Lab 4

12.08.2022

Verification of Uncertainty Principle

(Δx) & (Δp)

$U_n(x)$

$$\int_{x_{min}}^{x_{max}} |U_n(x)|^2 dx = I$$

Redefine $\frac{U_n(x)}{\sqrt{I}} \leftarrow U_n(x) \rightarrow$ Normalisation

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \langle (x - \langle x \rangle)^2 \rangle$$

$$(\Delta p)^2 = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2 = \langle (\hat{p} - \langle \hat{p} \rangle)^2 \rangle$$

where $\langle x^n \rangle = \int_0^L x^n |u(x)|^2 dx$ for infinite pot well

$= \int_0^L x^n \rho(x)\, dx$

$\rho(x) = |u(x)|^2$

$= (u(x))^2$

$\rightarrow$ Prob density

Also for a real wave fn

$\langle \hat{p} \rangle = \int_{-\infty}^{\infty} u^* \left(-i\hbar \frac{d}{dx}\right) u \, dx$ $\qquad u^* = u$

$= -i\hbar \int_0^L u u' \, dx = -\frac{i\hbar}{2} \int_0^L \frac{d}{dx}(u^2) \, dx$

$= -\frac{i\hbar}{2} [u^2]_0^L = 0$

Uncertainty Relation

$$(\Delta X)(\Delta P) \geq \frac{\hbar}{2}$$

In dimensionless units

$$X \rightarrow \frac{\xi}{L} ,$$

$$\hat{P} \equiv -i\hbar \frac{d}{dx} \rightarrow -i\frac{\hbar}{L}\frac{d}{d\xi}$$

Define $\zeta = \frac{L\hat{P}}{\hbar}$

$\frac{\hbar}{L}$ have dimensions of momentum

$$\therefore \hat{P} \rightarrow \zeta = -i\frac{d}{d\xi}$$

In program, use x & p in place of ξ & ζ respectively

Thus, in nth state

$$\langle x \rangle_n = \int_0^L x\, u_n^2(x)\, dx$$

$$\langle x^2 \rangle_n = \int_0^L x^2\, u_n^2(x)\, dx$$

$$\langle \hat{p} \rangle_n = \int u_n(x) \Big(\underbrace{-i \frac{d}{dx}}_{\text{dimensionless}} \Big) u_n(x)\, dx = 0$$

$$\langle \hat{p} \rangle_n^2 = \int_0^L u_n \left(- \frac{d^2}{dx^2} \right) u_n\, dx$$

Approximate u'' by

$$\left.\frac{d^2 u_n}{dx^2}\right|_{x=x_j} = \frac{u_n(x_{j+1}) - 2u_n(x_j) + u_n(x_{j-1})}{h^2}$$

$$h = \frac{x_N - x_0}{N} = \frac{1-0}{N}$$

Use Simpson$_{1/3}$ to compute these integrals for the discrete data of $u_n(x)$ for each state

Define $D_n = (\Delta X)_n \, (\Delta \hat{P})_n$

and print the table

n	D_n	D_n (analytical)
1		
2		
⋮		

D_1 will be minimum but $D_1 > \frac{1}{2}$

Compare with analytical values

Numerov Method

Special Problem -- Without the first order derivative term y' -- appear in various physical problems

Such problems can be solved by a better algorithm -- the Numerov Meethod

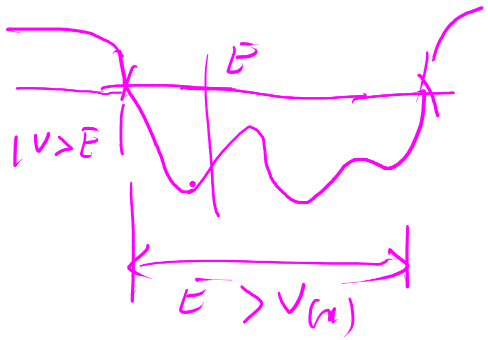
$$y'' + \underbrace{[k(x)]^2}_{k^2(x)} y = F(x) \qquad a \le x \le b$$

$$y(a) = \alpha , \quad y(b) = \beta$$

We can interpret F(x) as an inhomogenous driving force

k(x) is a real function.
If it is positive the solutions y(x) will be oscillatory functions, and
if negative they are exponentionally growing or decaying functions.

