EE2703 : Applied Programming Lab
Assignment 4

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EE20B118

February 2022

1 Abstract

The goal of the assignment is the following:

- To fit two functions e^x and $cos(cos(x))$ over the interval [0,2π] using the Fourier series.
- To calculate the deviation of Fourier coefficients obtained using least squares method and direct integration.

2 Plotting functions e^x and $cos(cos(x))$

e^x is a non periodic function whereas $cos(cos(x))$ is periodic function with period 2π. Given below are the plots of the above mentioned functions over the interval $[2\pi,4\pi)$ in Figure 1 and 2 respectively.
Function generated by the Fourier series are periodic extensions of the original functions. So $cos(cos(x))$ remains same whereas e^x will be different.

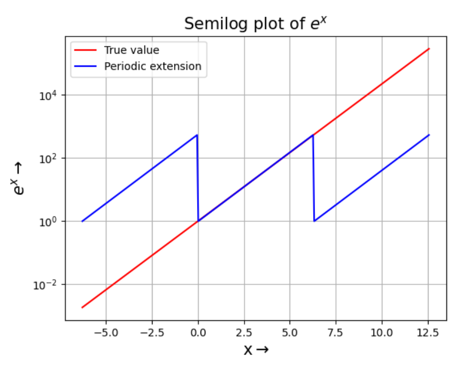

Figure 1: Data plot of e^x

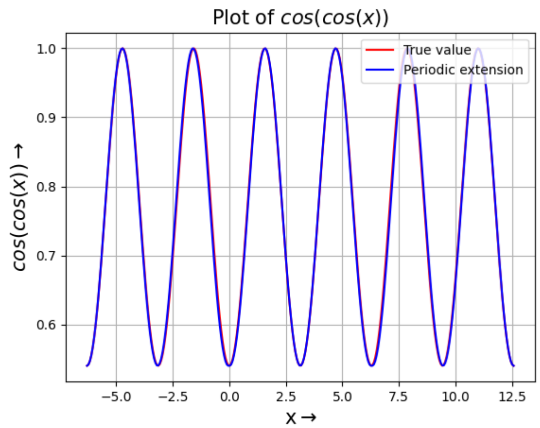

Figure 2: Data plot of $cos(cos(x))$

3 Fourier coefficients of e^x and $cos(cos(x))$

The Fourier coefficients an and bn of functions with period 2π are computed as follows:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} f(x) * cos(nx)dx$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi} f(x) * sin(nx)dx$$

The first 51 coefficients for the two functions above are generated using *integrate.quad()* function and equations mentioned above and stored as $\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ \ldots \\ a_{25} \\ b_{25} \end{bmatrix}$

The Semilog and loglog plots of the Fourier coefficients of e^x and $cos(cos(x))$ is as shown:

