16. A family has two children. Given that one of the children is a boy and that he was born on a Tuesday. What is the probability that both children are boys?

Assume that the probability of being born on a particular day of the week is 1/7 and is independent of whether the child is boy or girl?

(a) 1/3

(b) ½

(c) 13/27

(d) 11/27

Solution:

Let B be the event that the family has one child who is a boy on Tuesday.

A be the event that both children are bags.

∴ Given that there are 7 days of the week, there are 49 possible combinations for the days of the week the two boys were born on. 13 of these have boy who was born on a Tuesday.

$P(\frac{B}{A}) = \frac{13}{49}$

$P(A) = \frac{1}{4}$

$We\ need\ to\ find\ P(\frac{A}{B})$

$P(\frac{A}{B}) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)P(\frac{B}{A})}{P(B)}$

$P(B) = P(one\ child\ who\ is\ a\ boy\ born\ on\ Tuesday)$

$\therefore 14^2$ possible ways to select the gender and the day of the week the child was born on.

⇒ First child can be boy or girl and can be born on every any day of week.

$7 + 7 = 2 \times 7 = 14$

$\therefore Only\ one\ way\ this\ child\ can\ be\ born\ on\ tuesday.$

⇒ This appears to the 2^{nd} child also again

$\therefore Only\ one\ way\ this\ child\ can\ be\ born\ on\ tuesday.$

$14\ way$

$\therefore Total\ number\ of\ way = 14 \times 14 = 196$

13^2 ways which don't have a boy born on Tuesday.

⇒ First child can be boy or girl who didn't born on Tuesday

$14 - 1 = 13$

⇒ Same way as first child

$\therefore Number\ of\ ways = 14 - 1 = 13$

$\therefore Total\ number\ of\ ways\ which\ don't\ have\ a\ boy\ born\ on\ Tuesday = 13 \times 13 = 169$

$\therefore Number\ of\ ways\ one\ child\ is\ a\ boy\ born\ on\ Tuesday = 196 - 169 = 27$

$\therefore P(B) = \frac{27}{196}$

$$P(\tfrac{A}{B}) = \frac{P(A) \cdot P(\frac{B}{A})}{P(B)}$$

$$= \frac{\frac{13}{49} \times \frac{1}{4}}{\frac{27}{196}}$$

$$= \frac{13}{27}$$

17. Box 1 contains 1 red ball and 2 white balls. Box 2 contains 2 red balls and 1 white balls. One ball is drawn randomly from box 1 and transferred to box 2.

Then a ball is drawn randomly from box 2 and it is red. What is the probability that the transferred ball was white?

(a) 7/12

(b) 3/4

(c) 4/7

(d) 1/2

Solution:

Let W be that the transferred ball is white.

Let the event R be that the ball is drawn is red.

$\therefore P(W) = \frac{2}{3}\ and\ P(\frac{R}{W}) = \frac{2}{4} = \frac{1}{2}$

It is given that the ball drawn is red.

∴ and P(R) is dependent on which ball is transferred.

$$P(R) = P(W)P(\frac{R}{W}) + P(\neg W)P(\frac{R}{\neg W})$$

$$= \frac{2}{3} \times \frac{2}{4} + \frac{1}{3} \times \frac{3}{4}$$

$$= \frac{7}{12}$$

we need to find,

$$P(\frac{W}{R}) = \frac{P(\frac{R}{W})P(W)}{P(R)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{3}}{\frac{7}{12}}$$

$$= \frac{4}{7}$$

18. There are four boxes. Each contains 2 balls the first box has a red and a white ball in it. The remaining three boxes each have two white balls in them.

A ball is picked at random from box 1 and put in box 2. Then a ball is picked at random from box 2 and put into 3. Then a ball is picked from box 3 and put into box 4. Finally a ball is picked from box 4.

