1)

Let $(x_i, y_i) \Rightarrow$ Data points (row)

$n = 10$

$\bar{x} = \sum x_i / n = -0.9$

$\bar{y} = \sum y_i / n = 1.4$

Let, $S_{xx} = \sum (x_i - \bar{x})^2 = 160.9$

$S_{yy} = \sum (y_i - \bar{y})^2 = 164.4$

$\Rightarrow \sigma_x = \sqrt{S_{xx}/(n-1)} = 4.2282$

$\Rightarrow \sigma_y = \sqrt{S_{yy}/(n-1)} = 4.274$

Let, $z_{x_i} = (x_i - \bar{x}) / \sigma_x$

$z_{y_i} = (y_i - \bar{y}) / \sigma_y$

i	x_i	y_i	z_{xi}	z_{yi}
1	−2	1	−0.260	−0.094
2	−5	−4	−0.970	−1.263
3	−3	1	−0.497	−0.094
4	0	3	0.213	0.374
5	−8	11	−1.679	2.246
6	−2	5	−0.260	0.842
7	1	0	0.449	−0.328
8	5	−1	1.395	−0.562
9	−1	−3	−0.024	−1.029
10	6	1	1.632	−0.094

Standard form Data $\Leftarrow$

Let, $\Sigma = Z^T Z (n-1)^{-1}$

Since the data is standardized,

$\Sigma = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$ where r is correlation coeff.

Let, $S_{xy} = \Sigma (x_i - \overline{x})(y_i - \overline{y}) = -66.4$

$$\Rightarrow r = \frac{S_{xy}}{(n-1)\sigma_x \sigma_y} = -0.4083$$

$$\Rightarrow \Sigma = \begin{bmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{bmatrix}$$

$$\det(\Sigma - \lambda I) = 0$$

$$\Rightarrow (1-\lambda)^2 - r^2 = 0$$

$$\Rightarrow \lambda = 1 \pm r$$

$\Rightarrow \lambda_1 = 1.4083$ $\qquad$ $\Rightarrow \lambda_2 = 0.5917$

$$(\Sigma - \lambda_1 I) V_1 = 0$$

$$\Rightarrow \begin{bmatrix} 1-\lambda_1 & -0.4083 \\ -0.4083 & 1-\lambda_1 \end{bmatrix} \begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix} = 0$$

$\Rightarrow -0.4083\, V_{1x} - 0.4083\, V_{1y} = 0$

Let, $V_{1x} = 1 \Rightarrow V_{1y} = -1$

$\Rightarrow V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is first eigenvector

$\Rightarrow u_1 = 2^{-1/2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Similarly,

$u_2 = 2^{-1/2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b)

$PC1_i = u_1 z_i = 2^{-1/2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} [z_{ix} \quad z_{iy}]$

$= (z_{ix} - z_{iy})\, 2^{-1/2}$

=

i	PC1_i
1	−0.118
2	0.208
3	−0.285
4	−0.114
5	−2.776
6	−0.780
7	0.549
8	1.384
9	0.711
10	1.22