THE MILLIKAN OIL-DROP EXPERIMENT

(2 weights)

INTRODUCTION

This experiment is one of the most fundamental of the experiments in the undergraduate laboratory. The experimental apparatus is patterned after the original apparatus, made and used by R.A. Millikan to show that electric charge exists as integral multiples of *"e"* the charge on a single electron. Historically, this experiment ranks as one of the greatest experiments of modern physics.

GETTING STARTED

This experiment first described in 1913, is based on the fact that different forces act on an electrically charged oil drop moving in the homogeneous electric field of a plate capacitor. When the plate capacitor's electric field intensity is E, the following forces act on a droplet of charge Q:

- gravitational force $m_{oil}\mathbf{g}$, where m_{oil} is the mass of the oil droplet,

- buoyant force $m_{air}\,\mathbf{g}$, where m_{air} is the mass of air displaced by the oil drop,

- electric force $Q\mathbf{E}$,

- and, only if the droplet, considered in this case a sphere, moves against the ambient air: Stokes' resistance force. Stokes' Law states that for a spherical object of radius r moving through a fluid of viscosity η at a speed v under the laminar flow conditions, the viscous force $\mathbf{F}$ on the object is given by $F \approx 6\pi r \eta v$. The viscous force always opposes the motion and is, of course, responsible for the steady terminal velocities observed when a drop falls in air.

The Leybold-Heraeus apparatus (see the diagram in Appendix III) allows you to determine the elementary electronic charge by two different methods:

Method I. By measuring the voltage at which a charged oil droplet is floating in the Millikan chamber when the voltage is applied, and by measuring the velocity of the droplet falling in the free-field space upon switching off the voltage. The charge of the droplet is determined by the expression

$$Q = 2 \cdot 10^{-10} \frac{v_1^{3/2}}{U} \quad \text{(C)}, \qquad (1)$$

where v_1 is the fall velocity at zero voltage and U is the floating voltage.

Method II. By measuring the fall velocity of a droplet in the field-free space and the rise velocity of a droplet at a definite voltage. The charge of the droplet is determined by the expression

$$Q = (v_1 + v_2) \cdot \frac{\sqrt{v_1}}{U} \cdot 2 \cdot 10^{-10} \quad \text{(C)} \qquad (2)$$

where v_1 is the fall velocity in the free-field space and v_2 is the rise velocity in the electric field (when a non-zero voltage is applied to the Millikan chamber).

These relationships used in the two methods can be derived from the analysis of forces that acts on a charged oil droplet (see above). The formulae for both methods and the details of their derivations for each method can be found in the Appendices.

Familiarize yourself with the apparatus before taking actual measurements. Make sure that the spray nozzle of the oil atomizer is positioned before the small holes in the acrylic glass cover of the Millikan chamber. Try both methods and decide which one you prefer to follow.

In case you decide to proceed with the first method, set the voltage so that the chosen droplet is held stationary. Try to suspend the droplet in the lower part of the observation field. Read off the voltage. Simultaneously switch off the voltage and start the stop-clock to measure the terminal velocity of the droplet falling in the field–free space. We suggest that you select drops with similar charge.

In case you choose the alternative method**,** measure the terminal velocity v_1 at a zero voltage and the rise velocity v_2 at a definite voltage for each droplet. Perhaps the easiest method to measure the velocity of drops using only one stopwatch is to monitor the time required for the drop to cover the distance between certain chosen divisions of the objective scale.

Once you complete taking data, calculate the charge values for each experiment according to the method you have chosen. Represent the results in the form of a histogram (number of measurements within a range of 10^{-20} C versus $Q/(10^{-20}$ C)) and extract the value of the electron charge. The elementary electron charge e is obtained by forming the largest common divisor from the different charge values.

HINTS AND SUGGESTIONS

We suggest that you do this experiment in pairs.

You do not need two stopwatches to do this experiment (disregard Section 4.3. in the Instruction Manuals for the Leybold-Heraeus apparatus).

The main objective of the experiment is to demonstrate the quantization of charge. Since some of the values of the physical constants above are approximate, you should not worry too much if the value of *e* you obtain is outside the error range you predict from your measurement errors. Is the buoyancy of airsignificant in this experiment?

