University of Toronto Mississauga

Mathematical and Computational Sciences
MAT133Y5Y- Term Test 1
Duration - 110 minutes
No Aids Permitted

Surname/Family Name: _____

First/Given Name: _____

Student Number: _____

UTORid: _____

Email: _____

This exam contains 11 pages (including this cover page) and 5 problems. Check to see if any pages are missing and ensure that all required information at the top of this page has been filled in. No aids are permitted on this examination. Examples of illegal aids include, but are not limited to textbooks, notes, calculators, or any electronic device.

Each part of Problem 1 includes five multiple choice answers, of which only a single response is correct. All answers should be indicated on the included scantron sheet (page 11). When filling out the scantron:

- Use a #2 pencil to bubble answers (do not use ink).
- Ensure that circles are completely filled and make complete erasures.
- You will **not** be penalized for incorrect answers.

Your solutions to Problems 2 - 5 should be written on the front page of the test only. Do not, under any circumstances, write on the QR code in the top right corner of each page. Doing so will result in you receiving a zero (0) on this test. **Any work written on the back of each page will not be marked**, and can therefore be used for rough work.

Unless otherwise indicated, you are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work, will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	5	
3	5	
4	5	
5	5	
Total:	30	

1. Each question is worth 1 mark. Indicate your answer in the given box. No part marks will be awarded for this question, and you are not required to show your work.

   (i) (1 point) Define the sets

   - $A = \{x \in \mathbb{R} : -4 < x < 4\}$
   - $B = \{1, 3, 5, 7, 9, \ldots\} = \{2n + 1 : n \in \mathbb{N}\}$ (the positive odd numbers)
   - $C = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\} = \{2n : n \in \mathbb{Z}\}$ (all even numbers).

   Which statement is correct?

   A. $A \cap C = \{-4, -2, 0, 2, 4\}$
   B. $B \cup C = \mathbb{Z}$
   C. $A \setminus B = (-4, 1) \cup (1, 3) \cup (3, 4)$
   D. $B \setminus C = C$
   E. None of the above options are correct.

   (ii) (1 point) Suppose you are told that $\sum_{k=1}^{10} k^3 = 3025$. What is the value of $\sum_{k=3}^{9} k^3$?

   A. 3025
   B. 2038
   C. 2025
   D. 2022
   E. 2016

   (iii) (1 point) Aziz and Bo are planning a big graduation party for their friends in May 2022. They decide to invest some money today to cover the costs of the party. Both determine they will receive an APR of 3% from their investments and need \$10 000 to cover the costs, but Aziz's investment compounds monthly while Bo's compounds weekly.

   If P_A is the initial value of Aziz's investment, P_B is the initial value of Bo's investment, and both aim to finance the party with the least amount of capital, what is the relationship between P_A and P_B.

   A. $P_A < P_B$
   B. $P_A > P_B$
   C. $P_A = P_B$
   D. $P_A = 2P_B$
   E. There is no relationship between P_A and P_B.

   (iv) (1 point) Suppose a principal P is invested at an APR of r% compounded continuously over n years. Which statement is correct?

   A. It takes $\ln(3)/r$ years for P to triple in size.
   B. If P triples in 5 years, then $r = \sqrt[n]{e^5}$.
   C. The amount of time it takes for P to triple depends on P.
   D. It will take longer for this investment to triple than if it were compounded monthly.
   E. None of the above statements are correct.

(v) (1 point) Suppose an ordinary annuity consists of \$100 deposits made every week at an APR of 3%, compounding weekly. Which sum represents the future value of the annuity after 1 year?

A. $\displaystyle\sum_{k=0}^{51} 100\left(1+\frac{0.03}{52}\right)^{k}$.

B. $\displaystyle\sum_{k=1}^{52} 100\,(1+0.03)^{k}$

C. $\displaystyle\sum_{k=1}^{52} 100\left(1+\frac{0.03}{12}\right)^{4k}$

D. $\displaystyle\sum_{k=0}^{52} 100\left(1+\frac{0.3}{2}\right)^{2k}$

E. None of the above options represent the future value.

(vi) (1 point) Your rich aunt Hannelore is setting up a fund for victims of math tests. The fund will pay \$10 000 per year without end. If her financial advisors think they can get a 2% return (compounded yearly), how much money must be invested today to ensure the fund lasts forever?

A. \$2000

B. \$50 000

C. \$200 000

D. \$500 000

E. None of the above.

(vii) (1 point) Consider the linear system

$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 + 6x_2 + x_3 &= 8 \\ 3x_1 + 9x_2 + x_3 - x_4 &= 13. \end{aligned}$$

For $s, t \in \mathbb{R}$, indicate which tuple (x_1, x_2, x_3, x_4) is a solution to the given system.

