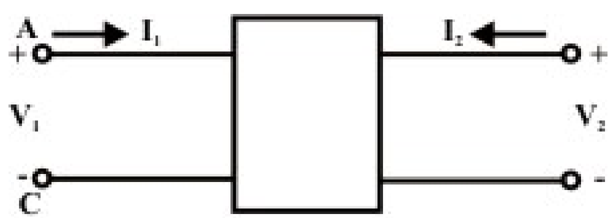


Fig. 6.1(b) Two port network

Fig.6.1.(b) can be described as a four terminal network, for convenience subscript 1 to refer to the variables at the input port (at the left) and the subscript 2 to refer to the variables at the output port (at the right). The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readially possible to consider the 'π *or* Δ' – network also as a three-terminal network in fig.6.1(c). Another frequently encountered circuit configuration that shown in fig.6.1(d) is approximately refered to as a three-terminal Y connected circuit as well as two-port circuit.

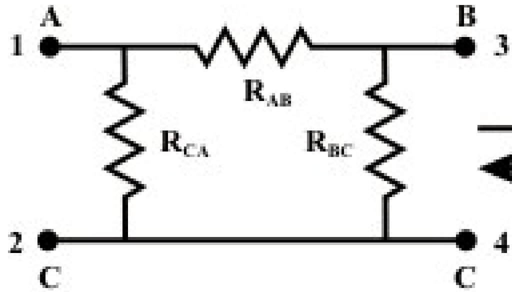


Fig. 6.1 (c)

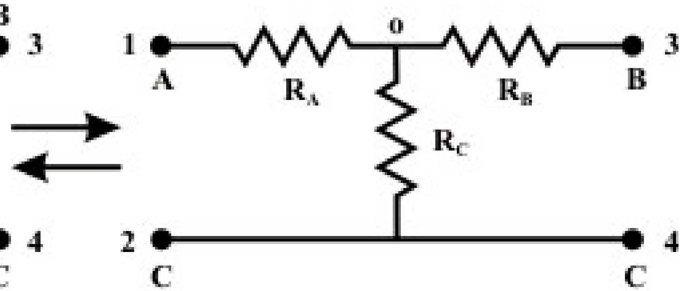


Fig. 6.1 (d)

The name derives from the shape or configuration of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter Δ.

L.6.1.1 Delta (Δ) – Wye (Y) conversion

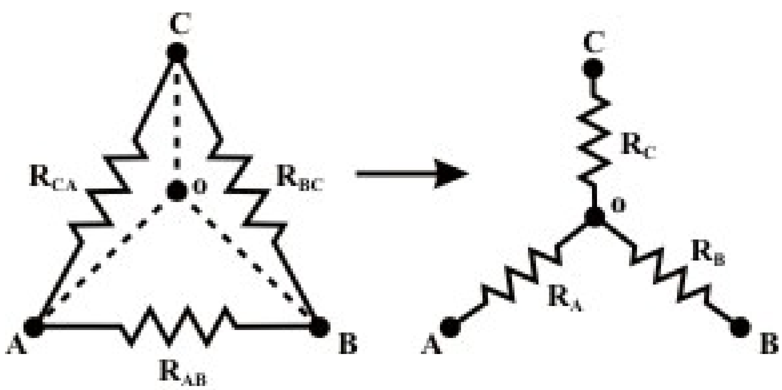


Fig. 6.1 (e)

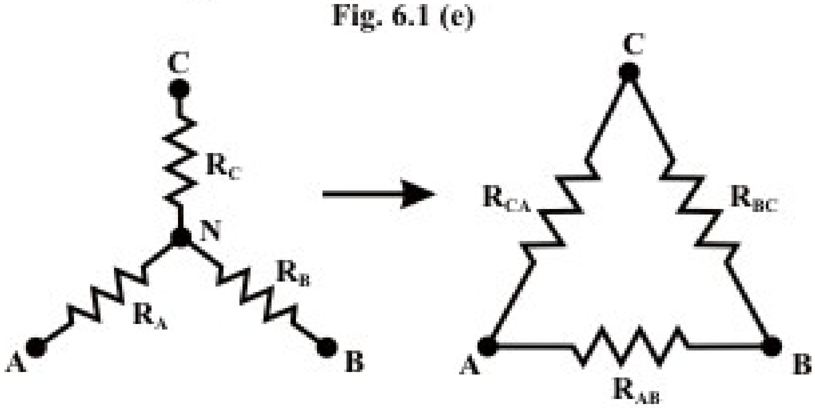


Fig. 6.1 (f)

These configurations may often be handled by the use of a $\Delta - Y$ or $Y - \Delta$ transformation. One of the most basic three-terminal network equivalent is that of three resistors connected in "Delta(Δ)" and in "Wye(Y)". These two circuits identified in fig.L6.1(e) and Fig.L.6.1(f) are sometimes part of a larger circuit and obtained their names from their configurations. These three terminal networks can be redrawn as four-terminal networks as shown in fig.L.6.1(c) and fig.L.6.1(d). We can obtain useful expression for direct transformation or conversion from Δ to Y or Y to Δ by considering that for equivalence the two networks have the same resistance when looked at the similar pairs of terminals.

L.6.2 Conversion from Delta (Δ) to Star or Wye (Y)

Let us consider the network shown in fig.6.1(e) (or fig. $6.1(c)\rightarrow$) and assumed the resistances (R_{AB} , R_{BC} , *and* R_{CA}) in Δ network are known. Our problem is to find the values of R_A, R_B, *and* R_C in Wye (Y) network (see fig.6.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.1)$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.2)$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.3)$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.4)$$

Subtracting equations (6.2), (6.1), and (6.3) from (6.4) equations, we can write the express for unknown resistances of Wye (Y) network as

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.5)$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.6)$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad (6.7)$$

L.6.2.1 Conversion from Star or Wye (Y) to Delta (Δ)

To convert a **Wye** ($\underline{Y}$) to a **Delta** (Δ), the relationships R_{AB}, R_{BC}, *and* R_3 must be obtained in terms of the **Wye** ($\underline{Y}$) resistances R_A, R_B, *and* R_C (referring to fig.6.1 (f)). Considering the Y connected network, we can write the current expression through R_A resistor as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad \text{(for } Y \text{ network)} \tag{6.8}$$

Appling KCL at 'N' for Y connected network (assume A, B, C terminals having higher potential than the terminal N) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \quad \Rightarrow V_N\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)$$

or, $$\Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)} \tag{6.9}$$

For Δ-network (see fig.6.1.(f)),
Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \text{(for } \Delta \text{ network)} \tag{6.10}$$

From equations (6.8) and (6.10),

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the V_N expression in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \Rightarrow \quad \frac{\left(\frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

or $$\frac{\left(\frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \tag{6.11}$$

Equating the coefficients of V_{AB} and V_{AC} in both sides of eq.(6.11), we obtained the following relationship.

$$\frac{1}{R_{AB}}=\frac{1}{R_A R_B\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AB}=R_A+R_B+\frac{R_A R_B}{R_C} \qquad (6.12)$$

$$\frac{1}{R_{AC}}=\frac{1}{R_A R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AC}=R_A+R_C+\frac{R_A R_C}{R_B} \qquad (6.13)$$

Similarly, I_B for both the networks (see fig.61(f)) are given by

$$I_B=\frac{\left(V_B-V_N\right)}{R_B} \text{ (for } Y \text{ network)}$$

$$I_B=\frac{V_{BC}}{R_{BC}}+\frac{V_{BA}}{R_{BA}} \text{ (for } \Delta \text{ network)}$$

Equating the above two equations and using the value of V_N (see eq.(6.9), we get the final expression as

$$\frac{\left(\frac{\left(\frac{V_{BC}}{R_C}+\frac{V_{BA}}{R_A}\right)}{\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)}\right)}{R_B}=\frac{V_{BC}}{R_{BC}}+\frac{V_{BA}}{R_{BA}}$$

Equating the coefficient of V_{BC} in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}}=\frac{1}{R_B R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{BC}=R_B+R_C+\frac{R_B R_C}{R_A} \qquad (6.14)$$

When we need to transform a Delta (Δ) network to an equivalent Wye (Y) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye (Y) to Delta (Δ) conversion.

Observations

In order to note the symmetry of the transformation equations, the Wye (Y) and Delta (Δ) networks have been superimposed on each other as shown in fig. 6.2.

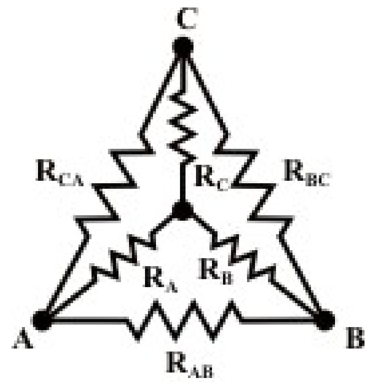


Fig. 6.2

- The equivalent star (Wye) resistance connected to a given terminal is equal to the product of the two Delta (Δ) resistances connected to the same terminal divided by the sum of the Delta (Δ) resistances (see fig. 6.2).
- The equivalent Delta (Δ) resistance between two-terminals is the sum of the two star (Wye) resistances connected to those terminals plus the product of the same two star (Wye) resistances divided by the third star (Wye (Y)) resistance (see fig.6.2).

L.6.3 Application of Star (Y) to Delta (Δ) or Delta (Δ) to Star (Y) Transformation

Example: L.6.1 Find the value of the voltage source (V_S) that delivers 2 Amps current through the circuit as shown in fig.6.3.

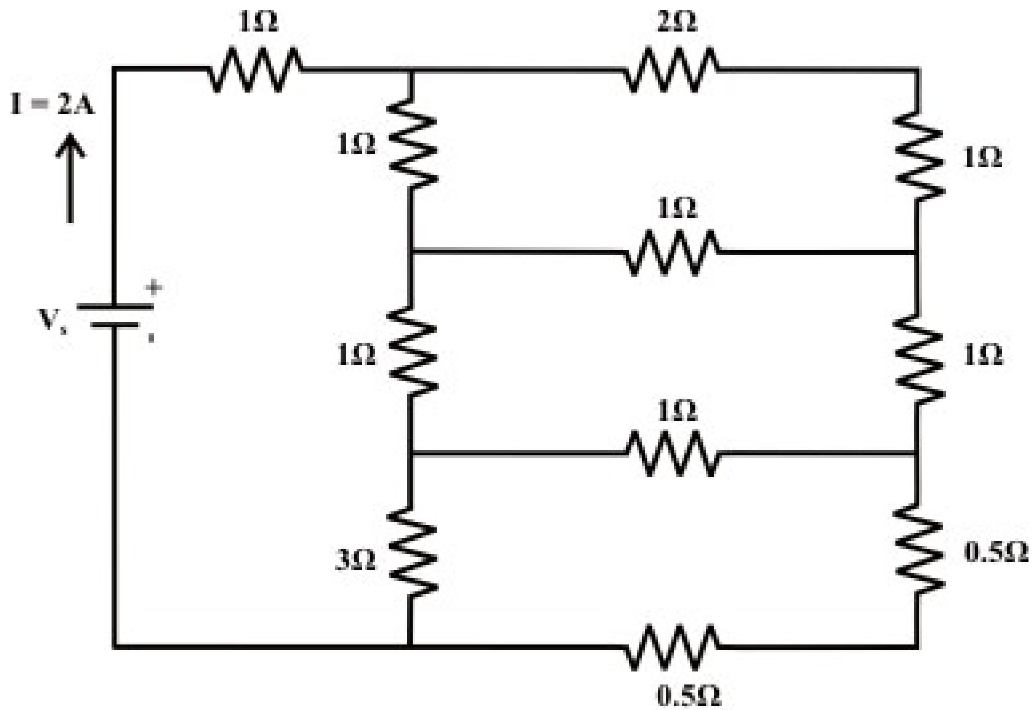
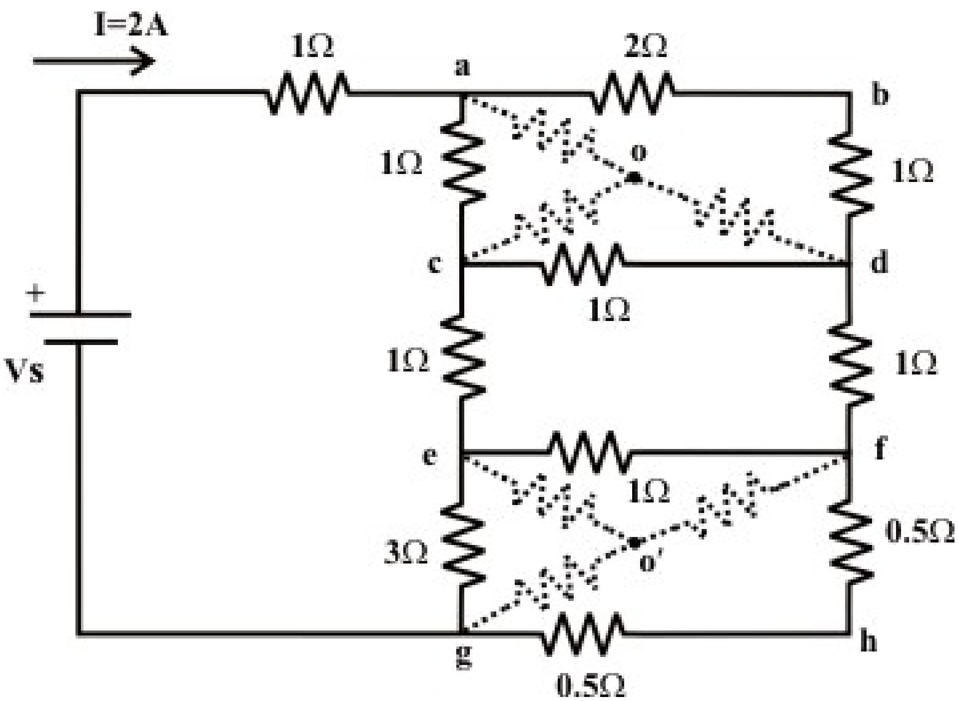


