PHYS 451 Final Paper

Anomalous Impact in Reaction-diffusion Financial Models

Utsa Poddar (30071131)

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1 Introduction

Before we start to explore the statistical physics aspects of this paper, we first need to understand a few terminologies, such as what are financial models, what are reaction-diffusion models and what is meant by anomalous impact. After gaining insight into these concepts we will be able to explore how the reaction-diffusion model can be implemented in financial models and how changing values slightly of certain variables in these financial models can cause entirely different outcomes, which relates to the anomalous impact.

In this paper, after we are done setting up the necessary key background concepts, where the required terms will be defined as they show up to explain the said concept. As well as use several examples, and algorithms in combination with code to convey a summarized version of the concept we are dealing with. Then this paper will show how all the concepts of financial models, reaction-diffusion and anomalous impact in the market (specifically share market) link together. In the end, also mention another research paper, which was able to fix the financial model made using the reaction-diffusion algorithm (which was inaccurate in some circumstances), using a modified version of the reaction-diffusion algorithm called fractional diffusion model.

2 Background

2.1 What are financial models?

As the term suggests this is a model, just like any other, where there are going to be variables and depending on the variables, we will be able to predict what the outcome of a financial decision is going to be if a company were to go through with the said decision[14]. But what are these models based on? These are based on the company's historical performance to predict the future performance of the company based on variables that are circumstantial, therefore, no financial model can predict the future performance with a hundred percent certainty. Even though, it may sound like this is where the statistical aspect of the financial models comes in or rather far from it. Usually, if we were to get into the nitty-gritty of financial models, we would need the company's income statement (a record of all the revenues and costs over a year from selling and making the products or services the company offers)[5], balance sheet (record showing all the company's assets, liabilities, capitals and income and expenses match up at the end of each year)[8], as well as cash flow statements (these are derived from the income statement, where the net income is adjusted with the company's cash activities, such as operating, investing and financial activities)[10]. However, note that all the definitions are just a small insight into the world of finance and by no means encapsulate the whole purpose of these statements, they are mentioned just to get the main idea of these statements across (as well as, the period of the statements is usually annually, however, they can be for different time periods as well, as such quarterly or semi-annually).

However, the main impact that we are going to observe in this paper is, say a company is to launch a new product with its competitors (or with its own pre-existing product). To determine the price point of the product we need to build the financial model and see if the price point is too low, if it is then the company needs to adjust some of the costs so that the product is sustainable for the company (it can launch and keep the product in the market for the long run). Therefore, the financial model is helping in making better-informed decisions to make changes, analyze new opportunities, attract investors, as well as get loans[14].

2.2 Reaction-diffusion models

Now as we move away for a bit from the financial topics. Let us discuss what reaction-diffusion models are, to do that this paper is going to explore one of the easier models (Gray-Scott model)[13] to go through a build an algorithm to show exactly what reaction-diffusion models are and how they work.

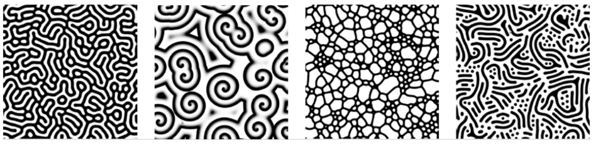

Figure 1: Diagram showing different patterns in reaction diffusion models [15]

The idea of reaction-diffusion is an image on a 2D plane if you were to pour chemicals into it, say chemicals A and B (choosing two chemicals for simplicity, there can be more than two variables or "chemicals"), where chemical A is white in color and chemical B is black. Now chemicals A and B are going to react and diffuse and based on how these chemicals react on the 2D plane and arrange themselves, we are going to have a density distribution of these chemicals on the screen, meaning regions with a lot of chemical B will be black in color and regions with a lot of chemical A will be white. But when both the chemicals are present then the color will be determined on a grayscale (a scale with all the shades of gray) depending on the amounts of A and B together in that region.

