# Student Information

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# Answer 1

## a.

We have K ={ $q_0, q_1, q_2, q_3, h$ } ,
$\Sigma$ = { a,b,$\sqcup$,$\triangleright$ } ,
s = $q_0$ ,
H = {h} ,
and transitions $\delta$ is the following :

| *State* | $\sigma$ | $\delta(q,\sigma)$ |
|---|---|---|
| $q_0$ | $\triangleright$ | ($q_0$,$\rightarrow$) |
| $q_0$ | a,b,$\sqcup$ | ($q_1$,$\rightarrow$) |
| $q_1$ | $\triangleright$ | ($q_1$,$\rightarrow$) |
| $q_1$ | a | ($q_2$,$\sqcup$) |
| $q_1$ | b | ($q_4$,$\sqcup$) |
| $q_1$ | $\sqcup$ | (h,$\sqcup$) |
| $q_2$ | $\triangleright$ | ($q_2$,$\rightarrow$) |
| $q_2$ | a,b | ($q_2$,$\leftarrow$) |
| $q_2$ | $\sqcup$ | ($q_3$,$\leftarrow$) |
| $q_3$ | $\triangleright$ | ($q_3$,$\rightarrow$) |
| $q_3$ | a,b | ($q_3$,$\leftarrow$) |
| $q_3$ | $\sqcup$ | (h,a) |
| $q_4$ | $\triangleright$ | ($q_4$,$\rightarrow$) |
| $q_4$ | a,b | ($q_4$,$\leftarrow$) |
| $q_4$ | $\sqcup$ | ($q_5$,$\leftarrow$) |
| $q_5$ | $\triangleright$ | ($q_5$,$\rightarrow$) |
| $q_5$ | a,b | ($q_5$,$\leftarrow$) |
| $q_5$ | $\sqcup$ | (h,b) |

## b.

i) Computation for the given string and head position is:

$$
\begin{aligned}
(q_0,\triangleright\sqcup\sqcup\mathrm{b}\underline{\mathrm{a}}\mathrm{b}) &\vdash (q_1,\triangleright\sqcup\sqcup\mathrm{ba}\underline{\mathrm{b}}) \\
&\vdash (q_4,\triangleright\sqcup\sqcup\mathrm{ba}\underline{\sqcup}) \\
&\vdash (q_5,\triangleright\sqcup\sqcup\mathrm{b}\underline{\mathrm{a}}\sqcup)
\end{aligned}
$$

$$
\begin{aligned}
&\vdash (q_5, \triangleright \sqcup \sqcup \underline{b}a\sqcup) \\
&\vdash (q_5, \triangleright \sqcup \underline{\sqcup}ba\sqcup) \\
&\vdash (h, \triangleright \sqcup \underline{b}ba\sqcup)
\end{aligned}
$$

The computation has 6 steps . $\diamondsuit$

ii) Computation for the given string and head position is:

$$
\begin{aligned}
(q_0, \triangleright a\underline{a}a) &\vdash (q_1, \triangleright aa\underline{a}) \\
&\vdash (q_2, \triangleright aa\underline{\sqcup}) \\
&\vdash (q_3, \triangleright a\underline{a}\sqcup) \\
&\vdash (q_3, \triangleright \underline{a}a\sqcup) \\
&\vdash (q_3, \underline{\triangleright} aa\sqcup) \\
&\vdash (q_3, \triangleright \underline{a}a\sqcup)
\end{aligned}
$$

As it is seen in the computation , the machine enters an infinite loop and never halts . $\diamondsuit$

iii) Computation for the given string and head position is:

$$
\begin{aligned}
(q_0, \triangleright \underline{a}\sqcup bb) &\vdash (q_1, \triangleright a\underline{\sqcup}bb) \\
&\vdash (h, \triangleright a\underline{\sqcup}bb)
\end{aligned}
$$

The computation has 2 steps . $\diamondsuit$

# Answer 2

We have the tape of the form $\triangleright \underline{\sqcup}$ babc at first . Let's trace its operation by analyzing teh Turing Machine.

We start with moving one square to right. New form of the tape is $\triangleright \sqcup \underline{b}$abc .

We read the symbol b . So we'll use a = b . And the head move to the right until it finds a blank . After that , we move to the left square .New form of the tape is $\triangleright \sqcup$ bab$\underline{c}$ .

Now , we read teh symbol c , so we have b = c . we go one square to the right and write the blank symbol . New form of the tape is $\triangleright \sqcup$ babc$\underline{\sqcup}$ .

