CSC358H5: Principles of Computer Networking — Winter 2025

Worksheet 3 Solution: Probability Review, Abstract Model of Link, Error Detection and Correction

Q0 Knowledge Check (from Week 03 Lecture)

0.a (Bandwidth, Throughput, and Goodput) Erfan claims that he transferred 1000 bytes in 1 second on a 100 Mbps point-to-point link. Determine the bandwidth, throughput, and goodput of this connection.

Bandwidth:	☐ 1000 bps	☑ 100 Mbps	☐ 8 Kbps	☐ Can't be determined
Throughput:	☐ 1000 bps	☐ 100 Mbps	☑ 8 Kbps	☐ Can't be determined
Goodput:	☐ 1000 bps	☐ 100 Mbps	☐ 8 Kbps	☑ Can't be determined

0.b (Error Detection and Correction Codes) In 2-3 sentences, describe Error Detection Codes and Error Correction Codes. Furthermore, specify the advantages and disadvantages of each of these two techniques.

Answer. Error Detection Codes are used to detect errors introduced during data transmission, allowing to invoke retransmission. Error Correction Codes not only detect errors but also attempt to correct them at the receiver without needing retransmission.

	Advantages	**Disadvantage**
Error Detection Codes	Simpler to implement and requires less overhead	Increases the network traffic. Increases latency and reduces throughput
Error Correction Codes	Reducing the network traffic. Improving throughput	Complex implementation and requires more overhead

0.c (ARQ vs. FEC) For each cases below, choose the more appropriate technique between Automatic Repeat reQuest (ARQ) and Forward Error Correction (FEC).

Case 1: Sending packets of size 1000 bits over a link with bit error rate of 0.001 with random errors.
☐ ARQ ☑ FEC

Case 2: Sending packets of size 1000 bits over a link with bit error rate of 0.001 with bursty error of 1000 bits.
☑ ARQ ☐ FEC

Case 3: Sending packets of size 1000 bits over a link with bit error rate of 0.001 for real-time application (*e.g.*, teleconference) .
☐ ARQ ☑ FEC

Q1 (Probability Review) The following ARQ-themed questions are designed to provide a concise review of the probability concepts you'll need for this course.

1.a A packet of size 1000 bits is transmitted over a link with an independent and identically distributed (i.i.d.) error model. Given a bit error rate of 0.001, **what is the probability that the packet is transmitted without any errors?** You can use the following estimation: $(1-\epsilon)^k \approx e^{-k\epsilon}$.
[Probability Theory Refresher: The random variable that can be used to model *the error in each bit* is the **Bernoulli random variable**. A Bernoulli random variable models a single experiment with two possible outcomes: success (usually represented as 1) or failure (usually represented as 0). In this case, a bit error can be considered a "success" (even though it's undesirable) and no error can be considered a "failure." In this scenario,, the probability of success (bit error) is given as $p = 0.001$, and the probability of failure (no error) is $q = 1 - p = 0.999$.
We can model each bit transmission as an independent Bernoulli trial with a probability of error (success) of $p = 0.001$. To find the probability that the entire packet is transmitted without error, we are essentially looking for the probability of 1000 consecutive "failures" (no errors).**]**

Answer. Let k denote the packet size and ϵ denote the bit error rate. Observe that $k = 1000$ bits and $\epsilon = 0.001$. Note that the probability of NO error in a single bit is $1 - \epsilon = 1 - 0.001 = 0.999$. Since the errors are independent, the probability of the entire packet being transmitted without error is the probability of a single bit having no error raised to the power of the number of bits in the packet:

$$\mathbb{P}[\text{No bit Error}] = (1-\epsilon)^k = (1-0.001)^{1000} \overset{(1)}{\approx} e^{-1000\times 0.001} = e^{-1} = 0.3679,$$

where the approximation above is due to the given estimation provided in the problem.

1.b Referring to the scenario in part **1.a**, suppose Stop-and-Wait ARQ is employed for reliable packet transmission. In Stop-and-Wait ARQ, the sender transmits a single packet and waits for an acknowledgment (ACK) from the receiver before sending the next packet. If no ACK is received due to a timeout, the sender retransmits the same packet. Calculate the average number of transmissions required to successfully deliver one packet. Assume the acknowledgments to be error-free for simplicity.
[**HINT:** The expectation of a geometric random variable with probability success of ρ is $\frac{1}{\rho}$.]
[Probability Theory Refresher: The random variable that can be used to model the number of transmissions for a packet in this Stop-and-Wait ARQ scenario is the **Geometric random variable**. A Geometric random variable models the number of independent Bernoulli trials needed to get the first "success." In this context, a "success" is a successful packet transmission (*i.e.*, the packet is received without errors, and an ACK is received).
We need to determine the probability of a single successful transmission. This involves the packet being transmitted without error and the ACK being received (which we assume to be error-free for simplicity in this problem). We already calculated this probability in part **1.a**.
As mentioned in the hint, the average number of independent Bernoulli trials needed to get the first "success" is $\frac{1}{\rho}$, where ρ is the probability of success in each trial. Deriving this formula is covered in introductory probability courses and can be reviewed in this Khan Academy video. In CSC358, you are **not** required to derive this expectation. You will be provided with the formula for the expected value of a geometric random variable whenever needed, as demonstrated in this question's hint.**]**

