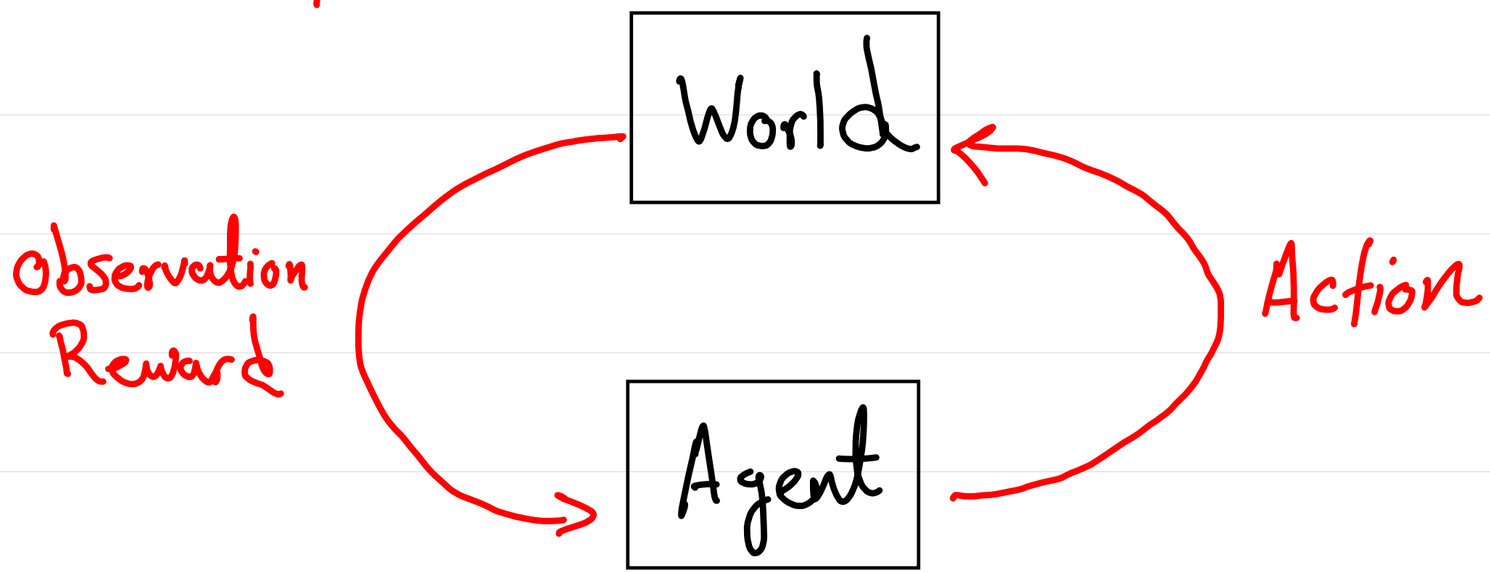
Week 11 - Part 2

- Outcome of this part
  - Sequential Decision process
  - Observation, history, and state
  - Markov Decision Process (MDP)
    - What is Markov about MDP? Why Markov assumption is common?
  - Dynamics Model & Reward Model
    - Transition Graph
  - Return / Utility
  - Policies
  - Finding the "best" policy (i.e., solving the MDP)