Schrodinger Equation is this kind of equation - no first order derivative



$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$F(x) = 0$$

$$\& \quad k^2(x) = \frac{2m}{\hbar^2}\left[E - V(x)\right]$$

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V\right)\psi = E\psi$$

Taylor's series

$$y(x+h) = y(x) + h\,y^{(1)}(x) + \frac{h^2}{2!}y^{(2)}(x) + \frac{h^3}{3!}y^{(3)}(x)$$

$$+ \frac{h^4}{4!}y^{(4)}(x) + \frac{h^5}{5!}y^{(5)}(x) + \cdots$$

$$y^{(n)}(x) \equiv \frac{d^n y}{dx^n}$$

$$y(x-h) = y(x) - h\,y^{(1)}(x) + \frac{h^2}{2}y^{(2)}(x)$$

$$- \frac{h^3}{3!}y^{(3)}(x) + \frac{h^4}{4!}y^{(4)}(x) - \frac{h^5}{5!}y^{(5)}(x) + \cdots$$

$$y(x+h) - y(x-h) = 2y(x) + h^2 y^{(2)}(x) + \frac{h^4}{12}y^{(4)}(x) + O(h^6)$$

$$y^{(2)}(x) = \frac{y(x+h) + y(x-h) - 2y(x)}{h^2} - \frac{h^2}{12} y^{(4)}(x) + O(h^4)$$

$$y^{(2)} + \frac{h^2}{12} y^{(4)}(x) = \left(1 + \frac{h^2}{12}\frac{d^2}{dx^2}\right) y^{(2)}(x)$$

To eliminate the fourth-derivative term we apply the operator $\left(1 + \frac{h^2}{12}\frac{d^2}{dx^2}\right)$

on the differential equation $y'' + k^2(x)\, y = 0$

$$y^{(2)}(x) + \frac{h^2}{12} y^{(4)}(x) + k^2(x)\, y(x) + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2 y\right] = 0$$

$$\Rightarrow y(x+h) + y(x-h) - 2y(x) + h^2 k^2 y(x) + \frac{h^4}{12}\frac{d^2}{dx^2}(k^2 y) \approx 0$$

approximate the second derivative of $k^2_{(x)}\, y(x)$

$$\frac{d^2}{dx^2}(k^2 y) = \frac{1}{h^2}\left[(k^2 y)\big|_{x+h} + (k^2 y)\big|_{x-h} - 2(k^2 y)\big|_{x} + O(h^2)\right]$$

$$\frac{d^2}{dx^2}(k^2 y) \simeq \frac{1}{h^2}\Big[\left\{k^2(x+h)\, y(x+h) - k^2(x)\, y(x)\right\}$$

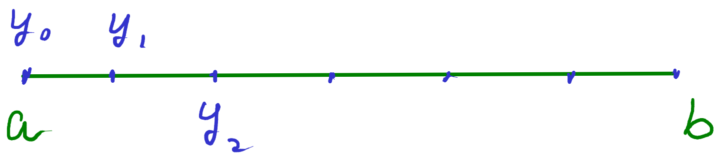
$$+ \left\{k^2(x-h)\, y(x-h) - k^2(x)\, y(x)\right\}\Big]$$

Writing $\quad x = x_i, \quad x+h = x_{i+1}, \quad x-h = x_{i-1}$

$y(x) = y_i, \quad y(x+h) = y_{i+1}, \quad y(x-h) = y_{i-1}$

$$\boxed{y_{i+1} = \frac{1}{\left[1 + \frac{h^2}{12} k^2_{i+1}\right]}\left[2\left(1 - \frac{5}{12} h^2 k_i^2\right) y_i - \left(1 + \frac{1}{12} h^2 k^2_{i-1}\right) y_{i-1}\right] + O(h^6)}$$

The Numerov method can be used to determine y_p for $p = 2, 3, 4, \cdots$, given two initial values, y_0 & y_1.
We need two initial values because we are solving a second order differential equation.



$$F(x) \neq 0$$

$$y_{i+1} = \frac{1}{\left[1+\frac{h^2}{12}k_{i+1}^2\right]}\left[2\left(1-\frac{5}{12}h^2k_i^2\right)y_i - \left(1+\frac{1}{12}h^2k_{i-1}^2\right)y_{i-1}\right]$$

$$+ \frac{h^2}{12}\left(F_{i+1} + F_{i-1} - 2F_i\right) + O(h^6)$$

The error in one x-step is $\mathcal{O}(h^6)$

However, the number of steps needed to integrate over a fixed range of x, from a to b is

$$\frac{b-a}{h} \propto \frac{1}{h}$$

One might expect that the errors at each step would be roughly comparable so so the total error in the Numerov method would be $\mathcal{O}(h^5)$

Thus it is a 5-th order method, one higher than RK4.

