

International Institute of Information Technology, Hyderabad  
MA4.101-Real Analysis (Monsoon-2025)

Quiz 2

Time: 45 Minutes

Total Marks: 30

**General Guidelines**

- Attempt **any three questions only**.
- No cheat sheet is allowed.
- Show **all steps clearly**; unsupported answers may not receive full credit.

**Question 1.** [10 marks] Prove the following statements.

- (a) [7 marks] A necessary condition for a series  $\sum_{n=1}^{\infty} u_n$  of real numbers to be convergent is that the corresponding sequence  $(u_n)_{n=1}^{\infty}$  converges to 0.
- (b) [3 marks] The condition  $\lim_{n \rightarrow \infty} u_n = 0$  is not sufficient for the series  $\sum_{n=1}^{\infty} u_n$  of real numbers to be convergent.

**Question (2).** [10 Marks] Answer following questions.

- (a) [4 Marks] Prove or disprove that the series  $\sum_{n=1}^{\infty} \frac{(n+1)^{1/n}}{n^2}$  is convergent.
- (b) [6 Marks] Prove or disprove that the series  $\sum_{n=1}^{\infty} a_n$ ,  $a_n = \frac{1}{2^n} \left( 1 + \frac{(-1)^n}{n} \right)$  is convergent. If convergent, compute the sum of the series.

**Question 3.** [10 marks] Let  $\sum_{n=1}^{\infty} u_n$  be a series of positive real numbers and suppose that

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l.$$

Then prove that

- (a) [5 marks] The series  $\sum_{n=1}^{\infty} u_n$  is convergent if  $l < 1$ ;
- (b) [5 marks] The series  $\sum_{n=1}^{\infty} u_n$  is divergent if  $l > 1$ .

**Question 4.** [3+6+1 Marks] Consider the series

$$\sum_{n=3}^{\infty} \frac{1}{(n+2)(n+4(-1)^n)}.$$

Show that the series is convergent/divergent. If it is convergent, then compute the sum. Can this series be convergent but not absolutely convergent?