MAT-CSC A67: Discrete Mathematics — Summer 2024

Practice Questions

Week 2, Friday

Q1. Let $P(x)$ be the statement "x has visited Prague," where the domain consists of the students in your class. Express each of these quantifications in English.

1.a. $\exists x P(x)$

There exists some student in class who had visited Prague.

1.b. $\forall x P(x)$

All students in class had visited prague

1.c. $\exists x \neg P(x)$

- There exist Some student who had not visited prague
- Not all students . . .

1.d. $\neg \exists x P(x)$

There's no student who had visited Prague.

Every student has not visited Prague

Q2. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

2.a. $\forall x (C(x) \to F(x))$

- Every person who is a Comedian is funny

2.b. $\forall x (C(x) \land F(x))$

- Every person is a comedian and funny

2.c. $\exists x (C(x) \to F(x))$

- In home practice

2.d. $\exists x (C(x) \land F(x))$

- There is someone who is a Comedian and funny
- There exists a Comedian who is funny.

Q3. Translate the statement $\forall x \forall y \exists z (x = y + z)$ into English, where the domain for each variable consists of all real numbers.

for all real numbers x and y, there exist some real number Z s.t. $x = y + Z$.

Q4. Let $Q(x, y)$ be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

4.a. $\exists x \exists y Q(x, y)$

home practice

4.b. $\forall x \exists y Q(x, y)$

for all student x, there exist some student y s.t. x has sent e-mail to y

4.c. $\exists x \forall y Q(x, y)$

home practice

Q5. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not a spider.
George is a spider.

$\therefore$ George has eight legs.

$$\neg p \to \neg q$$
$$q$$
$$\therefore p$$

Q6. What rule of inference is used in the following argument?

- If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

home practice

Q7. For the following set of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

- "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat."

$E(x)$: "you eat x"
$T(x)$: "x tastes good"
$H(x)$: "x is healthy"

1. $\forall x\ E(x) \to T(x)$
2. $\forall x\ H(x) \to \neg T(x)$
3. $H(\text{tofu})$
4. $H(\text{Tofu}) \to \neg T(\text{Tofu})$ by 2
5. $\neg T(\text{Tofu})$ by 3 & 4
6. $E(\text{Tofu}) \to T(\text{Tofu})$ by 1
7. $\neg E(\text{tofu})$ by 5 & 6 (but this statement was already provided as a hypothesis)
8. $\neg H(\text{cheesburgers})$

No conclusion more than what we already know

Q8. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express "Nancy can fool exactly two people."

home practice.

Q9. Determine whether $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are logically equivalent. Justify your answer.

domain: integers

$P(x)$: "x is even"

$Q(x)$: "x is odd"

$\underbrace{\forall x P(x)}_{F} \longrightarrow \underbrace{\forall x Q(x)}_{F} \longleftarrow$ Truth value: T

$\forall x (P(x) \to Q(x))$: $\longleftarrow$ Truth value F

$\underbrace{P(2)}_{T} \longrightarrow \underbrace{Q(2)}_{F}$: $\longleftarrow$ Truth value: F

Q10. Establish the logical equivalence, $\forall x(q \to P(x)) \equiv q \to \forall x P(x)$, where x does not occur as a free variable in q. Assume that the domain is nonempty.

home practice.