Optimizing Resource Allocation under Envy Constraints

Tharunkumar Amirthalingam (ta9@illinois.edu)

Utsav Majumdar (utsavm2@illinois.edu)

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Prof. Jugal Garg

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Abstract

Efficient resource allocation is imperative in numerous practical applications, often manifesting as minimum cost assignment dilemmas. Such scenarios necessitate optimal task distribution among agents to minimize overall completion time. However, neglecting individual preferences may engender diminished performance and morale among workers. This report presents an approach that integrates considerations of both cost and agents' valuations, seeking to achieve a harmonious balance between minimizing dissatisfaction and maintaining cost-effectiveness. Two primary strategies are examined: post-minimum cost assignment envy cycle elimination and a modified Round Robin method incorporating valuation per cost. Through systematic simulations across varied agent and task configurations, these approaches are assessed against established benchmarks. The findings contribute insights into tailored resource allocation strategies applicable across diverse operational contexts.

Introduction and Motivation

At its core, the optimization of resource allocation hinges upon the intricate task of assigning responsibilities among agents in a manner that minimizes costs while concurrently maximizing productivity and satisfaction. However, this endeavor is markedly complex, given the diverse preferences and valuations inherent to individual agents.

Neglecting the preferences of agents in real-world scenarios can precipitate adverse outcomes. Task assignment devoid of consideration for individual preferences may engender diminished motivation, reduced productivity, and a pervasive sense of dissatisfaction among workforces. Hence, our research is propelled by the compelling need to explore methodologies that seamlessly integrate agents' individual preferences into the resource allocation process alongside traditional cost-based optimization techniques.

Our pursuit is motivated by the recognition that while cost minimization is indisputably vital for organizational efficiency, it must not eclipse the significance of upholding high levels of agent morale and performance. This is pivotal for ensuring the sustained productivity of a firm, as employee morale exerts a profound butterfly effect on organizational performance. An organization fixated solely on cost optimization risks entering an unsustainable cycle where diminishing employee performance precipitates increased turnover rates, leading to escalated expenditure and man-hours in recruitment endeavors. Furthermore, there is a consequential decline in overall performance, compounded by the need for additional investment in training new recruits or enduring a period of suboptimal performance until proficiency is attained. This underscores the importance of prioritizing employee satisfaction, considering the long-term cost implications associated with low morale.

Consequently, our endeavor embarks on a quest to unravel the intricate relationship between cost optimization and agent satisfaction. Through meticulous analysis and simulation, our study aims to shed light on the delicate equilibrium between minimizing costs and maximizing agent contentment. By elucidating this multifaceted interplay, we aspire to furnish decision-makers with invaluable insights and actionable recommendations for refining resource allocation strategies across diverse operational landscapes. Ultimately, our research endeavors to empower decision-makers with the requisite tools to navigate the inherent trade-offs in resource allocation, thereby fostering more efficient and sustainable organizational practices.

Problem Formulation

Premise:

In a scenario where a manufacturing firm is seeking to efficiently allocate tasks among recruited employees, various factors come into play. Each employee possesses unique skills and efficiency levels, influencing how quickly they can complete assigned tasks. Naturally, the aim is to minimize the overall time required for task completion, a classic optimization challenge where time is the primary cost variable.

However, there's an added layer of complexity: the individual preferences of each employee, whom we'll refer to as 'agents'. We refer to the preference as ‘valuation’ and it is a score that can be obtained by a simple survey among the agents. If an agent isn't assigned their preferred task, it leads to dissatisfaction and a sense of envy within the system. Therefore, the overseer must also consider the subjective valuation that each agent places on their tasks.

This dual objective—minimizing time/cost while maximizing agent satisfaction—often creates a balancing act. Sometimes, the optimal allocation for time efficiency conflicts with satisfying agent preferences, necessitating a compromise that serves the best interests of the firm overall.

Assumptions:

Additivity: We consider the Costs and Valuations to be monotonic and additive.