Fig. 6.3

Solution:



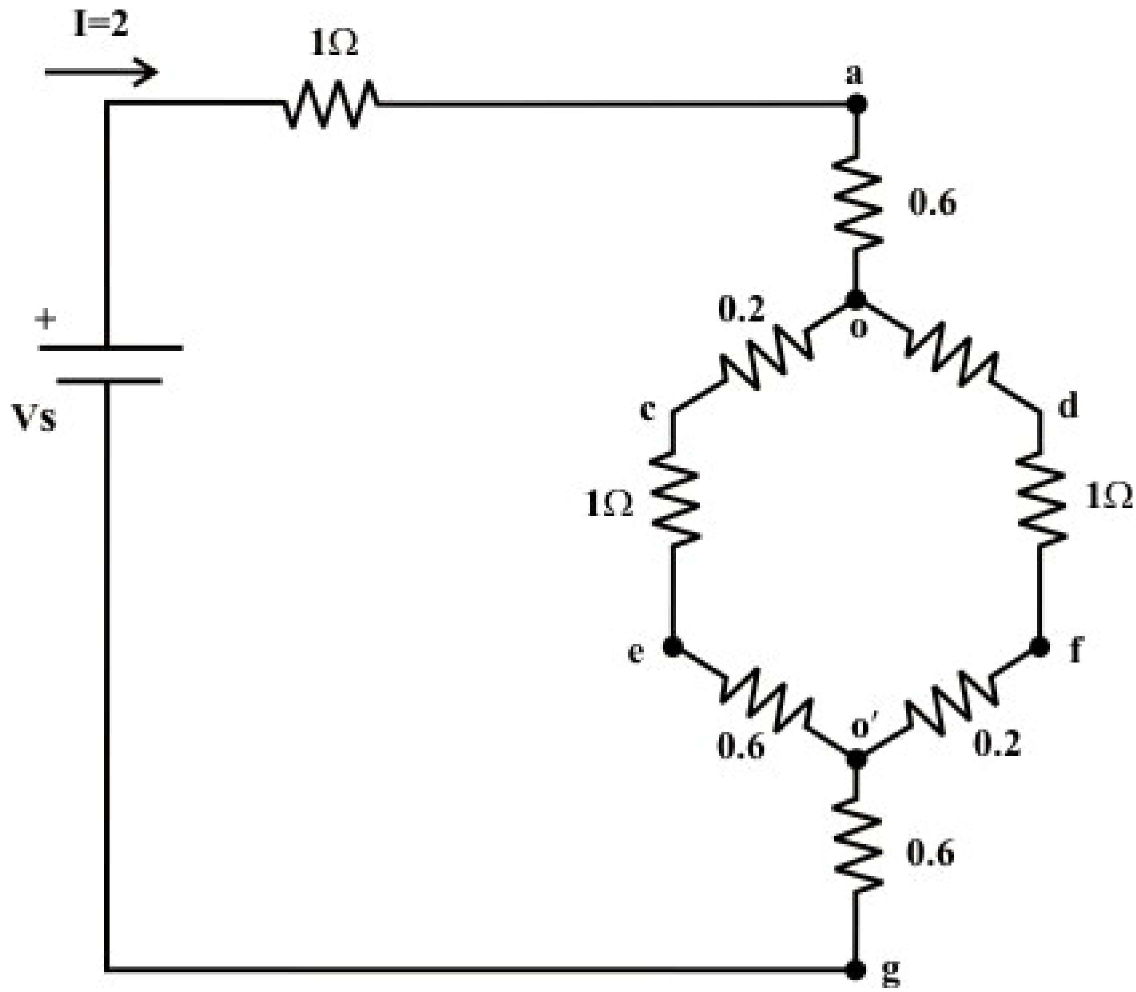
Convert the three terminals Δ-network (a-c-d & e-f-g) into an equivalent Y-connected network. Consider the Δ-connected network 'a-c-d' and the corresponding equivalent Y-connected resistor values are given as

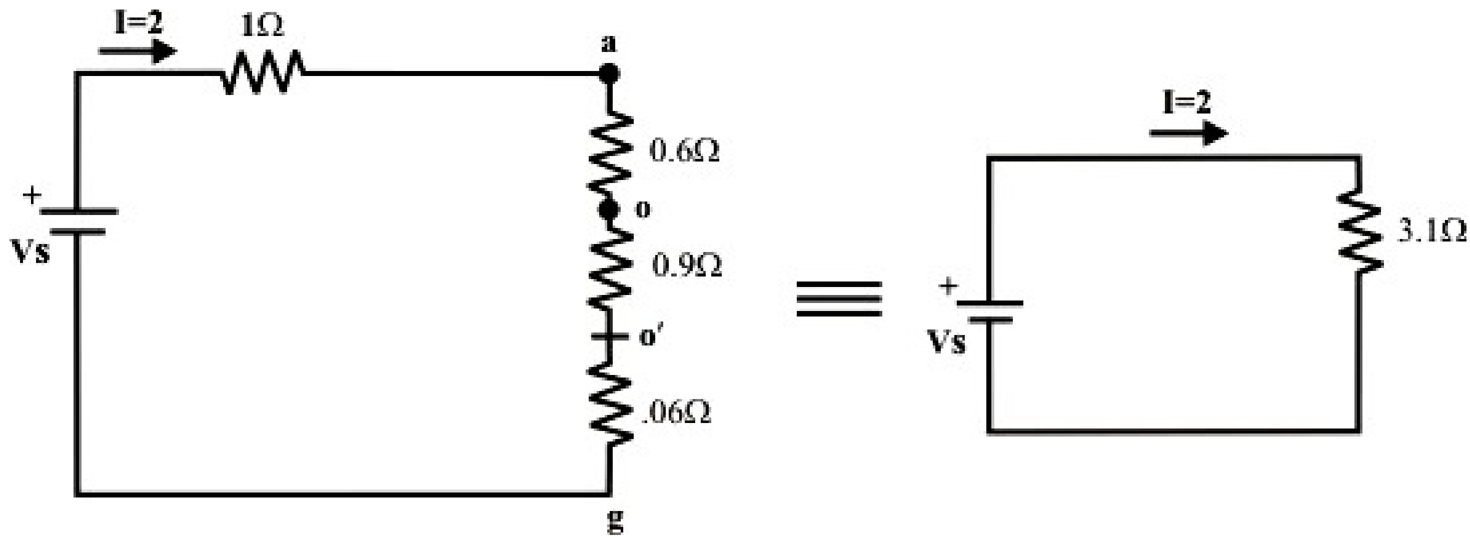
$$R_{ao} = \frac{3\times1}{5} = 0.6\,\Omega;\; R_{co} = \frac{1\times1}{5} = 0.2\,\Omega;\; R_{do} = \frac{3\times1}{5} = 0.6\,\Omega$$

Similarly, for the Δ-connected network 'e-f-g' the equivalent the resistances of Y-connected network are calculated as

$$R_{eo'} = \frac{3\times1}{5} = 0.6\Omega;\; R_{go'} = \frac{3\times1}{5} = 0.6\,\Omega; R_{fo'} = \frac{1\times1}{5} = 0.2\,\Omega$$

Now the original circuit is redrawn after transformation and it is further simplified by applying series-parallel combination formula.





The source V_s that delivers $2A$ current through the circuit can be obtained as $V_s = I \times 3.2 = 2 \times 3.1 = 6.2 Volts$.

Example: L.6.2 Determine the equivalent resistance between the terminals $A \text{ and } B$ of network shown in fig.6.4 (a).

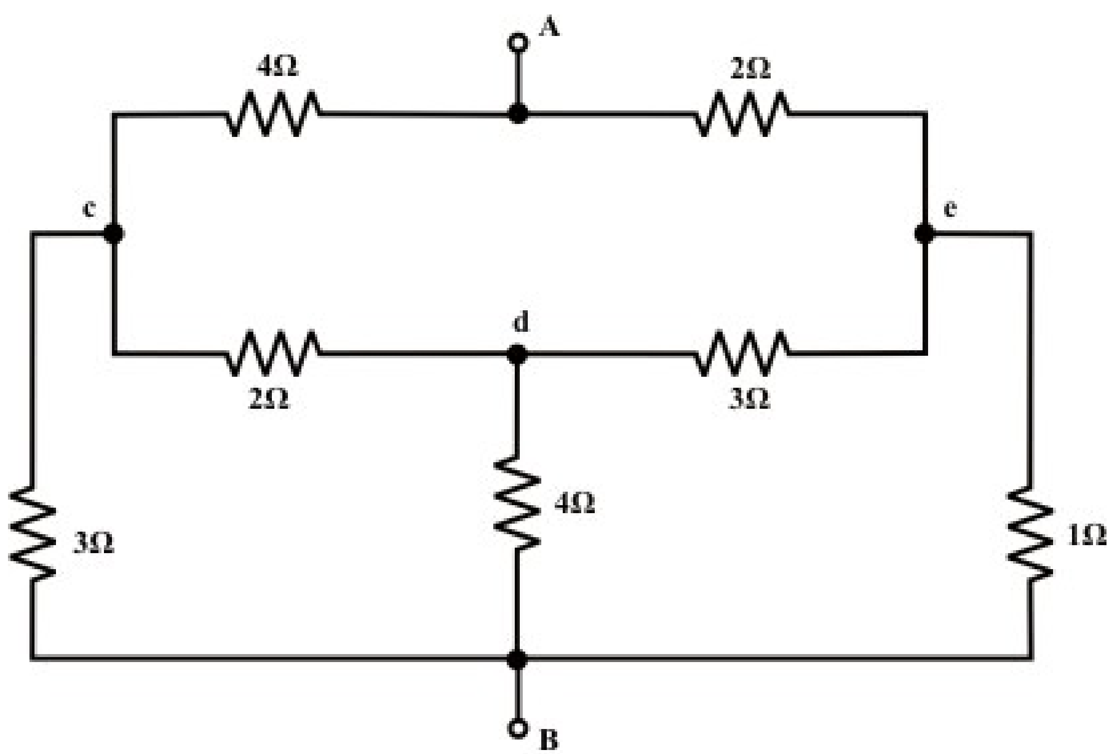


Fig. 6.4 (a)