The probability that the ball picked from box 2 is red, given that the final ball picked from box 4 is white is _______ [enter 3 decimal digits]

Solution:

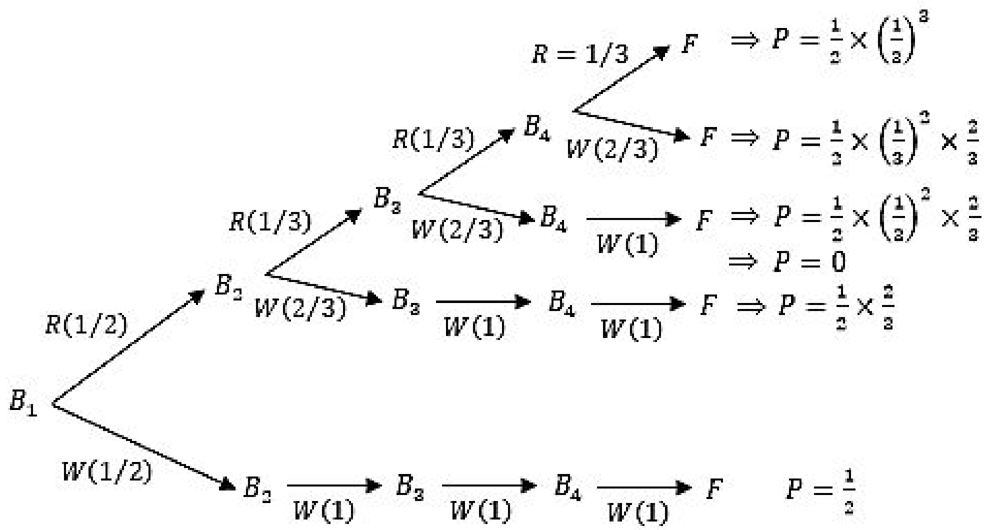
Let R_2 = Probability that a red ball was picked from the second box

Let W_4 = Probability that a white ball was picked out from the fourth box

We need to find,

$$P(\frac{R_2}{W_4}) = \frac{P(R_2 \cap W_4)}{P(W_4)}$$

$$= \frac{P(\frac{W_4}{R_2}) \times P(R_2)}{P(W_4)}$$



$\therefore P(W_4) = 1 - P(R_4)$

$= 1 - \frac{1}{2}\times(\frac{1}{3})^3$

$= \frac{53}{54}$

$P(W_4/R_2) = \frac{2}{3}\times 1 + \frac{1}{3}\times\frac{2}{3}$

$= \frac{8}{9}$

$P(R_2) = \frac{1}{6}$

$P(\frac{R_2}{W_4}) = \frac{\frac{8}{9}\times\frac{1}{6}}{\frac{53}{54}}$

$= \frac{8}{53}$

19. Harmoine granger is taking a question on a multiple choice test, she either knows the answer or guesses.

Let p be the probability that she knows the answer and 1 – p be the probability that she guesses.

Assume if she answers a question by guess, it will be correct with probability 1/m, where m is the number of multiple choice alternatives.

What is the conditional probability that she knew the answer to a question given that she answered it correctly?

(a) $\frac{mp}{1+mp}$

(b) $\frac{(m-1)p}{1+(m-1)p}$

(c) $\frac{mp}{1+(m-1)p}$

(d) $\frac{(m-1)p}{1+mp}$

Solution:

Let c and k denote, respectively the events that she answers question correctly and the event that she actually knows the answer.

$$p(\frac{k}{c}) = \frac{p(k \cap c)}{p(c)}$$

$$= \frac{p(\frac{c}{k})p(k)}{p(\frac{c}{k})p(k) + p(\frac{c}{k'})p(k')}$$

$$= \frac{p}{p + \frac{1}{m}(1-p)}$$

$p(k \cap c)$ is actually she knows the answer which is $'p'$

$$= \frac{mp}{1+(m-1)p}$$

20. Consider a medical practitioner pondering the following dilemma: "If I am atleast 80% certain that my patient has this disease, then I always recommend surgery, whereas if I am not quite certain, then I recommend additional tests that are expensive and sometimes painful. Now, initially I was only 60% certain that Emma had the disease. So I ordered the series A test, which always gives a positive when the patient has the disease and almost never does when she is healthy.

The test result was positive and I was all set to recommend surgery when Emma informed me, for the first time that she is a diabetic.

