計算科学・量子計算における情報圧縮

Data Compression in Computational Science and Quantum Computing

2022.12.1

#7: テンソルネットワーク表現への発展

Tensor network representation

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I put (URLs of) recordings of previous lectures on ITC-LMS.
You can also download lecture slide from ITC-LMS.

Today's topic

1. Computational science, quantum computing, and data compression
2. Review of linear algebra
3. Singular value decomposition
4. Application of SVD and generalization to tensors
5. Entanglement of information and matrix product states
6. Application of MPS to eigenvalue problems
7. Tensor network representation
8. Data compression in tensor network
9. Tensor network renormalization
10. Quantum mechanics and quantum computation
11. Simulation of quantum computers
12. Quantum-classical hybrid algorithms and tensor network
13. Quantum error correction and tensor network

Outline

- Exercises 2 and 3
- Breakdown of MPS representation
  - Critical system
  - Higher dimensional system
- Tensor Network for critical systems
  - Multi-scale Entanglement Renormalization Ansatz (**MERA**)
- Tensor Network for higher dimensions
  - Tensor Product State (**TPS**)

Exercise 2: Make MPS and approximate it

2: Make exact MPS and approximate it by truncating singular values

Try MPS approximation for a random vector, GS of spin model, or a picture image.
Let's see how the approximation efficiency depends on the bond dimensions and vectors.

Sample code: Ex2-1, Ex2-2, Ex2-3. ipynb, or .py

show help: *python Ex2-1.py -h*

These codes correspond to random vector, spin model and picture image, respectively.

I recommend *.ipynb because it contains an appendix part.

*If you run them at Goole Colab, please upload MPS.py in addition to the *.ipynb.

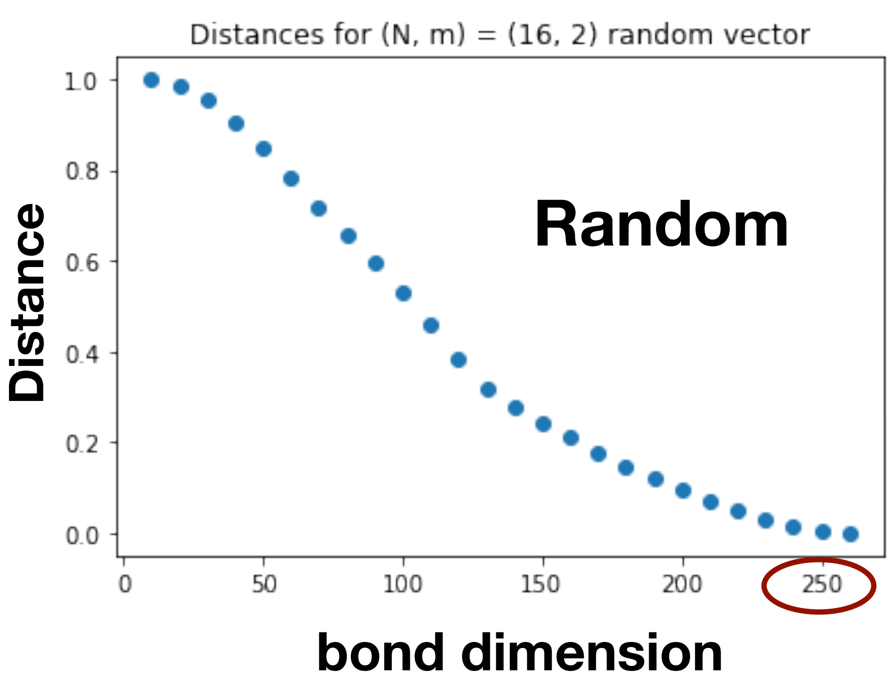
*In the case of Ex2-2 you also need ED.py.

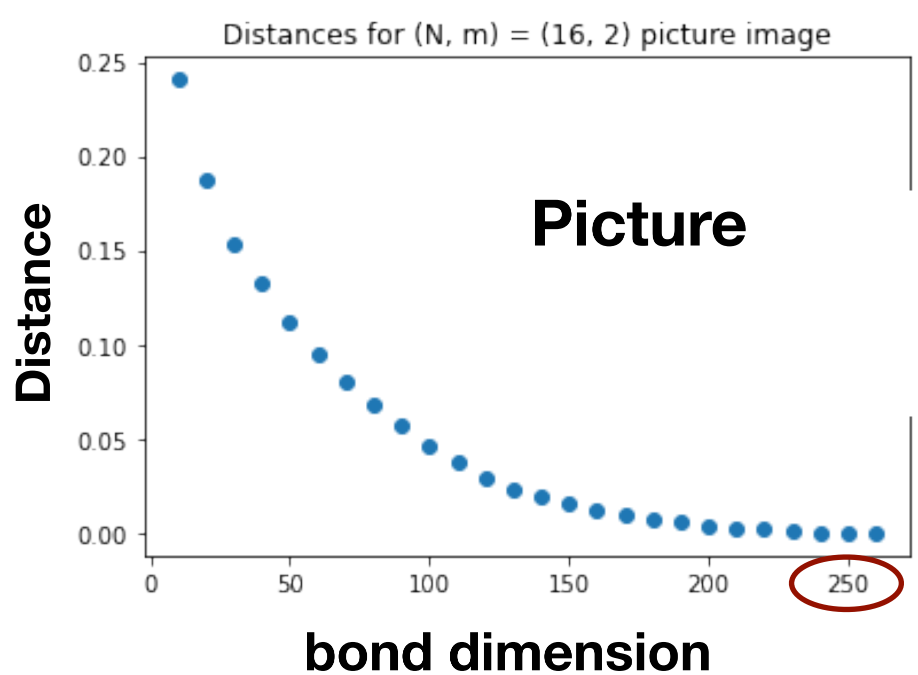
*In the case of Ex2-3 you also need picture file.

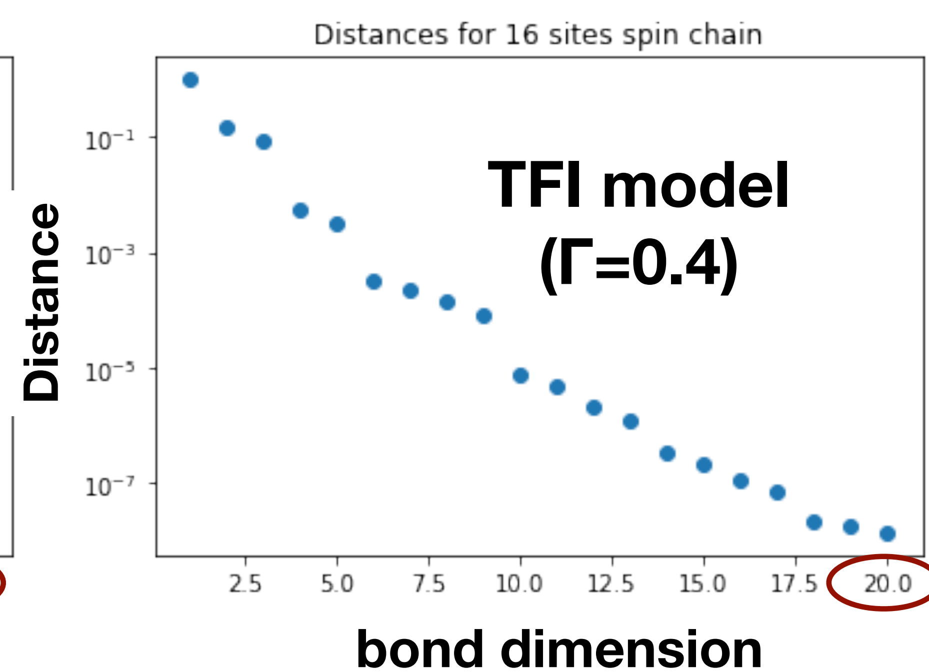
Exercise 2: Make MPS and approximate it

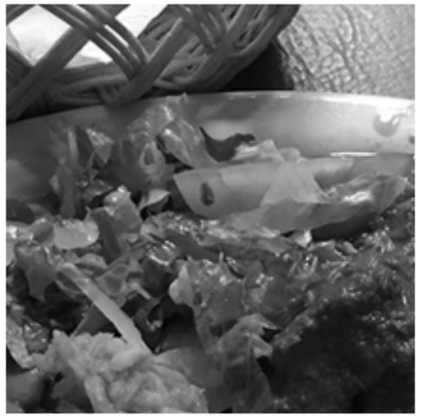
2^{16} dimensional vectors (=16-leg tensors)

Distance between the original and approximated vectors: $||\vec{v}_{ex} - \vec{v}_{ap}||$









$$\mathcal{H} = -\sum_{i=1}^{L-1} S_{i,z}S_{i+1,z} - \Gamma \sum_{i=1}^{L} S_{i,x}$$

Exercise 3: (TEBD and) iTEBD simulation (ITE)

3-1: TEBD simulation

Simulate small finite size system and compare energy with ED

Sample code: Ex3-1.py or Ex3-1.ipynb

3-2: iTEBD simulation

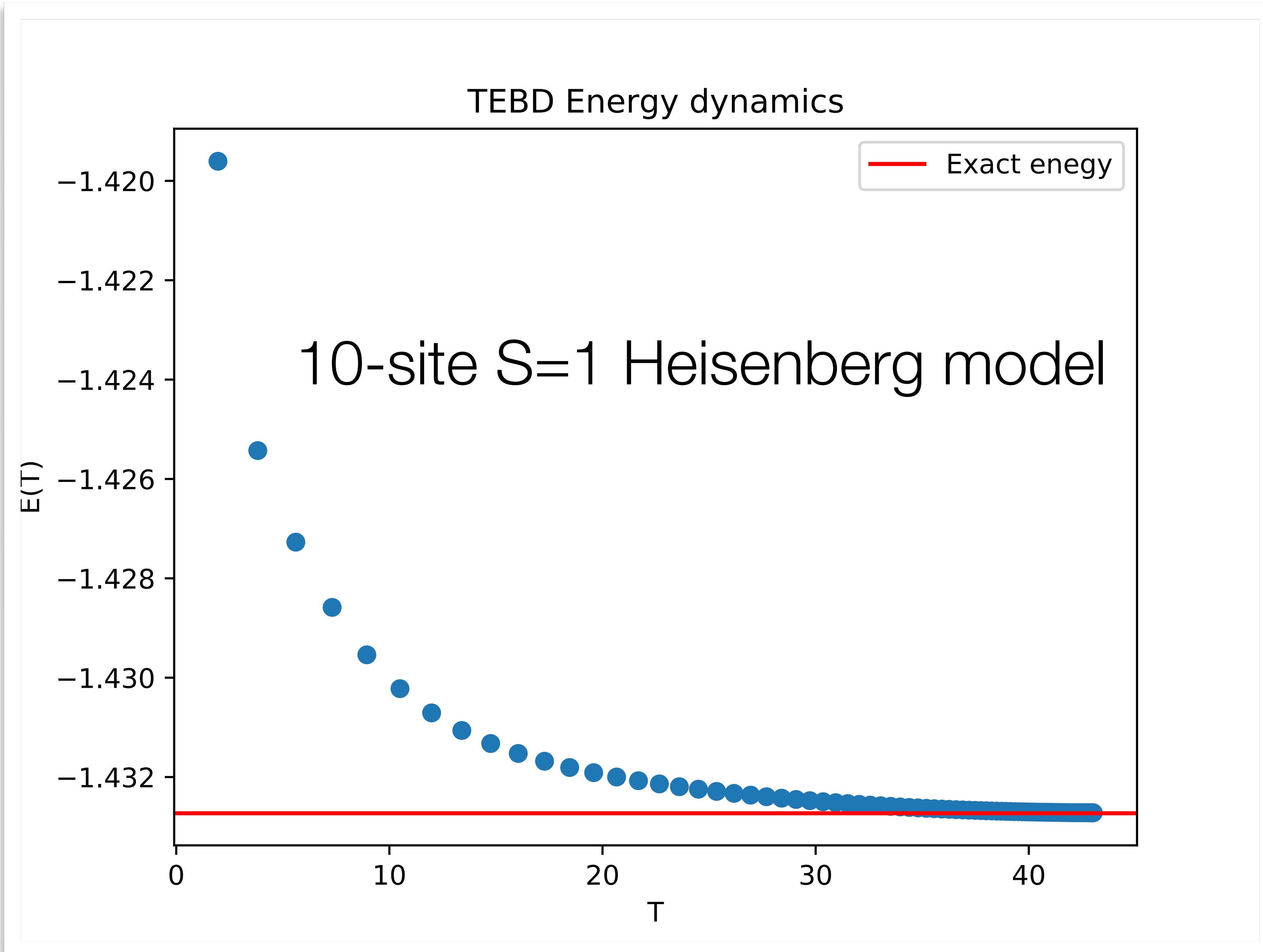
Simulate infinite system and calculate energy

Sample code: Ex3-2.py or Ex3-2.ipynb

*** Try simulation with different "chi_max", "T_step"**

*If you run them at Goole Colab, please upload ED.py and TEBD.py for Ex3-1.ipynb, and please upload TEBD.py and iTEBD.py for Ex3-2.ipynb.

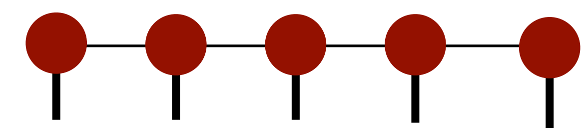
3-1: Energy dynamics in TEBD



Breakdown of MPS representation

Required bond dimension in MPS representation

$$\boxed{S_A = -\mathrm{Tr}\ \rho_A \log \rho_A \leq \log \chi}$$



The upper bound is independent of the "length".

length of MPS ⇔ size of the problem

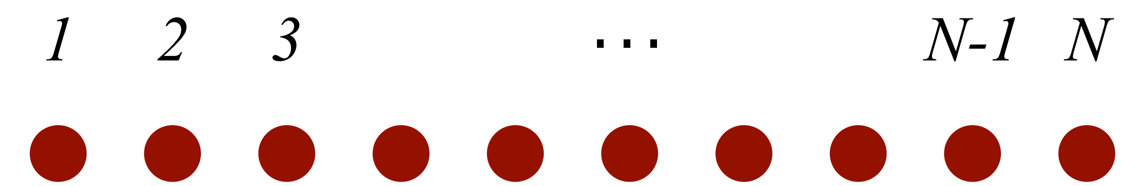
N $\quad\quad\quad$ a^N

EE of the original vector	Required bond dimension in MPS representation
$S_A = O(1)$	$\chi = O(1)$
$S_A = O(\log N)$	$\chi = O(N^\alpha)$
$S_A = O(N^\alpha)$	$\chi = O(c^{N^\alpha})$

Phase transition

Transverse field Ising chain:

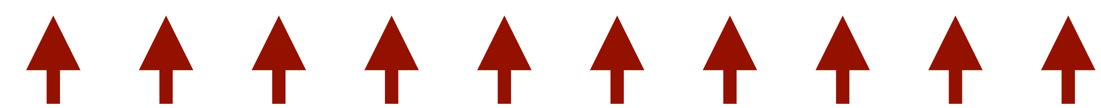
$$\mathcal{H} = -\sum_{i=1}^{N-1} S_i^z S_{i+1}^z - h\sum_{i=1}^{N} S_i^x$$



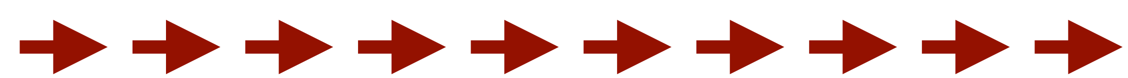
$(|\uparrow\rangle, |\downarrow\rangle)$

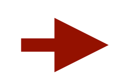
Ground state $|\Psi\rangle$

$h = 0$:Ferromagnetic state



$h \to \infty$:Disordered state
(Field induced ferro)



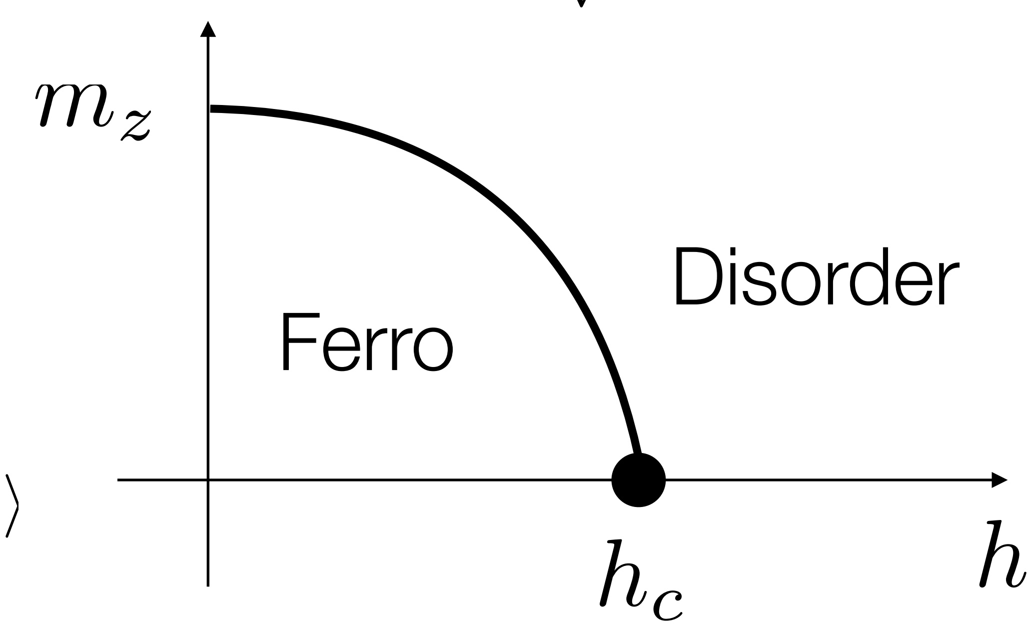
 $= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

In between these two limits, there is a phase transition.