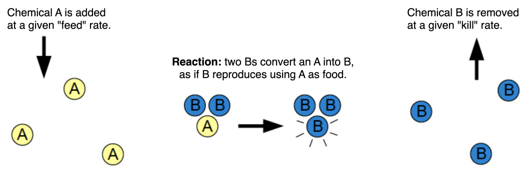

Figure 2: Diagram showing the reaction between chemical A and B [15]

Now if we refer to Figure 2, here initially the 2D plane is filled with chemical B and we will be adding chemical A at a given “feed” rate, which is how fast we are adding the chemical A. Now a combination of two chemical atoms or molecules with one chemical A atom or molecule will result in the chemical A getting converted into chemical B. Then there is also a rate at which the chemical B is being ejected out of the system (in this case the 2D plane) which is the “kill rate”. We are going to split the 2D plane into small boxes through (say 200x200 of these small boxes make up the 2D plane) called cells.

Therefore, each cell will have its own concentration of chemicals A and B, also known as the local concentrations of A and B. Now, these concentration values are how much in percentage is chemical A present in that cell, say chemical A has a concentration of 70% (concentration value of A=0.7), so the naturally the concentration of B in that cell would be 30% (concentration value of B=0.3). These concentration values can be used to set the color of each cell/pixel using a gray scale[13].

Now that we have the setup all the background to this model it is important to notice that this model is going to change in time as we keep adding chemical A and remove chemical B, as well as the reaction between these two chemicals calls for an iterative function to determine the present concentration of chemical A and B depending on the previous concentrations of these chemicals, as shown by equations (1) and (2).

To do that, to make this simpler, we are going to take the average of the two concentrations to determine the new concentrations. However, it is a bit more complicated than it sounds. Take a 3 by 3 matrix, for example, see figure 3:

To know the present color of the cell at

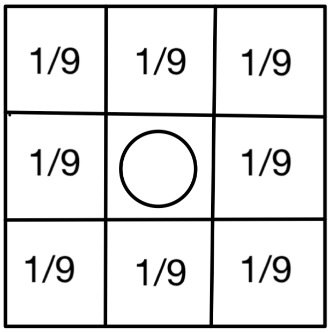

Figure 3: Diagram showing how the new concentration value in the center has to be determined by taking the average of a 3 by 3 matrix

the center we would have to know what the colors of the neighboring cells are, so the convolution of a 3 by 3 matrix would be to take 1/9 of every concentration value of every cell (including the middle one, the new color of which we are trying to determine) and add them up. However, note that for a 3 by 3 matrix it is pretty simple, for a 4 by 4 matrix, not all the cells are neighboring cells of the center cell, and the weight (which was 1/9 for the 3 by 3 matrix) is not the same for all the cells in a 4 by 4 matrix. This gets more and more complicated as we increase the number of cells or pixels. Also, note that if the number of cells/pixels is big enough the weight being multiplied at the edge of the matrix will be approximately 0.

Now we are going to implement the actual reaction-diffusion formula which accounts for the convolution for this model[13]:

$$A' = A + (D_A \nabla^2 A - AB^2 + f(1-A))\Delta t \quad (1)$$

$$B' = B + (D_B \nabla^2 B + AB^2 - (k+f)B)\Delta t \quad (2)$$

Where A' and B' are the new values, A and B are the old values, D_A and D_A are the diffusion rates for A and B respectively, $\nabla^2 A$ and $\nabla^2 B$ are Laplacian functions which account for the convolution, the middle term (AB^2) accounts for two Bs becoming one A, f(1-A) is the feed rate and -(k+f)B is the kill rate, and Δt is the change in time between iterations.

The above equations and the algorithm that we explored all this while was used to program this model into JavaScript. The 2D plane was modeled using an array of arrays with a resolution of 200 by 200 pixels. The values of the color of the pixels were determined over time using the reaction-diffusion equations. Then using the JavaScript the following images were obtained (Figure 4, 5, 6, and 7) (see Appendix A for the code used to generate these images).

