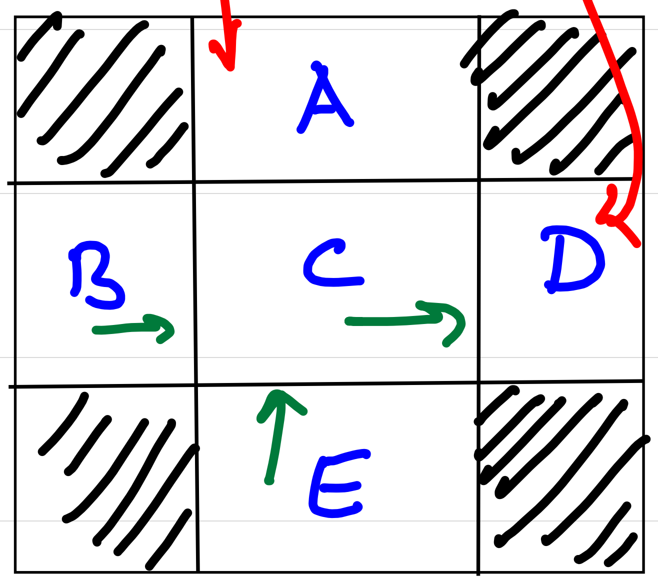
Reinforcement Learning (RL)

- The story So far:
  - MDP V.S. RL ☑
  - Active RL V.S. Passive RL ☑
    - Model based Learning ☑ : Estimate T, R; Passive RL
    - Model Free Learning
      - Direct Evaluation : Estimates $V_\pi(s)$; Passive RL
      - Temporal Difference Learning
      - Q-Learning

Review -- Example: Model-based Learning

Terminal states



Input policy π

(Assume $\gamma = 1$)

Episode 1:

B, east, C, -1,
C, east, D, -1,
D, exit, "end", +10

Episode 2

B, east, C, -1,
C, east, D, -1,
D, exit, "end", +10

Episode 3

E, east, C, -1,
C, east, D, -1,
D, exit, "end", +10

Episode 4

E, east, C, -1,
C, east, A, -1,
A, exit, "end", 10

Learned Model

$\hat{T}(B, east, C) = 1$

$\hat{T}(C, east, D) = 3/4$

$\hat{T}(C, east, A) =$

$\vdots$

$\hat{R}(B, east, C) =$

$\hat{R}(C, east, D) =$

$\hat{R}(C, east, A) =$

$\vdots$

Review -- Pros and Cons of Model-based Learning

- Pro:
  - Makes efficient use of experience (low sample Complexity)
- Con:
  - May not scale to large state space
    - Learns model on state_action pair at a time
    - Cannot Solve MDP for very large $|S|$
  - Much harder when the environment is partially observable

Reinforcement Learning (RL)

- The story So far:
  - MDP V.S. RL
  - Active RL V.S. Passive RL
  - Active RL
    - Model based Learning
    - Model Free Learning → Let's see the basic idea behind it
      - Direct Evaluation
      - Temporal Difference Learning
      - Q-Learning

Basic Idea Behind Model-Free Learning

- To approximate expectations w.r.t. a distribution, we can either
  - Estimate the distribution from samples, then compute the expectation based on the estimated distribution
  - or, bypass the distribution and estimate the expectation from the samples directly

Let's See an Example

- Consider the task of estimating the expected age of UofT students: $E[A]$
- If the probability distribution of A was known, we could find it easily: $\mathbb{E}[A] = \sum_a P(a) \cdot a$
- Without $P(A)$, we have to collect samples: $[a_1, a_2, \dots, a_N]$
  - Model-Based approach:
    - Estimat $P(A)$ first: $\hat{\mathbb{P}}[A=a] = \frac{N_a}{N}$
    - Then we use $\hat{\mathbb{P}}$ to estimate $\mathbb{E}[A]$: $\hat{\mathbb{E}}[A] = \sum_a \hat{\mathbb{P}}[A=a] \cdot a$
  - Model-Free approach:
    - Use the samples to estimate $\hat{\mathbb{E}}[A]$: $\frac{\sum_i a_i}{N}$

Basic Idea Behind Model-Free Learning

- In RL, our ultimate goal is to find an estimate of expected return in each state.
  - Model-based Learning: Estimate the probability distribution of return from the samples. then use it to calculate the expected return.
  - Model-Free: estimate the expected return directly from samples.

