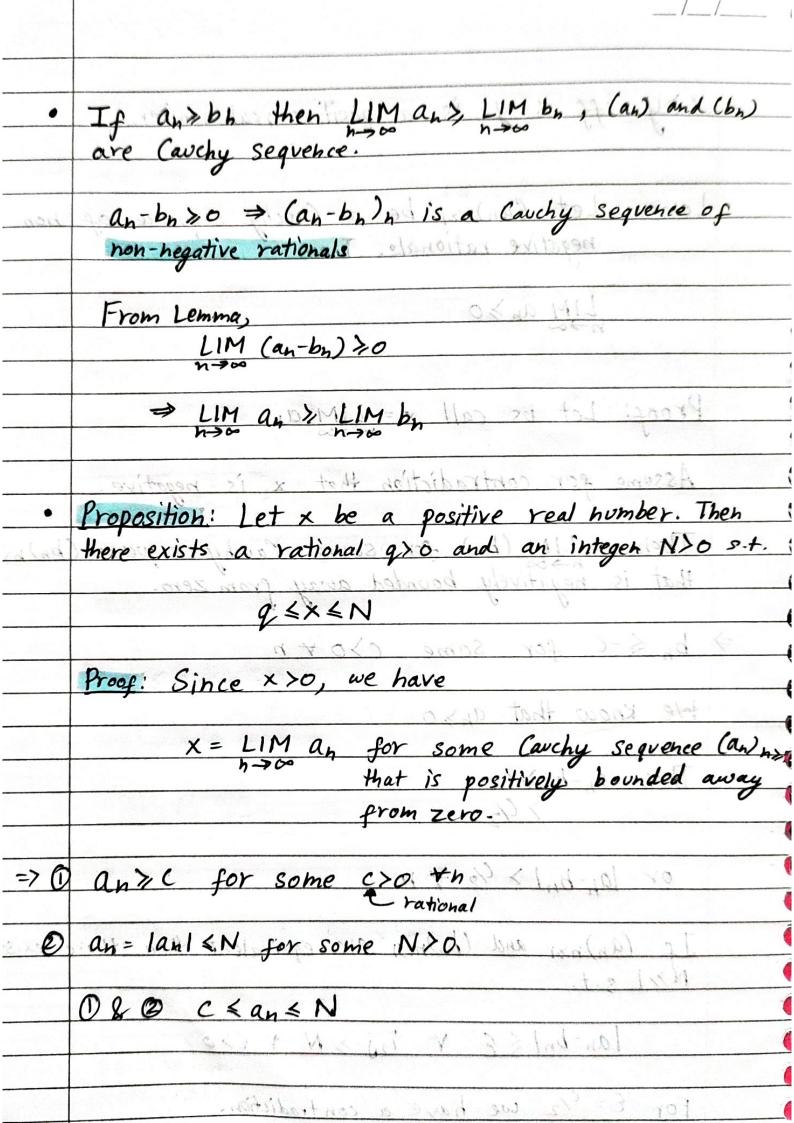
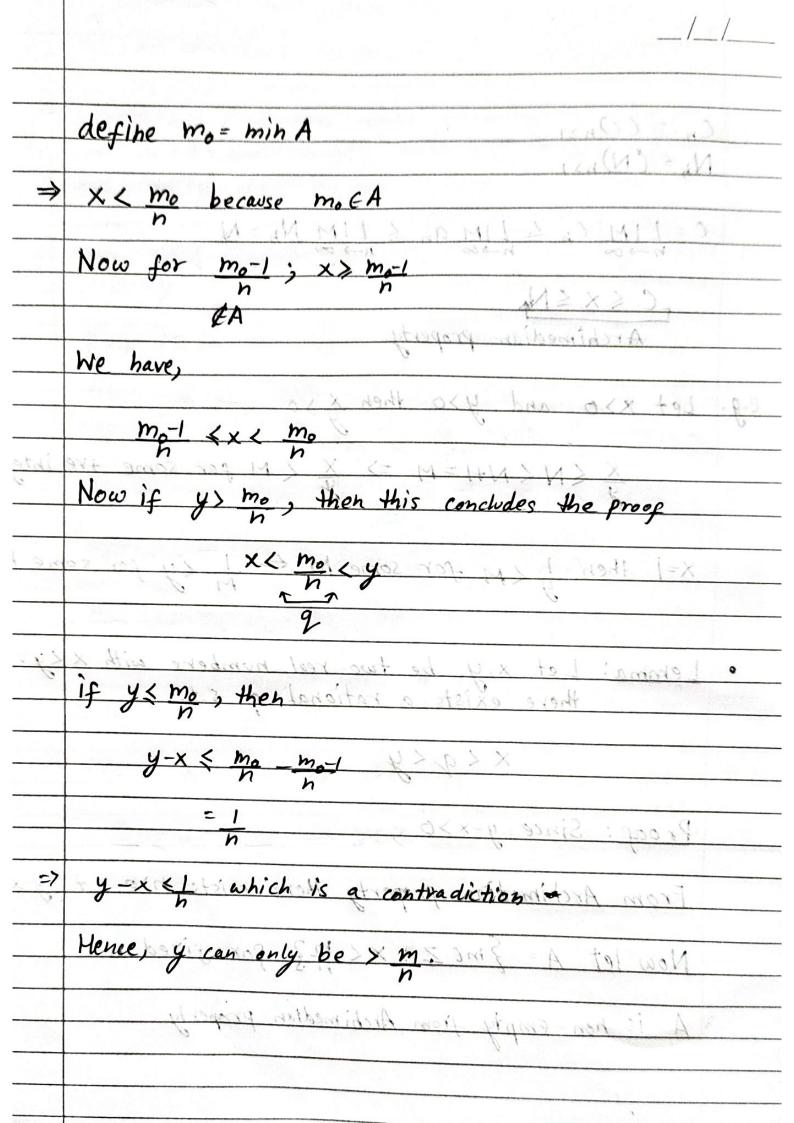
	1	1	
	/	/	
-	/	/	