Local Truncation Error $\sim \mathcal{O}(h^6)$

Global Error $\sim \mathcal{O}(h^5)$

We shall see that the Global error actually turns out to be $\sim \mathcal{O}(h^4)$ same as RK

there can be problems with roundoff errors in using algorithm so make sure you use double precision arithmetic

Solution of 1-d Time independent Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\Rightarrow \quad \frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\psi(x) = 0$$

of the form $\quad y'' + k^2(x)\, y = 0 \quad$ with $\quad k^2(x) = \frac{2m}{\hbar^2}\left[E - V(x)\right]$

Thus we can solve by Numerov method

write $f(x) = k^2(x)$

Two additional Complexities

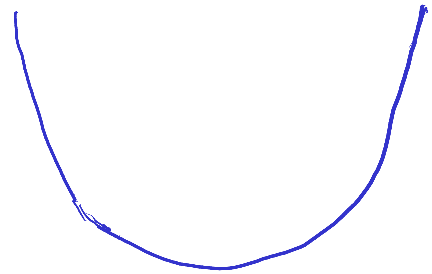
(i) Solution does not exist for all E. For Bound states, only certain discrete values of E give valid solution

Valid means -- satisying the conditions of continuity of ψ and ψ' and wave function should approach zero at $|x| \to \infty$

SO the numerical algorithm or code should be able to identify the valid values of E

(ii) In the classically forbidden regions the wavefunction should be exponentially decaying

Simple Harmonic Oscillator Using Numerov Method



Sch eq^n

$$u'' + k^2(x)\, u = 0$$

x is dimensionless

x_i

$$k^2(x) = \varepsilon - V(x) = f(x)$$

$$f_i = f(x_i) \qquad u_i = u(x_i)$$

$$x = x_0, \dots, x_N$$

x_0: x_{min}, x_N: x_{max}

$$u_{i+1} = \frac{1}{\left[1 + \frac{(\Delta x)^2}{12} f_{i+1}\right]} \left[2\left(1 - \frac{5}{12}(\Delta x)^2 f_i\right) u_i - \left(1 + \frac{1}{12}(\Delta x)^2 f_{i-1}\right) u_{i-1} \right] + O(\Delta x^6)$$

Define

$$c(x) = 1 + \frac{(\Delta x)^2}{12} f(x)$$

$$c_i = c(x_i)$$

$$c_i = 1 + \frac{(\Delta x)^2}{12} f_i$$

$$f_i = \frac{12 c_i - 12}{(\Delta x)^2}$$

$$-\frac{5}{12} = \frac{1-6}{12} = \frac{1}{12} - \frac{1}{2}$$

$$2\left(1 - \frac{5}{12}(\Delta x)^2 f_i\right) = 2\left(1 + \frac{(\Delta x)^2}{12} f_i\right) - (\Delta x)^2 f_i$$

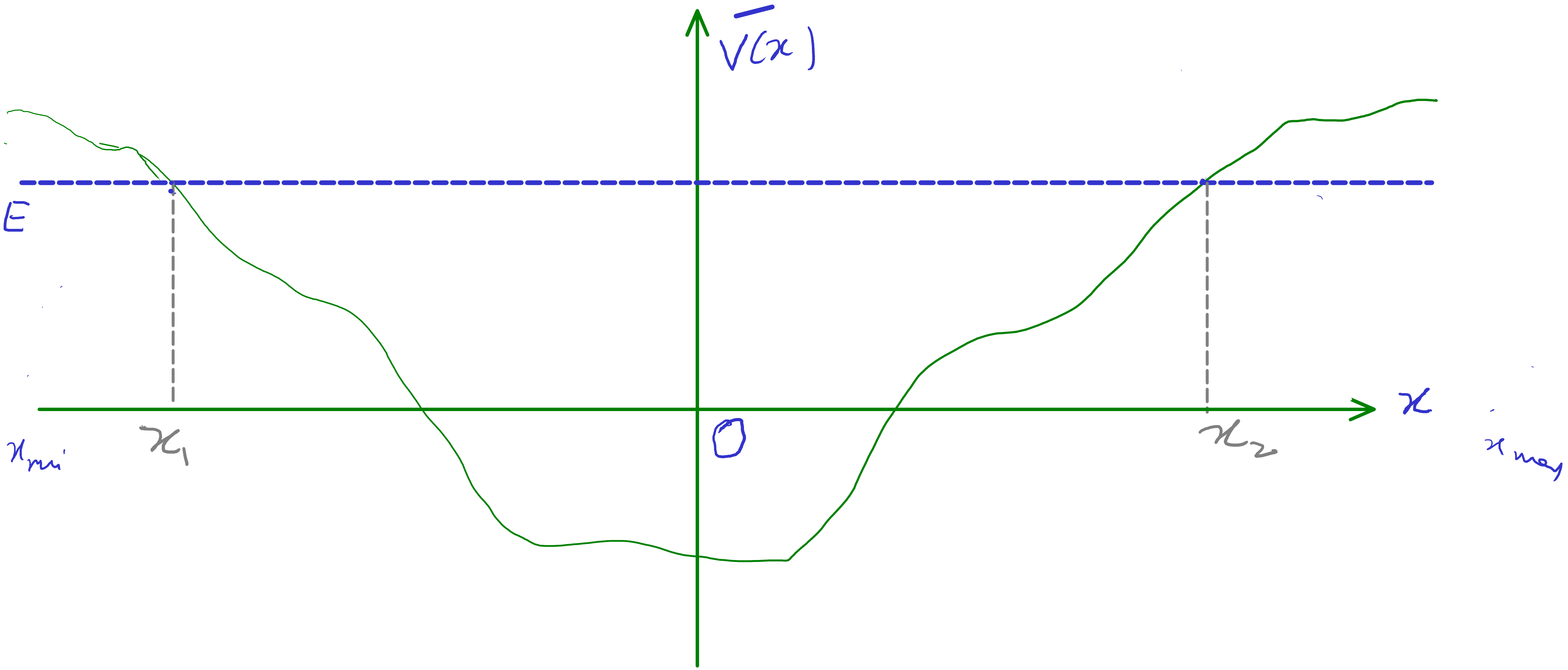
$$= 2\left(1 + \frac{(\Delta x)^2}{12} f_i\right) - (\Delta x)^2 \left(\frac{12 c_i - 12}{(\Delta x)^2}\right)$$

