# CSCD84: Artificial Intelligence:

## Problem Set 4: Bayesian Networks

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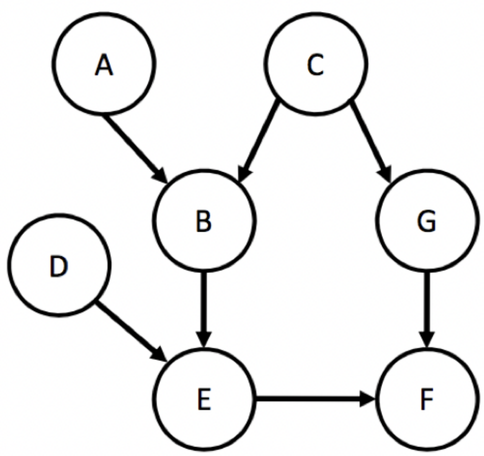
*By turning in this assignment, I agree by the University of Toronto honor code and declare that all of this is my own work.*

1. Given the Bayesian Network below, determine if:
a) A is independent of C given F.
b) G is independent of D given E.
c) C is independent of D.

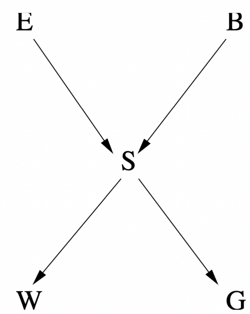



*Solution:* a) False. There is an unblocked (or not d-separated) path from A to B to E, and then thru F to G to C. Note that without information about F, the path from E to G is blocked.

b) False. There is an unblocked (or not d-separated) path from D to E to B, and then to G.

c)True. The fact that we have no information about E d-separates the path from D to B. No information about F d-separates E and G. So information about D is d-separated from paths to C both via F and E.

2. Consider the following Bayesian Network:
where the marginal probabilities are given as:
Find $P(G|W)$. (i.e., the four probability values $P(g|w)$, $P(-g|w), P(g|-w)$, and $P(-g|-w)$).
*Solution:* To solve this problem, we should do variable elimination. The query variable is G. For the first run of VE, evidence is $W = w$. For the second run of VE, the evidence is $W = -w$. We use the same ordering for both runs of VE: E, B, S, G. This helps reuse the factors between two runs of VE. Let's start with E, it affects $P(E)$




$P(E, S, B, W, G) = P(E)P(B)P(S|E, B)P(W|S)P(G|S)$

| P(E) | e | -e |
|---|---|---|
| | 1/10 | 9/10 |

| P(B) | b | -b |
|---|---|---|
| | 1/10 | 9/10 |

| P(S\|E,B) | s | -s |
|---|---|---|
| e ∧ b | 9/10 | 1/10 |
| e ∧ -b | 2/10 | 8/10 |
| -e ∧ b | 8/10 | 2/10 |
| -e ∧ -b | 0 | 1 |

| P(W\|S) | w | -w |
|---|---|---|
| s | 8/10 | 2/10 |
| -s | 2/10 | 8/10 |

| P(G\|S) | g | -g |
|---|---|---|
| s | 1/2 | 1/2 |
| -s | 0 | 1 |

and $P(S|E, B)$:

$$F_1(S, B) = \sum_E P(E) \times P(S|E, B) = P(e) \times P(S|e, B) + P(-e) \times P(S|-e, B). \tag{1}$$

$$F_1(-s, -b) = P(e)P(-s, e, -b) + P(-e)P(-s, -e, -b) = 0.1 \times 0.8 + 0.9 \times 1 = 0.98. \tag{2}$$

$$F_1(-s, b) = P(e)P(-s, e, b) + P(-e)P(-s, -e, b) = 0.1 \times 0.1 + 0.9 \times 0.2 = 0.19. \tag{3}$$

$$F_1(s, -b) = P(e)P(s, e, -b) + P(-e)P(s, -e, -b) = 0.1 \times 0.2 + 0.9 \times 0 = 0.02. \tag{4}$$

$$F_1(s, b) = P(e)P(s, e, b) + P(-e)P(s, -e, b) = 0.1 \times 0.9 + 0.9 \times 0.8 = 0.81. \tag{5}$$

Next variable is B. It affects $P(B)$ and $F_1(S, B)$:

$$F_2(S) = \sum_B P(B) \times F_1(S, B) = P(b)F_1(S, b) + P(-b)F_1(S, -b). \tag{6}$$

$$F_2(-s) = P(b)F_1(-s, b) + P(-b)F_1(-s, -b) = 0.1 \times 0.19 + 0.9 \times 0.98 = 0.901. \tag{7}$$

$$F_2(s) = P(b)F_1(s, b) + P(-b)F_1(s, -b) = 0.1 \times 0.81 + 0.9 \times 0.02 = 0.099. \tag{8}$$

Next variable is S, modifying $P(w|S)$, $P(S|G)$, and $F_2(S)$:

$$F_3(G) = \sum_S P(w|S) \times P(S|G) \times F_2(S) = P(w|s)P(s|G)F_2(s) + P(w|-s)P(-s|G)F_2(-s) \tag{9}$$

$$F_3(-g) = P(w|s)P(s|-g)F_2(s)+P(w|-s)P(-s|-g)F_2(-s) = 0.8\times0.5\times0.099+0.2\times1\times0.901 = 0.2198. \tag{10}$$

$$F_3(g) = P(w|s)P(s|g)F_2(s) + P(w|-s)P(-s|g)F_2(-s) = 0.8 \times 0.5 \times 0.099 + 0.2 \times 0 \times 0.901 = 0.0396. \tag{11}$$

The final variable is G, for which we should normalize $F_3(G)$:

$$P(-g|w) = \frac{0.2198}{0.2198 + 0.0396} = 0.8473. \tag{12}$$

$$P(g|w) = \frac{0.0396}{0.2198 + 0.0396} = 0.1527. \tag{13}$$

Now, we should consider $W = -w$ and repeat the above steps. We follow the same steps to eliminate E and B. To eliminate S, we have:

$$F_3(G) = \sum_S P(-w|S) \times P(S|G) \times F_2(S) = P(-w|s)P(s|G)F_2(s) + P(-w|-s)P(-s|G)F_2(-s) \tag{14}$$

$$F_3(-g) = P(-w|s)P(s|-g)F_2(s)+P(-w|-s)P(-s|-g)F_2(-s) = 0.2\times0.5\times0.099+0.8\times1\times0.901 = 0.7307. \tag{15}$$

$$F_3(g) = P(-w|s)P(s|g)F_2(s)+P(-w|-s)P(-s|g)F_2(-s) = 0.2\times0.5\times0.099+0.8\times0\times0.901 = 0.0099. \tag{16}$$

Similarly, for G, we should normalize $F_3(G)$:

$$P(-g|w) = \frac{0.7307}{0.7307 + 0.0099} = 0.9866. \tag{17}$$

$$P(g|w) = \frac{0.0099}{0.7307 + 0.0099} = 0.0134. \tag{18}$$