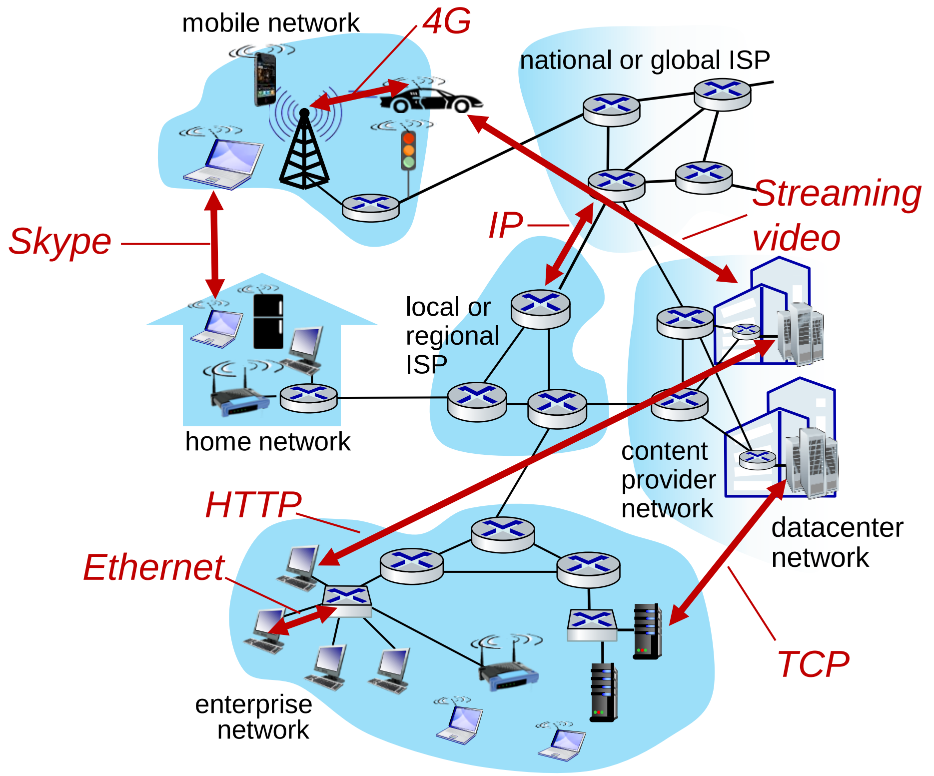
CSC358: Principles of Computer Networks

Week 3 Practical:

Network Sharing and Encoding



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EX0: True or False

- For most networks, the peak of aggregate demand for resources is greater than the aggregate of peak demands → False
- Circuit switching is easier to implement than packet switching → False
- How does Manchester Encoding make clock recovery easier?
  - By sending a sequence of alternating 0s and 1s before the main frame.
  - By sending a sequence of 1s at the beginning of each frame.
  - By embedding the clock cycle in the signal. → Answer
  - By sending the clock signal as a separate signal.
- Which encoding scheme do you suggest in order to provide high bandwidth?
  - Manchester
  - 4B/5B → Answer
- What is the maximum overhead in byte-stuffing algorithm (excluding the flags marking the start and end of the frame)?→100%

EX1: Encoding

- Fill out the blank in the following statement. Justify your answer.
  - In 4B/5B Encoding that we discussed in the lecture, which uses NRZI encoding as part of its procedure, a signal transition will occur at least every _____ bit times (*i.e.*, clock cycle).
  - Answer: 4
  - Every 4 bits is first encoded as 5 bits. Then, the transformed digital data is transmitted with NRZI.
  - 4B/5B carefully selects 16 5-bit chips such that no more than one leading 0 and no more than two trailing 0s in each 5-bit.
  - Hence, no pair of 5-bit codes results in more than three consecutive 0's. Hence, after encoding as 5 bit chips, there will be a bit of 1 in at least every 4 bits. Since it uses NRZI, it means that a signal transition will occur at least every 4 bit times.