We again go one square to the right and write a which is equal to b (since we have a = b) . New form of the tape is $\triangleright \sqcup$ babc $\sqcup$ $\underline{b}$ .

Head is still at the same position and we write b which is equal to c (since we have b = c) . New form of the tape is $\triangleright \sqcup$ babc $\sqcup$ $\underline{c}$ .

We go one square to the right and write a blank symbol . New form of the tape is $\triangleright \sqcup$ babc $\sqcup$ c $\underline{\sqcup}$ .

In the last step , we go one square to the right and write a which we know it is b from above . Last form of the tape is $\triangleright \sqcup$ babc $\sqcup$ c $\sqcup$ $\underline{b}$ .

After this form , the machine halts . $\diamondsuit$

# Answer 3

a) To find the language that M semidecides , we need to analyze the machine . What it does is , it reads a string and do some operations . One important thing is the machine never enters an infinite loop ,i.e. the machine halts somewhere . Empty string is also accepted . So , the language that M semidecides is :

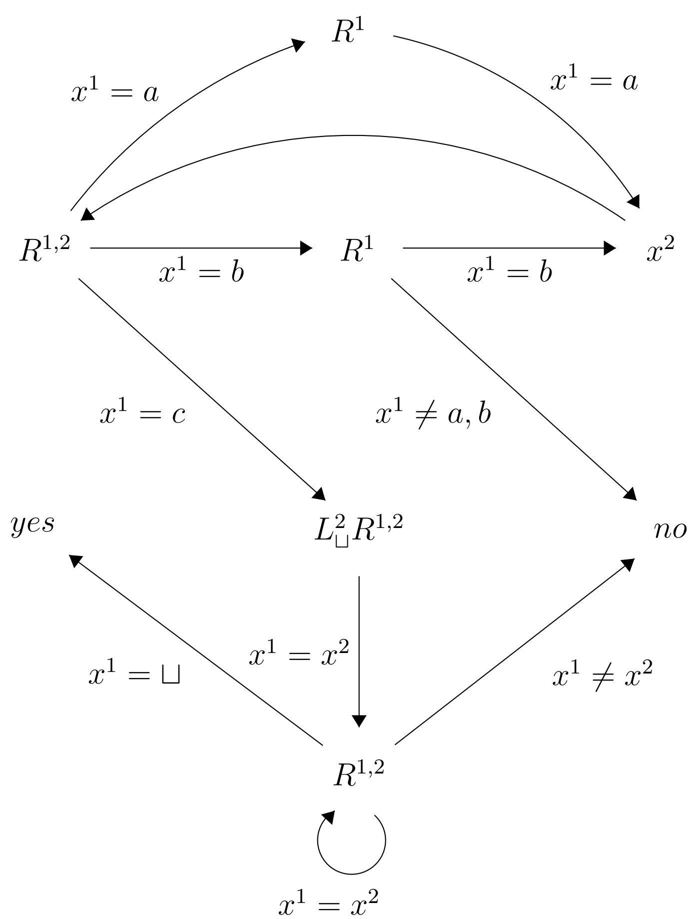
L = (a∪b)* . ♢

b) To find the function , we need to analyze the machine . What it does is , it reads a string and puts a's before b's . When it read the blank character ⊔ , the machine halts . For example , the string aababba becomes aaaabbb after the Turing Machine M applies its transitions . Empty string is also accepted , and number of a's and b's will be the same as the input string's a's and b's . By **Definition 4.2.2** , the function f can be defined is :