The constants from the equations such as

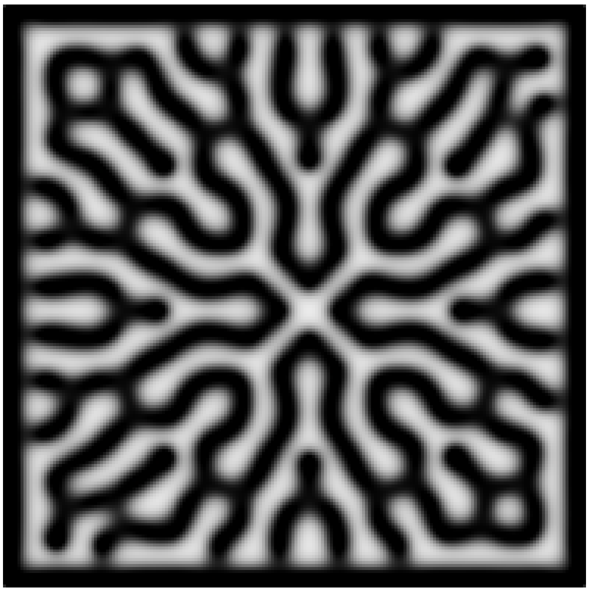

Figure 4: Diagram showing reaction diffusion pattern when D_A=1, D_B=0.5, feed (or f)=0.055, kill (or k)=0.062

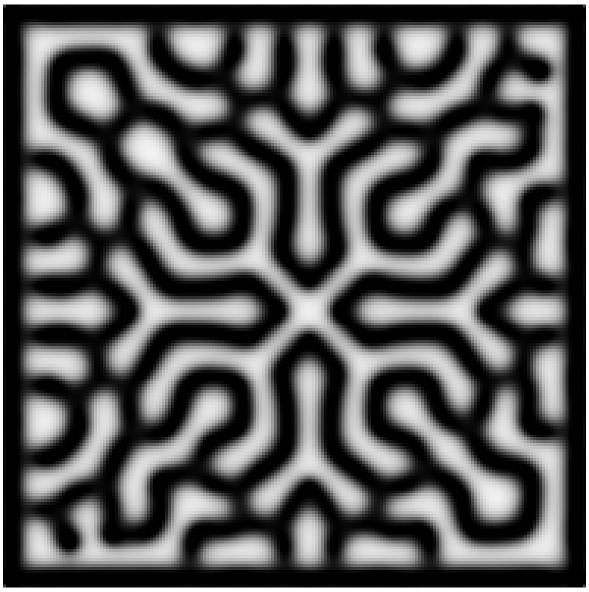

Figure 5: Diagram showing reaction diffusion pattern when D_A=1, D_B=0.5, feed (or f)=0.060, kill (or k)=0.062

Figure 6: Diagram showing reaction diffusion pattern when D_A=1, D_B=0.5, feed (or f)=0.055, kill (or k)=0.060

Figure 7: Diagram showing reaction diffusion pattern when D_A=0.9, D_B=0.5, feed (or f)=0.055, kill (or k)=0.062

D_A, D_B, feed rate and kill rate were slightly changed to see the effect. As seen in Figures 4, 5, and 6, changing the kill rate or feed rate ever so slightly produces vastly different results. This would be discussed in the next section when we are exploring anomalous impact.

However, reaction-diffusion by itself isn't useful, it is just a tool that has to be implemented while modeling some scenarios.

For example, the spread of the COVID-19 pandemic can be modeled using a reaction-diffusion equation, and by implementing it into a map of say, Canada we can observe, depending on the r_0 value (a measure of how many people will one affected person will infect), how the demographic will look like over time. Therefore, predicting how the masses are going to be affected if a new variant of the virus is detected. This was actually done by Henri Berestycki in 2021[4].

Therefore, as seen from the model that we developed before, as well as in the implementation of the reaction-diffusion function in the COVID-19 pandemic, there are a couple of crucial components to making a reaction-diffusion model. The first one is the function to determine the present values of the reactants (or the number of people unaffected and the number of people affected), using the past values of the reactants. These are kind of predator-prey models that can be used in many different circumstances.