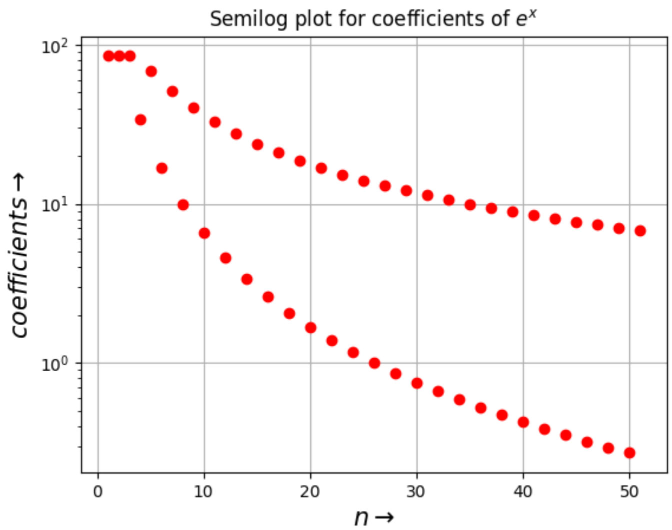

Figure 3: Semilog plot of the Fourier coefficients of e^x

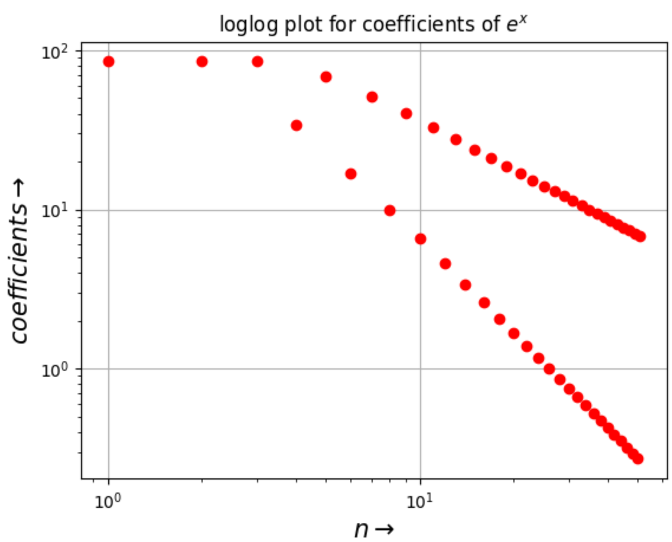

Figure 4: loglog plot of the Fourier coefficients of e^x

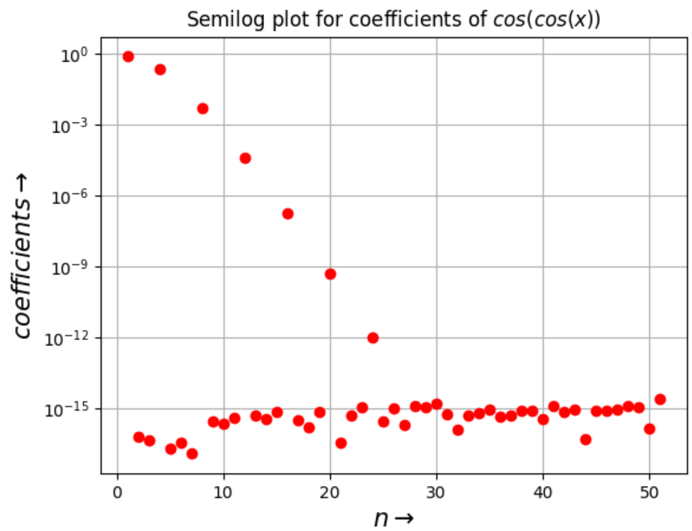

Figure 5: Semilog plot of the Fourier coefficients of $cos(cos(x))$

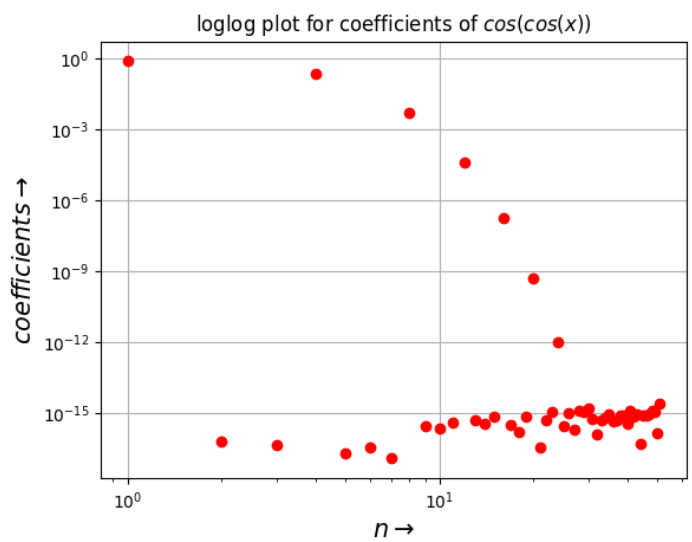

Figure 6: loglog plot of the Fourier coefficients of $cos(cos(x))$

a. As it is evident from the plots, b_n is nearly zero for $cos(cos(x))$. This is because $cos(cos(x))$ is an even function, hence in the Fourier series expansion, all the b_n terms should be zero for the series to be an even function.

b. The magnitude of the coefficients would represent how much of certain frequencies happen to be in the output. $cos(cos(t))$ does not have very many frequencies of harmonics, so it dies out quickly. However, since the periodic extension of e^t is discontinuous. To represent this discontinuity as a sum of continuous sinusoidal, we would need high frequency components, hence coefficients do not decay as quickly.
c. The loglog plot is linear for e^t since Fourier coefficients of e^t decay with $1/n$ or $1/n^2$. The Semilog plot seems linear in the $cos(cos(t))$ case as its Fourier coefficients decay exponentially with n.