Positivity: For our analysis we have considered, costs and valuations to be positive. However, the approaches we have taken can accommodate for negative values as well with minor tweaks.

Independence: We have considered the cost and valuation to be independent of each other. This essentially means that each agent behaves differently. In the context of the example given previously, we can have agents who prefer a task more because it takes them less time and we can also have agents who prefer to take more time in completing a task that they particularly enjoy doing. However, there may be special circumstances or assumptions where the cost and valuations are dependent on each other. We do not study these special cases in this report and work on a more general setting.

Normalization: We consider the cost and valuations as random values uniformly distributed between 0 and 1.

Elimination of triviality: The number of tasks is assumed to be more than the number of agents. This removes the trivial circumstance where every agent always gets their most preferred task automatically. We also assume that the number of agents is atleast more than 1.

Formulation:

We work with a cost matrix and a valuation matrix. Our objective is to come up with an assignment that minimizes total cost and maximizes every agent's self-valuation.

From the context of valuation, our ideal objective would be to reduce envy throughout the system. That is only possible if an agent's valuation of their own tasks is higher than their valuation for any other agent's bundle. Hence, it is always beneficial to study every agent's own valuation since it serves as a strong correlation with overall satisfaction.

Suppose there is n agents and m tasks, where $m>n>1$.

Let N be the set of agents and M be the set of tasks

Let cost matrix (nxm) be C, where each value c_{ij} denotes the cost of agent i for task j.

Let valuation matrix (nxm) be V, where each value v_{ij} denotes the value of agent i for task j.

Let A_i be the bundle allocated to agent i

The problem can be stated as :

$\min \Sigma_{i \in N, j \in M}\, c_{ij}x_{ij}$

s.t. $\Sigma_{i \in N}\, x_{ij} = 1 \quad \forall\ j \in M$

$\exists\ g \in A_k: v_i(A_i) \geq v_i(A_k \backslash g)\ \forall\ k \in N$

$x_{ij} \in \{0,1\}\ \forall\ i \in N, j \in M$

Approaches

We have taken 6 approaches and simulated their performance with respect to cost and value:

1. Random Assignment
2. Min Cost Assignment
3. Round Robin Assignment
4. Envy cycle elimination post Min Cost Assignment
5. Bang for Buck variation of Round Robin Assignment
6. Bang for Buck variation on Envy cycle elimination post Min cost assignment

Random Assignment

Here we randomly assign tasks to agents without taking into consideration the cost and valuations. We just make sure that each task is allocated to a single agent. However, an agent can have multiple tasks. We randomly assign tasks until there is no unassigned tasks left.

Min Cost Assignment

Here we consider the costs only. It becomes a standard BIP optimization problem which we solve using an optimization solver like Gurobi. This approach guarantees us the best possible cost. So we can use it as a standard benchmark to compare the costs of the other approaches.

$\min \Sigma_{i\in N,\, j\in M}\, c_{ij}x_{ij}$

s.t. $\Sigma_{i\in N}\, x_{ij} = 1 \quad \forall\ j \in M$

$x_{ij} \in \{0,1\}\ \ \forall\ i \in N, j \in M$

Round Robin Assignment

In this approach, we only focus on the values. The Round Robin algorithm is guaranteed to provide an EF1 allocation, which in turn ensures the best possible valuation for every agent's own goods and hence the least envy in the system.

Algorithm

1. Start with empty allocations
2. Arbitrarily order the agents
3. Let the remaining goods be R
4. Repeat while there are remaining goods:
    a. Let each agent pick one good from the remaining goods
    b. Assign to each agent $i \in N$, the following good : $\mathrm{argmax}_{j\in R}\, v_{ij}$

Envy Cycle Elimination post Min Cost Assignment

Here we initially run the min cost assignment approach which can be done using a standard BIP optimization solver (Gurobi).

Once the initial assignments are completed, we take the valuations into consideration.

Rest of the algorithm:

1. Construct an Envy Graph
2. If there are any envy cycles, swap the bundles along the cycle
3. Repeat until there is no cycle.

