# Digital Image Processing
# Project 6
# Wavelets and Multi-Resolution Processing

Omar Rawashdeh
Harshal Raut
Utsav Shah

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# 1 Part I

## 1.1 Discrete Wavelets

Discrete Wavelets extend the idea of wavelets to enable general wavelets to be used on sequences of numbers. This is very useful as there are many instances in which data is recorded as a discrete sequence (such as when dealing with images; which are typical stored as pixels) rather than as a function.[1]

## 1.2 Two-dimensional Wavelets

Two-dimensional wavelets are a natural extension from the single dimension case. As a concept they can be applied to many two-dimensional situations, such as 2D functional spaces. However they really come into there own when images are considered. In a world where digital images are processed by computers ever second, methods for condensing the information carried in an image are needed. Two-dimensional wavelets are used in a number of areas, most notably in image manipulation. 2D wavelets can be used in a few different ways when it comes to manipulating images.[1]

## 1.3 Denoising

This has been done by applying the 2D wavelet transform and then thresholding the coefficients that are produced so as to identify the areas where the noise has built up. These can then be altered to produce a better picture.Wavelets and wavelet transforms (including the DWT) have many advantages over rival multiscale analysis techniques, such as the Fast Fourier Transform (FFT).[1]

## 1.4 Advantages of Discrete Wavelets

The key advantages are listed below:

- Structure Extraction: wavelets can be used to analyse the structure (shape) of a function, that is the coefficients tell you how much of the corresponding wavelet makes up the function.
- Localisation: If the function f(x) has a discontinuity at x then only the wavelets which overlap the discontinuity will be affected and the associated wavelet coefficients.
- Efficiency: Wavelet transforms are usually much faster than other methods (or at least as good as). For example the Discrete Wavelet Transform is O(n) whereas the FFT is O(nlogn).[1]

## 1.5 Daubechies

Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1. For an input represented by a list of numbers, the Haar wavelet transform may be considered to simply pair up input values, storing the difference and passing the sum. This process is repeated recursively, pairing up the sums to provide the next scale, finally resulting in differences and one final sum.

Ingrid Daubechies, one of the brightest stars in the world of wavelet research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and 'db' the "surname" of the wavelet.[2]

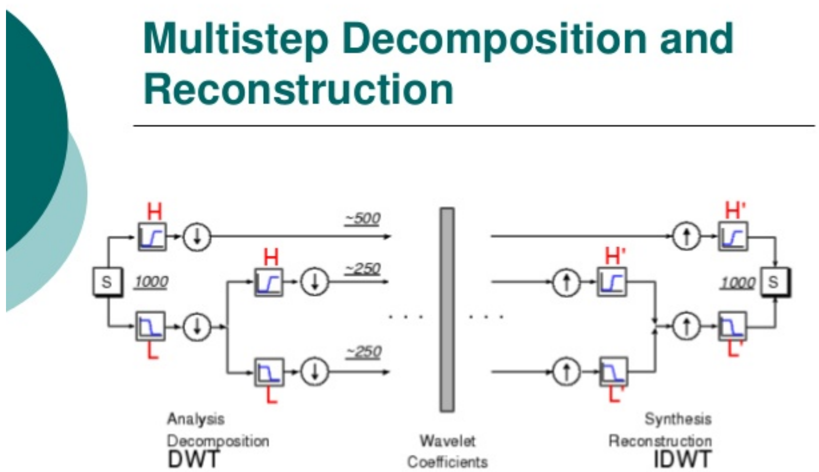


Figure 1: Multi-step Decomposition and Reconstruction

### General Characteristics

- Family: Daubechies
- Short name: db
- Order N: N strictly positive integer
- Examples: db1 or haar, db4, db15

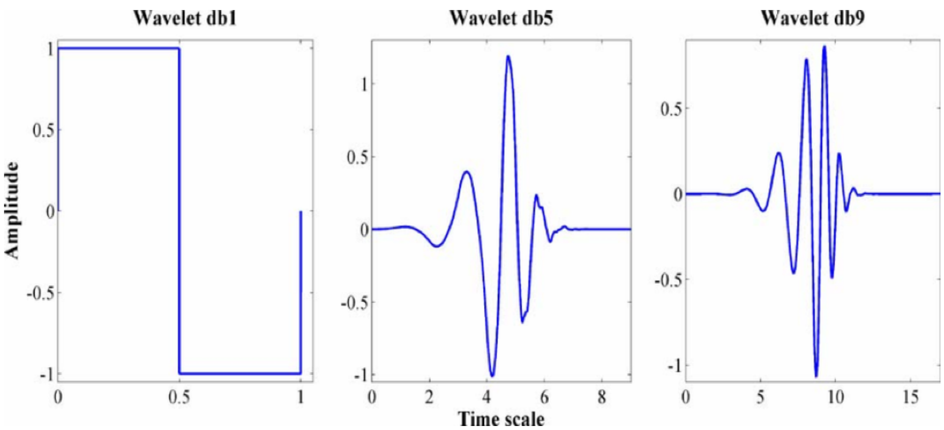


Figure 2: Wavelets: db1, db5, db9

## 1.6 Explanation

Discrete Wavelet Transform is applied on the input image given, and the output resulted in vertical, horizontal and diagonal coefficients. These coefficients represent noise in given image.

