University of Toronto Scarborough
Sample Final Exam, CSCD84H3 S: Artificial Intelligence
Aids: No aid-sheet is permitted. No electronic or mechanical computing devices are permitted.
Date, Time, Duration: 3h

- The University of Toronto Scarborough and you, as a student, share a commitment to academic integrity. You are reminded that you may be charged with an academic offence for possessing any unauthorized aids during the writing of an exam. Clear, sealable, plastic bags have been provided for all electronic devices with storage, including but not limited to: cell phones, smart watches, SMART devices, tablets, laptops, and calculators. Please turn off all devices, seal them in the bag provided, and place the bag under your desk for the duration of the examination. You will not be able to touch the bag or its contents until the exam is over.

  If, during an exam, any of these items are found on your person or in the area of your desk other than in the clear, sealable, plastic bag, you may be charged with an academic offence. A typical penalty for an academic offence may cause you to fail the course. Please note, once this exam has begun, you CANNOT re-write it.

- There are X questions and X pages in this exam, including this one.

Q1. Circle either True or False to indicate the truth of each of the following statements.

1.a. ☐ TRUE ☐ FALSE – A* with a heuristic that is not completely admissible may still find the shortest path from the start state to the goal state.

1.b. ☐ TRUE ☐ FALSE – Breadth first search is complete if the state space is infnite but the branching factor is finite.

1.c. ☐ TRUE ☐ FALSE – Every CSP with only unary constraints with non-empty domain has a feasible solution.

1.d. ☐ TRUE ☐ FALSE – On a chess board, the king can move one space in any direction (left, right, up, down, and diagonally). In this context, manhattan distance is an admissible heuristic for the problem of moving the king from square A to square B.

1.e. ☐ TRUE ☐ FALSE – Uniform-cost search will never expand more nodes than A* search.

1.f. ☐ TRUE ☐ FALSE – The minimum remaining value heuristic (MRV) provides a way to select the next value to assign a variable in a backtracking search for solving a CSP.

1.g. ☐ TRUE ☐ FALSE – Backtracking search only checks if a constraint has been violated once all variables in the problem are assigned a value.

1.h. ☐ TRUE ☐ FALSE – When using expectimax to compute a policy, re-scaling the values of all the leaf nodes by multiplying them all with 10 can result in a different policy being optimal.

1.i. ☐ TRUE ☐ FALSE – By using the most-constrained variable heuristic and the least-constraining value heuristic we can solve every CSP in time linear in the number of variables.

1.j. ☐ TRUE ☐ FALSE – If A and B are independent, then $\mathbb{P}(A \mid B) = \mathbb{P}(B \mid A)$.

1.k. ☐ TRUE ☐ FALSE – A* is guaranteed to have found the optimal path to the goal as soon as it generates a path to the goal.

1.l. ☐ TRUE ☐ FALSE – Squaring the values of all the terminals in a game tree may alter the move selections made by Expectimax search.

1.m. ☐ TRUE ☐ FALSE – If $h_1(n)$ and $h_2(n)$ are admissible heuristics, $h_3(n) = 2h_1(n) - h_2(n)$ will be admissible.

Q2. There are three different musicians: John, Mark, and Sam. They each come from different country; one comes from United states, one from Australia, and one from Japan. They each play a different musical instrument; one plays piano, one play saxophone, and one plays the violin. They tale turns playing a solo piece of music (that is, they take turns playing alone) and each musician plays a solo only once. The pianist plays first. John plays the saxophone and plays before the Australian. Mark comes from the United States and plays before the violinist.

Set up this problem as a CSP using the variables john, mark, sam , violin, sax, piano, aust, us, japan. Each of these variables has domain $1, 2$, and 3 except piano which has domain 1. The value of each variable indicates the order in which that variable plays in the band. For example, if aust is assigned the value of 1, then the Australian plays first if piano is assigned the value of 1, then the piano is played first if sam is assigned the value of 3, then Sam plays third.

2.a. Translate the problem into a set of binary constraints between variables, That is, each constraint must be between two variables. List all of the constraints, giving:

- The two variables in the scope of the constraint.
- The constraint definition. Use the symbols $=, \neq$. $<, >, \leq, \geq$, to specify each constraint between the two variables. [**HINT:** all constraints can be specified with these operators.]