EX2: Circuit Switching vs Packet

- Suppose several users share a 3 Mbps link and each user requires 150 Kbps when transmitting, but each user transmits only 10 percent of the time. Answer the following questions.
  a) When circuit switching is used, how many users can be supported?
     - Answer: 3Mbps / 150 Kpbs = 20
  b) For the remainder of this question, suppose packet switching is used.
     1. Find the probability that a given user is transmitting at any given point in time.
        - Answer: $p = 0.1$
     2. Suppose there are 120 users. Find the probability that at any given time, exactly n users are transmitting simultaneously.
        - Answer: $\Pr(\mathrm{n}) = \binom{120}{n} p^n (1-p)^{(120-n)}$
     3. Find the probability that there are 21 or more users transmitting simultaneously. How do you interpret the meaning of this probability?
        - Answer: $\sum_{n=21}^{120} \Pr(n) \approx 0.0079$

EX3: Statistical Multi-What?

- Consider three flows (F1,F2,F3) sending packets over a link.
- The sending pattern of each flow is described by how many packets it sends within each one-second interval
- This table shows their pattern for the first ten intervals.
  - Our three flows are very bursty,
  - with highly varying numbers of packets in each interval.

Time (s)	1	2	3	4	5	6	7	8	9	10
F_1	1	8	3	15	2	1	1	34	3	4
F_2	6	2	5	5	7	40	21	3	34	5
F_3	45	34	15	5	7	9	21	5	3	34

EX3: Statistical Multi-What? – Part 1

Time (s)	1	2	3	4	5	6	7	8	9	10
F_1	1	8	3	15	2	1	1	34	3	4
F_2	6	2	5	5	7	40	21	3	34	5
F_3	45	34	15	5	7	9	21	5	3	34

- What is the peak rate of F1? F2? F3? What is the sum of the peak rates?
  - F1 → 34, F2 → 40, F3 → 45
  - Sum of peak rates → 34+40+45=119

EX3: Statistical Multi-What? – Part 2

Time (s)	1	2	3	4	5	6	7	8	9	10
F_1	1	8	3	15	2	1	1	34	3	4
F_2	6	2	5	5	7	40	21	3	34	5
F_3	45	34	15	5	7	9	21	5	3	34

- Now consider all packets to be in the same aggregate flow. What is the peak rate of this aggregate flow?

Time (s)	1	2	3	4	5	6	7	8	9	10
Aggregate Flow	52	44	23	25	16	50	43	42	40	43

  - The peak happens at 1s, where it is 52.

EX3: Statistical Multi-What? – Part 3

- Which is higher - the sum of the peaks, or the peak of the aggregate?
  - The sum of the peaks is 119, whereas the peak of the aggregate is 52.
  - The sum of the peaks is much higher.
- This is the insight from Statistical Multiplexing!
  - The peak of the aggregate can only be at most the sum of the peaks,
  - but that only happens in the case that all of the peaks happen at the same time.
    - This is **very unlikely**. See the next example for a statistical analysis of this phenomenon.
  - So, usually, the peak of the aggregate is much lower than the sum of the peaks.

EX4: Statistical Multiplexing

- Consider N traffic flows. The arrivals from each flow are governed by a random variable X with a geometric distribution.
- Specifically, in any time slot, the probability that the number of arrivals from a single flow are equal to k kilobytes (kB) is given by

$$P(X = k) = \begin{cases} 0, & k = 0 \\ (1-p)^{k-1}p, & k = 1,2,\ldots \end{cases}, \text{ where } 0 < p < 1$$

- Assume that the N flows are i.i.d..

EX4: Statistical Multiplexing, cont’d

- In every time slot, the arrivals are entered into a buffer with a given size.
- If arrivals to the buffer exceed the buffer size, the excess traffic is lost.
- The buffered data is removed from the buffer before the start of the next time slot.
  - Hence, the buffer is empty at the beginning of each time slot.
- The objective is to design the size of the buffer such that the probability that a loss occurs does not exceed a given threshold value.