Bang for Buck variation on Round Robin Assignment

In this approach, we only focus on the values relative to the cost of a task. We run the Round Robin algorithm with a modification of taking valuation per cost.

Algorithm

1. Start with empty allocations
2. Arbitrarily order the agents
3. Let the remaining goods be R
4. Repeat while there are remaining goods:
    a. Let each agent pick one good from the remaining goods
    b. Assign to each agent $i \in N$, the following good : $\text{argmax}_{j \in R}\ v_{ij}/c_{ij}$

Bang for Buck variation on Envy cycle elimination post Min cost assignment

First we consider $c_{ij}' = c_{ij} / v_{ij}$. With the new cost, we run the min cost assignment approach which can be done using a standard BIP optimization solver (Gurobi).

Once the initial assignments are completed, we take the valuation / cost into consideration.

Rest of the algorithm:

1. Construct an Envy Graph where $v_{ij}' = v_{ij}/c_{ij}$
2. If there are any envy cycles, swap the bundles along the cycle

Repeat until there is no cycle.

Experiment: Setup

We consider the following number of agents:

$|N| = \{10, 20, 30, 40, 50, 60\}$

We consider the following number of tasks:

$|M| = \{60, 80, 100, 120, 140\}$

We iterate over every combination of number of agents and tasks for a total of 30 combinations.

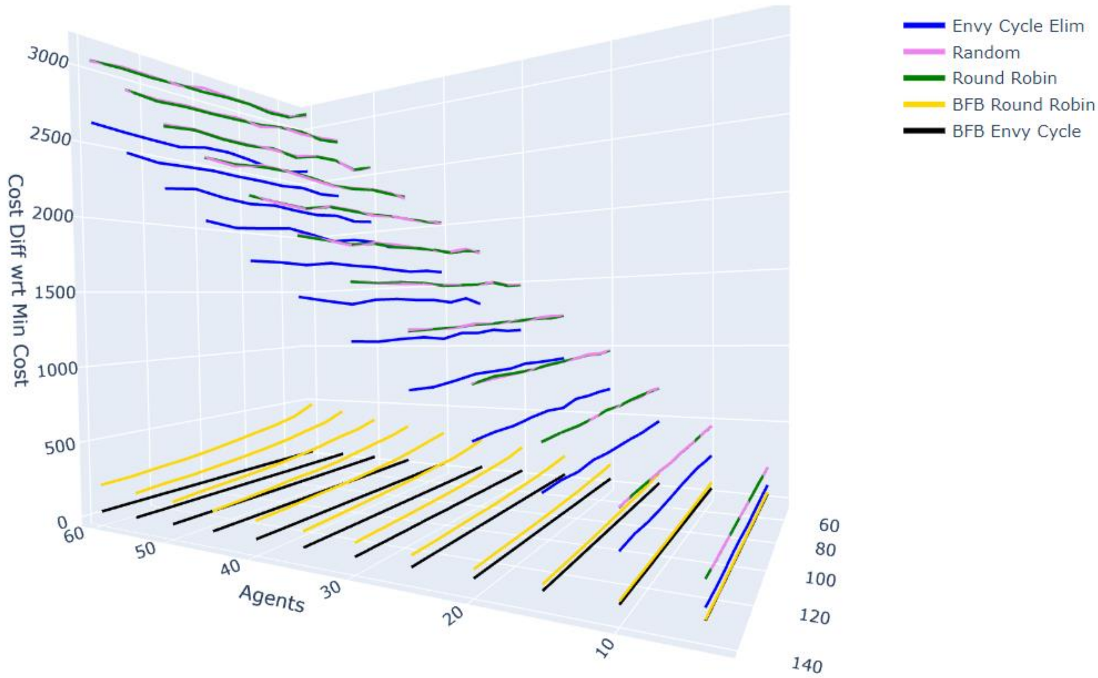
We use the terms 'goods' and 'tasks' inter-changeably.