At the phase transition,
order parameter becomes zero.
（秩序変数）

(Spontaneous) Magnetization （自発磁化）

$$m_z = \frac{1}{N}\sum_i \langle\Psi|S_i^z|\Psi\rangle$$



Critical point and correlation length

$h = h_c$:Critical point （臨界点）

Behavior of a correlation function:

$0 \leq h < h_c$:Ferromagnetic state

$$\langle \Psi | S_i^z S_{i+r}^z | \Psi \rangle \sim C e^{-\frac{r}{\xi}} + m_z^2$$

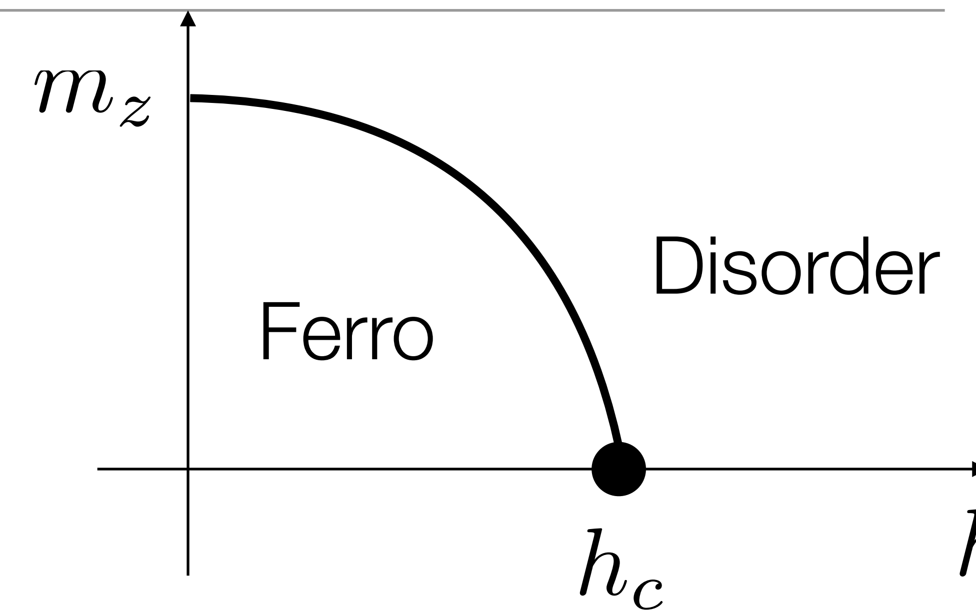
$h_c < h$:Disordered state

$$\langle \Psi | S_i^z S_{i+r}^z | \Psi \rangle \sim e^{-\frac{r}{\xi}}$$

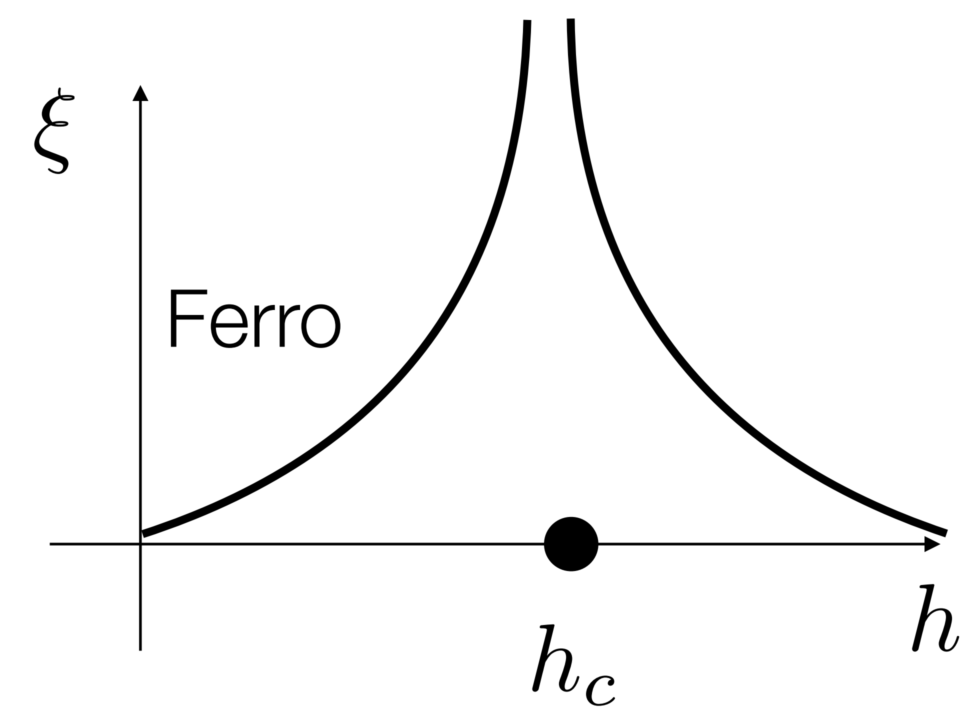
$h = h_c$:Critical point

$$\langle \Psi | S_i^z S_{i+r}^z | \Psi \rangle \sim r^{-2p}$$

Correlation length diverges at critical point!



$\xi = \xi(h)$:Correlation length (相関長)



Scale invariance at the critical point

$h = h_c$:Critical point （臨界点）

$$C(r) \equiv \langle \Psi | S_i^z S_{i+r}^z | \Psi \rangle \sim r^{-2p}$$

$\boxed{\text{Power low decay!}}$

After a scale transformation $r' = br$

$$\Rightarrow \quad C(r') = C(br) = b^{-2p} C(r)$$

Change in the correlation function is only a constant factor.

$\Rightarrow$ If we scale spins as $\tilde{S}_i^z = b^p S_i^z$

the correlation function becomes

$$\tilde{C}(r') \equiv \langle \Psi | \tilde{S}_i^z \tilde{S}_{i+r'}^z | \Psi \rangle = C(r)$$

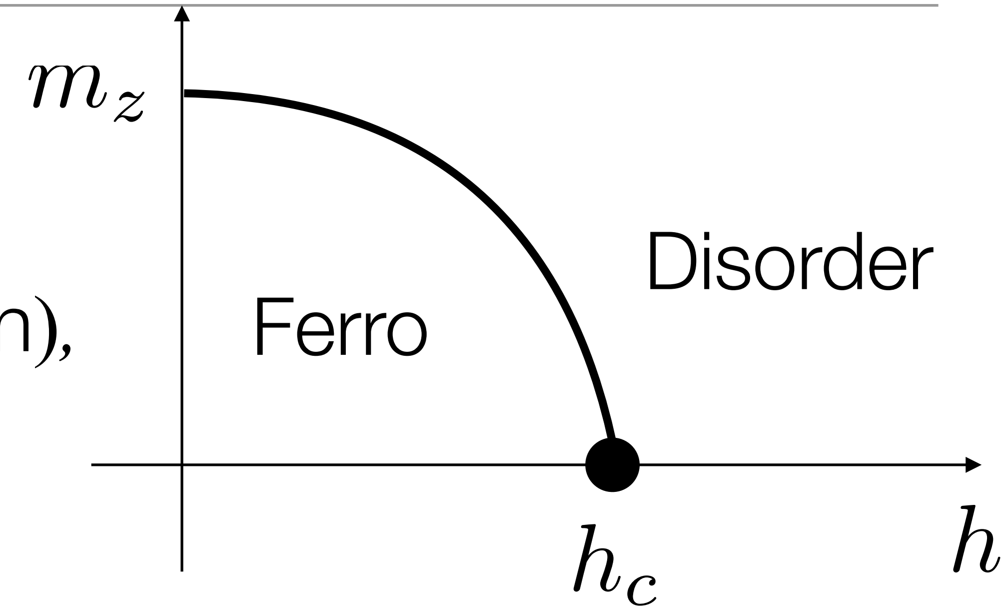
This property is called as "scale invariance".（スケール不変性）

$\boxed{\text{Physics (properties) in different scale is essentially same.}}$

DMRG (variational MPS) calculation of TFI model

Ö. Legeze, and G. Fáth, Phys. Rev. B **53**, 14349 (1996)

Relative errors of the ground and the 1st excited states energies varying system size N.



For a fixed bond-dimension m (= χ in our notation),

Ferro and disordered states:

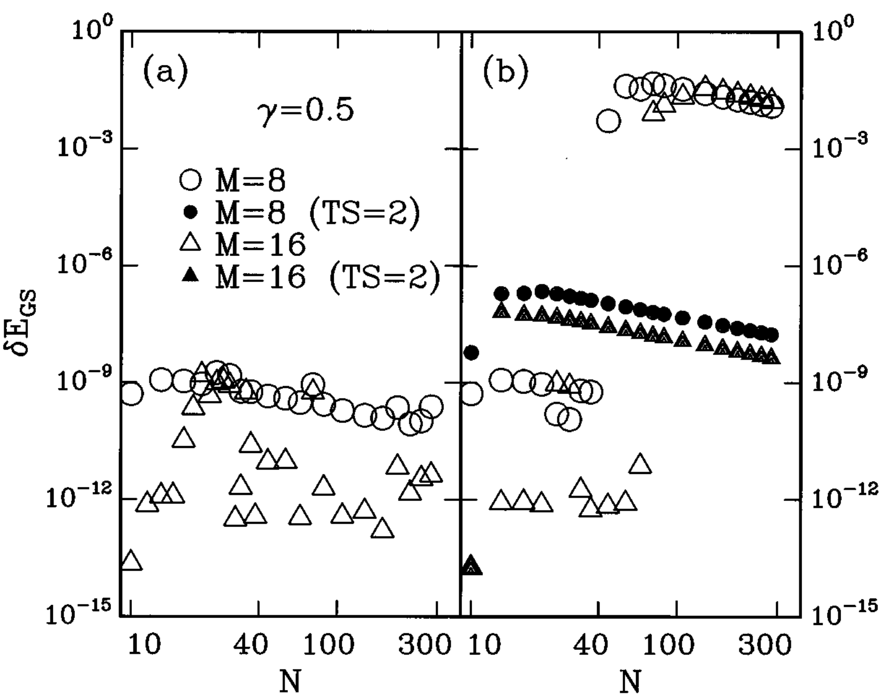
The errors are almost independent of N.

Critical point:

The errors gradually increases as increase N.

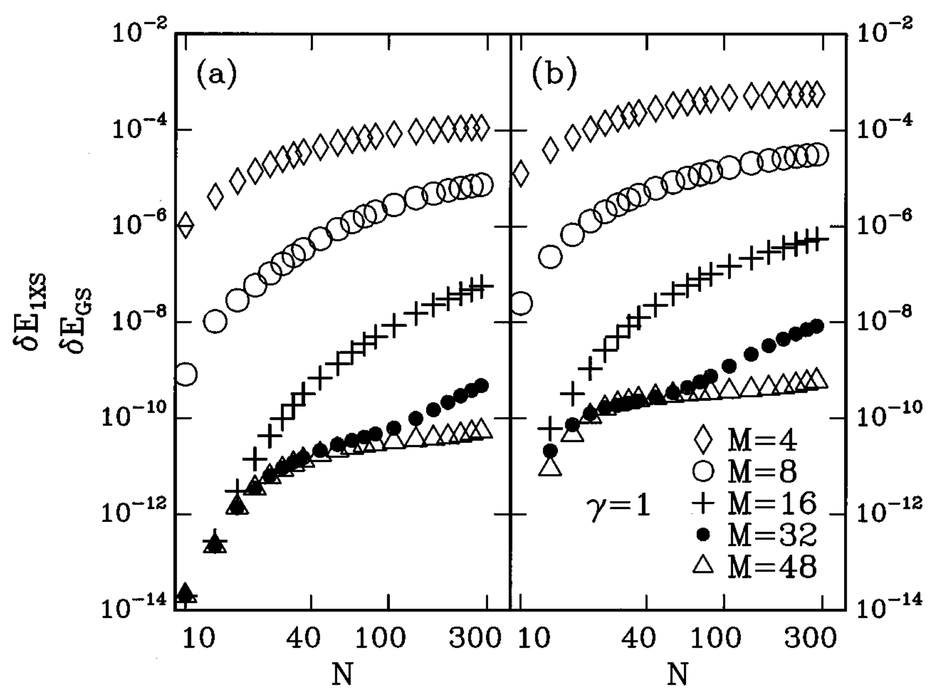
$0 \le h < h_c$

$h = 0.25$



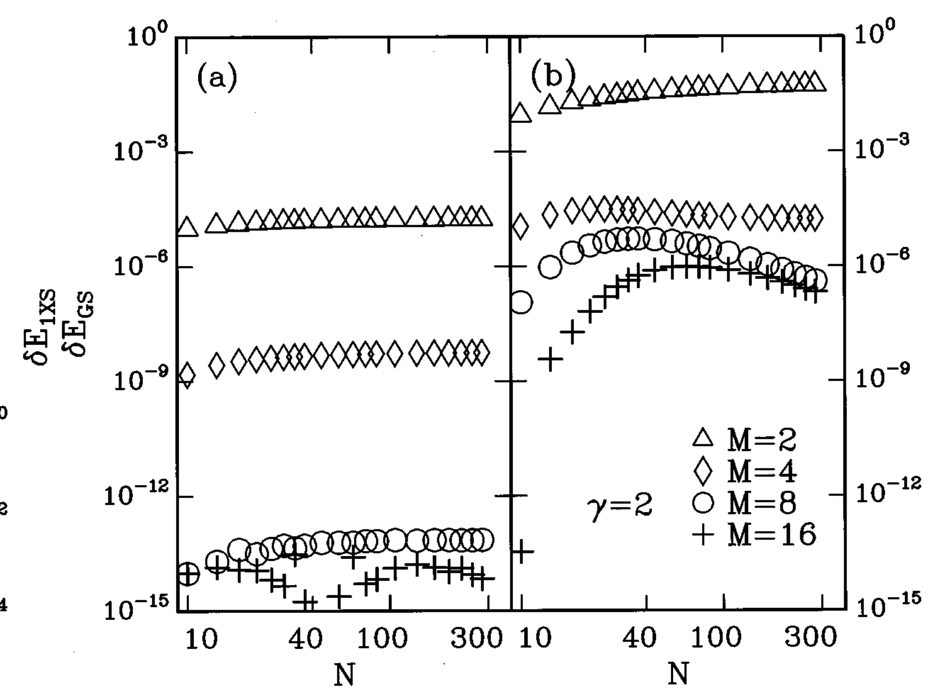
$h = h_c$

$h = 0.5$



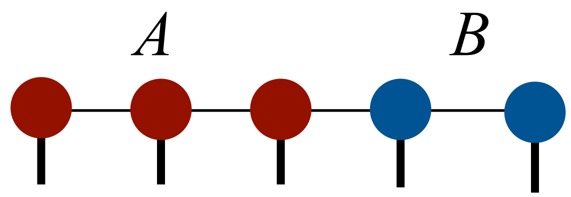
$h_c < h$

$h = 1.0$



Entanglement entropy of TFI model

Entanglement entropy: $\boxed{S_A = -\mathrm{Tr}\ \rho_A \log \rho_A}$



State	EE of the original vector	Required bond dimension
Ferro or Disordered	$S_A = O(1)$	$\chi = O(1)$
Critical	$S_A = O(\log N)$	$\chi = O(N^\alpha)$

We need polynomially large bond dimension for critical system!

More efficient tensor network for critical systems?

