Project Report
on

"Design of a Supersonic Nozzle using Method of Characterisitics"

submitted in partial fulfilment of VI Semester

Bachelor of Engineering

in

Mechanical Engineering

Visvesvaraya Technological University, Belgaum

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This is to certify that this Project Report titled

"Design of a Supersonic Nozzle using Method of Characterisitics"

has been successfully carried out by

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Certificate

Certified that this B.Tech Project Report titled ”Design of Convergent-Divergent nozzle using Method of Characterestics” by Achyut Rokkam, Bharath Shastry and C.P.Uttam is approved by me for submission. Certified further that, to the best of my knowledge, the report represents work carried out by the students.

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Abstract

A nozzle is a device designed to control the direction or characterstics of fluid flow. In the design of nozzles, the area ratio is employed to calculate the inlet, throat and exit areas of the nozzle which correspond to a particular exit mach number and a certain set of operating conditions. However, this method has one drawback. It does not permit the operator to get the actual contour of the nozzle which is approximated to a straight line by employing area ratios.

Owing to this drawback, a mathematical technique known as 'Method of Characterestics' has been employed in this project in order to map the contour of the divergent part of the nozzle by specifying the outlet mach number. This is a technique used to solve partial differential equations. The governing equations for compressible fluid flow reduces to the hyperbolic class of partial dif- ferential equations that do not have closed form solutions. Nozzles are employed for a variety of applications. One such application is accelerating fluid to supersonic speeds. In order to use the Method of Characterestics, it is essential that the nozzle is operating in design condition. Design condition is characterised by subsonic flow at inlet,'choking'(mach one) at the throat and super- sonic flow at the outlet without the presence of shocks in the divergengt section. Due to the applied constraints, the equations reduce to hyperbolic class of partial differential equations which is the requirement in order to employ this technique.

Nomenclature

Φ	Full velocity potential
ϴ	Angle made with respect to streamline co-ordinate axis
Ϻ	Mach Number
ν (Ϻ)	Prandtl Meyer Function
ν	Prandtl Meyer Angle
a	Speed of sound
V	Overall fluid velocity
γ	Ratio of specific heats
α	Mach angle
m	Slope of characterstic line
x	x co-ordinate of characteristic line
y	y co-ordinate of characteristic line

Chapter 1

1. Nozzles

Nozzles are devices used to control the characteristics of a fluid flow, mainly velocity and direction. By varying the cross section of a nozzle its characteristics are controlled. Nozzles are classified broadly based on the speed of flow as subsonic and supersonic nozzles. Supersonic nozzles are further classified as minimum length nozzle and gradual expansion nozzle. Gradual expansion nozzles are used when high quality flow conditions are required at the exit but has high weight considerations, hence minimum length nozzle is used in general cases.

Supersonic nozzles consist of two sections, converging section and diverging section. Converging section is for subsonic flow which increases the velocity of subsonic flow until it reaches speed of sound, ie Mach 1. This occurs at the intersection of converging and diverging section called the throat. This phenomena is called choking. The flow in diverging section is supersonic where it increases from Mach 1 to the required exit Mach number.

Nozzles are designed in a lot of different ways but the most prominent method is by using area ratios and by using method of characteristics. In the first method the area at the inlet throat and exit are calculated based on the exit Mach numbers and pressure ratios. Once the areas are obtained they are joined by straight lines to obtain the contours in between the areas. Whereas method of characteristics is a numerical method where the governing partial differential equations is converted to ordinary differential equations. Which is used to obtain the contour of the nozzle. In this method the area ratios get automatically satisfied because of the governing general equations.

Figure 1.0

1.1 Supersonic flow

The fluid flow through the nozzle is isentropic and quasi one dimensional. In the diverging part the flow is supersonic where the speed of fluid is more than speed of sound ie it is not pre-warned about the flow, hence any obstructions in the flow will be compensated with a “shock” which is basically an obstruction to supersonic flow. A shock in a supersonic flow is the phenomena where properties of the flow changes suddenly across it, generally supersonic flow becomes subsonic after passing through a shock, that is supersonic flow becomes subsonic. During the design of a nozzle it is ensured that there are no shocks produced in it so that the obstruction in flow is avoided.

Chapter 2

Mathematical Modelling

2.1 Method of Characterestics

Method of Characterestics is a technique which is employed to convert partial differential equations into ordinary differential equations. The governing differential equations of a compressible flow are non-linear equations that do not have closed form solutions. It is not possible to solve these equations analytically. To overcome this problem, numerical methods have been devised to solve the equations for a set of boundary conditions.

By employing this method, the co-ordinates can be changed to a new co-ordinate system which transforms a partial differential equation to an ordinary differential equation which can be solved. These PDEs reduce to ODEs only along certain curves which are known as characteristic curves.

Charactersitics are ‘lines’ in a supersonic flow oriented in specific directions along which disturbances (pressure waves) are propagated.

2.2 Properties of Characteristics

2.2.1 *Property 1*

A characteristic in a two-dimensional supersonic flow is a curve or line along which physical disturbances are propagated at the local speed of sound relative to the gas.