We simulate all 6 approaches for 1000 iterations each and study the cost and valuations of every approach compared against the best approach for cost (min cost assignment) and best approach for value (Round Robin assignment).

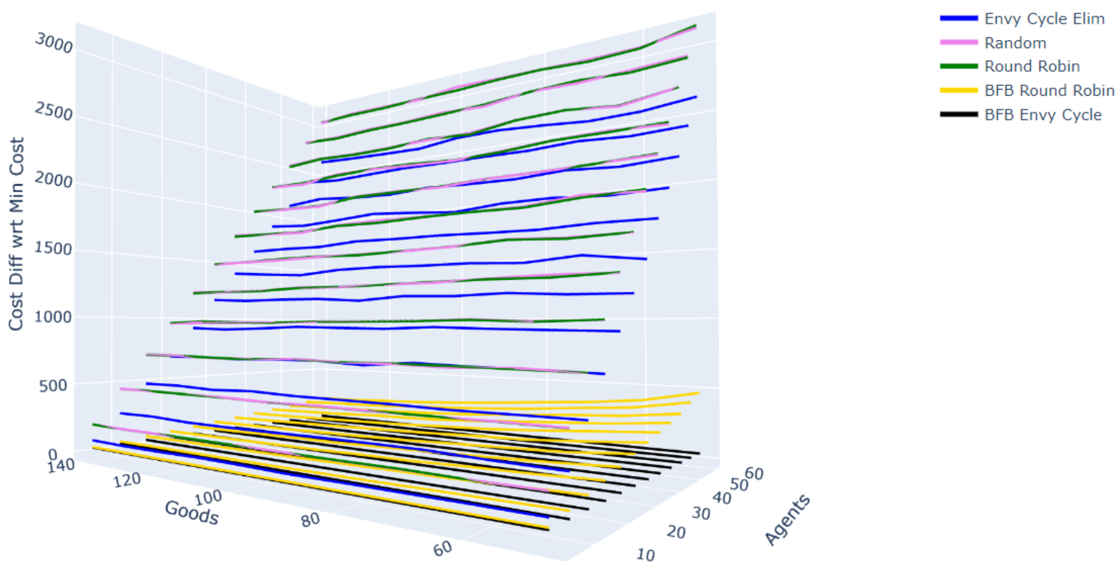
Experiment: Simulations

Cost Difference % w.r.t Min cost assignment

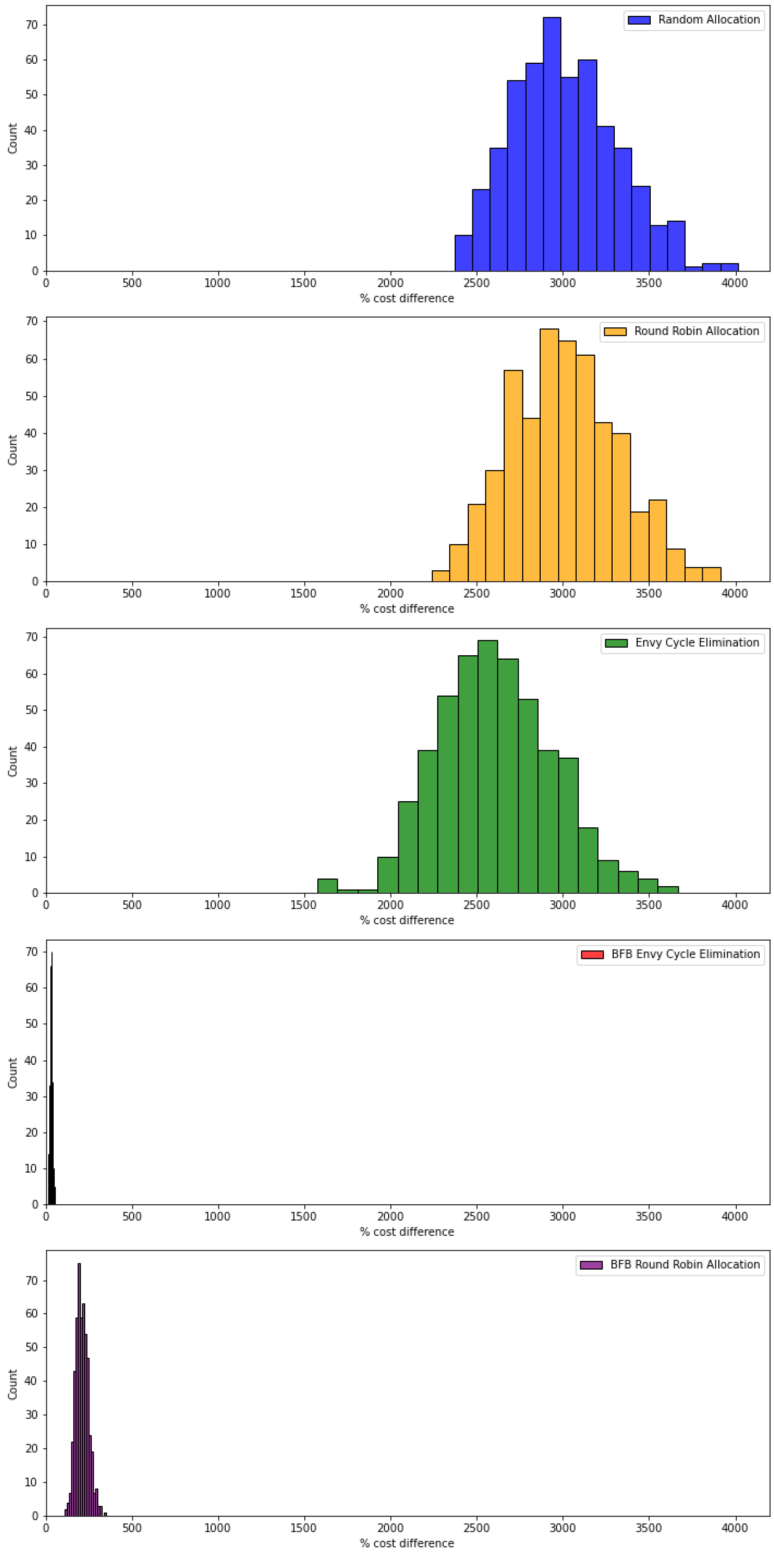