## 1.7 Output

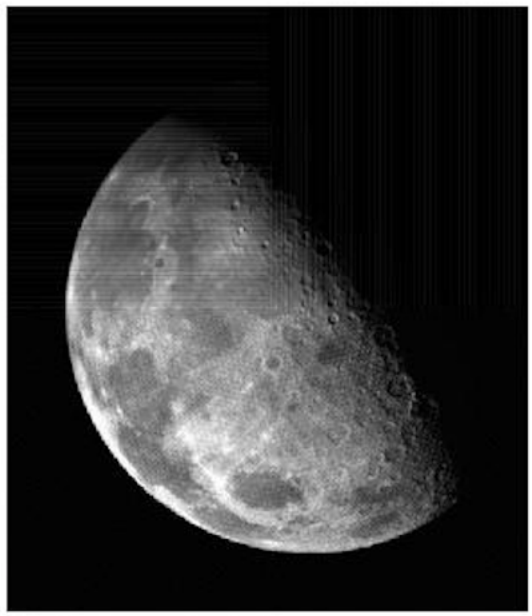

Figure 3: Given Input Image

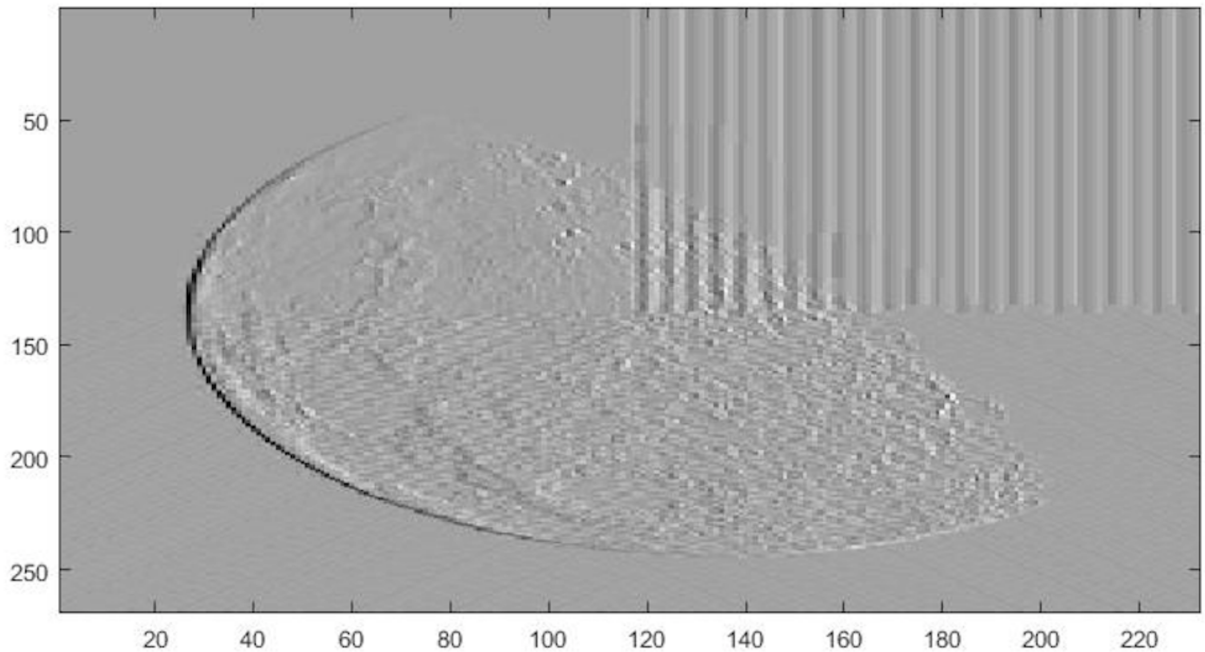


Figure 4: Vertical Coefficient

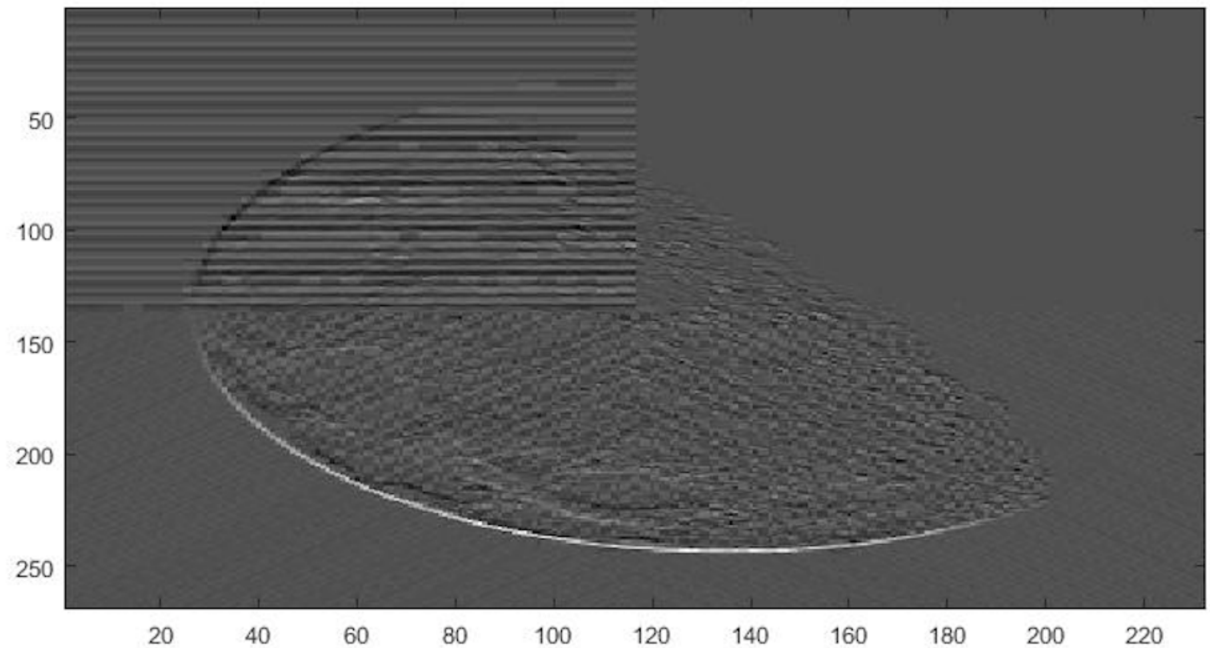


Figure 5: Horizontal Coefficient

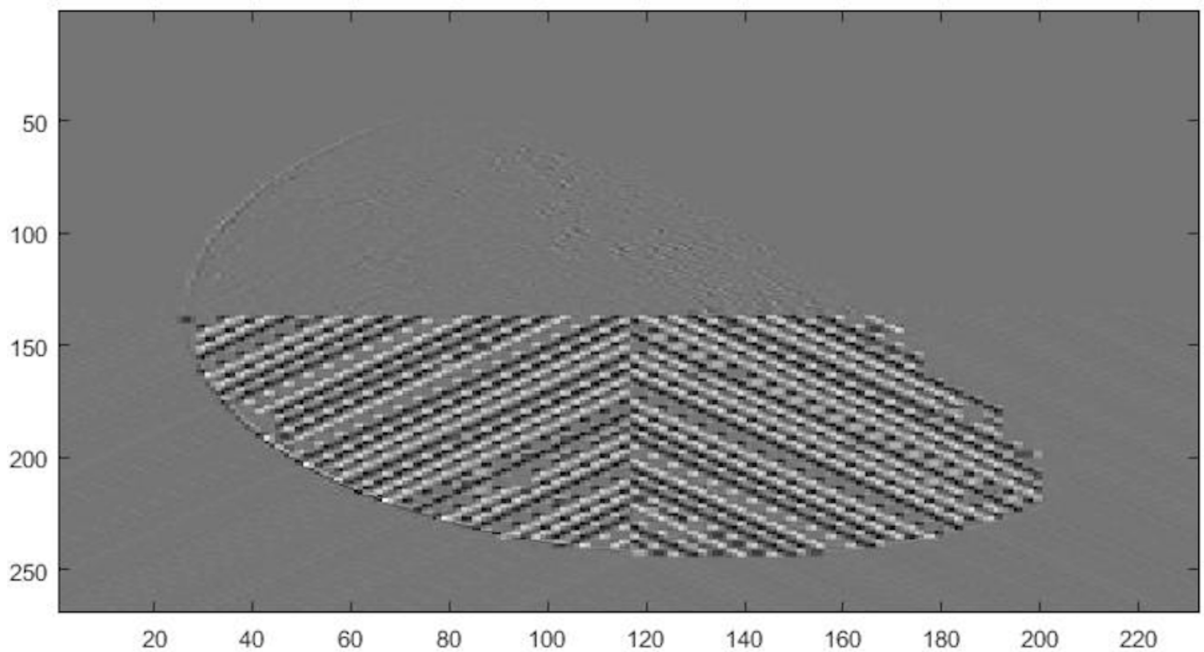


Figure 6: Diagonal Coefficient

# 2 Part II

## 2.1 Variance

The variance is a measure of the distribution, or range of pixel values, of an image. If an image is quite uniform throughout, the variance will be small. If pixel values vary widely from the mean, then the variance will be larger.[3]

## 2.2 Gaussian Noise

Gaussian Noise (Amplifier Noise) The term normal noise model is the synonym of Gaussian noise. This noise model is additive in nature and follow Gaussian distribution. Meaning that each pixel in the noisy image is the sum of the true pixel value and a random, Gaussian distributed noise value. The noise is independent of intensity of pixel value at each point.[4]

## 2.3 Explanation

Gaussian noise is added to a blank image (to obtain the same vertical, horizontal and diagonal components). Discrete wavelet transform is applied to this image and vertical, horizontal and diagonal coefficients are obtained. The variance of these coefficients is calculated and displayed on the image itself. The value of variance is the same for these three components.