However, the more difficult ones to determine and implement are the diffusion rates (D_A and D_B). These diffusion rates vary from system to system and only work when the right amount of diffusion rate is implemented in the model. For example, if both reactants diffuse at the same rate, then no pattern will form, again if the diffusion rates are too different then, one reactant will be more prevalent and no pattern will form (the whole 2D plane will be the same color, as shown by Figure 6). Even the reaction parameters such as the feed and kill rate have a similar effect on the model and are just as difficult to determine.

2.3 Anomalous impact

To get the idea of the anomalous impact, let us continue with an example, magnets. We have always known that opposite sides (North and South) of the magnets attract, and the same sides (North-North or South-South) repel (bear with me as I explain it in a crude but simplest possible way, as concepts like ferromagnetic state and paramagnetic state will not be as important in this paper and this is just an example). However, we also know how magnetism comes into effect, each magnetic material would be made out of atoms, these atoms have an intrinsic property called spin. We can think of these spins being an arrow, being up and down, for each atom. These are the two different spin states on an atom.

Now magnetic materials are made out of atoms that are in a lattice, so these associated spin states are also in a lattice, and we can deal with just the spins in the lattice. This lattice of spins is known as the Ising model[11], named after Ernst Ising. In the Ising model, each spin in the lattice can interact with its nearest neighbor. Now we would imagine that the Ising model is three-dimensional, however, it is very general and can be applied to two-, one- and four-dimensional materials. Nevertheless, magnetization in materials happens when there is a net alignment of the spins, for example, if there are more up spins than down spins, the material would be magnetized (be it temporarily or permanently, depending on a whole lot of parameters). If we heat up this magnetized material, the spin-lattice of the magnets gets disordered and there's more probability that the material loses its magnetism. On the other hand, if we were to cool the material the spins are more probable to align themselves and we have more probability of increasing the magnetism of the material.

It is well established through experimentation that as a material is heated[9], it becomes less and less magnetic until it reaches a certain temperature, known as the Curie temperature, at which point the magnetization drops to zero. Therefore, as there is a relation between magnetization and temperature, there must be a function of magnetization that is dependent on temperature. This determined and magnetization varied with temperature using a power law.

$$M \sim (1 - \frac{T}{T_c})^\beta \tag{3}$$

Now, these exponents had to be determined, but they discovered something strange, no matter what the substance, the power-law relation came out to be the same as well as the value of the exponent. For two-dimensional substances, for example, a sheet of graphene [1] that is about one atom thick, it was 1/8 or 0.125 (β value). The Ising model is therefore used to determine properties of materials, such as their Curie temperature and magnetization. Lars Onsager was able to find the exact solution to the two-dimensional Ising model[11]. That showed for any 2D substance, no matter what the chemistry of the substance was, the exponent for the magnetization power law will always be 0.125 (universality)[9]. Interestingly, it was not just magnetization that has this property, quantities like heat capacity also have power laws and their own value of exponents for each dimension, be it 1D, 2D, 3D, or 4D. These exponents are known as critical exponents, and these show us one of the important characteristics of materials of universality. Equation 4 shows an equation governing the curie temperature, where J is an eigenvalue for the stationary state, T_C is the curie temperature, K_B is the Boltzmann's constant[6].

$$\frac{K_B T_C}{J} = \frac{2}{ln(1+\sqrt{2})} \tag{4}$$

$$M = [1 - sinh^{-4}(2BJ)]^{\frac{1}{8}} \tag{5}$$

		values of the exponents		
Quantity	Critical behavior	$d = 2$ (exact)	$d = 3$	mean-field theory
specific heat	$C \sim \epsilon^{-\alpha}$	0 (logarithmic)	0.113	0 (jump)
order parameter	$m \sim \epsilon^{\beta}$	1/8	0.324	1/2
susceptibility	$\chi \sim \epsilon^{-\gamma}$	7/4	1.238	1
equation of state ($\epsilon = 0$)	$m \sim H^{1/\delta}$	15	4.82	3
correlation length	$\xi \sim \epsilon^{-\nu}$	1	0.629	1/2
correlation function $\epsilon = 0$	$G(r) \sim 1/r^{d-2+\eta}$	1/4	0.031	0

