CSCD84: Artificial Intelligence

Midterm Exam – 90 Minutes

The Department of Computer and Mathematical Sciences
University of Toronto Scarborough

Date: Monday, March 4, 9:10 – 10:40 AM

UTORid:

First and Last Name:

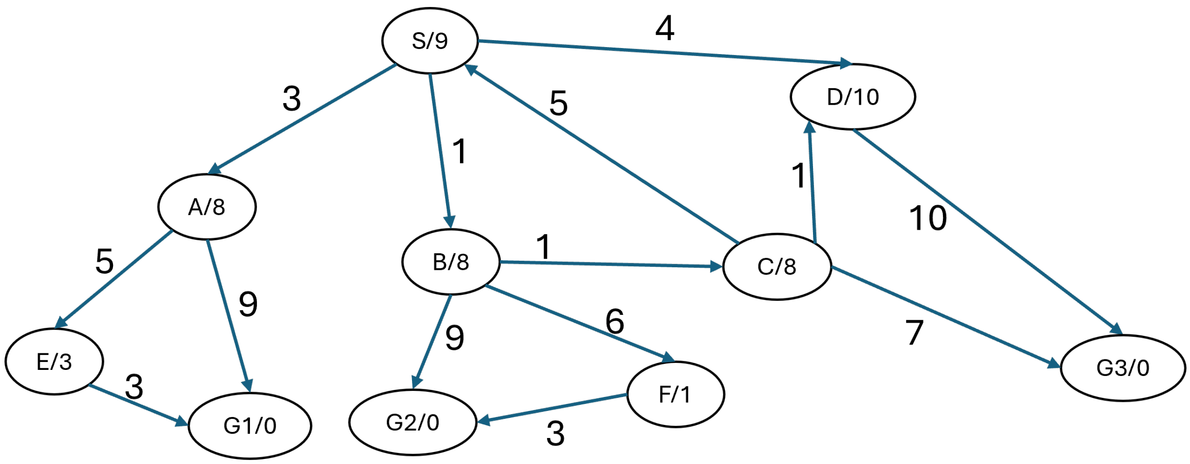
Instructions.

- This test has 4 questions. Make sure to skim through all the questions before starting. This will help you pace yourself. This exam has 45 points in total, and you have 90 Minutes.
- This exam is closed book and closed notes, and no calculator is allowed.
- Write your answers on this questions paper. Make sure to put your name and UTORid on every page.

Question	**Score**
Question 1	/13
Question 2	/9
Question 3	/11
Question 4	/12
Total	/45

Question 1. [Search – 13 points]

(a) **[5 points]** Consider the following search graph, where **S** is the start node and **G1**, **G2**, and **G3** are the goal states. Arcs are labeled with the cost of traversing them and the heuristic cost to a goal is shown inside the nodes. Answer the following questions. If tie-breaking is required, assume it is left-to-right, *e.g.,* if two nodes are tied, first expand the one that appears on the left of the other in the figure.



(i) **[1 points]** With **breadth-first search**, indicate which of the goal states is reached and the cost of the found path.

Reached Goal: Path cost:

(ii) **[1 points]** With **uniform cost search**, indicate which of the goal states is reached and the cost of the found path.

Reached Goal: Path cost:

(iii) **[1 points]** With **A*** search, indicate which of the goal states is reached and the cost of the found path.

Reached Goal: Path cost:

(iv) **[2 points]** Is the heuristic function illustrated in the above search graph a *consistent* heuristic? Why?

(b) **[2 points]** If h_1 and h_2 are both admissible heuristics, which of the below heuristics are guaranteed to be admissible? (Mark all heuristics that are guaranteed to be admissible).

☐ $g(n) = \max\{h_1(n), h_2(n)\}$. ☐ $q(n) = wh_1(n) + (1-w)h_2(n)$, for some $0 < w < 1$.

(c) **[2 points]** Suppose we are using A* search with heuristic function $h^*(n)$, where $h^*(n)$ is the optimal path cost of reaching goal from state n. Prove that whenever A* expands a node n, n must lie on an optimal path to a goal.

(d) **[4 points]** Suppose we are solving a search problem using A* search where actions have non-zero positive costs and the heuristic function is a so-called ϵ-admissible function, meaning that there is some $\epsilon > 0$ and $h(n) \leq h^*(n) + \epsilon$ for all nodes n. Here, $h^*(n)$ is the optimal path cost and the lowest cost solution is of cost C^*.

(i) **[1 points]** Will this version of A* be complete, *i.e.*, will it be guaranteed to find a solution if there is one? Why?

(ii) **[3 points]** When using an ϵ-admissible heuristic, how far from the optimal cost (C^*) might our final solution be? Why?

Question 2. [CSP – 9 points]

a) [**4 points**] The objective of the Sudoku game that is illustrated below is to place numbers $1, 2, 3, 4$ on the 4×4 grid such that each row, column, and 2×2 box constrains each number only once. In the illustration, some cells have assigned values; those that do not are labelled with letters (*e.g.*, A1, A2, etc.). Assume you are solving this puzzle as a CSP that contains only binary not-equals constraints.