## 2.4 Output

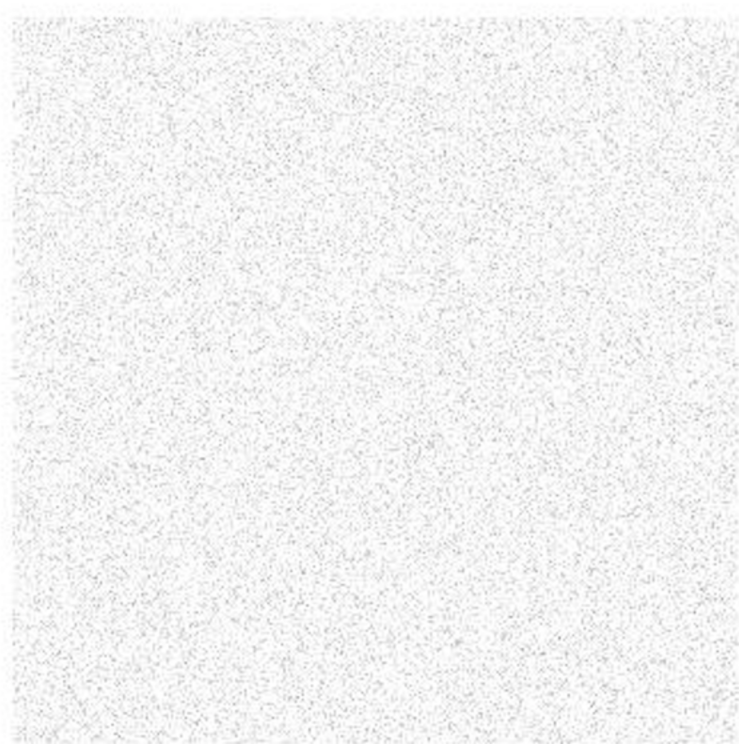

Figure 7: Blank Image

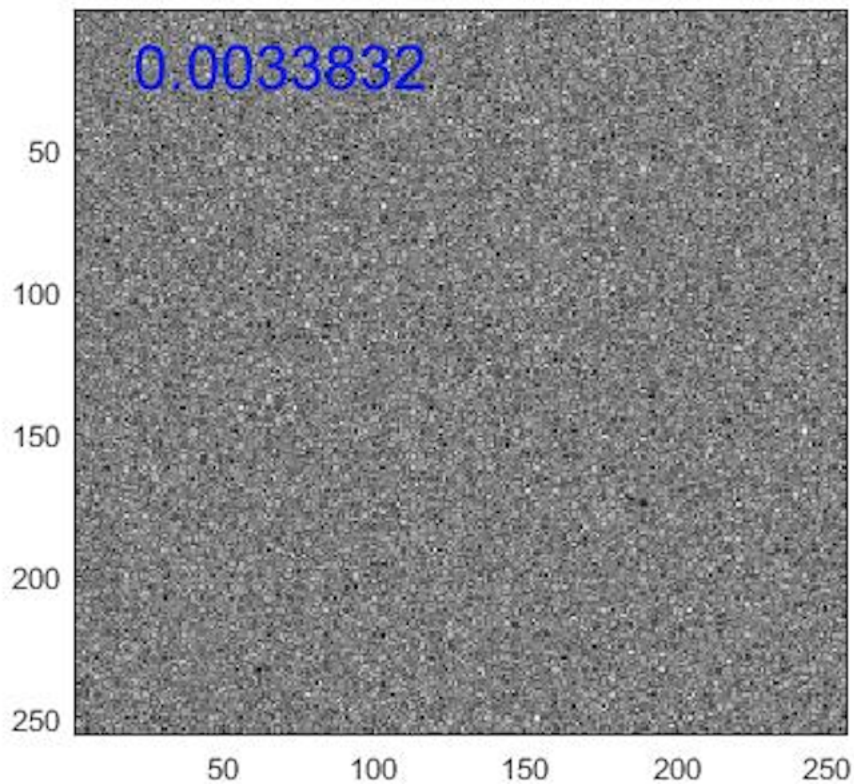


Figure 8: Vertical Coefficient, Variance displayed

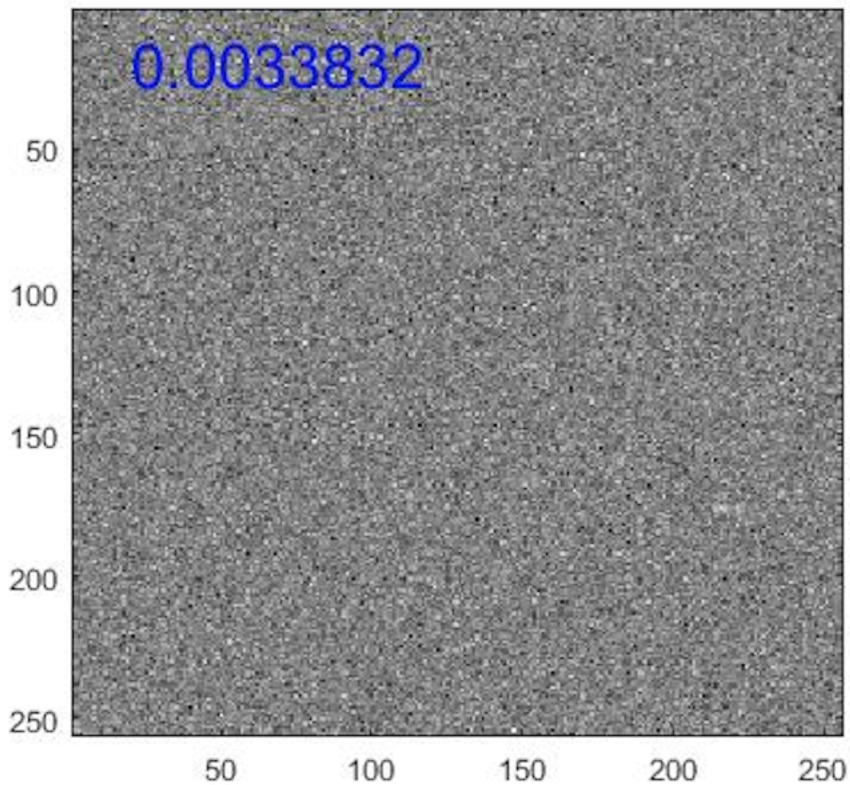


Figure 9: Horizontal Coefficient, Variance displayed

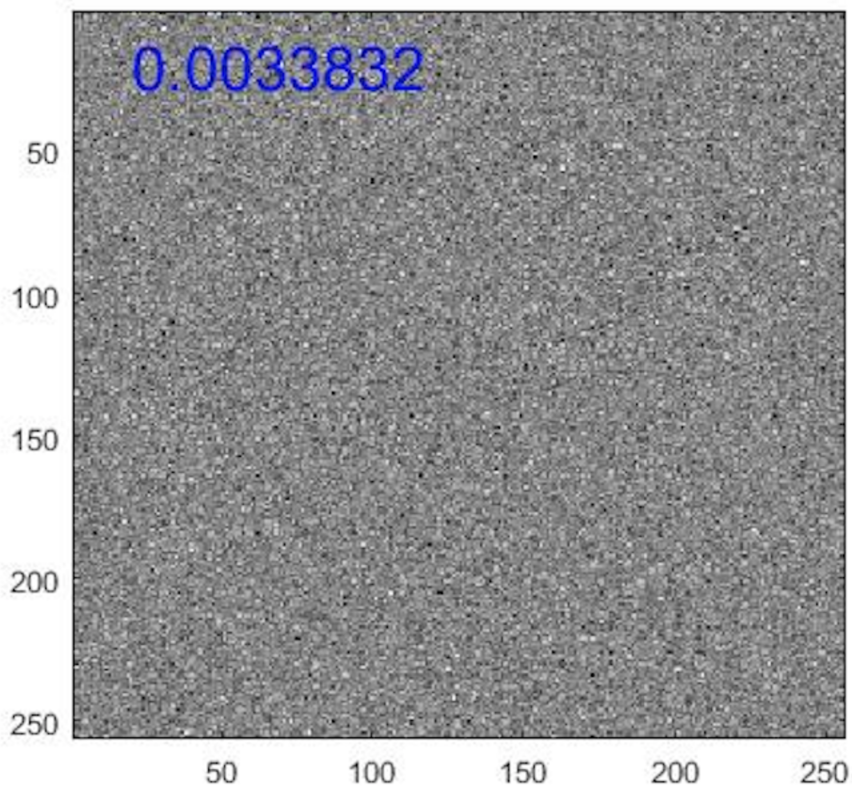


Figure 10: Diagonal Coefficient, Variance displayed

# 3 Part III

## 3.1 Soft Thresholding

Soft thresholding preserves the edges by less smoothening them. The soft thresholding shrinks the coefficients above the threshold in absolute value of wavelet.

Soft thresholding procedures is in the choice of the nonlinear transform on the empirical wavelet coefficients.

In addition, you we use a data driven procedure for choosing the threshold. This procedure is based on Stein's principle of unbiased risk estimation. The main idea of this procedure is the following. Since s(x)is a continuous function, given t, one can obtain an unbiased risk estimate for the soft thresholding procedure. Then the optimal threshold is obtained by minimization of the estimated risks.

## 3.2 Explanation

Added Gaussian noise to an image. Applied soft-thresholding on this noisy image. On the application of soft-thresholding, the noise in the noisy image is reduced significantly.

