Active Learning

- We saw how to learn $V_\pi(s)$ with TD Learning.
- That was good for passive Learning.
  - In passive Learning, we are given samples generated by policy π
  - Our task is to find $V_\pi(s)$. TD Learning is a model-free approach to Learn $V_\pi(s)$'s.
- Active Learning: We want to be able to do more than that?

We want to be able to update policy π (i.e., the sampling policy) so that not only we collect samples and learn the model, but also make decision that give us good rewards.

- Active Reinforcement Learning:
  - Actively Collecting data, while we learn the model
- Problem model:
  - We don't know T and R, and we choose the actions.
  - Goal: Learn the optimal policy / actions.

Active Learning

- In active reinforcement learning, Learner makes the choices.
- Fundamental tradeoff:
  - exploration v.s. exploitation

Recall: To Estimate an Expectation, use Running Average of Samples

- Let's first write the Bellman equation in terms of Values (i.e. $V^*(s)$):
  - $V^*(s) = \max_a \sum_{s'} T(s,a,s') [R(s,a,s') + \gamma V^*(s')]$
  - Value iteration finds $V^*(s)$ with dynamic programming, i.e.

    $$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') [R(s,a,s') + \gamma V_k(s')]$$

  - In reinforcement Learning, we don't have T and R.

★ That's why we usually use samples to estimate expectations.

- e.g., in TD we could estimate $V_\pi(s)$ w/ running average of samples

$$V_\pi(s) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_\pi(s')] \xrightarrow[\text{running average of samples}]{\text{So, we estimated it with}} V_{\pi,k+1}(s) = (1-\alpha) V_{\pi,k}(s) + \alpha [R(s,a,s') + \gamma V_{\pi,k}(s')]$$

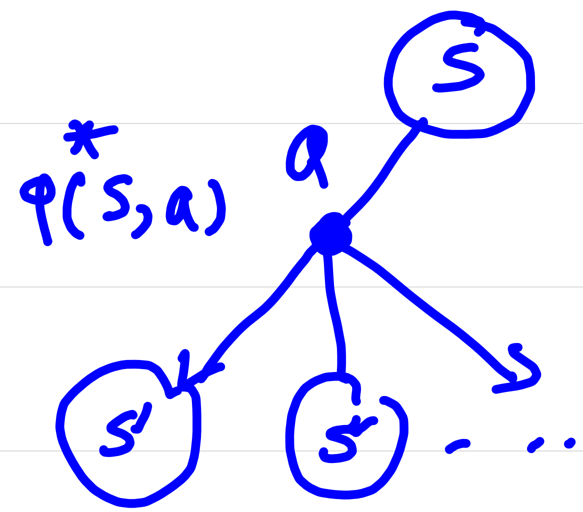
Can We Use Running Average to Estimate Optimal Values?

- Now, we want to do active learning,
  - i.e., Learning the optimal policy / value / q-value
- Can we use the same idea as in TD to find $V^*(s)$?
  - In other words, Can we use running averaging to find $V^*(s)$?
- Let's revisit the recursion for optimal values, i.e. $V^*(s)$:
  - $V^*(s) = \max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$
- Main question: Is $V^*(s)$ written in the above recursion an expectation? Yes ☐ No ☑
  why? Because of the max.

Can We Use Running Average to Estimate q-values?

- Now, let's write the recursion for $Q^*(s,a)$ in terms of Q^*

$$Q^*(s,a) = \underbrace{\sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]}_{\text{This is an expectation}}$$



- Nice! We can use the running average:

$$\text{Sample} = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

$$Q(s,a) \leftarrow (1-\alpha)\, Q(s,a) + \alpha \,\text{Sample}$$

Q-learning

- Q-Learning: Sample-based q-value iteration.

- We have: $Q^*(s,a) = \sum T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$
- Thus, we can estimate the above expectation by
  - Whenever you receive a new sample from a transition like $(s, a, s', R(s,a,s'))$,
  - By using running averaging you can update your estimate of $Q^*(s,a)$: $Q_{k+1}(s,a) \leftarrow (1-\alpha_t)\, Q_k(s,a) + \alpha_t \underbrace{\left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]}_{\text{new sample}}$

Q-Learning Properties

- Q-learning converges to optimal q-values, even if you're acting suboptimally!
- This is called off-policy learning.
- what it means is that in the limit it doesn't matter how you select the action. Regardless of action selection, you will converge to the optimal q-values (hence optimal policy) in the limit.
- But,
  - You have to explore enough (Needs to visit all states often enough)
  - You have to eventually make the learning rate small enough, but not decrease too quickly. $\sum \alpha_t = \infty$, $\sum \alpha_t^2 < \infty$

Exploration vs. Exploitation

- It's not a good idea to always exploit the action that you consider to be the best action you've seen so far.

- You have to try many actions. You must explore.
  - They may not be good for you. But you won't know unless you try them.

How to Explore

- Several scheme for forcing exploration.
  - random actions (ε-greedy) $0 < \varepsilon \ll 1$
    - Every step, flip a coin.
    - with (small) ε probability, act randomly
    - with $(1-\varepsilon)$ probability, act based on the current policy (i.e. estimated q-values)
  - with ε-greedy, we eventually explore the space $\rightarrow$ hence, find the optimal values

    But, it keeps randomly choosing actions even when the learning is done
  - Solutions: i) lower ε over time ii) use exploration function

i) Lower ε Over Time

- One option is to set $\varepsilon = 1/t$.
- i.e., at time step t, with probability $1/t$, act randomly
- with probability $(1 - 1/t)$, act on optimal policy.

- It does eventually converge, but
  - It can be slow
  - It explore all states with similar probability

ii) Exploration Functions

- Idea: to explore areas whose badness is not (yet) established, and eventually stop exploring

- We can implement this idea by modifying our estimate of the q-values.

→ increase the estimate q-values based on how unexplored they are:

- previously, new sample was: $R(s,a,s') + \gamma \max_{a'} Q(s',a')$
- with exploration function, we modify the new sample to

$$R(s,a,s') + \gamma \max_{a'} f\big(Q(s',a'), N(s',a')\big)$$

- $N(s', a')$: # times we had seen the pair (s', a') before
- $f\left(Q(s', a'), N(s', a')\right) = Q(s', a') + \dfrac{K}{N(s', a')}$ ← some constant

exploration function

- So, the update rule with exploration function will be

$$Q_{k+1}(s,a) \leftarrow (1-\alpha)\, Q_k(s,a) + \alpha\left[R(s,a,s') + \gamma \max_{a'} f\left(Q(s',a'), N(s',a')\right)\right]$$

- Note: This propagates the "bonus" back to states that lead to unknown states, as well.