The 5 Steps for Evaluating Line Integrals !!

How to parametrize?
(i) Line segments: $\vec{r}(t) = P_i \cdot (1-t) + P_f \cdot t$ for $0 \le t \le 1$ [fixed]
(ii) Try, $x = t$, $y = \dots$
(iii) More complex, think and make smart move.

1. parameterize using t.
2. $x = f(t)$ and $y = g(t)$ and $\underline{a \le t \le b}$ → find.
3. Find ds OR dx, dy, dz (if question has them)

$$ds = \boxed{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt.} \qquad x'(t)\,dt \qquad y'(t)\,dt \qquad z'(t)\,dt$$

4. Plug in the x, y in $f(x,y)$ in the integral

5. Eval the integral from

$|r'(t)|\,dt$

ds

$$\int_{t=a}^{t=b} f(f(t), g(t), h(t)) \cdot \underbrace{\sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\; dt}_{ds}$$

Independent & Conservative

The 4 Conditions of Ch-16.

① $\int_C \bar{F} \cdot d\bar{r}$ is Independent of path $\overset{\text{i.f.f.}}{\Longleftrightarrow}$ $\int_C \bar{F} \cdot d\bar{r} = 0$

$P\hat{i} + Q\hat{j}$ $\quad \boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$ ✓

② If we know, $\int_C \bar{F} \cdot d\bar{r}$ is independent of path, THEN, $\bar{F}$ (vec. feild) is CONSERVATIVE.

$\Rightarrow$

③ If $\bar{F}$ is Conservative, THEN, $\exists$ a func. f s.t. $\bar{F} = \nabla f$.

$\Rightarrow$

④ FTC for Line $\int$: $\int_C \nabla f \cdot d\bar{r} = f(\bar{r}(b)) - f(\bar{r}(a))$

Conservative Vector Field

let $F = P\hat{i} + Q\hat{j}$ be a vector field on an open Simply-Connected region (set) D. Suppose that P and Q have continuous 1st order partial derivatives

How to check Conservative? $\bar{F} = Pi + Qj$ Then, $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x} \Rightarrow$ Conservative $\bar{F}$.

$$\int_c \bar{F} \cdot d\bar{r}$$

① $\bar{F}$ is Conservative | ② $\bar{F}$ is NOT conservative

$a \le t \le b$

$$\boxed{\int_c \bar{F} \cdot dr = \underline{f}(r(b)) - \underline{f}(r(a))}$$

first find

$\because$ F is conservative,

$\therefore$ $\bar{F} = \nabla f$.

$\underline{\underline{f}} = \int \nabla f$

once you get it then, use the FTC Thm. :)

$$\int_c \bar{F} \cdot dr = \int_{t=a}^{t=b} \underbrace{F(r(t))}_{\text{vector}} \cdot \underbrace{r'(t)}_{\text{vector}} dt$$

dot product