Key point: Scale invariance of the system

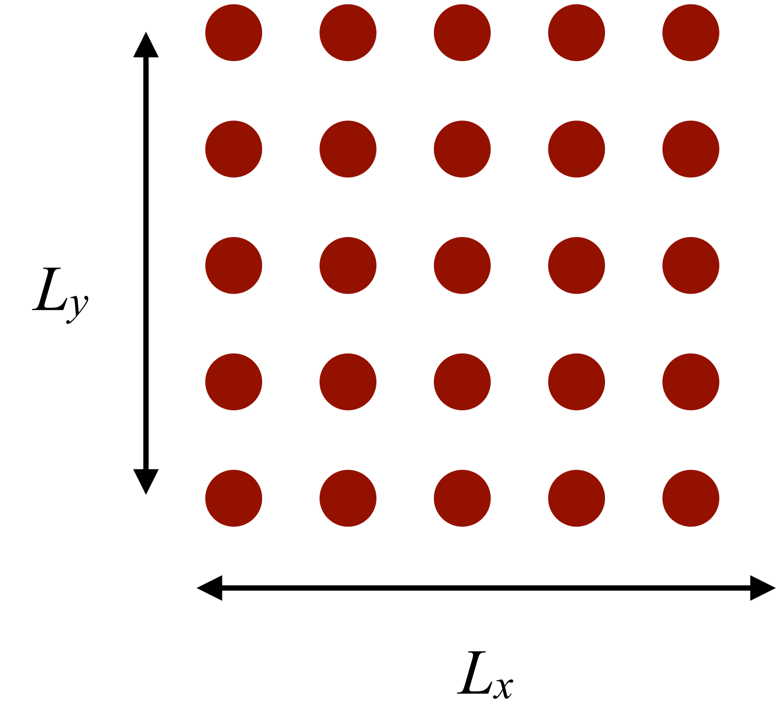
Higer dimensional system

Transverse field Ising model on square lattice:

$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z - h \sum_{i=1}^{N} S_i^x$$

$\sum_{\langle i,j \rangle}$:Summation over the nearest neighbor pair

Two-dimensional array



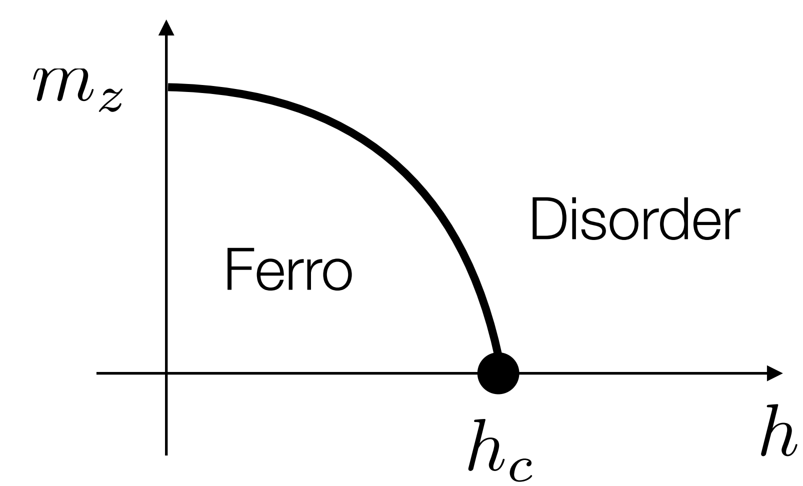
$$N = L_x \times L_y$$

Area law

Even in ferro and disordered phases,
the entanglement entropy depends on size N.

$$S_A \sim \sqrt{N} = L$$

Phase diagram



MPS for two-dimensional system

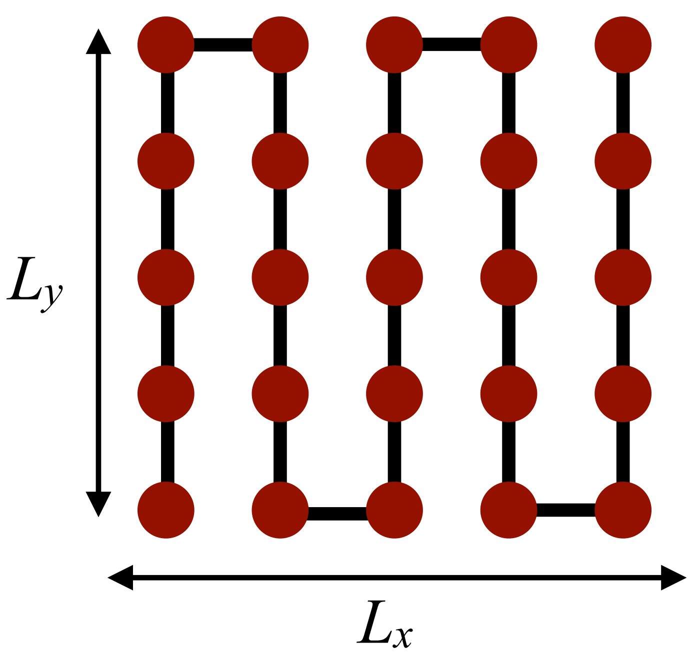
When we apply MPS representation for a square lattice system:

Setting **(1)** $S_A \leq L_x \log \chi$:Satisfying area law?

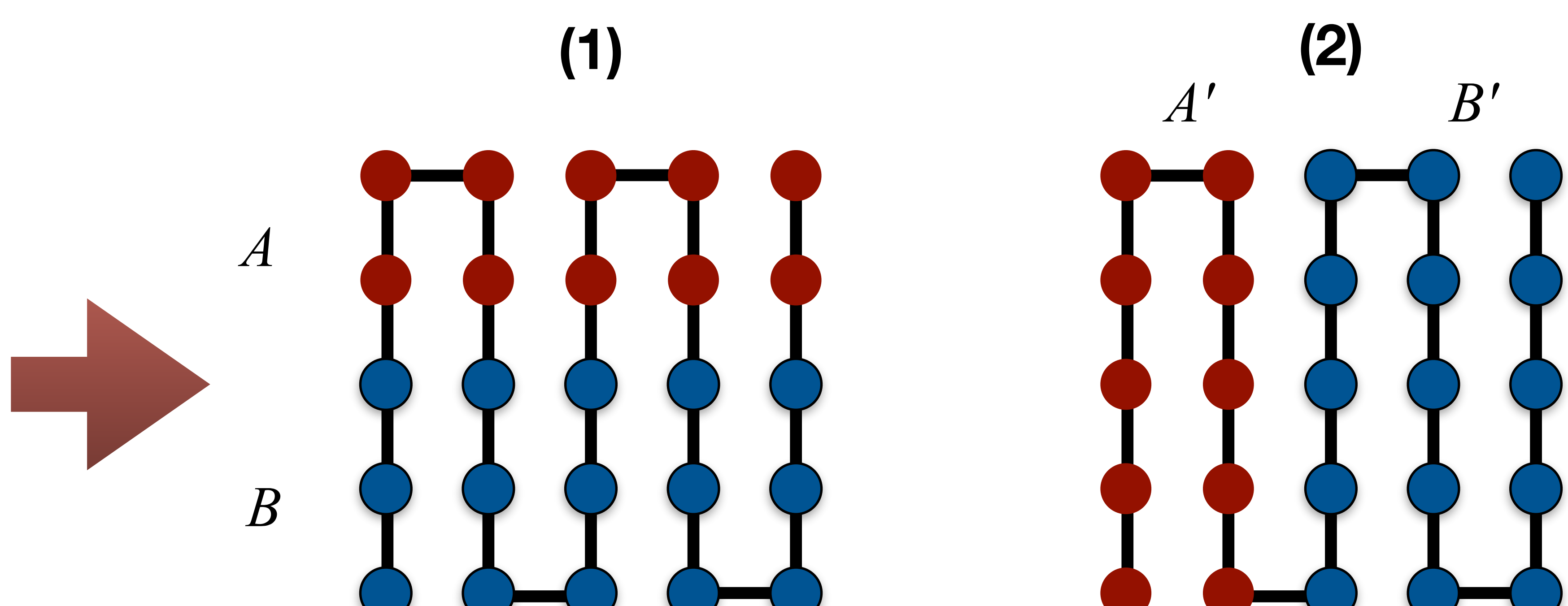
Setting **(2)** $S_{A'} \leq \log \chi$:Break down of the area law!

MPS cannot cover the area law of the entanglement entropy in higher (d =2,3, ...) dimensions.

Possible MPS (Snake form)



Two settings of system and environment



MPS for two-dimensional system: comment

MPS can treat "**rectangular**" or "**quasi one dimensional**" lattice.

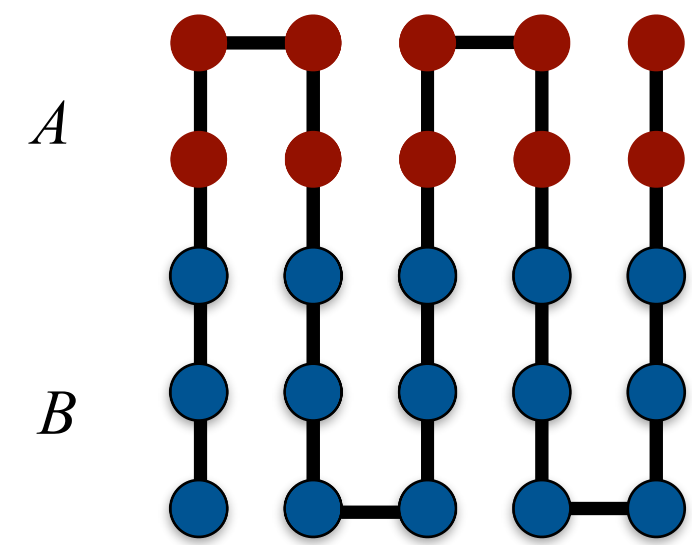
In setting (1), MPS can satisfy the area low partially.

→ We can increase L_x easily with keeping L_y constant.

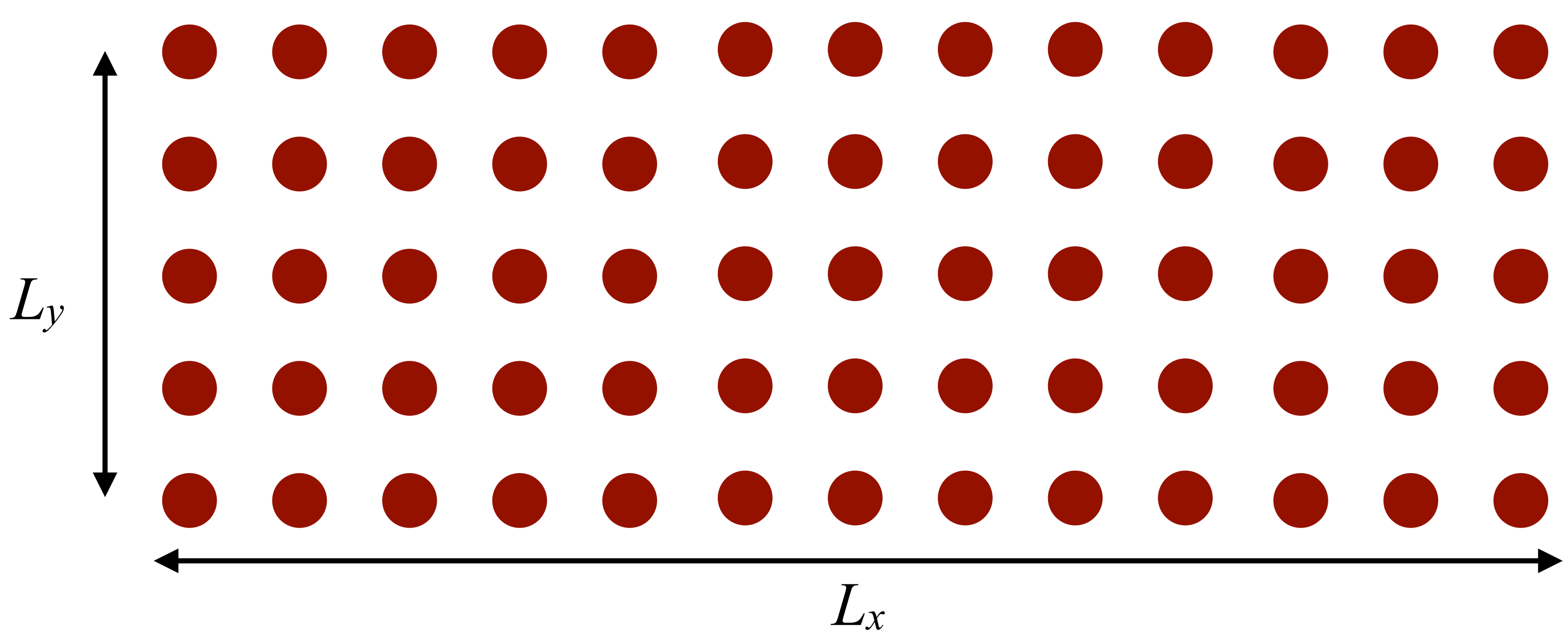
$$\chi = O(e^{L_y})$$

$$L_y \lesssim 10, L_x \gg L_y$$

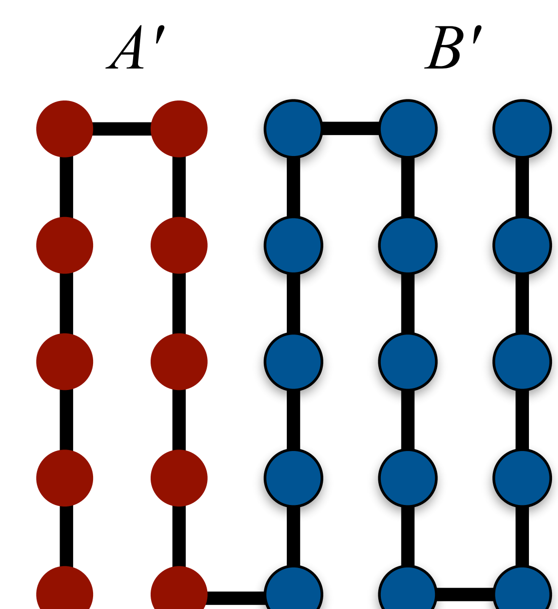
(1) $S_A \leq L_x \log \chi$



Quasi one dimensional system ("strip" or "cylinder")



(2) $S_{A'} \leq \log \chi$



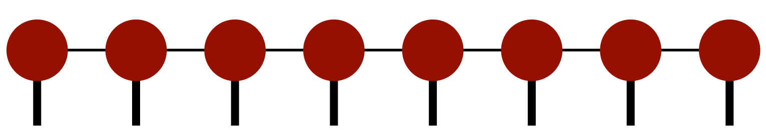
Tensor network for critical systems:
Multi-scale Entanglement Renormalization Ansatz

Hierarchical structure: tree tensor network

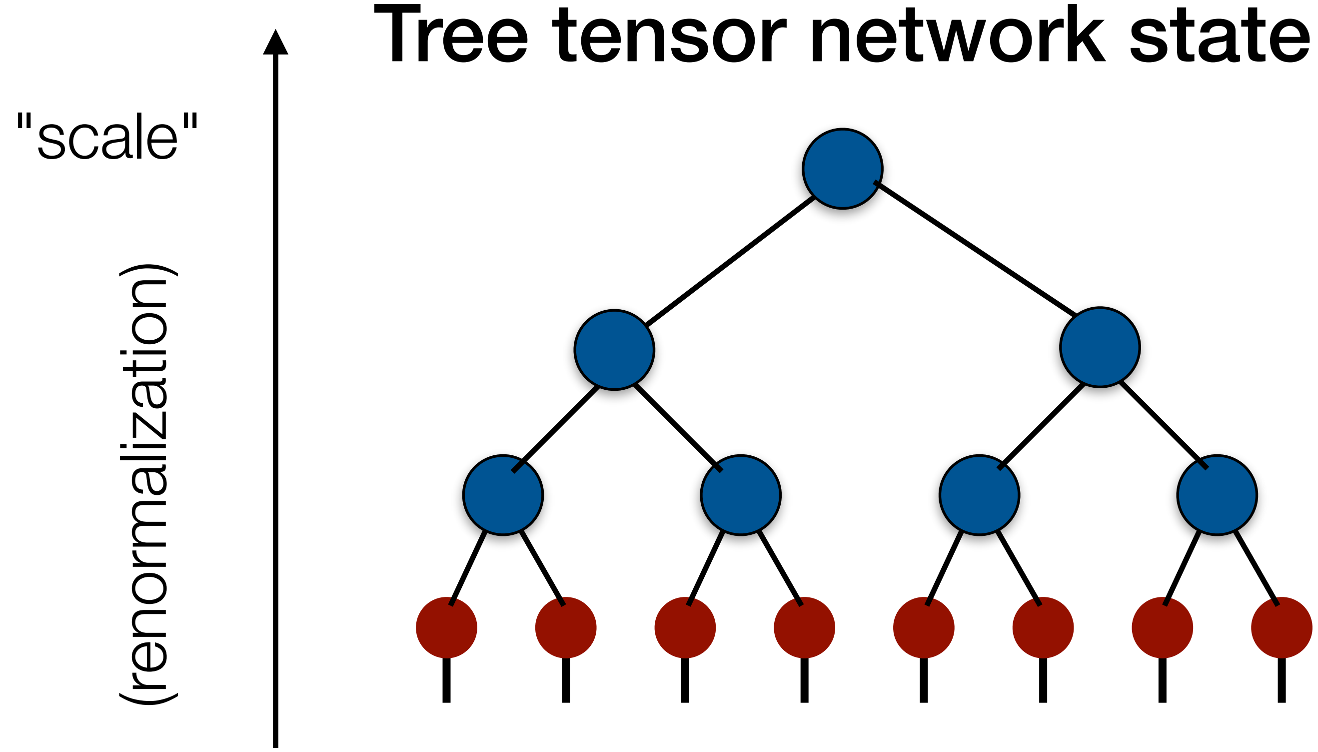
Critical system $\Longleftrightarrow$ Scale invariance