Simplified Passive RL Model

- Simplified Passive RL model
  - Input: stream of transitions produced by following some fixed policy $\pi(s)$
    - E.g., we are given the following episodes:

      $(s, \pi(s), s', r, \pi(s'), r', \ldots, end)$

      $(s, \pi(s), s'', r, \pi(s''), r'', \ldots, end)$

      $\vdots$
  - Output: estimate of the state values $\hat{V}_\pi(s)$
  - Note: we don't know T and R

Direct Evaluation

- Consider the passive learning model described before, with the goal of estimating $V_\pi(s)$, i.e. expected total discounted reward from s onward. $V_\pi(s) = \mathbb{E}[R(s,\pi(s),s_1) + \gamma R(s_1,\pi(s_1),s_2) + \cdots]$
- Direct Evaluation: It uses returns, the actual sums of discounted reward from s onward, to estimate its expectation.
  - Anytime you observe a transition from $s,\pi(s),\ldots$, you calculate the sum of discounted reward for it. This will be our sample.

$$U_i(s) = R(s,\pi(s),s') + \gamma R(s',\pi(s'), s'') + \gamma^2 \cdots$$

← the ith time that we observed a transition from state "s", onward.

$$\hat{V}_\pi(s) = \frac{\sum U_i(s)}{N}$$

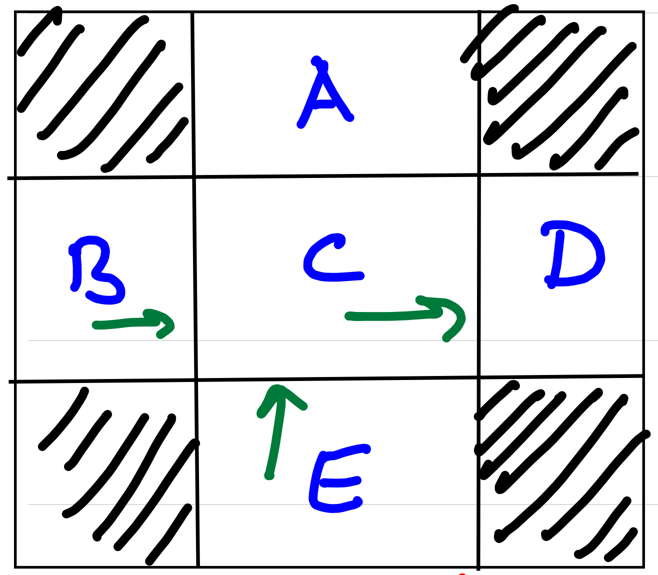
← total number of samples for "s"

This is also known as "direct utility estimation" or "Monte-Carlo evaluation"

Example: Direct Evaluation

Input policy π

(Assume $\gamma=1$)



$\hat{V}_\pi(D) = \frac{10+10+10}{3}$

$\hat{V}_\pi(A) = \frac{-10}{1}$

$\hat{V}_\pi(E) = \frac{\left((-1)+\gamma(-1)+\gamma^2(10)\right) + \left((-1)+\gamma(-1)+\gamma^2(-10)\right)}{2} = -2$

Episode 1:

B, east, C, −1 ,
C, east , D, −1 ,
D, exit, "end", +10

Episode 2

B, east, C, −1 ,
C, east , D, −1 ,
D, exit, "end", +10

Episode 3

E , east, C, −1 ,
C , east , D, −1 ,
D , exit, "end", +10

Episode 4

E , east, C, −1 ,
C , east , A, −1 ,
A , exit, "end", −10

Output Values

	−10	
+8	+4	10
	−2	

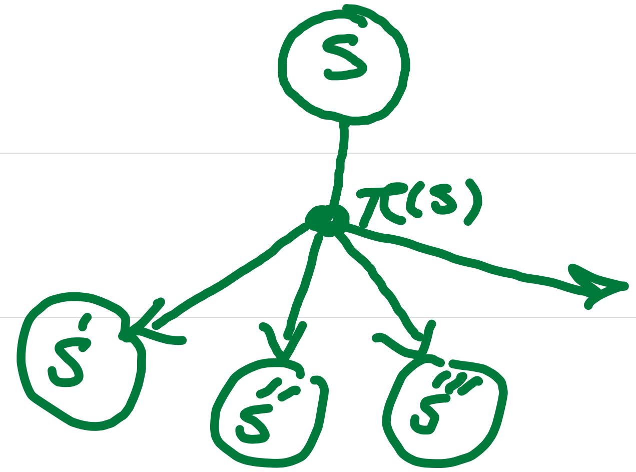
Do you see anything off putting with these estimated value?

