· Claim: Let $(a_n)_{n=0}^{\infty}$ be a sequence of rationals then

LIMan = $\lim_{n\to\infty} a_n$ Proof: let L= LIM LER We want to show that (an) nom converges to L. YEYO FN>M, s.t. Y n>N, lan-LISE Suppose for contradiction that (an) n=m doesn't converge to 1. J €>0, Y N>m, J n>N 5.+. |an-L1>€ Contrapositive: Let E>0 be such a positive number For each N>m there is always some n=N s.t. lan-Ll>E There are infinitely many now s.t. L-E>andL+E => Either there are infinitely many n s.t. and L+ & or there are infinitely many n s.t. and 1- &. (Pigeonhole principle)

Men	We also know that (an) n=m is a Cauchy sequence
	YESO IN st. Y Nm >N
7	$ a_n - a_m \leq \varepsilon$
	Maria Maria Maria Maria
	For E = 80/2
	(at except to about that (as), converges to 1
	lan-aml < Ed > Yn, m> No 5
	Then the Vita MINI OSS
	let n,> No be the index s.t. any 1+80
1000	Server in the line that (as) in their
	From (3) an > an - 80/2
	am 49/2 (an + am + 60/2)
	121/26
	LIM an > LIM (1+60/2)
	Let EXO de each a positive success
1.4	1 > 1 + 600
1/4	L? L+ 20/2 for some co>o [(ontradiction]
	Drang (Casa): 10+ 111
	Proof (Case 2): Let n2>No be an index s.t.
	There are institify notify and state 1-6 >0.
	From 3 . a. (a. + 40)
2 2	From 3 an 4 an + 40/2
	in - 1 so CL To Robert platinioni ava aradi
	L= 11M Q, \leq 1-\frac{20}{n}
	L= LIM an ≤ L-20/2 (signing stantage)
⇒	L \le L - \(\frac{\chi_0}{2}\) for some \(\epsilon\) [Contradiction]

D	(an) n=m is a convergent sequence converging to L or lim an=L n->00
	h->00 an=L
	Bounded sequences: A sequence is said to be bounded iff I some MEIR st. lanl & M Y n>m.
•	Bounded sequences: A sequence is said to be
	bounded iff I some MEIR s.t. lan1 & M + n>m.
	\$60 - (661)) 301 (38)
•	Lemma: Every Cauchy sequence is bounded.
	Proof: (a, azan, an, an, an,
	Cauchy sequence of reals.
•	(Sup(E) Least upper bound on [ip ECR : swat timil
	(2) If I contains too i.e. toof I then sup (I) = too
	Limit faws: $(a_n)_{n=m}^{\infty} \text{ and } (b_n)_{n=m}^{\infty} \text{ s-t.}$ Let $(a_n)_{n=m}^{\infty} \text{ and } (b_n)_{n=m}^{\infty} \text{ s-t.}$
	$x = \lim_{n \to \infty} a_n, y = \lim_{n \to \infty} b_n$
	Then,
0	lim (an t bn) = lim an t lim bn
Ø	lim (an bn) = (lim an) (lim bn)
3	If bn to & y to then or too bon monory
	$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \lim_{n\to\infty} a_n$ $\lim_{n\to\infty} b_n$
	Sup(A) = 12 but 12 # A

__/_/___

ct	(1,2,3,)
	Extended real number system
9-A	$R^{\dagger} = R \cup \S + \infty, -\infty \S$
	Define $S = (+\infty) = -\infty$ $Z = (-\infty) = \infty$
	Supremum on extended real line R*
	Let ECR* then
0	Sup(E) = Least upper bound on E if ECR
	If E contains +00, i.e. +00 + E then sup (E) = +00
	If + \sigma \notin \in \in \in \in \in \in \in \in \in \
	sup(t) = sup(t/2-02)
	Inf(E) = - Sup(-E), where
	-E= \{-x: x \neq R \neq \} if E= \{x: x \neq R \neq \}
	Supremum need not to be in the set.
eg.	{x:xeR, x2/23 = A sil = (1) ail
	$sup(A) = \sqrt{2}$ but $\sqrt{2} \notin A$

