B.Sc.(Hons.) Physics
32221501
Teacher: Mamta

S.G.T.B. Khalsa College
Quantum Mechanics (2022-23)
Lab Assignment # 11
Time Evolution of Wave Packets

Due Date and Time :
25.09.2022, 11:59PM
Max. Marks : 20

The objective of this assignment is to

- understand that a wave packet can represent a free particle
- study the time evolution of a given wave packet numerically

1. (8 marks) **Theory**

   (a) Write down the Schrödinger Equation for a free particle in dimensionless form and determine the stationary states.

   (b) Discuss why the stationary states cannot represent a physical state.

   (c) What is a wave packet. Show that the group velocity of the wave packet corresponds to the speed of free particle.

   (d) How does the wave packet evolve with time?

   (e) Given that at $t = 0$, a quantum particle of mass m is described by the wave function

   $$\psi(x,\,0) \quad = \quad \begin{cases} A & \text{for } |x| < b \\ 0 & \text{for } |x| > b \end{cases},$$

   normalise the wave function and determine the fourier components $a(k)$ given by

   $$\psi(x,0) \quad = \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a(k)\, \exp\{ikx\}\, \mathrm{d}k \tag{1}$$

   (f) Use these $a(k)$ to write down the expression for wave function and the probability density at time t, $\psi(x,\,t)$ as an integral. Express these integrals in terms of dimensionless quantities.

2. (10 marks) **Programming**

   (a) Write a Python code to

   i. Plot the probability density for finding the particle in position space at $t = 0$.

   ii. Plot the probability density for the momentum of the particle at $t = 0$.

   (b) Extend the code to determine the probability density in position space at time t by evaluating the required integral numerically at $\tau = 0, 0.1, 0.2, \ldots 2.0$ where τ is the time in dimesnionless units.

   (c) Extend the code further to plot the probability density in position space at $\tau = 0, 0.1, 0.5,\ 1.0,\ \ldots 2.0$. Also plot the probability of finding the particle in the range $|x| < \frac{b}{2}$ as a function of τ.

   (d) Write another code to study the time evolution of a Gaussian wave packet and plot

   i. the wave packet at various times

   ii. the uncertainty in position and momentum as a function of time.

3. (2 marks) **Discussion**
   Discuss your results and compare with those of the Finite Difference Method.