How come $\hat{V}_\pi(B) \neq \hat{V}_\pi(E)$

Direct Evaluation Pros and Cons

- Pros:
  - It does not require any knowledge of T and R
  - It converges to the right answer in the limit.
- Cons:
  - It ignores information about state connections.
  - Each state must be learned separately
  - So, slow to learn.

How Can We Incorporate Information About state Connections?



- Direct Evaluation waits for each episode to end
  - Then it finds the sum of discounted return of state "s" for that episode
  - Then, it updates $\hat{V}_{\pi}(s)$
- why don't we use a similar approach to Policy Evaluation?
  - Policy Evaluation uses the state Connection

$$V_{\pi,k+1}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')\left[R(s,\pi(s),s') + \gamma V_{\pi,k}(s')\right]$$

Policy Evaluation incorporates states Connection by using Bellman-like equations.

  - Sadly, we don't have $R(\cdot)$ and $T(\cdot)$:(
    - Temporal Difference (TD) Learning to the rescue

Before TD, Let's See Some Naive Ideas

- Before we present TD, let's study some naive ideas to exploit state connections
  - Idea 1: Use actual samples to estimate the expectation
  - Idea 2: Update value of s after each transition s, a, s', r
- These two naive ideas help us better understand the design principle behind TD

Idea 1

- Let's take a second look at the state value recursion relation: $V_\pi(s) = \sum_{s'} T(s,\pi(s),s')\left[R(s,\pi(s),s') + \gamma V_\pi(s')\right]$

$$= \mathbb{E}_{s'|s,\pi(s)}\left[R(s,\pi(s),s') + \gamma V_\pi(s')\right]$$

- Hence, to estimate $V_\pi(s)$, we must estimate the expectation abave.
  - Just like what we saw earlier, estimate the expected value of a random variable by finding average of its realizations

idea 1, cont'd

- **Idea 1:** use actual sample to estimate the expected return.

$s, \pi(s), s', \ldots \longrightarrow \text{sample}_1 = R(s,\pi(s),s') + \gamma \hat{V}_\pi(s')$

$s, \pi(s), s'', \ldots \longrightarrow \text{sample}_2 = R(s,\pi(s),s'') + \gamma \hat{V}_\pi(s'')$

$\vdots$

$s, \pi(s), s''', \ldots \longrightarrow \text{Sample}_N = R(s,\pi(s),s''') + \gamma \hat{V}_\pi(s''')$

- $\hat{V}_\pi(s) \longleftarrow \dfrac{\sum_i \text{Sample}_i}{N}$

- Issue: In reality, we do not have the luxury of putting our robot is state "s" over and over again, and re-run.

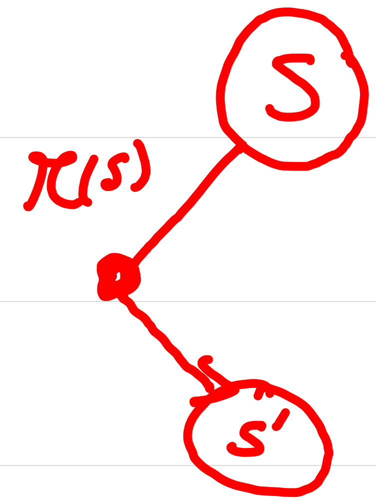
Idea 2

- Idea 1 didn't work, because we cannot generate N samples whenever we want.
- Instead, we should learn to use the possible sample that we may observe in an episode.
- Idea 2: update value of s after each observed transition $s, a, s', \dots$

$\Rightarrow$ Upon seeing this transition, we have a sample:

- Sample of $V_\pi(s)$: $\text{Sample} = R(s, \pi(s), s') + \gamma \hat{V}_\pi(s')$
- update $\hat{V}_\pi(s)$: $\hat{V}_\pi(s) \leftarrow (1-\alpha)\hat{V}_\pi(s) + \alpha\,$ Sample

running average



Running Average

- How can we compute the average of $x_1, x_2, \ldots, x_n$ numbers?
  - Method 1: Add them up and divide by n.
  - Method 2: Keep a running average μ_k and a running count k.
    - $k=0$, $\mu_0 = 0$
    - $k=1$, $\mu_1 = (0 \times \mu_0 + x_1)/1 = x_1$
    - $k=2$, $\mu_2 = (1 \times \mu_1 + x_2)/2 = \dfrac{x_1 + x_2}{2}$
    - $k=3$, $\mu_3 = (2 \times \mu_2 + x_3)/3 = \dfrac{x_1 + x_2 + x_3}{3}$
    - General formula: $\mu_n = \left((n-1)\mu_{n-1} + x_n\right)/n$

