Linear Algebra Quiz Practice

Practice Questions

Q: Which of the following systems of equations has no solution?

a) $x - 2y = 4$, $2x - 4y = 8$
b) $x + y = 6$, $x - y = 2$
c) $x + y = 2$, $3x + 3y = 9$
d) $x + y = 3$, $2x + 2y = 6$

Correct Answer: c

Explanation: In system (c), the second equation $(3x+3y = 9)$ can be simplified to $x+y = 3$. This contradicts the first equation, $x + y = 2$. Therefore, the system is inconsistent and has no solution. Options (a) and (d) have infinitely many solutions, and option (b) has a unique solution.

Q: What is the solution set for the system: $x - y + z = 0$, $y - 2z = -5$, $2z = 4$?

a) No solution
b) A unique solution
c) Infinitely many solutions
d) None of the above

Correct Answer: b

Explanation: From the third equation, $2z = 4$, we get $z = 2$. Substituting $z = 2$ into the second equation, $y - 2(2) = -5$, we get $y = -1$. Substituting $y = -1$ and $z = 2$ into the first equation, $x - (-1) + 2 = 0$, we get $x = -3$. Thus, there is a unique solution $(x, y, z) = (-3, -1, 2)$.

Q: Which of the following matrices is in row echelon form (REF) but not reduced row echelon form (RREF)?

a) $\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Correct Answer: b

Explanation: A matrix in RREF must have every leading entry as the only non-zero entry in its column. Matrix (b) is in REF because the leading entries move to the right and rows of zeros are at the bottom. However, it is not in RREF because of the '3' above the leading '1' in the second column.

Q: What is the rank of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$?

a) 0

b) 1

c) 2

d) 3

Correct Answer: c

Explanation: To find the rank, we perform row reduction: $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$ gives $\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$. $R_3 \to R_3 - R_2$ gives $\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. There are two non-zero rows (two pivots), so the rank is 2.

Q: For which value of k is the system represented by the augmented matrix $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & k & 0 \\ 0 & 0 & k-2 & 5 \end{array}\right)$ inconsistent?

a) $k = 0$

b) $k = 1$

c) $k = 2$

d) $k = 3$

Correct Answer: c

Explanation: The system is inconsistent if it leads to a contradiction. The last row corresponds to the equation $(k-2)z = 5$. If $k = 2$, this equation becomes $0 \cdot z = 5$, which is impossible. Thus, the system is inconsistent for $k = 2$.

Q: The determinant of a skew-symmetric matrix of order $n \times n$ is always zero when...

a) n is even
b) n is odd
c) n is a prime number
d) $n > 1$

Correct Answer: b

Explanation: For a skew-symmetric matrix A, we have $A^T = -A$. This implies $\det(A^T) = \det(-A)$. We know $\det(A^T) = \det(A)$ and $\det(-A) = (-1)^n \det(A)$. So, $\det(A) = (-1)^n \det(A)$. If n is odd, this becomes $\det(A) = -\det(A)$, which implies $2\det(A) = 0$, so $\det(A) = 0$.

Q: If the eigenvalues of a 4x4 symmetric matrix A are -1, 1, 2, and 4, what are the eigenvalues of $A^2 - I$?

a) -1, 1, 2, 4
b) 1, 1, 4, 16
c) 0, 0, 3, 15
d) -2, 0, 1, 3

Correct Answer: c

Explanation: If λ is an eigenvalue of A, then $f(\lambda)$ is an eigenvalue of $f(A)$. Here, $f(\lambda) = \lambda^2 - 1$. The eigenvalues of $A^2 - I$ are: $(-1)^2 - 1 = 0$; $1^2 - 1 = 0$; $2^2 - 1 = 3$; $4^2 - 1 = 15$. So the eigenvalues are 0, 0, 3, 15.

Q: For which value of x will the matrix $\begin{pmatrix} 3 & 6 \\ x & 4 \end{pmatrix}$ be singular?

a) 1
b) 2
c) 3
d) 4

Correct Answer: b

Explanation: A matrix is singular if its determinant is 0. $\det(\begin{pmatrix} 3 & 6 \\ x & 4 \end{pmatrix}) = (3)(4) - (6)(x) = 12 - 6x$. Setting the determinant to zero: $12 - 6x = 0 \implies 6x = 12 \implies x = 2$.