4 Least squares approach

The Fourier series used to approximate a function is as follows:

$$a_0 + \sum_{n=1}^{\infty} a_n \cos(nx_i) + b_n \sin(nx_i) \approx f(x_i)$$

We choose 400 equally spaced values in the range $[0,2\pi]$ using *linspace()* function. Using matrices, we can say

$$Ac = b$$

where A= $\begin{bmatrix} 1 & cosx_1 & sinx_1 & ... & cos25x_1 & sin25x_1 \\ 1 & cosx_2 & sinx_2 & ... & cos25x_2 & sun25x_2 \\ ... & ... & ... & ... & ... & ... \\ 1 & cosx_{400} & sinx_{400} & ... & cos25x_{400} & sin25x_{400} \end{bmatrix}$, b = $\begin{bmatrix} f(x_1) \\ f(x_2) \\ ... \\ f(x_{400}) \end{bmatrix}$

and c is the matrix of Fourier coefficients that is $\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ ... \\ a_{25} \\ b_{25} \end{bmatrix}$

The function *c=lstsq(A,b)[0]* finds the 'best fit' numbers that will satisfy the approximation equation at exactly the points we have evaluated *f(x)*.
The plots of the true and predicted values of the Fourier coefficients are shown below:

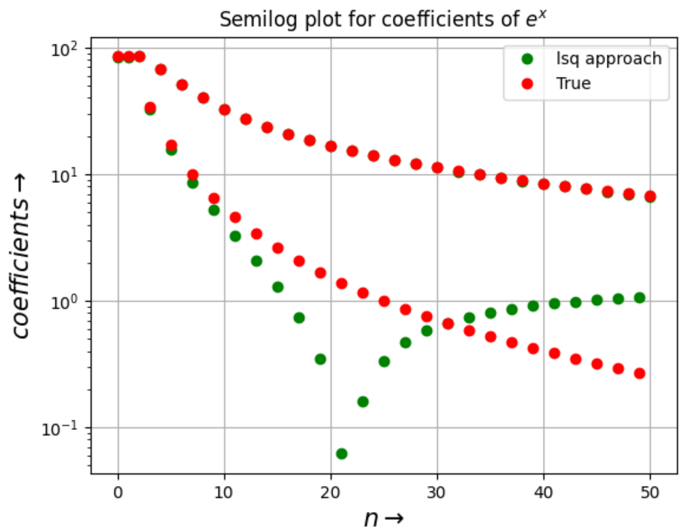

Figure 7: Semilog plot of the Fourier coefficients of e^x

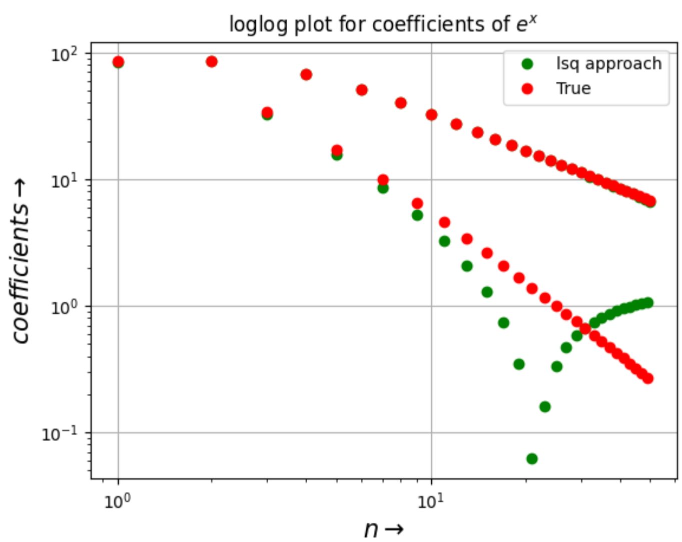

Figure 8: loglog plot of the Fourier coefficients of e^x

5 Deviations from True plot

The maximum deviation in the case of e^x is 1.332730870335368
The maximum deviation in the case of $cos(cos(x))$ is 2.553315730564766e-15