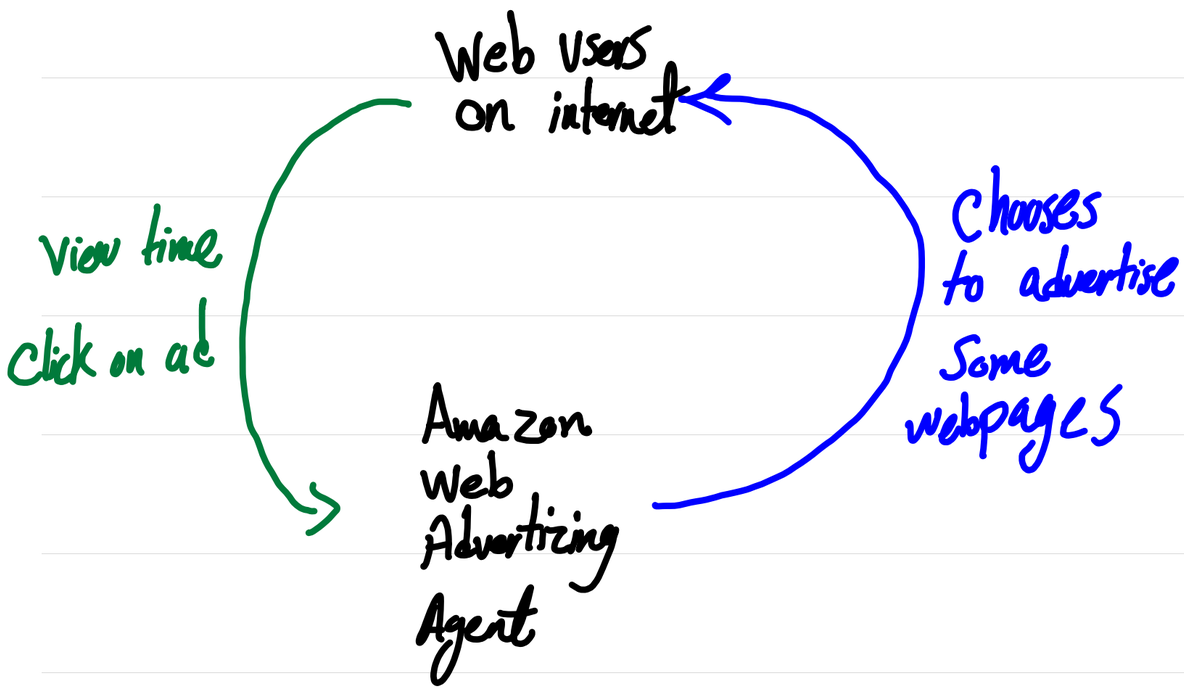
← Next part

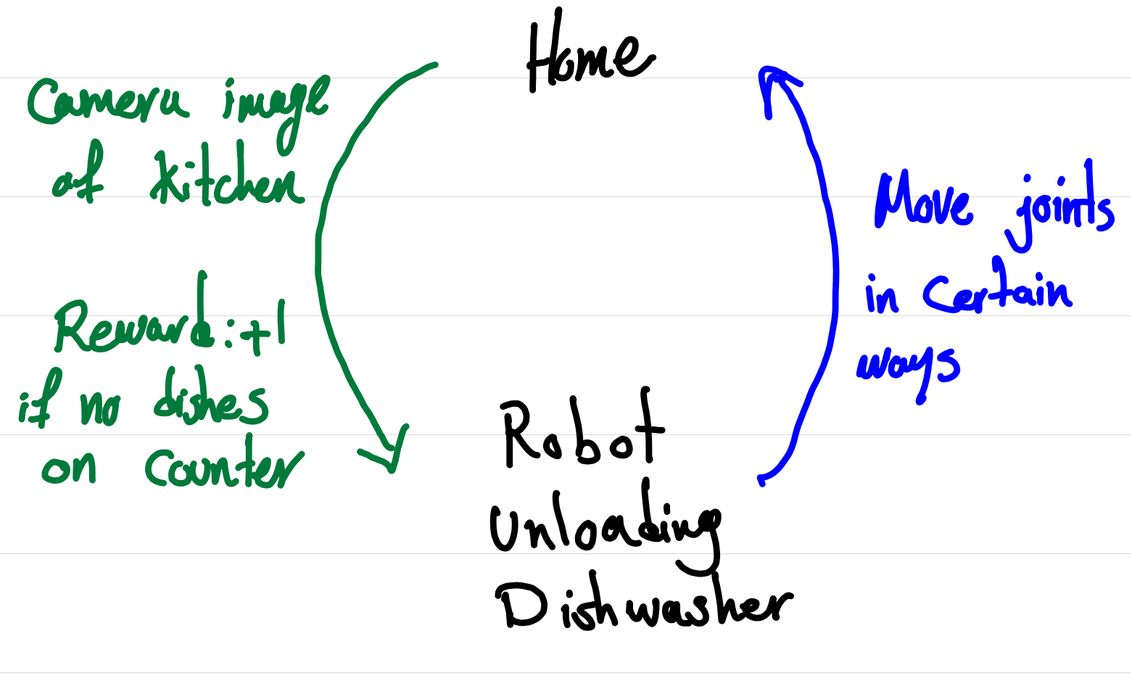
Sequential Decision Making



- **Goal:** Select actions to maximize total expected future reward
