

$$1) \theta = [\theta_0, \theta_1]^T$$

$$y = \theta_0 + \theta_1 x$$

$$\text{Let, } X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_{10} \end{bmatrix}$$

Let,  $y$  be  $10 \times 1$  vector

$$\hat{\theta} = (X^T X)^{-1} X^T y$$

$$S_x = \sum_{i=1}^{10} x_i = -9 \quad S_y = \sum_{i=1}^{10} y_i = 14$$

$$S_{xx} = \sum_{i=1}^{10} x_i^2 = 169 \quad S_{xy} = \sum_{i=1}^{10} x_i y_i = -79$$

$$X^T X = \begin{bmatrix} 10 & S_x \\ S_x & S_{xx} \end{bmatrix} = \begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix}$$

$$\begin{aligned} \text{Determinant } (X^T X) &= 10 \cdot 169 - (-9)(-9) \\ &= 1609 \end{aligned}$$

$$(X^T X)^{-1} = \frac{1}{1609} \begin{bmatrix} 169 & 9 \\ 9 & 10 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} S_y \\ S_{xy} \end{bmatrix} = \begin{bmatrix} 14 \\ -79 \end{bmatrix}$$

$$\begin{aligned} \hat{\theta} &= \frac{1}{1609} \begin{bmatrix} 169 & 9 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} 14 \\ -79 \end{bmatrix} \\ &= \begin{bmatrix} 1609^{-1} (169 \cdot 14 + 9(-79)) \\ 1609^{-1} (9 \cdot 14 + 10 \cdot (-79)) \end{bmatrix} \\ &= \begin{bmatrix} 1.0288 \\ -0.4126 \end{bmatrix} \end{aligned}$$

$$\hat{y} = 1.0288 - 0.4126x$$


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$$2) a) \text{ Let, } J = u^2 \quad \Rightarrow u = x_1 + x_2 - 2$$

$$\Rightarrow \frac{dJ}{du} = 2u \quad \Rightarrow \frac{\partial u}{\partial x_1} = 1 = \frac{\partial u}{\partial x_2}$$

$$\Rightarrow \frac{\partial J}{\partial x_1} = 2u \times 1 = 2u = 2(x_1 + x_2 - 2)$$

$$\Rightarrow \frac{\partial J}{\partial x_2} = 2u \times 1 = 2(x_1 + x_2 - 2)$$

$$\nabla J = \begin{bmatrix} 2x_1 + 2x_2 - 4 \\ 2x_1 + 2x_2 - 4 \end{bmatrix}$$

$$2) c) 2x_1 + 2x_2 - 4 = 0$$

$$\Rightarrow x_1 + x_2 = 2 \Rightarrow x_1 = 2 - x_2$$

$$\text{For, } x_2 = 0, x_1 = 2 - 0 = 2$$

$$x_2 = 1, x_1 = 1$$

$$x_2 = 2, x_1 = 0$$

$$\Rightarrow \text{values are } (2, 0), (1, 1), (0, 2)$$