# Mean, Median, and Mode: Questions and Solutions

Below you will find 9 questions in three categories (**Mean**, **Median**, **Mode**), along with detailed solutions. Questions range from simple ungrouped data to discrete and grouped frequency distributions.

## I. Mean (3 Questions)

### Question 1 (Simple, Ungrouped Data)

You have the following daily temperatures (in °C) recorded over 5 days:

$$18, \quad 22, \quad 19, \quad 23, \quad 21.$$

(a) Find the arithmetic mean of these temperatures.

(b) Interpret the result in context (average temperature).

#### Solution (Q1)

- **(a) Calculate the Mean:**

$$\text{Mean} = \frac{18 + 22 + 19 + 23 + 21}{5} = \frac{103}{5} = 20.6\text{ °C}.$$

- **(b) Interpretation:**
  The average daily temperature over these 5 days is about 20.6°C. This is a central measure indicating a comfortable overall temperature.

### Question 2 (Discrete Frequency Distribution)

A store manager records the number of customers arriving each hour during a single day:

| Customers (x) | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Frequency (f) | 1 | 2 | 2 | 3 | 4 | 2 |

(a) Calculate the mean number of customers per hour.

(b) Comment on whether the store seems busy or quiet on average.

**Solution (Q2)**

- **(a) Mean Calculation:**

  First, sum the frequencies:

  $$n = 1 + 2 + 2 + 3 + 4 + 2 = 14.$$

  Next, compute $\sum x\,f$:

  $$(2 \times 1) + (3 \times 2) + (4 \times 2) + (5 \times 3) + (6 \times 4) + (7 \times 2).$$

  Calculate step by step:

  $$2 + 6 + 8 + 15 + 24 + 14 = 69.$$

  Hence, the mean number of customers per hour is

  $$\text{Mean} = \frac{69}{14} \approx 4.93.$$

- **(b) Interpretation:** On average, about $\approx 5$ customers arrive each hour. This indicates a moderate flow of customers, not particularly high, but steady.

## Question 3 (Grouped Frequency Distribution)

The following table shows grouped data on weights (in kg) of 50 gym members:

| Weight Interval | No. of People (f) |
|---|---|
| 50–55 | 4 |
| 55–60 | 8 |
| 60–65 | 12 |
| 65–70 | 14 |
| 70–75 | 7 |
| 75–80 | 5 |

(a) Compute the mean weight using class midpoints.

(b) Interpret the result in context of the group's average weight.

**Solution (Q3)**

- **(a) Calculate Mean:**

First, find each class midpoint (m):

$$52.5, \quad 57.5, \quad 62.5, \quad 67.5, \quad 72.5, \quad 77.5.$$

Corresponding frequencies: 4, 8, 12, 14, 7, 5. The total number of people is $n = 50$.

Compute $\sum f \times m$:

$$4(52.5) + 8(57.5) + 12(62.5) + 14(67.5) + 7(72.5) + 5(77.5).$$

Step-by-step:

$$4 \times 52.5 = 210, \quad 8 \times 57.5 = 460, \quad 12 \times 62.5 = 750,$$

$$14 \times 67.5 = 945, \quad 7 \times 72.5 = 507.5, \quad 5 \times 77.5 = 387.5.$$

Summation:

$$210 + 460 + 750 + 945 + 507.5 + 387.5 = 3260.$$

Hence,

$$\text{Mean weight} = \frac{\sum f \times m}{\sum f} = \frac{3260}{50} = 65.2 \text{ kg}.$$

- **(b) Interpretation:** The average (mean) weight of the 50 gym members is about 65.2 kg. This suggests that, overall, participants weigh around 65 kg on average.

---

# II. Median (3 Questions)

## Question 4 (Simple, Ungrouped Data)

A small class of 7 students took a quiz. Their scores (out of 10) are:

$$6, \quad 9, \quad 7, \quad 10, \quad 5, \quad 8, \quad 5.$$

(a) First, sort the data.

