MAT-CSC A67: Discrete Mathematics — Summer 2024

Quiz 5

Due Date: Monday, June 17, 11:59 PM, on Crowdmark

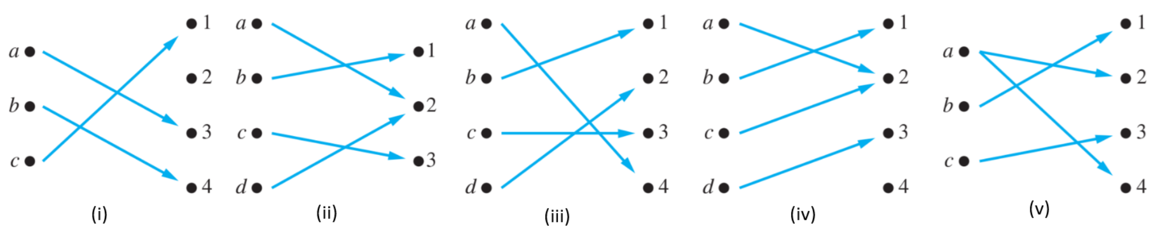
Q1. Let A be an arbitrary non-empty set, and consider the statement $(\{x\} \in \mathcal{P}(A)) \to (x \in A)$. The next paragraph claims to prove this statement. Is this proof correct?

> Suppose x is an arbitrary element of A. Then, by definition of the power set, there will be a singleton set containing x in $\mathcal{P}(A)$, that is, $\{x\} \in \mathcal{P}(A)$. Therefore, $\{x\} \in \mathcal{P}(A) \to x \in A$.

Q2. Fill in the blanks with the proper function properties "injective", "not injective", "surjective", and "not surjective".

1. Suppose that $f : A \to B$. To show that f is ______, show that if $f(x) = f(y)$ for arbitrary $x, y \in A$, then $x = y$.
2. Suppose that $f : A \to B$. To show that f is ______, find a particular elements $y \in B$ such that $f(x) \neq y$ for all $x \in A$.
3. Suppose that $f : A \to B$. To show that f is ______, consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.
4. Suppose that $f : A \to B$. To show that f is ______, find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

Q3. Consider the following five mappings illustrated in the image below. Choose the best option that describes these mappings.



1. (i) is one-to-one but not onto
   (ii) is onto but not one-to-one
   (iii) is one-to-one and onto
   (iv) is neither one-to-one nor onto
   (v) is not a function
2. (i) is one-to-one and onto
   (ii) is neither one-to-one nor onto
   (iii) is onto but not one-to-one
   (iv) is not a function
   (v) is one-to-one but not onto
3. (i) is one-to-one and onto
   (ii) is one-to-one but not onto
   (iii) is onto but not one-to-one
   (iv) is onto and one-to-one
   (v) is one-to-one but not onto
4. (i) is onto but not one-to-one
   (ii) is one-to-one but not onto
   (iii) is one-to-one and onto
   (iv) is neither one-to-one nor onto
   (v) is neither one-to-one nor onto

Q4. Prove that for any sets A and B, if $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

Q5. For a finite set A, $f : A \to A$ is a bijection if there is an inverse function $g : A \to A$ such that $\forall x \in A$ $g(f(x)) = x$.