Last Lecture Recap:

- Logical Functions, predicates $P(x)$
- Quantification, existential & universal $\exists x$ $\forall x$
- Argument Forms, Valid
- Rules of Inference for logical propositions

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Plan for Today:

- Rules of Inference for Quantifications
- Practice Questions

Rules of Inference for Quantified Statements

universal Instantiation: $\forall x\, P(x) \rightarrow P(c)$, for arbitrary c

universal generalization: $P(c)$, for arbitrary c $\rightarrow \forall x\, P(x)$

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

c is any arbitrary member of the domain

$$\frac{P(c)}{\therefore \forall x\, P(x)}$$

$P(a)$

$\exists x\, P(x) \rightarrow P(a)$ for some specific a in the domain

$$\frac{\exists x\, P(x)}{\therefore P(a) \text{ for some specific } a}$$

$$\frac{P(a) \text{ for some specific } a}{\therefore \exists x\, P(x)}$$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

Table from the Textbook, Sec 1.6

EX. "Each of five roommate, Jane, John,
- * Joe, Julia, and Jen, has taken a
a course in discrete math"
- * "Every student who has taken a course
D(x) in discrete math can take a course
in algorithm" A(x)
* Therefore, "all five roommate can take
algorithm"

D(x): "x has taken a course
in Discrete Math"

A(x): "x can take algorithm"

$\forall x (D(x) \rightarrow A(x))$

R(x): "x is one of the listed Roomates"

$\forall x \; R(x) \rightarrow D(x)$

	Reason
1. $\forall x (R(x) \rightarrow D(x))$	by hypo
2. $R(c) \rightarrow D(c)$, for arbitrary c	by universal inst.
3. $\forall x (D(x) \rightarrow A(x))$	by hypo
4. $D(c) \rightarrow A(c)$, for arbitrary c	by universal inst.
5. $R(c) \rightarrow A(c)$, for arbitrary c	(by rule of inference on 2, 4)
6. $\forall x (R(x) \rightarrow A(x))$	by universal generaliza