Solution:

A 'Δ' is substituted for the 'Y' between points c, d, and e as shown in fig.6.4(b); then unknown resistances value for Y *to* Δ transformation are computed below.

$$R_{cB} = 2+4+\frac{2\times 4}{3} = 8.66\,\Omega;\; R_{eB} = 3+4+\frac{4\times 3}{2} = 13\,\Omega;\; R_{ce} = 2+3+\frac{2\times 3}{4} = 6.5\,\Omega$$

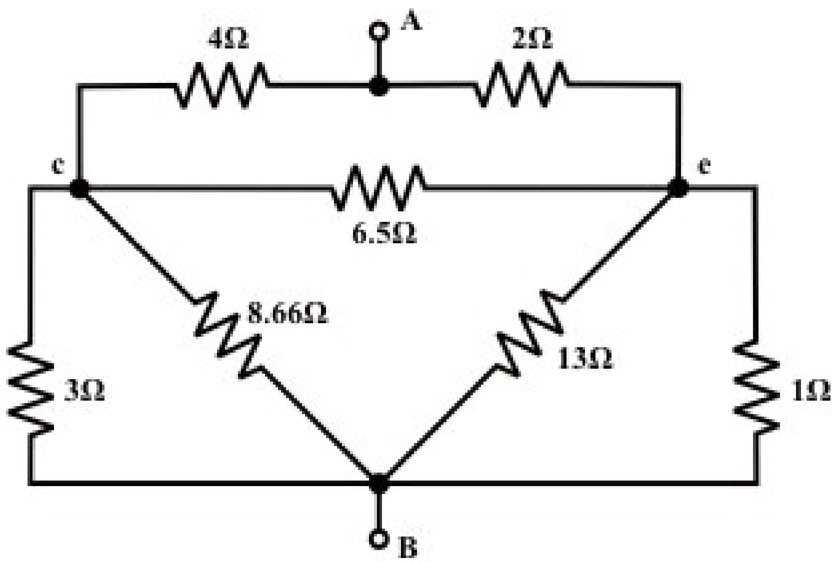


Fig. 6.4 (b)

Next we transform 'Δ'connected 3-terminal resistor to an equivalent 'Y' connected network between points 'A'; 'c' and 'e' (see fig.6.4(b)) and the corresponding Y connected resistances value are obtained using the following expression. Simplified circuit after conversion is shown in fig. 6.4(c).

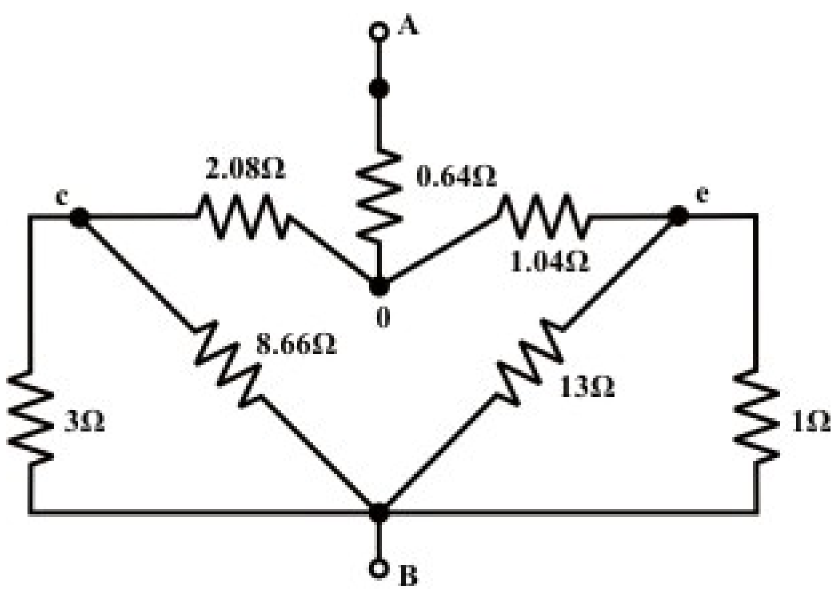


Fig. 6.4 (c)

$R_{Ao} = \dfrac{4 \times 2}{4+2+6.5} = 0.64\,\Omega$; $R_{co} = \dfrac{4 \times 6.5}{4+2+6.5} = 2.08\,\Omega$; $R_{eo} = \dfrac{6.5 \times 2}{4+2+6.5} = 1.04\,\Omega$;

The circuit shown in fig.6.5(c) can further be reduced by considering two pairs of parallel branches $3 \,||\, 8.66$ *and* $13 \,||\, 1$ and the corresponding simplified circuit is shown in fig.6.4(d).

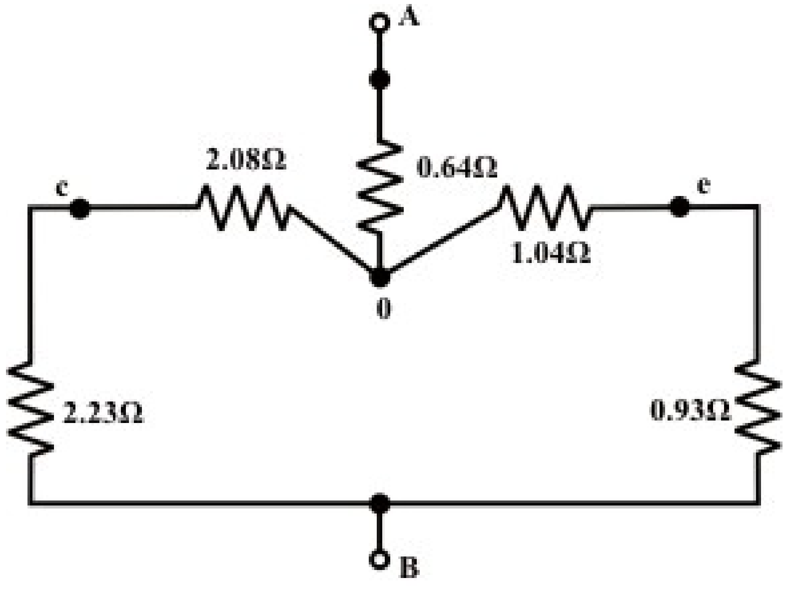
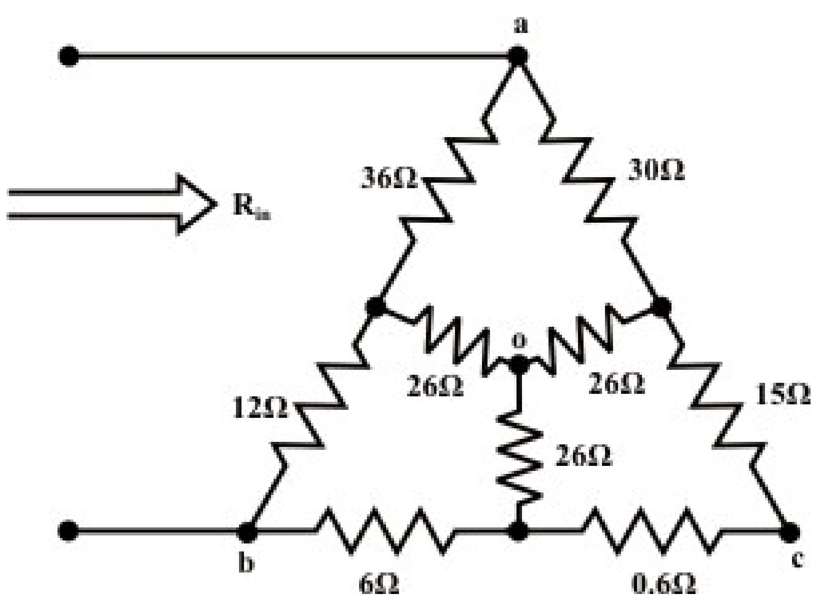


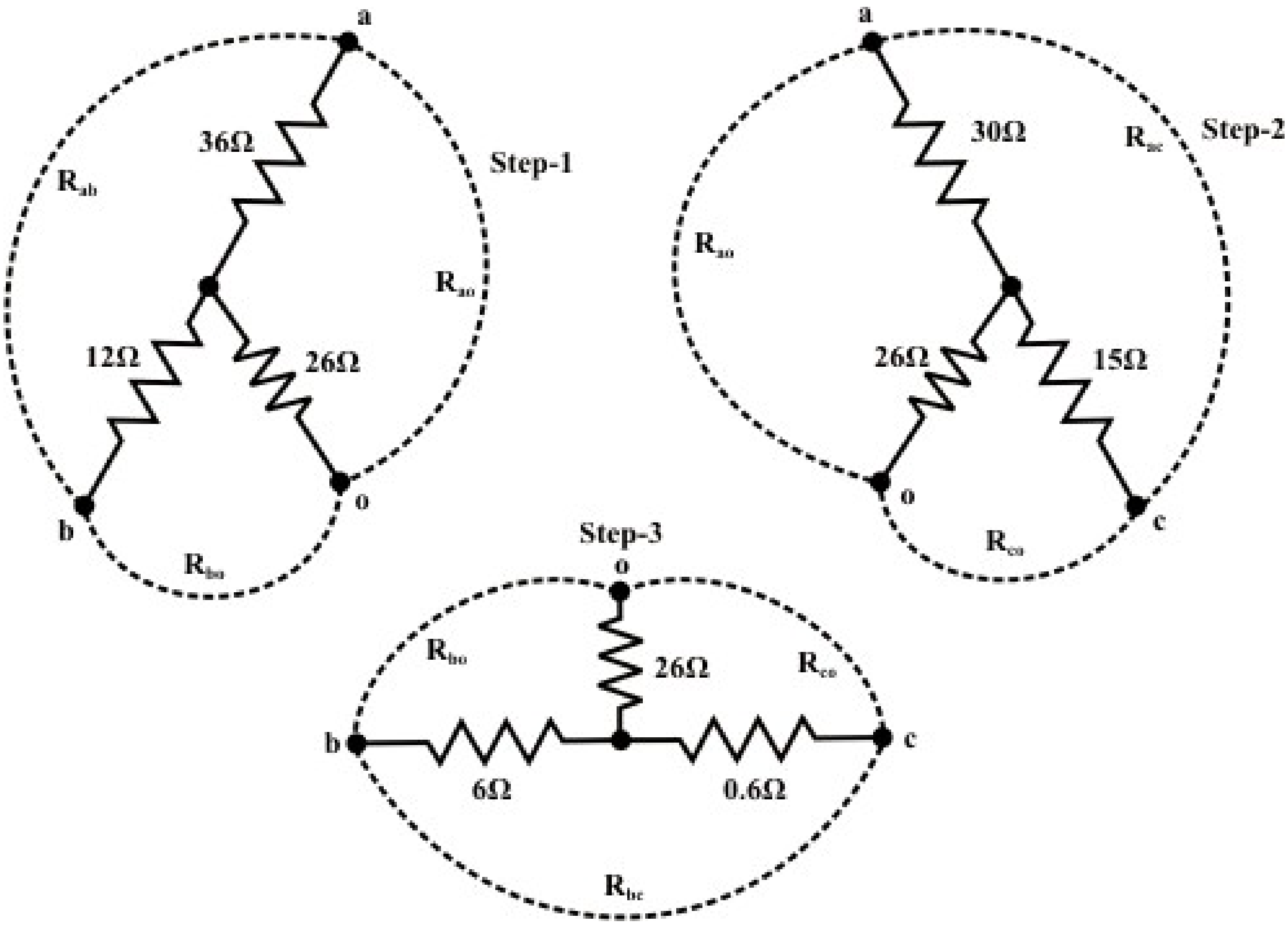
Fig. 6.4 (d)

Now one can find the equivalent resistance between the terminals 'A' and 'B' as $R_{AB} = \left(2.23 + 2.08\right) \,||\, \left(1.04 + 0.93\right) + 0.64 = 2.21\,\Omega$.