	emma: Let x to be a real number, then x= LIM and cor some Cauchy sequence (an)nx, that is bounded away from zero.
1	for some Cauchy sequence (an)ny that is bounded
10	away from zero. MIL WILL THE
7	
	emma: If (an) no, is a Cauchy sequence bounded away
+ +	emma: If (an) no, is a Cauchy sequence bounded away rom zero, then (an') no, is a Cauchy sequence.
1	for all x to, we can define
	where x= LIM an = LIM by MIJ = 1 DIMIL 10
	( a,, an,) = (an)
-	
16	A= (LIM On) (LIM On) (max on) = 18 K lipla
+	la; -ajl ≤ E + ij > N
+	(1) (2) (111 bn) (111 bn) = x-1
+	(bn) = (88, a,an,)
+	<u> </u>
+	(an) & (bn) are equivalent.
+	
	Positive I Nagative real numbers
de	X LIM On is a positive Inegative real no
	if (asking is a partially projectly bounded a
No La	a morand that the track of mondey as

Claim: radious loss of oix to 1 : results for some Careby sequence (as) my that i If x = LIM an, y = LIM by for some cauchy sequences (an) & (bn) that are bounded away many prom zero. If x=y onigh des on OTX He vol Then x'= yten (as) grandes 'as MI] = x or LIMan = LIM by MI = 10 MI = x 913 dis Proof: (4)= (40) A = (LIM an) (LIM an) (LIM bni) = LIM (an xanxin) = yt = ( $\lim_{n\to\infty} a_n^{-1}$ ) ( $\lim_{n\to\infty} b_n$ ) ( $\lim_{n\to\infty} b_n^{-1}$ ) =  $\chi^{-1}$ Hence, proved. (an) & (in) one enviratent Positive/ Negative real numbers x = LIM an is a positive/negative real number iff (an), is a positively/negatively bounded away from zero Cauchy sequence.

x>y	liff x-y is	a positive real nu	mber:
Lemm	negative rations	be a Cauchy sequels. Then,	vence of non-
	LIM an >0	LIM (an-ba) >0	1 0011
Prop	fi Let us call ?	= LIM an MI	
Assu	me for contradict	on that x is n	egative.
Ther	is negatively boun	ded away from ze	sequence (bn) no.
> b <sub>n</sub>	≤-c for some	c>o+n	Bone -
1 3365 44	Know that anxo  an-bnxc  742	K LIM an for	
or	lan-bn1> 42 7 n	162 Sano 2 13}	7890 DZ=
Ic N>/	(an) nzi and (bn)nz	, are equivalent	hen there exist
	lan-bnl≤€ ¥	isj > N + E>O	W 7 W
For	E= 4/2 we have		





	-co = least upper bound of set &
	136 papernoques of some section for period as con
P.	For reals, we defined
	AND PRICE UPPER EDUNG IF IT HIS UP UPPER DOUNG.
-	Addition, subtraction, inverse, multiplication, absolute value
->	from the need to show that there is at least one
-/	All the laws of algebra that are true for rationals
	or caise true for reals.
- 9	tot M be an upper bound on I (Given is promis
<i>y</i> 1	Upper bound on subset of R
100	Then for any note we can find integer & s.t. &
21410	Let E = IK Then E is said to have at support have
	MEK Iff X < M for all X E E
	MER iff X < M for all X E E
	Least upper bound on E:-
	MER is a least upper bound on Eiff
::)	1 Is an upper bound
וו	M is an upper bound M'on E satisfies M'>M
	Claim A is not an upper bound one
	Claim: A subset E of IR can have at most two
	least upper bound.
	Proof: Assume M, & M2 ER are two least upper
	bounds on E.
0	Since M1 is a least upper bound, M2 < M1,
0	alca. M \ M
0	also, $M_1 > M_2$
	C. M. = M.