Figure 8: Table showing different values of the critical exponents for the Ising model in two and three dimensions [9]

However, how do critical exponents relate to anomalous impact? Generally changing a variable in a system would result in the outcome of the system being changed by an equal amount, this is said to be a linear response. However, there are some systems that don't in the same manner, a small change in one of the variables makes a catastrophic change to the output, these fragile systems are known as critical systems. Up till now, we were discussing magnetism to show that this is one of the critical systems. As mentioned before, at low enough temperatures if a material is in an external magnetic, we may be able to magnetize the material. When the external magnetic field is removed, if the material loses all of its magnetic properties, then the material was in a ferromagnetic phase, however, at high enough temperatures the material will lose all of its magnetization, which is the paramagnetic phase. Here the magnetic susceptibility (the amount by which a material will get magnetized when an external magnetic field is applied) changes drastically when a material is transitioning from a paramagnetic phase to a ferromagnetic phase, and as we can see from the table magnetic susceptibility has its own critical exponent. This drastic change is the anomalous impact that we have been chasing all this while.

3 Application

Now that we have been introduced to all the puzzle pieces, it is time to put them all together (while keeping in mind that this exact topic seems to have not been explored by many researchers, and the resources available are not exhaustive). There is a term called market impact in finance that is just as fragile, as our example of magnetic susceptibility[12]. Market impact, in simple terms, is when the price of an asset (for example a company's selling or buying shares) changes when it is being sold or bought. This change in price depends on the number of transactions over a given period of time, as well as the number of items (shares) being sold in each transaction. Furthermore, each of these transactions has costs involved with them, for example, taxes, brokerage commissions, etc. Therefore, market impact is an important quantity that these shareholders are interested in because market impact directly relates to the trading costs, moreover, it also relates to one of the most important questions in economics, why and how are the prices changing? After all, that is what market impact is. Usually in financial models the average price change ($\mathcal{I}$) is assumed to be a linear function of the total volume of contacts (Q) (or the proposed number of sales of stocks of a company, for example). If the volume of contracts increases, the price of the shares should decrease by the same amount. However, more and more people in the field of Econophysics [7] believe (and their research suggests) that the relationship is rather in the domain of power law, given by Equation 6 [12]. Where δ is in the range 0.4-0.7, σ_D is the daily price fluctuations, V_D is the daily traded volume and Y is a coefficient in the order of 1.

$$\mathcal{I} = Y\sigma_D(\frac{Q}{V_D})^{\delta} \tag{6}$$

Therefore, we can see from this equation in this equation that if V_D is much smaller than Q, the average price change would be huge. Here is the anomalous effect that we have already introduced before in this paper. Moreover, other properties of the critical exponents (universality) also carry over, the σ_D value is the same regardless of the size of the company, where the company is, etc.

Keeping this in mind, a rough model was built by [12]. Where they established a few key features to get started, orders will be placed and updated/removed to buy (sell) at the lowest (highest) price possible (this is our feed and kill rate from reaction-diffusion equations), there will be no pattern if the buying and the selling price are equal. Here the buying and selling of the products are modeled using particles B and A, where the number of B particles represents the number of shares to be bought and A represents the number of shares to be sold. Where the price levels are the cells or pixels that we have already established the concept of previously. The only thing different, however, in this model they allowed the particles to hop from one cell to another the only allowed direction being left or right (1D) with a probability of D, as well as the particles have a probability (λ) of reacting with each other (instead of having a definitive reaction of two Bs and one A forming three Bs). This probability (λ), however, is a function of time, as more time passes by the probability of the particles reacting increases. Using this model [12] created equations that changed continuously depending on the variables:

$$\frac{\partial\langle b(x,t)\rangle}{\partial t} = D\frac{\partial^2\langle b(x,t)\rangle}{\partial x^2} - \lambda u_A\langle a(x,t)b(x,t)\rangle \tag{7}$$

$$\frac{\partial\langle a(x,t)\rangle}{\partial t} = D\frac{\partial^2\langle a(x,t)\rangle}{\partial x^2} - \lambda u_B\langle a(x,t)b(x,t)\rangle \tag{8}$$