Example: L.6.3 Find the value of the input resistance R_{in} of the circuit.



Solution:



Y connected network formed with the terminals a-b-o is transformed into Δ connected one and its resistance values are given below.

$R_{ab} = 36 + 12 + \dfrac{36 \times 12}{26} = 64.61\Omega$; $\quad R_{bo} = 12 + 26 + \dfrac{26 \times 12}{36} = 46.66\Omega$

$R_{ao} = 26 + 36 + \dfrac{26 \times 36}{12} = 140\Omega$

Similarly, Y connected networks formed with the terminals 'b-c-o' and 'c-a-o' are transformed to Δ connected networks.

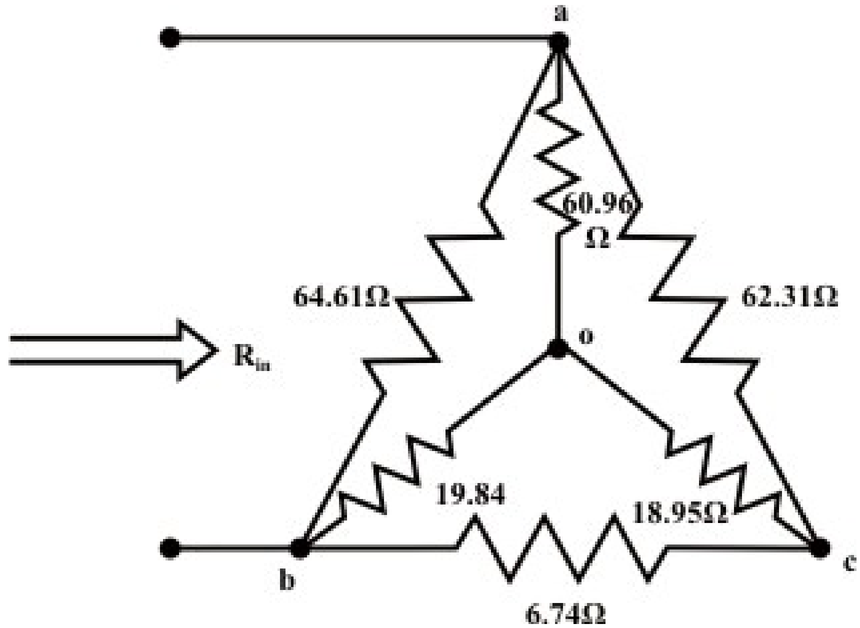
$R_{bc} = 6 + 0.6 + \dfrac{6 \times 0.6}{26} = 6.738\Omega$; $R_{co} = 0.6 + 26 + \dfrac{0.6 \times 26}{6} = 29.2\Omega$

$$R_{b0} = 6 + 26 + \frac{6 \times 26}{0.6} = 34.60\Omega$$

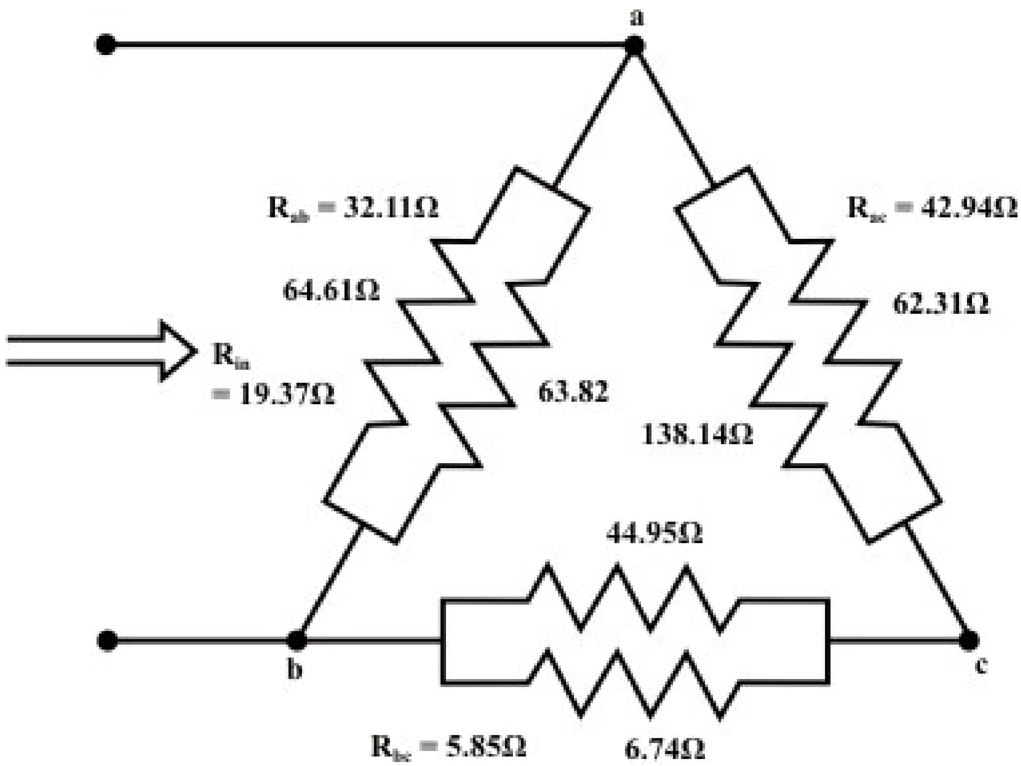
and, $R_{co} = 15 + 26 + \dfrac{15 \times 26}{30} = 54.00\Omega$; $\quad R_{ao} = 30 + 26 + \dfrac{30 \times 26}{15} = 108\Omega$

$$R_{ac} = 30 + 15 + \frac{30 \times 15}{26} = 62.31\Omega$$

Note that the two resistances are connected in parallel ($140 \| 108$) between the points 'a' and 'o'. Similarly, between the points 'b' and 'o' two resistances are connected in parallel ($46.66 \| 34.6$) and resistances $54.0\,\Omega$ and $29.2\,\Omega$ are connected in parallel between the points 'c' and 'o'.



Now Y connected network formed with the terminal 'a-b-c' is converted to equivalent Δ connected network.



Now, $R_{in} = \dfrac{(R_{ac} + R_{bc})R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = 19.37\Omega$

Remarks:

- If the Δ *or* Y connected network consists of inductances (assumed no mutual coupling forms between the inductors) then the same formula can be used for Y *to* Δ *or* Δ *to* Y conversion (see in detail 3-phase ac circuit analysis in Lesson-19).
- On the other hand, the Δ *or* Y connected network consists of capacitances can be converted to an equivalent Y *or* Δ network provided the capacitance value is replaced by its reciprocal in the conversion formula (see in detail 3-phase ac circuit analysis in Lesson-19).

Example: L.6.4 Find the equivalent inductance R_{eq} of the network (see fig.6.5(a)) at the terminals ‘a’ & ‘b’ using $Y - \Delta$ & $\Delta - Y$ transformations.

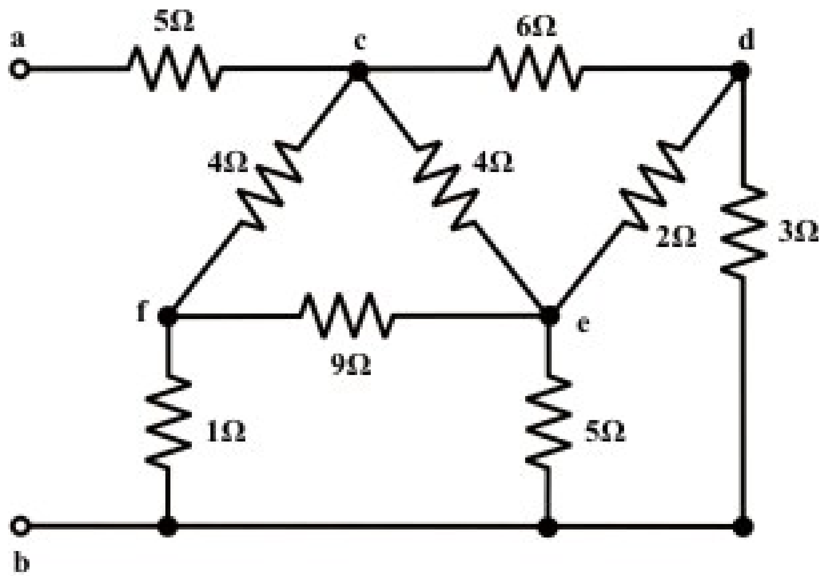


Fig. 6.5(a)

Solution: Convert the three terminals (c-d-e) Δ network (see fig.6.5(a)) comprising with the resistors to an equivalent Y-connected network using the following $\Delta - Y$ conversion formula.

$$R_{co} = \frac{6\times 4}{12} = 2\,\Omega;\ R_{do} = \frac{6\times 2}{12} = 1\,\Omega;\ \ and\ \ R_{eo} = \frac{2\times 4}{12} = 0.666\,\Omega$$

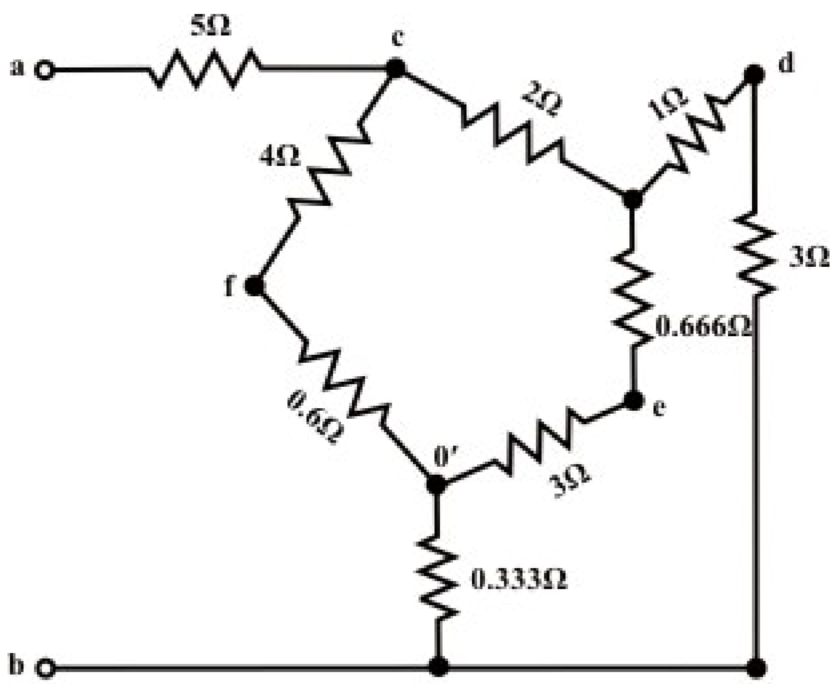


Fig. 6.5(b)