A. $(x_1, x_2, x_3, x_4) = (4 - 4s + 2t, 2 + s, -2 - 2t, 3t - 5)$.

B. $(x_1, x_2, x_3, x_4) = (5 - 3s + t, s, -2 - 2t, t)$.

C. $(x_1, x_2, x_3, x_4) = (3s + t, 2 - s, 2t, 5 - t)$.

D. $(x_1, x_2, x_3, x_4) = (s + 3t, s, 2t, 3t)$.

E. No given option is a solution to the system.

(viii) (1 point) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 15 & 0 \\ 0 & -3 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -3 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which statement is correct?

A. A and C are in row echelon form, but only C is in reduced row echelon form.

B. A and B are in row echelon form, but only A is in reduced row echelon form.

C. A and C are both in reduced row echelon form.

D. B and C are both in reduced row echelon form.

E. None of the above statements are correct.

(ix) (1 point) Consider a linear system with ten (10) variables and seven (7) equations. Which statement is correct?

A. The system must have a solution.

B. If the system has solutions, the solution set can have at most three (3) parameters.

C. If the system has solutions, the solution set must have at least three (3) parameters.

D. If the system has solutions, the solution set has exactly three (3) parameters.

E. If the system has solutions, it's possible for the solution set to have exactly five (5) parameters.

(x) (1 point) What is the rank of the matrix

$$\begin{bmatrix} 2 & 0 & 1 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & -1 \end{bmatrix}?$$

A. 0

B. 1

C. 2

D. 3

E. 4

2. For this question, write down a formula for the desired quantity. You are not expected to evaluate the expression.

   Suppose you have purchased a bond with face value \$1000. The bond pays \$20 coupons quarterly and expires in one year. Assume the risk free rate is an APR of 2%.

   (i) (2 points) Suppose we assume quarterly compounding periods. Express the present value of the bond in terms of an *annuity*.

   (ii) (3 points) If we assume a weekly compounding period (13 weeks per quarter), find the present value of the bond.

3. Consider the sets

$$A = \{10^n : n \in \mathbb{N}\} = \{1, 10, 100, 1000, \ldots\} \qquad C = [-1, 1]$$

$$B = (0, 3) \setminus \{1, 2\} \qquad D = \left\{x \in \mathbb{R} : x^2 < 4\right\}.$$

Each of the following can be written as an interval. Indicate that interval in the box below. If the set is empty, write $\emptyset$.

(i) (1 point) D

Answer:

(ii) (1 point) $A \cap B$

Answer:

(iii) (1 point) $C \setminus A$

Answer:

(iv) (1 point) $B \cup D \cup \{2\}$

Answer:

(v) (1 point) $B \setminus C^c$

Answer:

4. (5 points) Find all solutions to the following linear system:

$$\begin{aligned} x_1 - 2x_2 \qquad\quad + \;\; 4x_4 &= \;\; 3 \\ 3x_1 + \;\; x_2 + \;\; 7x_3 - \;\; 2x_4 &= 16 \\ x_1 + 5x_2 + \;\; 7x_3 - 10x_4 &= 10 \\ 3x_1 + 8x_2 + 14x_3 - 16x_4 &= 23 \end{aligned}$$

5. (5 points) Consider the linear system whose augmented matrix is given by

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 1 & 0 & 9 & -5 \\ 2 & -1 & a+11 & a+1 \end{array}\right].$$

Determine the values of a such that the system

i. Has a unique solution,

ii. Has no solutions,

iii. Has infinitely many solutions.

Clearly indicate which scenario corresponds with which values of a, and justify why you have chosen those values of a in each case.

This page is for additional work and will not be marked.

Formula Sheet

You are required to know and understand when each formula below applies. This includes the interpretation of the variables which appear within each formula. This information will not be clarified during the test.

Simple Compounding Interest: $$S = P\left(1 + \frac{r}{t}\right)^{nt}$$

Effective Annual Rate: $$E_a = \left(1 + \frac{r}{t}\right)^{t} - 1$$

Continuous Compounding Interest: $$S = Pe^{rn}$$

Effective Continuous Rate: $$E_c = t \ln\left(1 + \frac{r}{t}\right)$$

Annuity (Future Value): $$S = R\left[\frac{(1 + r/t)^{nt} - 1}{r/t}\right]$$

Annuity (Present Value): $$A = R\left[\frac{1 - (1 + r/t)^{-nt}}{r/t}\right]$$

Perpetuity (Present Value): $$A = \frac{R}{r}$$

Sum of a Finite Geometric Series: $$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

Sum of an Infinite Geometric Series: $$\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}$$

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$