  - ↳ May require balancing immediate & long-term rewards

Some Examples

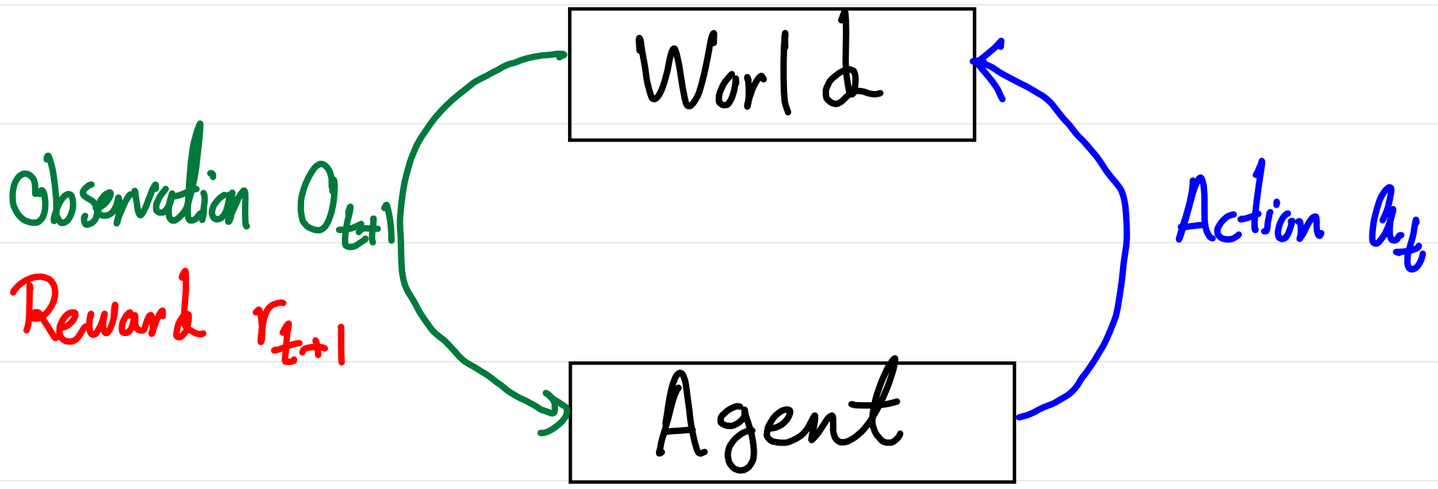




The robot can break all dishes by throwing them on the floor.