2.b. Find the *first* solution to this CSP using the forward checking algorithm. Use the Minimum Remaining Value (MRV) heuristic to order the variables selected for assignment at each step. the variable piano must therefore be the first variable instantiated during search as its domain contains only one value. After piano, always instantiate next variable with the least number of remaining domain values at this point in the search. Break ties by always instantiating the variable that lies first in this ordering:

$$\text{jhon} > \text{mark} > \text{sam} > \text{violin} > \text{sax} > \text{aust} > \text{us} > \text{japan}$$

When exploring possible values of a variable, always explore the lowest numeric value first.

Draw the search tree explored by the above algorithm. **At each node of the search tree indicate:**

- The variable being instantiated, and the value it is being assigned.
- A list of the variables that have had at least one of their values pruned by the new assignment and a list of remaining legal values for each of these variables. Follow the forward checking algorithm precisely: only prune values that would be pruned by the algorithm!
- Mark any node where a dead-end occurs because of domain wipe out (use the symbol DWO).
- What is a solution to this CSP?

Q3. Suppose there is a resource that two agents may want to fight over. Each agent chooses to act as a hawk or as a dove. Suppose the resource is worth R units, where $R > 0$. If both agents act as doves, they share the resource (*i.e.*, each gets $\frac{R}{2}$). If one agent acts as a hawk and the other as a dove, the hawk agent gets the resource and the dove gets nothing. If they both act like hawks, there is destruction of there source and the reward to both is $-D$, where $D > 0$.

3.a. Show the payoff matrix for this dove hawk game.

3.b. Is there any pure Nash Equilibrium (NE) in this game? If yes, specify them.

3.c. Is there any mixed NE in this game different from the pure NE? If yes, specify them.

Q4. You are playing a peculiar card game, but unfortunately you were not paying attention when the rules were described. You did manage to pick up that for each round you will be holding one of three possible cards [Ace, King, Jack] ([A, K, J], for short) and you can either Bet or Pass, in which case the dealer will reward you points and possibly switch out your card. You decide to use Q-Learning to learn to play this game, in particular you model this game as an MDP with states [A, K, J], actions [Bet, Pass] and discount $\gamma = 1$. To learn the game you use $\alpha = 0.25$.

4.a. Say you observe the following rounds of play (in order):

s	a	s'	r
A	Bet	K	4
J	Pass	A	0
K	Pass	A	-4
K	Bet	J	-12
J	Bet	A	4
A	Bet	A	-4

What are the estimates for the following Q-values as obtained by Q-learning? All Q-values are initialized to 0.

$Q(J, Pass) =$ $\qquad\qquad$ $Q(J, Bet) =$

4.b. For this next part, we will switch to a feature based representation. We will use two features:

$$f_1(s,a) = 1,$$
$$f_2(s,a) = \begin{cases} 1, \text{if } a = \text{Bet} \\ 0, \text{if } a = \text{Pass} \end{cases}$$

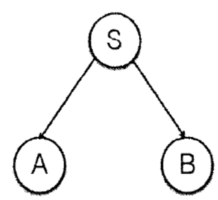
Starting from initial weights of 0, compute the updated weights after observing the following samples:

s	a	s'	r
A	Bet	K	8
K	Pass	A	0

What are the weights after the first update (*i.e.*, after using the first sample), and after the second update (*i.e.*, after using the second sample). Show your process below. Only correct answer with correct process will receive full mark.

Q5. Spacefleet Collegiate has decided to create a class of cyborg students. 90% of these cyborgs study hard for their exams. Out of tlie cyborgs who study hard for an exam, 80% gets a mark of A on the exam. Out of the cyborgs who do not study, only half get a mark of A. Cyborgs who study hard have a 75% probability of causing their battery to die in a day. Cyborgs who do not study hard have a longer battery life only 10% of them will have their battery die in a day.

Let A, B, and S be binary random variables. A – cyborg gets a mark of A, B – battery dies in a day, and S – cyborg studies. The Bayes Net for this problem is as follows:



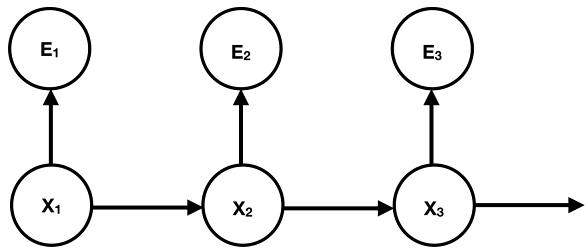
5.a. Using the information provided, construct the conditional probability tables for this Bayes Net.