2.2.2 *Property 2*

A characteristic is a curve across which flow properties are continuous, although they may have discontinuous first derivatives, and along which the derivatives are indeterminate.

2.2.3 *Property 3*

A characteristic is a curve along which the governing partial differential equations(s) may be manipulated into an ordinary differential equation(s).

2.3 2D irrotational equations

The governing differential equation for 2D, irrotational, isentropic flow is given by

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}-\frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy}=0 \tag{2.1}$$

In the region immediately after sonic throat where the flow is turned away from itself that the air expands into supersonic velocity. This expansion happens rather gradually over the initial expansion region. In the Prandtl Meyer expansion scenario, it is assumed that the expansion takes place across a centred fan originating from an abrupt corner. This phenomenon is typically modelled as a continuous series of expansion waves, each turning the airflow an infinitesimal amount along the contour of the channel wall.

These expansion waves can be thought of as the opposite of shock compression waves, which slow air flow. This is governed by the Prandtl-Meyer function:

$$d\theta = \pm\sqrt{M^2-1}\,\frac{dV}{V}$$

Where the change in flow angle (relative to its original direction) is represented by dθ. Integrating the above equation to give the following:

$$\nu(M)=\sqrt{\frac{\gamma+1}{\gamma-1}}tan^{-1}\sqrt{\frac{\gamma-1(M^2-1)}{\gamma+1}}.\tan^{-1}\sqrt{M^2-1}$$

2.4 Physical meaning of Method of Characterestics

Characterstics give an insight into the PDEs. The crossings of these characterestics can be used to determine ‘shocks’ for potential flow in a compressible fluid. Each characteristic line can imply a solution to the function. Thus, when two characteristic lines cross, the function becomes multi varied at that point resulting in a non physical situation. This contradiction is removed by the formation of a shock wave, a tangential discontinuity or a weak discontinuity and can result in non-potential flow violating the initial assumptions.

Chapter 3

Gas Dynamics

3.1 Governing equations

The three universal governing equations are the following

3.1.1 *Mass*

3.1.2 *Momentum*

3.1.3 *Energy*

These equations are can be derived in two ways

3.1.4 *Integral approach*

This approach employs integral equations and can be used for discontinuous domains.

3.1.5 *Differential approach*

This approach employs differential equations and is applicable to only continuous systems because it evaluates the function at a point.

Gas dynamics is the branch of fluid mechanics that deals with fluids having compressible change in density. The distinguishing factor between compressible and incompressible fluids is done with the help of Mach number which is the ratio of speed of flow to the speed of sound in the medium. For Mach numbers that are greater than 0.3, the fluid can be considered as compressible.

3.2 Isentropic Flow

Isentropic flow is an idealized flow condition which is adiabatic and the work transfers of the system are frictionless; there is no transfer of heat or matter and the process is reversible. Such an idealized process is useful in engineering as a model of and basis of comparison for real processes.

3.3 Varying area flow

The point of difference between incompressible and compressible fluids is the flow through a varying area duct. There are considerable changes in quantities such as pressure, temperature and density when a compressible fluid moves through a varying area duct.

There are certain equations governing this flow which is given by

$$dp = \frac{\rho V^2}{g_c}\left(\frac{1}{1 - M^2}\right)$$

(3.1)

$$\frac{d\rho}{\rho} = \left(\frac{M^2}{1 - M^2}\right)\frac{dA}{A}$$

(3.2)

$$\frac{dV}{V} = -\left(\frac{1}{1 - M^2}\right)\frac{dA}{A}$$

(3.3)

These equations relate the changes in area to the changes in the quantities – pressure, density and volume. As it was mentioned in the beginning of the chapter that Mach number is a good indicator of compressible flow, it is also a determinant of the quantity variations.

As an example, consider a decreasing area for which the ratio of change in area to original area is negative. In case the Mach number is greater than 1 which means it is a supersonic flow, then the pressure will increase according to the above equation. This is the situation of a diffuser with supersonic inlet.

Chapter 4

Programming for nozzle contour

The contour for the nozzle is obtained by generating a grid and obtaining the wall points from the interior points and the initial boundary conditions. The diverging section needs to carefully designed and is obtained in this manner as the flow in this region is supersonic and shocks are to be avoided whereas the flow in converging section is subsonic and any section which can produce choking conditions can be used.

The grid is generated using a program in MATLAB. An algorithm is followed where a new keypoint is generated from existing two keypoints on characteristic lines where the flow properties are known. The keypoints initially known are at the throat and axis of the nozzle. Using these keypoints the keypoints in the interior of the nozzle are determined which are used to determine the wall points of the nozzle which basically give the contour of the nozzle which is the objective at hand.