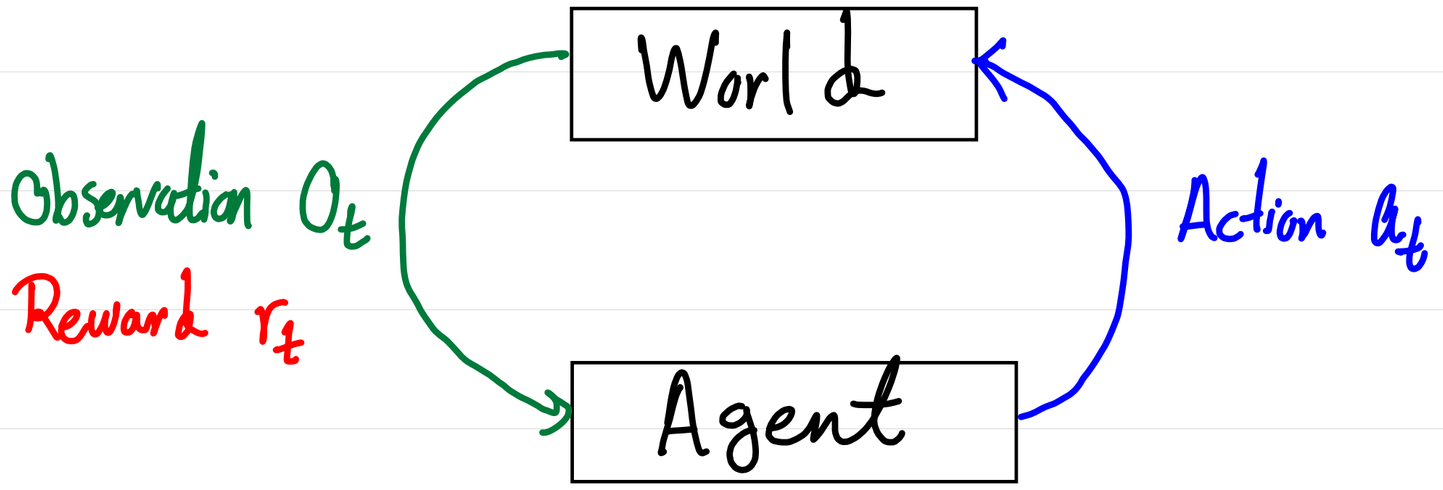
This issue is known as Rewark Hacking

Sequential Decision Process: Agent & the World (Discrete Time)



- Each time step t:
  - Agent take an action a_t
  - Given action a_t, the world updates and emits observation O_{t+1} and reward r_{t+1}
  - the agent receives O_{t+1} and r_{t+1}

History: Sequence of Past Observations, Actions, & Rewards



- History: $h_t = (a_0, O_1, r_1, a_1, O_2, r_2, \ldots, a_{t-1}, O_t, r_t)$
- Agent chooses action based on history
- State: information assumed to determine what happens next
  - State is a Function of history: $S_t = \psi(h_t)$
    - the simplest State function is $S_t = h_t$

$$S_t = O_t$$

Observation V.S. History V.S State

- Can anyone tell me what is the difference between observation, history, and state?

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- Consider the Atari game. How would you design/choose obeservation, history, and state?
  - observation is the last image snapshot of the game
  - state would be defined as the last few observation
    - This allows us to find the speed & direction of the ball

Markov Assumption

- Often, to make problems tracktable, and because it is not a terrible assumption in reality, we will make Markov Assumption.

$$P(S_{t+1} \mid S_t, a_t) = P(S_{t+1} \mid S_t, a_t, S_{t-1}, a_{t-1}, S_{t-2}, a_{t-2}, \ldots, S_1, a_1)$$

- Future (S_{t+1}) is independent of past ($S_{t-1}, a_{t-1}, \ldots, S_1, a_1$) given the present ($S_t, a_t$)
- with markov assumption

$$P(S_{t+1}, S_t, \ldots, S_0 \mid a_t, \ldots, a_1) = P(S_{t+1} \mid S_t, S_{t-1}, \ldots, S_1, a_t, \ldots, a_1) \times \rightarrow P(S_{t+1} \mid S_t, a_t)$$
$$P(S_t \mid S_{t-1}, S_{t-2}, \ldots, a_t, \ldots a_1) \times \rightarrow P(S_t \mid S_{t-1}, a_{t-1})$$
$$P(S_0)$$

Why is Markov Assumption Popular?

