MAT135Y5 Calculus

Test 1

26 October 2018

Time: 1 hour 45 min

No calculators or other aids are allowed

Instructions:

1. Calculators and other aids are NOT permitted.
2. Please have your student card ready for inspection. Turn off cell phones and other electronic equipment.
3. The test has 2 Parts; Part A and Part B. Please read the instructions at the beginning of each part.
4. In Part A, there are several multiple choice questions. **You must indicate the correct answers for the Multiple Choice questions on the last page of the test.**
5. In Part B, unless otherwise stated, you must provide full solutions and justify all your work. A correct answer obtained with false reasoning or no/little justifications will not receive any marks.
6. Please make sure that this test has 16 pages, including this front page.
7. **Please do not tear off any pages.**
7. Page 14 includes a formula sheet.
8. Unless otherwise stated you should simplify your answers as much as possible.

GOOD LUCK!

Part A: (32 marks)

For Part A, you should **clearly indicate your answers in the table on the last page of the test**. Answers that are not indicated on the last page of the test will not be graded. There is ONE correct answer for each question. Each question is worth **4 marks**.

1. Find the domain of the function $f(x) = \dfrac{1}{4-x} + \dfrac{\sqrt{x+1}}{x}$.

   (a) $[-1, 0) \cup (0, 4) \cup (4, \infty)$

   (b) $(-1, 0) \cup (0, 4) \cup (4, \infty)$

   (c) $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

   (d) $[-1, \infty)$

   (e) All real numbers

2. Find an exact value for $\arccos\left(-\frac{1}{2}\right)$.

   (a) $\frac{\pi}{3}$

   (b) $-\frac{\pi}{3}$

   (c) $\frac{\pi}{6}$

   (d) $\frac{2\pi}{3}$

   (e) $\frac{5\pi}{6}$

3. Use the following table to evaluate $(f^{-1} \circ g)(5)$.

x	1	2	3	4	5
$f(x)$	3	1	5	4	2
$g(x)$	2	3	4	5	1

   (a) 2

   (b) 5

   (c) 1

   (d) 3

   (e) 4

4. Find all vertical asymptote(s) of the function $f(x) = \dfrac{x(x^2-2)}{(x+\sqrt{2})(\sqrt{2}\,x+1)}$.

   (a) $x = -\sqrt{2}$ and $x = \sqrt{2}$

   (b) $x = -\sqrt{2}$

   (c) $x = -\sqrt{2}$ and $x = -1/\sqrt{2}$

   (d) $x = 0$, $x = \sqrt{2}$ and $x = -\sqrt{2}$

   (e) $x = -1/\sqrt{2}$

5. Compute the following limit: $\lim\limits_{x\to 4^-} \dfrac{9-x^2}{|1-x|(4-x)}$.

   (a) 0

   (b) $-\infty$

   (c) $-7/3$

   (d) ∞

   (e) The limit does not exist

6. Let $f\,(x)$ be a function that satisfies

$$\frac{1}{(x-1)^4+2} \le f(x) \le \frac{1}{2(x-1)^2+1}$$

   for all real numbers x. With the given information, only one of the limits below can **NOT** be determined by using the Squeeze Theorem. Which one is it?

   (a) $\lim\limits_{x\to 0} f(x)$

   (b) $\lim\limits_{x\to 1} f(x)$

   (c) $\lim\limits_{x\to 2} f(x)$

   (d) $\lim\limits_{x\to \infty} f(x)$

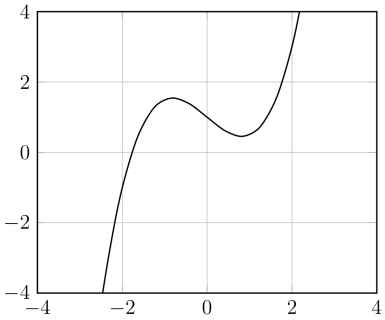
   (e) $\lim\limits_{x\to -\infty} f(x)$

7. Find the equation of the tangent line to the curve $y = 2 - x^2$ at the point where $x = 1$.

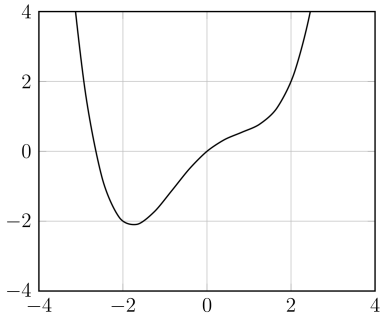
   (a) $y = -x + 3$
   (b) $y = -x - 3$
   (c) $y = -2x + 3$
   (d) $y = -2x - 3$
   (e) None of the above