A simple scale invariant tensor network: tree tensor network

MPS



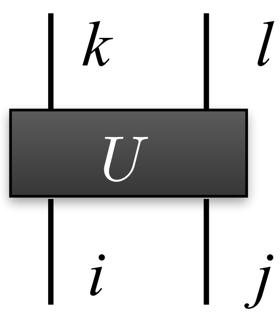
Tree tensor network state



Notice:

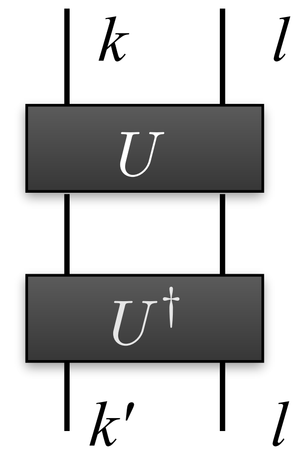
Unitary tensors

Unitary tensor

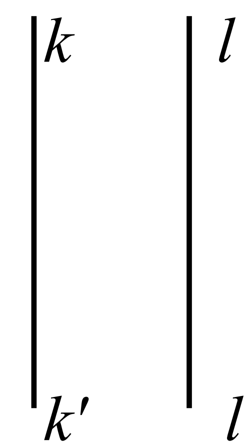
$$U_{ij}^{kl} =$$ 

$$\boxed{(U^\dagger)^{ij}_{kl} = (U^{kl}_{ij})^*}$$

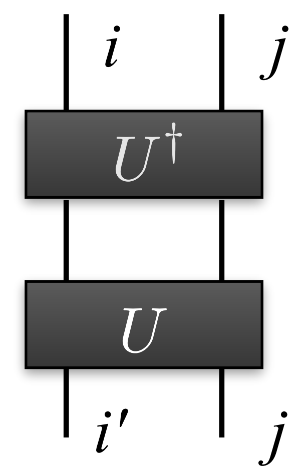
$$\sum_{i,j} U_{ij}^{kl} (U^\dagger)^{ij}_{k'l'} = \delta_{kk'}\delta_{ll'}$$



$=$



$$\sum_{k,l} (U^\dagger)^{ij}_{kl} U^{kl}_{i'j'} = \delta_{ii'}\delta_{jj'}$$



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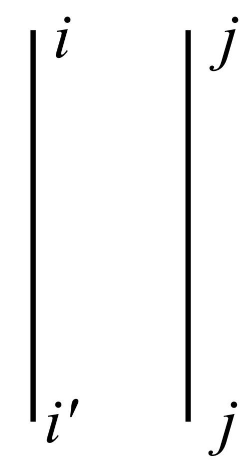
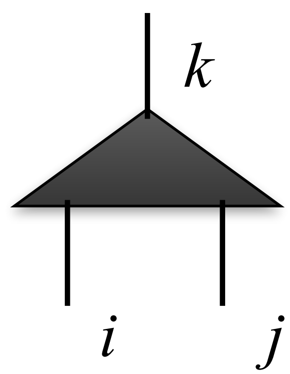


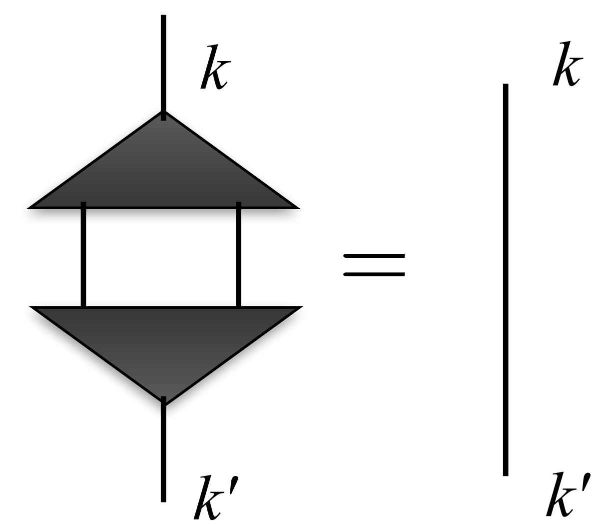
Isometric tensors

Isometric tensor (half unitary tensor) = Isometry

$$W_{ij}^{k} =$$



$$\sum_{i,j} W_{ij}^{k} (W^{\dagger})_{k'}^{ij} = \delta_{kk'}$$



Unitarity condition only for "bottom" legs.

Isometry works as a "projector" from the bottom space to the top space.

$$\mathrm{dim}(\mathrm{bottom}) \geq \mathrm{dim}(\mathrm{top})$$

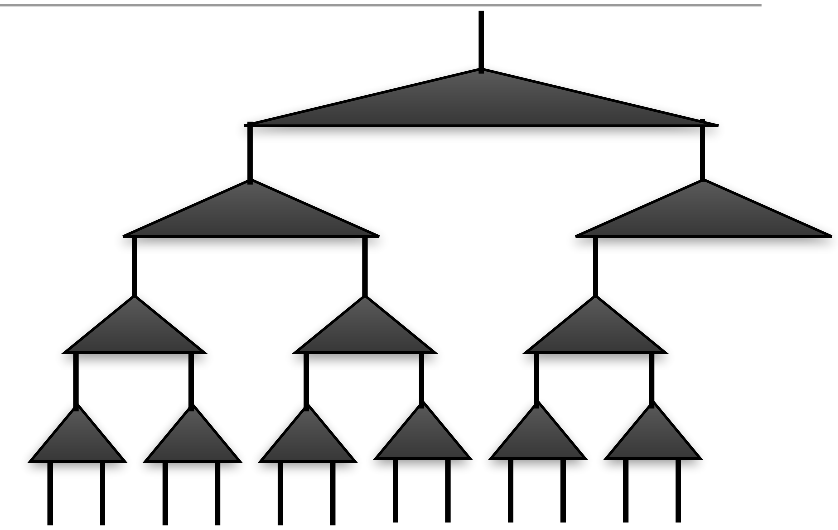
It is also related to the "renormalization" of degree of freedoms.

We pick up "important" degree of freedoms by isometries.

Isometric tree tensor network and its scale invariance

Consider an (infinite) tree tensor network consists of identical isometries as a wave function.

$|\Psi\rangle =$



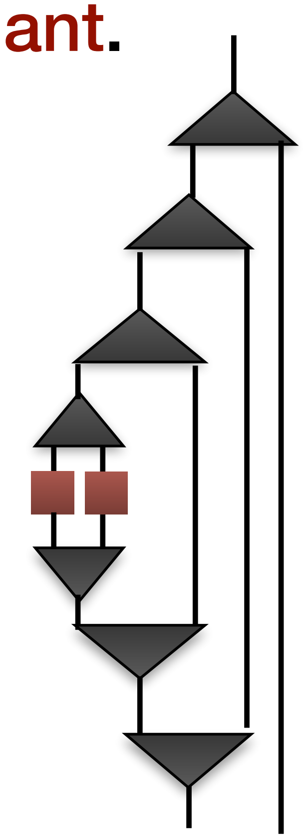
Properties:

1. It is normalized as $\langle \Psi | \Psi \rangle = 1$

(Trivial from the definition of the isometry)

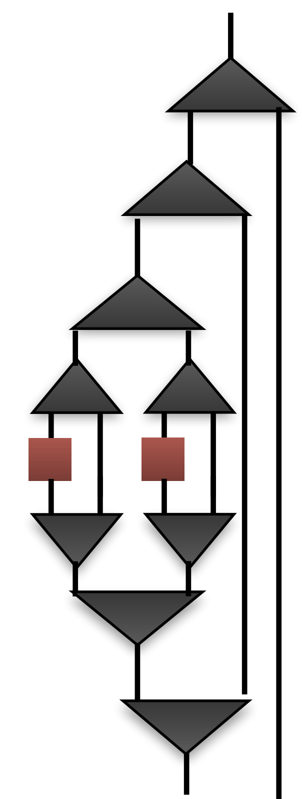
2. It can be scale invariant.

$C(1) \equiv \langle \Psi | S_1^z S_2^z | \Psi \rangle =$



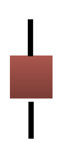
$\cdots$

$C(2) \equiv \langle \Psi | S_1^z S_3^z | \Psi \rangle =$



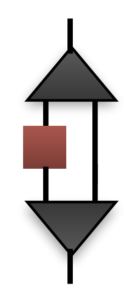
$\cdots$

$S_i^z =$



spin

$(S_i^z)' =$



"renormalized" spin

If $(S_i^z)' = 2^{-p} S_i^z$, then $C(2r) = 2^{-2p} C(r)$

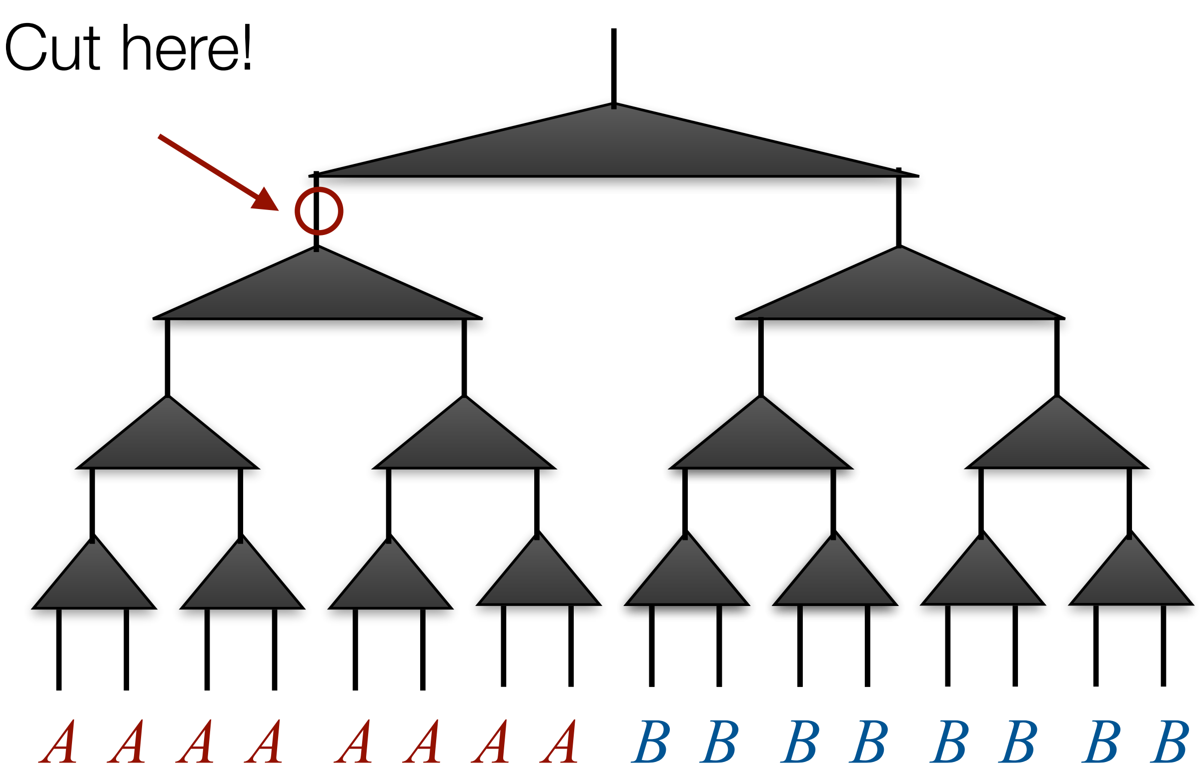
Scale invariant!

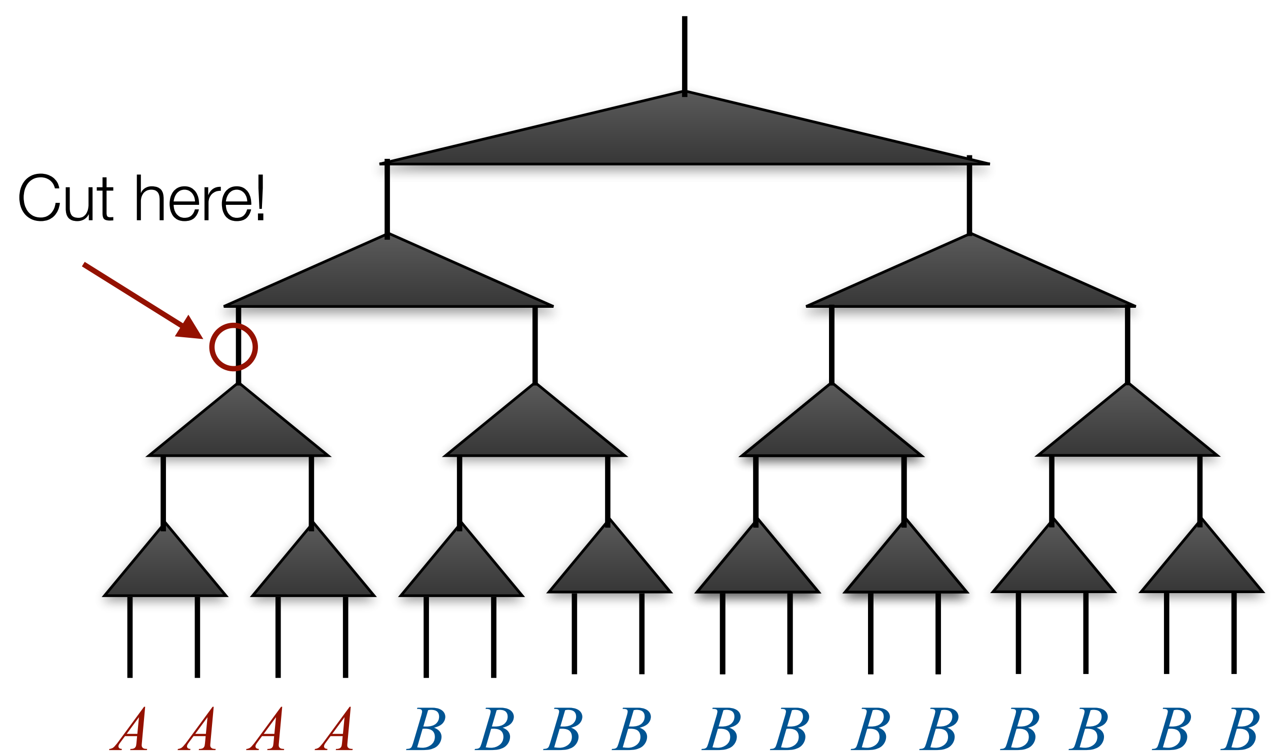
Entanglement entropy of TTN

Entanglement entropy of tree tensor networks (TTN):

Due to the tree structure, two regions are connected by only "one bond".
(or a few)

$$\Rightarrow \quad \boxed{S_A = -\mathrm{Tr}\ \rho_A \log \rho_A \le \log \chi}$$



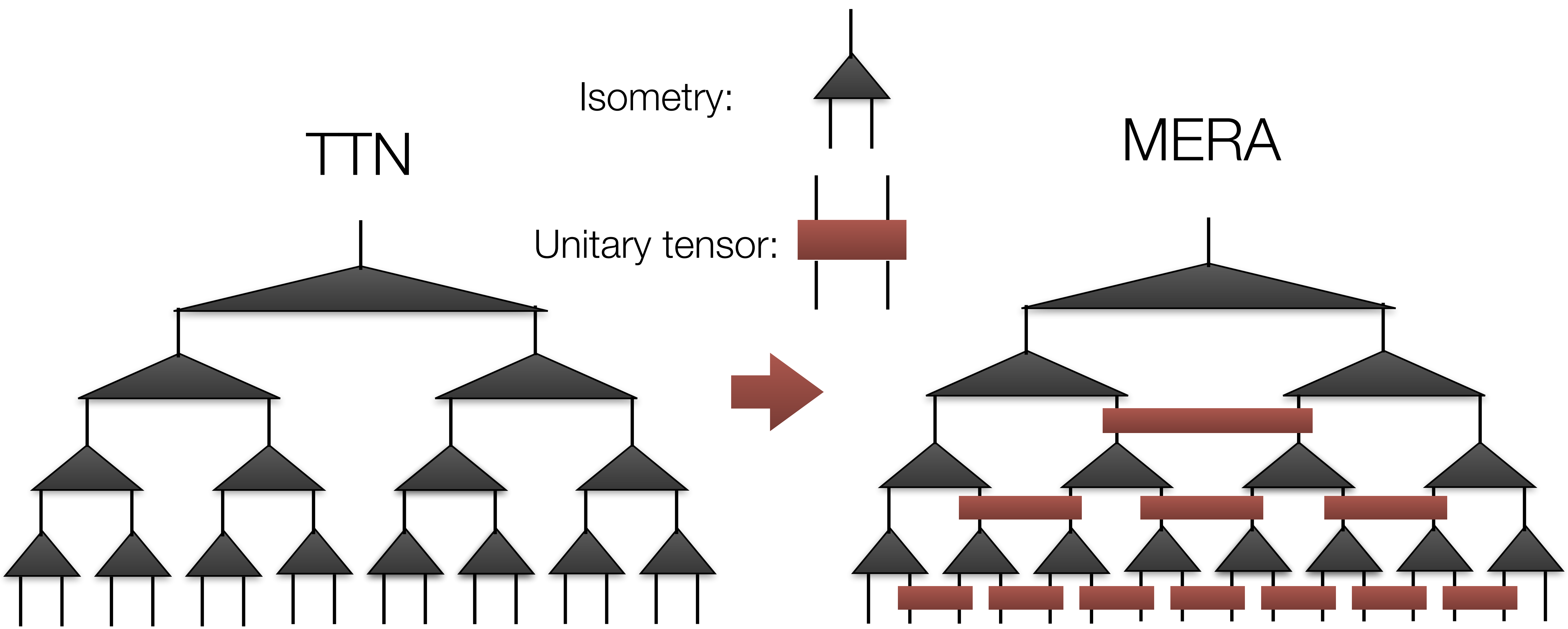


MERA

(G. Vidal, Phys. Rev. Lett. **99**, 220405 (2007))
(G. Vidal, Phys. Rev. Lett. **101**, 110501 (2008))

Multi-scale Entanglement Renormalization Ansatz (**MERA**)

Before applying isometry, insert a unitary tensor.