3	A2	A3	A4
B1	B2	B3	B4
C1	**3**	C3	C4
D1	D2	D3	**4**

(i) [**2 points**] What does it mean for a CSP to be path-consistent (*i.e.*, 3-consistent)?

(ii) [**2 points**] Given the current assignment, the cell labeled D3 cannot be a 1 or 2. By enforcing path-consistency on the current CSP, will these values be pruned? Why?

b) **[5 points]** Consider an undirected and unweighted graph $G = (V, E)$, where V is the set of vertices and $E \subseteq \{(v_i, v_j) \mid (v_i, v_j) \in V^2 \text{ and } v_i \neq v_j\}$ is the set of edges. The Hamiltonian Circuit Problem asks if there is a cyclic ordering of V such that every pair of successive nodes in the ordering are adjacent in G. In other words, A Hamiltonian circuit is a circuit that visits every vertex once with no repeats. Being a circuit, it must start and end at the same vertex.

Formulate the Hamiltonian Circuit Problem as a CSP with binary constraints. Clearly specify the variables of your CSP and their domains, and formulate the constraints explicitly.

Question 3. [Game Tree Search – 11 points]

(a) **[1 point]** When using alpha-beta pruning, it is possible to get an incorrect value at the root node by choosing a bad ordering when expanding children. (Check one of the following):

☐ True ☐ False

(b) **[1 point]** When using alpha-beta pruning, the computational savings are independent of the order in which children are expanded. (Check one of the following):

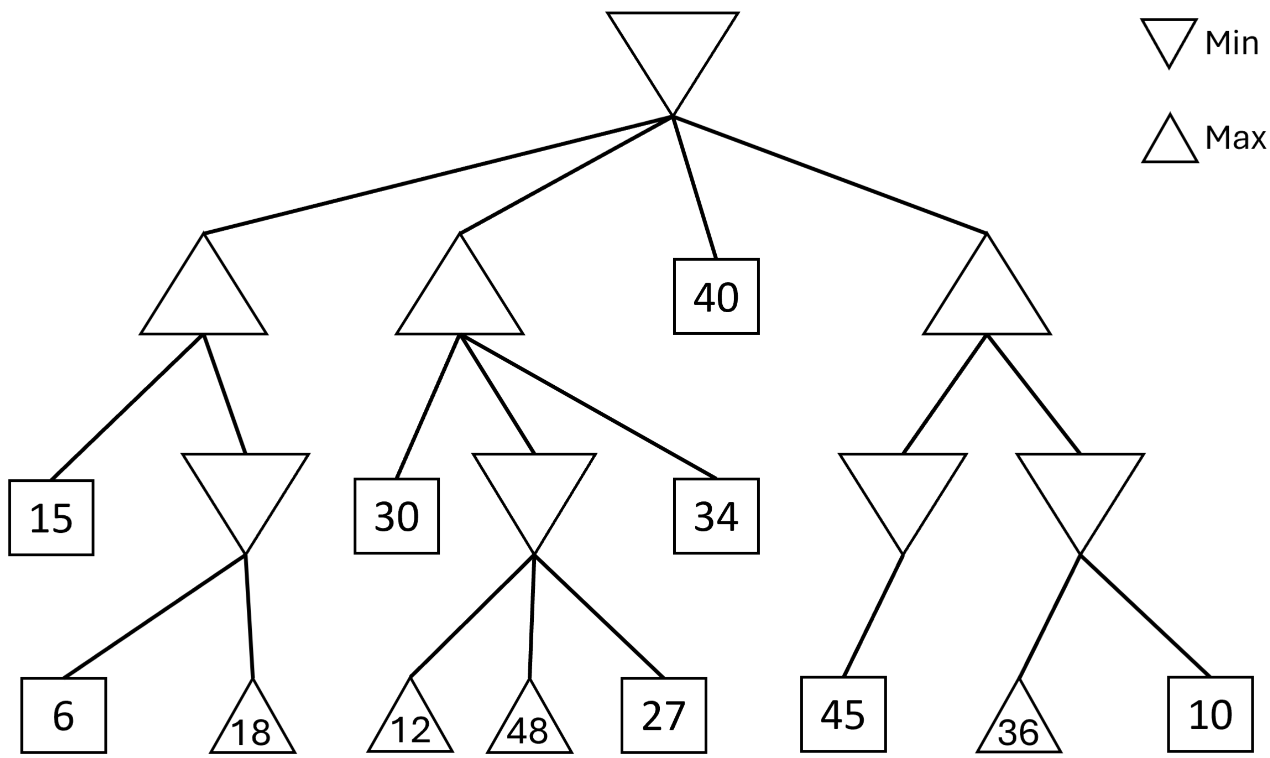
☐ True ☐ False

(c) **[3 point]** We have an evaluation function $f(s)$ that we want to use to evaluate game states. For all s, $f(s) > 0$. Which of the following is/are true? (Check all that apply.):

☐ Replacing $f(s)$ with $\log f(s)$ may change the action chosen by **minimax** search.

☐ Replacing $f(s)$ with $100f(s)$ may change the action chosen by **expectimax** search.