Q: Let $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$. Is S a subspace of $\mathbb{R}^3$?

a) Yes
b) No, because it is not closed under addition.
c) No, because it is not closed under scalar multiplication.
d) No, because it does not contain the zero vector.

Correct Answer: d

Explanation: A subspace must contain the zero vector. The zero vector in $\mathbb{R}^3$ is $(0, 0, 0)$. For this vector, $x + y + z = 0 + 0 + 0 = 0$, which is not equal to 1. Since the zero vector is not in S, S is not a subspace of $\mathbb{R}^3$.

Q: Which of the following sets of vectors is an orthogonal set in $\mathbb{R}^3$?

a) (1,-1,0), (1,1,1), (0,0,1)
b) (2,1,-1), (1,-1,-3), (1,-1,1)
c) (1,2,2), (2,1,-2), (2,-2,1)
d) (1,0,0), (1,1,0), (1,1,1)

Correct Answer: c

Explanation: Two vectors are orthogonal if their dot product is 0. Let $v_1 = (1, 2, 2)$, $v_2 = (2, 1, -2)$, $v_3 = (2, -2, 1)$. $v_1 \cdot v_2 = (1)(2) + (2)(1) + (2)(-2) = 2 + 2 - 4 = 0$. $v_1 \cdot v_3 = (1)(2) + (2)(-2) + (2)(1) = 2 - 4 + 2 = 0$. $v_2 \cdot v_3 = (2)(2) + (1)(-2) + (-2)(1) = 4 - 2 - 2 = 0$. All pairs are orthogonal.

Q: Suppose applying the Gram-Schmidt process to a set of non-zero vectors $\{v_1, v_2, v_3\}$ yields an orthogonal set where the third vector, u_3, is the zero vector. What does this imply about the original set?

a) v_3 is a linear combination of v_1 and v_2.
b) The set $\{v_1, v_2, v_3\}$ is linearly independent.
c) v_1 and v_2 are orthogonal.
d) v_3 is the zero vector.

Correct Answer: a

Explanation: In the Gram-Schmidt process, u_3 is found by taking v_3 and subtracting its projections onto u_1 and u_2. If $u_3 = 0$, it means v_3 is entirely composed of its projections onto u_1 and u_2. Since u_1 and u_2 are in the span of v_1 and v_2, this means v_3 is a linear combination of v_1 and v_2.

Q: Which of the following is an orthonormal basis for $\mathbb{R}^2$?

a) (1,1), (-1,1)

b) $(\frac{3}{5}, \frac{4}{5})$, $(-\frac{4}{5}, \frac{3}{5})$

c) (1,0), (0,2)

d) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

Correct Answer: d

Explanation: An orthonormal basis consists of vectors that are mutually orthogonal and each have a norm (length) of 1. The vectors in option (d) are orthogonal: $(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}) = \frac{1}{2} - \frac{1}{2} = 0$. Both vectors have a norm of 1. Option (b) is also a valid orthonormal basis.

Q: Which of the following statements about inner products $\langle u, v\rangle$ is always FALSE for some vectors in a real vector space?

a) $\langle u + v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$

b) $\langle u, u\rangle \geq 0$

c) $\langle u, v\rangle^2 = \langle u, u\rangle\langle v, v\rangle$

d) $\langle ku, v\rangle = k\langle u, v\rangle$

Correct Answer: c

Explanation: Option (c) represents the equality condition of the Cauchy-Schwarz inequality. The inequality itself is $|\langle u, v\rangle| \leq ||u||||v||$, which is $\langle u, v\rangle^2 \leq \langle u, u\rangle\langle v, v\rangle$. Equality holds only when the vectors are linearly dependent. It is not true for all vectors, making the statement false in general.