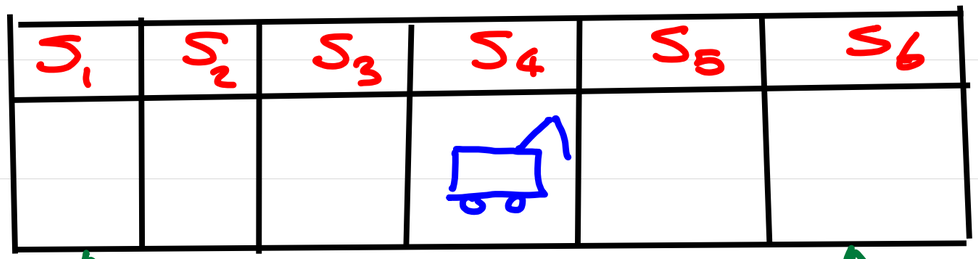
- It is simple and often can be satisfied if includes some previous observations as part of state
- There are many problems that we can satisfy Markov assumption by simply setting $S_t = O_t$.

- Markov assumption significantly simplifies working with the problem.

- Just like all ML problems we studied so far, there is this trade-off between the expressive power of state representation and how long it takes to train your model.

MDP Example: Mars Rover

locations on Mars →



s_1 and s_6 are the terminal locations, you obtain 1 and 5 samples from Soil in locations s_1 and s_6, respectively, and then the robot will shut down

- Mars Rover is the robot that you designed to collect samples on Mars.
- Your robot has to choose from going to right or left, to move between locations.
- Mars is strangely small. Only 6 locations on Mars.

State set: $S = \{s_1, \ldots, s_6\}$

Action Set: $A = \{\text{"Try Left"}, \text{"Try Right"}, \text{"exit"}\}$

Rewards: reward 1 and 5 when you get to s_1 and s_6, respectively.
reward 0, otherwise.

Dynamics Model & Reward Model

- Transition / dynamics model: predicts next state
- In other words, dynamics model specifies the distribution of outcomes when the agent makes a decision

→ $T(s, a, s') = \mathbb{P}[S_{t+1} = s' \mid S_t = s, a_t = a]$

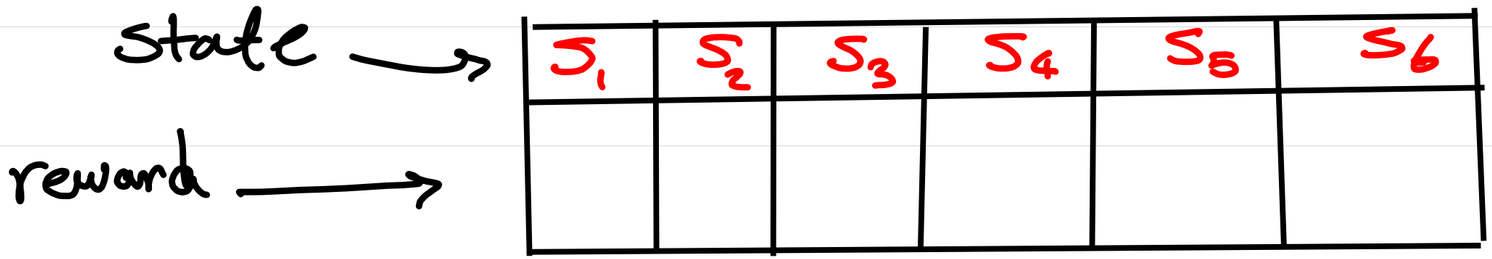
In this course, we assume stationary transition / dynamics model.

i.e., $\mathbb{P}[S_{t+1} = s' \mid S_t = s, a_t = a] = \mathbb{P}[S_{k+1} = s' \mid S_k = s, a_k = a]$

for all t and k.

- Reward model: predicts immediate reward: $R(s, a, s')$
  - Sometimes just $R(s)$, $R(s, a)$, or $R(s')$, depending on the model
  - Note that rewards can be zero, negative, or positive.