For aid in visualizing cost, please click **HERE** .



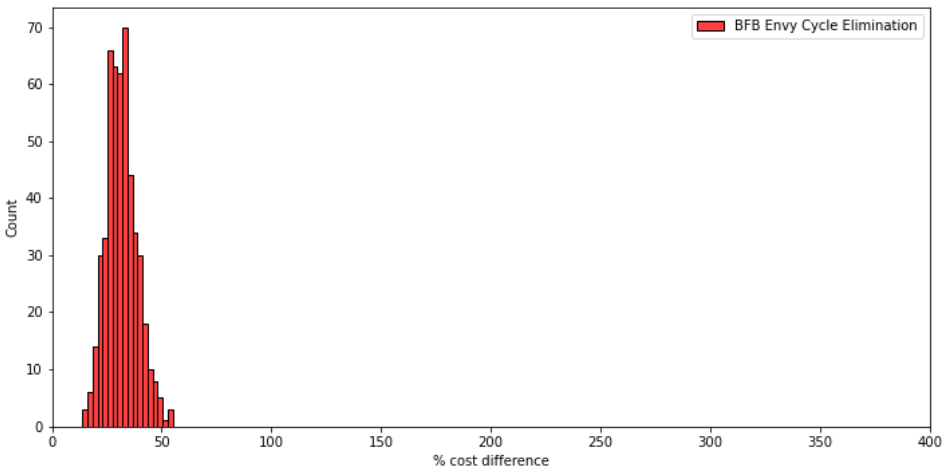
Right off the bat, we notice that with more agents, cost is worsening for all approaches, except Bang for Buck variation of Envy cycle elimination post min cost allocation. It seems to be relatively the most stable among the lot, followed by the Bang for Buck variation of Round-Robin approach.

