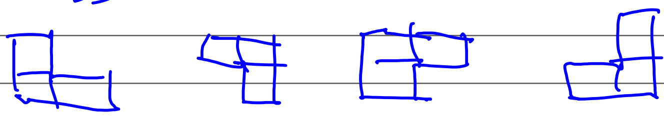
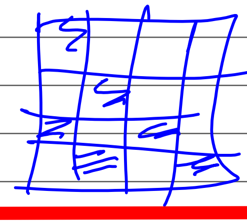
- Can you perfectly tile a 4x4 chessboard with triominos?





NO. Since $3 \nmid 16$.

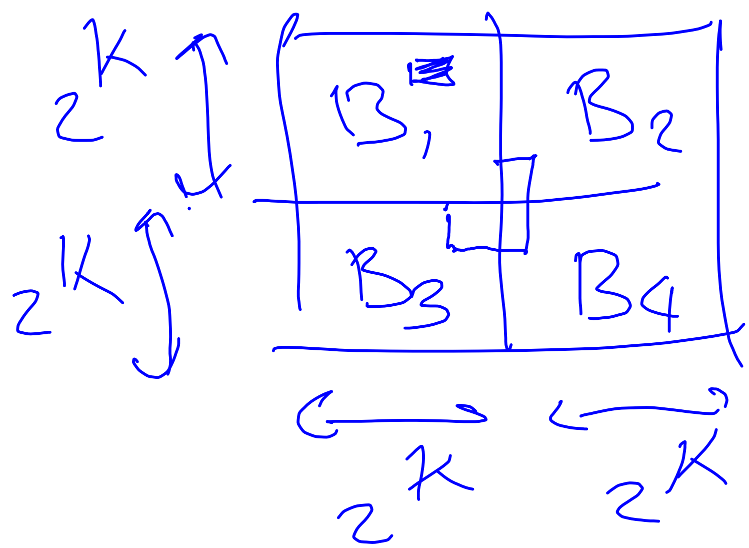
- For any $n \geq 1$, every $2^n \times 2^n$ chessboard with one squared removed can be perfectly tiled w/ triominos?

- Base case: $n = 1$

WLG, assume that bottom right corner is removed. It is trivial that you can tile this board.

- Inductive step. Assum arbitrary $k \geq 1$. Assume $2^k \times 2^k$ board w/ one squared removed can be tiled.

observe that the $2^{k+1} \times 2^{k+1}$ can be divided into 4 $2^k \times 2^k$ board.



WLG, assume that the removed square is in the top left quadrant (since we can rotate the board). Name the quadrant as B_1, B_2, B_3, and B_4 as suggested by the figure above.

By the inductive step assumption, B_1 can be perfectly tiled w/ triominos. To cover the other quadrants, we first place a triominos in the center such that it covers one square from B_2, B_3, and B_4. Observe that due to the induction assumption, all other squares of B_2, B_3, and B_4 can be perfectly covered by triominos. Hence, we can perfectly tile a $2^{k+1} \times 2^{k+1}$ board with an arbitrary square removed, as desired

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