B.Sc.(Hons.) Physics
32221501
Teacher: Mamta

S.G.T.B. Khalsa College
Quantum Mechanics (2022-23)
Lab Assignment # 12
Screened Coulamb Potential

Due Date and Time :
16.10.2022, 11:59PM
Max. Marks : 20

This assignment is based on the Problem 2 of the syllabus.

1. (6 marks) **Theory**

   (a) The electron in an atom is subjected to the screened Coulomb Potential given by

   $$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}e^{-r/a} = V_c e^{-r/a}$$

   V_c being the Coulamb potential.
   Write down the Schrödinger Equation in spherical polar coordinates. Use separation of variables to obtain the radial part of the Schrödinger Equation for a given value of ℓ.

   (b) Now consider the radial equation for $\ell = 0$ and convert it into dimensionless form by redefining $r = \xi a_0$, a_0 being the Bohr radius.

   (c) Plot the Coulamb potential V_c (in dimensionless form) and on the same graph, plot the potential $V(\xi) = V_c e^{-\xi/\alpha}$ as a function of ξ for different values of $\alpha = \frac{a}{a_0}$, say $\alpha = 2, 5, 10, 20, 100$. Discuss what do you expect for the bound state eigen values.

2. (12 marks) **Programming** Write a Python code to solve the s-wave ($\ell = 0$) Schrödinger Equation for an atom (in dimensionless form) for the screened Coulamb Potential

   $$V(\xi) = V_{\text{coul}} e^{-\xi/\alpha}$$

   The code should

   (a) obtain the bound state energy eigen values. Is the number of bound state finite?

   (b) obtain the the energy (in eV) of the ground state of the atom to an accuracy of three significant digits for the values of α mentioned above. Take $e = 3.795(eV\mathring{A})^{1/2}$, $m = 0.511$ MeV/c^2. In these units $\hbar c = 1973(eV\mathring{A})$.

   (c) plot the corresponding normalised wavefunctions. Also plot the wavefunctions for the Coulamb potential on the same graph.

   (d) plot the probability densities

   (e) plot the ground state energy as a function of α

3. (2 marks) **Discussion**
   Read the article shared with you. Interpret and discuss your results.