CSC358H5: Principles of Computer Networking — Winter 2025

Worksheet 4 Solution: Error Detection and Correction, MAC Approaches, Ethernet Collision Recovery, Ethernet Protocols Feild

Q0 Knowledge Check (from Week 04 Lecture)

0.a **(Hamming Distance Between Two Binary Strings)** Find the Hamming distance between the two codewords 00001010 and 01000110.

Answer. 3.

0.b **(Hamming Distance of A Coding Scheme)** Consider the $(8, 7)$ coding which appends a parity bit to each 7-bits, *i.e.*, if $(d_1, D_2, d_3, d_4, d_5, d_6, d_7)$ is the message, then it calculates the parity bit as $p_1 = d_1 \oplus d_1 \oplus d_3 \oplus d_4 \oplus d_5 \oplus d_6 \oplus d_7$ and the final 8 7-bit codeword is $(d_1, D_2, d_3, d_4, d_5, d_6, d_7, p_1)$. Find the Hamming distance of this coding scheme.

Answer. 2.

0.c **(Hamming Theorem)** Fill in the blanks:

> **Hamming Theorem:** Let d be the Hamming distance of a coding scheme. The coding scheme can detect up to any $\underline{(d-1)}$ bits of error and correct up to any $\underline{\lfloor \frac{d-1}{2} \rfloor}$ bits of error.

0.d **(True/False)** It is more difficult to detect collision in shared wired LANs compared to shared wireless LANs. ☐ True ☑ False

0.e **(Medium Access Control)** In lecture, we studied three Medium Access Control (MAC) approaches to share a broadcast medium. Explain these three approaches and name some examples for each of these approaches.

Answer.

1. **Channel partitioning:** Divides channel into smaller pieces and allocates each piece to a node for exclusive use.
   - **Examples:** Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), and Code Division Multiple Access (CDMA)
2. **Taking turns:** Nodes take turns, but nodes with more to send can take longer turns.
   - **Examples:** Token-passing protocol, Polling protocol
3. **Random access:** Channel not divided. Allows collisions, but then recovers from collision.
   - **Examples:** ALOHA, Slotted ALOHA, CSMA/CD, CSMA/CA

Q1

Consider the Ethernet frame fields. For each field, name the task that it is needed for. Choose from the following list of tasks:

(i) Get the frame to the destination and get responses back to source
(ii) Tell host what to do with the frame once arrived
(iii) Indicates the length of the data field
(iv) Carries the actual data being transmitted
(v) Synchronization
(vi) Ensures a minimum frame size of 64 bytes
(vii) Signals the start of the actual frame
(viii) Detects errors
(ix) Corrects errors

Field	Task
Preamble	(v)
Start Frame Delimiter (SFD)	(vii)
MAC addresses	(i)
Length / Type	(iii) / (ii)
Data	(iv)
Padding	(vi)
Frame Check Sequence (FCS)	(viii)

Q2 (Internet Checksum as (N, K) Block Coding) In this problem, we define a (N, K) coding scheme, named *CSC358 block coding*, which relies on the Internet Checksum. *CSC358 block coding* transforms any 10-byte message into a codeword by appending its Internet Checksum to the end of the original message.

2.a Specify the value of N and K for this (N, K) coding scheme, *i.e.*, CSC358 block coding.

$N = 96$ $\qquad\qquad$ $K = 80$

2.b What is the Hamming distance of this coding scheme? Justify your answer.

Answer. It has a Hamming distance of 2.

To prove this, it suffices to show that HD > 1 (*i.e.*, there does not exist any two valid codewords with distance 1) and HD ≤ 2 (*i.e.*, there exists two valid codewords with distance 2) in the set of codewords.

It is trivial that there does not exists any two valid codewords with distance 1, as changing any bit of a valid codeword, either from the checksum part or the original message part, would result in an *invalid* codeword since the checksum won't match the remaining part in the changed codeword.

