## MAT-CSC A67: Discrete Mathematics — Summer 2024

**Practice Questions** Week 2, Friday

Q1.	Let $P(x)$ be the statement " $x$ has visited Prague,"	where the domain consists of the students in your class
	Express each of these quantifications in English.	

**1.a.**  $\exists x P(x)$ 

There exists some of dent in closs who had visited prague.

**1.b.**  $\forall x P(x)$ 

All students in class had visited prague

**1.c.**  $\exists x \neg P(x)$ 

- There exist some student who had not visited prague

- Not all students · -

There's no student who had visited progue.

Every student has not visited Prague

**Q2.** Translate these statements into English, where  $\underline{C(x)}$  is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people.

**2.a.**  $\forall x (C(x) \rightarrow F(x))$ 

- Every person who is a Comedian

**2.b.**  $\forall x (C(x) \land F(x))$ 

- Every person is a comedian and funny

**2.c.**  $\exists x (C(x) \rightarrow F(x))$ 

- In home practice

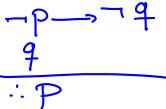
**2.d.**  $\exists x (C(x) \land F(x))$ 

- There is someone who is a Comedian and

- There exists a Comedian who is funny.

<b>Q3.</b> Translate the statement $\forall x \forall y \exists z (x = y + z)$ into English, where the domain for each variable consists of all real numbers.			
for all real numbers or and y, there exist some v-	w		
Number $Z = S.L$ . $x = y + Z$ .			
<b>Q4.</b> Let $Q(x,y)$ be the statement "x has sent an e-mail message to y" where the domain for both $x$ and $y$ consists of all students in your class. Express each of these quantifications in English.			
<b>4.a.</b> $\exists x\exists y Q(x,y)$			
home practice			
-4.b. $\forall x \exists y Q(x,y)$ for all student $\chi$ , there exist some student $\chi$ s.t. $\chi$ has sent e-mail to $\chi$			
<b>4.c.</b> $\exists x \forall y Q(x,y)$			
home practice			

- $\mathbf{Q5}$ . Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?
  - If George does not have eight legs, then he is not a spider.
  - George is a spider.
    - ... George has eight legs.



- **Q6.** What rule of inference is used in the following argument?
  - If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.



- Q7. For the following set of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
  - "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat."

E(m): "You eat or" 1. You E(n)\_> Timo: " x tustes good"

2 Vx H(n) \_\_\_\_ T(n) H(n): x is healthy"

3. H (totu)

4. H(Tofu) -> - T(Tofu)

一て(てより)

6. E(Tofu) -> T(Tofu)

7. TE (tofu)

by 3 & 4

by 5 & 6 (but this statement was already provided as a hypothesis)

8. - H(cheesburgers)

we already

 ${\sf Q8.}$  Let F(x,y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express "Nancy can fool exactly two people."

home practice.

**Q9.** Determine whether  $\forall x(P(x) \to Q(x))$  and  $\forall xP(x) \to \forall xQ(x)$  are logically equivalent. Justify your answer.

domain: integers

p(n): "n is even"

Qm: "x is odd"

In p(n) > Yn Q(n) Truth value: T F

F

F

Truth value F

P(2) -> Q(2): E

Truth value: F

Q10. Establish the logical equivalence,  $\forall x(q \to P(x)) \equiv q \to \forall x P(x)$ , where x does not occur as a free variable in q. Assume that the domain is nonempty.

home practice.