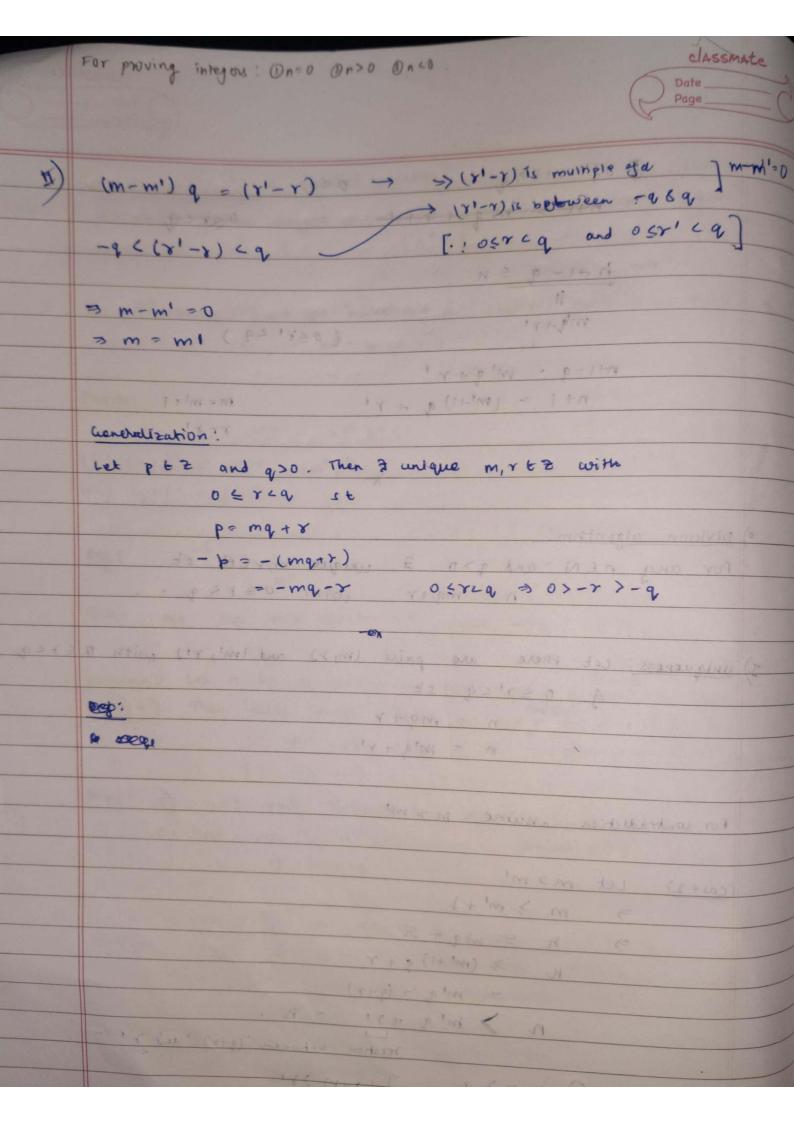
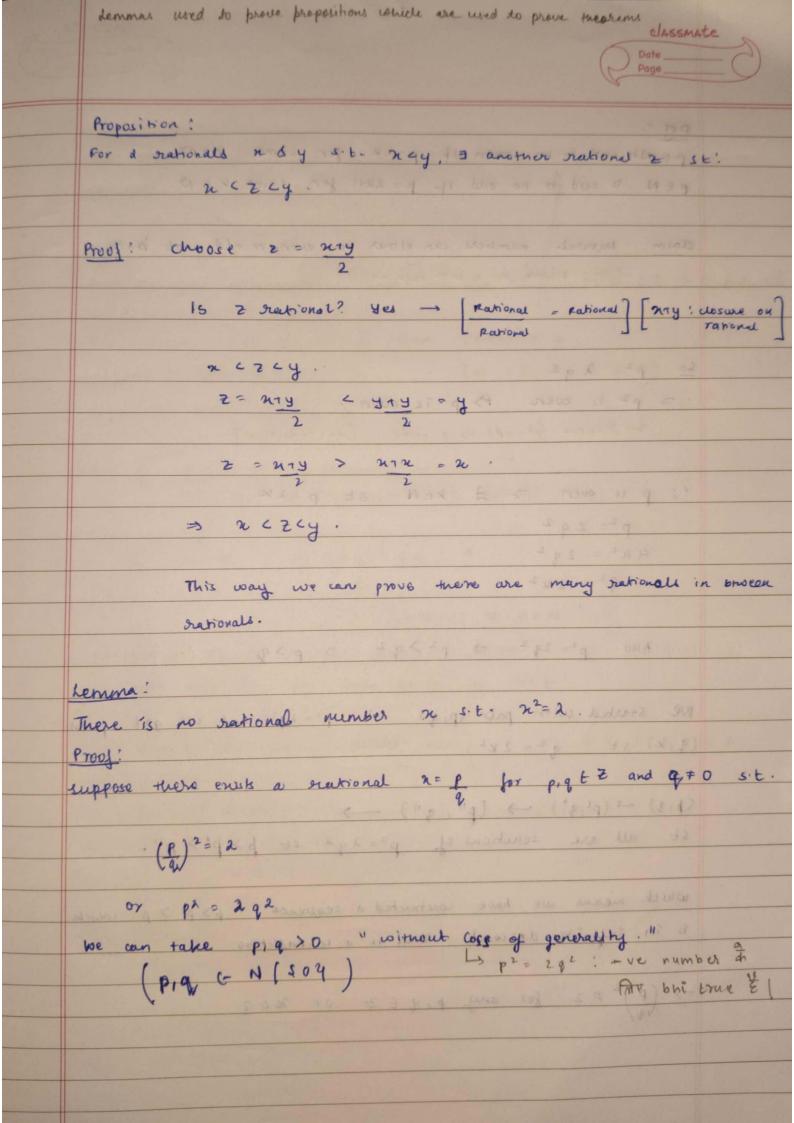
o) Divkion algorithm! for any nEN and g>0 } unique m, rEN sE n = matr with 0 < r < q. 3) uniqueness: Let there are pairs (m, r) and (m', r') with 0 < r < q A 0 6 21 CQ st n = mq+r n = m'q+r' For contradiction assume m + m' (cove 1) Let m>m' 3 m 3 m'+1 3 n = mq+ 92 'n > (m'+1) q+ ~ = m' q + (q+r) n > m'q + r! = h. relation between (gor) and r1 1 99x 3 9] (94x) >81 from unqueries > @ q > 81 3 n > n [contradiction] =) m = m1



