1. A total of 46% of the voters in a certain city classify themselves as Independents, where as 30% classify themselves as Liberals and 24% say they are Conservatives. In s recent local election, 35% of the Independents, 62% of the Liberals and 58% of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, the probability that he or she is an Independent is _________ [enter up to 3 decimal points]

Solution:

Given that,

P(J) = 46% Independents

P(L) = 30% Liberals

P(C) = 24% Conservatives

⇒ And also given that 35% of Independents are voted

P (V/J) = 35%

P (V/L) = 62%

P (V/C) = 58%

∴ We have to find P (J/V).

According to the Bays theorem,

$$P\left(\frac{J}{V}\right) = P\left(\frac{\mathrm{P(I \cap V)}}{\mathrm{P(V)}}\right)$$

$$= \frac{P\left(\frac{V}{I}\right)P(I)}{P\left(\frac{V}{I}\right)P(I)+P\left(\frac{V}{L}\right)P(L)+P\left(\frac{V}{C}\right)P(C)}$$

$$= \frac{0.35\times0.46}{0.35\times0.45+0.62\times0.30+0.58\times0.24}$$

$$= 0.331$$

2. Mr. Elliot has just had a biopsy or a possibly cancerous tumor. Not wanting to spoil a weekend family event, he does not want to hear any bad news in the next few days. But if he tells the doctors to call only if the news is good, then if the doctor doesn't call, Mr. Elliot can conclude that the news is bad. Being a student of probability Mr. Elliot instructs the doctor to flip a coin. If it comes up head, the doctor is to call if the new is good and not call if the new is bad. If the coin comes up tails, the doctor is not to call whether the news is good or bad.

Solution: Let α be the probability that the tumor is cancerous. Let β be the probability that the tumor is cancerous given that the doctor doesn't call then β = __________________

(a) $\frac{a}{1+a}$

(b) $\frac{2a}{1+3a}$

(c) $\frac{1}{1+2a}$

(d) $\frac{2a}{1+a}$

Solution:

Let C be the event that tumor is cancerous. Let N be the event that the doctor doesn't call.

So we need to find $P\left(\frac{C}{N}\right)$

$$\beta = P\left(\frac{C}{N}\right) = \frac{P(C\cap N)}{P(N)} = \frac{P\left(\frac{N}{C}\right)P(C)}{P\left(\frac{N}{C}\right)P(C)+P(\frac{N}{C^1})P(C^1)}$$

It is given that he has cancer.

If coin turns to be head $\left[p(n) = \frac{1}{2}\right]$ he wants call.

If coin turns to be tail $\left[p(t) = \frac{1}{2}\right]$ he want call.

$\therefore$ In both cases doctor won't call him $P\left(\frac{C}{N}\right)$ = 1

$P\left(\frac{N}{C^1}\right)$:

It is given that he doesn't have cancer.

$\therefore$ Doctor won't call him only when the coin turns to be tail (t) = $\frac{1}{2}$

$$P\left(\frac{N}{C^1}\right) = \frac{1}{2}$$

$$P\left(\frac{C}{N}\right) = \frac{P\left(\frac{N}{C}\right)P(C)}{P\left(\frac{N}{C}\right)P(C)+P\left(\frac{N}{C^1}\right)P(C^1)}$$

$$= \frac{1\times\alpha}{1\times\alpha+\frac{1}{2}\times(1-\alpha)}$$

$$= \frac{1\times\alpha}{\alpha+\frac{1}{2}(1-\alpha)}$$

$$= \frac{2\alpha}{2\alpha+(1-\alpha)}$$

$$= \frac{2a}{1+a}$$

$$\therefore\ \beta = \frac{2a}{1+a}$$

3. According to the question 2, which should be bigger α or β?

(a) α > β

(b) α < β

(c) α = β

(d) Can't say

Solution:

$\frac{2a}{1+a} \geq a$[which is strictly inequal unless $\alpha = 1$]

$\beta \geq \alpha \Rightarrow \beta > \alpha$

4. A family has children with probability Pi where $P1$ = 0.01, $P2$ = 0.25, $P3$ = 0.35, $P4$ = 0.3. A child from this family is randomly chosen. Given that this child is the eldest child in the family, the conditional probability that the family has 4 children _______.

(a) 0.24

(b) 0.18

(c) 0.26

(d) 0.16

Solution: Let E be the event the child selected is the eldest. Let Fi be the event that the family has j children. We need to find

$P\left(\frac{F^4}{E}\right) = P\frac{(F^4 \cap \mathrm{E})}{P(E)}$

$P(F^4)$= $P\left(\frac{F^4}{E}\right)$P($F^4$)

$= \frac{1}{4} \times 0.3$

$= \frac{0.3}{4}$

= 0.075

$P(E) = \sum P(\mathrm{F})\mathrm{P}\left(\frac{E}{F_i}\right) = P(F_1)\mathrm{P}\left(\frac{E}{F_1}\right) + P(F_2)\mathrm{P}\left(\frac{E}{F_2}\right) + P(F_3)\mathrm{P}\left(\frac{E}{F_3}\right) + P(F_4)\mathrm{P}\left(\frac{E}{F_4}\right)$

$P(E) = 0.1 \times 1 + 0.25 \times \frac{1}{2} + 0.35 \times \frac{1}{3} + 0.3 \times \frac{1}{4}$

= 0.4155

Therefore, $P\left(\frac{F_4}{E}\right) = \frac{0.075}{0.4155} = 0.18$

5. A gambler has in his pocket a fair coin and a two-head coin. He selects one of the coins at random; when he flips it, it shows head. What is the probability that it is the fair coin?