8. Based on the graph of $y = f(x)$, which graph best represents its derivative?

$y = f(x)$



(a)



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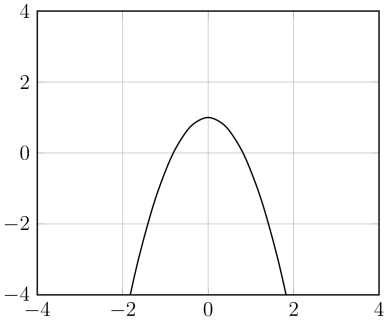

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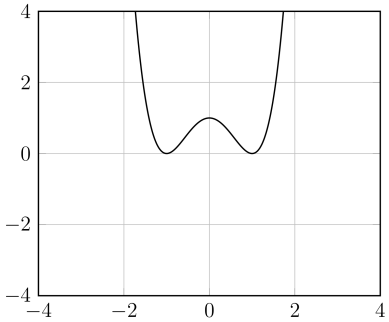

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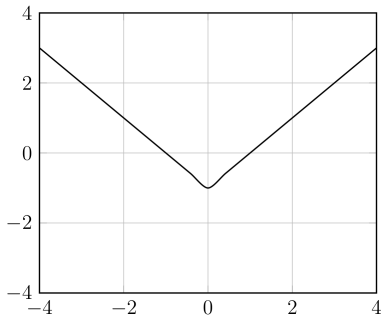

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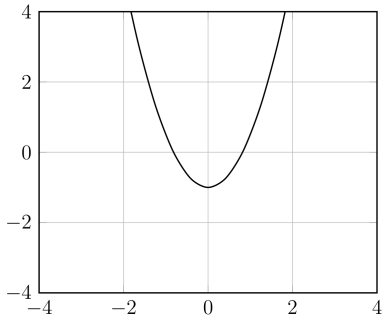


Part B: (68 marks)

In Part B, you must fully justify all your answers. Provide full solutions and show all your work. Part marks may be given. A correct answer with little or no justification will not receive any marks. Simplify your answers as much as possible.

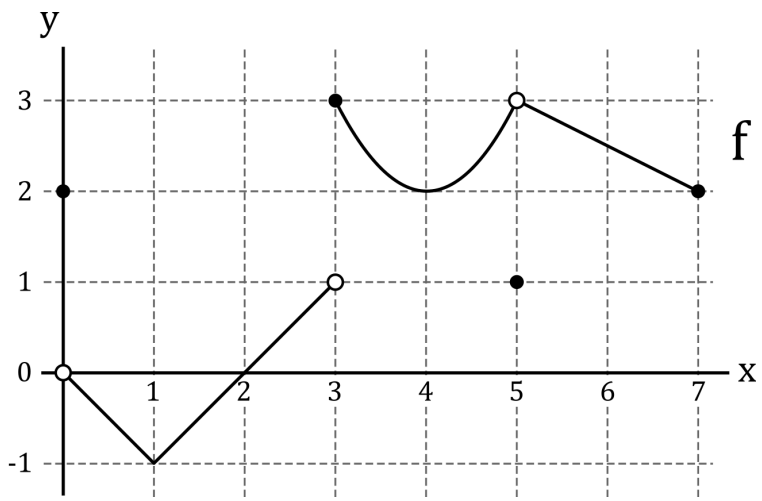
1. (10 marks) Using the limit definition of the derivative, compute $f'(x)$ if

$$f(x) = \sqrt{2x + k}$$

where k is a constant.

2. (10 marks) Find the inverse of the function $f(x) = \dfrac{\ln(2x)}{5 - \ln(2x)}$. You can assume that the inverse exists.

3. (2 marks each) Suppose that the graph of f with domain $[0, 7]$ is as shown:



Answer the following questions. No explanations are required.

(a) Using interval notation, find the range of f.

Answer:

(b) Find a value of x in $[0, 7]$ for which $g(x) = \dfrac{1}{f(x)}$ is **NOT** defined.

Answer:

(c) What is $\lim_{x \to 5} f(x)$?