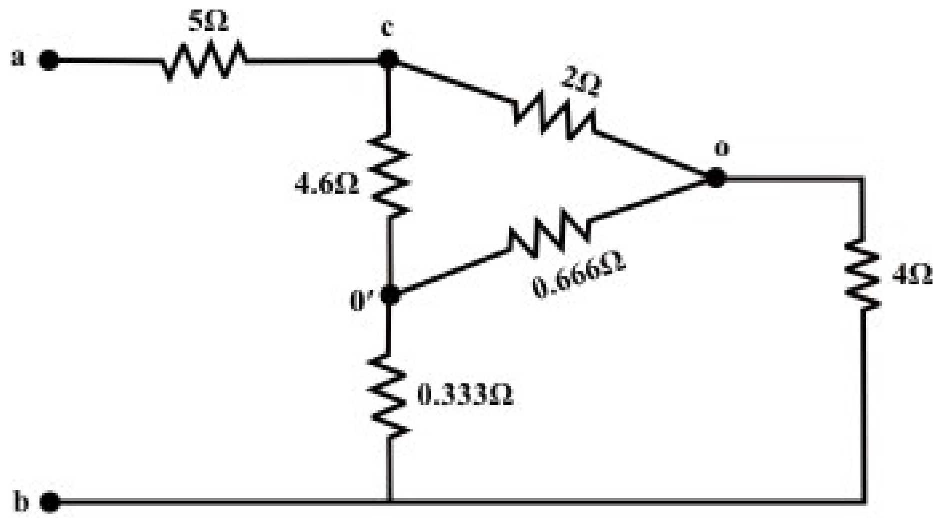


Fig. 6.5(c)

Similarly, the Δ-connected network (f-e-b) is converted to an equivalent Y-connected Network.

$$R_{fo'} = \frac{1\times 9}{15} = 0.6\,\Omega;\ R_{eo'} = \frac{5\times 9}{15} = 3\,\Omega;\ and\ R_{bo'} = \frac{1\times 5}{15} = 0.333\Omega$$

After the $\Delta - Y$ conversions, the circuit is redrawn and shown in fig.6.5(b). Next the series-parallel combinations of resistances reduces the network configuration in more simplified form and it is shown in fig.6.5(c). This circuit (see fig.6.5(c)) can further be simplified by transforming Y connected network comprising with the three resistors ($2\,\Omega$, $4\,\Omega$, and $3.666\,\Omega$) to a Δ-connected network and the corresponding network parameters are given below:

$$R_{co'} = 2 + 3.666 + \frac{2\times 3.666}{4} = 7.5\,\Omega;\ R_{cb} = 2 + 4 + \frac{2\times 4}{3.666} = 8.18\,\Omega;$$

$$and\ R_{bo'} = 4 + 3.666 + \frac{4\times 3.666}{2} = 15\,\Omega$$

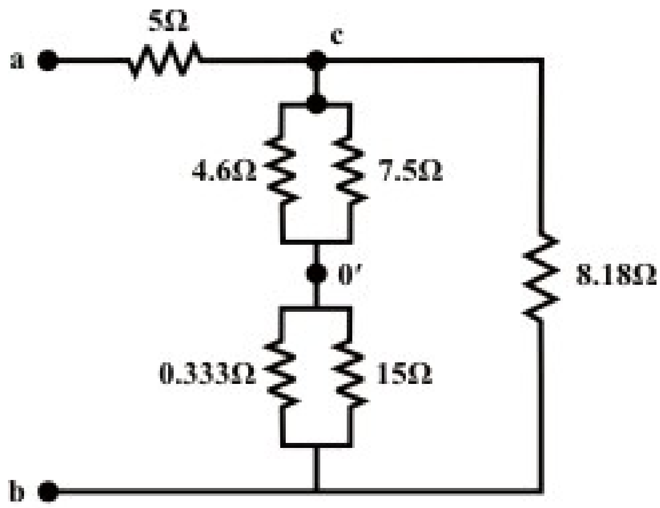


Fig. 6.5(d)

Simplified form of the circuit is drawn and shown in fig.6.5(d) and one can easily find out the equivalent resistance R_{eq} between the terminals 'a' and 'b' using the series-parallel formula. From fig.6.5(d), one can write the expression for the total equivalent resistance R_{eq} at the terminals 'a' and 'b' as

$$
\begin{aligned}
R_{eq} &= 5+\left[\left(4.6 \,\|\, 7.5\right)+\left(0.333 \,\|\, 15\right)\right] \| \, 8.18 \\
&= 5+\left[2.85+0.272\right] \| \, 8.18 = 5+\left(3.122 \,\|\, 8.18\right) \\
&= 7.26\,\Omega
\end{aligned}
$$

L.6.3 Test Your Understanding

[Marks: 40]

T.1 Apply $Y-\Delta$ *or* $\Delta-Y$ transformations only to find the value of the Current I that drives the circuit as shown in fig.6.6. [8]

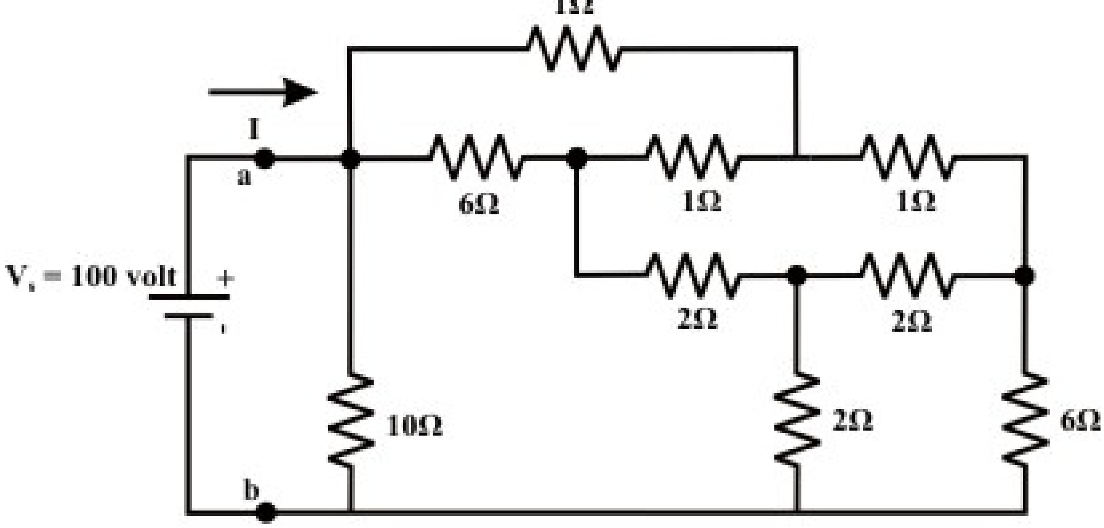


Fig. 6.6

(ans: 10.13Ω)

T.2 Find the current I through 4Ω resistor using $Y-\Delta$ or $\Delta-Y$ transformation technique only for the circuit shown in fig.6.7. [10]

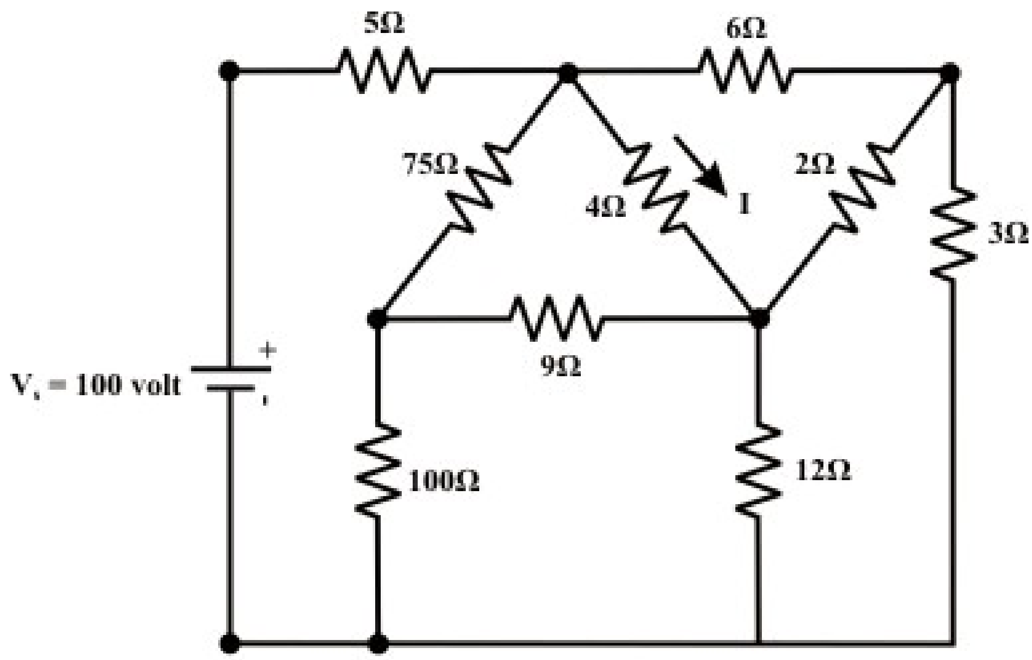


Fig. 6.7

(ans: $7.06\,A$)

T.3 For the circuit shown in fig.6.8, find R_{eq} without performing any conversion. [4]

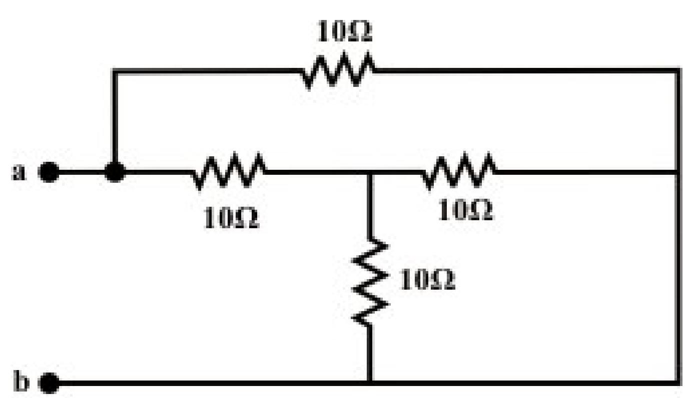


Fig. 6.8

(Ans. $6\ \Omega$)

T.4 For the circuit shown in fig.6.9, calculate the equivalent inductance R_{eq} for each circuit and justify your answer conceptually. [6]

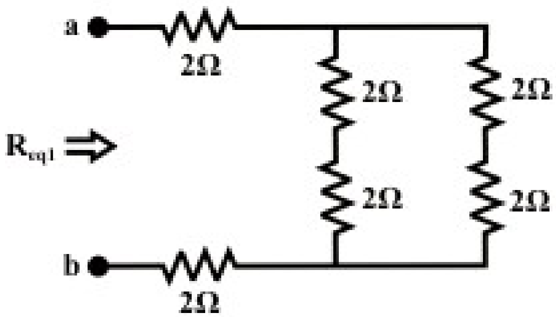


Fig. 6.9(a)

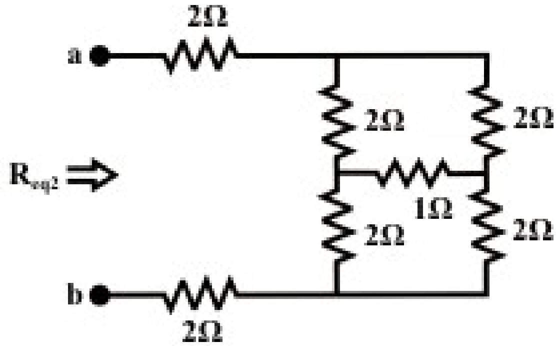


Fig. 6.9(b)

(ans. $R_{eq_1} = R_{eq_2}$)

T.5 Find the value of R_{eq} for the circuit of fig.6.10 when the switch is open and when the switch is closed. [4]

(Ans. $R_{eq} = 8.75\Omega$; $R_{eq} = 7.5\Omega$)

