Real Analysis

Reference :-
Intro to Real Analysis
by Bartly and Shirbert

Abstract Proofs

What is Real Analysis	why
Rigorous study of real numbers, sequences, series and....	why the below examples are problematic

Q What is the largest Real Number?

Q what is the largest Natural Number?

Q when is a function differentiable or bounded?

Eg:- ① $S = 1 + \frac{1}{2} + \frac{1}{4} \dots$

$2S = 2 + \underbrace{1 + \frac{1}{2} + \frac{1}{4} \dots}_{S}$

$S = 2$

② $S = 1 + 2 + 4 + 8 \dots$

u cannot blindly do the same thing

$2S = \underbrace{2 + 4 + 8 \dots}_{S-1} \dots$

$\boxed{\cancel{S = -1}}$

③ $S = 1 - 1 + 1 - 1 + 1 - 1$

$(1-1) + (1-1) \dots$

?

$\cancel{S = 0}$

$\cancel{S = 1 - (1-1) - (1-1) \dots}$

$\cancel{S = 1}$

Number systems

N - Natural
Z - Integers
Q - Rational
R - Real
C - Complex.

Axiomatic Number System (Done by Peano)

Recent

Peano axioms for system of Natural numbers

Axiom 1: 0 is a natural number i.e $0 \in N$

Axiom 2: If $n \in N$, then "the increment '++'" of n, given by $n{+}{+}$ also belongs to N

Eg: $0 \in N$
$1 = 0{+}{+} \in N$
$2 = (0{+}{+}){+}{+} \in N$
⋮ ⋮

Based on 1 and 2, let us say
$N' = \{\dots\}$ where
$0 \in N'$, $0{+}{+} = 1 \in N'$, $1{+}{+} = 0 \in N'$
This is not wrong acc. to above 2 Axioms.
So we need more Axioms

Axiom 3: 0 is not increment of any Natural Number

Axiom 4: If $m \neq n \Rightarrow m{+}{+} \neq n{+}{+}$
and if $m = n \Rightarrow m{+}{+} = n{+}{+}$

Eg: $N'' = \{0, 0.5, 1, 1.5 \dots\}$ we will need more Axioms

Axiom 5 :- (Principal of Mathematical Induction)

a) Let $P(n)$ be some property of $n \in N$

b) Let $P(0)$ be true. $(n=0)$

c) If we suppose $P(n)$ is true for some arbitrary $n \in N$

then $P(n{+}{+})$ is also true.

$\boxed{\text{If}}$

Then $P(n)$ is true for all $n \in N$

Eg :- $P(n) =$ n is not an half integer

b) $P(0) = 0$ is not a half integer

$P(0)$ is true

c) Let $P(n) = n$ is not a half integer

$P(n{+}{+}) = n{+}{+} \in N$

Remove what we dont want. मान लो की n is not x हमें og N tak Pohucha hai we can make a numbersystem any way we want.

Assumption :- There exists a set N following all 5 Peano Axioms, whose elements we will call natural numbers.

Hindu Arabic	Roman
$\{0,1,2\ldots\}$	$\{0, \mathrm{I}, \mathrm{II}\ldots\}$

Both follow all 5 they just look different

Addition of Natural Numbers

Def 1 :- Adding $0 \in N$ to any other $n \in N$ means

$$\underbrace{0+n}_{\text{symbol}} := n$$

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: means define

② Assume $m+n$ is Known / defined intuitively for any $(m++)+n = (m+n)++$

Eg $0+n=n$

$(0++)+n = (0+n)++ = n++ = n+1$ (getting there)

and so on $= n+2$

Eg $n+0$ Proove $n+0=n$ given def 1

$P(n): \quad n+0=n \ldots i$

a) from def 1 $0+n=n$

if $n=0$

$0+0=0$

~~then sub 0 into i def 1~~

b) If $n+0=n$

Proove :- $(n++)+0 = n++$

$\downarrow$

$(n+0)++ = n++$

Using Axiom 5

Hence Prooved

Ex :- Proove that $m+n = n+m \ \forall \ n,m \in N$

$(m+n)+l = m+(n+l)$