Example: Mars Rover Dynamics Model (Reward Model)



States & Actions: $S = \{s_1, \ldots, s_6\}$, $A = \{a_L, a_R, a_e\}$

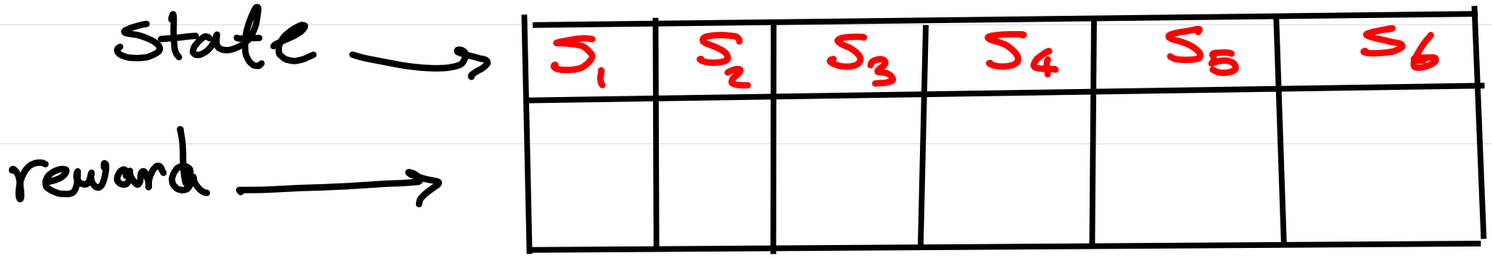
Rewards Model:

$$R(s,a,s') = \begin{cases} 5 & \text{if } s = s_5, a \in \{a_L, a_R\}, s' = s_6 \\ 1 & \text{if } s = s_2, a \in \{a_L, a_R\}, s' = s_1 \\ 0 & \text{otherwise} \end{cases}$$

$$R(s_6, a_e, s_6) = R(s_1, a_e, s_1) = 0$$

It's Mars we are talking about. It's different from our Earth. Our robot is not Completely predictible there. When the robot decides to go toward a direction, with 0.2 probability it may go the opposite direction.

Example: Mars Rover Dynamics Model (Dynamics Model)



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Transition Model:

$$T(S_i, a_L, S_{i-1}) = T(S_i, a_R, S_{i+1}) = 0.8 \quad \forall i \in \{2, \ldots, 5\}$$

$$T(S_i, a_L, S_{i+1}) = T(S_i, a_R, S_{i-1}) = 0.2$$

$$T(S_1, a_e, S_1) = T(S_6, a_e, S_6) = 1$$

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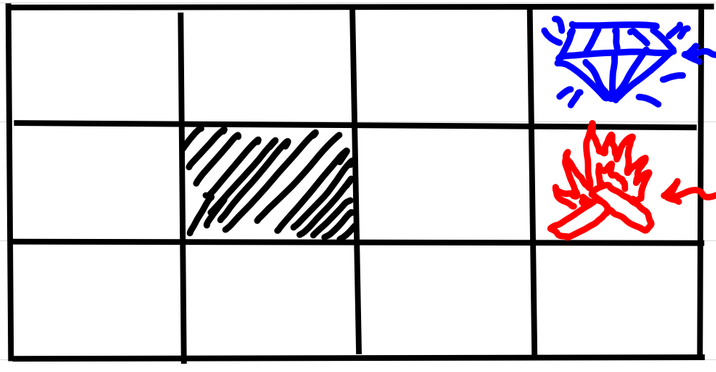
Example: Mars Rover In Grid World

- Consider the following Grid Model of Mars



Mars Rover gets a reward of (+1) if gets to this location

Mars Rover gets a reward of (-1) if it gets to this location

$R(s, a, s') = -0.03$ for all non-terminal states s'

- Noisy Movements on Mars: actions do not always go as planned
  - with prob. 0.8, the action takes the agent to the desired direction
  - with prob. 0.1, the actions takes the agent to a direction prependicular to the desired direction.
  - If there's a wall in the direction would have been taken to, it stays put

Homework: Mars Rover In Grid World

- formulate this MDP.