- [x] Normalization
- [x] Scale invariance (if we set the identical tensors)

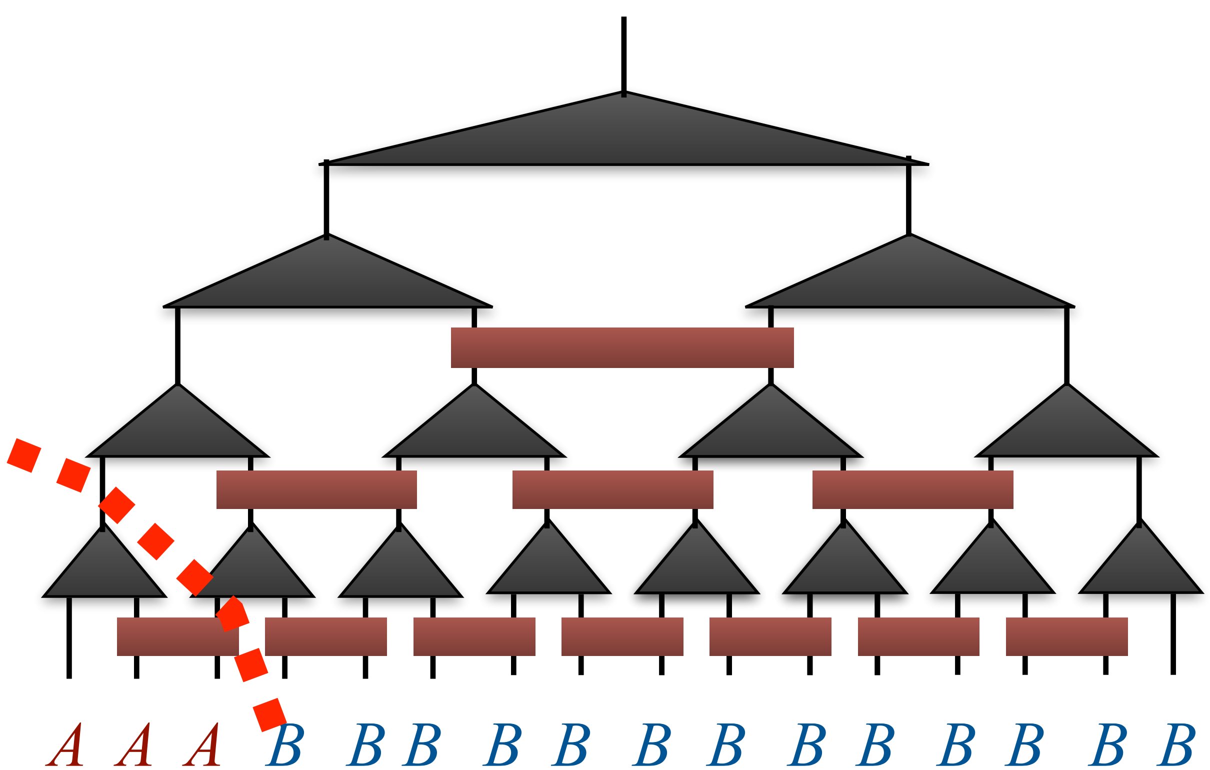
Entanglement entropy of MERA

Due to the unitary matrices, # of bonds connecting two regions logarithmically increase.

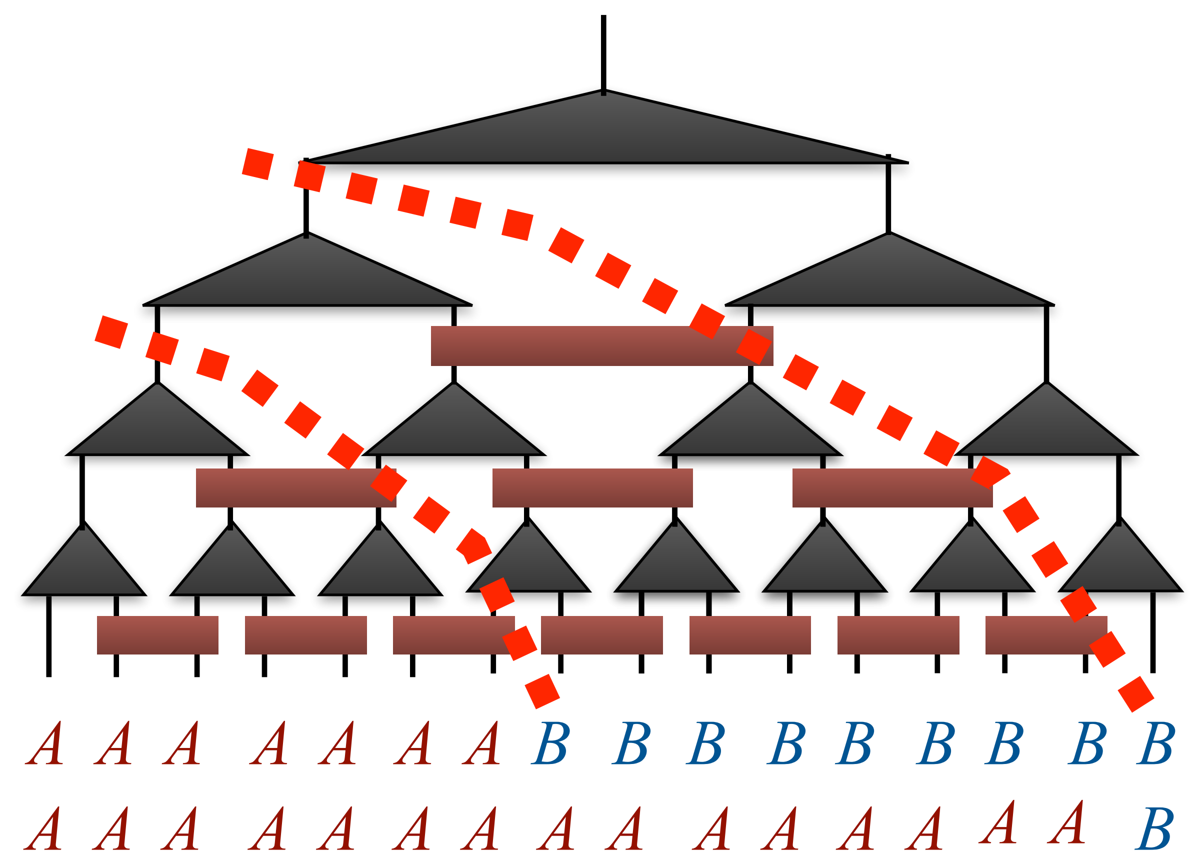
$$\mathrm{rank}\ \rho_A \le \chi^{N_c(N)} \sim \chi^{\log N}$$
$$S_A = -\mathrm{Tr}\ \rho_A \log \rho_A \le (\log \chi) \log N$$

$N_c(N)$:# of minimum cut for a N-site region

$$N = 2^{N_c} - 1$$
$$N_c \sim \log_2 N$$



Minimum # of cuts = 2



Minimum # of cuts = 3

Minimum # of cuts = 4

Application of MERA

Transverse field Ising chain:

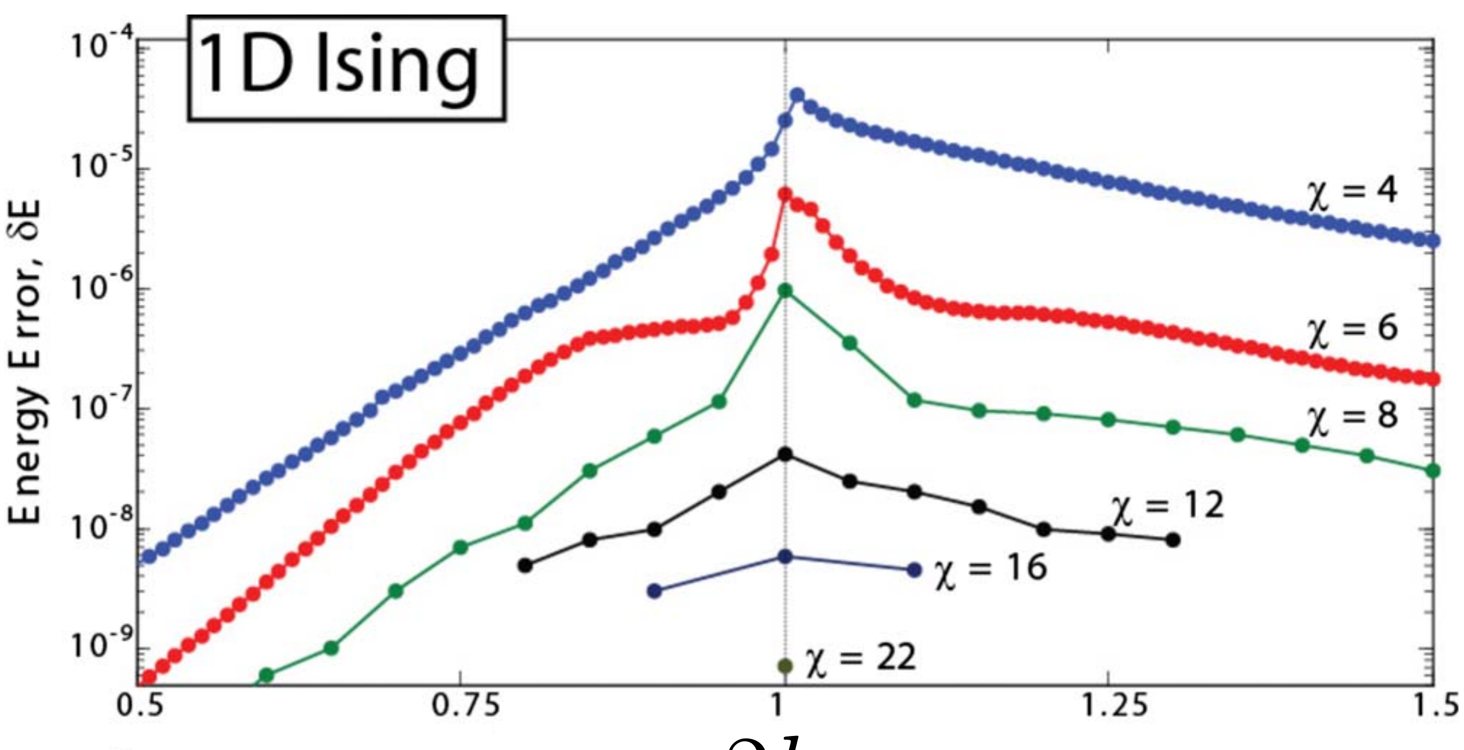
$$\mathcal{H} = -\sum_{i=1}^{N-1} S_i^z S_{i+1}^z - h\sum_{i=1}^{N} S_i^x$$

MERA can represent very large (Infinite) critical system!

Energy errors:

MERA (Infinite chain)

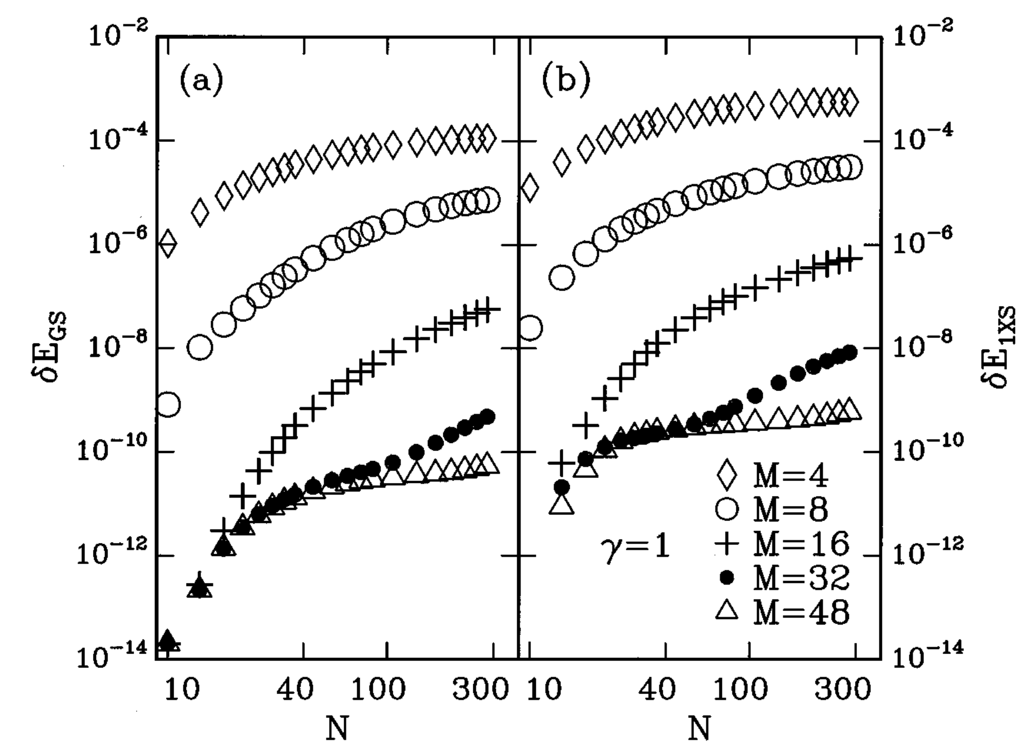
(G. Evenbly and G. Vidal, Phys. Rev. B. **79**, 144108 (2009))



$2h$

DMRG (finite chain)

Ö. Legeze, and G. Fáth (1996)



$h = h_c$

Interesting topics related to MERA

- By using scale invariance of MERA, we can calculate properties of critical system accurately.
  (G. Evenbly and G. Vidal, Phys. Rev. B. **79**, 144108 (2009))
  (R.N.C. Pfeifer, (G. Evenbly and G. Vidal, Phys. Rev. A. **79**, 040301(R) (2009))
  - Critical exponents and Operator product expansion coefficients in the Conformal Filed Theory (CFT)
- We can consider MERA in higher dimensions
  - It is scale invariant but satisfies the area low
    (G. Evenbly and G. Vidal, Phys. Rev. Lett. **102**, 180406 (2009))
  - For the system with logarithmic correction in the EE, such as metal, "branching MERA" has been proposed.
    (G. Evenbly and G. Vidal, Phys. Rev. Lett. **112**, 220502 (2014))
    (G. Evenbly and G. Vidal, Phys. Rev. B. **89**, 235113 (2014))
- Relation between MERA and other fields
  - Wavelet transform (G. Evenbly and S. R. White, Phys. Rev. Lett. **112**, 140403 (2016))
  - AdS/CFT (quantum gravity, black hole)
    (M. Nozaki, S. Ryu, and T. Takayanagi, J. High Energy Phys. **10**, 193 (2012))

Tensor network for higher dimensional systems:

Tensor Product State

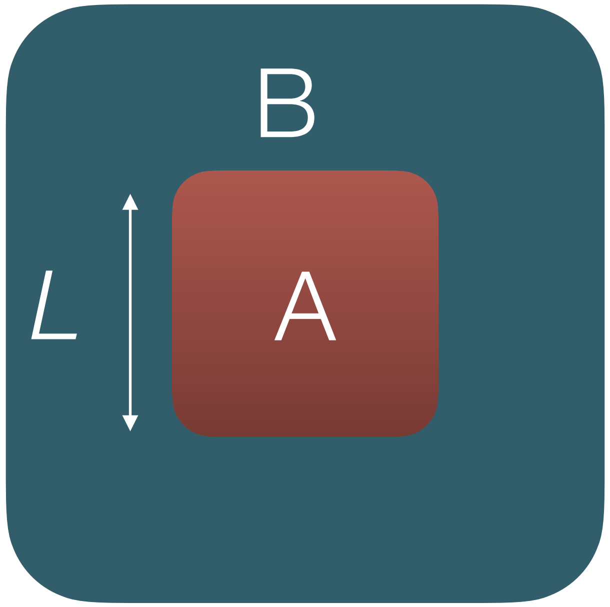
(Projected Entangled Pair State)

Entanglement entropy in higher dimensions

Ground state wave functions:

For a lot of ground states, EE is proportional to its area.