Answer. Let X be the random variable representing the number of transmissions required to successfully deliver one packet. Since we are using Stop-and-Wait ARQ and assuming error-free acknowledgments, X follows a geometric distribution. The probability of a packet being transmitted without any errors is derived in the previous part as 0.3679. Thus, X has a probability of success $\rho = 0.3679$. The hint states that the expectation of a geometric random variable with probability of success ρ is $\frac{1}{\rho}$. Therefore, the average number of transmissions required is $\mathbb{E}(X) = \frac{1}{\rho} = \frac{1}{0.3679} \approx 2.718$. Therefore, the average number of transmissions required to successfully deliver one packet using Stop-and-Wait ARQ in this scenario is approximately 2.718.

1.c Consider the problem described in part **1.b**. Assume that the link bandwidth is infinity and the one-way delay is 1 ms. Furthermore, assume that the time-out (*i.e.*, the time between sending the last bit of a packet and waiting for its ACK) is set to one round-trip-time (RTT). What is the average time it takes between transmitting the first bit of the packet for the first time and receiving its acknowledgment.

Answer. Given the information provided in the question, we can approximate RTT as twice the one-way delay. Thus, RTT $= 2$ ms. As we derived in the previous part, the average number of transmissions required to successfully deliver one packet is 2.718. Since each transmission takes one RTT, the average time it takes between transmitting the first bit of the packet for the first time and receiving its acknowledgment is $2.718 \times 2 = 5.436$ ms.

1.d Now, assume that we are sending a message of 10 packets, each with size 1000 bits, with Stop-and-Wait ARQ over the link described in part **1.c**. What is the average time between transmitting the first bit of the first packet for the first time and receiving the acknowledgment for the last packet.

Answer. From the previous part, we found that the average time for a single packet transmission is approximately 5.436 ms. Since we are transmitting 10 packets, the average time to send all these packets and receive their acknowledgments is $10 \times 5.436 = 54.36$ ms.

Q2 (Throughput and Message End-to-end Latency Timeline) Suppose we have a file of 12.5 Kb to be transmitted over a point-to-point connection. Assume that the propagation delay is 5 ms and packet size is 2.5 Kb. Furthermore, assume that:

- The packets' header size and queuing delay are negligible.
- Prior to sending the file, we have to spent one round-trip-time (RTT) for "initial handshake," which is a process for establishing the connections.
- After receiving the acknowledgment for the last packet, we have to spent two RTT for "connection termination", which is a process for terminating the established connection between two hosts.
- The size of packets during "initial handshake" and "connection termination" is negligible.

2.a Assume that the bandwidth is 1 Mbps and we must wait for the acknowledgment of each packet before start sending the next one. Assume that the acknowledgment size is negligible and non of the packets and acknowledgments are lost. Draw the message end-to-end timeline diagram and Calculate the link throughput.

Answer. Observe that the total number of packets to be transmitted is $\frac{12.5 \text{ Kb}}{2.5 \text{ Kb}} = 5$. Furthermore, the propagation delay is 5 ms. Thus, RTT $= 2 \times 5 = 10$ ms. Moreover, the transmission delay per packet is $\frac{2.5 \text{ Kbit}}{1 \text{ Mbps}} = 2.5$ ms. Therefore, the packet latency is $5 + 2.5 = 7.5$ ms. Thus, the time between start sending the packet to receiving ACK is: Packet latency $+$ propagation delay $= 7.5 + 5 = 12.5$ ms. Therefore,

$$
\begin{aligned}
\text{Transfer Time} = {} & \text{RTT}+ && \text{(due to initial handshake)}\\
& 5 \times 12.5+ && \text{(due to transmitting the five packets)}\\
& 2 \times \text{RTT} && \text{(due to connection termination)}\\
= {} & 92.5 \text{ ms.}
\end{aligned}
$$

Consequently, the link throughput is $\frac{\text{Transfer Size}}{\text{Transfer Time}} = \frac{12.5 \text{ Kb}}{92.5 \text{ ms}} \approx 135$ Kbps. Observe that the throughput is much less than the bandwidth in this example. This is due to propagation delay dominating the transmission delay.