-ve numbers.

Del: A sequence do, ai ... of N or 2 or 8 is said to be infinite descent of a0>a1>q2 Lemma ! For natural numbers, we can't have any sequence that is infinite descent. Proof: suppose that an is a sequence of natural numbers s.t an 7 anti V new > an > an+1 +1 => anti = an-1 | + m = s = tot an $\leq a_0 - n$ $\forall n$ From induction 9 a < n > a 20 which can't happen as an EN. [CONTRADICTION] Is we can't have a respect sequence of natural numbers in infinite descent proposition: [In tel persong of trationals inside in tegers] tet n be a natural number. Then I a unique integer nt 2 st n 5 20 < n+1 In particular, I a neutral number N st. nen dereg Proof! Since nto, we have 2= P/q for some pt 2 and q>0 (= 2 (4 =0) Ly o: pt 2 so we can form born toed

	Drulaien Algorithm:
as with	p=mq+a for mire z wim 0 ≤ x < q
	n=P=mq+r
	n = m + r
aling	For noticed numbers se cam Sun nuy reprience that It
	Also 2 c m+1 [" 1/9 c 1]
	m 5 % cm+1 for some integer m
	lupper that as to assure of natural numbers set
	Uhrquenen: RADA Y TANDERO
	Let us say 7 m' & 2 s. t.
	m1 & n & m1 + 1 1 - n = 1411 6
	m & n < m + 1 n - 0 > 40
	=> ncm'+1 8 n/ ncm+1 g n> m1
	[MITJOASTINES]
	m < n < m'+1 m+1 > n > m1
dossan	3 (m-m) (m-m) >-1 ~- 0
	combine O and O
	=> Im-mile < 1 in a partial modification
42 4	(: m, m' EZ)
	\$tn2 & 2 n
	· m+1>0
	N = m+1
	If Infinitely so many rationals between m and mil (mt 2) . Proveit
	x 212 for some per



penis said to be even if p=2k for some ken.

pen is said to be odd if p=2k+1 for some k t N

Proof: Msume no. is both even and odd.

50 p2 = 2 q2

through the top and

3 p² is even > p is even.

La Assume p2-odd => p = odd [contradiction]

1; p is even => 3 ken st p=2k

p²= 2 q²

4 K2 = 2 9 2

Monte q2 = 2 k2 see arout some that you find that

Also p2 292 => p2>q2 => p>q (41, p, 9 >0)

We started with pair (p,q) st $p^2=2q^2$ and we got pair (q,k) st $q^2=2k^2$.

 $(p,q) \rightarrow (p',q') \rightarrow (p'',q'') \rightarrow \dots$

st all are solutions of p2= 292 st p>p1>p1.

is in infinite descent. Yets 75 a contradiction

 $\Rightarrow \left(\frac{P}{q}\right)^2 \neq 2$ for any $P, q \in \mathcal{Z}$ or $26 \in Q$

o) Proposition:

For every rational 800 there exists rational 20,0000 s.t.

22 C 2 C (21 E)2

Proof:

suppose for contradiction there does not exist n>0 EQ S: E for all E>0, $n^2<2<(n+E)^2$

This means that if n2 <2 then (x+E)2 < 2

We can't have (2014)2=2 [as a (natrioral)2 = 2]

> (n+E) 2 < 2

Now if x=0

02 = 0 < 2 3 (0+E)2 < 2 => E2 < 2

If N° € ⇒) (2 €) 2 < 2 €

A

(n E)2 C2 Y DEN

But nE is a tre trational number

For each & we can chose n s.t

dene

choose $n = \frac{1}{2} \cdot t \cdot \frac{1}{12} \cdot \frac{1}{2} \cdot \frac{1}{2}$

>> 4 < 2 (Impossible) >> (x+t) 2 >2