This information complicates matters because although it doesn't change my original 60% estimate of her chances of having disease, it does effect the interpretation of the results of the A test. This is so because the A test while never yielding a positive result when the patient is healthy, does unfortunately yield appositive result 30% of the time in the case of diabetic patients not suffering from the disease. Now what to do? More tests or immediate surgery?

(a) More tests

(b) Surgery

(c) Any one of this

(d) None of the above

Solution:

Let D denote the event that Emma has the disease.

E the event of a positive A test result, we need to find $P(\frac{D}{E})$

$$P(\frac{D}{E}) = \frac{P(D\cap E)}{P(E)}$$

$$= \frac{P(\frac{E}{D})P(D)}{P(\frac{E}{D})P(D)+P(\frac{E}{D'})P(D^{'})}$$

$$= \frac{1\times 0.6}{1\times 0.6+0.3\times 0.4}$$

$$= 0.833$$

∵ It is given 30% of the time in the case of diabetic patients not suffering from disease.

$P(\frac{E}{D'}) = 30\%$

$P(D^{'}) = 1 - P(D)$

$= 1 - 60\%$

$= 0.4$

21. A letter is known to have come either from LONDON or CLIFTON; on the postmark on the two consecutive letters ON are legible. The probability that it come from LONDON is

(a) $\frac{5}{17}$

(b) $\frac{12}{17}$

(c) $\frac{17}{30}$

(d) $\frac{3}{5}$

Solution:

We need to find

$$P(\frac{LONDON}{ON}) = \frac{P(LONDON\cap ON)}{P(ON)}$$

$$= \frac{P(\frac{ON}{LONDON})P(LONDON)}{P(\frac{ON}{LONDON})P(LONDON)+P(\frac{ON}{CLIFTON})P(CLIFTON)}$$

$P(\frac{ON}{LONDON}) = \frac{2}{5}[5 \Rightarrow Two\ consecutive\ letters\ can\ appear\ in\ 5\ ways$

$\{LO,\ ON,\ ND,\ DO,\ ON\}\ there\ is\ only\ one\ ways\ can\ ON\ legible]$

$P(\frac{ON}{CLIFTON}) = \frac{1}{6}[Two\ consecutive\ letters\ can\ appear\ in\ 6\ ways$

$\{CL,\ LI,\ IF,\ FT,\ TO,\ ON\}\ there\ is\ only\ one\ ways\ can\ ON\ legible]$

$\therefore \frac{\frac{2}{5}\times\frac{1}{2}}{\frac{2}{5}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{2}}$

$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{12}}$

$= \frac{\frac{1}{5}}{\frac{17}{12\times5}}$

$= \frac{12}{17}$

22. A certain disease effects about 1 out of 10,000 people. There is a test to check whether the person has the disease. The test is quite accurate. In particular, we know that

- the probability that the result is positive (suggesting the person has the disease), given that the person does not have the disease, is only 2%.
- the probability that the test result is negative (suggesting the person does not have the disease), given that the person has the disease is only 1%.

A random person gets tested for the disease and the result comes back positive. What is the probability that the person has the disease?

(a) $< 50\%$

(b) $> 50\%$

(c) 50%

(d) $0\%\ or\ 100\%$

Solution:

Let D be the event that the person has the disease, T be the event that the result be positive

$P(D) = \frac{1}{10{,}000}$

$P(\frac{T}{D'}) = 0.02$

$P(T'/D) = 0.01$

We want to compute $P(D/T)$

$P(\frac{D}{T}) = \frac{P(\frac{T}{D})P(D)}{P(\frac{T}{D})P(D)+P(\frac{T}{D'})P(D')}$

$$= \frac{(1-0.01)\times 0.0001}{(1-0.01)\times 0.0001+0.02(1-0.0001)}$$

$$= 0.0049$$

$\therefore$ < 50%

23. In a market, they sells eggs in egg holders they share 10 of them in each. There is 60% chance, that all of the eggs are ok, 30% chance, that exactly 10 of them is broken, and 10% chance that exactly 2 of them are broken.

We buy an egg holder and after we grab our first egg, we are sad because it is broken, what is the probability that there is one more broken egg in our holder?

(a) $\frac{3}{5}$

(b) $\frac{1}{5}$

(c) $\frac{2}{5}$

(d) $\frac{1}{4}$

Solution:

Let B be the event where the first egg we observe is broken.

