0.1 Exercise 1: Wave splitting with STARFiSh

Main topics: Simulation of wave propagation in compliant pipe with STARFiSh. Reflection factor. Wave splitting. Analysis and visualization in MATLAB.

About STARFiSh. STARFiSh is a freeware Python toolbox, abbreviated from STochastic ARterial Flow Simulations, able to perform calculations of flow in arterial networks. For the interested student, this can be downloaded to run simulations on your own using Python. In this problem set, however, the results from such simulations are simply given.

Wave splitting The pressure and flow in a compliant vessel can be split into contributions from forward and backward propagating waves, as follows:

$$p = p_f + p_b \tag{1}$$

$$Q = Q_f + Q_b \tag{2}$$

Furthermore, the characteristic impedance is defined:

$$Z_c \equiv \frac{p_f}{Q_f} = -\frac{p_b}{Q_b} \tag{3}$$

Note that It is assumed that the pressure used is expressed relative to the diastolic; i.e. as the difference between the systolic and diastolic pressure.

a) Based on Equations (1) - (3), show that the expressions for the pressure and flow of the forward and backward propagating waves can be written:

$$p_f = \frac{p + Z_c Q}{2} \qquad p_b = \frac{p - Z_c Q}{2} \tag{4}$$

$$Q_f = \frac{Q + p/Z_c}{2} \qquad Q_b = \frac{Q - p/Z_c}{2} \tag{5}$$

Reflection from bifurcation: Assume that we have an artery that bifurcates, as illustrated in Figure 1. The change in the artery at the bifurcation results in a reflection of the original wave. Figure 2 shows the pressure and volume flow in the aorta based on the bifurcation system estimated by STARFiSh, when the period of the incoming pulse is T=0.2s. Figure 3 shows the same results from the bifurcation.png, when it is exposed to an incoming pulse with period T=0.5s. Assume the characteristic impedance, $Z_c = array([[1022602.00367211]])$.

b) Propose an estimate of the reflection factor that this bifurcation results in, based on this plot.

c) By studying Figure 2, we see that the flow is $-1m^3$ from the reflected wave (-1 means that it travels in opposite direction). The flow of the incident wave is $2m^3$. Thus the reflection factor is $\Gamma = \frac{1m^3}{2m^3} = 0.5$.

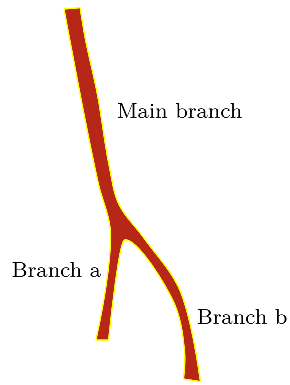

Figure 1: Bifurcation of artery.

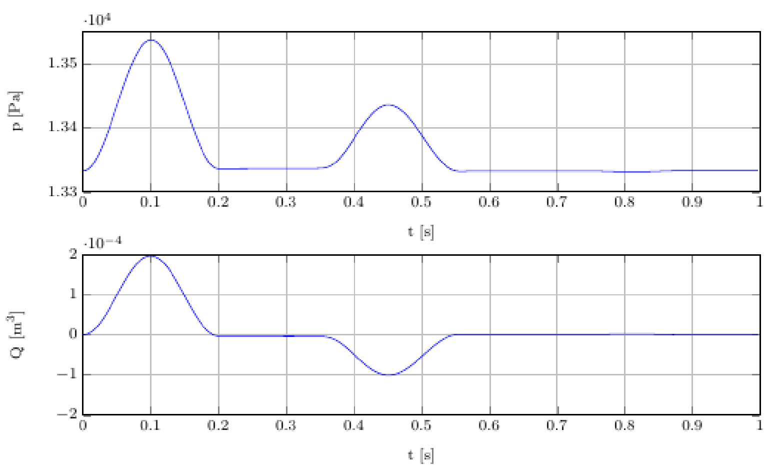

Figure 2: Pressure and flow in aorta for $T = 0.2s$

d) As seen in Figure 3, the estimation of an reflection factor is not as straightforward when the pulse period increases; the forward going and reflected waves are mixed more together. Perform a wave split calculation, using the data found in `bifurcation.csv`, to estimate the reflection factor. Present the resulting plot as well.

