MAT-CSC A67: Discrete Mathematics — Summer 2024

Assignmnet 1: Logic

Due Date: Sunday, 26 May, 11:59 PM, on Crowdmark

Q0. [0 **pt (Warm-up)]** This question is for you to check your understanding of the core concepts. Do not submit your answer to this question on Crowdmark. This question will not be graded, has 0 credit, and has no bonus credit. Feel free to discus your solutions to this question with the teaching team during the office hours and with the other students over Piazza. Do not share your solutions to Q1-8 with the other students or over Piazza.

0.a. Give the truth table for the following compound propositions.

0.a. i. $\neg(p \wedge q) \vee (p \vee q)$.

0.a. ii. $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$.

0.b. Assume x is a real number. Use DeMorgan's Laws to write the negation of the statement:$-3 \leq x < 1$.

0.c. Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ true when p, q, and r do not have the same truth value (*i.e.*, they are not all true or all false) and it is false otherwise.

0.d. For each of the following statements, (1) rewrite the statement formally using quantifiers and variables, and (2) write the negation for the statement.

0.d. i. Everybody trusts somebody.

0.d. ii. Any even integer equals twice some integer.

0.e. For each of the following statements, determine whether the proposed negation is correct. If it is not, write a correct negation.

0.e. i.

Statement: The sum of any two numbers is irrational. (domains: irrational numbers)

Proposed negation: The sum of any two numbers is rational. (domains: irrational numbers)

0.e. ii.

Statement: For all n, if n^2 is even then n is even. (domain: integers)

Proposed negation: For all n, if n^2 is even, then n is not even. (domain: integers)

0.f. For each of the following, (1) write a new statement by interchanging the symbols $\forall$ and $\exists$ (*i.e.*, changing $\forall x \exists y$ to $\exists y \forall x$ and changing $\exists x \forall y$ to $\forall y \exists x$). Then (2) state which is true: the given statement, the interchanged statement, neither, or both. The domains of x and y are real numbers.

0.f. i. $\forall x \exists y (x < y)$

0.f. ii. $\exists x \forall y (y < 0 \rightarrow x > y)$

Q1. [2 **pts]** Use truth tables to determine whether the two statements is each part are logically equivalent. Clearly state your conclusion about whether they are logically equivalent or not, and how your truth table justifies your conclusion.

1.a. [1 **pt]** $p \vee (p \wedge q)$ and p.

1.b. [1 **pt]** A conditional statement and its inverse.

Q2. [2 **pt]** Use truth tables to establish whether the statement $(\neg p \vee q) \vee (p \wedge \neg q)$ is a tautology or a contradiction.

Q3. [4 **pts]** Determine whether the following two statements in each part are logically equivalent. Justify your answer.

3.a. [2 **pts]** $\forall x (P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$

3.b. [2 **pts]** $\exists x (P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$

Q4. [2 **pts]** Consider the following sequence of digits: 0204. A person claims that all of the 1's in this sequence are to the left of a 0 in this sequence. Is this true? Justify your answer.
[**HINT:** write the claim formally and write a formal negation for it. Is the negation true or false?]

Q5. **[2 pts]** For each of the following statements, determine whether the proposed negation is correct. If it is not, write a correct negation.

5.a. [1 pt]

Statement: For all n, if n is prime then n is odd or $n = 2$. (domain: integers)

Proposed negation: There exists n, such that if n is prime then n is even and $n \neq 2$. (domain: integers)

5.b. [1 pt]

Statement: $\forall x \exists y (x + y = 1)$ (domain: integers)

Proposed negation: $\forall x \exists y (x + y \neq 1)$. (domain: integers)

Q6. **[2 pts]** Use a truth table to show that the following argument (also known as Proof by Cases) is valid. Indicate the premises and conclusion on your table. Explain how your truth table supports your conclusion.

$$\begin{array}{l} p \vee q \\ p \to r \\ \underline{q \to r} \\ \therefore r \end{array}$$

Q7. **[2 pts]** The following incorrect type of reasoning is called the **fallacy of denying the hypothesis**. Establish that the following argument is invalid. Rewrite the argument using symbols to help establish its invalidity. You can use a truth table or a counterexample to establish invalidity.

If at least one of the two numbers is divisible by 6, their product is divisible by 6.

Neither of the two numbers is divisible by 6.

$\therefore$ The product of these two numbers is not divisible by 6.

Q8. **[3 pts]** What can you conclude about the validity or invalidity of the following argument forms? Explain your answer.

8.a. [1 pt]

If an infinite series converges, then its terms go to 0.
The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.

$\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

8.b. [1 pt]

If an infinite series converges, then its terms go to 0.
The terms of the infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ do not go to 0.

$\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ does not converge.

8.c. [1 pt]

$\forall x \; (P(x) \to Q(x))$.
$\neg P(a)$ for a particular a.

$\therefore$ $\neg Q(a)$.