You will need to take measurements on about 50 drops to demonstrate the quantization of electric charge and extract the elementary charge value; plot the results on a histogram (i.e.number of measurements versus the charge value).

We suggest that you select drops with more or less the same radius (same terminal velocity). If in doubt, ask your demonstrator.

Before starting a longer series of measurements, decide whether it is better to choose droplets that have small or large charge. How would you decide which these are?

Avoid using drops of very small radii. Why? Estimate the radius of typical droplets in your experiment. Avoid using drops that move very fast. Why?

EXPERIMENTAL MAGNIFICATION OF THE MICROSCOPE OBJECTIVE

The telescope eyepiece should be adjusted to bring the scale into a sharp focus.

The smallest division on the scale covers about 1/20 mm in real space. When an oil drop moves along a distance x of micrometer scale division (=x 10^{-4} m), the actual distance traveled *s,* taking into account the objective magnification m:

is $s = \frac{x}{M} 10^{-4}$ m.

Our instrument has a magnification of approximately M=2.2. Using the transparent ruler with mm divisions you can determine the microscope magnification yourself. Remove the Millikan chamber with its acrylic glass cover from the top of the stand rod. Place the ruler with mm graduation vertically against the now visible centering rod. Adjust the microscope by means of knurled screw, so that the mm graduation of the transparent ruler is sharply visible. The magnification can be calculated by comparing the micrometer scale in the eyepiece (0.1 mm between the divisions) with the mm graduation of the transparent ruler. Replace the Millikan chamber with the acrylic glass cover again in position.

QUESTIONS TO CONSIDER

- Why is it difficult to prove the quantization for larger values of charge?
- Which one of the described two methods introduces lesser error?
- Justify your choice of the method.
- Is the motion of the smallest of the oil drops at all unusual? Do you have any explanation?
- This apparatus is very sensitive even to the air motion in the room. How to account for this effect?

APPENDIX

I. The following values are substituted for η, ρ, d and g in your experiment:

the density of the oil ρ_o = 875.3 kg/m^3
the density of air ρ_a =1.29 kg/m^3
the acceleration due to gravity g=9.80 m/s^2
the viscosity of air at room temperature and 1 atm η= 1.81 × 10^{-5} N·s/m^2
the separation of the parallel plates d=6.0 mm.

II. Motion of charged oil drops in air

(a) For a droplet falling through a field-free space with the terminal velocity v_1 the following rule of forces applies (Figure 1):

$$V(\rho_o - \rho_a)g - 6\pi r v_1 \eta = 0$$

or

$$(4/3)\pi r^3 (\rho_o - \rho_a) g - 6\pi r v_1 \eta = 0 \qquad \text{(A1)}$$

resulting in
$$r = \sqrt{\frac{9 v_1 \eta}{2 g \rho}} \qquad \text{(A2)}$$

(where $\rho = \rho_o - \rho_a$.).

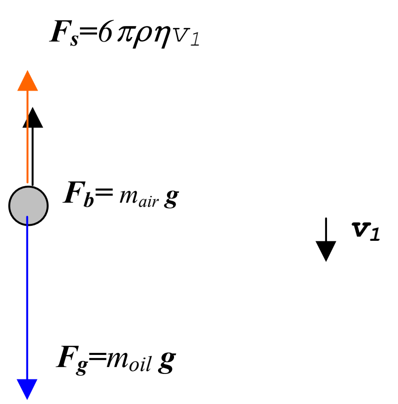

Figure 1

The forces on a droplet falling through a field-free space with the terminal velocity $\boldsymbol{v_1}$

(b) With U= voltage between the plates of the Millikan chamber, d=plate spacing, for a droplet floating in the chamber under the influence of an electric field $E=U/d$ the following relation applies:

$$V(\rho_o-\rho_a)g-QE=0$$

or

$$(4/3)\pi r^3(\rho_o-\rho_a)g-QU/d=0 \qquad \text{(A3)}$$

(c) For a droplet moving upward with the terminal velocity v_2 under the influence of an electric field E (Figure 2):

