

# The Velocity of Ultrasonic Waves in Water by the Debye-Sears Effect

**Goal:** This experiment features the use of an ultrasonic wave as an optical diffraction grating. You'll study the diffraction of light by density fluctuations, measure the speed of sound in water and determine the bulk modulus of water.

## Theoretical background

### The Debye-Sears effect

Sound waves in liquids determine density changes, caused by the pressure nodes and antinodes. The spacing of these periodic fluctuations in density is determined by the frequency  $f_s$  and the speed of the sound wave  $v_s$ . In a transparent medium, variations in density correspond to variations in the index of refraction and therefore a monochromatic parallel light beam traveling perpendicular to the sound direction is refracted as if it had passed through a diffraction grating of spacing  $d = \lambda_s$ :

$$m\lambda_L = d \sin \theta \quad (1)$$

Where:  $m$  is the order of diffraction,  $\theta$  the angle of diffraction and  $\lambda_L$  the wavelength of the light source.

The sound field is set up by driving a piezo-electric crystal at frequency  $f_s$  of  $\sim 1$  MHz. The main goal of the experiment is deduce the velocity of sound in water:

$v_s = f_s \lambda_s$  by measuring  $d$  from Eq. (1).

### The bulk modulus of water

The speed of sound depends on both an inertial property of the medium (to store kinetic energy) and an elastic property (to store potential energy):

$$v_s = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \quad (2)$$

For a medium such as water, the bulk modulus  $B$  accounts for the extent to which an element from the medium changes in volume when a pressure is applied:

$$B = - \frac{\Delta p}{\frac{\Delta V}{V}} \quad (3)$$

Here  $\Delta V/V$  is the fractional change in volume produced by a change in pressure  $\Delta p$ . The signs of  $\Delta p$  and  $\Delta V$  are always opposite. The unit for  $B$  is the pascal (Pa).

Therefore, the speed of sound in water can be expressed as:

$$v_s = \sqrt{\frac{B}{\rho}} \quad (4)$$

where  $\rho$  is water density.

## Experiment

1) In order to gain some familiarity with diffraction gratings, set up the spectroscope with only the diffraction grating provided. Let the sodium lamp to warm up for at least 10 minutes. Remove the cell from the goniometric table and replace it by the grating. Observe the many orders of diffraction. Using the vernier scale on the angle display window, measure the angles corresponding to  $m = 0$ ,  $m = \pm 1$  and  $m = \pm 2$ . Use the data to find the wavelength of the sodium lamp  $\lambda_L$ .

2) Fill the cell with water and set it on the goniometric table of the spectrometer. Adjust the slit and cell to obtain a sharp image of the slit and crosshairs in the telescope, using the Na light source.

Connect the ultrasound transducer to the oscillator through the “Generator” box and make the connections to the oscilloscope and frequency counter. DO NOT connect the scope directly to the oscillator as its output voltage is of order 300 volts. Switch on the ultrasonic beam and adjust the frequency to  $\sim 2$  MHz

The challenge of this experiment is to measure  $d$  which is  $\lambda_s$  from Eq. (1)



*Note 1: You must have the cell normal to the incident light.*

### 3) → Python requirement<sup>1</sup>

Measure the diffraction due to the ultrasound for several frequencies. Note that the vernier scale on the goniometric table is in degrees and arc minutes (60 minutes to a degree). Estimate the experimental errors.

Do a linear fit and output the velocity of sound and the bulk modulus of water.

Examine the goodness of fit. Do your results agree with a linear dependency of  $d$  vs.  $1/f_s$ ? If not, you may have evidence for dispersion of sound waves in water. Is this the case? If they do agree, how strong is this as evidence for no dispersion?

➔ Compare the results with data from the literature.

*Note 2:* Velocity of sound in liquids is temperature dependent.

*Note 3:* From this experiment we are determining *the bulk modulus for adiabatic compression* because there is no energy exchanged with the region next to the sound wave. This should be distinguished from the isothermal bulk modulus.

### **Just for Fun: the ultrasonic fountain**

When an ultrasonic wave strikes a liquid-air interface and reflects back to the liquid, momentum is transferred to the liquid, which is levitated and looks like a fountain. At higher ultrasonic powers *cavitation* is also produced, which can cause the appearance of a “boiling liquid”.

Use the independent transducer; arrange it on a flat horizontal surface and place a couple of water drops on its surface. Connect it to the oscillator and tune the frequency until the water forms a pointy meniscus. Any froth would be due to cavitation.

This application is used in mist generators.

*Thanks to Robert Smidrovskis for developing this part of the experiment.*

### **References:**

Most optics texts should have the Debye-Sears effect; all optics texts will discuss diffraction:

Born, M. and Wolf, E., “Principles of Optics”, Cambridge University Press 7th (expanded) edition 1999. Chapter XII, p. 674-694.

Most introductory physics textbooks should have a discussion on the speed of sound in liquids:

Halliday/Resnick/Walker: Fundamentals of Physics, Wiley, 7<sup>th</sup> edition 2005, Ch. 17.

You may also read about the dielectric dispersion. The following reprint is available upon request from the Resource Centre:

K. E. Oughstun, N. A. Cartwright “On the Lorentz-Lorenz formula and the Lorentz model of dielectric dispersion” OSA Optics Express 11(13), p1541-1546 (2003).

*This guide sheet was written by Ruxandra M. Serbanescu in 2006 (updated in 2009). Previous versions: Bryan Statt 2002*

**<sup>1</sup>Python requirement is only for PHY224/324 students**