The equations used to generate are displayed below, which basically involve solving for the coordinates which also involve solving for the slope angles and the prandtl mayer function(nu).

$$m_I = \tan\left(\frac{(\theta-\alpha)_A+(\theta-\alpha)\mathrm{p}}{2}\right) \quad \textbf{(5.1)}$$

$$m_{II} = \tan\left(\frac{(\theta-\alpha)_B+(\theta-\alpha)p}{2}\right) \quad \textbf{(5.2)}$$

$$y_p = y_A + m_1(x_P - x_A) \quad \textbf{(5.3)}$$

$$y_p = y_B + m_{11}(x_P - x_B) \quad \textbf{(5.4)}$$

$$x_P = \frac{y_1 - y_B + m_{11}x_B - m_1 x_A}{m_{11} - m_1} \quad \textbf{(5.5)}$$

The MATLAB code written incorporating these equations and corresponding output obtained is shown below:

```
tic
clearvars
clc
%% Input Parameters
Me=3;
n=10;
G=1.4;
se=5*Me+30;
%se=0.375;
%% Finding PMF for the first Left running char
Gp=G+1;
Gm=G-1;
nul=(sqrt(Gp/Gm)*atand(sqrt((Me^2-1)/(Gp/Gm)))-atand(sqrt(Me^2-1)));
nuf=se;                 % Initial assumption of First Expansion Wave.
nul=se+(nul/2);                 % Nu is half of Theta max or Nu max
dnu=(nul-nuf)/(n-1);
nu=nuf-se+0.375:dnu:(nul-(se-1));
theta=nu;

%% M and Mu
Mb=sym('Mb',[n+1,1]);
for i=1:n
    eqn=nu(i)==(180/pi)*(sqrt(2.4/.4)*atan(sqrt((Mb(i).^2-1)/(2.4/0.4)))-
atan(sqrt(Mb(i).^2-1)));
    M(i)=abs(double(solve(eqn,Mb(i))));
    mu(i)=asind(1/M(i));
end
%% First Sym Point
syms x y
y0=1;               %Point on throat
x0=0;
eqn1 = y - tand(270 + nuf)*x == y0 - x0*tand(270 + nuf);
eqn2 = y == 0;
[A,B] = equationsToMatrix([eqn1, eqn2], [x, y]);
X =double((linsolve(A,B)));

%% First Characteristic
x1=sym('x1',[n+1,1]);
y1=sym('y1',[n+1,1]);
x1(1)=X(1);
y1(1)=X(2);
s1=tand(270+se+theta);
for k=2:n
    s2(k)=tand(0.5*(theta(k-1)+mu(k-1)+theta(k)+mu(k)));
    eqn1=y1(k)-x1(k)*s2(k)==y1(k-1)-x1(k-1)*s2(k);
    eqn2=y1(k)-x1(k)*s1(k)==y0-x0*s1(k);
    [A,B] = equationsToMatrix([eqn1, eqn2], [x1(k), y1(k)]);
    Y =double((linsolve(A,B)));
    x1(k)=Y(1);
    y1(k)=Y(2);
end

%% First Wall Point
% Equating certan values at the wall point from the nearest known point
```

```
nu(n+1)=nu(n);
M(n+1)=M(n);
mu(n+1)=mu(n);
theta(n+1)=theta(n);

% Find intersection of a left running characteristic and streamline
theta0=theta(n);
Dys=tand((theta0+theta(n+1))/2);            %Slope of streamline
Dyc=tand(0.5*((theta(n)+mu(n))+(theta(n+1)+mu(n+1))));  %Slope of
Charectiristic Line
eqn1=(Dys)*x1(n+1)-y1(n+1)==x0*Dys-y0;
eqn2=(Dyc)*x1(n+1)-y1(n+1)==(Dyc)*double(x1(n))-double(y1(n));
[A,B] = equationsToMatrix([eqn1, eqn2], [x1(n+1), y1(n+1)]);
Z=double((linsolve(A,B)));
x1(n+1)=Z(1);
y1(n+1)=Z(2);

x1=double(x1);
y1=double(y1);


%% Interior Points
xs=sym('xs',[n,1]);
ys=sym('ys',[n,1]);
xi=sym('xi',[n+1,n+1]);
yi=sym('yi',[n+1,n+1]);
xs(1)=x1(1);
ys(1)=y1(1);



xi(1,:)=x1;
yi(1,:)=y1;
thetai(1,:)=theta;
nui(1,:)=nu;
mui(1,:)=mu;


count=2;
Mi=sym('Mi',[n+1,n+1]);
for i = 2:n
    for j = 1:n-i+1    %Ensures that column size reduces as you
progress(Others filled with zeros)

        if j==1
            thetai(i,j)=0;
            nui(i,j)=thetai(i-1,j+1)+nui(i-1,j+1);
            eqn=nui(i,j)==(180/pi)*(sqrt(2.4/.4)*atan(sqrt((Mi(i,j).^2-
1)/(2.4/0.4)))-atan(sqrt(Mi(i,j).^2-1)));
            Mi(i,j)=abs(double(solve(eqn,Mi(i,j))));
            mui(i,j)=asind(double((1/Mi(i,j))));

            Dyc_s=tand(0.5*(thetai(i-1,j+1))-mui(i-1,j+1)+thetai(i,j)-
mui(i,j));         %slope of right runnning char
