

University of Toronto Scarborough

Sample Term Test 1, MAT-CSCA67: Discrete Mathematics, Summer 2024

Section: LEC01, Erfan Meskar

Aids: No aid-sheet is permitted. No electronic or mechanical computing devices are permitted.

Date: June 7, Time: 2:10 PM, Duration: 50 minutes

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

- The University of Toronto Scarborough and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, smart watches, SMART devices, tablets, laptops, and calculators. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over. If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag, you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course.
- There are 4 questions and 4 pages in this exam, including this one. When you receive the signal to start, please make sure that your copy of the examination is complete.
- Answer each question directly on the examination paper, in the space provided.

	Q1	Q2	Q3	Q4	Total
Max	9	4	7	10	30
Score					

Q1. (9 pts) Mark either True or False to indicate the truth value of each of the following statements and provide a brief one or two sentence explanation.

1.a. (3 pts) The compound propositions $(p \leftrightarrow q) \rightarrow r$ and $r \vee (\neg p \leftrightarrow \neg q)$ are logically equivalent.

☐ TRUE ☐ FALSE

1.b. (3 pts) The proposition $(\exists x P(x)) \wedge (\forall y \forall z (P(y) \wedge P(z) \rightarrow y = z))$ means that there is exactly one element x in the domain such that $P(x)$ is true.

☐ TRUE ☐ FALSE

1.c. (3 pts) The compound proposition $(p \rightarrow F) \vee (p \rightarrow T)$ is a tautology, where T and F are true and false.

☐ TRUE ☐ FALSE

Q2. (4 pts) Use existential and universal quantifiers to express the statement “Everyone has exactly two biological parents” using the propositional function $P(x, y)$, which represents “ x is the biological parent of y .”

Q3. (7 pts) Prove that there exists some integer k such that $\sum_{i=1}^n i^2 < n^3$ for any $n \geq k$.

Q4. (10 pts) The arithmetic mean of two numbers x and y is defined as $A(x, y) = \frac{(x+y)}{2}$. The geometric mean of two nonnegative numbers x and y is defined as $G(x, y) = \sqrt{xy}$. Prove that $G(a, b) \leq A(a, b)$ for any positive a and b , and $G(a, b) < A(a, b)$ unless $a = b$.