To show that there exists two valid codewords with distance 2, consider message m_1 which is 80 bits of 0's and message m_2 which is a bit of 1 followed by 79 bits of 0's. The checksum for m_1 is all 1's and the checksum for m_2 is a bit of 0 followed by 15 bits of 1's. It is not difficult to see that the corresponding codeword of these two messages (*i.e.*, message appended by checksum) have a distance of 2.

Q3 (Ethernet Collision and Backoff) Suppose Hosts A and B share the same Ethernet link to send their frames. Assume that they have unlimited number of frames to send. Suppose that A and B attempt to send their frames at the same time and as a result, there will be a collision detected by both. Assume that this is the first detected collision for their frames. Thus, A and B should select a backoff time uniformly at random form $0 \times T$ and $1 \times T$, where $T = 51.2$ μs is the Slot Time. Assume that A selects 0 and B selects $1 \times T$ for backoff. Thus, A wins and sends the frame while B is waiting. Assume that the transmission delay of A and B's frames are equal to T.

3.a When A is done with sending its frame, both A and B tries to transmit again and there will be a collision. Thus, A selects timeout randomly from 0 and T, and B selects from 0, T, $2T$, or $3T$. Find Probability of A winning this and transmit before B.

Answer. Let us show A and B's selected backoff as $(k_A \times T, k_B \times T)$.
The list of possible cases for the selected backoff times is: $(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3)$.
The list of cases in which A wins is: $(0,1),(0,2),(0,3),(1,2),(1,3)$.
Thus, the probability of A winning is $\frac{5}{8}$.

3.b Suppose A wins the second round (*i.e.*, the round in part **3.a**). When A is done with sending its frame, both A and B tries to transmit again and there will be a collision. Find the probability of A winning third round and transmit before B.

Answer. Let us show A and B's selected backoff as $(k_A \times T, k_B \times T)$. Observe that k_A can be 0 or 1, and k_B can be $0, 1, \ldots,$ or 7. Thus, there are 16 possible cases in total.
Note that A wins if $k_A = 0$ and $k_b > 0$, or $k_A = 1$ and $k_B > 1$. Thus, there are 13 cases in which A wins. Therefore, the probability of A winning is $\frac{13}{16}$.

3.c Assume the same procedure repeats, *i.e.*, A wins the k^{th} round and sends its frame, and then both A and B tries to transmit again and there will be a collision. Find the probability of A winning the $(k+1)^{\text{th}}$ round and transmit before B, in terms of k. Show that this probability is larger than $1 - \frac{1}{2^k}$.

Answer. Let us show A and B's selected backoff as $(k_A \times T, k_B \times T)$. Observe that k_A can be 0 or 1, and k_B can be $0, 1, \ldots,$ or $2^{k+1} - 1$. Thus, there are $2 \times 2^{k+1} = 2^{k+2}$ possible cases in total.

Note that A wins if $k_A = 0$ and $k_b > 0$, or $k_A = 1$ and $k_B > 1$. Thus, there are $(2^{k+1} - 1) + (2^{k+1} - 2) = 2^{k+2} - 3$ cases in which A wins. Therefore, the probability of A winning is $\frac{2^{k+2}-3}{2^{k+2}} = 1 - \frac{3}{2^{k+2}}$.

It is not difficult to see that $\frac{3}{2^{k+2}} < \frac{4}{2^{k+2}}$. Thus, $\frac{3}{2^{k+2}} < \frac{1}{2^k}$. Hence, $1 - \frac{3}{2^{k+2}} > 1 - \frac{1}{2^k}$. Therefore, the probability of A winning is larger than $1 - \frac{1}{2^k}$.

3.d Find the probability of A winning every round after the third round.

Answer. The probability of A winning every round after the third round is:

$$\prod_{i=3}^{\infty}(1 - \frac{3}{2^{k+2}}) > \prod_{i=3}^{\infty}(1 - \frac{1}{2^k}) \approx \frac{3}{4}.$$[1]

This phenomenon is called the capture effect.

[1] For interested readers: This approximation is due to Euler function.