Where $a(x,t)$ and $b(x,t)$ are the concentration of the particles of A and B respectively, $u_A = 1 - p^{\frac{1+m}{2}}$ and $u_B = 1 - p^{\frac{1-m}{2}}$, where m is a parameter for bias and p is a parameter of bid (when buying) or ask (when selling) particle. However, solving these equations in one dimension is quite challenging as the equations heavily rely on the particles' positions, therefore approximation and numerical simulations were used to get a solution. Using which the following relationship between the Price change ($\mathcal{I}$) and Time (T) is shown in Figure 9.

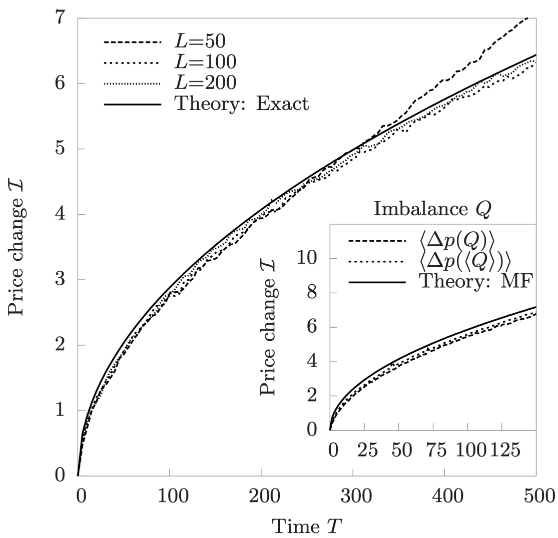

Figure 9: The graph compares different solutions from simulations and analytical solution. Here the parameter were $D = 1$, $p = 0.5$, $m = 0.75$, $\lambda = 10^3$ and different values of L (which is the length of the 1D model). For the solid line the parameters were $D = 1$, $\lambda = 1000$, $p = 1$ and $m = 0.5$[12]

This was done by establishing the square root impact law. Which was determined to be Equation 9 (please view reference [12] for a detailed explanation of how this equation was derived).

$$y_\tau^* = 2\alpha(u_B/u_A)\tau^{1/2} \tag{9}$$

In that research [12] they were able to present an analysis of how the changes in the market parameters result in price changes using the square root effect. They were able to parameterize the changes in prices using analytical means rather than numerical which led to more accurate results. However, note that although the exact prediction of the model structure they were working with may vary depending on the response parameters used, their results suggested that in a single-dimensional particle system, the focus should be more on the dependence of the actual horizontal interface. This also means that sufficient common characteristics (diffusion and market equity) are sufficient to explain the unusual (anomalous) price behavior in terms of volume equity which has also been predicted in other previous studies [16].

However, [12] was able to determine the equations and model the market impact for long-range frequency (for trading over months or years), but when the same model is used for high-frequency trading (for trading over hours to minutes), it fails to predict market impact accurately. This is why in 2017, another group of researchers set out to make a rectified model [3] which would be able to predict the market in the long as well as the short time ranges. They, however, also used a modified version of the reaction-diffusion algorithm called fractional reaction-diffusion, where a normal reaction term is combined with a fractional diffusion term [2]. They believed if a model is to be realistic, then it has to be able to predict results in the short and long-time scales, which is why they used a fractional reaction-diffusion model.

Moreover, several key components in their model could be determined using analytical means. They were able to find that the volume of the transactions related to the market impact in a concave function, which confirmed the square root law from the other paper [12]. However, they were also able to confirm using the fractional reaction-diffusion model that the value of β, which was thought to be a constant of 1/2 in the previous paper [12], was actually dependent on time. This should that the value of β was in the range [0,1/2], instead of being a constant. Furthermore, when this theoretical result matched the numerical model, they had created and was instead able to predict market impact in the short and long-time scales, they were able to conclude that fractional reaction-diffusion is the better model to create a financial model for the market impact. However, the other characteristics of power-law relations still got carried over, such as universality and, not-so-conveniently, anomalous effect.