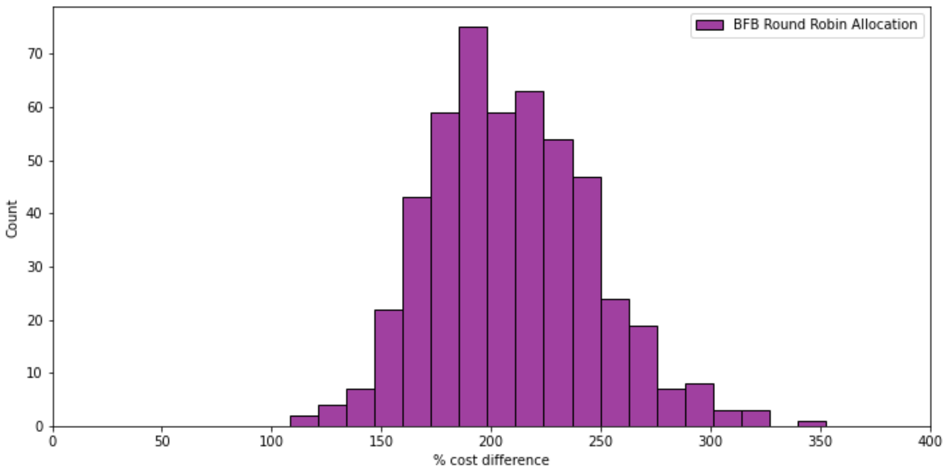


With respect to goods, we notice that Bang for Buck variation of the Round Robin algorithm performs significantly better for higher number of goods. For the other approaches, the increase in number of goods is not quite affecting the cost much.



The Bang for Buck variations on both Envy Cycle Elimination and Round Robin approaches perform much better than the other 3 approaches.