Q: The Cauchy-Schwarz inequality states that for any two vectors x and y in an inner product space...

a) $|\langle x, y\rangle| \leq ||x|| + ||y||$

b) $|\langle x, y\rangle| \geq ||x|| \cdot ||y||$

c) $|\langle x, y\rangle| \leq ||x|| \cdot ||y||$

d) $|\langle x, y\rangle| = ||x|| \cdot ||y||$

Correct Answer: c

Explanation: The Cauchy-Schwarz inequality provides an upper bound on the absolute value of the inner product of two vectors in terms of the product of their norms. It is a fundamental result in linear algebra and analysis.

Q: If $\{u, v, w\}$ is a basis for a vector space V, what is the dimension of V?

a) 1

b) 2

c) 3

d) Cannot be determined

Correct Answer: c

Explanation: The dimension of a vector space is defined as the number of vectors in any of its bases. Since $\{u, v, w\}$ is a basis and contains 3 vectors, the dimension of V is 3.

Q: What is the maximum number of non-zero singular values a 8x3 matrix can have?

a) 8
b) 5
c) 3
d) 11

Correct Answer: c

Explanation: The number of non-zero singular values of a matrix is equal to its rank. For an $m \times n$ matrix, the rank is at most $\min(m, n)$. Here, $m = 8$ and $n = 3$, so the maximum rank is $\min(8, 3) = 3$. Therefore, there can be at most 3 non-zero singular values.

Q: If $A = \begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}$, what are the eigenvalues of A?

a) 5, 0
b) 3, 2
c) 4, 1
d) 6, -1

Correct Answer: b

Explanation: The characteristic equation is $\det(A - \lambda I) = 0$. $\det(\begin{pmatrix} 5-\lambda & -3 \\ 2 & -\lambda \end{pmatrix}) = (5 - \lambda)(-\lambda) - (-3)(2) = -5\lambda + \lambda^2 + 6 = \lambda^2 - 5\lambda + 6 = 0$. Factoring gives $(\lambda - 3)(\lambda - 2) = 0$. So the eigenvalues are $\lambda = 3$ and $\lambda = 2$.

Q: The vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix $A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$. What is the corresponding eigenvalue?

a) 1
b) 2
c) 3
d) 4

Correct Answer: c

Explanation: If v is an eigenvector of A, then $Av = \lambda v$. $Av = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4(2) - 2(1) \\ 1(2) + 1(1) \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. So, $Av = 3v$. The eigenvalue is 3.

Q: For a 3x3 matrix B, the characteristic polynomial is $p(\lambda) = -\lambda^3 + 2\lambda^2 + 5\lambda - 7$. What is the determinant of B?

a) 2
b) 5
c) -7
d) 1

Correct Answer: c

Explanation: The determinant of a matrix is the constant term of its characteristic polynomial, $p(\lambda) = \det(B - \lambda I)$. To see this, set $\lambda = 0$, which gives $p(0) = \det(B - 0 \cdot I) = \det(B)$. For the given polynomial, the constant term is -7.

Q: What is the relationship between the singular values of A, σ_i, and the eigenvalues of $A^T A$, λ_i?

a) $\sigma_i = \lambda_i$
b) $\sigma_i = \sqrt{\lambda_i}$
c) $\sigma_i = \lambda_i^2$
d) $\sigma_i = 1/\lambda_i$

Correct Answer: b

Explanation: By definition, the singular values of a matrix A are the square roots of the eigenvalues of the matrix $A^T A$. This is a fundamental concept in Singular Value Decomposition (SVD).

Q: In the Singular Value Decomposition (SVD) of a matrix $A = U\Sigma V^T$, what property must the matrices U and V have?

a) They are symmetric.
b) They are orthogonal.
c) They are invertible.
d) They are diagonal.

Correct Answer: b

Explanation: In the SVD, both U and V are orthogonal matrices. U contains the orthonormal eigenvectors of AA^T, and V contains the orthonormal eigenvectors of $A^T A$.

Q: A 3×4 matrix A has singular values 8, 4, and 2. What is the spectral norm of A?

a) 4
b) 2
c) 14

d) 8

Correct Answer: d

Explanation: The spectral norm of a matrix, denoted $||A||_2$, is equal to its largest singular value. In this case, the largest singular value is 8.

