

Last Lecture Recap:

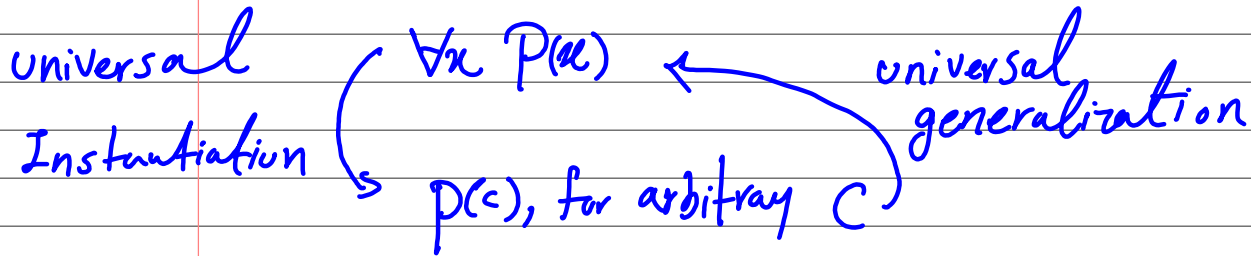
- Logical Functions, predicates $P(n)$
- $\exists x \quad \forall x$ - Quantification, existential & universal
- Argument Forms, Valid
- Rules of Inference for logical propositions

$$\begin{array}{r} P \rightarrow q \\ P \\ \hline \therefore q \end{array}$$

Plan for Today:

- Rules of Inference for Quantifications
- Practice Questions

Rules of Inference for Quantified Statements

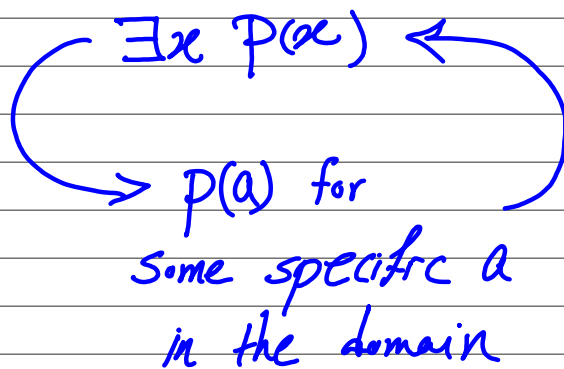


$$\frac{\forall x P(x)}{\therefore P(c)}$$

c is any arbitrary member of the domain

$$\frac{P(c)}{\therefore \forall x P(x)}$$

$P(a)$



$$\frac{\exists x P(x)}{\therefore P(a) \text{ for some specific } a}$$

$$\frac{P(a) \text{ for some specific } a}{\therefore \exists x P(x)}$$

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	Name
$\rightarrow \frac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\rightarrow \frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal <u>generalization</u>
$\rightarrow \frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\rightarrow \frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

Table from the Textbook, Sec 1.6

EX. "Each of five roommate, Jane, John,

- * Joe, Julia, and Jen, has taken a
a course in discrete math"

- * "Every student who has taken a course
in discrete math can take a course
in algorithm" $A(n)$

$D(n)$

* Therefore, "all five roommate can take
algorithm"

$D(n)$: "x has taken a course
in Discrete Math"

$A(n)$: "x can take algorithm"

$\forall x (D(x) \rightarrow A(x))$

$R(n)$: "x is one of the listed Roommates"

$\forall x R(x) \rightarrow D(x)$

1. $\forall x (R(x) \rightarrow D(x))$

Reason
by hypo

2. $R(c) \rightarrow D(c)$, for arbitrary c

by universal inst.

3. $\forall x (D(x) \rightarrow A(x))$

by hypc

4. $D(c) \rightarrow A(c)$, for arbitrary c

by universal int.

5. $R(c) \rightarrow A(c)$, for arbitrary c

(by rule of inference
on 2, 4)

6. $\forall x (R(x) \rightarrow A(x))$

by universal generaliza