

University of Toronto Scarborough

MAT-CSCA67: Discrete Mathematics, Summer 2024, Erfan Meskar

Sample Term Test 2

Duration: 50 minutes

Aids: No aid-sheet is permitted. No electronic or mechanical computing devices are permitted.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

- The University of Toronto Scarborough and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, smart watches, SMART devices, tablets, laptops, and calculators. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over. If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag, you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course.
- There are X questions and Y pages in this exam, including this one. When you receive the signal to start, please make sure that your copy of the examination is complete.
- Answer each question directly on the examination paper, in the space provided. **Please write carefully and clearly, in complete English sentences.**
- This exam includes two pages of scratch paper at the end, which must be submitted, but will not be graded. Do not under any circumstances unstaple the exam.

Q1 Set and Function

- 1.a [8 pts]** Suppose that $f : A \rightarrow B$ is an injective function (*i.e.*, one-to-one function) and $S \subseteq A$. Prove that $f(A - S) = f(A) - f(S)$.

[Write your answer to 1.a here.]

Q2 Cardinality

2.a [8 pts] Prove or disprove: Cartesian product of countably infinitely many countably infinite sets is countable.

[Write your answer to 2.a here.]

Q3 Number Theory

3.a [4 pts] Find the remainder of 34^{82248} divided by 83. **[HINT: 83 is a prime.]**

[Write your answer to 3.a here.]

3.b [4 pts] Prove that $\gcd(ka, kb) = k \gcd(a, b)$.

[Write your answer to **3.b** here.]

3.c [4 pts] Let F be the set of all primes p such that $p \equiv 3 \pmod{4}$. Prove that F has an infinite number of primes. **[HINT:** Considering the number $4p_1p_2 \dots p_n - 1$ may be helpful.]

[Write your answer to 3.c here.]