### 3.3 Output

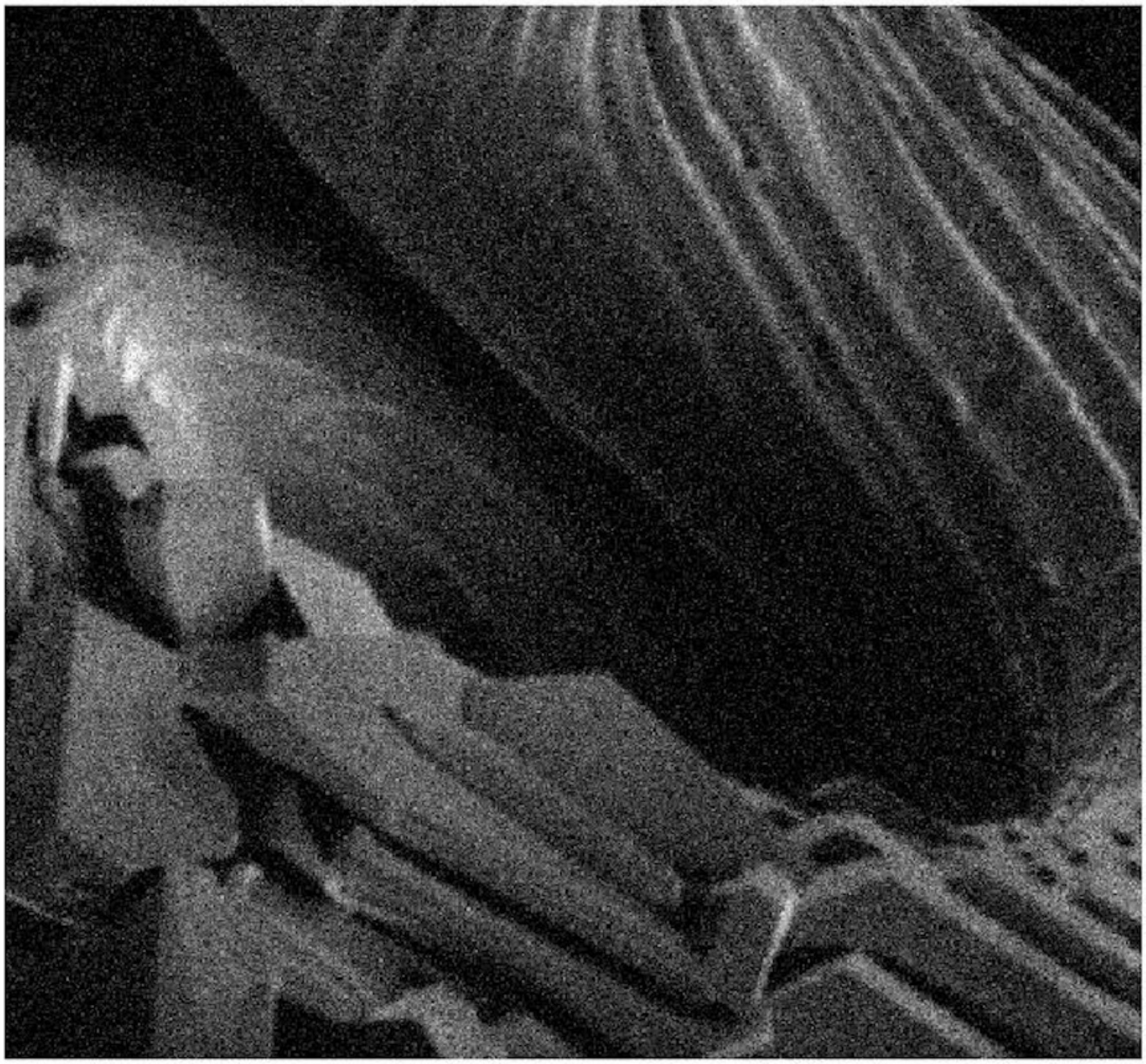

Figure 11: Noisy Image

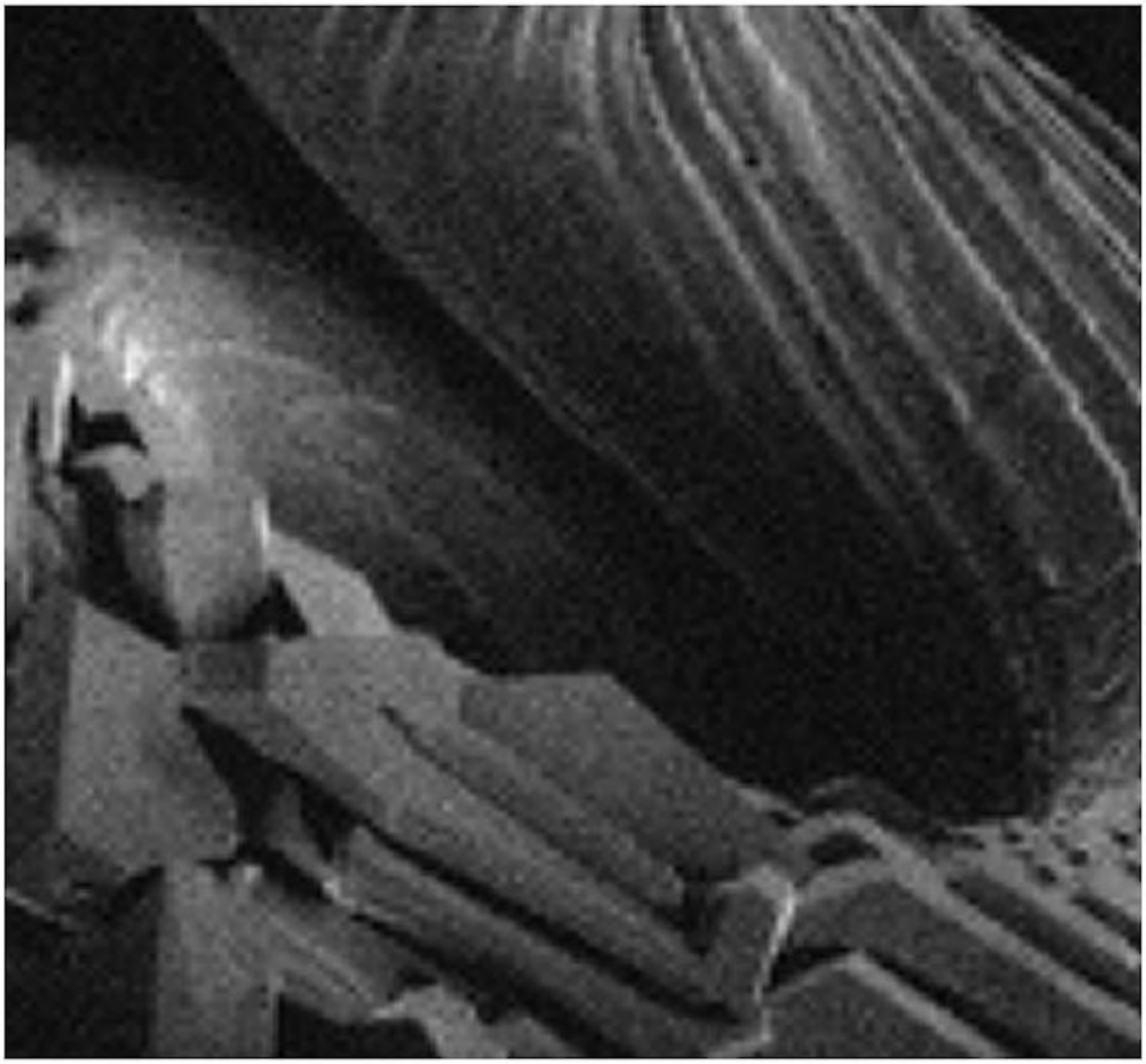

Figure 12: After Soft-Thresholding

# 4 Part IV

## 4.1 Explanation

We used function : $NC = wthcoef2('type', C, S, N, T, SORH)$

Some of the noise components in the vertical coefficient of noise are visible Figure 15.