H_1 be the event our holder has one broken egg.

H_2 be the event where our holder has two broken eggs.

Since there's one broken egg out of ten in the H_1 case.

$P(\frac{B}{H_1}) = \frac{1}{10}$

likewise $P(\frac{B}{H_2}) = \frac{2}{10}$

$\therefore P(B) = \frac{1}{10}\times\frac{5}{10} + \frac{2}{10}\times\frac{1}{10} = \frac{5}{100}$

$P(\frac{B}{H_1})P(H_1) + P(\frac{B}{H_2})P(H_2)$

We need to find,

$$P(\frac{H_2}{B}) = \frac{P(\frac{B}{H_2})P(H_2)}{P(B)}$$

$$= \frac{\frac{2}{10}\times\frac{1}{10}}{\frac{5}{100}}$$

$$= \frac{2}{5}$$

24. I'm on a farm with six cows; three are white, two are white, two are black and one is completely black on one side and completely white on the other. I see one cow from one side which appears to be black. What's the probability that the cow's black?

Solution:

Let B be the event that the cow I observe is completely black, and S be the event that atleast one side is black. S is given, so I need to find $P(B/S)$.

$P(\frac{B}{S}) = \frac{P(\frac{S}{B})*P(B)}{P(S)}$

$P(\frac{S}{B}) = 1$, since if the entire cow is black then atleast one side is black. $P(B) = 1/3$ since there are two black cows among a group of six.

$P(S) = 5/12$ since there are 12 sides you can see, all equally likely, and 5 of the sides are black. So, $P(S) = 5/12$.

This yields $P(\frac{B}{S}) = \frac{(\frac{1}{3})}{(\frac{5}{12})} = \frac{4}{5}$

25. A person has two boxes A and B. First one A has 4 white balls and 5 black balls and the second or B has 5 white balls and 4 black balls. This person takes randomly one ball from the first box and put into the second box. After that he takes a ball from the second box. Find the probability of taking balls of the same color in this process?

Solution:

The probability of choosing white from the first box and then from the second box is:

$(\frac{4}{9})\cdot(\frac{6}{10}) = \frac{24}{90}$

The probability of choosing black from the first box and then from the second box is:

$(\frac{5}{9})\cdot(\frac{5}{10}) = \frac{25}{90}$

So, the probability of choosing the same color is:

$\frac{24}{90} + \frac{25}{90} = \frac{49}{90}$

26. Prof X is crossing the atlantic ocean on a plane, on his way to a conference. The captain has just announced that on unusual engine taught has been signaled by the plane is computer. This indicates a fault that only occurs once in 10,000 flights. If the fault report is true then there is a 70% chance the plane will have to crash land in the ocean, which means certain death for the

passengers. However the reports are completely reliable. There is a 2% chance of a false positive; and there is a 1% chance of the same fault occurring without the computer flagging the error report.

Solution:

$Baye's\ law:$

$$P(\tfrac{A}{B}) = \frac{P(\frac{B}{A}) \cdot P(A)}{P(B)}$$

$$P(\tfrac{A}{B}) = \frac{P(\frac{B}{A}) \cdot P(A)}{P(B)}$$

$$P(\frac{Fault}{Positive\ Indication}) = \frac{P(\frac{Positive\ Indication}{Fault}) \cdot P(Fault)}{P(Positive\ Indication)}$$

$$P(\frac{Positive\ Indication}{Fault}) = 0.99$$

$$P(Fault) = 0.0001$$

$P(Positive\ Indication)\ is\ a\ little\ trickier.\ So,\ lets\ make\ this\ table.$

	Fault	NoFault	Total
Positive	$0.99 * 0.0001$	$0.02 * 0.9999$	0.020097
Negative	$0.01 * 0.0001$	$0.98 * 0.9999$	0.979902
Total	0.00001	0.9999	

$$P(\frac{Fault}{Positive\ Indication}) = \frac{0.000099}{0.020097} = 0.0049$$

$$P(Survival) = 1 - P(Death) = (1 - 0.0049) * 0.70 = 0.9965$$