            eqn1 = (Dyc_s)*xs(i)-ys(i)==(Dyc_s)*double(xi(i-1,j+1))-
double(yi(i-1,j+1));
            eqn2 = ys(i) == 0;
            [A,B] = equationsToMatrix([eqn1, eqn2], [xs(i), ys(i)]);
            Xs=double((linsolve(A,B)));
            xs(i)=Xs(1);
```

```
            ys(i)=Xs(2);
            xi(i,j)=xs(i);
            yi(i,j)=ys(i);

        else

        thetai(i,j)=0.5*(thetai(i-1,j+1)+nui(i-1,j+1)+thetai(i,j-1)-
nui(i,j-1));                           %Instead of simultaneous eqns
        nui(i,j)=0.5*(thetai(i-1,j+1)+nui(i-1,j+1)-thetai(i,j-1)+nui(i,j-
1));                              %Instead of simultaneous eqn
        eqn=nui(i,j)==(180/pi)*(sqrt(Gp/Gm)*atan(sqrt((Mi(i,j).^2-
1)/(Gp/Gm)))-atan(sqrt(Mi(i,j).^2-1)));
        Mi(i,j)=abs(double(solve(eqn,Mi(i,j))));
        mui(i,j)=asind(double((1/Mi(i,j))));

        Dyrc_i(i,j)=tand((thetai(i-1,j+1)-mui(i-1,j+1)+thetai(i,j)-
mui(i,j))/2);              %Slope of right characteristic
        Dylc_i(i,j)=tand(0.5*(thetai(i,j-1)+mui(i,j-
1)+thetai(i,j)+mui(i,j)));  %Slope of left Charectiristic Line
        eqn1=yi(i,j)-Dyrc_i(i,j)*xi(i,j)==double(yi(i-1,j+1))-
Dyrc_i(i,j)*double(xi(i-1,j+1));
        eqn2=yi(i,j)-Dylc_i(i,j)*xi(i,j)==double(yi(i,j-1))-
Dylc_i(i,j)*double(xi(i,j-1));
        [A,B] = equationsToMatrix([eqn1, eqn2], [xi(i,j), yi(i,j)]);
        Z=double((linsolve(A,B)));
        xi(i,j)=Z(1);
        yi(i,j)=Z(2);
        end

    end

    thetaw(count) = thetai(i,j);
    muw(count) = mui(i,j);
    nuw(count) = nui(i,j);
    Miw(count) = double(Mi(i,j));
    x_fw(1)=x1(n);
    y_fw(1)=y1(n);
    x_fw(count)=double(xi(i,j));
    y_fw(count)=double(yi(i,j));
    count=count+1;
end
%% REMAINING WALL POINTS
xw=sym('xw',[n,1]);
yw=sym('yw',[n,1]);
xw(1)=x1(n+1);
yw(1)=y1(n+1);
thetaw(1)=theta(n+1);
for i=2:n
    Dys(i)=tand((thetaw(i-1)+thetaw(i))/2);                     %Slope
of streamline
    Dyc(i)=tand(thetaw(i)+muw(i));                              %Slope
of left Charectiristic Line is half of sum of 2 identical things ie wall
point and internal point before it)
    eqn1=(Dys(i))*xw(i)-yw(i)==Dys(i)*xw(i-1)-yw(i-1);
    eqn2=(Dyc(i))*xw(i)-yw(i)==(Dyc(i))*(x_fw(i))-(y_fw(i));
    [A,B] = equationsToMatrix([eqn1, eqn2], [xw(i), yw(i)]);
    Z=double((linsolve(A,B)));
    xw(i)=Z(1);
    yw(i)=Z(2);
end
```

```
xw=double(xw);
yw=double(yw);



%%  OUTPUT
wall_1=[x0 y0; x1(n+1) y1(n+1)];
firstwave=[x0 y0; x1(1) y1(1)];
plot(firstwave(:,1),firstwave(:,2),'-b')
plot(wall_1(:,1),wall_1(:,2),'-b.')
axis equal
axis([0 xw(n)+0.5 0 yw(n)+0.5])
hold on
plot(x1,y1,'-b')
plot(xw,yw,'-b.')

% plot of interior points
i=n;
for q=2:n
    for j = 1:n-i+1
        xpi(j)=double(xi(i,j));
        ypi(j)=double(yi(i,j));
    end

    xpi(j+1)=xw(i);
    ypi(j+1)=yw(i);

    plot(xpi,ypi,'-b');
    i=i-1;
end

%plot of waves
for i=2:n
    xy1=[x0 y0 ; double(xi(i,1)) double(yi(i,1))];
    plot(xy1(:,1),xy1(:,2),'-b');
end
str=sprintf('Nozzle contour for Mach: %4.2f',Me);
title(str)
hold off
%%
toc
```

Chapter 5

Simulation and verification of the validity of nozzle contour

The contour that was obtained from the MATLAB code is presented in the image given below. From this, it can be observed that setting a higher number of characteristic lines can improve the smoothness of the contour since there will be more number of points to interpolate. The outlet Mach number was set to 3 and the calculations were made.