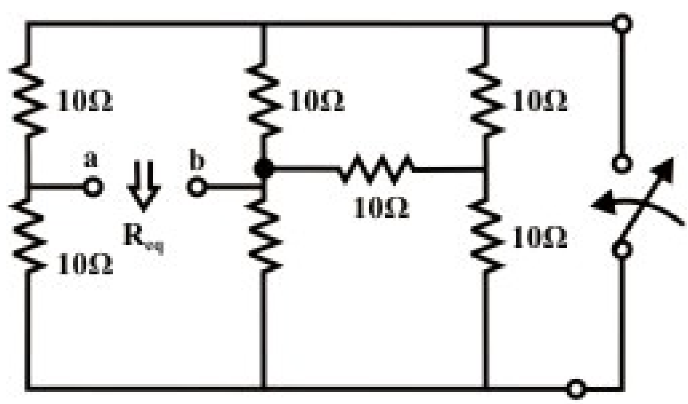


Fig. 6.10

T.6 For the circuit shown in fig.6.11, find the value of the resistance 'R' so that the equivalent capacitance between the terminals 'a' and b' is $20.57\,\Omega$. [6]

(Ans. $30\,\Omega$)

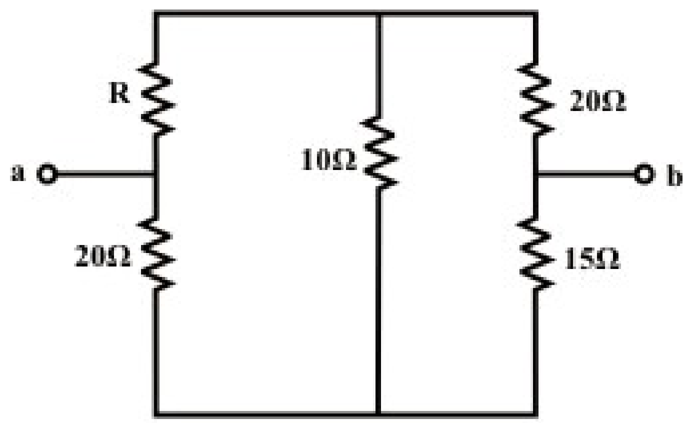


Fig. 6.11

T.7 $Y-\Delta$ *or* $\Delta - Y$ conversion is often useful in reducing the ------------ of a resistor network ---------- to the beginning nodal or mesh analysis. [1]

T.8 Is it possible to find the current through a branch or to find a voltage across the branch using $Y-\Delta/\Delta-Y$ conversions only? If so, justify your answer. [1]

Chapter 6

Circuit Transients

6.1 Introduction

The V – i relationships across the inductors and capacitors involve integral and differential relationship. By applying KCL and KVL to circuits containing L and C we get equations called integro differential equations. Differential equations contain functions and their derivatives. Integral equations contain functions and their integrals.

Consider the differential equation

$$a_2 \frac{d^2x(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_0\, x(t) = f(t) \qquad \ldots(1)$$

This equation involved a relationship between two variables *i.e.*, $x(t)$ and t. t is independent variable where as $x(t)$ is dependent variable

$f(t)$ is called forcing function.

The order of a differential equation represents the highest derivative present in the equation. In the equation (1), second derivation $\frac{d^2x(t)}{dt^2}$ is the highest derivative. Hence equation (1) is the second – order differential equation. The coefficients of the terms (a_0, a_1, a_2) on the left hand side of the equation (1) are constants. So it is considered as time – invariant.

A linear differential equation is one in which the dependent variable and its derivatives appear in first power only. For example, equation (1) is a linear differential equation.

If $f(t)$ is zero, then it is homogeneous differential equations, otherwise it is non homogeneous.

6.2 Transients

If a circuit containing one or more energy – storage elements (such as L and C) is excited by a source which abruptly changes its value, the energy state of the circuit is disturbed. This disturbance may also be due to changes of L or C or both and also due to switching conditions. After a certain time the circuit settles down to a new steady state operation and the node voltage and element currents reach their final steady state values. The time required for the network to change from one steady state to another steady state is called transient period. The values of voltages and currents during the transient period are known as transient responses or simply transients. Transient means short lived. In electrical circuit transients do not occur if it is purely resistive. For the transients to take place there must be presence of L or C or both in the circuit.

...s, the resp... related to what has happened in the ... words, the circuit can have non–zero response even if there is no input signal.

6.5 Zero – input response :

The zero – input response of a system is response obtained when the input is identically zero. Such response need not be zero, because there may be initial charges on the capacitors and/or the initial fluxes in the inductors.

6.6 Zero – state response :

Zero – state response is the response obtained for an arbitrary input when all initial conditions are identically zero.

For a linear system, the complete response is equal to the sum of zero – input and zero – state response.

6.7 Initial (Boundary) conditions for inductance

For an inductor, we know that

$$v_L = L\frac{di_L}{dt}$$

$$\therefore i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L\,dt \qquad \text{...(1)}$$

Let

$$i_L(0-) = \text{the value of } i_L \text{ at } t = 0-$$

$$i_L(0+) = \text{the value of } i_L \text{ at } t = 0+$$

Therefore at $t = 0+$

$$i_L(0+) = \frac{1}{L}\int_{-\infty}^{0} v_L\,dt$$

$$= \frac{1}{L}\int_{-\infty}^{0-} v_L\,dt + \frac{1}{L}\int_{-0}^{0+} v_L\,dt$$

$$= i_L(0-) + \frac{1}{L}\int_{-0}^{0+} v_L\,dt \qquad \text{...(2)}$$

$i_L(0-)$ is the initial state of the inductor. It gives the past history. (before $t = 0$) of the inductor.

The interval from 0– and 0+ is almost zero. If V_L is finite and does not contain impulses

$$\frac{1}{L}\int_{0-}^{0+} v_L\,dt = 0 \qquad \text{...(3)}$$

$$i_L(0+) = i_L(0-)$$

That is, the current through the inductor just after the switch is closed must be same as that just before closing the switch.

If an inductor has no initial current, then $i_L(0-) = 0$. From equation (4), $I_L(0+) = 0$. Hence an inductor with no initial current behaves like an open circuit at $t = 0+$.

An inductor with initial current $i_L(0-) = I_{L0}$ is considered as a current source I_{L0} at $t = 0+$ as shown in fig 6.1 Remember that $V_L(0-) \neq V_L(0+)$

6.8 Initial (Boundary) conditions for capacitors :

$$\text{We know } i_c(t) = C\frac{dV_c(t)}{dt} \qquad \ldots(5)$$

To determine the capacitor voltage in terms of capacitor current, we write.

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)dt \qquad \ldots(6)$$

The above expression states that the voltage at any time t, is proportional to the integral of the current through the capacitor.

At $t = 0+$.

$$V_c(0+) = \frac{1}{C}\int_{-\infty}^{0-} i_c\, dt + \frac{1}{C}\int_{-0}^{0+} i_c\, dt$$

$$= V_c(0-) + \frac{1}{C}\int_{0-}^{0+} i_c\, dt \qquad \ldots(7)$$

The interval from 0– to 0+ is almost zero. If i_c is finite and does not contain impulses or derivatives.

$$\frac{1}{C}\int_{0-}^{0+} i_c\, dt = 0 \qquad \ldots(8)$$

From equation (7) and (8)

$$V_c(0+) = V_c(0-) \qquad \ldots(9)$$

$$\text{Since } V_c = \frac{q}{c}$$

$$q(0-) = q(0+) \qquad \ldots(10)$$

We conclude that the voltage and hence the charge across the capacitor can not change instantaneously. But the capacitor current has no such restriction.

$$i_c(0+) \neq i_c(0-)$$

...al voltage ...

...ort circuit at $t = 0+$.

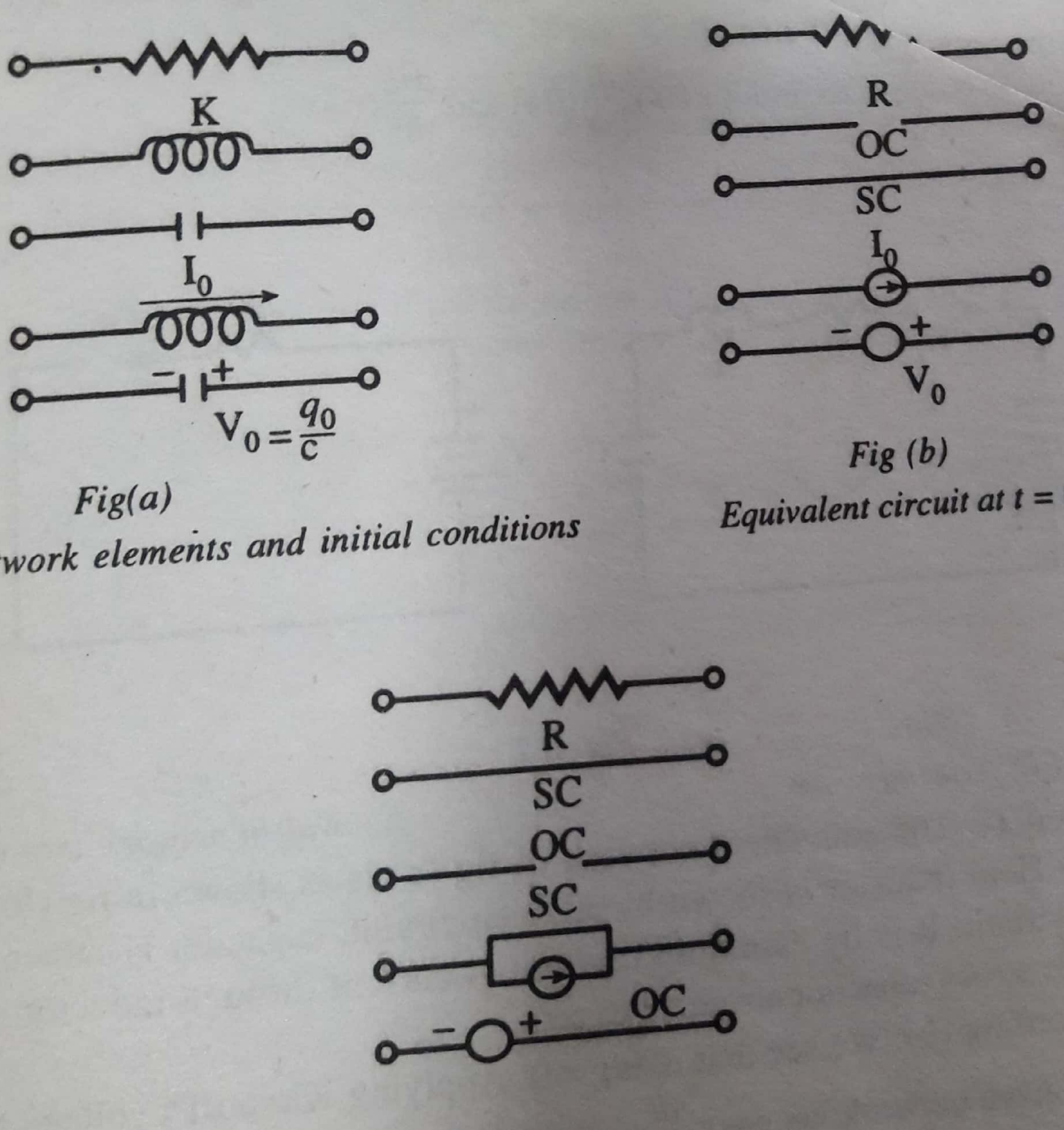
The capacitor with an initial charge $q_c(0-) = Q_0$ is equivalent to a voltage source of value $V_{c0} = \frac{Q_0}{c}$ at $t = 0+$.

Effect of switching on resistor

We ... w that for a linear resistor $V_R = Ri$. so, theoretically, the current throug... or the voltage across the resistor can change instantaneously, *i.e..*, ... zero time. Hence, there will be no transients in purely resistive networks. ... network changes from one steady state to another without any transient.