## 4.2 Output

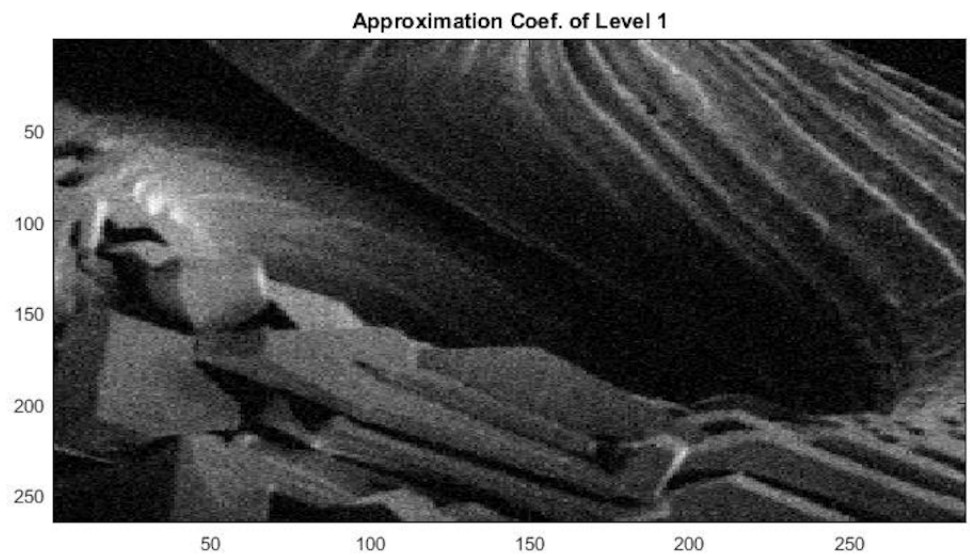


Figure 13: Bonus Image Output

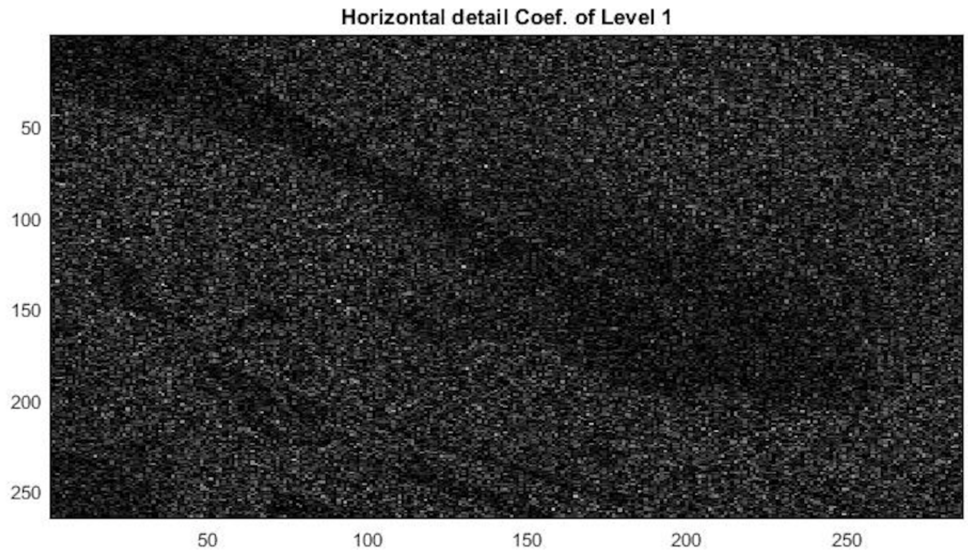


Figure 14: Bonus Image Output

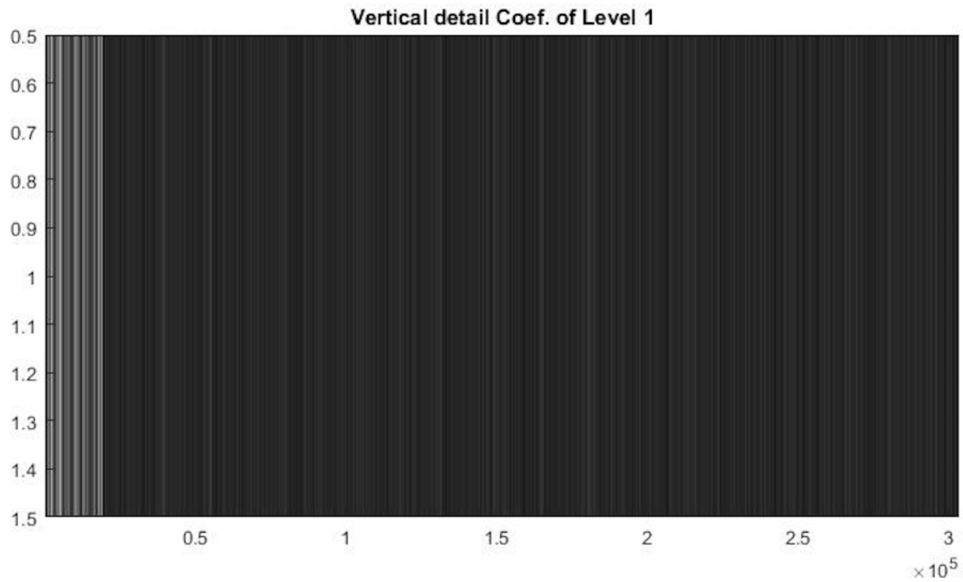


Figure 15: Bonus Image Output

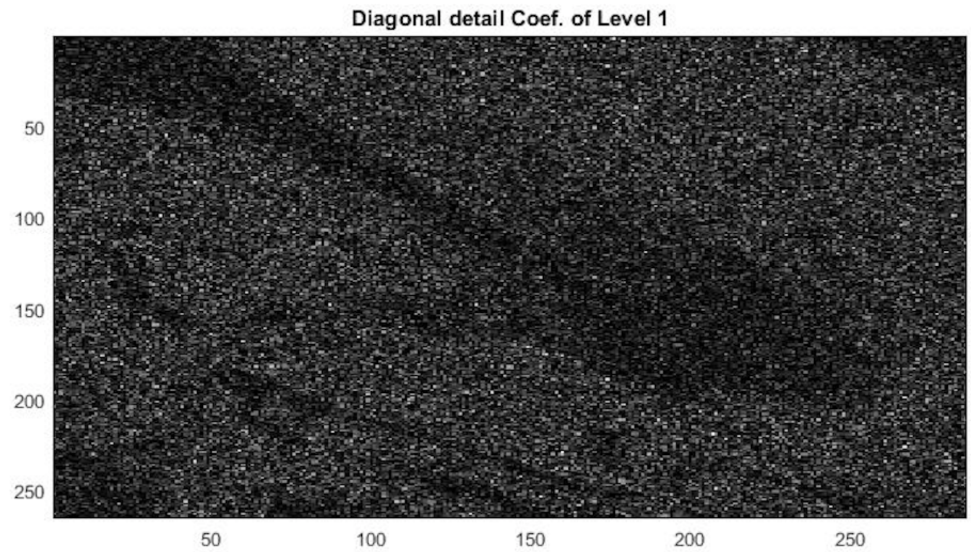


Figure 16: Bonus Image Output

# 5 Source Code

```matlab
%##########################################################################
%##                      Students names :                               ##
%##                      Omar Rawashdeh                                  ##
%##                      Harshal Raut                                    ##
%##                      Utsav Shah                                      ##
%##########################################################################
%##########################################################################
%##                Digital Image Processing HW6                          ##
%##                      Wavelet Transform                               ##
%##                   Matlab R2016a was used                             ##
%##                        v9.0.0.341360                                 ##
%##########################################################################

%close all figures
close all;
clear all;

%prepare folders
warning('off', 'MATLAB:MKDIR:DirectoryExists');
mkdir(fullfile(pwd,'results'));
warning('on', 'MATLAB:MKDIR:DirectoryExists');

%first problem
%load the image
MoonImage_With_PeriodicNoise = imread(fullfile(pwd, 'mooncorrupt.jpg'), 'jpg' );

```

```

[cA,cH,cV,cD] = dwt2(MoonImage_With_PeriodicNoise,'db1');

Fig1 = figure('Name', 'locating periodic noise using DWT', 'Visible', 'On', 'units','normalized','outerposition',[0 0 1 1]);
subplot(221);
imshow(MoonImage_With_PeriodicNoise);
subplot(222);
imagesc(cV);
colormap gray;
subplot(223);
imagesc(cH);
colormap gray;
subplot(224);
imagesc(cD);
colormap gray;

%second problem
BlankImage = ones(512, 512);
BlankImage = BlankImage * 128;
BlankImageWithNoise = imnoise(BlankImage,'gaussian',0,0.01);

[cA,cH,cV,cD] = dwt2(BlankImageWithNoise,'db1');

Fig2 = figure('Name', 'locating periodic noise using DWT', 'Visible', 'On', 'outerposition',[0 0 1024 1024]);
subplot(221);
imshow(BlankImageWithNoise);

subplot(222);
imagesc(cV);
Variance = var(double(cV(:)));
text(20, 20, num2str(Variance), 'Color','white','FontSize',20);
colormap gray;

subplot(223);
imagesc(cH);
Variance = var(double(cV(:)));
