University of Toronto Scarborough
MAT-CSCA67: Discrete Mathematics, Summer 2024, Erfan Meskar
Term Test 1: June 7, 2:10 PM
Duration: 50 minutes
Aids: No aid-sheet is permitted. No electronic or mechanical computing devices are permitted.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

- The University of Toronto Scarborough and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, smart watches, SMART devices, tablets, laptops, and calculators. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over. If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag, you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course.

- There are 4 questions and 8 pages in this exam, including this one. When you receive the signal to start, please make sure that your copy of the examination is complete.

- Answer each question directly on the examination paper, in the space provided. **Please write carefully and clearly, in complete English sentences.**

- This exam includes two pages of scratch paper at the end, which must be submitted, but will not be graded. Do not under any circumstances unstaple the exam.

	Q1	Q2	Q3	Q4	Total
Max	4	4	8	8	32
Score					

Q1 **(4 pts)** Determine if the two compound propositions $(\neg p \vee q) \wedge (p \wedge (p \wedge q))$ and $(p \wedge q)$ are logically equivalent.

☐ Equivalent   ☐ Not Equivalent

[**Write your answer to question Q1 here.**]

Q2 **(4 pts)** Translate the following sentences from English to predicate logic. The domain that you are working over is the set of people. You may use the functions $S(x)$, meaning that "x has taken A67," $A(x)$, meaning that "x has gotten an A in A67," and $T(x)$, meaning that "x is a TA of A67."

- All people who are A67 TAs and have taken A67 got A in A67.

[**Write your answer to Q2 here.**]

Q3 **(8 pts)** Prove that $8 \mid 3^{2n} - 1$ for any $n \geq 1$.

[**Write your answer to Q3 here.**]

[**Write your answer to Q3** here.]
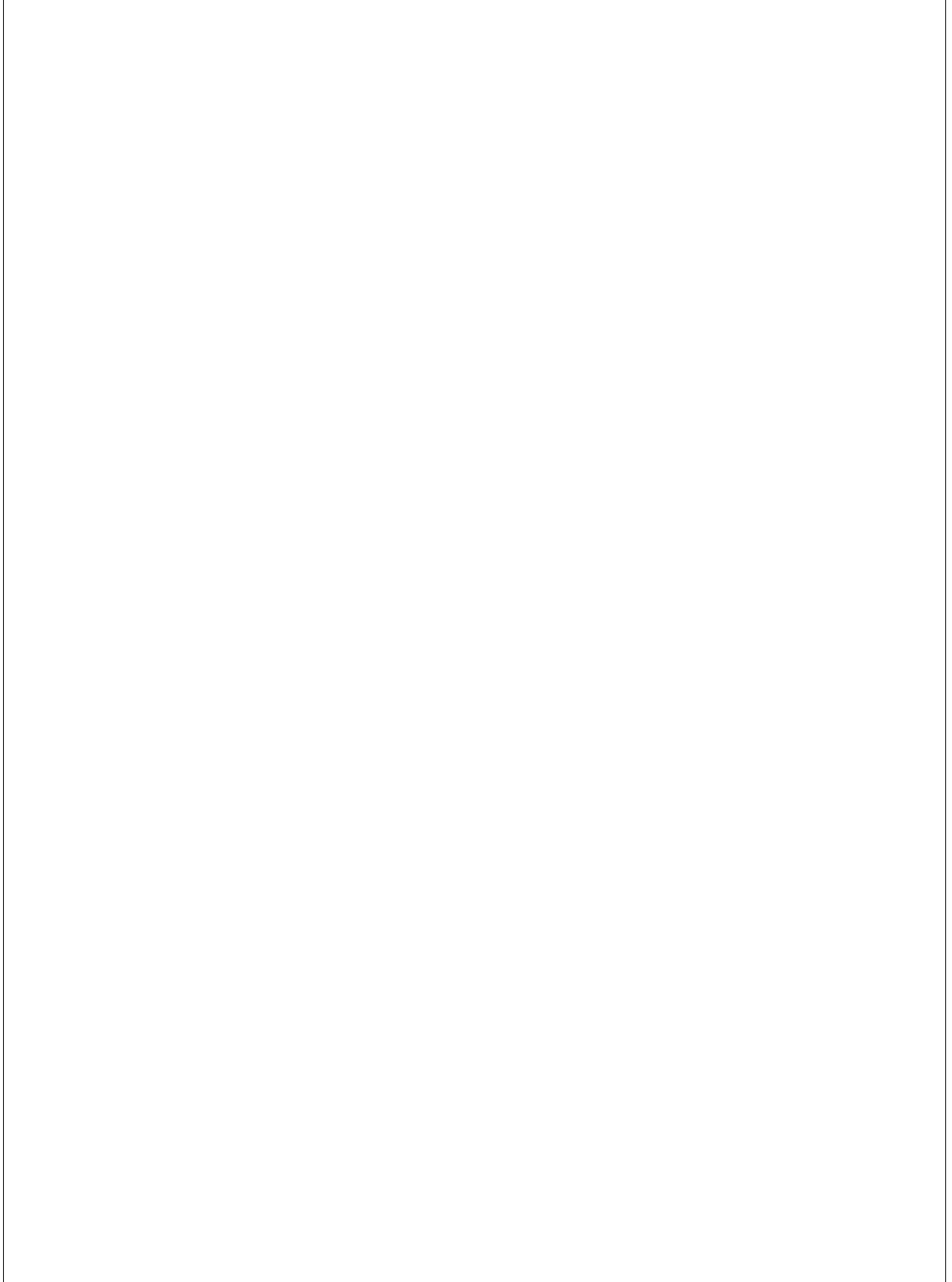
Q4 (**8 pts**) For any positive integers w, x, y, and z that $z^2 = w^2 + x^2 + y^2$, if z is even, then w, x, and y are even.

[**HINT:** It may be helpful to represent even integers as $2i$ and odd integers as $2j + 1$, where i and j are integers.]

[**Write your answer to Q4 here.**]

[**Write your answer to Q4** here.]

[Scratch Paper 1]

[Scratch Paper 2]