$$= 2c_i - 12 c_i + 12$$

$$= -10 c_i + 12$$

$$\boxed{u_{i+1} = \frac{1}{c_{i+1}} \left[(12 - 10 c_i) u_i - c_{i-1} u_{i-1} \right]}$$

$$c_{i+1} = 1 + \frac{\Delta x^2}{12} f_{i+1} \qquad c_{i-1} = 1 + \frac{1}{12}(\Delta x)^2 f_{i-1}$$



$\pm\infty$?

$[x_{min}, \quad x_{max}]$

classically forbidden

$x < x_1$ & $x > x_2$

$u(x) \longrightarrow 0$ as $x \rightarrow \pm\infty$

Lab 5

Harmonic Oscilaltor

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + \frac{1}{2}m\omega^2 x^2 u = Eu$$

classical turning pt

for gd state $\rightarrow$ $x_{cl} = \sqrt{\frac{\hbar}{m\omega}}$

Define

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\, x \qquad \& \qquad \xi_{cl} = \sqrt{\frac{m\omega}{\hbar}}\, x_{cl} = 1$$

$$\frac{d}{dx} = \sqrt{\frac{m\omega}{\hbar}}\,\frac{d}{d\xi} \quad , \quad \frac{d^2}{dx^2} = \frac{m\omega}{\hbar}\frac{d^2}{d\xi^2}$$

$$-\frac{\hbar^2}{2m}\cdot\frac{m\omega}{\hbar}\frac{d^2u}{d\xi^2} + \frac{1}{2}m\omega^2\left(\frac{\hbar}{m\omega}\right)\xi^2 = Eu$$

$$-\frac{\hbar\omega}{2}\frac{d^2u}{d\xi^2} + \frac{\hbar\omega}{2}\xi^2 = Eu$$

$$u''(\xi) - \xi^2 u = -\frac{2E}{\hbar\omega} u$$

$$\varepsilon = \frac{E}{\hbar\omega}$$

$$u''(\xi) + f(\xi)\, u(\xi) = 0$$

$$f(\xi) = (2\varepsilon - \xi^2) = 2(\varepsilon - v) \qquad v = \frac{1}{2}\xi^2$$

For prog call ξ as x only & ε as e

i.e. $u''(x) + f(x)\, u(x) = 0$; $f(x) = 2(e - v(x))$; $v(x) = \frac{1}{2}x^2$

Potential is even function of x

Wave functions are of definite parity - so we determine the wavefunction for only x>0 and use parity property to find wavefunction for x<0

Discrete non-degenerate energy eigenvalues

Wavefunctions have a fixed number of nodes

u_0 — 0 nodes — even fn

u_1 — 1 node — odd fn

$u_n(x)$ has $n-1$ number of nodes

The probability to find particle in classically forbidden region is expected to decrease exponentially

The classical turning points determine the classically allowed region: $-x_{cl} < x < x_{cl}$

x_{cl} depends upon energy of particle

$$\frac{1}{2} m\omega^2 x_{cl}^2 = E_n$$

The classical turning point for ground state $n=0$ is

$$\frac{1}{2} m\omega^2 x_{cl}^2 = E_0 = \frac{1}{2}\hbar\omega$$

$$x_{cl} = \sqrt{\frac{\hbar}{m\omega}} \Rightarrow \xi_{cl} = 1 \quad \text{for gd state}$$

$$\varepsilon_n = n + \frac{1}{2}$$

for nth state, $\frac{1}{2}\xi_{cl}^2 = \varepsilon_n = n + \frac{1}{2}$ or $\xi_{cl}(n) = \sqrt{2n+1}$

