

MAT-CSC A67: Discrete Mathematics — Summer 2024

Quiz 6

Due Date: Monday, June 24, 11:59 PM, on Crowdmark

Q1. Determine which of the following statements are true:

- (i) $(a < b \wedge c < d) \rightarrow (a + c < b + d)$
- (ii) $(a < b \wedge c \leq d) \rightarrow (a + c \leq b + d)$
- (iii) $(a < b \wedge c \leq d) \rightarrow (a + c < b + d)$
- (iv) $(a \leq b \wedge c \leq d) \rightarrow (a + c \leq b + d)$
- (v) $(a < b \wedge c < d) \rightarrow (a - c < b - d)$
- (vi) $(a < b \wedge -c < -d) \rightarrow (a - c < b - d)$
- (vii) $(a < b \wedge c > d) \rightarrow (a - c < b - d)$
- (viii) $(a < b \wedge c > 0) \rightarrow (ac < bc)$
- (ix) $(a < b \wedge c < 0) \rightarrow (ac < bc)$
- (x) $(a < b \wedge c < 0) \rightarrow (ac > bc)$

Q2. How many of the following 14 statements are true.

- (i) $3 \in \{1, 2, 3\}$
- (ii) $1 \subseteq \{1, 2, 3\}$
- (iii) $\{2\} \in \{1, 2, 3\}$
- (iv) $\{3\} \in \{\{1\}, \{2\}, \{3\}\}$
- (v) $1 \in \{1\}$
- (vi) $\{1\} \subseteq \{1, 2, 3\}$
- (vii) $\{2\} \subseteq \{\{1\}, \{2\}, \{3\}\}$
- (viii) $\{1\} \subseteq \{1\}$
- (ix) $\emptyset \subseteq \{1\}$
- (x) $\{\emptyset\} \subseteq \emptyset$
- (xi) $\emptyset \in \emptyset$
- (xii) $0 \in \emptyset$
- (xiii) $2 = \{2\}$
- (xiv) $|\{1, \{1\}, \{1, \{1\}\}\}| = 3$

Q3. Find $n \bmod d$ for $n = -45$ and $d = 11$.

Q4. Prove that for any positive integers a and b and any integer $n \geq 2$, $a \equiv b \pmod{n} \rightarrow a^2 \equiv b^2 \pmod{n}$.