

## Announcements:

- + Lectures will be recorded and live-streamed
- + Quizzes are published on Mondays or Fridays after the lecture. They are due by the end of their publishing day
- + Depending on the quiz content, you may need to submit it on Quercus (for auto-grading) or submit it on Crowdmark for manual grading. This would be clarified in the description of the quiz.
- + Assignments will be posted on Quercus and they are due at least one week after their publishing date. The exact due date and submission instruction will be specified upon publishing each assignment.

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Last lecture: I) propositional logics  
II) logical operator  
III) logical equivalence

Plan for today: Extra examples

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**EX**: state the converse of "If it snows, then I will stay at home"

$p$ : "it snows"

$q$ : "I will stay at home"

$P \rightarrow q$

Converse:  $q \rightarrow p$  "If I stay at home, then it snows"

contrapositive:  $\neg q \rightarrow \neg p$  "If I don't stay at home, then it won't snow"

Inverse:  $\neg p \rightarrow \neg q$   
"If not snow, I won't stay home"

**EX** State the following proposition as a compound proposition with  $p, q$ , and conditional operator

To pass this course, It is necessary to get at least 40% on final

$p$ : pass this course  
 $q$ : get at least 40%  
 $p \rightarrow q$

(If someone tells you that they have passed the course, then you conclude that they must have received at least 40% on their final.)

**EX** "When I stay up late, it is necessary that I sleep until noon"

$p$ : "I stay up late"       $p \rightarrow q$   
 $q$ : "sleep until noon."

**EX** What is  $p$ , only if  $q$ .

$$\square p \rightarrow q \quad \square q \rightarrow p$$

A good way to remember this is by examining  $p \leftrightarrow q$ . It is read as  $p$  if and only if  $q$ .

We know that  $p \leftrightarrow q \equiv (q \rightarrow p) \wedge (p \rightarrow q)$

$$\begin{aligned} p \text{ if and only if } q &\equiv (p \text{ if } q) \text{ and } (p \text{ only if } q) \\ &\equiv (q \rightarrow p) \wedge (p \rightarrow q) \end{aligned}$$

**EX** Consider the proposition "You can see the movie only if you are 18 years old or you have the permission of a parent."

Express this compound proposition in terms of

$m$ : "You can see the movie"

$e$ : "You are over 18 years old"

$p$ : "You have the permission of a parent"

$$m \rightarrow (e \vee p)$$

**EX** Using a truth table show the logical equivalency of  $\neg(p \vee q) \equiv \neg p \wedge \neg q$

| p | q | $\neg p$ | $\neg q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p \wedge \neg q$ |
|---|---|----------|----------|------------|------------------|------------------------|
| T | T | F        | F        | T          | F                | F                      |
| T | F | F        | T        | T          | F                | F                      |
| F | T | T        | F        | T          | F                | F                      |
| F | F | T        | T        | F          | T                | T                      |

The same truth value

- Similarly one can show that

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

These two pairs of equivalencies are known as De Morgan's Law.

**EX** Is the statement  $(p \rightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$  a tautology, contradiction, or contingency?

☐ tautology    ☐ Contradiction    ☒ Contingency

| p | q | $p \rightarrow \neg q$ | $p \leftrightarrow q$ | $(p \rightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|------------------------|-----------------------|--|
| T | T | F                      | T                     | F  |
| T | F | T                      | F                     | F  |
| F | T | T                      | F                     | F  |
| F | F | T                      | T                     | T  |