As the resistor does not store energy, ... neral

$$i_R(0+) \neq i_R(0-) \text{ and } V_R(0+) \neq V_R(...)$$



Fig(a)
Network elements and initial conditions

Fig (b)
Equivalent circuit at $t = 0$

Fig (c) Equivalent circuit at $t = \infty$

Fig 6.1

$$\therefore \frac{di}{dt}(0+) = \frac{V_0}{L} \qquad ...(12)$$

Again differentiating equation (11), we get

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = 0 \qquad ...(13)$$

Therefore, $$L\frac{d^2i}{dt^2}(0+) + R\frac{di}{dt} + (0+)\frac{i(0)}{c} = 0$$

$$\Rightarrow \qquad L\frac{d^2i}{dt^2}(0+) + R\frac{V_0}{L} + 0 = 0$$

$$\therefore \qquad \frac{d^2i}{dt^2}(0+) = -\frac{RV_0}{L^2} \qquad ...(14)$$

6.11 Laplace Transformation Technique

For a given circuit, we can obtain the differential equations by the applications of Kirchoff's law. Such equations can be solved by *a)* classical method *b)* Laplace transformation method and so on.

If the differential equation is of higher order, classical method is very tedious.

Laplace transformation solves differential equation systematically and incorporates both transient and steady state as well as the initial conditions.

6.12 DC Transients

Here we have three types of circuit in each case the voltage applied (excitation) is assumed to be step voltage denoted by $Eu(t)\cdot u(t)$ is the unit step voltage.

6.12 (a) Case 1 : (*a*) R – L Transients : (Rise of current)

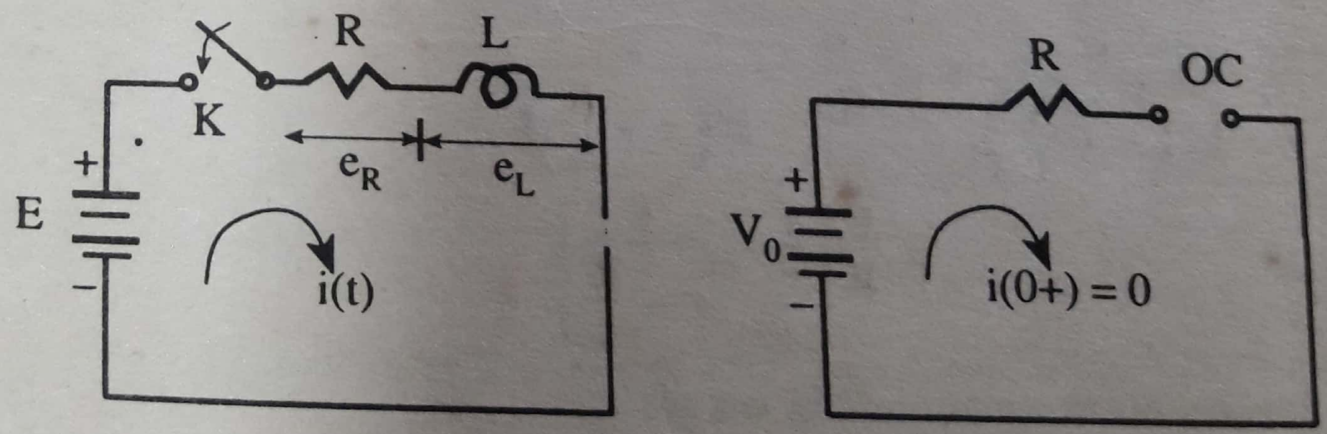


Fig (a) *Fig 6.3* *Fig (b)*

Equivalent circuit at t= 0+

Let the R – L series combination be impressed upon the d.c. voltage E by closing the switch K. Assume that the current through the inductor before closing the switch be zero. let K be closed at the instant $t = 0$

The equivalent circuit at $t = 0+$ is shown in fig 3(*b*). The inductor is shown an open circuit. Hence $i(0-) = i(0+) = 0$

Applying KVL to the circuit in fig 3(*a*), after t seconds of closing K,

$$\text{we get, } Ri + L\frac{di}{dt} = E \quad \ldots(15)$$

Taking Laplace Transformation on both sides, we get

$$RI(s) + L\{SI(s) - i(O_+)\} = \frac{E}{s}$$

$$\Rightarrow RI(s) + L\{SI(s) - 0\} = \frac{E}{s}$$

$$\Rightarrow (R + Ls\ I(s) = \frac{E}{s}$$

$$\therefore I(s) = \frac{E}{s(Ls + R)}$$

$$= \frac{E}{Ls\left(s + \frac{R}{L}\right)} \quad \ldots(16)$$

Let

$$\frac{E}{Ls\left(s + \frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{\left(s + \frac{R}{L}\right)} \quad \ldots(17)$$

$$A = \left.\frac{E}{L\left(s + \frac{R}{L}\right)}\right|_{s=0} = \frac{E}{R}$$

$$B = \left.\frac{E}{Ls}\right|_{s=-\frac{R}{L}} = -\frac{E}{R}$$

$$\frac{E}{Ls\left(s + \frac{R}{L}\right)} = \frac{E}{Rs} - \frac{E}{R\left(s + \frac{R}{L}\right)}$$

$$= \frac{E}{R}\left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right]$$

putting this in equation (16), we get

$$I(s) = \frac{E}{R}\left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right] \quad \ldots(18)$$

Now taking inverse Laplace transform on both sides we get

$$i(t) = L^{-1} I(s)$$

$$= \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$$

$$i(t) = I\left[1 - e^{-\frac{Rt}{L}}\right] \quad \ldots(19)$$

$I = \frac{E}{R}$ is the steady current.

Steady current is the value of $i(t)$ for $t = \infty$.

$\frac{L}{R}$ is called time constant of the RL circuit and is denoted by T.

Hence equation (19) can be written as

$$i = I\left[1 - e^{-\frac{t}{T}}\right] \quad \ldots(20)$$

The above equation shows that as t increases i increases exponentially. At $t = \infty$, the current reaches steady state value $I = \frac{E}{R}$.

The transient voltage e_R and e_L across R and L respectively, during the rise of current in the inductive can be expressed as below :

$$e_R = Ri$$

$$= RI\left[1 - e^{-\frac{Rt}{L}}\right]$$

$$e_R = E\left[1 - e^{-\frac{t}{T}}\right] \quad \ldots(22)$$

From the above expression, we can say that the voltage across R increases exponentially from zero to E, during rise of current

By KVL,

$$e_R + e_L = E, \text{ at any time}$$

$$\therefore e_L = E - e_R = E - E\left[1 - e^{-\frac{t}{T}}\right]$$

$$e_L = E e^{-\frac{t}{T}} \quad \ldots(23)$$

It shows that the voltage across L decreases with time, exponentially

$\left[\textit{Note} : e_L = -L \frac{di}{dt}\right]$

The variation of $i(t)$ with time is graphically shown in fig 6.4(a) and that of e_R and e_L is shown in fig 6.4(b).

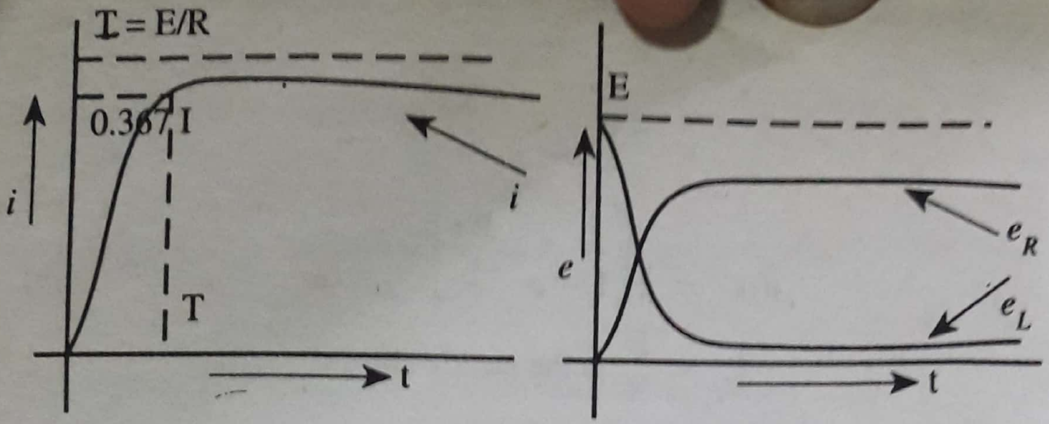


Fig (a) *Fig 6.4* *Fig (b)*

Definition of time constant T

Substituting $t = T$ in equation (2) we get

$$i = I(1 - e^{-1}) = 0.632\, I$$
$$= 63.22\% \text{ of I}$$

Thus the time constant of RL series circuit is defined as the period during which the current rises to 63.22% of its final value (OR steady value)

b) RL – decaying transients :

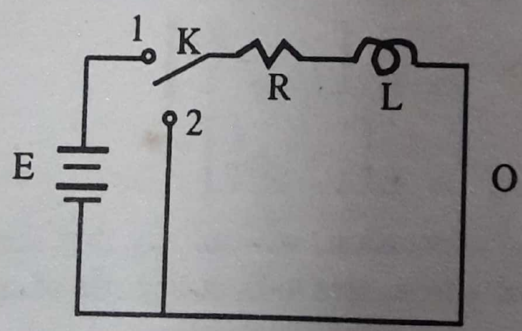


Fig 6.5

Assume that the switch K be kept connected to position 1 for sufficiently longer period. Then the current reaches steady state value given by $I = \frac{E}{R}$. After this instant, let the switch be moved from position 1 to position 2. Let this instant be taken as $t' = 0$. After t' seconds of closing the switch to position, applying KVL,

$$R_i + L\frac{di}{dt'} = 0 \qquad \text{...(21)}$$

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Taking Laplace Transformation on both sides, we get

$$RI(s) + L[sI(s) - i(0+)] = 0 \qquad \ldots(22)$$

$$\text{Here } i(0+) = I = \frac{E}{R}$$

$$\therefore\ RI(s) + Ls\,I(s) - \frac{LE}{R} = 0$$

$$\text{or } [R + Ls]\,I(s) = L\frac{E}{R} = LI$$

$$I(s) = \frac{LI}{Ls + R}$$

$$= \frac{I}{\left(S + \frac{R}{L}\right)}$$

$$i(t') = L^{-1}\,I(s) = I\,e^{-\frac{Rt'}{L}}$$

$$= i(t)' = I\,e^{-\frac{t'}{L}} \qquad \ldots(23)$$

Thus i decays exponentially from I to zero, as t' increases from zero to infinity.

The variation of $i(t')$ with t' is graphically shown in fig 6.6

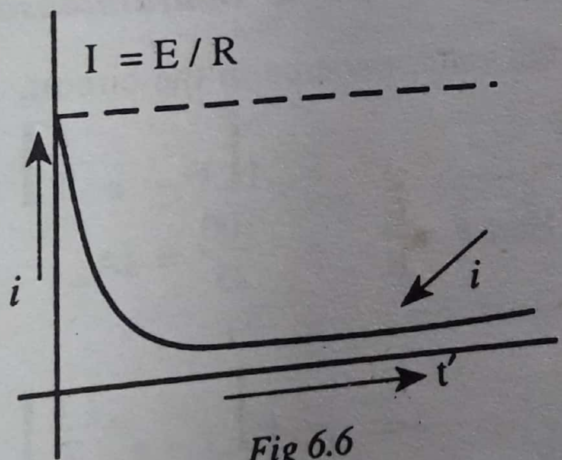

Fig 6.6

Decaying current in an R – L circuit

$$e_R = R_i = RI\,e^{-\frac{t'}{T}}$$

$$e_R = E\,e^{-\frac{t'}{T}} \qquad \ldots(24)$$

$$e_R + e_L = 0$$

$$\Rightarrow e_L = -e_R = -E\,e^{-\frac{t'}{T}} \qquad \ldots(25)$$

The variation of e_R and e_L with t' is graphically shown in fig 6.7

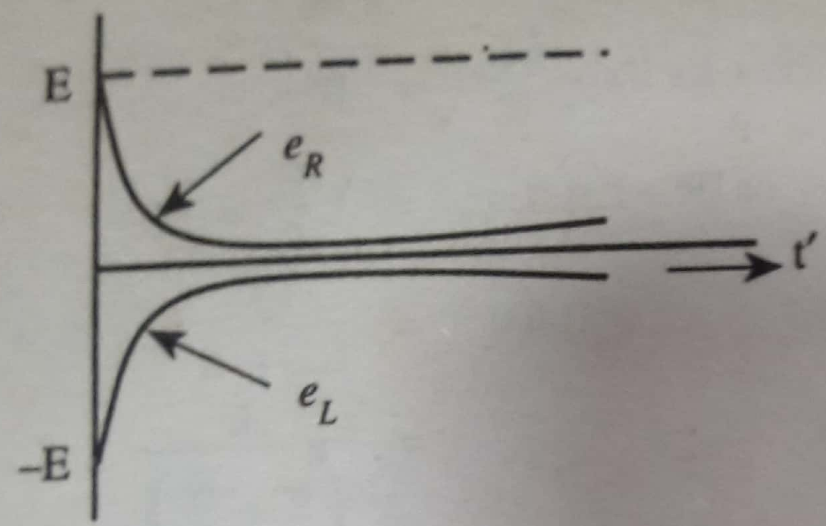

Fig 6.7

WORKED EXAMPLES

D.C TRANSIENTS IN RL CIRCUIT

Example 1 : A d.c. voltage of 100 volts is applied to a series RL circuit with R = 25Ω. What will be the current in the circuit at twice the time constant.

