Birla Institute of Technology & Science, Pilani
Work Integrated Learning Programmes Division
Second Semester 2024-2025

Mid-Semester Test
EC-2 Regular

Course No.	: AIMLCZC416
Course Title	: Mathematical Foundations for Machine Learning
Nature of Exam	: Closed Book
Weightage	: 30%
Duration	: 2 Hours
Date of Exam	: 28-06-2025, AN

No. of Pages = 2
No. of Questions = 6

Note to Students:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.
4. Notation used here are as per the text book.

Q1. Consider the matrices $A = \begin{bmatrix} 1 & 3 & -1 & 4 \\ 2 & 6 & -2 & 8 \\ 3 & 9 & -3 & 12 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(a) Find the Echelon form of A and hence find the rank of A. [2M]

(b) Find all the solutions of AX = b. [2M]

(c) Suppose if the matrix $b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ what can you conclude about the linear system of equations AX = b, Where A is the same matrix given in (a). [1M]

Q2. Consider the matrix $A = \begin{bmatrix} m & 1 & 1 \\ 1 & m & 1 \\ 1 & 1 & m \end{bmatrix}$, where m ≥ 2, a positive integer.

(a) Prove that A is a positive definite matrix. [3M]

(b) Using this matrix A, find an inner product on R^3. [1M]

(c) Using the inner product defined in (b) find the norm of the vector $\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$. [1M]

Q3. (a) Is the set $\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} \right\}$ a basis for R^4. [3M]

(b) Write $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of the basis elements given in (a). [2M]

Q4. (a) Suppose $A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 2 & -2 \end{bmatrix}$. Find the diagonal matrix D and the invertible matrix P

Such that $(A^T A)P = PD$ [4M]

(b) Why for any mxn matrix A, A^TA is always diagonalizable [1M].

Q5. Consider the function $f(x, y, z) = x^2 + y^2 + z^2 + 3xyz$.

(a) Find ∇**f**(x,y,z) at (1, -1, 2). [2M]

(b) Is the vector $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ orthogonal to ∇**f**(x,y,z) at (1, -1, 2). Why?[1M]

(c) Suppose if you further consider x, y, z as functions of t given by $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$.

Using the chain rule find $\frac{df}{dt}$. [2M]

Q6. (a) Find the Taylor polynomial of degree 3 of the function $f(x) = e^{x^2}$ about the point 1. [3M]

(b) Find the Hessian matrix of the function $f(x, y) = xy$. [2M]