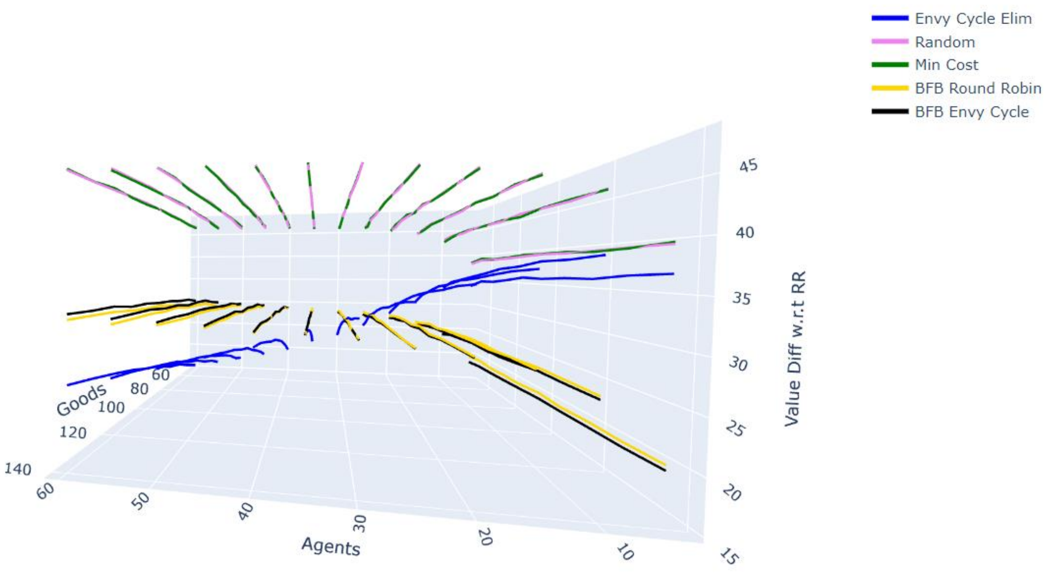




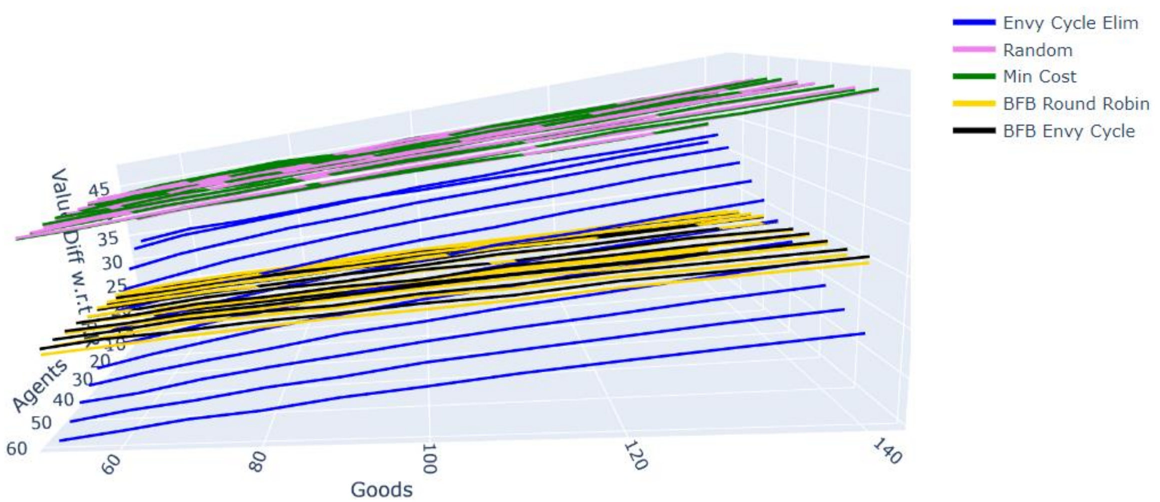
If we take a closer look at the 2 Bang for Buck approaches we notice that BFB Envy Cycle Elimination gives a much better performance.