J. Eisert, M. Cramer, and M. B. Plenio, Rev. Mod. Phys, 277, **82** (2010)



Area low:

$$S = -\mathrm{Tr}\left(\rho_A \log \rho_A\right) \propto L^{d-1}$$

In d=1, MPS satisfies the area low.

Q. What is a simple generalization of MPS to d > 1?

A. It is Tensor Product State (TPS)!

Tensor Product State (TPS)

TPS (Tensor Product State) (AKLT, T. Nishino, K. Okunishi, …)

PEPS (Projected Entangled-Pair State)

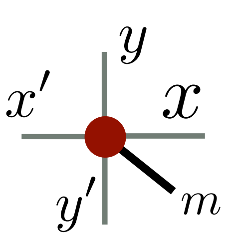
(F. Verstraete and J. Cirac, arXiv:cond-mat/0407066)

d-dimensional tensor network representation
for the wave function of a d-dimensional quantum system

$$|\Psi\rangle = \sum_{\{m_i=1,2,\cdots,m\}} \mathrm{Tr}\, A_1[m_1]A_2[m_2]\cdots A_N[m_N]|m_1m_2\cdots m_N\rangle$$

Tr: tensor network "contraction"

$A_{x_i x_i' y_i y_i'}[m_i]$: Rank 4+1 tensor

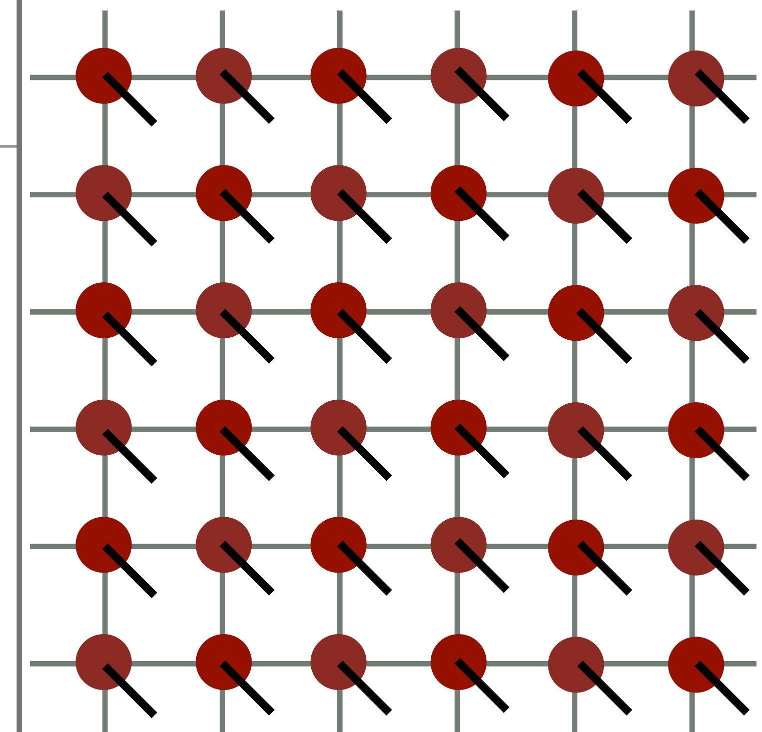


$x,y,x',y' = 1,2, \ldots D$ D = "bond dimension"

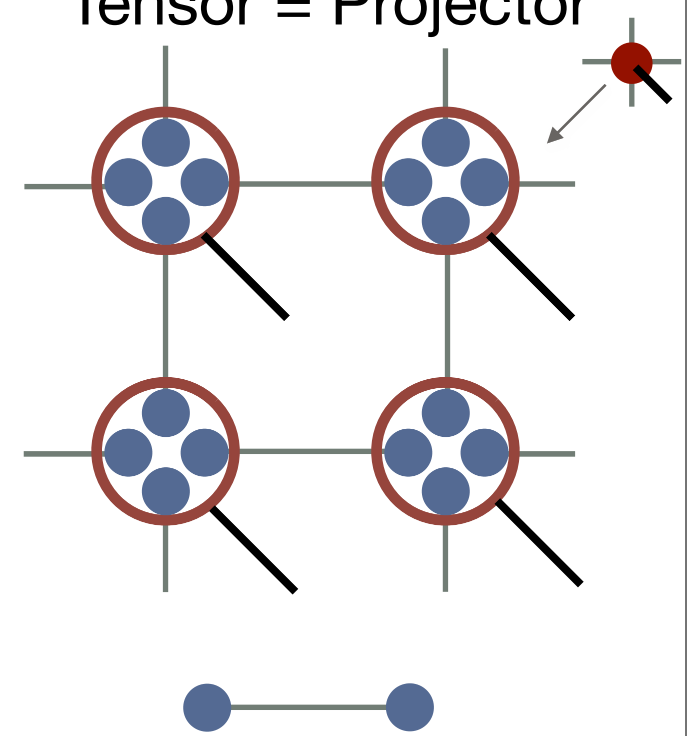
$m_i = 1,2, \ldots m$ m = dimension of the local Hilbert space

*D can be larger than m. "Virtual state "

TPS on square lattice

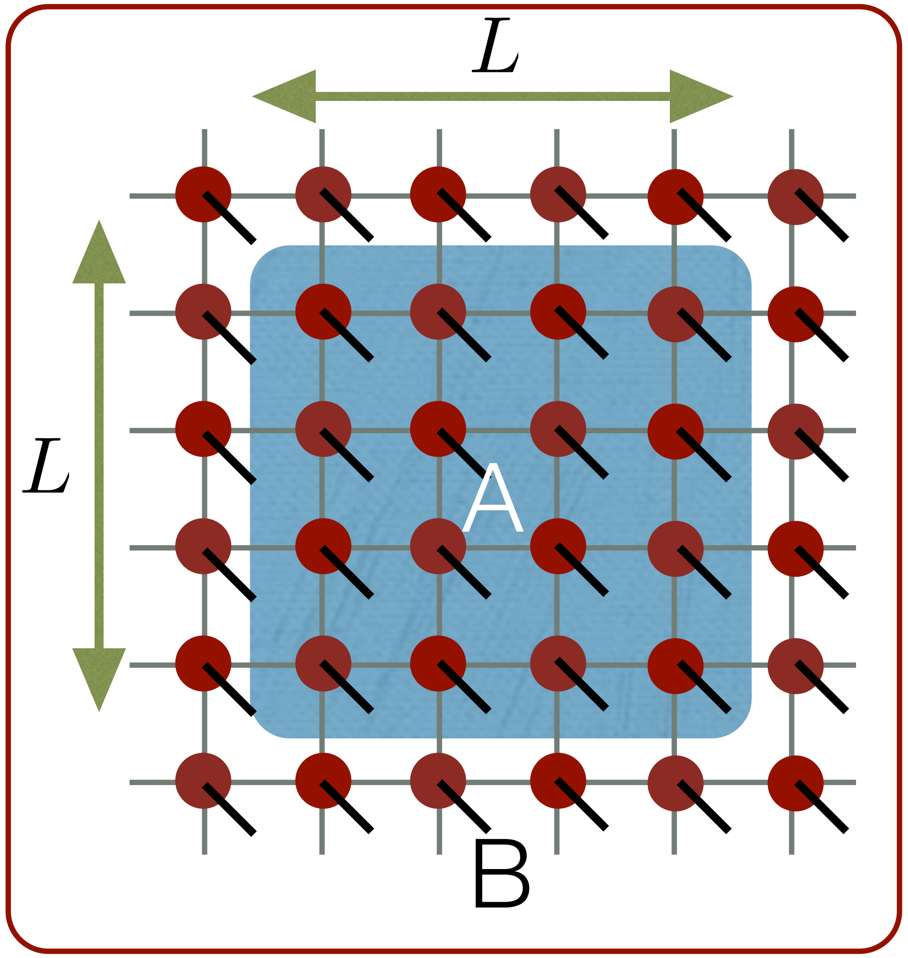


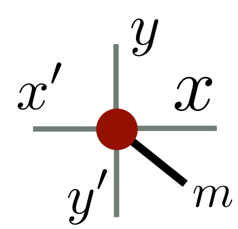
Tensor = Projector



Maximally entangled state between D-state spins

Entanglement entropy of TPS (PEPS)





Bond dimension = D

# of bonds connecting regions A and B

$N_c(L) = 4L$ (square lattice)

$N_c(L) = 2dL^{d-1}$ (d-dimensional hyper cubic lattice)

$$\mathrm{rank}\ \rho_A \leq D^{N_c(L)} \sim D^{2dL^{d-1}}$$

$$S_A = -\mathrm{Tr}\ \rho_A \log \rho_A \leq 2dL^{d-1} \log D$$

TPS can satisfies the area law even for d >1.

We can efficiently approximate vectors in higher dimensional space by TPS.

* Similar to the MPS in 1d, TPS can approximate infinite system!

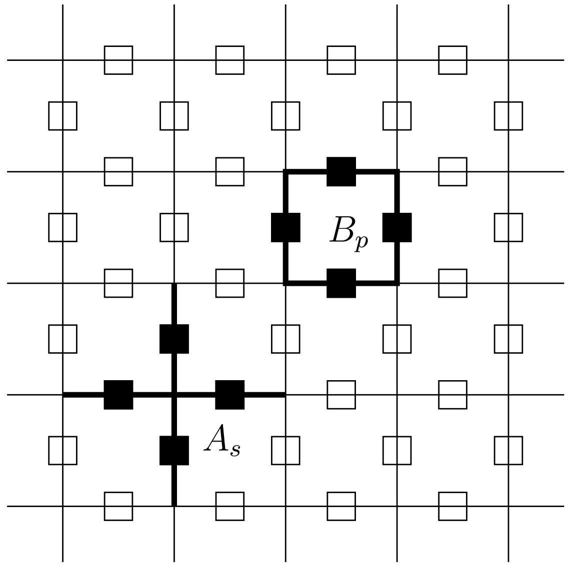
Example: Ground state represented by TPS (optional)

Toric code model

(A. Kitaev, Ann. Phys. **303**, 2 (2003).)

$$\mathcal{H} = -\sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{j \in \mathrm{star}(s)} \sigma_j^x \qquad B_p = \prod_{j \in \partial p} \sigma_j^z .$$



Its ground state is so called Z_2 spin liquid state.

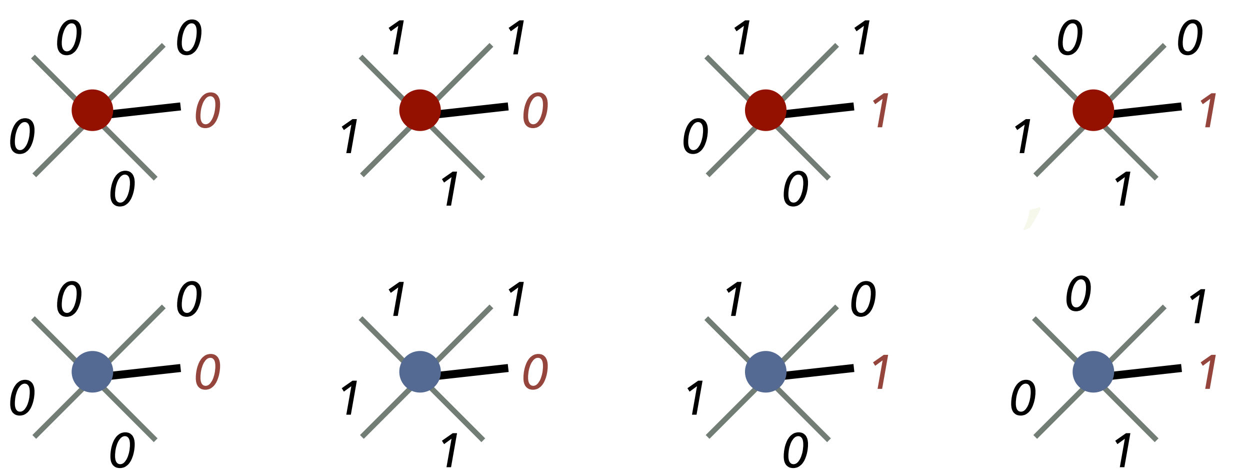
"Spin liquid" is a novel phase different from conventional magnetic orders.

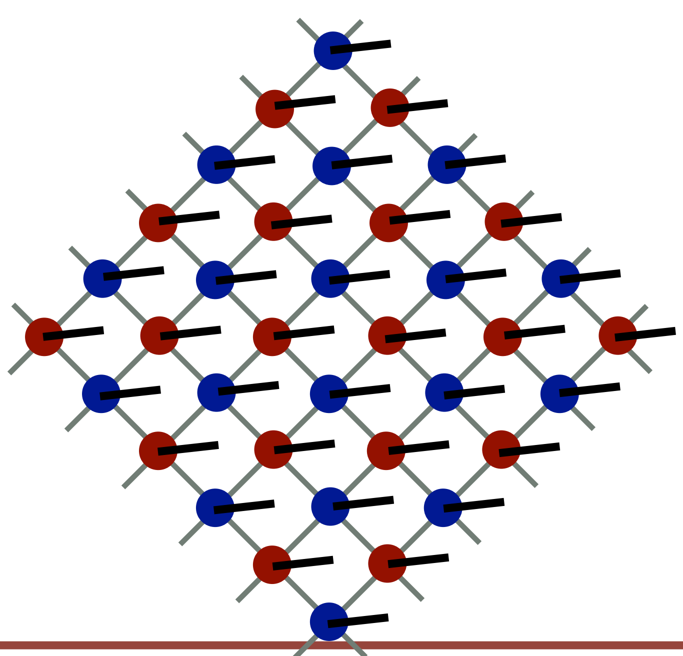
It can be represented by D=2 TPS.

(F. Verstraete, et al, Phys. Rev. Lett. **96**, 220601 (2006).)

0,1: eigen state of σ_x

(Non-zero elements of tensor)





Difference between MPS and TPS

Cost of tensor network contraction:

d-dimensional cubic lattice $N = L^d$

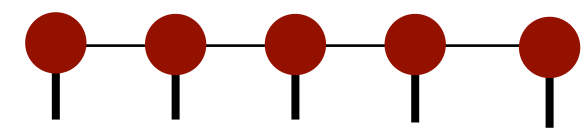
MPS: $O(N)$

TPS: $O(e^{L^{d-1}})$

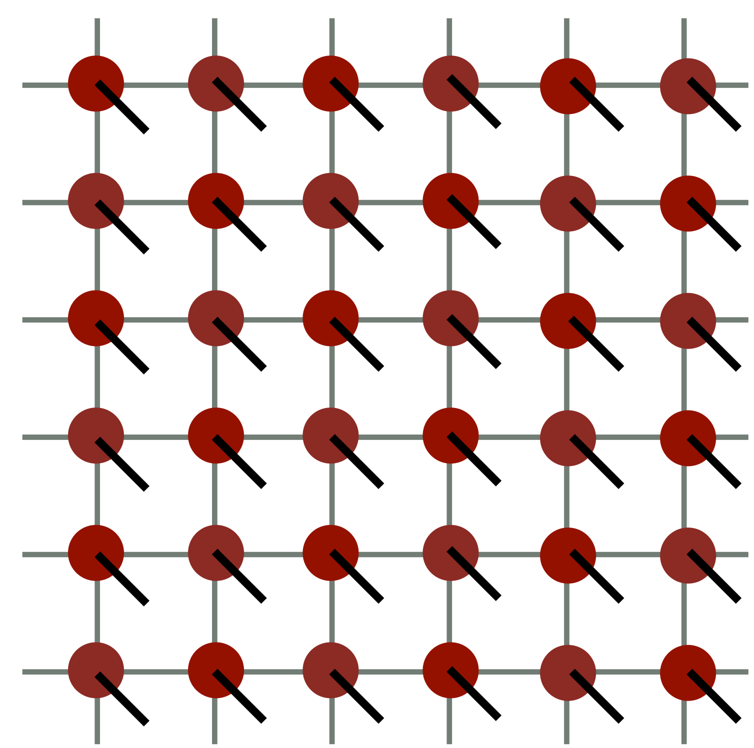
It is impossible to perform exact contraction even if we know local tensors in the case of TPS.