text(20, 20, num2str(Variance), 'Color','white','FontSize',20);
colormap gray;

subplot(224);
```

```
imagesc(cD);
Variance = var(double(cV(:)));
text(20, 20, num2str(Variance), 'Color','white','FontSize',20);
colormap gray;


%third problem
OriginalImage = imread(fullfile(pwd, 'Fig3-43a.jpg'), 'jpg');
% NoisyImage = OriginalImage + 15*randn(size(OriginalImage));
NoisyImage = imnoise(OriginalImage,'gaussian',0,0.01);

[thr,sorh,keepapp] = ddencmp('den','wv',NoisyImage);
denoised_image = wdencmp('gbl',NoisyImage,'sym4',2,thr,'s',keepapp);

Fig3 = figure('Name', 'locating periodic noise using DWT', 'Visible', 'On', 'units','normalized','outerposition',[0 0 1 1]);
subplot(121);
imshow(NoisyImage, []);
subplot(122);
imshow(denoised_image, []);

% %bonus problem
% Image = imread(fullfile(pwd, 'Fig2-21a.jpg'), 'jpg');
% NoisyImage = imnoise(Image,'gaussian',0,0.01);
%
%
% [c,s]=wavedec2(Image,2,'haar');
%
% [thr,sorh,keepapp] = ddencmp('den','wv',NoisyImage);
% DcV = wthcoef2('v', c, s, 1 , thr, 's');
%
% [H1,V1,D1] = detcoef2('all',c,s,1);
% A1 = appcoef2(c,s,'haar',1);
% V1img = wcodemat(DcV,255,'mat',1);
% H1img = wcodemat(H1,255,'mat',1);
% D1img = wcodemat(D1,255,'mat',1);
% A1img = wcodemat(A1,255,'mat',1);
%
% Fig4 = figure('Name', 'locating periodic noise using DWT', 'Visible', 'On', 'units','normalized','outerposition',[0 0 1 1]);
```

```
% subplot(2,2,1);
% imagesc(A1img);
% colormap gray;
% title('Approximation Coef. of Level 1');
%
% subplot(2,2,2);
% imagesc(H1img);
% title('Horizontal detail Coef. of Level 1');
%
% subplot(2,2,3);
% imagesc(V1img);
% title('Vertical detail Coef. of Level 1');
%
% subplot(2,2,4);
% imagesc(D1img);
% title('Diagonal detail Coef. of Level 1');
```

# Bibliography


[1] Robert Maidstone. *Wavelets in a Two-Dimensional Context*. Tech. rep. Lancaster University, May 28, 2012. URL: `www.lancs.ac.uk/~maidston/FinalReport.pdf`.

[2] Ms. Sonam Malik and Mr. Vikram Verma. *1) Comparative analysis of DCT, Haar and Daubechies Wavelet for Image Compression*. Tech. rep. International Journal of Applied Engineering Research, ISSN 0973-4562 Vol.7 No.11 © Research India Publications, 2012.

[3] H. E. Burdick. *Digital Imaging: Theory and Applications*. Tech. rep. McGraw-Hill, 1997.

[4] Mr. Rohit Verma and Dr. Jahid Ali. *A Comparative Study of Various Types of Image Noise and Efficient Noise Removal Techniques*. Volume 3, Issue 10, International Journal of Advanced Research in Computer Science and Software Engineering, October 2013.