Q: Let $A = \begin{pmatrix} 5 & -1 & 8 \\ 3 & 10 & 2 \\ 1 & 4 & -3 \end{pmatrix}$. What is the trace of A?

a) 12
b) 2
c) 18
d) 15

Correct Answer: a

Explanation: The trace of a square matrix is the sum of the elements on its main diagonal. tr(A) = 5 + 10 + (-3) = 12.

Q: For what range of values of c is the matrix $A = \begin{pmatrix} 2 & c \\ c & 8 \end{pmatrix}$ positive definite?

a) $c > 4$
b) $c < 4$
c) $-4 < c < 4$
d) $c = 4$

Correct Answer: c

Explanation: A symmetric matrix is positive definite if all its leading principal minors are positive. 1. The first minor is the top-left entry, which is 2 (and 2 ¿ 0). 2. The second minor is the determinant: det(A) = (2)(8) - c^2 = $16 - c^2$. For the determinant to be positive, $16 - c^2 > 0$, which means $c^2 < 16$. This is true for $-4 < c < 4$.

Q: Are the vectors $v_1 = (1, 0, 1)$, $v_2 = (0, 1, 1)$, and $v_3 = (1, 1, 2)$ in $\mathbb{R}^3$ linearly independent?

a) Yes, they are linearly independent.
b) No, because $v_3 = v_1 + v_2$.
c) No, because their determinant is non-zero.
d) Cannot be determined.

Correct Answer: b

Explanation: A set of vectors is linearly dependent if one vector can be written as a linear combination of the others. Here, we can see by inspection that $v_3 = (1, 1, 2) = (1, 0, 1) +$

$(0, 1, 1) = v_1 + v_2$. Therefore, the set is linearly dependent. Alternatively, the determinant of the matrix formed by these vectors is 0.

Q: Which of the following is an eigenvector of the matrix $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$ corresponding to the eigenvalue $\lambda = 1$?

a) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

c) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Correct Answer: b

Explanation: We need to find a vector x such that $(A - \lambda I)x = 0$. With $\lambda = 1$, we have $(A-I)x = \begin{pmatrix} 3-1 & -1 \\ 2 & 0-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. This gives the equation $2x_1 - x_2 = 0$, or $x_2 = 2x_1$. If we let $x_1 = 1$, then $x_2 = 2$. So, the eigenvector is a multiple of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Q: Which of the following sets is a subspace of $\mathbb{R}^2$?

a) $W_1 = \{(x, y) : x \geq 0, y \geq 0\}$
b) $W_2 = \{(x, y) : xy = 0\}$
c) $W_3 = \{(x, y) : x + y = 1\}$
d) $W_4 = \{(x, y) : 3x - 5y = 0\}$

Correct Answer: d

Explanation: A set is a subspace if it contains the zero vector, is closed under addition, and is closed under scalar multiplication. W_1 is not closed under scalar multiplication (e.g., multiply by -1). W_2 is not closed under addition (e.g., (1,0) + (0,1) = (1,1), which is not in W_2). W_3 does not contain the zero vector. W_4 represents a line through the origin and satisfies all three properties.

Q: A 3x3 matrix has eigenvalues 2, 3, and -5. What is the determinant of the matrix?

a) 0
b) -30
c) 30
d) 6

Correct Answer: b

Explanation: The determinant of a matrix is the product of its eigenvalues. det(A) = (2)(3)(-5) = -30.

Q: Which of the following matrices is an orthogonal matrix?

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

Correct Answer: b

Explanation: A matrix A is orthogonal if its columns are orthonormal vectors, which is equivalent to $A^T A = I$. For $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $A^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. $A^T A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$.

Q: What is the projection of vector $u = (3, 4)$ onto vector $v = (1, 1)$?

a) $(\frac{7}{2}, \frac{7}{2})$

b) $(3, 3)$

c) $(\frac{3}{2}, 2)$

d) $(7, 7)$

Correct Answer: a

Explanation: The projection of vector u onto vector v is given by the formula $\text{proj}_v u = \frac{u \cdot v}{||v||^2} v$. $u \cdot v = (3)(1) + (4)(1) = 7$. $||v||^2 = 1^2 + 1^2 = 2$. $\text{proj}_v u = \frac{7}{2} v = \frac{7}{2}(1, 1) = (\frac{7}{2}, \frac{7}{2})$.