Conservation of Momentum in One-Dimension

1. OBJECTIVE

The objective of this laboratory experiment is to test the validity of the laws of conservation of momentum and conservation of energy. The collision of carts of the same or different masses as they travel in a straight line will be analyzed.

Learning outcomes: Understand the difference between energy and momentum conservation and when they are conserved or not conserved.

2. THEORY

The momentum (p) of an object is defined as the product of its mass (m) and velocity (v):

$$p = mv \tag{1}$$

Momentum is a vector quantity. For a system that is not influenced by outside forces, the total momentum of the system is conserved. For example, for a system of two masses that collide (or interact), the total initial momentum is equal to the total final momentum. In a collision, energy may be conserved (elastic collision) or energy may not be conserved (inelastic collision).

Elastic collisions occur when two objects collide with no loss of total energy (to internal energy) in the interaction. The collision of billiards balls is a good example of such a process. Inelastic collisions occur when two objects collide and, in the collision process, there is a loss of total kinetic energy; the energy is transferred to internal energy of the materials. Examples include objects sticking together or deforming during the collision without restoration of stored energy after the interaction such as two clay balls colliding, sticking, and then moving as one object or two objects colliding and deforming in a collision as in an automobile collision.

Linear momentum is transferred from one object to the next, and if the two objects are the same mass, all of the momentum of the first is transferred to the second.

Developing the equation for momentum and assuming no changes in mass, we find that force is equal to the time rate of change of momentum.

$$p = mv = m\frac{dx}{dt} \tag{2}$$

$$F = ma = \frac{dp}{dt} \tag{4}$$

We also recall the definition of kinetic energy

$$KE = \frac{1}{2}mv^2 \tag{3}$$

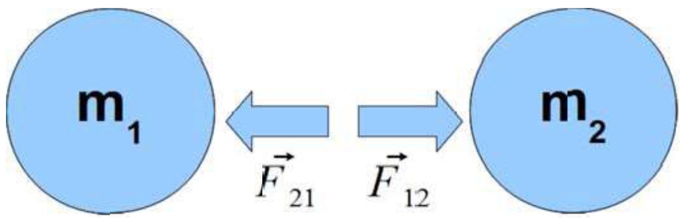
Conservation of linear momentum

For a system of particles, the forces that particles exert on each other are termed internal forces, while the forces exerted on the system by agents outside the system are termed external forces. If there are no external forces exerted on the system, the system is called an isolated system. Consider an isolated system comprised of just two masses, m1 and m2 that collide along a straight line.

The free body diagrams of the two interacting particles are shown in the figure. From Newton's Third Law:

$$\vec{F}_{12} = \vec{F}_{21}$$

Thus, the net force on the system is zero.



Combining this result with Newton's Second Law,

$$\vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}}{dt} = 0$$

so the momentum, $\vec{p}$, is a constant. Thus, when two particles collide, with no net external force present, the total linear momentum of the particles is conserved, i.e., the momentum of the two particles *before* the collision is equal to the net momentum of the two particles *after* the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Kinetic energy considerations – Elastic vs Inelastic collisions

It was shown above that for collisions for which net external forces are equal to zero, momentum is conserved. However, the total kinetic energy of a system is not necessarily conserved, even if the net external force on the particles is zero because the interaction can result in a change in the internal energy of the system. That is

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 \text{ may or may not equal } \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

- If the Total Kinetic Energy (the sum of the kinetic energy of all the particles) of a system is conserved (equal before and after the collision, the system is elastic.
- partially inelastic
- fully inelastic

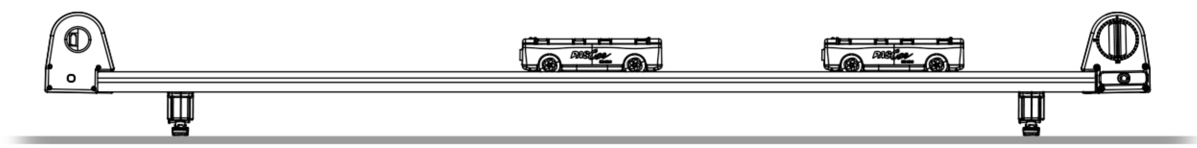
In most realistic situations, collisions are neither perfectly elastic nor perfectly inelastic. In addition, external forces, such as frictional forces, may be present as well in a collision. In this lab, losses due to frictional forces are minimized by using carts of different masses moving on nearly frictionless wheels.

In this lab, you will analyze data that demonstrates that the total linear momentum of a system consisting of two carts on a flat track is conserved in both elastic and inelastic collisions (implying that the total momentum of the system does not change in either collision type), but the total kinetic energy of the system is only constant in elastic collisions.

<u>3. Experimental Set-up</u>

Carts ride along a track with motion sensors at both ends that will detect instantaneous position (measured 50 times/sec). Analysis software provides data on position, velocity and acceleration as a function of time for each cart.

1. The track is set on a level surface with one motion sensor attached to each end. The track is leveled on the surface by rotating the adjusting screws on the feet.



2. Both motion sensors are connected to the data collection system.

> *NOTE: The motion sensors are pointed in opposite directions. In this orientation, one sensor by default will measure positive velocity for an object moving left to right, while the other will measure negative velocity for the same object. This must be corrected within your data collection system so that both sensors report the same velocity for the same object at the same time. The lab write up contains information on how to address this issue.*

3. The carts will travel towards each other and collide. After colliding, they may travel towards the ends of the track. The ends of the track contain a magnet that will interact with the cart to prevent the cart from hitting the sensor and going off the track. Be sure to not analyze this interaction as we are only interested in the carts colliding with each other.