11. A disease test is advertised as being 91% accurate. If you have the disease you will test positive 99% of the time, and if you don’t have the disease you will test negative 99% of the time. If 1% of all people have this disease and you feel positive, what is the probability that you actually have disease?

(a) 2/3 | (b) 1/3

(c) 1/2 | (d) 1/4

Solution:

Let D be the event having disease + be the event that the testing positive.

Let $\overline{D}$ be the event not having disease.

We need to find

$$P(D/+) = \frac{P(D \cap +)}{P(+)}$$

$$= \frac{P(D)\cdot P(+/T)}{P(D)P(+/T)+P(\overline{D})P(+/\overline{D})}$$

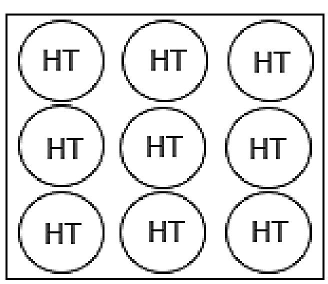
$$= \frac{1\%\cdot 99\%}{1\%\cdot 99\%+99\%\cdot 1\%}$$

$$= \frac{99/100}{99/100+99/100}$$

$$= \frac{1}{2}$$

$$= 0.5$$

12. Emma’s coin box contains 8 fair, standard coins (head and tails) and 1 coin which has head on both sides. He selects a coin randomly and flips it 4 times, getting all heads. If he flips the coin again, what is the probability it will be heads?



(a) 1/6 | (b) 1/3

(c) 5/6 | (d) 2/3

Solution:

Let F and H_4 be the events of having the fair coin and flipping 4 heads respectively.

Let F be the event having unfair coin.

$P(\ H_5) = P(F/\ H_4)P(H/F) + P(\overline{F}/\ H_4)P(H/\overline{F}\ )$

We need to find, $P(F/\ H_4)$ and $P(\overline{F}/\ H_4)$

$$P(\overline{F}/\ H_4) = 1 - P(F/\ H_4)$$

$$P(F/\ H_4) = \frac{P(F \cap H_4)}{P(H_4)}$$

$$= \frac{P(H_4/F) \cdot P(F)}{P(H_4/F) \cdot P(F) + P(H_4/\overline{F}) \cdot P(\overline{F})}$$

$$= \frac{\frac{1}{16} \times \frac{8}{9}}{\frac{1}{16} \times \frac{8}{9} + 1 \times \frac{1}{9}}$$

$$= \frac{1}{3}$$

$\therefore P(F/H_4) = \frac{2}{3}$

$P(H_5) = \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times 1 = \frac{5}{6}$

13. In general, the probability that it rains on Saturday is 25%. If it rains on Saturday the probability that it rains on Sunday is 50%. If it doesn't rain on Saturday, the probability that it rains on Sunday is 25%.

Given that it rained on Sunday, what is the probability that it rained on Saturday?

(a) 40% (b) 35%

(c) 60% (d) 35.7%

Solution:

Is is given that

$P(Su/Sa) = 50\%$

and $P\left(Su/\overline{Sa}\right) = 25\%$

Is is given that

$P(Sa) = 25\%$

$\therefore P(\overline{Sa}) = 75\%$

We need to find

$$P(Su/Sa) = \frac{P(Sa \cap Su)}{P(Su)}$$

$$= \frac{P(Sa)\ P(Su/Sa)}{P(Sa)P(Su/Sa)+P(\overline{Sa})P(Sa/\overline{Sa})}$$

$$= \frac{25\% \times 50\%}{25\% \times 50\% + 75\% \times 25\%}$$

$$= 40\%$$

14. 1% of people have a rare cancer and there is a test of this cancer which is "90% accurate" that is

If you have the cancer there is a 90% chance the test will be positive.
If you don't have the cancer there is a 90% chance the test will be negative.

If you take the test and positive, what is the approximate probability that you have the cancer?

(a) 10% (b) 90%

(c) 50% (d) None of these

Solution:

We need to find

$$P(C/+) = \frac{P(C \cap +)}{P(+)}$$

$$= \frac{P(C)P(+/C)}{P(C)P(+/C)+P(\overline{C})P(+/\overline{C})}$$

Given data

$P(C) = \frac{1}{100}$

$P(+/C) = \frac{90}{100}$

$$\therefore P(C/+) = \frac{\frac{1}{100} \times \frac{9}{10}}{\frac{1}{100} \times \frac{9}{10} + \frac{99}{100} \times \frac{1}{10}}$$

$= \frac{9}{108}$

$= \frac{1}{12}$

$\approx 10\%$

15. A family has two children. Given that one of the children is a boy. What is the probability that both children are boys?

(a) 1/3

(b) 1/2

(c) 13/27

(d) 11/27

Solution:

There are 4 ways {bb, bg, gb, gg}

It is given that the family already has a boy.

∴ The probability of both children is 1/3.