Choose the range of integration $[-\xi_{max}, \xi_{max}]$ s.t. $\xi_{max} \gg \xi_{cl}(n_{max})$ if interested in finding eigenstates for $n = 0, 1, 2, \ldots, n_{max}$

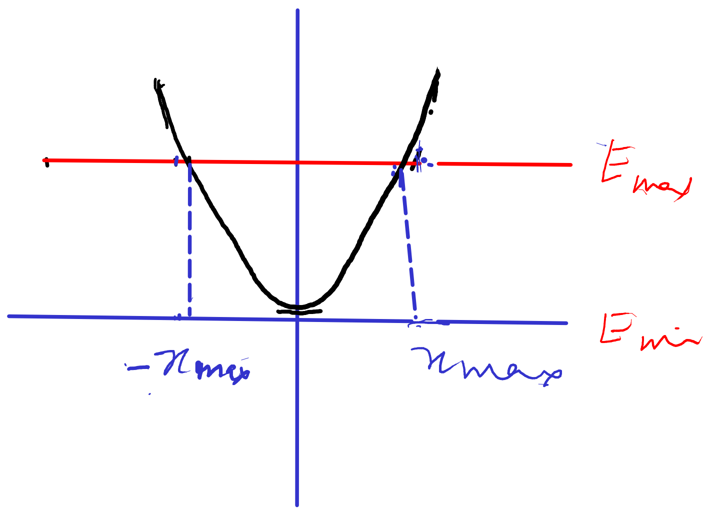
Determination of Energy Eigen values

use shooting method with Bisection

Search for solutions $u_n(x)$ for a given number of nodes $n\_nodes = n-1$

Start with a range $[E_{min}, E_{max}]$

→ should include the actual energy E_{actual}



For HO:

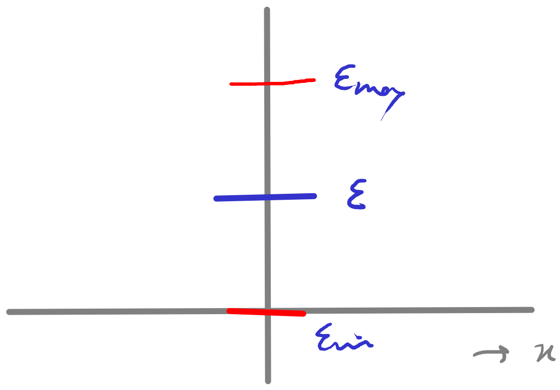
$E_{min} = 0$

$E_{max} = \frac{1}{2} m \omega^2 x_{max}^2$

$\Rightarrow \quad \varepsilon_{min} = 0, \quad \varepsilon_{max} = \frac{1}{2} \xi_{max}^2$

Start with guess value

$$\varepsilon = \frac{\varepsilon_{min} + \varepsilon_{max}}{2}$$



Use this ε and integrate from $x=0$ towards right to find $u_n(x)$

with $n = n\_nodes - 1$

While integrating count no of nodes, say nc

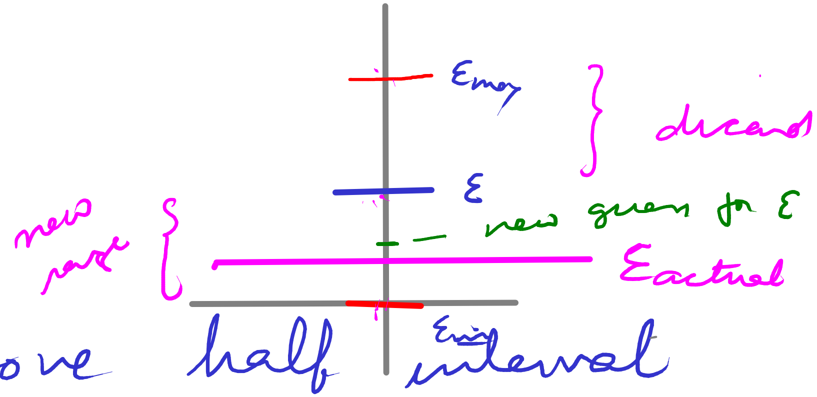
if $nc = n\_nodes$ – we get the energy

In general $nc < n\_nodes$

or $nc > n\_nodes$

1) If $nc > n\_nodes$

$\Rightarrow \quad \varepsilon > \varepsilon_{actual}$



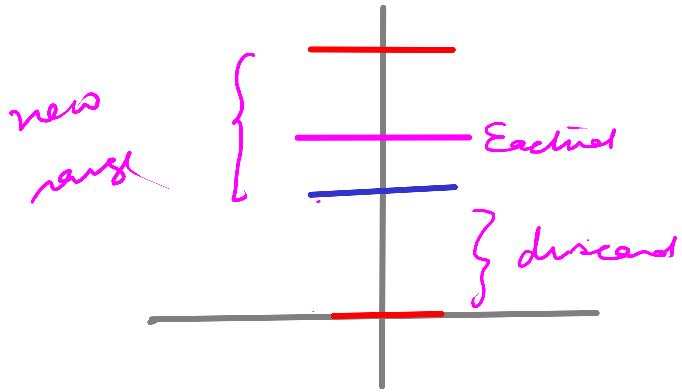
$\therefore$ discard the above half interval of energy