EX4: Statistical Multiplexing – Some Notes

- A random variable with Geometric distribution

$$P(X = k) = \begin{cases} 0, & k = 0 \\ (1-p)^{k-1}p, & k = 1,2,\ldots \end{cases}, \text{ where } 0 < p < 1,$$

has $\mathrm{E}[X] = 1/p$ and $\mathrm{VAR}[X] = (1-p)/p^2$

- Let $\Phi(.)$ denote the cumulative distribution function of the standard normal distribution.
  - To get $1 - \Phi(z) \approx \epsilon$, you may use the approximation $z = \sqrt{|\ln(2\pi\epsilon)|}$

EX4: Statistical Multiplexing – Part 1

- Consider that there is one buffer for each flow, *i.e.*, we have N buffers.
  - This is similar to circuit switching resource sharing.
- Express the total size of all N buffers needed so that the loss probability of each flow does not exceed ϵ.
  - Consider flow i.
  - $P(X_i > k) \leq \epsilon \Rightarrow \sum_{l=k+1}^{\infty} P(X_i = l) \leq \epsilon \Rightarrow \sum_{l=k+1}^{\infty} (1-p)^{l-1} p \leq \epsilon$
  - Hence, $p \sum_{l=k+1}^{\infty} (1-p)^{l-1} = p \frac{(1-p)^k}{p} = (1-p)^k \leq \epsilon \Rightarrow k \geq \frac{\ln \epsilon}{\ln(1-p)}$.
  - With this approach, the minimum total buffer size needed for N flows so that the loss probability of each flow does not exceed ϵ is $\frac{N \ln \epsilon}{\ln(1-p)}$.

EX4: Statistical Multiplexing – Part 2

- Consider that there is a single buffer that is shared by all N flows.
  - This is similar to packet switching resource sharing.
- Use the Central Limit Theorem (CLT) to derive the buffer size needed so that the loss probability does not exceed ϵ.
  - Let X_i's be i.i.d. random variables with $\mathrm{E}[X] = \mu$ and $\mathrm{VAR}[X] = \sigma^2$.
  - By CLT, the random variable $Z_N = \frac{X_1+\cdots+X_N - N\mu}{\sigma\sqrt{N}}$ converges in distribution to the standard normal random variable.
    - In our problem, $\mu = \frac{1}{p}$ and $\sigma = \frac{\sqrt{1-p}}{p}$.

$$P(X_1 + \cdots + X_N > k) \leq \epsilon \Rightarrow$$
$$P(X_1 + \cdots + X_N - N\mu > k - N\mu) \leq \epsilon \Rightarrow$$
$$P\left(Z_N = \frac{X_1 + \cdots + X_N \ - N\mu}{\sigma\sqrt{N}} > \frac{k - N\mu}{\sigma\sqrt{N}}\right) = 1 - \Phi\left(\frac{k - N\mu}{\sigma\sqrt{N}}\right) \leq \epsilon \Rightarrow$$
$$\frac{k - N\mu}{\sigma\sqrt{N}} \geq \sqrt{|\ln(2\pi\epsilon)|} \Rightarrow k > \frac{N + \sqrt{(1-p)N|\ln(2\pi\epsilon)|}}{p}$$

  - With this approach, the minimum total buffer size needed is $\frac{N+\sqrt{(1-p)N|\ln(2\pi\epsilon)|}}{p}$

EX4: Statistical Multiplexing – Part 3

- Considering parts (a) and (b),
- using p = 0.25, $\epsilon = 10^{-3}$, and N = 1000.
  - $\mathrm{SMG} = \dfrac{\frac{N \ln \epsilon}{\ln(1-p)}}{\frac{N+\sqrt{(1-p)N|\ln(2\pi\epsilon)|}}{p}} \approx 5.65$