$$= \left(1 - \frac{1}{n}\right)\mu_{n-1} + \frac{1}{n} x_n$$

Running Average

- What if we use a weighted averag with a fixed weight?
  - $\mu_n = (1-\alpha)\mu_{n-1} + \alpha x_n$

$$E[\mu_n] = \frac{\cancelto{\mu}{E[x_n]} + (1-\alpha)\cancelto{\mu}{E[x_{n-1}]} + \cdots}{1 + (1-\alpha) + \cdots} = \mu \frac{1 + (1-\alpha) + \cdots}{1 + (1-\alpha) + \cdots} = \mu$$

  - $$\mu_n = \frac{x_n + (1-\alpha)x_{n-1} + (1-\alpha)^2 x_{n-2} + \cdots}{1 + (1-\alpha) + (1-\alpha)^2 + \cdots}$$

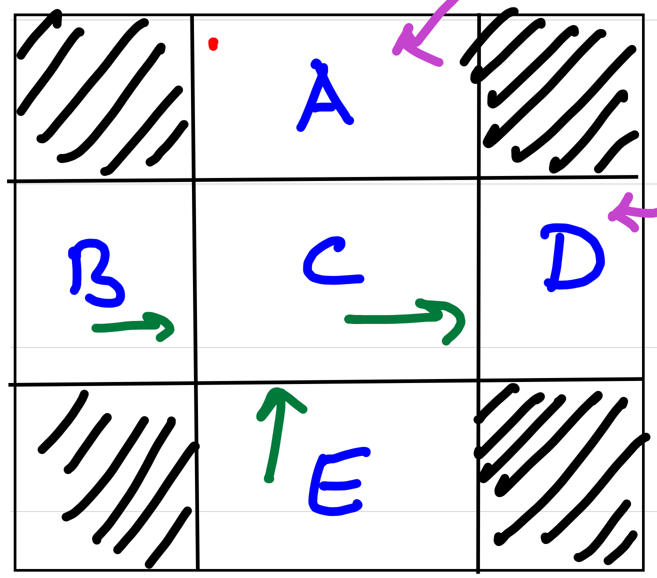
- This way of running average makes recent samples more important.
- Also, it forgets about the past (distant values were wrong anyway)

Temporal Difference Learning

- TD updated the values by maintaining a running average
  - upon seeing the transition s, a, s', R, we have a new sample: $R(s,a,s') + \gamma V_\pi(s')$
  - $V_\pi(s) \leftarrow (1-\alpha) V_\pi(s) + \alpha . \text{Sample}$ (i.e., $V_\pi(s) \leftarrow (1-\alpha) V_\pi(s) + \alpha[R(s,a,s') + \gamma V_\pi(s')]$)
    - We can also write it as: $V_\pi(s) \leftarrow V_\pi(s) + \alpha\,[\text{Sample} - V_\pi(s)]$
- This is Temporal Difference Learning Rule.
  - $[\text{Sample} - V_\pi(s)]$ is the TD error.
  - α is the learning rate
  - We observe a sample, move $V_\pi(s)$ a little bit to make it more consistent with its neighbor $V_\pi(s')$

Example: TD

Terminal states with Known Values



Input policy π

(Assume $\gamma=1$)

($\alpha = 1/2$)

Episode 1:

B, east, C, −2,
C, east, D, −2,
D, exit, "end", +10

Initially set to zero

	0	
0	0	8
	0	

B, east, C, −2

	0	
−1	0	8
	0	

C, east, D, −2

	0	
−1	3	8
	0	

$$\text{Sample} = (-2) + \gamma \hat{V}_\pi(C) = (-2) + 0 = (-2)$$

$$V_\pi(B) \leftarrow (1-\alpha) V_\pi(B) + \alpha\, \text{Sample} = -1$$

$$\text{Sample} = -2 + \gamma \hat{V}_\pi(D) = +6$$

$$V_\pi(C) \leftarrow (1-\alpha) \hat{V}_\pi(C) + \alpha\, \text{Sample} = 0 + \tfrac{1}{2}(6) = 3$$

Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation
  - ↳ mimicking Bellman updates with running sample averages.

- But we can't use the value function, or improve the policy without T.

$$Q(s,a) = \sum_{s} T(s,a,s')\left[R(s,a,s') + \gamma V(s')\right]$$

- What can we do?
  - ↳ Learn q-values (i.e. $Q(s,a)$), not values (i.e., $V(s)$)
  - ↳ makes action selection model-free, too!