$\Rightarrow$ new range is $[\varepsilon_{min}, \varepsilon]$

i.e. replace $\varepsilon \leftarrow \varepsilon_{max}$

2) If $nc < n\_nodes$

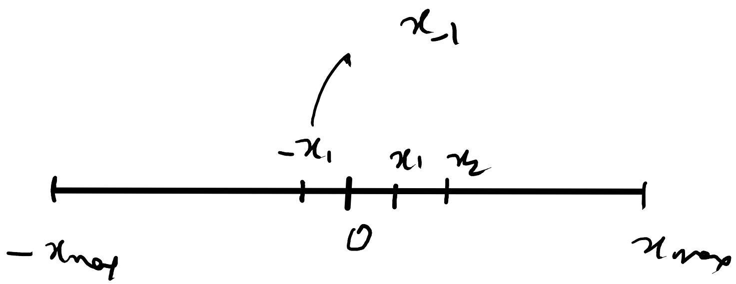
$\Rightarrow \quad \varepsilon < \varepsilon_{actual}$



$\varepsilon \leftarrow \varepsilon_{min}$

new range $[\varepsilon, \varepsilon_{max}]$

Thus energy range reduces



Again $\displaystyle \mathcal{E} = \frac{E_{min} + E_{max}}{2}$

determine $u_n(x)$ by integration (call numerov fn)

compare nc with n_nodes

Repeat $|E_{max} - E_{min}| <$ tolerance $\sim 10^{-8}$

$\Rightarrow$ the energy eigen value is $\mathcal{E}$ for n_nodes no of nodes and $u_n(x)$ is corresponding wavefn. for $x > 0$

use $u_n(-x) = (-1)^n u_n(x)$

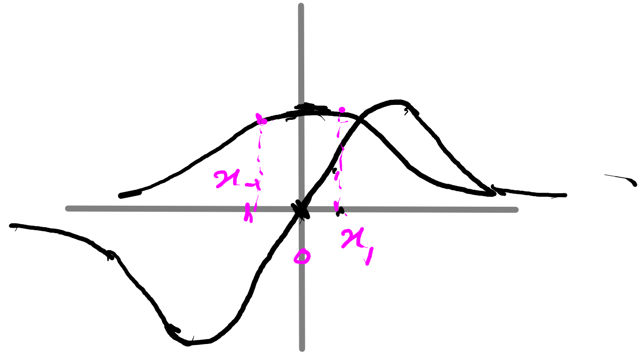
$n =$ n_nodes

$$u_{-i} = \begin{cases} u_i & n \text{ is even} \\ -u_i & n \text{ is odd} \end{cases}$$

$u_{-i} = u(-x_i)$

normalise $u_n(x)$ and plot alongwith the analytical solⁿ

Initial Conditions u_0 & u_1



use parity

If n is odd

$u_{ni} = u_n(x_i)$

$u_{n0} = u_n(x=0) = 0$

$u_{n1} = u_n(x = \Delta x) = \Delta x$ arbitrary finite value

$u_0' = 1$

$u_1 = u_0 + \Delta x \, u_0'$

If n is even

$u_{n0} =$ arbitrary finite value

and

$$u_{n1} = \frac{1}{c_1}\left[(12 - 10 c_0) u_0 - c_{-1} u_{-1} \right]$$

$c_{-1} = c_1$ & $u_{n_{-1}} = u_{n1}$ as u_n is even fn.

$$\therefore \; u_{n1} = \left(\frac{6 - 5c_0}{c_1} \right) u_{n0}$$

Input parameters

- x_{max}
- No of grid points N

$$\Delta x = \frac{x_{max}}{N}$$

$n = 0, 1, 2, \ldots$

- n for nth wave fn $\rightarrow$ n_nodes $= n$
- Trial energy range $E_{min} = 0$, $E_{max} = \frac{1}{2} x_{max}^2$
- File name to store the wave fn.
- Potential fn vpot(x) $\rightarrow$

$n = 0$, $n = 1$ $\Big\}$ $\rightarrow$ obtain E & $u(x)$
Plot $u(x)$ along with actual wave fn