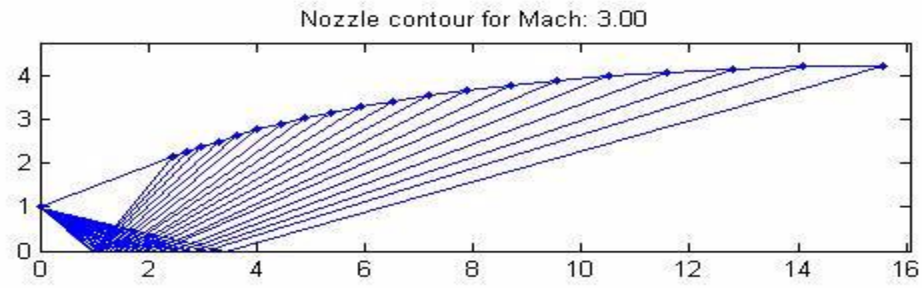

Figure 5.1

Once this was done, the contour and the output was validated using a CFD software. The CFD software used was fluent. The contour plots of Mach number, pressure and density was recorded and is displayed below.

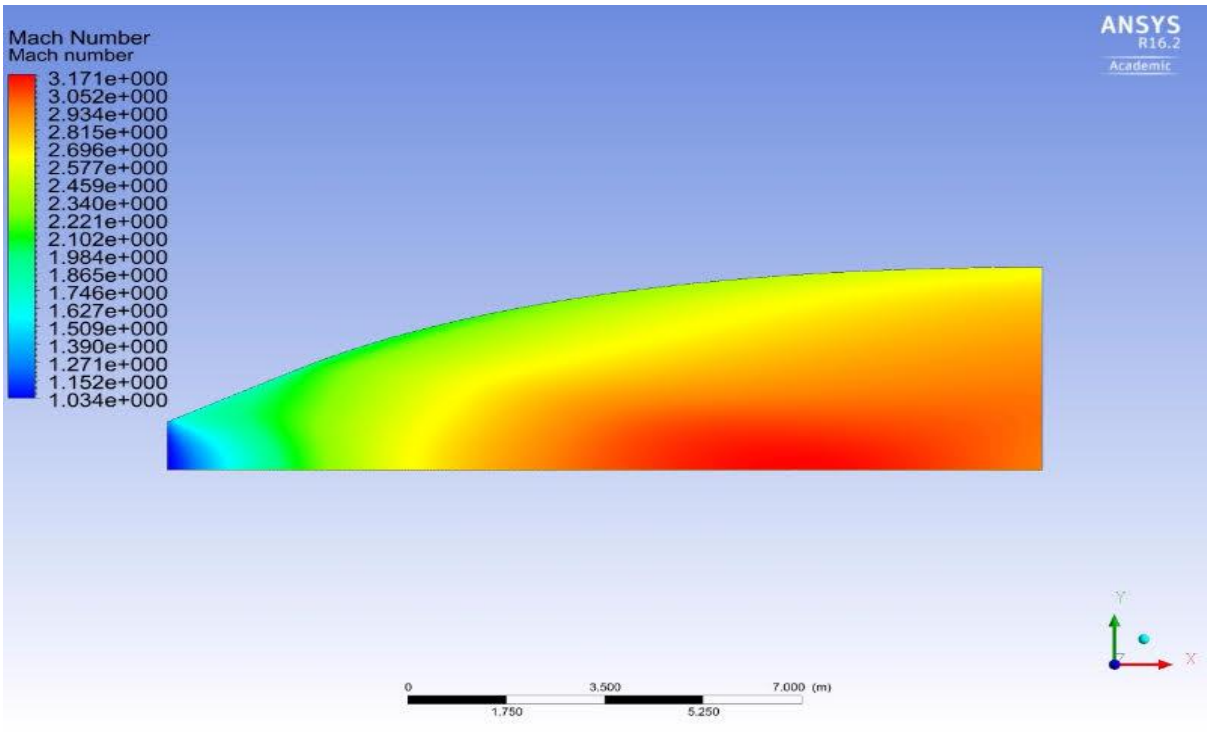

Figure 5.2

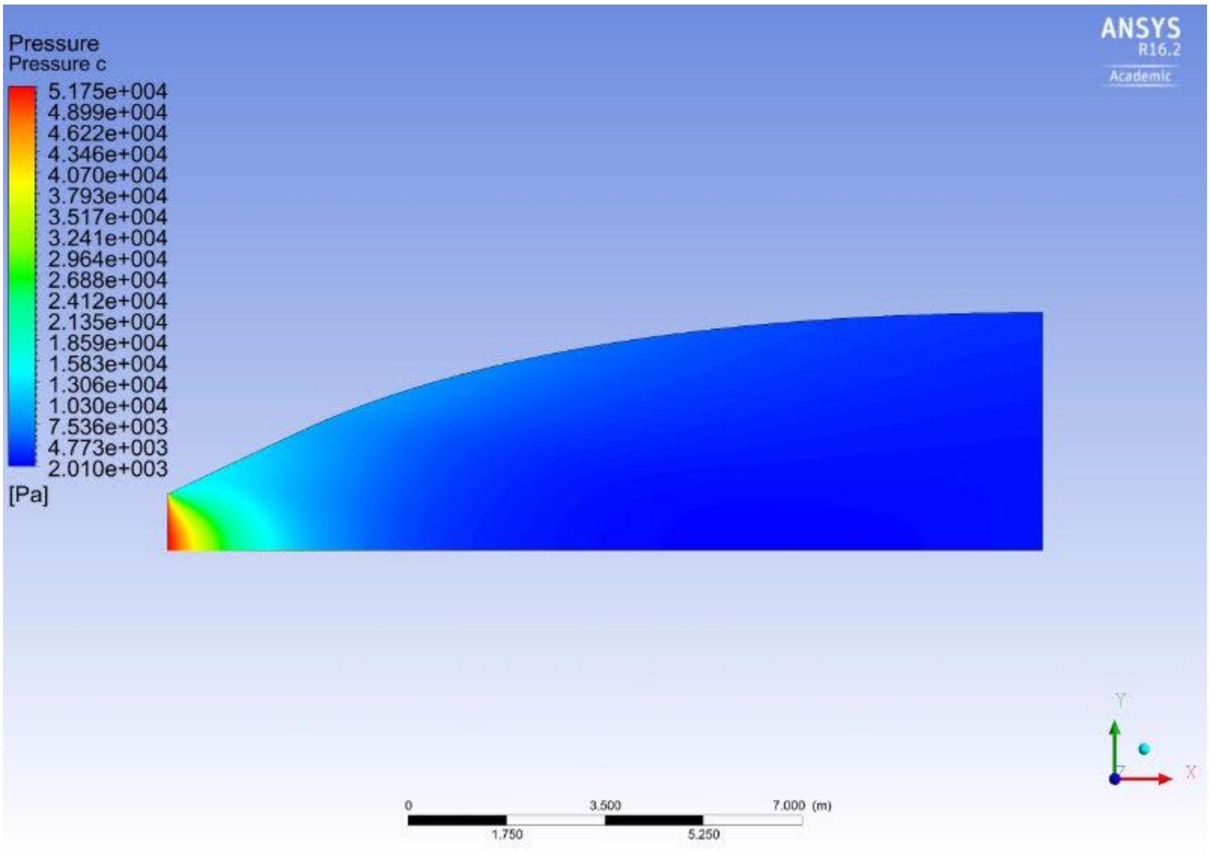

Figure 5.3

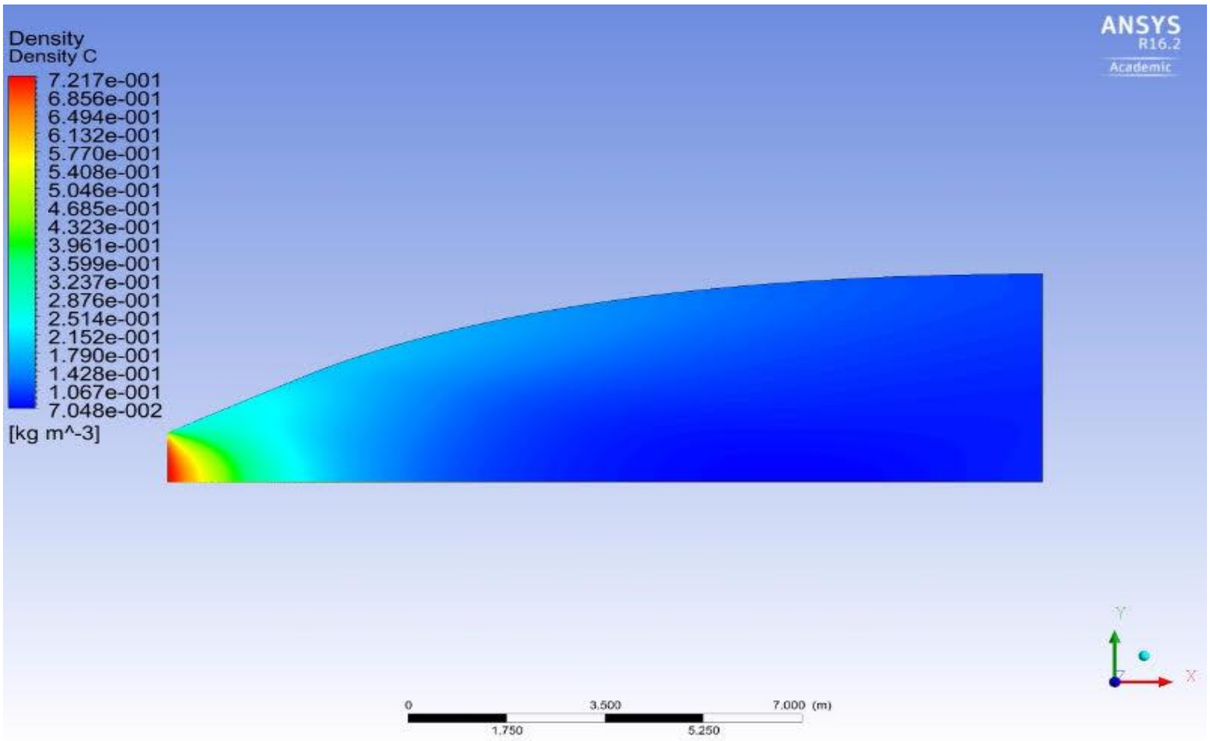

Figure 5.4

These contour plots give an indication that the contour of the nozzle obtained