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{5}$

Solution:

It is given that, the win shows head

We need to find

$P\left(\frac{Fair}{h}\right) = \frac{P(Fair \cap h)}{P(h)}$

$$= \frac{P\left(\frac{h}{fair}\right)P(fair)}{P\left(\frac{h}{fair}\right)P(fair)+P\left(\frac{h}{not\ fair}\right)P(not\ fair)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+1\times\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

6. A gambler has a fair coin and a two headed coin. He selects one of the coins at random flipped it twice and got two heads. Now the probability that it is the fair coin is _______.

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{5}$

Solution:

$$P\left(\frac{Fair}{hh}\right) = \frac{P(Fair \cap hh)}{P(hh)}$$

$$= \frac{P\left(\frac{hh}{fair}\right)P(fair)}{P\left(\frac{hh}{fair}\right)P(fair)+P\left(\frac{hh}{not\ fair}\right)P(no\ fair)}$$

$$P\left(\frac{hh}{fair}\right) = 1/4[(hh),(hT),(Th),(TT)]$$

$$P\left(\frac{hh}{no\ fair}\right) = 1$$

$$P\left(\frac{Fair}{hh}\right) = \frac{\frac{1}{4}\times\frac{1}{2}}{\frac{1}{4}\times\frac{1}{2}+1\times\frac{1}{2}}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{2}}$$

$$= \frac{1}{5}$$

7. A gambler has a fair coin and a two headed coin. He selects one of the coins at random. He flipped the same coin thrice. In the first time it showed head, in the second time also head and in the third time it showed tails. The probability that the coin selected is fair coin is ________.

Solution:

The given sequence of outcomes is HHT. It is given that there is a tail in the outcomes which means it is not a two-head coin we can say it is fair coin

∴ Answer = 1

8. English and American spellings are "rigour" and "rigor" respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel.

If the 40% of the English speaking men at the hotel are English and 60% are Americans. What is the probability that the writer is an Englishman?

(a) $\frac{5}{13}$

(b) $\frac{4}{9}$

(c) $\frac{5}{11}$

(d) $\frac{2}{5}$

Solution:

We have to find

$$P\left(\frac{E}{V}\right) = \frac{P\left(\frac{V}{E}\right)P(E)}{P\left(\frac{V}{E}\right)P(E)+P\left(\frac{V}{A}\right)P(A)}$$

$$= \frac{\frac{3}{6}\times\frac{40}{100}}{\frac{3}{6}\times\frac{40}{100}+\frac{2}{5}\times\frac{60}{100}}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{2}{5}\times\frac{3}{5}}$$

$$= \frac{5}{11}$$

9. You randomly choose a treasure chest to open and then randomly choose a coin from that treasure chest. If the coin you choose is gold then what is the probability that you choose chest A?

Treasure 1 = 100 gold coins

Treasure 2 = 50 gold coins + 50 silver coins

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

Solution:

Let G be the event of selecting a Gold coin.

⇒ Let A be the event selecting Treasure 1.

⇒ Let B be the event selecting Treasure 2.

$$P\left(\frac{G}{A}\right) = 1$$

$$P\left(\frac{G}{B}\right) = \frac{1}{2}$$

We need to find

$$P\left(\frac{A}{G}\right) = \frac{P(A \cap G)}{P(G)}$$

$$= \frac{P(A).P\left(\frac{G}{A}\right)}{P(A).P\left(\frac{G}{A}\right)+P(B).P\left(\frac{G}{B}\right)}$$

$$= \frac{\frac{1}{2}.1}{\frac{1}{2}.1+\frac{1}{2}.\frac{1}{2}}$$

$$= \frac{2}{3}$$

10.A machine learning models M1 predicts raining, the chance of raining is 60%, when the other model M2 predicts raining, the chance of raining is 60%. If both predicts to rain (assuming they did the prediction independently), what is the chance of raining? [choose the most appropriate answer]

1. it will rain for sure   3. Anything between 0 and 1

2. Incomplete data   4. 36% chance

Ans:

Let X, Y, Z be the events that M1 predicts rain, M2 predicts rain, it rains respectively. We are given that X, Y are independent and $P(Z|X) = P(Z|Y) = 0.6$.

$P(X \cap Z)/P(X) = P(Y \cap Z)/P(Y)$

It could be that $P(X)=P(Y)=0.6$ and $Z=X \cap Y$. In that case $P(Z|A, B)=1$.

It could also be that $P(X)=P(Y)=0.4$ and $Z=(X \cap Y)'$. In that case $P(Z|A, B)=0$.

And anything in between is possible.