[MKU Nov 94 & Bharathidasan Uni. Nov 95]

Solution : As the voltage source in the circuit,

$$i = I\left[1 - e^{-\frac{t}{T}}\right]$$

$$\text{Here } I = \frac{E}{R} = \frac{100}{25} = 4A$$

$$\text{Given that } t = 2T$$

$$\therefore i = 4\left[1 - e^{-\frac{2T}{T}}\right]$$

$$= 4\,[1 - e^{-2}]$$

$$= 3.46 \text{ A}$$

Ans

Example 2 : Sketch the current given by $i(t) = 5 - 4$

$$\text{Let} \quad \frac{\ldots}{s(5s+6)} = \frac{K_1}{s} + \frac{K_2}{5s+6}$$

$$K_1 = \left.\frac{5s+36}{(5s+6)}\right|_{s=0} = \frac{36}{6} = 6$$

$$K_2 = \left.\frac{5s+36}{s}\right|_{s=-1.2} = \frac{30}{-1.2} = -25$$

$$\therefore I(s) = \frac{6}{s} - \frac{25}{5s+6} = \frac{6}{s} - \frac{5}{s+1.2} \qquad \ldots(iv)$$

$$\therefore i(t) = L^{-1} I(s)$$

$$\boxed{i(t) = 6 - 5\,e^{-1.2t}} \quad \textbf{Answer}$$

6.12.2 Case 2 : (a) R – C Transients (charging)

Consider the R – C series combination. A DC voltage E is applied to this combination through the switch as shown in fig. 6.13

Let Q_0 = initial charge on the capacitor so, the initial voltage on the capacitor $= V_0 = \frac{Q_0}{C}$. Let the polarities of V_0 be as shown :

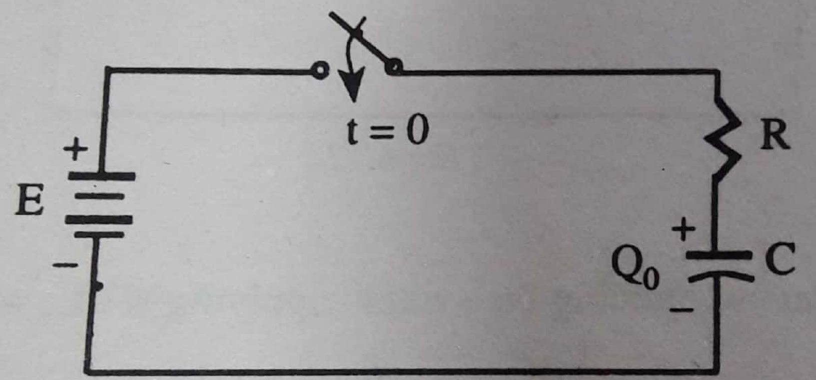

Fig 6.13

After *t* seconds of closing the switch, applying kirchoff's voltage law to the circuit we get,

$$iR + \frac{1}{C}\int_{-\infty}^{t} i\,dt = E \qquad \ldots(i)$$

$$\Rightarrow iR + \frac{1}{C}\int_{-\infty}^{0} i\,dt + \frac{1}{C}\int_{0}^{t} i\,dt = E \qquad \ldots(ii)$$

Here, $\frac{1}{C}\int_{-\infty}^{0} i\,dt = \text{initial voltage} = V_0$

Therefore, the equation (*ii*) can be written as

$$iR + V_0 + \frac{1}{C}\int_0^t i\,dt = E \qquad \ldots(iii)$$

[***Note*** : If the polarities of V_0 are opposite to that shown in the fig then replace V_0 in the above equation by $-V_0$]

In the present case let us assume that the initial charge on the capacitor is 0. Hence $V_0 = 0$.

Substituting $V_0 = 0$ in equation (*iii*) we get, the final equation as

$$iR + \frac{1}{C}\int_0^t i\,dt = E \qquad \ldots(iv)$$

Take, Laplace transform on both sides, we get

$$R\,I(s) + \frac{1}{C}\cdot\frac{I(s)}{s} = \frac{E}{s}$$

$$\text{or } \left(R + \frac{1}{sC}\right) I(s) = \frac{E}{s}$$

$$\text{or } I(s) = \frac{E}{s\left(R + \frac{1}{sC}\right)} = \frac{E}{sR + \frac{1}{C}}$$

$$I(s) = \frac{E}{R\left(s + \frac{1}{RC}\right)}$$

$$i(t) = L^{-1}\,I(s) = \frac{E}{R}e^{-\left(\frac{t}{RC}\right)} \qquad \ldots(v)$$

$\frac{E}{R}$ is the initial value of the current denoted by I, RC is called time constant denoted by λ.

Therefore, we can express the charging current as

$$i(t) = Ie^{-\left(\frac{t}{\lambda}\right)} \qquad \ldots(vi)$$

This charging current seems to be exponentially decreasing as is observed from the equation (*vi*)

The initial current = the value of i when $(t = 0) = I$.

The steady current = the value of i when $(t = \infty) = 0$

The variation of the current with time is graphically shown in the figure 6.14

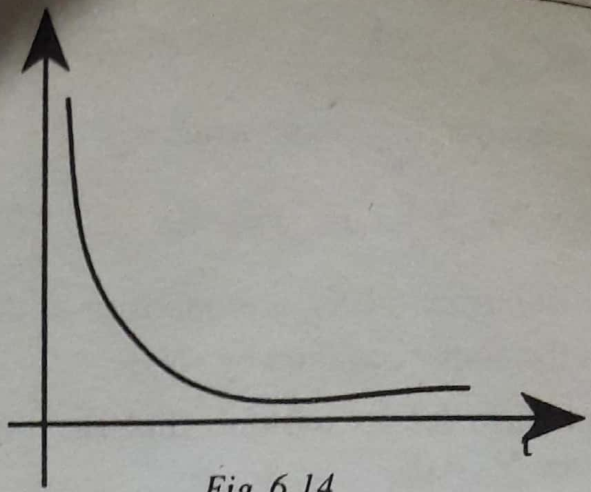

Fig 6.14

The transient voltages e_R and e_c are obtained as below :

$$e_R = Ri = R \times Ie^{-\left(\frac{t}{\lambda}\right)} = E\, e^{-\left(\frac{t}{\lambda}\right)} \qquad \ldots(vii)$$

$$\text{But } e_R + e_c = E$$

$$\text{Therefore } e_c = E - e_R$$

$$= E\left[1 - e^{-\left(\frac{t}{\lambda}\right)}\right] \qquad \ldots(viii)$$

e_R exponentially decreases from E to 0 as t increases from 0 to $\propto$.

e_C exponentially increases from 0 to E as t increases from 0 to ∞. So, these observations are made from equations (*vii*) and (*viii*).

Graphically, the variations of e_R and e_C can be plotted as shown in the fig.6.15

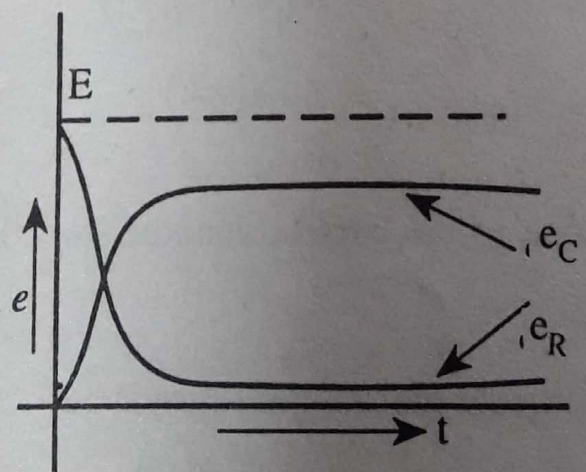

Fig 6.15

Definition of Time – constant (λ) :

In the equation (*viii*), substitute that $\lambda = t$.

Then $i = I \times e^{-1} = 0.368 \times I$

$= 36.8\%$ of the initial current.

Thus, the time constant for the RC circuit is the time during which the current falls to 36.8% of initial current

[Note : We can also get e_C by the following formula $e_C = \frac{1}{C}\int_0^t i(t)\,dt$]

6.12.3 Case (b) R – C decaying Transient (discharging)

Consider the circuit of the fig 6.16. The switch has been in position 1 for sufficient time to establish steady state condition. Let this switch be moved to position 2 at the instant $t = 0$.

Under steady state condition, the capacitor is fully charged. The voltage across C is E with the polarities as shown in the fig (16). The differential equation of the circuit is

$$iR + \frac{1}{C}\int i\,dt + E = 0 \qquad \ldots(ix)$$

Taking Laplace transform and rearranging the term, we get

$$I(s)\left(R + \frac{1}{sC}\right) = \frac{-E}{s}$$

$$\text{Therefore } i(t) = \frac{-E}{R} e^{\left(-\frac{t}{cR}\right)}$$

$$= -I e^{-\frac{t}{\lambda}} \qquad \ldots(x)$$

This current is called discharging current.

The current curve is as shown in the fig 6.16

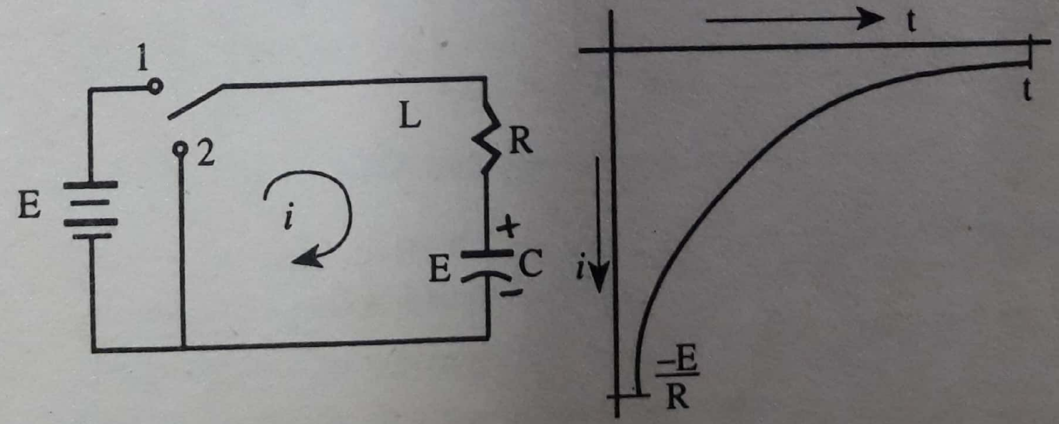

Fig (a) Fig 6.16 Fig (b)