Practice Problems

Problem 1: Single Coin Toss (Very Basic)

Problem

A fair coin is tossed **once**. What is the probability of getting heads?

Solution

Solution:
A fair coin has two equally likely outcomes (Heads or Tails). Probability of heads is:

$$P(\text{Heads}) = \frac{1}{2} = 0.5.$$

Problem 2: Fair Die (Basic)

Problem

A fair six-sided die is rolled **once**. Find the probability of getting:

(a) A number less than 4.

(b) An odd number.

Solution

Solution:
(a) Numbers less than 4 are {1,2,3}, so 3 outcomes out of 6. Probability = $3/6 = 1/2$.
(b) Odd faces are {1,3,5}, again 3 out of 6, so $1/2$.

Problem 3: A Coin Is Tossed 4 Times

Problem

A fair coin is tossed **4 times**. What is the probability that **at least one tail** appears?

Solution

Solution:
Total outcomes $= 2^4 = 16$.
It's easier to do $1 - P(\text{no tails})$. "No tails" means all heads: Probability $= (1/2)^4 = 1/16$.
Hence

$$P(\text{at least one tail}) = 1 - \frac{1}{16} = \frac{15}{16}.$$

Problem 4: Balls Without Replacement

Problem

We have 6 green and 4 red balls in a box. Two balls are drawn one after the other, **without replacement**. What is the probability that the **first** is green and the **second** is red?

Solution

Solution:
Initially 10 balls: 6 green, 4 red.

$$P(\text{1st is green}) = \frac{6}{10}.$$

Then 5 green, 4 red left (9 total). Probability second is red: 4/9.
Multiply:

$$P(\text{1st green, 2nd red}) = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90} = \frac{4}{15} \approx 0.2667.$$

Problem 5: Two-Card Draw from a Deck

Problem

From a standard 52-card deck, two cards are drawn **without replacement**. What is the probability that both cards are spades?

Solution

Solution:
There are 13 spades in 52 cards. Probability first card is a spade: $13/52 = 1/4$. Then 12 spades left out of 51. Probability second is spade: $12/51$.
Thus

$$P(\text{both spades}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204} = \frac{3}{51} \approx 0.0588.$$

Problem 6: At Least 2 Heads in 3 Tosses

Problem

A fair coin is tossed **3 times**. Find the probability that **at least 2 heads** appear.

Solution

Solution:
Let $X=$ number of heads. Probability of each outcome is $(1/2)^3 = 1/8$.
"At least 2 heads" means $X = 2$ or $X = 3$.

$$P(X = 2) = \binom{3}{2}\left(\frac{1}{2}\right)^3 = 3 \times \frac{1}{8} = \frac{3}{8}.$$

$$P(X = 3) = \binom{3}{3}\left(\frac{1}{2}\right)^3 = 1 \times \frac{1}{8} = \frac{1}{8}.$$

Hence

$$P(X \geq 2) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}.$$

Problem 7: Sum of Two Dice (7 or 11)

Problem

Two fair dice are rolled. Find the probability that the sum is either 7 or 11.

Solution

Solution:
Total outcomes $= 6 \times 6 = 36$.
Sum $= 7$ has $\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} = 6$ ways.
Sum $= 11$ has $\{(5,6),(6,5)\} = 2$ ways.
Total favorable $= 6 + 2 = 8$. Probability $= 8/36 = 2/9 \approx 0.2222$.

Problem 8: Selecting People (Combinatorics)

Problem

We have 5 men and 5 women in a group of 10 people. If we choose 3 people at random, what is the probability that **exactly 2 are men**?

Solution

Solution:
Total ways to choose 3 out of 10: $\binom{10}{3} = 120$.
Number of ways to get exactly 2 men (and 1 woman):

$$\binom{5}{2} \times \binom{5}{1} = 10 \times 5 = 50.$$

Hence

$$P(\text{exactly 2 men}) = \frac{50}{120} = \frac{5}{12} \approx 0.4167.$$

Problem 9: Card Probability (At Least 1 Ace)

Problem

Two cards are drawn from a standard 52-card deck without replacement. What is the probability that **at least one** is an ace?

Solution

Solution:
It's simpler to use the complement:

$$P(\text{at least one ace}) = 1 - P(\text{no aces}).$$

Number of aces = 4. Non-aces = 48.

$$P(\text{no aces in 2 draws}) = \frac{\binom{48}{2}}{\binom{52}{2}}.$$

Compute combinations:

$$\binom{48}{2} = \frac{48 \times 47}{2} = 1128, \quad \binom{52}{2} = \frac{52 \times 51}{2} = 1326.$$

Hence

$$P(\text{no aces}) = \frac{1128}{1326} \approx 0.8507.$$

Thus

$$P(\text{at least one ace}) = 1 - 0.8507 = 0.1493,$$

or about 14.93%.