HW

might not lead to correct asymptotic behavior

$$-\frac{\hbar^2}{2m} u'' + V(x) u = E u \Rightarrow u'' + \frac{2m}{\hbar^2}\left(E - V(x)\right) u = 0$$

$$\boxed{u''(x) + f(x)\, u(x) = 0}$$

$$x \leftarrow \sqrt{\frac{m\omega}{\hbar}}\, x$$

$$f(x) = 2\left(e - v(x)\right) = 2\left(e - \frac{1}{2} x^2\right)$$

$$e = \frac{E}{\hbar\omega}$$

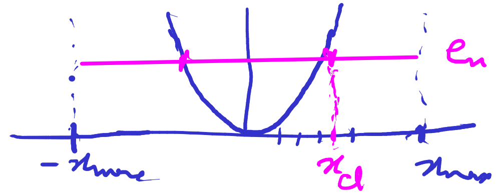
$$v = \frac{1}{2} x^2$$

Numerove algorithm

$$\boxed{u_{i+1} = \frac{1}{c_{i+1}}\left[(12 - 10 c_i) u_i - c_{i-1} u_{i-1}\right]}$$

$$c_{i+1} = 1 + \frac{\Delta x^2}{12} f_{i+1} \qquad c_{i-1} = 1 + \frac{1}{12} (\Delta x)^2 f_{i-1}$$

$e_n = n + \frac{1}{2}$



Algorthm we discussed till now:

$u_n(x)$ to be determined from x_0 to $x_0 + x_{max}$

For HO, $x_0 = 0$

Use Parity to find $u(x)$ in $[x_0 - x_{max}, x_0]$

$u_n(x_0 - x) = (-1)^n u(x_0 + x)$

$x_{max} \gg x_{cl}$

$tol = 0.5 \times 10^{-8}$

center of potential. Take default value to zero

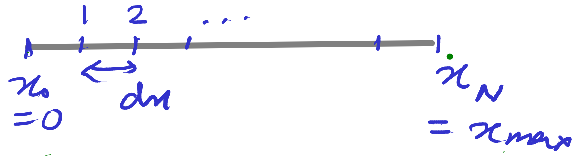
1. Input $x_0 = 0$, x_{max}, N, maxnodes, n_iter, tol
   - N → no of intervals, no of grid pts $= N+1$
   - maxnodes → highest wavefn to be determined; if this is 5 then first six wavefn $u_0, u_1, \ldots, u_5$ will be determined
   - n_iter → max no of iter

   x_0, x_{max}, vpot are floats
   N, n_nodes → integers

2. $dx = \dfrac{x_{max}}{N}$

3. $ddx12 = \dfrac{dx * dx}{12}$ $\qquad \dfrac{(\Delta x)^2}{12}$

4. Make the array of size $(N+1)$ x-values s.t
   $x_i = x_0 + i * dx \qquad i = 0, \ldots, N$



5. Define vpot array by calling potential fn $V(x)$
   $vpot_i = V(x_i)$ → must be even fn $vpot(x_0 - x) = vpot(x_0 + x)$

6. Give name of the file where data will be stored

7. n_nodes = 0

8. if n_nodes > max nodes → stop

9. emax = max(vpot)
emin = min(vpot)
} Trial energy range

$emax = \frac{1}{2} x_{max}^2$
emin = 0

10. $e = \frac{emin + emax}{2}$

If you want your code to run for a single trial energy add an optional guess value here
→ here n_iter = 1

iterations for shooting →

11. For k = 1, n_iter

a) icl = -1

b) for i = 0 to N

Set up the fn required in Numerov
f > 0 → classically allowed regions
f < 0 → classically forbidden
f_0 should be +ve

(i) Construct the f-array
$f_i = 2(e - vpot_i)$

(ii) if $f_i = 0$, $f_i = 1 \times 10^{-20}$

(iii) if f_i has sign opposite to f_{i-1}, icl = i

check sign of $(f_i * f_{i-1})$
x_{cl} is bet x_{i-1} & x_i
approx x_{cl}

c) if icl ≥ N - 10 → stop → need to change x_{max}
else if icl < 1 → stop → No turning pt → something wrong

d) Start with numerov

$h\_nodes = int\left(\frac{n\_nodes}{2}\right)$

we are integrating in half the region
if n_nodes is even there are
2* hnodes no of nodes
if n is odd there are 2*hnodes + 1
nodes (as one node is at $x=0$)

$c_i = 1 + dx^2/12 * f_i$

e) if 2*hnodes = n_nodes → even

$u_0 = 1$

$u_1 = (6 - 5c_0)/c_1$

else

$u_0 = 0$

$u_1 = dx$

end if

Check for even & odd

ib separate fn call Numerov

(f) Start integrating from 0 to xmax counting the no of times fn crosses zero

ncross = 0

For i = 1, N-1

$u_{i+1} = \left((12 - 10c_i)u_i - c_{i-1}u_{i-1}\right)/c_{i+1}$

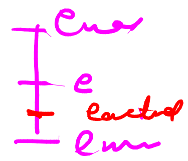
write a separate fn numerov

if u_{i+1} & u_i have diff sign   check the sign of $(u_i * u_{i+1})$

    $ncross = ncross + 1$

end for loop.