	Let (an) n=m be a sequence of reals then
	Def: Sup (an) n=m := Sup(E)
	E= {aner: n>m3
	L- Tayer. nsmg
	$\inf_{n \in \mathbb{N}} (a_n)_{n=m}^{\infty} = \inf_{n \in \mathbb{N}} E$
	$\inf(a_n)_{n=m} = \inf E$
_	be event to show that labbers courries in
	Every convergent and/or Cauchy sequence is bounded
	V () () () () () () () () () (
	Is converse true? No
	3=17-40
	§ 1,-1, 1, -1, } ← Bounded but not convergent
120	Let us fix some exo then 1-6 cannot be in by
	But we can impose more conditions on bounded sequence
	to get convergence! Yes.
	= 3 scme Nosm for which aust-E
•	Proposition:
	For no Na
@	Let $(a_n)_{n=m}^{\infty}$ be a sequence that is upper bounded by some MEIR and $a_{n+1} > a_n \forall n > m$ then $(a_n)_{n=m}^{\infty}$ is
	some MEIR and any an & nom then (an) 13
	convergent. In particular,
	lim a = sunta 200 x M
	lim an = sup(an) sm < M
(b)	Let (bn) be a sequence that is lower bounded by
	Let $(bn)_{n=m}^{\infty}$ be a sequence that is lower bounded by some MCIR and $a_{n+1} \leq a_n + n > m$ then $\lim_{n \to \infty} b_n = \lim_{n \to \infty} (b_n)^{\infty}$
	\$ \$\frac{1}{2} \tau \tau \tau \tau \tau \tau \tau \tau
	Lie> an> 1-6
	OF 10-L1 CE 4 0> No

	Preof:
@	Since (an) n=m has an upper bound, this means sup(an) n=m exists
	5 - p county = m ex 1313
	$L := \sup(a_n)_{n=m}^{\infty} = \sup(E), \text{ where } E = \{a_n : n\}_m \}$
	The sale of the sa
	We want to show that (an) n=m converges to L
19	· Exercise conversely andloss Carely conversely borns
	YE>o, ∃N≥m s.t. Yn≥N
	Is convenie to a No
	lan-L1 & E
	91,-1,1,-1, Evended but not convergent
	Let us fix some E>0 then L-E cannot be an Upper
01131162	bound on E. Williams sent stognil in set toll
	to not conservence. Yes.
→	3 some Norm for which and 1-E
	· Proposition:
	For n> No
61 p	@ lot law be a sequence that is upper bounds
21	anzanoxiLE-O sec so ban sight and
	Constraent. In particulars
	But I is an upper bound on E
	anel (n>No)
- 10	6) Let Chi), be a seque of that 3+1/20 bearing
11 41	some Mell and and and and have the
(=)	0 6 2 3
	L+E> an> L-E
	or lan-L1< & + n>No

	//
3	4 E>O, I some No s.t. Y n>N.
	lan-L1∠E → lim an = L
•	Upper Bounded + Increasing => convergent Lower Bounded + Decreasing => convergent
•	Not convergent > Not upper bounded or Not increasing
Ex:	
0	h→∞
6	$\lim_{n\to\infty} x^n = +\infty \forall x>1$
→	$x^{n+l} = x^{n} \cdot x < x^{n}$
113777	It is a decreasing sequence
	$1 + 6 \times 1 = \times 4 = 1$
	Also, xn >0 (because x>0)
	(1) = (100(X) × (1100 (1))
⇒	$(x^{\eta})_{n=1}^{\infty} \text{ is convergent} \xrightarrow{(x)_{n=1}^{\infty}} (x^{\eta})_{n=1}^{\infty} \text{ is convergent} \xrightarrow{(x)_{n=1}^{\infty}} (x^{\eta})_{n=1}^{\infty} \text{ is convergent}$
	> lim xh:=1 n→∞ taxo travot ab mil = 10-11
	TOLKS ELICIONO MIS COLOR
	lim xn+l = (lim xn) (lim x) h->00 h->00
	h->00
=>	L=Lx
	since XLI, L=0

