MAT232 - Tutorial 10

1. Without evaluating the integral, show that

$$\iint_R \sin(2x)\cos(4y)dA \leq 4\pi^2$$

where R is the rectangle with vertices $(0,0), (4\pi, 0), (4\pi, \pi)$, and $(0, \pi)$.

2. Find the area of the region bounded by the curves $y = e^x, x = 0$, and $y = e^3$.

3. The rest of this tutorial will be review for test 2. TAs will select topics from:

   (a) Multivariable limits.

   (b) The multivariable chain rule.

   (c) Implicit differentiation.

   (d) Finding tangent planes.

   (e) Finding directional derivatives, and directions of maximal rate of change.

   (f) Finding and classifying crticial points.

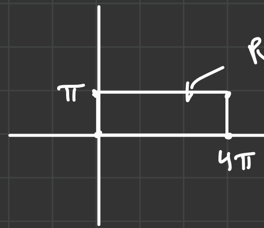
   (g) Finding absolute maximum and minimum over closed/bounded regions.

   (h) Using the method of Lagrange multipliers.

   (i) Evaluating double integrals.

1.

W.T.S $\iint_R \sin(2x)\cos(4y)\,dA \le 4\pi^2$



$$= \int_0^{\pi}\int_0^{4\pi} \sin(2x)\cos(4y)\,dx\,dy$$

$$= \int_0^{\pi} \sin(2x)\,dx \cdot \int_0^{4\pi} \cos(4y)\,dy$$

$\sin(2x) \le 1$ and $\cos(4y) \le 1$

we know from MAT136 if $f \le g \Rightarrow \int f \le \int g$

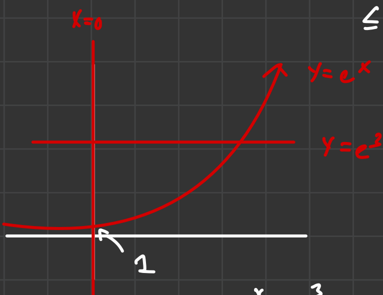
$$\Rightarrow \int_0^{\pi} \sin(2x)\,dx \le \int_0^{\pi} 1\,dx \quad \text{and} \quad \int_0^{4\pi} \cos(4y)\,dy \le \int_0^{4\pi} 1\,dy$$

$$\le \pi \qquad\qquad \le 4\pi$$

$$\therefore \int_0^{\pi} \sin(2x)\,dx \cdot \int_0^{4\pi} \cos(4y)\,dy \le \int_0^{\pi} 1\,dx \cdot \int_0^{4\pi} 1\,dy$$

$$\le \pi \cdot 4\pi = 4\pi^2$$

2.



$$A = \int_1^{e^3}\int_0^{\ln(y)} 1\,dx\,dy = \int_1^{e^3} \ln(y)\,dy$$

$$= \left[y\ln(y) - y\right]_1^{e^3}$$

$$= e^3\ln(e^3) - e^3 - (1\ln(1) - 1)$$

$$= 3e^3 - e^3 + 1$$

$$= 2e^3 + 1$$

$e^x = e^3 \Rightarrow x = 3$

or $$A = \int_0^3 \int_{e^x}^{e^3} 1\,dy\,dx$$

$$= \int_0^3 (e^3 - e^x)\,dx = \left[e^3x - e^x\right]_0^3$$

$$= 3e^3 - e^3 - (0 - 1)$$

$$= 2e^3 + 1$$

3.

Tut9 Q 1.

1.

a) $f(x,y) = 4x^2 + 10y^2$ $g(x,y) = x^2 + y^2$ where $g(x,y) = 4$

$\nabla f(x,y) = \langle 8x, 20y \rangle$ $\nabla g(x,y) = \langle 2x, 2y \rangle$

$\nabla f(x,y) = \lambda \nabla g(x,y)$

$$\begin{cases} 8x = 2x\lambda & (a) \\ 20y = 2y\lambda & (b) \\ x^2 + y^2 = 4 & (c) \end{cases}$$

(a) $2x(4-\lambda) = 0$

$x = 0$ or $\lambda = 4 \to$ (b) $y = 0 \Rightarrow x = \pm 2$

$\hookrightarrow$ (c) $y = \pm 2$

$(0,-2), (0,2), (2,0), (-2,0)$

$f(0,-2) = f(0,2) = 40$

$f(2,0) = f(-2,0) = 16$