2.b Assume the bandwidth is infinite and in each one RTT, only 20 packets are allowed to be sent. Assume that the acknowledgment size is negligible and non of the packets and acknowledgments are lost. Draw the message end-to-end timeline diagram and Calculate the link throughput.

Answer. Observe that we are allowed to send all five packets in the first RTT. Furthermore, since the link bandwidth is infinity, the transmission delay is zero. Therefore, the time between sending the packets to receiving their ACKs is $2 \times$ propagation delay $=$ RTT $= 10$ ms. Thus,

$$
\begin{aligned}
\text{Transfer Time} = {} & \text{RTT}+ && \text{(due to initial handshake)}\\
& \text{RTT}+ && \text{(due to transmitting the five packets)}\\
& 2 \times \text{RTT} && \text{(due to connection termination)}\\
= {} & 40 \text{ ms.}
\end{aligned}
$$

Consequently, the link throughput is $\frac{\text{Transfer Size}}{\text{Transfer Time}} = \frac{12.5 \text{ Kb}}{40 \text{ ms}} \approx 312$ Kbps. The key takeaway is that, regardless of bandwidth improvements, propagation delay sets a fundamental limit on link throughput.

2.c Assume the bandwidth is infinite. During the first RTT we can send 1 packet and receive its ACK. During the second RTT we can send up to 2 packets and receive their ACKs. During the third we can send up to 4 packets and receive their ACKs and so on.[1] Assume that the acknowledgment size is negligible and non of the packets and acknowledgments are lost. Draw the message end-to-end timeline diagram and Calculate the link throughput.

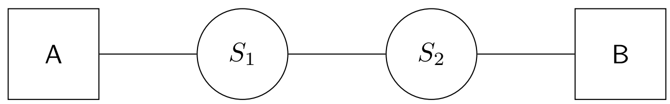
[1] Why do we study this bizarre way of sending packets? In the second half od the course, we'll see that this is closely related to Transmission Control Protocol (TCP) congestion control mechanism.

Answer. Observe that we are allowed to send the first packet in one RTT, the second and third packets in the next RTT, and the fourth and fifth packets in the next RTT. Thus,

$$\begin{aligned}\text{Transfer Time} = \ & \text{RTT}+ && \text{(due to initial handshake)}\\ & \text{RTT}+ && \text{(due to transmitting the first packets)}\\ & \text{RTT}+ && \text{(due to transmitting the second and third packets)}\\ & \text{RTT}+ && \text{(due to transmitting the fourth and fifth packets)}\\ & 2\times\text{RTT} && \text{(due to connection termination)}\\ = \ & 60 \text{ ms.}\end{aligned}$$

Consequently, the link throughput is $\frac{\text{Transfer Size}}{\text{Transfer Time}} = \frac{12.5 \text{ Kb}}{60 \text{ ms}} \approx 208$ Kbps.

Q3 **(Optimal Packet Size)** Consider sending a large file of F bits from Host A to Host B using packet switching in the network below. There are three hops of links between Host A and Host B, *i.e.*, therefore two switches



along the path that connects Host A to Host B. Assume that

- The propagation delay is negligible.
- Each link has a transmission rate of R bps.
- The links are *uncongested*, *i.e.*, no queuing delays.
- The two switches are store-and-forward switches.

Host A splits the file into chunks of S bits each and adds 80 bits of Transport, Network, and Link layer header to each chunk, forming *frames* of size $L = 80 + S$ bits. Assume that if the last chunk is smaller than S bits, Host A pads it with zeros to ensure it is exactly S bits in size.

3.a Suppose you are the network administrator to decide on the value of S. Discuss, qualitatively, the pros and cons of choices of S values that are very large or very small.

3.b Now do the math. Find the value of S that minimizes the delay of moving the file from Host A to Host B.

Answer. There are F/S packets. The time at which the last packet is received at the first switch is $(F/S)(S+80)/R$. At this time, the first $F/S-2$ packets are at the destination, and the $(F/S-1)^{\text{th}}$ packet is at the second router. The last packet must then be transmitted by the first router and the second router. Each transmission takes $(S+80)/R$ seconds. Thus, the the message end-to-end latency is $(F/S+2)(S+80)/R$. To find the optimal value for S, we should take the derivative and set to zero, *i.e.*,

$$\frac{\partial(F/S+2)(S+80)/R}{\partial S} = \frac{2}{R} - 80\frac{F}{R}S^{-2} \Rightarrow S^{\star} = \sqrt{40F}.$$

Interestingly, the optimal S only depends on F and does not depend on R at all.