Mathematical and Computational Sciences - University of Toronto Mississauga

MAT134Y5Y - Calculus for Life Sciences - 2018-2019

Term Test 1 - October 26, 2018 - Version 182256

Aids Permitted: None

Time Allotted: 110 minutes

Full Name (IN PEN): _____
Last First

Student ID (IN PEN): _____

Tutorial Section (fill in completely the appropriate circle):

- ◯ Sara Khan, Tue. 5-6pm (TUT0101).
- ◯ Anhadvir Singh, Tue. 6-7pm (TUT0102).
- ◯ Moses Cook, Tue. 7-8pm (TUT0103).
- ◯ Thai-Son Tang, Wed. 9-10am (TUT0104).
- ◯ Thai-Son Tang, Wed. 10-11am (TUT0105).
- ◯ Marc De Benedetti, Wed. 11am-12pm (TUT0106).
- ◯ Oyindamola Adepoju, Wed. 5-6pm (TUT0108).
- ◯ Thai-Son Tang, Wed. 5-6pm (TUT0109).
- ◯ Daniyal Ahmad, Wed. 6-7pm (TUT0110).
- ◯ Eman Abo Alya, Wed. 7-8pm (TUT0111).
- ◯ Fayez Habach, Thu. 9-10am (TUT0112).
- ◯ Sara Khan, Thu. 10-11am (TUT0113).
- ◯ Fayez Habach, Thu. 12-1pm (TUT0115).
- ◯ Moses Cook, Thu. 5-6pm (TUT0116).
- ◯ Dennis Fernandes, Thu. 6-7pm (TUT0117).
- ◯ Neel Mistry, Fri. 9-10am (TUT0118).
- ◯ Neel Mistry, Fri. 10-11am (TUT0119).
- ◯ Anhadvir Singh, Fri. 11am-12pm (TUT0120).
- ◯ Marc De Benedetti, Tue. 6-7pm (TUT0122).
- ◯ Oyindamola Adepoju, Wed. 10-11am (TUT0123).
- ◯ Daniyal Ahmad, Thu. 10-11am (TUT0124).

Instructions

- Please have your student card ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- This test contains two parts: Part A (36 marks) contains **nine** short questions, and Part B (64 marks) contains **four** questions. All answers are to be given in this booklet.
- Check that this test has **11 pages**, including this cover page.
- There is a formula sheet on page 11, and space for rough work. You can also use page 10 and the back of page 11 for rough work. If you wish to tear off the formula sheet, **fill in your details** at the top of the page, and **submit it** together with the rest of the test.
- Make sure to provide **exact answers**, using symbols such as $\sqrt{\ }$, e and π, if needed.

Question	Part A	B1	B2	B3	B4	TOTAL
Marks	/36	/16	/16	/16	/16	

GOOD LUCK!

PART A (36 marks)

In this part, **clearly indicate your final short answer in the appropriate box**.
You must show your work (if any), even though **only the final answer will be graded**.
Simplify your answers **as much as possible**. Each question is worth **4 marks**.

1. Find the **natural domain** and the **range** of the function $f(x)=\sqrt{9-2|x|}$.
   Use **the interval notation**.

Answer for 1.

Domain: ; Range:

2. If $g(x)=x^4\cdot e^x$, what is $g'(1)$? Use 'e' in your answer, if needed.

Answer for 2.

$g'(1)=$

Continued on page 3 ...

3. **Calculate** the limit $\lim_{x\to 0} \dfrac{\tan(2x)}{5x}$. (Hint: Use $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$.)

   Your answer should be **a number**, $+\infty$, $-\infty$ or **DNE** if the limit does not exist.

Answer for 3.

4. **Simplify** the following expression **as much as possible**: $\log_4\left(2^{6\sin^2 x}\right) + 3\ln\left(\dfrac{e^{\cos^2 x}}{e^{3x}}\right)$.

Answer for 4.

5. Calculate the limit $\lim_{x\to -\infty} \dfrac{\sqrt{4x^2+1}}{3x+1}$.

   Your answer should be **a number**, $+\infty$, $-\infty$ or **DNE** if the limit does not exist.

Answer for 5.

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6. Suppose we perform the following transformations on the graph of $y=\sqrt{x}$:

   Shift **two units to the right**, then **reflect about the x-axis**, and then **shift three units upwards**.

   What is the equation of the resulting graph?

Answer for 6.

7. If f is a **continuous** function, and $\lim_{x\to 1}\left[e^{-5\cdot f(x)+3^x\cdot f(x)}\right]=3$, then what is $f(1)$?

   Use 'e' and/or 'ln()' in your answer, if needed.

Answer for 7.

$f(1)=$

Continued on page 5 ...

8. Where does the curve given by $y=\dfrac{\sqrt{x}}{9+4x}$ have **a horizontal tangent line**?

   Provide x **and** y **coordinates** of each such point. Use the square root symbol $\sqrt{\ }$ if needed.

Answer for 8.

9. The position of an object, moving on the number line, is given by $s(t)=t^3$ for $0\le t\le 2$.

   When is **the instantaneous velocity** equal the **average velocity** (on the time interval $[0\,,2]$) ?

   Use the square root symbol $\sqrt{\ }$ if needed.

Answer for 9.

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PART B (64 marks)

In this part you are required to provide **full solutions** and to **show all your work**.

A correct answer obtained with false reasoning or with no reasoning will not receive any marks.

Each question is worth **16 marks**.

1. Use the **Intermediate Value Theorem** to show that the equation

$$\sqrt{x-5} - \frac{x}{(x+3)^2} = 0$$

has **at least one solution**.

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2. Find **the equations** of all the **horizontal and vertical asymptotes** of the function

$$f(x) = \frac{1}{x+3} + \cos\left(\frac{1}{x}\right) \ .$$

Continued on page 8 ...

3. Let $g(x) = \sin^{-1}(\cos x)$.

   (a) **Calculate** $g(2\pi)$ and $g\left(\frac{\pi}{3}\right)$.

   (b) Find the **domain** and **range** of g . No explanation is needed here.

   (c) Is g an **even function**, an **odd function**, or **neither**? **Explain**.

   (d) Give **an example of an interval**, of length π , on which g is a **one-to-one function**. **Explain your answer**.

Continued on page 9 ...

4. Let $f(x) = \sqrt{|x^2 - 3x|}$.

(a) **Calculate** the one-sided limit $\lim_{h \to 0^+} \dfrac{f(h) - f(0)}{h}$.

(b) Is the function f **differentiable** at $x = 0$? **Explain**.

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Last Name (Family name) | First Name (Given name) | Student #

FORMULA SHEET

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\left(\dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

$\sin\left(\dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$\sin\left(\dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

SPACE FOR ROUGH WORK

THE END.