By using Equations (4) and (5), the given simulation data is used to calculate the pressure and flow for the forward and backward propagating waves. The pressure supplied is clearly the total pressure (has a static pressure contribution). Therefore, $p_{total} - p_{diastolic}$ is used. This gives the plot seen in Figure 4. Using this plot, the reflection factor is estimated in the same manner to $\Gamma = \frac{1m^3}{2m^3} = 0.5$.

Wave splitting from simulated flow and pressure in arterial network The arterial network of a human being is illustrated in Figure 5. STARFiSh

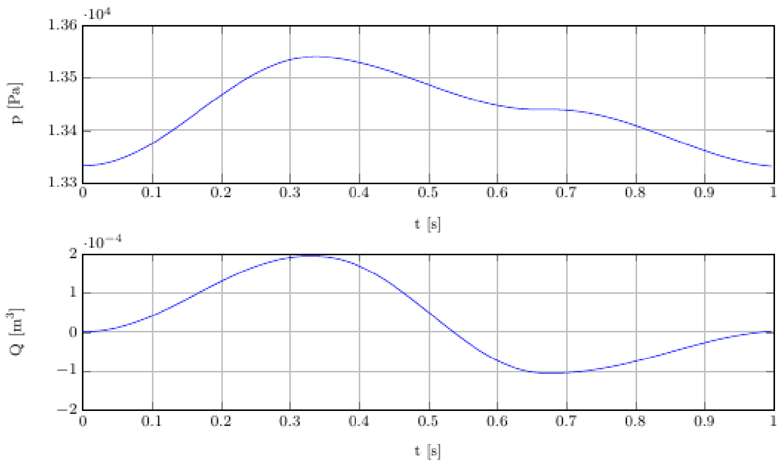

Figure 3: Pressure and flow in aorta for $T = 0.5s$.

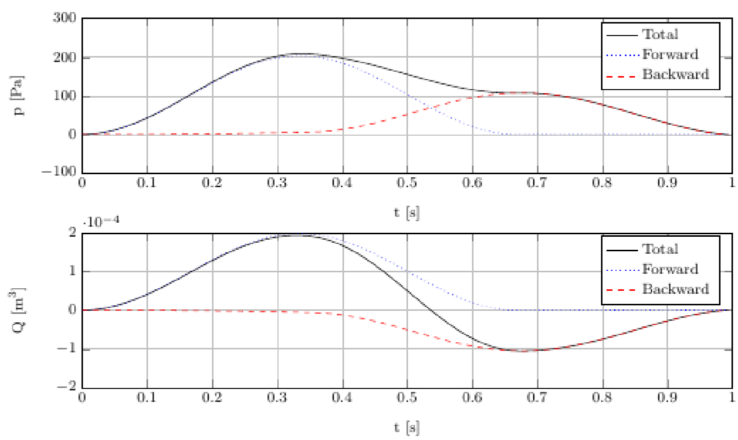

Figure 4: Pressure and flow in aorta for $T = 0.5s$, divided into forward propagating and backward propagating contributions.

is used to run a simulation of a model of the complete arterial network, to yield time series of pressure, flow, wave speed and compliance. This is performed for two different models; one representing a 19 year old test subject and one representing a 75 year old test subject, with results found in the files `age19.csv` and `age75.csv`, respectively. For the 19 year old $Z_c = 6591781.70212634$ and for the 75 year old $Z_c = 10755188.41381322$