☐ Replacing $f(s)$ with $(f(s))^2$ may change the action chosen by **expectimax** search.



(d) **[1 point]** Fill the evaluation values for all the empty states in the game tree above. Assume that the minimax algorithm is being used, according to the labels on the top right.

(e) **[5 point]** On the same diagram above, indicate which states will not be explored if alpha-beta pruning is used. Circle all unvisited subtrees.

Question 5. [MDP – 12 points]

There's a potential for a strike from the Unit 1 workers' union at UofT. The union's negotiation team seeks assistance from CSCD84 students to analyze the negotiation process with their employers, treating it as an MDP.

This MDP has states 10 states: 0, 100, 200, 300, 400, 500, 600, 700, 800, and *Done*. Each state corresponds to the annual increase in workers' salary offered by the employer, and also a *Done* state where the negotiation ends. The negotiating team had received an offer of \$200, *i.e.* the MDP starts at state 200. The union has two actions: *Stop* and *negotiate* for one more week, and is forced to take the *Stop* action at states 0, 100, and 800.

Choosing the *Stop* action transitions the union to the terminal state *Done* and rewards them with the amount corresponding to the state they transitioned from. For instance, *Stop* at state 300 yields \$300. The negotiation process concludes upon reaching the *Done* state.

The *Negotiate* action is viable between states 200 and 700. After each negotiation round, the union may receive offers of \$100, \$200, \$300, or \$400 with a probability of $\frac{1}{8}$ each, and \$500 or \$600 with a probability of $\frac{1}{4}$ each.

If the union chooses to *Negotiate* from state s and receives an offer of o, it transitions to state $s + o - 200$, provided that $s + o - 200 \leq 800$. Here, s represents the current state's dollar amount, o is the offered amount, and the deduction of 200 accounts for the price of negotiating for one week. If $s + o - 200 > 800$, the union fails and transitions directly to the *Done* state without receiving any reward.
The provided diagram offers a partial illustration of the MDP, depicting transition probabilities and rewards for a better understanding. Note that the complete MDP graph is not displayed. The two numbers next to each edge represent the probability and reward of a transition. For instance, $\frac{1}{8}$ / 0 denotes that the probability of this transition is $\frac{1}{8}$ and the reward for it is 0.

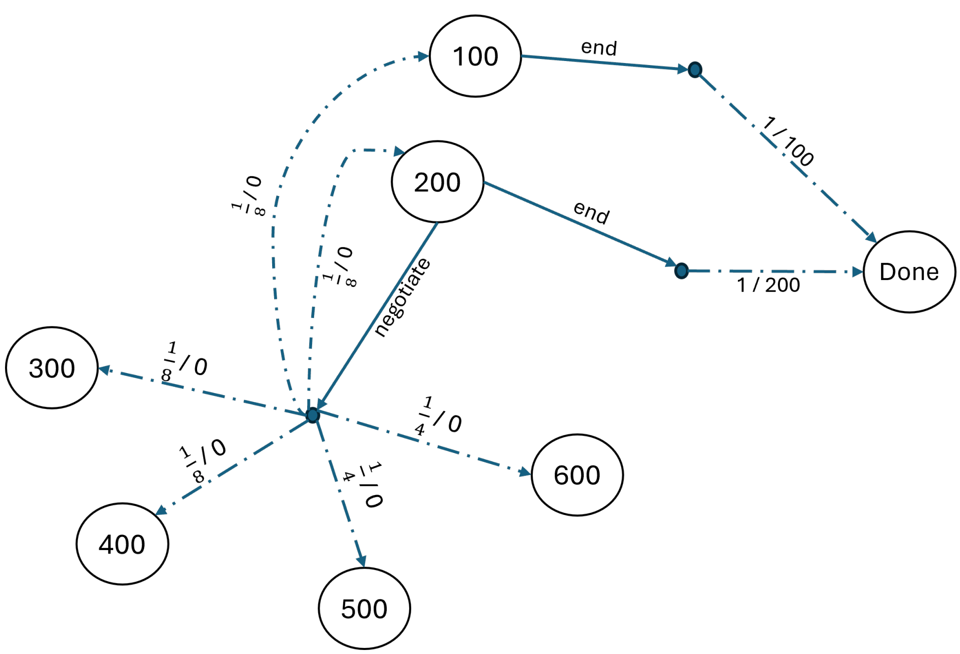


Figure 1: Note that this figure does not show the complete MDP graph.

a) **[7 points]** The union's initial policy π^i is given in the table below. Use policy iteration to evaluate the policy at each state, with $\gamma = 1$. Note that the actions at state $0, 100$, and 800 are *Stop* and are fixed into the rule.

State	200	300	400	500	600	700
$\pi^i(s)$	*Negotiate*	*Negotiate*	*Stop*	*Stop*	*Stop*	*Stop*
$V_{\pi^i}(s)$						

b) **[5 points]** One of the members of union claims that the strategy given in the table in part a, *i.e.*, π^i, is the an optimal strategy. Is that true? Prove why?

THIS IS AN EXTRA PAGE FOR ROUGH WORK.