(b) Find the median score.

(c) Compare this median to the mean (you may quickly estimate the mean if you wish).

**Solution (Q4)**

- **(a) Sort the scores:**
  The scores in ascending order:
  $$5,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10.$$
- **(b) Median:**
  Since there are 7 data points (odd number), the median is the $(n+1)/2 = (7+1)/2 =$ 4th value. Here, that is the 4th score in the sorted list, which is $\boxed{7}$.
- **(c) Quick Mean Check:**
  The sum of all scores is $5+5+6+7+8+9+10 = 50$. The mean is $50/7 \approx 7.14$. The median (7) is close to this mean (7.14), showing that the distribution is fairly symmetric (no large skew).

## Question 5 (Discrete Frequency Distribution)

The following table shows daily production (in units) for a machine over 15 days:

| Units (x) | 8 | 9 | 10 | 11 |
|---|---|---|---|---|
| Frequency (f) | 2 | 5 | 4 | 4 |

(a) Find the median units produced.

(b) Suggest what this median says about typical production.

**Solution (Q5)**

- **(a) Median:**
  The total number of days $n = 2+5+4+4 = 15$. The median day is the $(n+1)/2 = (15+1)/2 = 8$th observation.

  Compute the cumulative frequencies:
  $$\text{Units} = 8 \quad (\text{c.f.} = 2), \quad 9 \quad (\text{c.f.} = 2{+}5 = 7), \quad 10 \quad (\text{c.f.} = 7{+}4 = 11), \quad 11 \quad (\text{c.f.} = 11{+}4 = 15).$$
  The 8th observation lies in the class $x = 10$ (because the c.f. up to 9 is 7, and up to 10 is 11, so observations 8 through 11 are all 10).
  $$\text{Median} = 10.$$

- **(b) Typical Production:** A median of 10 units implies that on half of the days, the machine produces at least 10 units, which is a good central measure of typical daily output.

## Question 6 (Grouped Frequency Distribution)

A set of 40 employees was surveyed for their monthly salaries (in \$). The grouped data are:

| Salary Interval | No. of Employees |
|---|---|
| 2000–2499 | 5 |
| 2500–2999 | 11 |
| 3000–3499 | 10 |
| 3500–3999 | 9 |
| 4000–4499 | 5 |

Use the $\frac{k\,(n+1)}{4}$ formula (with $k = 2$ for the median) and **interpolate** if the rank falls inside a class. Then briefly comment on the typical monthly income.

**Solution (Q6)**

- **(a) Finding the Median (Q2):**
  Let $n = 40$. For the median, we set $k = 2$:

  $$\text{rank}(Q_2) = \frac{2\,(n+1)}{4} = \frac{40+1}{2} = 20.5.$$

  Construct the cumulative frequencies:

  $$\text{c.f. up to } 2000\text{–}2499 = 5, \quad \text{up to } 2500\text{–}2999 = 16, \quad \text{up to } 3000\text{–}3499 = 26, \quad \text{up to } 3500\text{–}3999 =$$

  Since $16 < 20.5 \leq 26$, the 20.5th observation is in the class 3000–3499.

  **Interpolation** within that class:

  - Lower boundary = 3000.
  - Class width = $3500 - 3000 = 500$.
  - Frequency of this class = 10.
  - Cumulative frequency before this class = 16.
  - Offset within class = $20.5 - 16 = 4.5$.
  - Fraction = $\frac{4.5}{10} = 0.45$.

Thus,

$$Q_2 = 3000 + 0.45 \times 500 = 3000 + 225 = \$3225.$$

- **(b) Typical Monthly Income:**
  The median salary is about \$3225, meaning half the employees earn less than \$3225 and half earn more (within these intervals). This is a good central measure of "typical" monthly pay.