In the case of TPS, usually we approximately calculate the contraction.

MPS

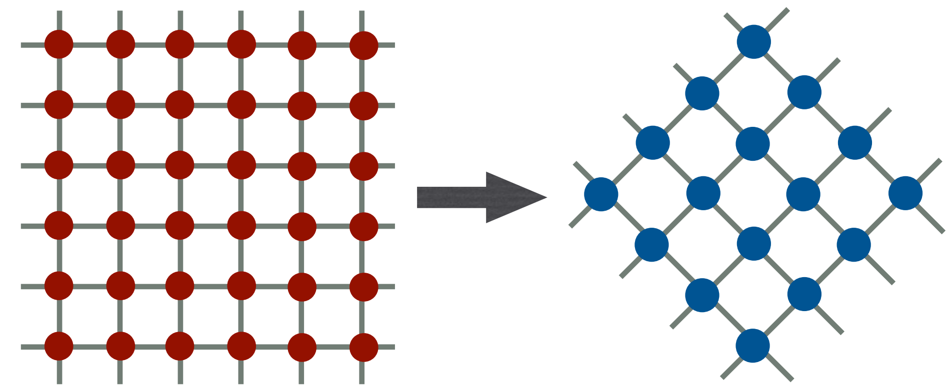


TPS (PEPS)

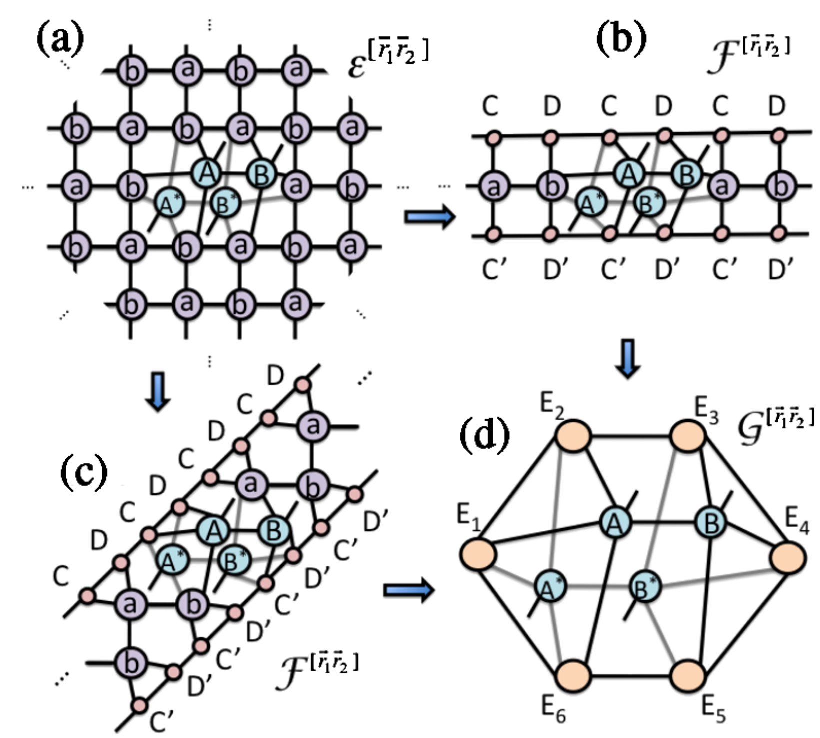


Contraction of infinite TPS (iTPS)

Methods for approximate contraction of iTPS:



- Tensor network renormalizations
  - TRG, HOTRG, SRG, TNR, loop-TNR, ...
- Boundary MPS
  - (Y. Hieida *et al* (1999) , J. Jordan *et al*, Phys. Rev. Lett. **101**, 250602 (2008))
- Corner transfer matrix
  - T. Nishino and K. Okunishi, JPSJ **65**, 891 (1996), R. Orús *et al*, Phys. Rev. B **80**, 094403 (2009).

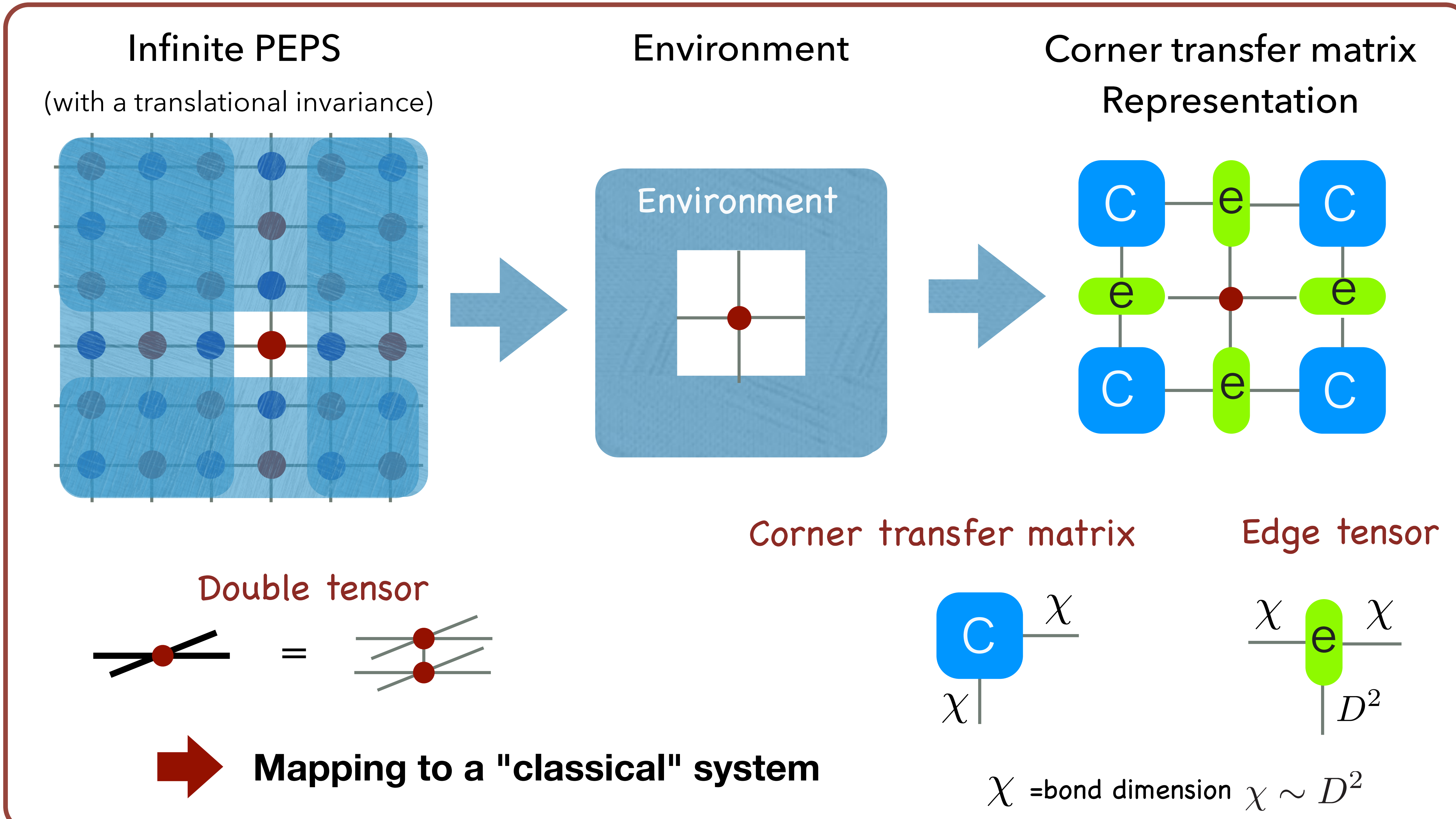


- Single layer approaches
  - bMPS: H. J. Liao *et al,* PRL **118**, 137202 (2017), Z. Y. Xie *et al*,PRB **96**, 045128 (2017).
  - CTM: Chih-Yuan Lee *et al*, PRB **98**, 224414 (2018) .
- Mean-field environment
  - S. Jharomi and R. Orús, PRB **99**, 195105 (2019).

Example of approximate contraction: CTM method

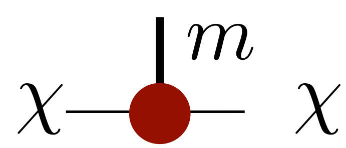
For (infinite) open boundary system

(T. Nishino and K. Okunishi, JPSJ **65**, 891 (1996))
(R. Orus *et al*, Phys. Rev. B **80**, 094403 (2009))

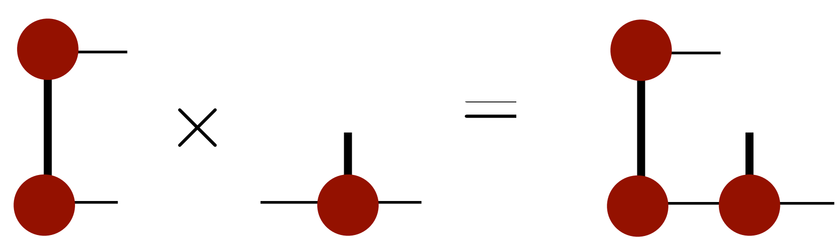


Cost of (approximate) contraction

MPS:

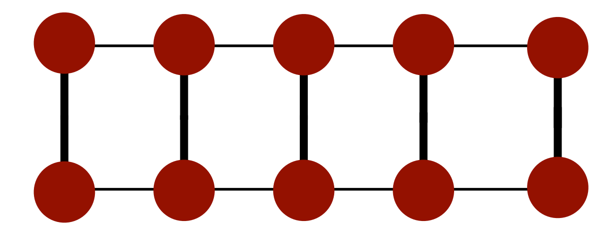


$(m \ll \chi)$

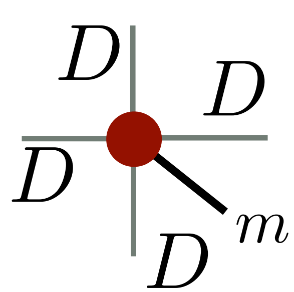


$\boxed{O(\chi^3)}$

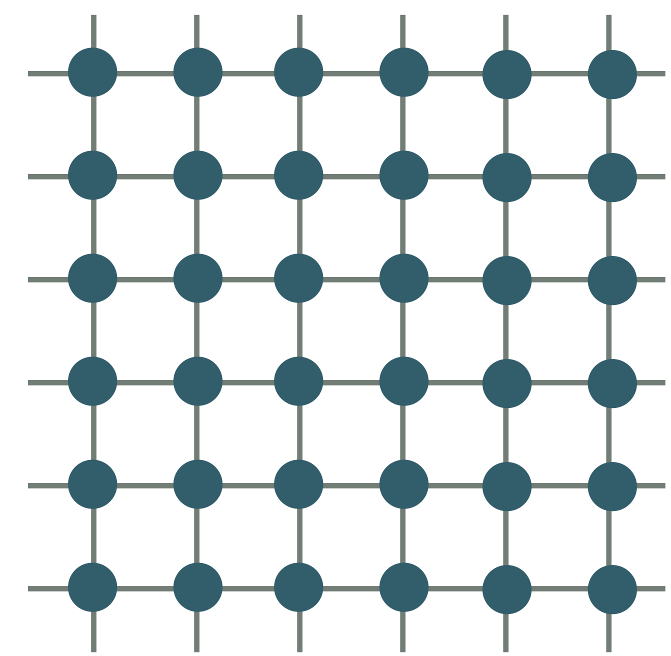
$\langle \Psi | \Psi \rangle =$



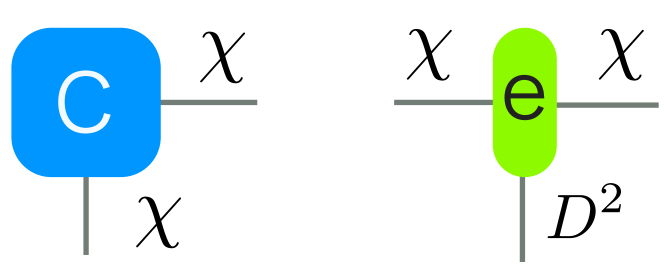
TPS:



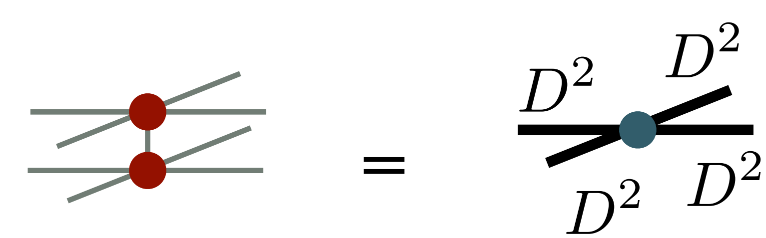
$\langle \Psi | \Psi \rangle =$



When we use CTM environment in 2D,



$$\boxed{O(\chi^2 D^6), O(\chi^3 D^4) \sim O(D^{10}) \quad (\chi \sim D^2)}$$

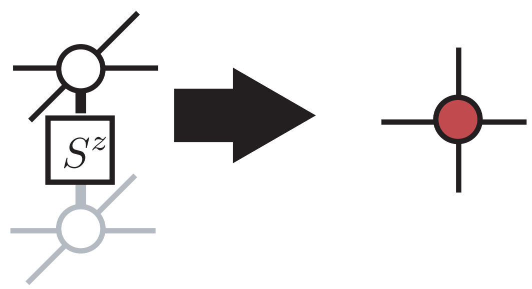


We can treat very small bond dimensions in TPS!

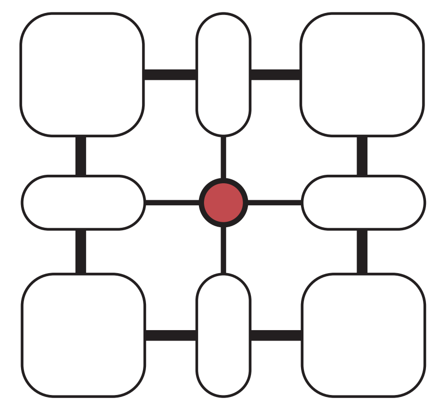
Physical quantities by CTMs

We can calculate physical quantities by CTM environments

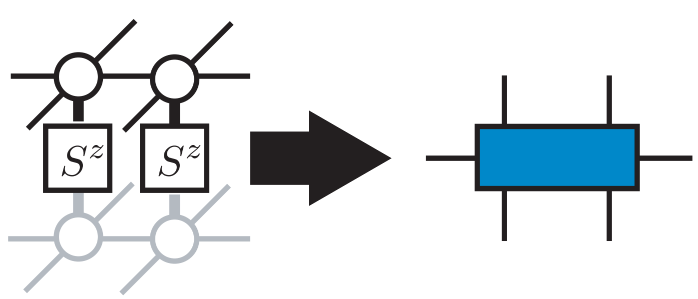
1-site quantity:



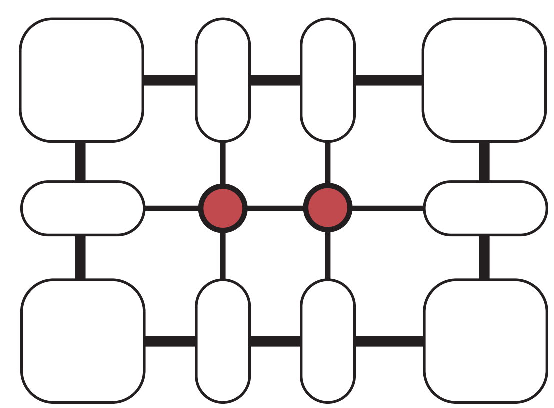
, $\langle \Psi | S^z | \Psi \rangle =$



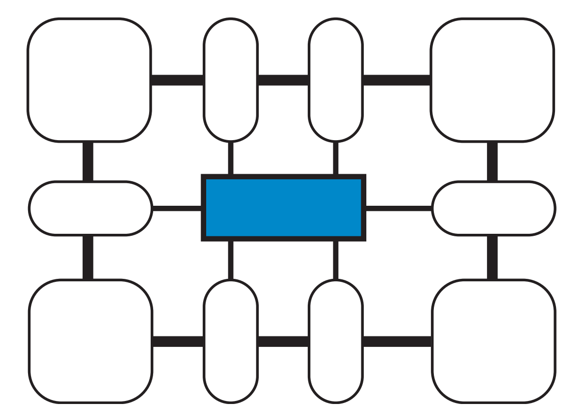
2-site quantity:



, $\langle \Psi | S_i^z S_{i+1}^z | \Psi \rangle =$



$=$



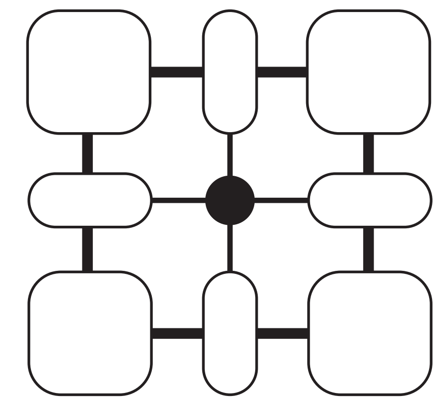
＊The computation cost increases when the size of the diagram is enlarged.