5.b. How many of the factors in the original Bayes Net have S as one of the variables?

5.c. What is the probability of a cyborg getting an A if it does not study hard?

5.d. Perform variable elimination to determine the probability of the battery dying (or not dying) given that the cyborg gets an A. Show your work.
(It is fine to leave your arithmetic computations in expanded form. E.g., if you find the arithmetic too time consuming, it's fine to leave a computation as (0.2 * 0.7) + 0.3 rather than computing the final answer.)

Q6. (10 pts) In the Hidden Markov Model below, state variables X_t are Boolean and emission variables E_t are drawn from the domain $\{1, 2, 3\}$. The CPTs to specify the model are below.



State	$P(X_1)$
$X_1 = True$	1/2
$X_1 = False$	1/2

State	X_{t+1}	$P(X_{t+1}\|X_t)$
$X_t = True$	$True$	1/4
$X_t = True$	$False$	3/4
$X_t = False$	$True$	1/5
$X_t = False$	$False$	4/5

State	E_t	$P(E_t\|X_t)$
$X_t = True$	1	1/10
$X_t = True$	2	3/10
$X_t = True$	3	3/5
$X_t = False$	1	3/5
$X_t = False$	2	3/10
$X_t = False$	3	1/10

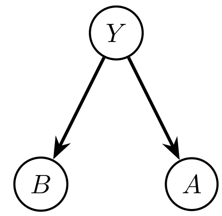
6.a. List three conditional independence relationships that exist in this HMM.

6.b. Calculate $\mathbb{P}(X_4 = \text{True} \mid E_1 = 1, E_2 = 2, E_3 = 3)$. You may leave your answer as products and/or sums of fractions.

Q7. The following parts of this question are independent.

7.a. In this question, we will train a Naive Bayes classifier to predict class labels Y as a function of input features A and B. Y , A, and B are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

A	1	1	1	1	0	1	0	1	1	1
B	1	1	0	1	1	1	1	0	0	0
Y	1	1	1	0	1	1	1	0	0	1



7.a. i. What are the maximum likelihood estimates for the tables $\mathbb{P}(Y)$, $\mathbb{P}(A \mid Y)$, and $\mathbb{P}(B \mid Y)$?

7.a. ii. Consider a new data point $(A = 1, B = 0)$. What label would this classifier assign to this sample?

7.b. Consider the problem of classifying text items into three topic areas: AI, Medicine, and Hiking. We have four training examples:

$\mathbf{e_1}$: Hill climbing informs gradient descent. (AI)

$\mathbf{e_2}$: Alzheimers is a gradual descent. (Medicine)

$\mathbf{e_3}$: Hill climbing helps. (Medicine)

$\mathbf{e_4}$: Gorp helps hill climbing. (Hiking)

7.b. i. Convert the four training examples into the (vector, category) form indicated by the table. (You'll use the reference vocabulary given in the table.)

$\mathbf{e_i}$	Alzheimers	climbing	descent	gorp	gradient	gradual	helps	hill	(category)
$\mathbf{e_1}$									
$\mathbf{e_2}$									
$\mathbf{e_3}$									
$\mathbf{e_4}$									

7.b. ii. We will start with a weight vector for each of the 3 categories ($\mathbf{W}_\mathsf{A}$ for AI, $\mathbf{W}_\mathsf{M}$ for Medicine, and $\mathbf{W}_\mathsf{H}$ for Hiking), with all weights 0 except 1 for the bias on the AI vector.

$\mathbf{W}$	bias	Alzheimers	climbing	descent	gorp	gradient	gradual	helps	hill
$\mathbf{W}_\mathsf{A}$	1	0	0	0	0	0	0	0	0
$\mathbf{W}_\mathsf{M}$	0	0	0	0	0	0	0	0	0
$\mathbf{W}_\mathsf{H}$	0	0	0	0	0	0	0	0	0

Perform one epoch of training showing the resulting changed vectors whenever there is a change made to a weight vector. When each training example is processed, if any weight vector does not change, do not rewrite that vector. Whenever a vector is updated, write the new vector on its own line below, clearly indicating which category it belongs to. For example, $W_H = \ldots$

7.b. iii. Training could take many epochs to converge. See if you can skip the training and manually provide a set of three weight vectors that will correctly handle the training examples.