CSCD84: Artificial Intelligence

Worksheet: Variable Elimination, HMM, and Particle Filtering

Q1. Bayes Nets: Variable Elimination

$\mathbb{P}(A)$	
$+a$	0.25
$-a$	0.75

$\mathbb{P}(B \mid A)$	$+b$	$-b$
$+a$	0.5	0.5
$-a$	0.25	0.75

$\mathbb{P}(C \mid A)$	$+c$	$-c$
$+a$	0.2	0.8
$-a$	0.6	0.4

$\mathbb{P}(D \mid B)$	$+b$	$-b$
$+b$	0.6	0.4
$-b$	0.8	0.2

$\mathbb{P}(E \mid B)$	$+c$	$-c$
$+b$	0.25	0.75
$-b$	0.1	0.9

1.a. Using the Bayes' Net and conditional probability tables above, calculate the following quantities:

1.a. i. $\mathbb{P}(+b \mid +a)$

1.a. ii. $\mathbb{P}(+a, +b)$

1.a. iii. $\mathbb{P}(+a \mid +b)$

1.b. Now we are going to consider variable elimination in the Bayes' Net above.

1.b. i. Assume we have the evidence $+c$ and wish to calculate $\mathbb{P}(E \mid +c)$. What factors do we have initially?

1.b. ii. If we eliminate variable B, we create a new factor. What probability does that factor correspond to and what is the equation to calculate that factor?

1.b. iii. After eliminating variable B, what are the new set of factors? As in **1.b. ii.**, write the probabilities that the factors represent. For each factor, also provide its size.

1.b. iv. Now assume we have the evidence $-c$ and are trying to calculate $\mathbb{P}(A \mid -c)$. What is the most memory efficient elimination ordering? If more than one ordering is most memory efficient, provide any one of them.

1.b. v. Once we have run variable elimination and have $f(A, -c)$, how do we calculate $\mathbb{P}(+a \mid -c)$?

Q2. Consider an HMM with the following Initial Distribution, Transition Probabilities, and Emission (Hidden Variable) Conditional Probability Table (CPT).

Initial Distribution

State	
$A_1 =$ true	0.99

Transition Probabilities

	$A_{t+1} =$ true	$A_{t+1} =$ false
$A_t =$ true	0.99	0.01
$A_t =$ false	0.01	0.99

Emission Probabilities

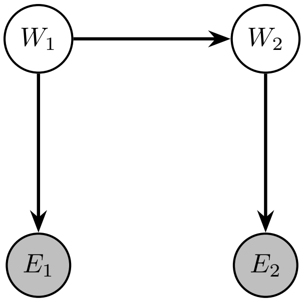
State	E_t	$\mathbb{P}(E_t \mid A_t)$
$A_t =$ true	1	0.2
$A_t =$ true	0	0.8
$A_t =$ false	1	0.9
$A_t =$ false	0	0.1

2.a. Draw the HMM implied by the CPTs above.

2.b. Calculate $\mathbb{P}(A_4 = \text{true} \mid E_1 = 0,\ E_2 = 1,\ E_3 = 0)$.

2.c. What is the probability of observing the emission sequence $(E_1 = 0,\ E_2 = 1,\ E_3 = 0)$?

Q3. Consider the following Hidden Markov Model. E_1 and E_2 are supposed to be shaded.



W_1	$\mathbb{P}(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$\mathbb{P}(W_{t+1} \mid W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	E_t	$\mathbb{P}(E_t \mid W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

Suppose that we observe $E_1 = a$ and $E_2 = b$. Using the forward algorithm, compute the probability distribution $\mathbb{P}(W_2 \mid E_1 = a, E_2 = b)$ one step at a time.

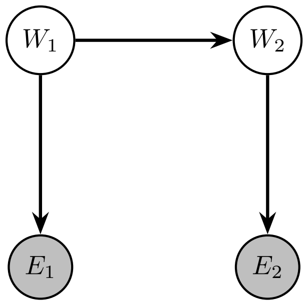
3.a. Compute $\mathbb{P}(W_1, E_1 = a)$.

3.b. Using the previous calculation, compute $\mathbb{P}(W_2, E_1 = a)$.

3.c. Using the previous calculation, compute $\mathbb{P}(W_2, E_1 = a, E_2 = b)$.

3.d. Finally, compute $\mathbb{P}(W_2|E_1 = a, E_2 = b)$.

Q4. Let's use Particle Filtering to estimate the distribution of $\mathbb{P}(W_2|E_1 = a, E_2 = b)$. Consider the following Hidden Markov Model. E_1 and E_2 are supposed to be shaded.



W_1	$\mathbb{P}(W_1)$
0	0.3
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W_t	W_{t+1}	$\mathbb{P}(W_{t+1} \mid W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	E_t	$\mathbb{P}(E_t \mid W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

We start with two particles representing our distribution for W_1.

- $P_1 : W_1 = 0$
- $P_2 : W_1 = 1$

Use the following random numbers to run particle filtering:

$$[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]$$

4.a. Observe: Compute the weight of the two particles after evidence $E_1 = a$.

4.b. Resample: Using the random numbers, resample P_1 and P_2 based on the weights.

4.c. Predict: Sample P_1 and P_2 from applying the time update.

4.d. Update: Compute the weight of the two particles after evidence $E_2 = b$.

4.e. Resample: Using the random numbers, resample P_1 and P_2 based on the weights.

4.f. What is our estimated distribution for $\mathbb{P}(W_2 \mid E_1 = a, E_2 = b)$?