Conclusion

A lot of attention has been brought to other econophysicists as well as shareholders who are actively participating in trading, which is universal anomalous impact due to being more and more known. Market impact (anomalous price changes) is important not only for experts in economics and finance but also for shareholders, for whom the market impact directly translates to trading costs. Moreover, it also solves one of the most important questions in economics such as, how and why prices change.

Research is still being done on market impact where the parameters of the market can be absorbed in the model will spit out what the price should be if the said parameters are to change in the future. However, the drastic changes in prices caused by small changes in the parameters of the market means markets structures are fragile. Therefore, the failure of this kind of mechanism due to anomalous effect may have significant outcomes for society, such cause uncertainty in the market, or in severe cases, market failure. Nevertheless, more research is being done and more and more accurate interpretations of the market, just like [12], [16] and [3] are yet to come.

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Appendix A

JavaScript code used to plot the reaction diffusion model in a 2D plane:

```
var grid;
var next;

var dA = 1;
var dB = 0.5;
var feed = 0.055;
var k = 0.062;

function setup() {
  createCanvas(200, 200);
  pixelDensity(1);
  grid = [];
  next = [];
  for (var x = 0; x < width; x++) {
    grid[x] = [];
    next[x] = [];
    for (var y = 0; y < height; y++) {
      grid[x][y] = {
        a: 1,
        b: 0
      };
      next[x][y] = {
        a: 1,
        b: 0
      };
    }
  }

  for (var i = 100; i < 110; i++) {
    for (var j = 100; j < 110; j++) {
      grid[i][j].b = 1;
    }
  }

}
```

```
function draw() {
  background(51);

  for (var x = 1; x < width - 1; x++) {
    for (var y = 1; y < height - 1; y++) {
      var a = grid[x][y].a;
      var b = grid[x][y].b;
      next[x][y].a = a +
        (dA * laplaceA(x, y)) -
        (a * b * b) +
        (feed * (1 - a));
      next[x][y].b = b +
        (dB * laplaceB(x, y)) +
        (a * b * b) -
        ((k + feed) * b);

      next[x][y].a = constrain(next[x][y].a, 0, 1);
      next[x][y].b = constrain(next[x][y].b, 0, 1);
    }
  }


  loadPixels();
  for (var x = 0; x < width; x++) {
    for (var y = 0; y < height; y++) {
      var pix = (x + y * width) * 4;
      var a = next[x][y].a;
      var b = next[x][y].b;
      var c = floor((a - b) * 255);
      c = constrain(c, 0, 255);
      pixels[pix + 0] = c;
      pixels[pix + 1] = c;
      pixels[pix + 2] = c;
      pixels[pix + 3] = 255;
    }
  }
  updatePixels();


  swap();


}
```

```
}
```

```
function laplaceA(x, y) {
  var sumA = 0;
  sumA += grid[x][y].a * -1;
  sumA += grid[x - 1][y].a * 0.2;
  sumA += grid[x + 1][y].a * 0.2;
  sumA += grid[x][y + 1].a * 0.2;
  sumA += grid[x][y - 1].a * 0.2;
  sumA += grid[x - 1][y - 1].a * 0.05;
  sumA += grid[x + 1][y - 1].a * 0.05;
  sumA += grid[x + 1][y + 1].a * 0.05;
  sumA += grid[x - 1][y + 1].a * 0.05;
  return sumA;
}

function laplaceB(x, y) {
  var sumB = 0;
  sumB += grid[x][y].b * -1;
  sumB += grid[x - 1][y].b * 0.2;
  sumB += grid[x + 1][y].b * 0.2;
  sumB += grid[x][y + 1].b * 0.2;
  sumB += grid[x][y - 1].b * 0.2;
  sumB += grid[x - 1][y - 1].b * 0.05;
  sumB += grid[x + 1][y - 1].b * 0.05;
  sumB += grid[x + 1][y + 1].b * 0.05;
  sumB += grid[x - 1][y + 1].b * 0.05;
  return sumB;
}



function swap() {
  var temp = grid;
  grid = next;
  next = temp;
}
```