$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{1,0}$		$S_{1,2}$	$2_{1,3}$
$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$

Example: The undergrad student's life at UofT

- Erfan wants to thrive in his undergrad
- He has three states: happy, tired, burnt-out
  - burnt-out is the terminal state.
- He has two actions: "Work hard", "party hard"
  - Working hard gets double reward: reward 2 for working hard

S	a	S'	T(S, a, S')	R(S, a)
	Work hard		0.5	2
	Work hard		0.5	2
	party hard		1	1
	Work hard		1	2
	party hard		0.5	1
	party hard		0.5	1
	end		1	0

We can summarize the dynamics and reward model with a table

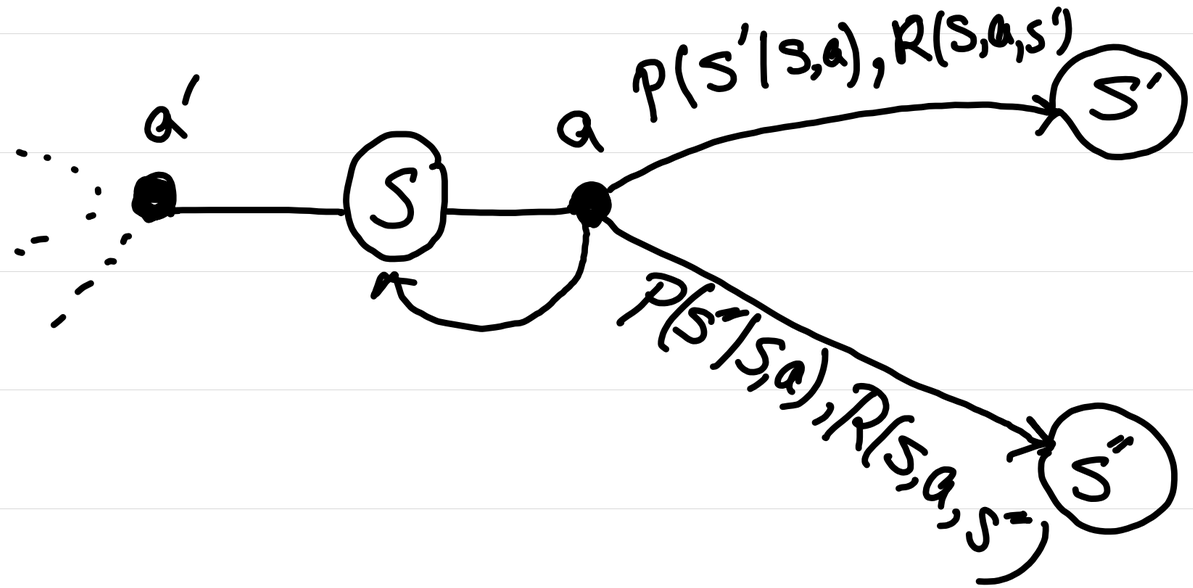
Transition Graph

- Transition Graph is a useful way of summarizing the dynamics of a finite MDP
- There are two types of nodes
  - state nodes: There is a state node for each possible state
    - Typically a large circle labeled by the name of the state, e.g., (s)
  - Action nodes (also called q-state node): There is an action node for each state-action pair.
    - Typically a small solid circle labeled by the name of action or (state, action) pair and connected by a line to the state node, e.g., $\bullet^{a}$ or $\bullet^{(s,a)}$

Transition Graph

- Starting in state "s" and taking action "a" move you along the line from state nods "S" to action node (s, a).

- Then the environment responds with a transition to the next state's node.



Activity: Draw the transition Graph for Erfan the Undergrad