---

# III. Mode (3 Questions)

## Question 7 (Simple, Ungrouped Data)

Find the mode(s) of the following data set representing daily fruit consumption (in pieces of fruit) by 10 individuals:

$$1,\ 2,\ 0,\ 2,\ 3,\ 2,\ 1,\ 4,\ 2,\ 2.$$

(a) Identify the value(s) with highest frequency.

(b) If more than one mode exists, report all of them.

### Solution (Q7)

- **Data Tally:**

  $$0 \to 1, \quad 1 \to 2, \quad 2 \to 5, \quad 3 \to 1, \quad 4 \to 1.$$

  Wait—let's carefully count the frequency of 2: The data are $1, 2, 0, 2, 3, 2, 1, 4, 2, 2$. Actually, that yields 2 appearing 5 times? Let's see: - 2 appears at positions 2, 4, 6, 9, 10. That is indeed 5 times.

- The highest frequency is 5, for the value 2. No other value has frequency near that.

  $$\boxed{\text{Mode} = 2.}$$

- No other value ties that frequency, so there is just one mode.

## Question 8 (Discrete Frequency Distribution)

A teacher records the shoe sizes (integer values) of 20 students in a class:

| Shoe Size | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| Frequency | 4 | 8 | 5 | 3 |

(a) Determine the modal shoe size in this set of students.

(b) Briefly comment on why the mode is more descriptive here than, say, the mean shoe size.

### Solution (Q8)

- **(a) Identify the Mode:**
  The frequency distribution:

$$\text{Size} = 5\,(f = 4), \quad \text{Size} = 6\,(f = 8), \quad \text{Size} = 7\,(f = 5), \quad \text{Size} = 8\,(f = 3).$$

  The highest frequency is 8 for shoe size 6. Thus,

$$\text{Mode} = 6.$$

- **(b) Significance of the Mode:**
  The mode (6) indicates the most common shoe size in the class. Since shoe sizes are discrete, the mode is often more intuitive as the "typical" size than the mean, which might be a non-integer like 6.35 (less directly meaningful in the context of actual shoe sizes).

## Question 9 (Grouped Frequency Distribution)

The table below shows the distribution of test scores for 50 students, grouped into intervals of width 10:

| Score Interval | Frequency |
|---|---|
| 0–9 | 5 |
| 10–19 | 8 |
| 20–29 | 14 |
| 30–39 | 15 |
| 40–49 | 8 |

(a) Identify the modal class (the class with the highest frequency).

(b) Estimate the modal score using the standard grouping formula for the mode:

$$\text{Mode} = L + \left( \frac{f_m - f_{m-1}}{2\,f_m - f_{m-1} - f_{m+1}} \right) \times\, h,$$

where $L$ is the lower boundary of the modal class, $f_m$ its frequency, $f_{m-1}$ the previous class's frequency, $f_{m+1}$ the next class's frequency, and $h$ the class width.

**Solution (Q9)**

- **(a) Modal Class:**
  Frequencies are: 5, 8, 14, 15, 8. The highest frequency is 15 in the interval 30–39. Thus the modal class is 30–39.

- **(b) Approximate Modal Score:**
  Use the given formula. Let:

  $$L = 30, \quad f_m = 15, \quad f_{m-1} = 14, \quad f_{m+1} = 8, \quad h = 10.$$

  Then the fraction:

  $$\frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} = \frac{15 - 14}{2 \times 15 - 14 - 8} = \frac{1}{30 - 14 - 8} = \frac{1}{8} = 0.125.$$

  Hence,

  $$\text{Mode} = L + 0.125 \times 10 = 30 + 1.25 = 31.25.$$

  So the modal score (estimate) is about 31.25.

---

**Final Note:** These 9 questions illustrate the main approaches to **Mean**, **Median**, and **Mode** with different types of data: simple, discrete frequency, and grouped frequency. By practicing all three measures across data formats, one gains a deeper understanding of descriptive statistics.