```
Respectational number
       my 3 reciprocal is defined as the (240, 440)
. If he is an integer then what is my ?
        9: 9 - 8
        9(=>2)= 7
    hen a rational number x/ly is nonzero
  When a
         If 2/1/y = 0 then x=0, y +0
         If 8/12 $0 Then 240, 400 6
       DEER
  Let
       yea be a non zero rational number then
             x/y = x. y's reciprocal of y
        Then for x= 21/1 21, y=2 NSZEQ
            21/1/ = 2/11, x 1/1/2 0 = 12
               2 2/1/y Willame iiit aci
=> Let a,b,c & & ST b & o, c & o then all be = alb
           (0/b)(c/c) = ac/bc
         per a se cache at being brooms
* Law of Algebra for Rationals:
  . Let 2, y, 2 € Q (x+) - 6 x 2 x
   i) 72+y = y+x
   ii) n+ (y+2) = (x+y)+2
    n+(-2e) = (-2e) + 2e = 0
   in) n+0=0+x=x
      x.y=y.x. met come ? 1 1 5 soul
   v)
   vi) n(y 2) = (xy) =
      n. 1 3 1. 22 = 22
   vii)
     n(y+2) = ny + n2
   viii)
   ix) (n+y) = n2+ y2
    X) 20 2.2 = 2 x=1
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value a let need then absolute value
        of ne 15 denoted as 121 4 is
 Absolute
          defined as a series and a series as
       121 = f or 1+ 200
              -x If me o
E-closeness of two rationals . Let x, y e Q . Then y is said
to be & - Close to 22 Hor rational Exo iff
           Ly distance blu n 2 y
         d (x,y) = 1x-91 5 8
Prop. of absolute values
 ) for each neQ 12120 12120 iff 2c=0
 2) pr | ≤ 14 # (3) = 76 ≤ 76 |
    for any ned -2 121 52 × 121 62
    Case 1: If 220 then 12/32 (12/52)
    case 2: It 2 <0 then 121 = -2
  case 1 of then then the
             121 = 26
          First - 120 & 0 & 2 ( ) ( )
         second part nex x x x \ |21
 Case 2 200 3 121 = 72 101 0100 10 1000
                ス= - 121 6 121)
スミス => - (-21) られ
            2) - | n | & n 2 + p = [ + x ()
                            ( +30 = (E+8) +30 (F
€ Let us assume - y ≤ n ≤ y
 case 1 is If 270 then 121 = 2540
       Case 2 =) If 200 then 1201 = -2 < y
                   2 (821) = (2 E/14 (14
=> Assume 1201 6 y
    case 1 220 then from assumption (1)
            121 = 2 = y => 2 & y (5+4) = (711)
         Also 4 > 121 > 0 + 535 & 5 (6+36)
          - 7 - 7 - 7 - 7 - 7 - Cx
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(:1

(4)

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Case 2: 2 20
  17 17+41 5 17x1 + 14)
        Proof = -1021 & 22 & 1061
               - 1A1 & A < 1A1
            Add
                  -121-181 < 26+4 < 1261+181
                 - (1x1+1/1) < x+y < (1x1+1/1)
 (v) 19/1701 = 1709)
    12-1) = 121-1
     1-21 2 Town of plants by hands in the 13
  * d(>c,y) := 1>c-y1
   1) d (21,4) 30 + 21, y 60 4 d (21,4) 50 iff 21=4
                       (non-negativity prop)
2) d(n,y): d(y,2)

3) d(x,y) \( \alpha \) (x,w) \( \alpha \) (w, \( \alpha \)) for all we \( \alpha \) (510 inequality)
* Prop of E-closeness
 D If or 15 E-close to y the vice-versa is also true
                 The see see see see and the
 3 If d(2c, y) < E
                         6,8,0
     as land (y,2) 48 g bas on lowled a sid a 721 of owns
then d(2,2) & E+S

3) Let d(2,3) & E & d(2,2) & E Then for lang w blw
     y to zonther or no well-whom place the ow a tour
         way & wat & bes for ware the
                        or so or way the
                                 proof is immediate
    Dycwcz Te at heading 2 2 wey
      Then w = (1-t) y + t & where t is blu 0, and 1
                w= y+ + (2-y) +6Q
               M- X > + (5-8) 1-4 # Tass
                 t= w-9/2-y
     Then
            d(n,w) = |n-w)
                    = lx- (1-t) y-t 21
                     = (+ x + (1-+)2 - (1-t)y -tz)
```

said

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= 1+(2-2) + (1-4)(2-4))
              = + (x-2) + 161-6) (x-y)
             = + 2(2, 2) + 1, 4) 2 (21.8)
              2 1 E + (1- E) E = E
  * Exponentiation to a natural number
         Let se ea then define
   If x" is defined inductively for new then
     properties + 1) set sem = sen = sem = sem sem
               2) (21)m = 21nm = (21m)n/2 (216 = (2 00))
               3) 1201 = pel"
There one can talk about n^n where n \in \mathbb{Z}
Lonna > Let n be a natural no and q be tre natural no.
  Then I unique m + & with of req st
              n= mq +r (div, algo) = ( = 16 +1)
Proof - we will apply induction or n for fixed 220
     a) Base case for n=0 if we choose m=0
                    id end 5 th 4 p.th.
         7=0 <9 0= 02+0
    stallamin at tong
     b) assume that n= mq+r with 0 < 7 < 2
          Then we need to st saway
           F mi r' with osr's 2 st
          19 3 - n+1/2 - m/5 + 21
        case 1 If n+1 < 2 then take
                ( 1'= n+1 and m'=0 i.e?
                  n+1= 0xq + (n+1)
         Case 2 St n+1 22 009
                    n+1-2 = n
                             057'69
                    m'2+ 7'
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