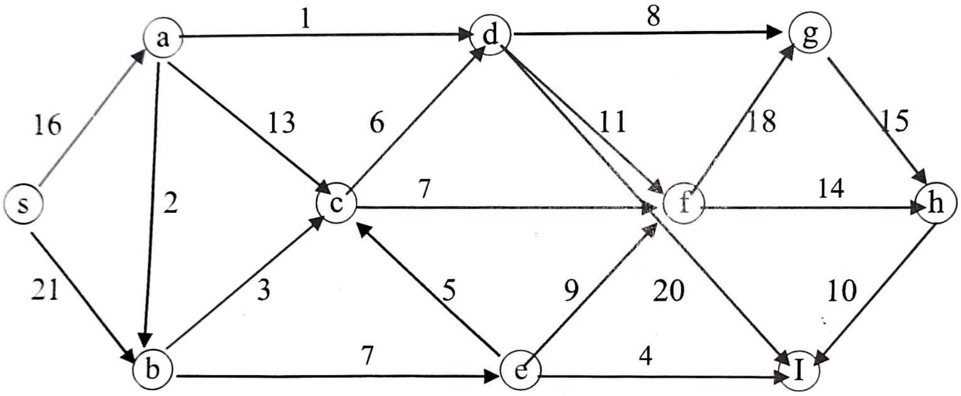
ABV-Indian Institute of Information Technology & Management, Gwalior

Major Examination

Sub: Design and Analysis of Algorithms (ITIT-2202)
Class: IPG-M.Tech. IV Sem
Date & Time: 28th April 2022 {10:00-12:00}
Max. Marks: 65

NOTE: Attempt all the **FIVE** questions. Answer **all parts** of the same question together otherwise the part done separately will not be corrected.

Q. 1. Consider the following directed graph. Find the shortest path between source **s** and all destinations using Greedy algorithm discussed in the class. [10]

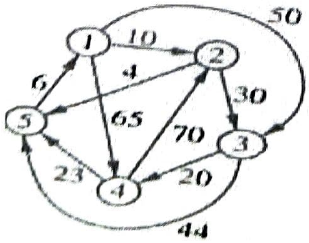


(b) Explain the two implementations of **Dijkstra's** single source shortest path algorithm. Which implementation will be applicable for the graph of part (a)? Also, give the time complexity of your implementation. [3]

Q. 2. (a) Suppose $T(1) = 1$, and for all $n \geq 2$, a power of 2, solve the following recurrence relation exactly. [5]

$$T(n) = 3T(n/2) + n^2 - n$$

(b) Find the all-pair shortest path in the following figure. [8]



Q.3. (a) For $n = 14$ the weights and values of the objects are given in the following table. Fill the knapsack as full as possible using fractional parts of the objects so that the **value is maximized** such that total weight does not exceed W = 300. [6]

Weight(w)	10	20	30	40	50	60	70	80	90	100	110	120	130	140
Value(v)	45	200	50	70	130	60	120	80	65	98	210	189	145	120

(b) A legend stated that the Tower of Hanoi problem cannot be solved before the End of Universe. Compute the number of disks for which the Legend's statement would be true. Derive the formula used in your computation, mention the assumption (**if any**) clearly. [4]

(c) Prove that if the algorithm discussed in the class is used to find UNION of sets, then, every tree with k nodes has at most $\lfloor \log k \rfloor + 1$ layers. How many nodes will be there in a 8 layer tree? [3]

Q. 4 (a) Find the MCST on the following graph of Fig. 4(a) using Kruskal's algorithm. Show all the iterations of the algorithm along with the termination criterion. [4]

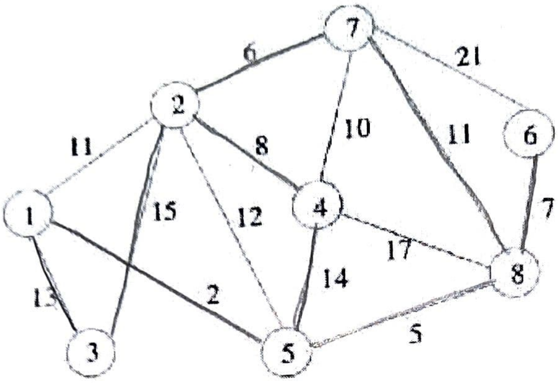


Fig. 4(a)

(b) Consider the problem of exhaustively generating all permutations on things. Suppose $A[1..n]$ contains n distinct values. The following algorithm generates all permutations of $A[1..n]$ in the following sense: A call to procedure permute (n) will result in procedure process(A) being called once with $A[1..\ n]$ containing each permutation of its contents. [5]

```
procedure permute($n$)
  if $n = 1$ then process($A$) else
     $B := A$
     permute($n$)
     for $i := 1$ to $n - 1$do
       swap $A[n]$ with $A[1]$
       permute($n - 1$)
     $A := B$ cyclically shifted one place right
```

Compute the running time of this algorithm.

(c) Give the average case analysis of operations in BST. [4]

Q. 5. (a) Describe the steps to prove that the new problem is NP complete. [5]

(b) Identify the polynomial time and exponential time problems. [4]

1. The Ramsay Number
2. Minimum cost spanning tree
3. The independent set problem
4. The Hamiltonian cycle problem
5. All pair shortest path
6. Transitive closure
7. Optimal binary search trees
8. The clique problem

(c) If $A \leq_p B$ and $B \leq_p C$ then prove that $A \leq_p C$. [4]

Good Luck