- Long-range correlation function along the diagonal direction.
- Many-body correlations on a larger cluster.

Correlation length with CTM

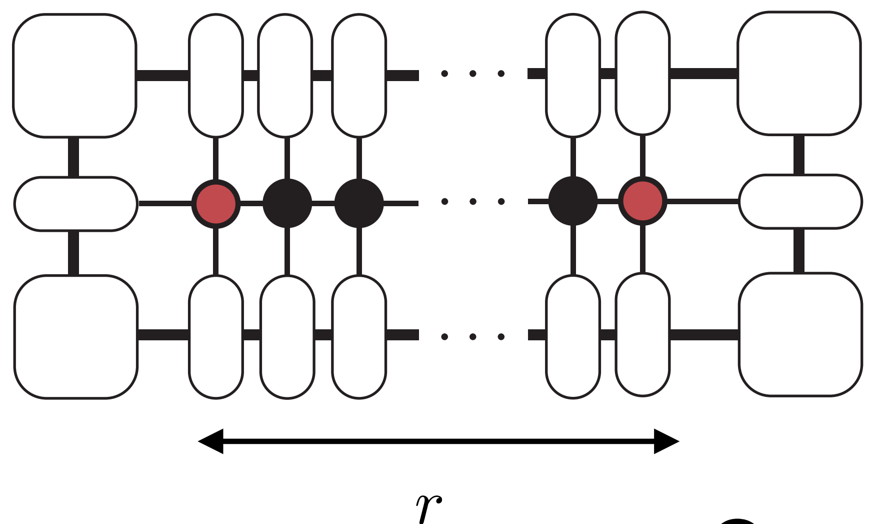
We can also calculate correlation lengths.

CTM environment

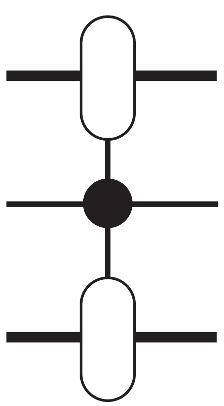


Correlation function

$$\langle \Psi | S_i^z S_{i+r}^z | \Psi \rangle =$$



Transfer matrix



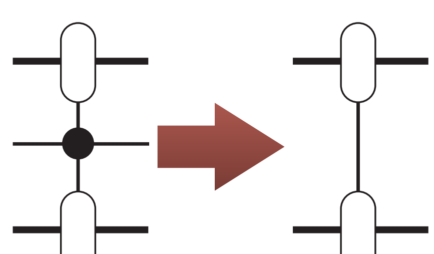
Eigenvalues: λ_i

$$|\lambda_0| \geq |\lambda_1| \geq |\lambda_2| \geq \cdots$$

Correlation length

$$\frac{1}{\xi} = -\ln \frac{|\lambda_1|}{|\lambda_0|}$$

* CMT represents the infinite environment. So we can neglect the center tensor.



Application of TPS to eigenvalue problem

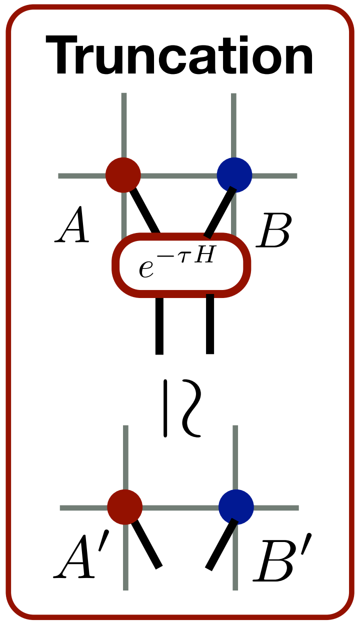
For the calculation of minimum eigenvalues and their eigenvector, we can use similar techniques to those in MPS.

1. Imaginary time evolution

(H. G. Jiang *et al*, Phys. Rev. Lett. **101**, 090603 (2008))
(J. Jordan *et al*, Phys. Rev. Lett. **101**, 250602 (2008))

$$\lim_{M\to\infty}\left(e^{-\tau\mathcal{H}}\right)^M|\psi\rangle = \text{ground state}$$

Suzuki-Torotter decomposition: $e^{-\tau H} \simeq e^{-\tau H_x}e^{-\tau H_y}e^{-\tau H'_x}e^{-\tau H'_y} + O(\tau^2)$



* By operating the time evolution operator, the bond dimension increases from the original *D*.

We need a "truncation." cf. iTEBD for iMPS

- Full update : consider global environment →Accurate but higher cost ($O(D^8)$~$O(D^{10})$)
- Simple update: consider only local environment →lower cost ($O(D^5)$)

2. Variational optimization

Cost function: $F = \dfrac{\vec{\psi}^\dagger(\mathcal{H}\vec{\psi})}{\vec{\psi}^\dagger\vec{\psi}}$

P. Corboz, Phys. Rev. B **94**, 035133 (2016).
L. Vanderstraeten *et a*l , Phys. Rev. B **94**, 155123 (2016).
H.-J. Liao *et a*l, Phys. Rev. X **9**, 031041 (2019).

cf. DMRG for MPS

Example of application: Honeycomb lattice Kitaev Model

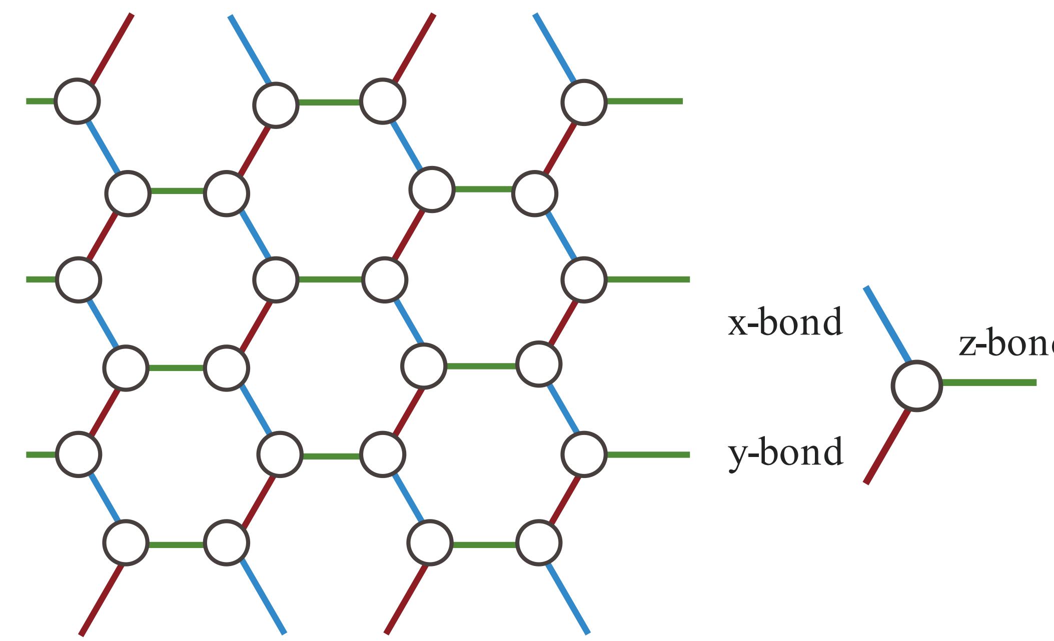
A. Kitaev, Annals of Physics 321, 2 (2006)

Kitaev model

$$\mathcal{H} = -\sum_{\gamma,\langle i,j\rangle_\gamma} J_\gamma S_i^\gamma S_j^\gamma$$

γ:bond direction

Depending on the bond direction, only specific spin components interact.



Exactly solvable by introducing Majorana fermion

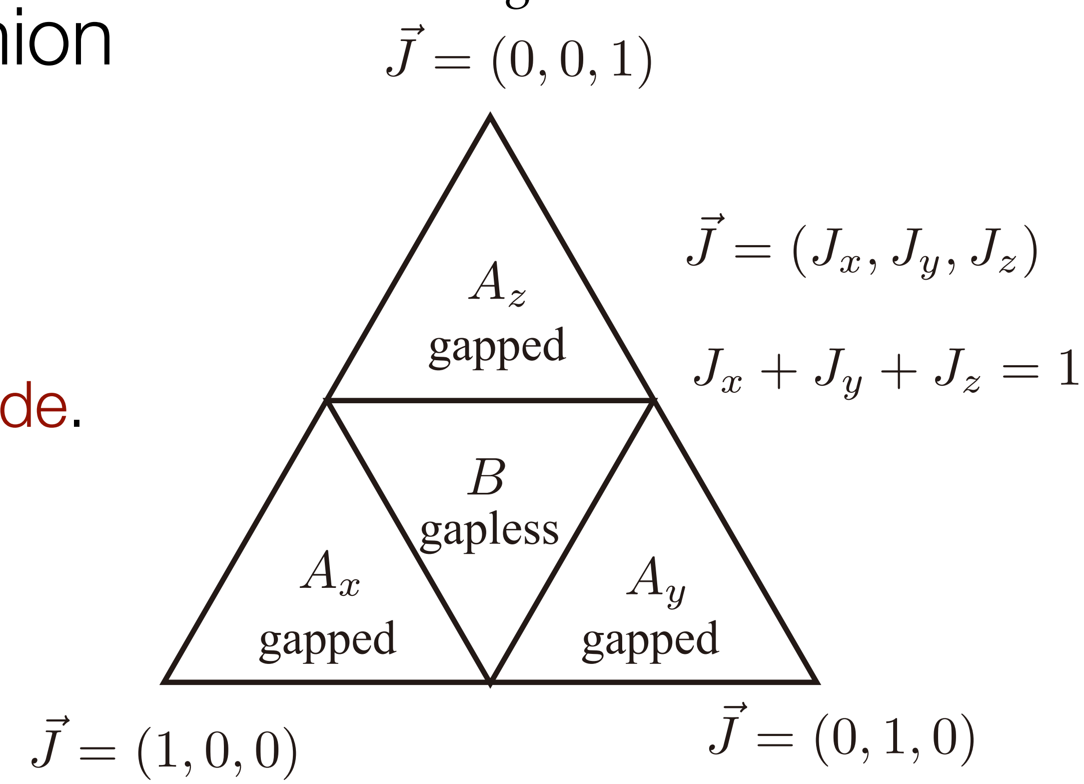
Isotropic region (B) : gapless spin liquid

Anisotropic region (A) : gapped spin liquid

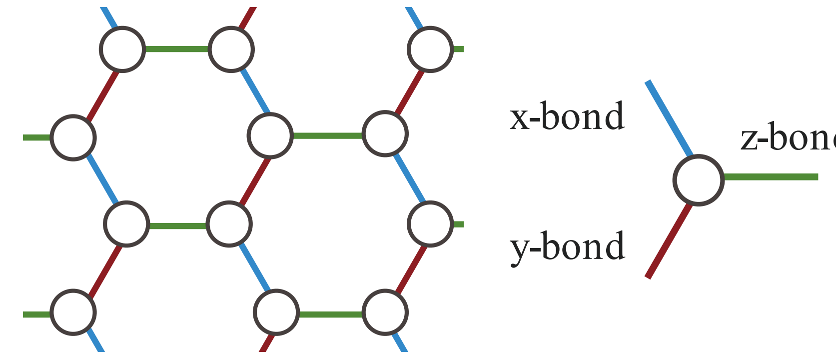
Cf. The anisotropic limit corresponds to the Toric code.

*Recently, researchers have realized that this type of models might appear in real materials. Hot topic!

Phase diagram



Application : Kitaev spin liquid



Honeycomb lattice Kitaev model

$$\mathcal{H} = -\sum_{\gamma,\langle i,j\rangle_\gamma} J_\gamma S_i^\gamma S_j^\gamma$$
$$(\gamma = x, y, z)$$

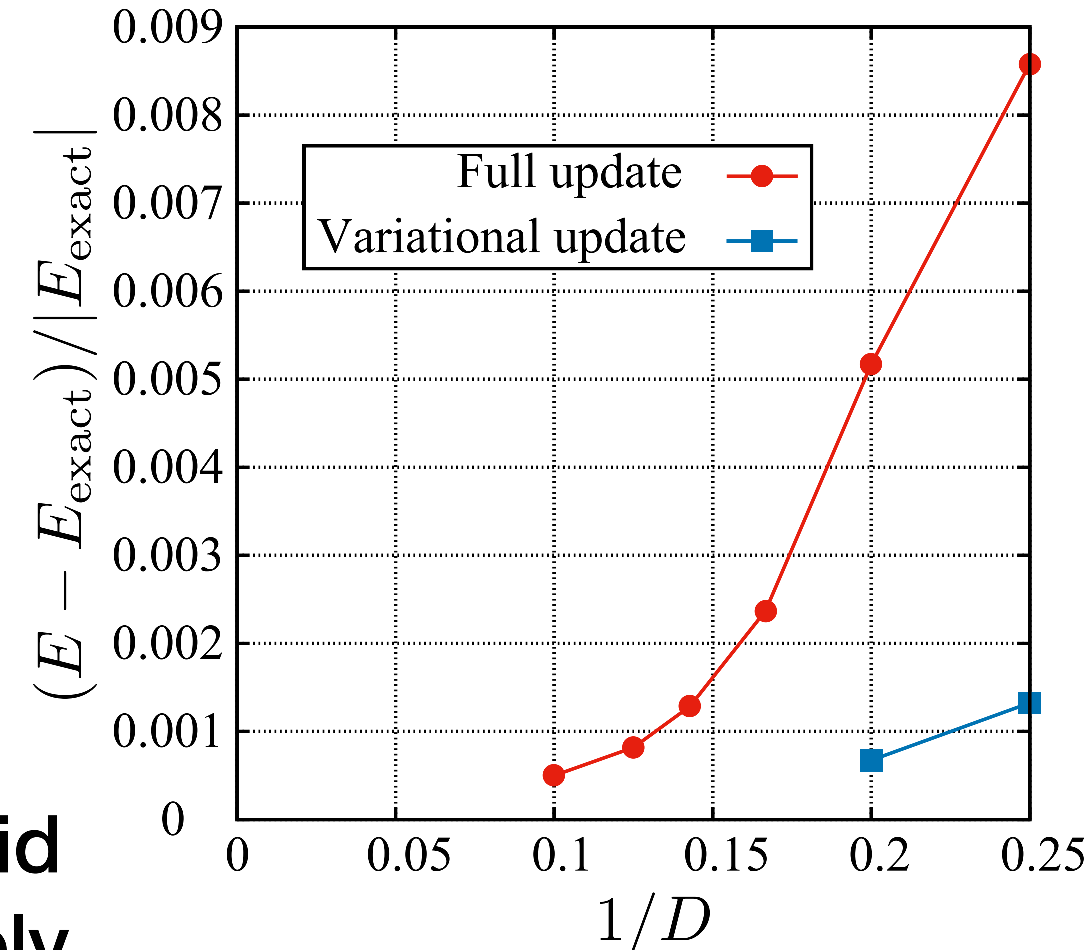
At $J_x = J_y = J_z$, the ground state is **a gapless spin liquid.**

In the present (super)computers,
we can access around D=10 (maybe 16)
by using massively parallel code.

The error of the ground state energy density is **less than 10^{-3}** for the infinite system!

Energy error obtained by iTPS

(T. okubo et al, unpublished)



iTPS can represent Kitaev spin liquid in the thermodynamic limit accurately.

Interesting topics related to TPS

- Application to itinerant electron system, which may break the area law

(P. Corboz et al, Phys. Rev. B. **81**, 165104 (2010))

(P. Corboz, Phys. Rev. B. **93**, 045116 (2016))

- Characterization of topologies in wave function
    - Symmetric tensor network and modular matrix

(J.-W. Mei et al, Phys. Rev. B. **95**, 235107 (2017))

- Application to three dimensions
    - So far, there is no practical calculations for non-trivial models.
    - Mainly, due to the scaling: $O(D^{18})$?
- Calculation of thermal states
    - Tensor network decomposition of the density matrix.

(A. Kshetrimayum et al, PRL **122**, 070502 (2019))
(Czarnik et al, PRB **99**, 035115 (2019))

Next (Dec. 8)

Notice

- No classes on Nov. 3, Nov. 17, and Nov. 22
- Classes will also be held on Jan. 5 and Jan. 19

1. Computational science, quantum computing, and data compression
2. Review of linear algebra
3. Singular value decomposition
4. Application of SVD and generalization to tensors
5. Entanglement of information and matrix product states
6. Application of MPS to eigenvalue problems
7. Tensor network representation
8. Data compression in tensor network
9. Tensor network renormalization
10. Quantum mechanics and quantum computation
11. Simulation of quantum computers
12. Quantum-classical hybrid algorithms and tensor network
13. Quantum error correction and tensor network