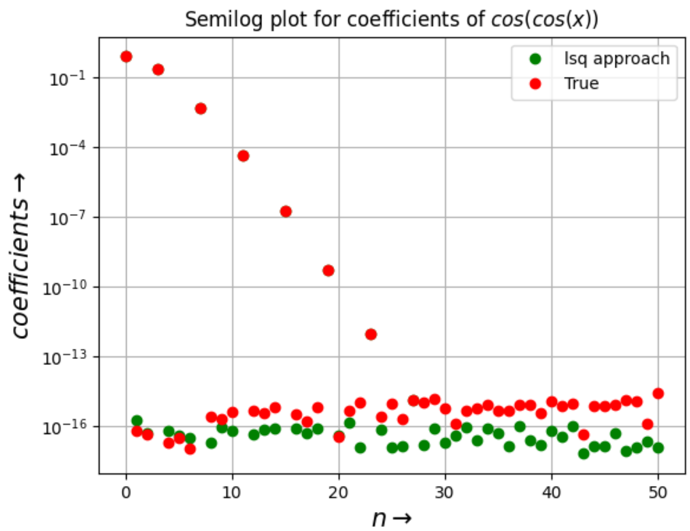

Figure 9: Semilog plot of the Fourier coefficients of $cos(cos(x))$

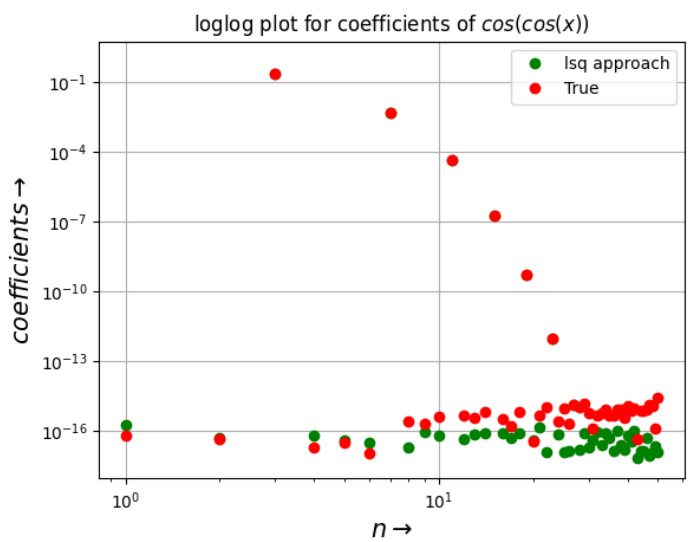

Figure 10: loglog plot of the Fourier coefficients of $cos(cos(x))$

6 Estimated Functions

It can be seen that the coefficients for cos(cos(x)) are in much closer agreement than the coefficients for e^x and is also expected as the former is periodic and the periodic extension of the latter has an increasing gradient and would require higher sinusoids for a better representation. Since the least square method is approximate, deviation is expected. We can also find the index for which the maximum deviation in coefficients is found as shown in the code. The least squares estimation method provides a faster way of evaluating a large number of Fourier coefficients. In this case of evaluating 51 coefficients, the time taken to evaluate the coefficients in the direct approach was almost 10 times that taken for the least squares approach.

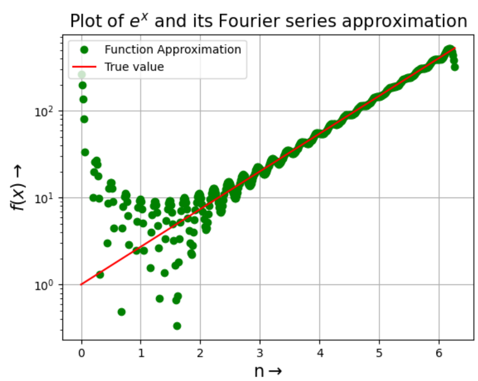

Figure 11: Estimated function of e^x

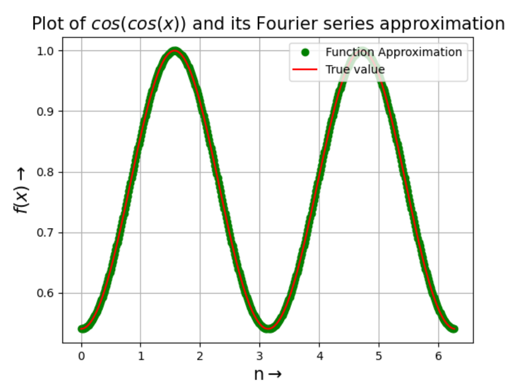

Figure 12: Estimated function of $cos(cos(x))$