from the MATLAB code that was developed is in close conformity to the values of Mach number from these plots. Slight variations that occur is due to approximation errors that usually arise in such CFD packages.

Plot of Mach number Vs Nozzle length is obtained. A gradual change in Mach number throughout the length of the nozzle is observed which explains the fact that there is no shock produced inside the divergent section of the nozzle. This is the objective of the project where shock was expected to be avoided to ensure agreement of nozzle design condition.

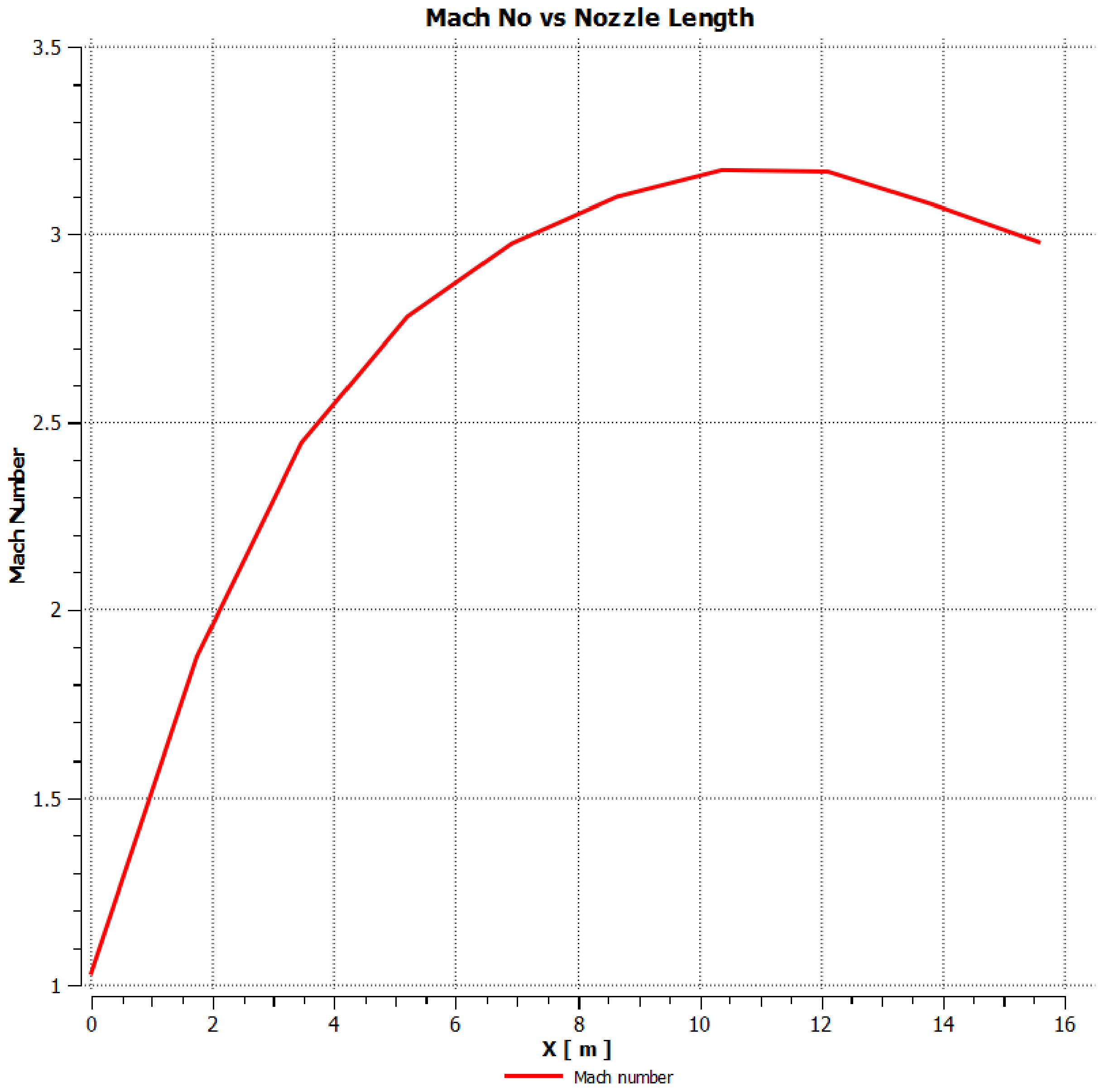

Figure 5.5

The final plot is the variation of pressure with the nozzle length. It can be observed that the pressure reduces gradually without any sudden jump which shows that there is no shock in the divergent section. Also, this graph conforms with the isentropic area variation relations which indicate that with an increase in the velocity of the fluid, there is a subsequent reduction in the pressure.