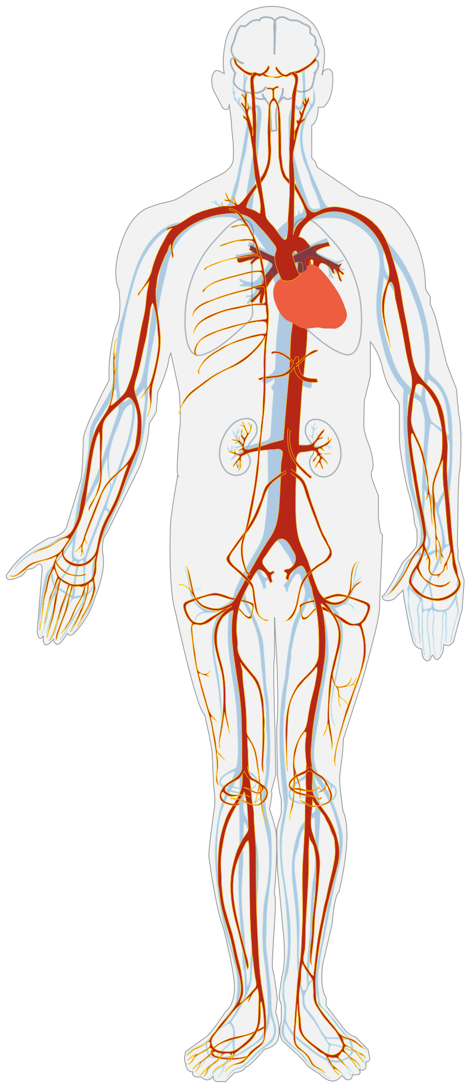

Figure 5: Arterial network of human being. Figure is based on "Arterial System" by LadyofHats, Mariana Ruiz Villarreal - Licensed under Public domain via Wikimedia Commons.

e) Calculate the pressures for the forward and backward propagating waves. Present the results in plots containing the time series of p, p_f and p_b, one for each of the two test subjects.

Solution. The procedure is identical as for the last task. The resulting pressures are shown in Figures 6 and 7.

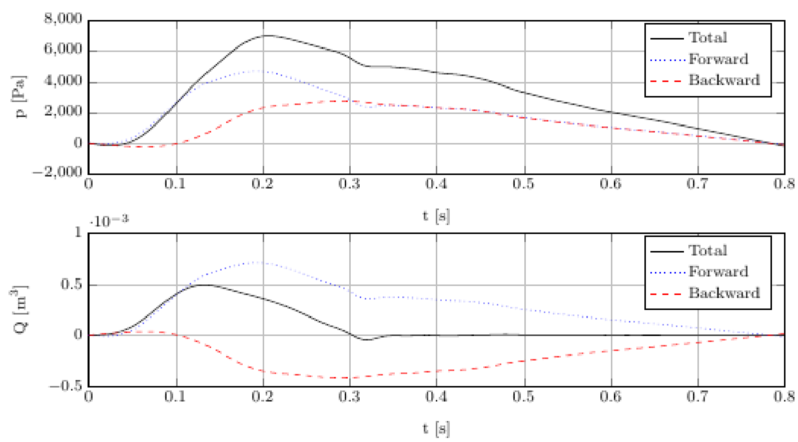

Figure 6: Pressure in aorta from the full arterial network simulation for the 19 years old test subject, divided into forward propagating and backward propagating contributions.

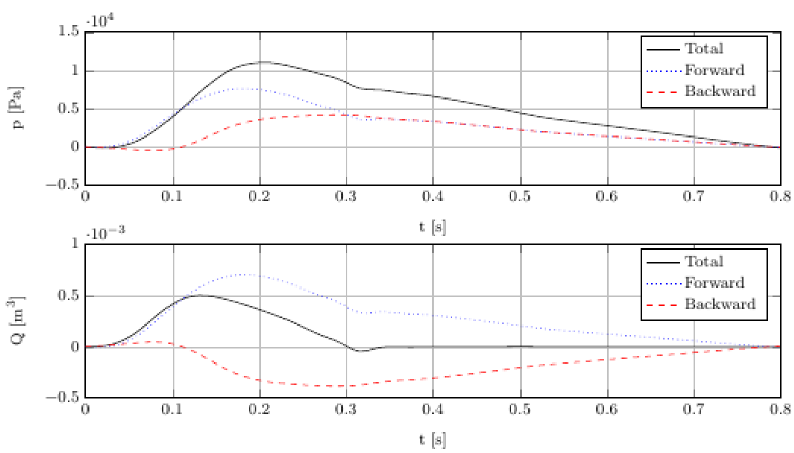

Figure 7: Pressure in aorta from the full arterial network simulation for the 75 years old test subject, divided into forward propagating and backward propagating contributions.

f) How does age influence the reflected (backward) pressure wave?

Solution. The pressure wave propagates faster due to the stiffer arteries, leading to a bigger contribution in late systole by the reflected waves (reflected waves get back earlier), compared to what is the case for younger patients. This contributes to a higher total pressure.