Value Difference % w.r.t Round Robin assignment

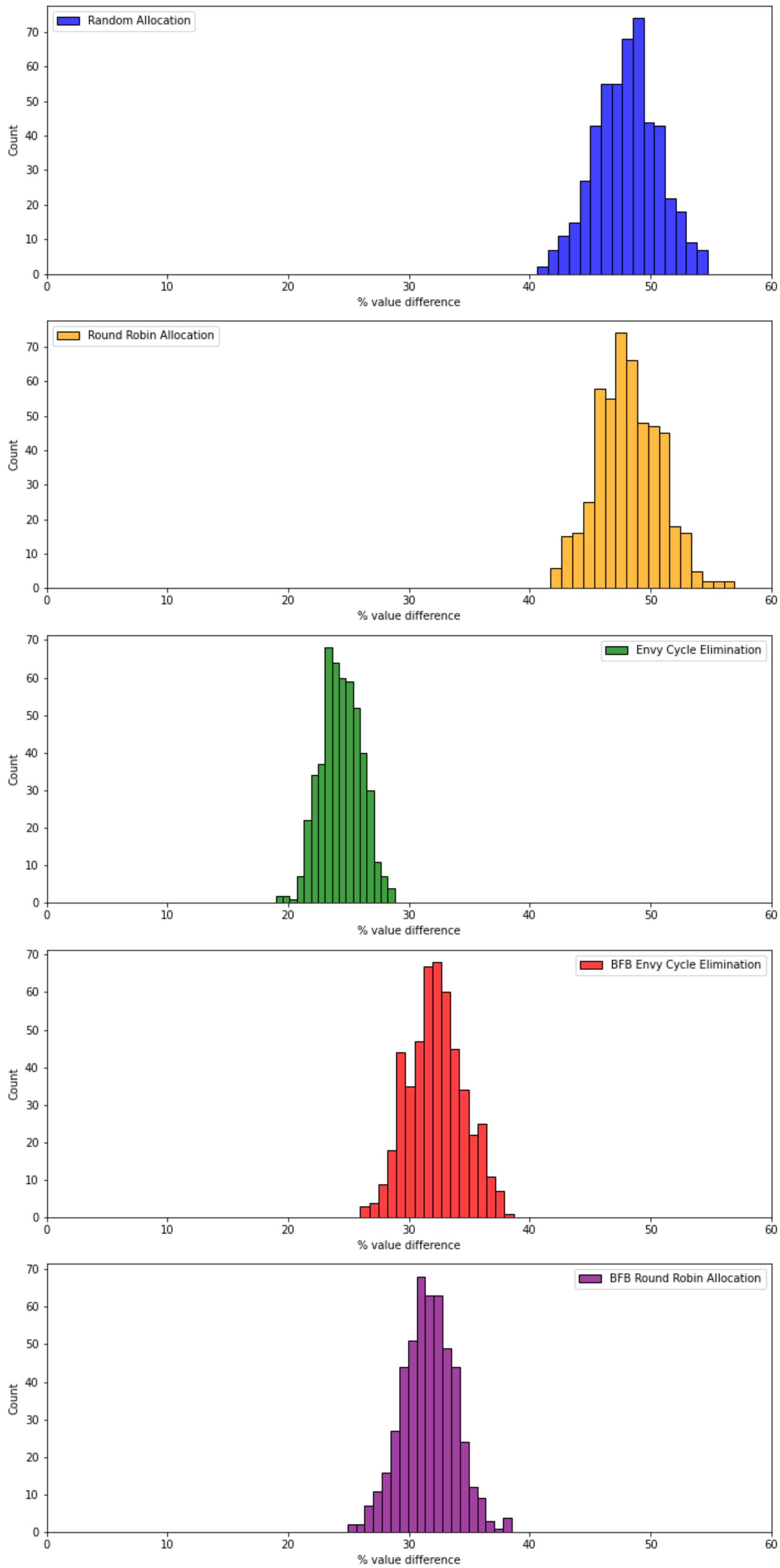
For aid in visualizing cost, please click **HERE** .



The Envy cycle elimination post min cost assignment approach shows a very interesting behaviour where it performs well for higher number of agents while surpassing all other approaches.



All approaches perform worse with increase in the number of goods. We note that Bang for Buck Envy cycle elimination performs better than Bang for Buck Round Robin approach for less number of agents.


Random Allocation
Round Robin Allocation
Envy Cycle Elimination
BFB Envy Cycle Elimination
BFB Round Robin Allocation
Count
% value difference
0
10
20
30
40
50
60
70

Envy cycle elimination seems to be performing the best with respect to valuation. However, if we take costs into consideration, it falls out of favour for higher number of agents, drastically.

Results and Observations

We make the following observations:

1. The approach where we considered the cost/value for min cost assignment followed by value/cost for Envy cycle elimination performs the best with respect to cost and value in most cases.
2. The Bang for Buck variation on Round Robin performs the best for less number of agents and high number of tasks.
3. The envy cycle elimination (without any variation) post min cost assignment performs the best in terms of values with increased number of agents and reduced number of goods. We even see that it surpasses the performance of Bang for Buck envy cycle elimination approach and the Bang for Buck Round Robin assignment as well in some cases (especially where agents exceed 30 or goods are less than 100)
4. The Bang for Buck Round Robin is stable and is as good as Bang for Buck Envy cycle elimination in terms of value, however doesn't perform as good as the Bang for Buck Envy cycle elimination in terms of cost.

Conclusion and Further Work

We have seen a few heuristic approaches toward balancing cost and valuation, based on algorithms that yield EF1 allocations. The positive experimental results suggest a good algorithm which can yield the Pareto frontier for cost and envy freeness, or one with approximation guarantees on these parameters, can indeed be found. Approaches that build similar versions of the envy cycle from the empty allocation set could be considered. We would want to study special cases where the values of cost and valuation are interdependent on each other.