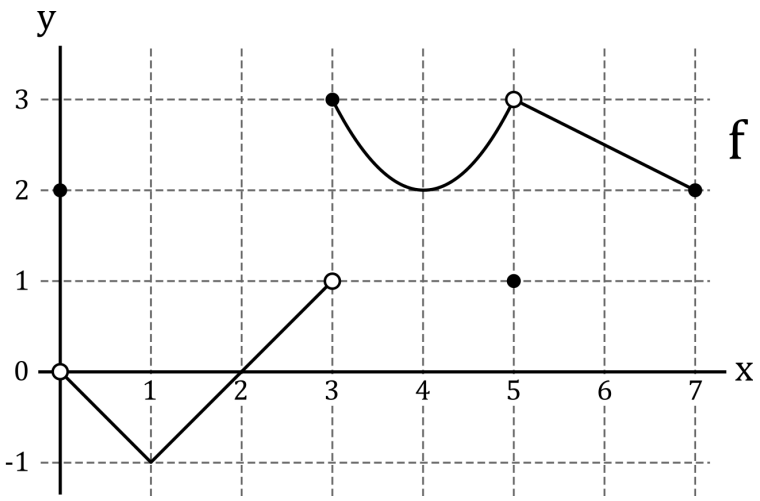
Answer:

(d) Find a value of c for which $\lim_{x \to c^+} f(x) = 2$. If no such c exists, write "No such point."

Answer:

(e) Find a closed interval of the form $[a, b]$ for which the Intermediate Value Theorem does **NOT** apply to f.

Answer:



(f) Find a value of c in $(0, 7)$ for which f has a removable discontinuity at $x = c$. If no such c exists, write "No such point."

Answer:

(g) Find a value of c for which f is continuous but not differentiable at $x = c$. If no such point exists, write "No such point."

Answer:

(h) Find a value of c for which f is differentiable but not continuous at $x = c$. If no such point exists, write "No such point."

Answer:

(i) Find a value of c for which $\lim_{x \to c} f(x)$ exists but f is not continuous at $x = c$. If no such point exists, write "No such point."

Answer:

4. (8 marks) Is the function

$$f(x) = \begin{cases} \dfrac{1-\sin x}{2+e^{1/x}} & \text{if } x \neq 0 \\ \\ 0 & \text{if } x = 0 \end{cases}$$

continuous or discontinuous at $x = 0$? If it has a discontinuity, is it a jump, infinite or removable discontinuity?

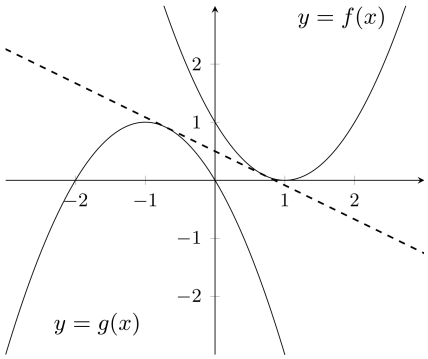
5. (8 marks) Evaluate the limit (or explain why it doesn't exist):

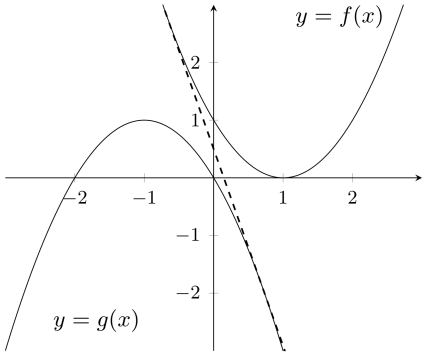
$$\lim_{t \to 1^-} \left(\frac{1}{1-t} - \frac{t}{1-t^2} \right)$$

6. (8 marks) Evaluate the limit (or explain why it doesn't exist):

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + 5x^4} - 1}{1 - 3x^2 - x}$$

7. (6 marks) There are two tangent lines to the graph of $f(x) = x^2 - 2x + 1$ that are also tangent to $g(x) = -x^2 - 2x$. Determine the slopes of these lines.





THIS PAGE IS EMPTY. USE IT FOR SCRAP WORK.
What you write on this page will not be marked.

MAT135 October 2018 Test 1 v1

Formula Sheet

Trigonometry:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

$$\sin(mx)\cos(nx) = \frac{1}{2}\left(\sin((m-n)x) + \sin((m+n)x)\right)$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left(\cos((m-n)x) - \cos((m+n)x)\right)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left(\cos((m-n)x) + \cos((m+n)x)\right)$$

Derivatives:

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\sec^{-1}x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\csc^{-1}x\right) = \frac{-1}{x\sqrt{x^2-1}}$$

Integrals:

$$\int \sec x\tan x\,dx = \sec x + C$$

$$\int \csc x\cot x\,dx = -\csc x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$