B.Sc.(Hons.) Physics 32221501 Teacher: Mamta	**S.G.T.B. Khalsa College** **Quantum Mechanics (2022-23)** **Lab Assignment # 5** **Harmonic Oscillator - I**	Due Date and Time : 21.08.2022, 11:59PM Max. Marks : 20

The objective of this assignment is to

- numerically solve the Schrödinger Equation for "particle in a HO potential" problem with Numerov method and determine the energy eigenvalues and corresponding normalised wavefunctions for bound states.

1. (8 marks) **Theory**
   A particle of mass m is subjected to the harmonic oscillator potential given by $V(x) = \frac{1}{2}m\omega^2 x^2$.
   (a) Convert the Schrödinger Equation for a quantum harmonic oscillator in dimensionless Form
   (b) Obtain the classical turning points. Write down the analytical expressions for classical turning points and Energy Eigenvalues e_i in the dimensionless form.
   (c) Write down the expressions of first five normalised Energy eigenfunctions u_i as a function of the dimensionless variable defined above.
   (d) Interpret the non-zero value of the ground state energy.

2. (10 marks) **Programming**
   (a) Write a Python code to solve the Schrödinger Equation for an electron subjected to the harmonic potential based on the Numerov algorithm as discussed in the class.
      i. Take $e_n = n + \frac{1}{2} + \Delta e$ (in units of $\hbar\omega$) with $\Delta e = 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$ and $n = 0$ and integrate from $x = 0$ to $x = x_{\max}$ using Numerov method to determine $u_0(x)$.
      ii. Use the even and odd property of the wave function to extend it in the range $[-x_{\max} : 0]$.
      iii. Plot the corresponding normalised wavefunction vectors $u_0(\xi)$ (as scatter plot) for all values of Δe along with the analytical wavefunction (as continuous curves).
      iv. Repeat for the first three excited states with $\Delta e = 10^{-6}$
   (b) Plot the probability densities (as scatter plots) along with the analytical ones (as continuous curves) for all the four states in one plot with $\Delta e = 10^{-6}$.
   (c) Convert energies to eV for an electron subjected to harmonic oscillator of frequency $\omega = 5.5 \times 10^{14}$ s and print a table of energy eigenvalues in eV along with the analytical values.
   (d) Extend your program to compute the probability of finding electron in the classically forbidden region when it is in the ground state.

3. (2 marks) **Discussion**
   Interpret and discuss your results.