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Probability and Statistics

Major Examination

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1. The following are the weight losses of certain machine parts (in milligrams) due to friction when three different lubricants were used under controlled conditions:

Lubricant A	12.2	11.8	13.1	11.0	3.9	4.1	10.3	8.4
Lubricant B	10.9	5.7	13.5	9.4	11.4	15.7	10.8	14.0
Lubricant C	12.7	19.9	13.6	11.7	18.3	14.3	22.8	20.4

   Test at 0.01 level of significance whether the differences among the means can be attributed to chance.

2. The following data concern the amount of adhesion (y) and its relation to the amount of additive (x_1) and temperature (x_2) of reaction:

x_1	0	70	35	0	70	70	0	35	35
x_2	100	100	140	180	180	140	140	100	180
y	10	48	41	40	39	44	24	31	44

   Fit an equation of the form $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ to the given data and use it to estimate the amount of adhesion when the amount of additive is 40 and the temperature is 130.

3. What are nonparametric tests. Write a summary of each test by explaining why we use that test, what is the null hypothesis and test statistic in each case.

4. Tests of the fidelity and the selectivity of 190 radio receivers produced the results shown in the following table:

		Fidelity		
		Low	Average	High
Selectivity	Low	6	12	32
	Average	33	61	18
	High	13	15	0

   Use the 0.01 level of significance to test whether there is a relationship (dependence) between fidelity and selectivity.

[Please Turn Over]

5. Let $X_1, X_2, \ldots, X_n$ be the observed values of a random sample of size n from the exponential distribution $f(x;\beta) = \beta^{-1}e^{-x/\beta}$ for $x > 0$. Find the maximum likelihood estimator of β. Also, find the maximum likelihood estimator of the probability that the next observation is greater that 1.

6. Two discrete random variables X and Y have the joint pmf:

$$p(x,y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!(x-y)!},\ y = 0, 1, 2, \ldots, x; \quad x = 0, 1, 2, \ldots \infty,$$

where, λ, p are constants with $\lambda > 0$ and $0 < p < 1$. Find:

(a) The marginal probability density functions of X and Y.

(b) The conditional distribution of Y given X and of X given Y.

7. If you wish to estimate the proportion of the engineers and scientists, who have studied probability theory, to be correct within 2% with probability 0.95 or more, how large a sample would you take: (a) if you have no idea what the true proportion is, and (b) if you are confident that the true proportion is less than 0.2?

8. A study shows that 16 of 200 tractors produced on one assembly line required extensive adjustment before they could be shipped, while the same was true for 14 of 400 tractors produced on another assembly line. At the 0.01 level of significance, does this support the claim that the second production line does superior work?

BEST OF LUCK