f(w) = t , where t = { $a^n b^m$ | n ≥ 0 , m ≥ 0} . ♢

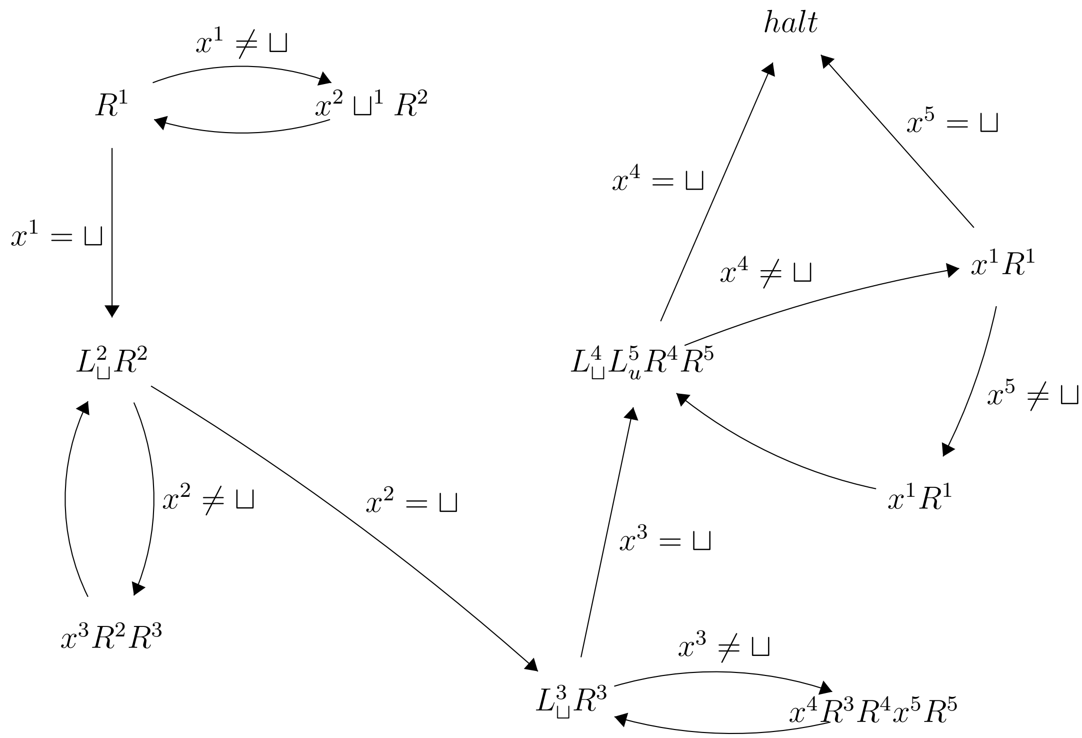
# Answer 4

The following machine has 2 tapes and decides the given language L :



# Answer 5

The following machine has 5 tapes and computes the given function f :




# Answer 6

# Answer 7

## a.

What we have to do is the following for deletion of the front and rear positions of the input string on the tape :

Assume we are at the somewhere middle of the input string . By using two heads on one tape , we go to the leftmost and rightmost square which is not $\sqcup$ . When we delete those two , we do the delete operation .

For the insert operation , by using the head , we find the correct position and insert the new square .

**b.**

**c.**

**d.**

The Turing Machine in this question has multiple heads . According to the **Theorem 4.3.2** , any function computed by a Turing Machine with several tapes , heads , etc. can be computed by a standard Turing Machine . Therefore , there is a standard TM that does insert-delete operations like the insert-delete TM .

# Answer 8

# Answer 9

Let's define tuple notations for Turing Machines of recursively enumerable languages $L_1, L_2$ and $L_3$ :

$M_1 = \{K_1, \Sigma_1, \delta_1, s_1, H_1\}$ ,

$M_2 = \{K_2, \Sigma_2, \delta_2, s_2, H_2\}$ ,

$M_3 = \{K_3, \Sigma_3, \delta_3, s_3, H_3\}$ .

After concatenate $L_1$ and $L_2$ we will have a new form of quintuple where ;

$K_A = K_1 \cup K_2$ ,

$s_A = s_1$ ,

$H_A = H_2$ ,and we must set a new transition to use $H_2$ as the set of halt states ,

$\delta(h_1, \Sigma_1) = (s_2, \Sigma_2)$ .

After that we do the union intersection with $L_3$ . Start state will be the same with previous one , i.e. $s_1$ . The alphabet will consist of common elements of $(L_1 \cup L_2)$ and $L_3$(because of the intersection). Set of finite states will be the intersection of $(K_1 \cup K_2)$ and $K_3$ . Set of halting states will be the intersection of $H_2$ and $H_3$ . And we will have a new transition to use $M_1$ :

$\delta(h_2, \Sigma_1 \cup \Sigma) = (s_3, \Sigma_3)$ .

When the first machine $M_1$ halts , it goes to the start state of $M_2$ follow its operations . After $M_2$ halts ,it goes to $M_3$ and $M_3$ operates according to its rules . When $M_3$ halts , the complete turing machine halts .

As we have formed , there is a non-deterministic turing machine with a quintuple which semi-decides L utilizing TM's for recursively enumerable languages $L_1, L_2$ and $L_3$ . $\diamondsuit$