CSCD84: Artificial Intelligence

Problem Set 2: CSP

Due: 11:59 PM EST, Sunday Feb 4

Q1: Formulating k-Clique problem as a CSP

Consider an undirected and unweighted graph $G = (V, E)$, where V is the set of vertices and $E \subseteq \{(v_i, v_j) \mid (v_i, v_j) \in V^2 \text{ and } v_i \neq v_j\}$ is the set of edges. A clique, C, in the graph G is a subset of the vertices, $C \subseteq V$, such that every two distinct vertices in C are adjacent.

In the k-clique problem, the input is the undirected and unweighted graph $G = (V, E)$ and a number k. The output is a clique with k vertices, if one exists, or a special value (*e.g.*, "FAIL") indicating that there is no k-clique otherwise.

Formulate the k-clique problem as a Constraint Satisfaction Problem (CSP). Clearly specify the variables of your CSP and their domains, and formulate the constraints explicitly.

[**HINT:** The CSP can be constructed with binary constraints for most parts, except for one constraint.]

Solution

- Variables: $\{v_i \mid v_i \in V\}$
- Domains: $D = \{0, 1\}$ (Where 1 indicates a vertex v_i is in the clique, and 0 indicates it is not)
- Constraints:
  1. $(v_i, v_j) \in \Big\{(0,0), (0,1), (1,0)\Big\}$, $\forall (v_i, v_j) \in \Big\{(v_k, v_m) \;\Big|\; v_k \neq v_m \text{ and } (v_k, v_m) \in V^2 - E\Big\}$
  2. $\sum_{i \in V} v_i = k$