g) print k, e, ncross, hnodes.



h) if $n\_iter > 1$ then → if n_cross = hnodes

  if (n_cross > hnodes) then   means $e > e_{actual}$

    $e_{max} = e$

  else   means $e < e_{actual}$

    $e_{min} = e$

  end if

  New trial value

  $e = 0.5 * (e_{min} + e_{max})$

  if $(e_{max} - e_{min}) < etol$ exit the k-For loop

end if

k_iter = k

end k-for Loop   (go to 11)

12   if k_iter = n_iter

  print "Required tolerance could not be achieved in max no of iterations"

else
print " Reqd tolerance achied in " k_iter "no of iterations"

13 Normalisation

norm = 0

a) $p_i = u_i^2$ for $i = 0$ to icl
$p_i = 0$ otherwise

b) A = integral (p)

c) $u_i = \frac{u_i}{\sqrt{2A}}$ for each i

d) compare with classical prob fn.
p_{cl}

The wavefn obtained is not normalised

Calculate the prob density

$$p(x_i) = [u(x_i)]^2$$

$$\int_{-x_{max}}^{+x_{max}} p_i \, dx = 1$$

$$u(-x_i) = \pm u(x_i)$$

$$\int_0^{x_{cl}} p \, dx = 0.5$$

If $\int_0^{x_{max}} p \, dx = A$

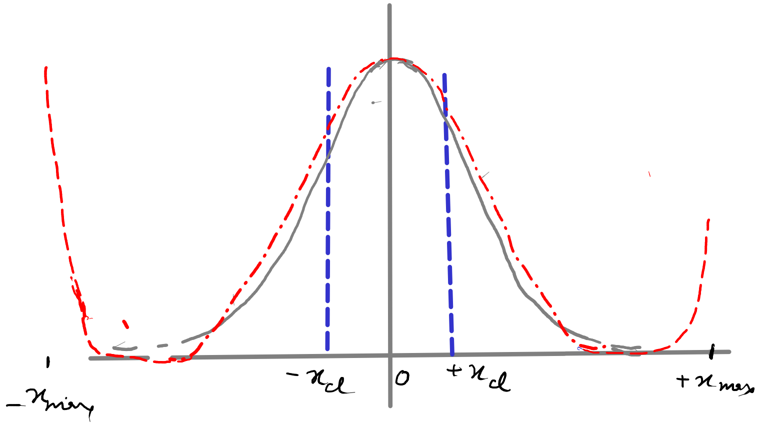
$$u \leftarrow \frac{u}{\sqrt{2A}}$$

14 Define wavefn from $[-x_{max} : 0]$ using parity

15 Write x_i, u_i, p_i in a file

16) Plot u_i using pts & u_{anal} as continuous curve
→ remember to use the dimensionless form

17) Plot p_i and p_{cl}

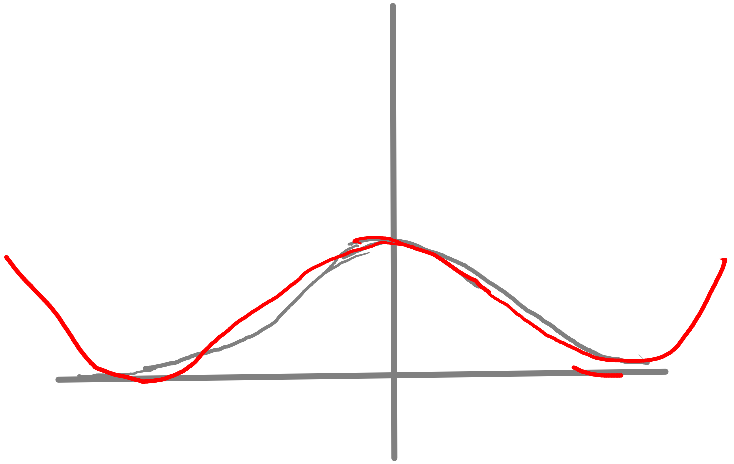


$x_{max} = 10$ $N = 100$

For ground state $e = \frac{1}{2}$

$\frac{1}{2} x_{cl}^2 = \frac{1}{2} \Rightarrow x_{cl} = \pm 1$

x_i u_i p_i p_{cl} v_i



Assignment 4a

- Take a fixed value of e (say e= 1/2 for graound state and solve the Schrodinger equation with Numerov method

Assignment 4b

- Implement the above algorithm and get the eigenvalues and eigenfuntions for first five (n=0,1,2,3,4) eigenstates
- Plot the wavefunctions and show that though the eigenvalues obtained are quite accurate the wavefunctions do not show the correct asymptotic behaviour.