$$V(\rho_o-\rho_a)g-QE+6\pi r v_2\eta=0$$

or

$$(4/3)\pi r^3(\rho_o-\rho_a)g-QU/d+6\pi r v_2\eta=0 \qquad \text{(A4)}$$

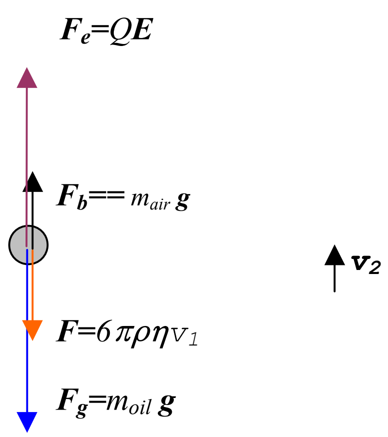

Figure 2

The forces on a droplet moving upward with the terminal velocity $\boldsymbol{v_2}$
under the influence of an electric field $\boldsymbol{E}$

Method I:
Write down the equation for an oil drop falling in air under the gravity (A1) and that for a charged oil-drop whose weight is exactly balanced by an applied electric field (A3); eliminate the radius of the drop from these two equations to obtain the expression for the charge on the drop in terms of measurable quantities and the constants given below.

You should obtain:

$$Q = \eta \frac{6\pi d v_1}{U} \sqrt{\frac{9\eta v_1}{2\rho g}} \qquad \text{(A5)}$$

When substituting the values *for* η, ρ, d and g indicated already, one obtains the following final equation (1) which enables relatively quick calculation of the charge values:

$$Q = 2 \cdot 10^{-10} \frac{v_1^{3/2}}{U} \qquad \text{(C)}$$

The alternate method (Method 2):
Combining the equations (A1) and (A4) and solving for Q gives:

$$Q = (v_1 + v_2) \frac{\sqrt{v_1}}{U} \eta^{3/2} \frac{18\pi d}{\sqrt{2g\rho}} \qquad \text{(A6)}$$

which results in the following final equation (2):

$$Q = (v_1 + v_2) \cdot \frac{\sqrt{v_1}}{U} \cdot 2 \cdot 10^{-10} \qquad \text{(C)}$$

==> Python Programming(PHY224/324 only). Do the analysis by using the plt.hist function (building a histogram). Output the error bars over the histogram bins.

III. Leybold-Heraeus apparatus

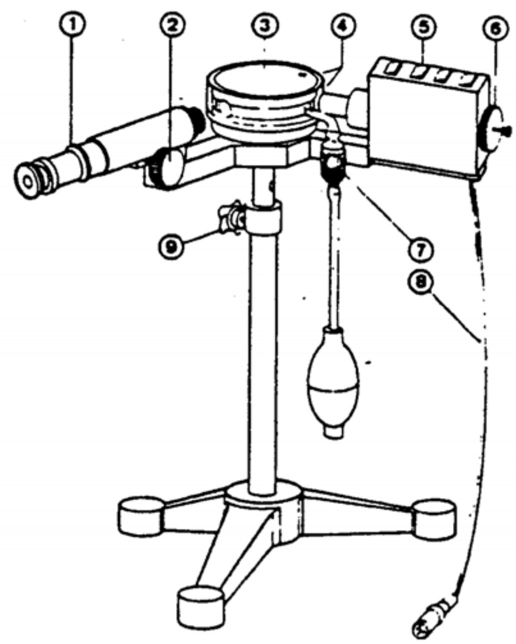

Fig. 1

(1) Measuring microscope with micrometer eyepiece
(2) Knurled knob for microscope adjustment
(3) Millikan chamber (plate capacitor) with acrylic glass cover
(4) Socket pair to connect the d. c. voltage for the plate capacitor (can be tapped from socket pair (11), adjustable via knob (14)
(5) Illumination device
(6) Knurled knob for lamp adjustment
(7) Oil atomizer with rubber ball in resilient holder (one bottle with oil included in scope of delivery)
(8) Connecting cable for lamp voltage (from multiple socket (15))
(9) Screw for height adjustment (to adapt the microscope to the eye level of the experimenter)

REFERENCES

1.R.A. Millikan, The Electron (photocopied excerpts available at the Resource Centre).

2.Instruction Manuals for Leybold-Heraeus apparatus (available at the Resource Centre).

(gmg-1990,jbv-1994,tk-1996.1998, ta-2000, 2001)