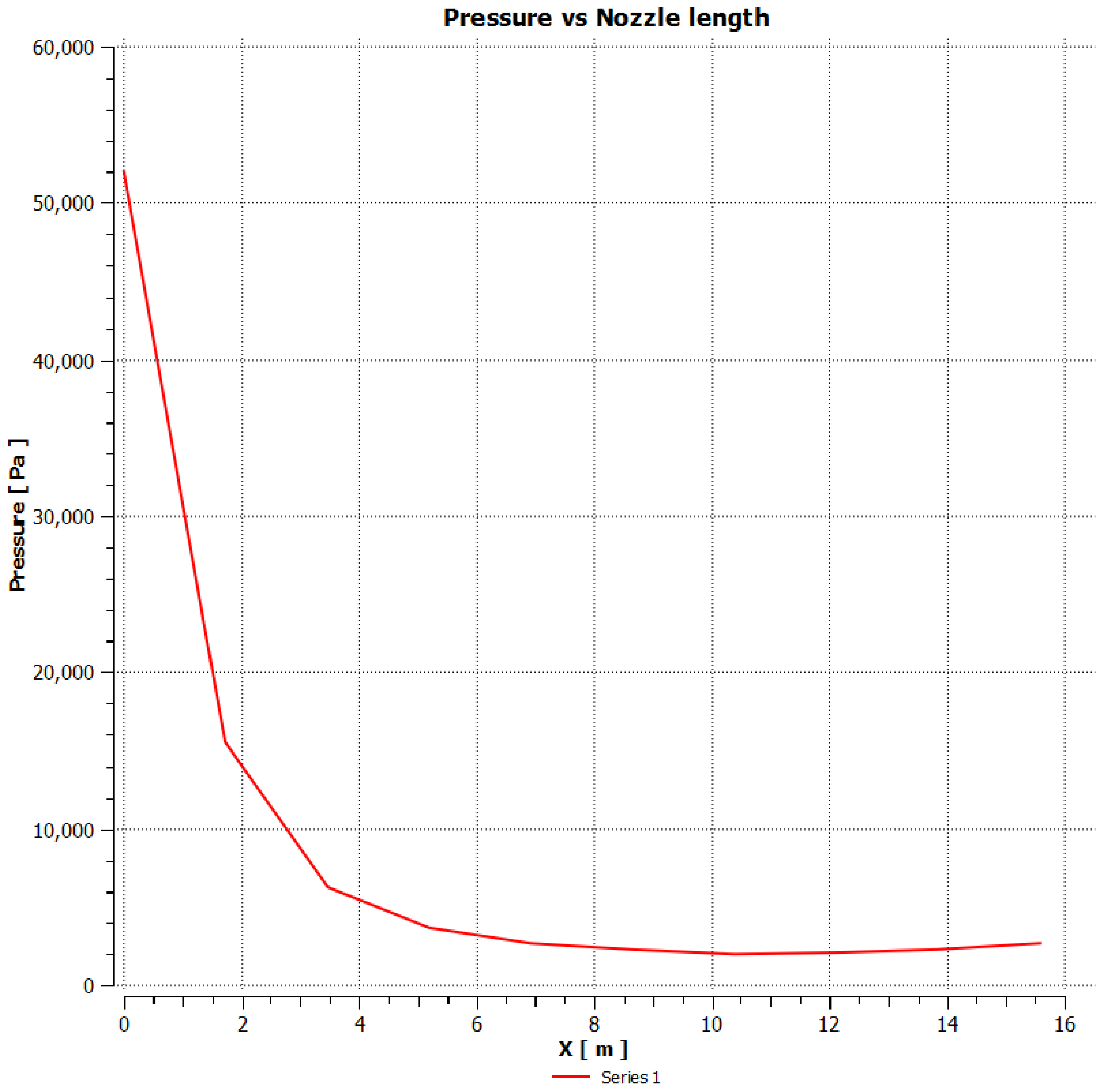
Pressure vs Nozzle length
60,000
50,000
40,000
30,000
20,000
10,000
0
Pressure [Pa]
0
2
4
6
8
10
12
14
16
X [m]
Series 1

Figure 5.6

Chapter 6

Conclusion

Contour for a minimum length nozzle is obtained by an algorithm, which is executed on MATLAB. Using the code written on MATLAB, contours can be obtained for any exit Mach number. The contour for exit Mach number 3 is simulated on fluent and the results are successfully verified.

The nozzle can be manufactured and can be used to generate the required supersonic Mach numbers.

Chapter 7

References

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