EE2703 : Applied Programming Lab
Assignment 9

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EE20B118

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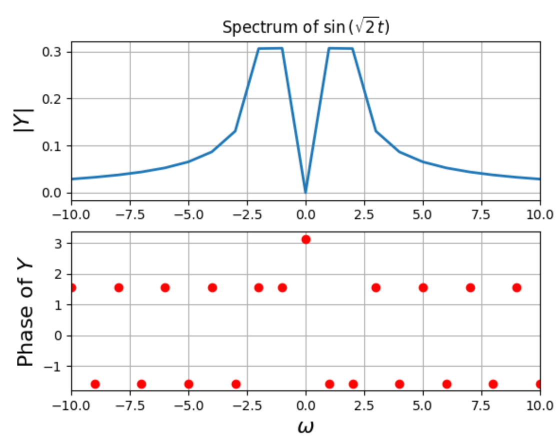
1 Abstract

In this assignment, we study about the DFTs of non periodic signals. The discontinuities causes fourier components in frequencies other than the sinusoid frequency which decays with $1/\omega$, due to Gibbs phenomenon. We resolve this problem using a hamming window. We also perform a sliding DFT on a chirped signal and plot the results.

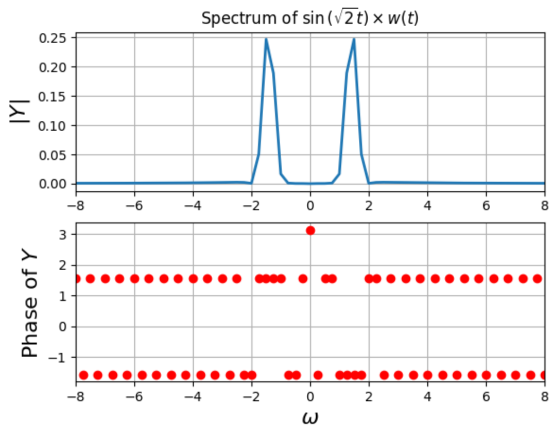
2 Questions

2.1 Question 1: Worked example

The worked example in the assignment is for $\sin(\sqrt{2}t)$. Spectrum of $\sin(\sqrt{2}t)$ is given below:

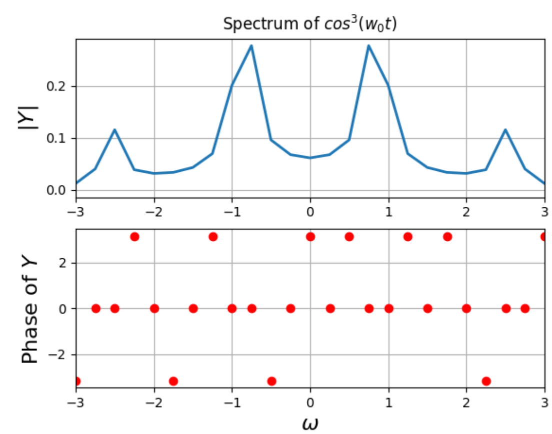


The spectrum that is obtained with a time period 8π has a slightly sharper peak as given below:

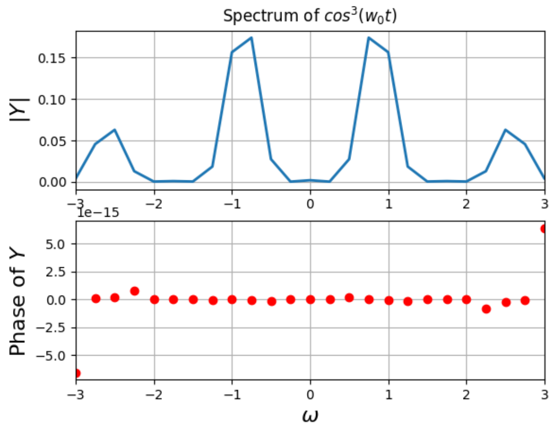


2.2 Question 2: Spectrum of $cos^3(0.86t)$

In this question, we shall plot the FFT of $cos^3(0.86t)$. The FFT without hamming window is given below: The FFT with the hamming window is: We notice

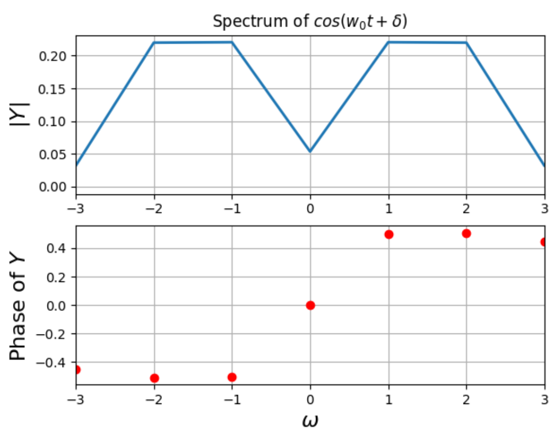


that a lot of energy is stored in frequencies that aren't a part of the signal. After windowing, these frequencies are attenuated and hence the peaks are sharper in the windowed function. It is still not an impulse because the convolution with fourier transform of the windowed function smears out the peak.



2.3 Question 3: Digital spectrum of $\cos(\omega t + \delta)$

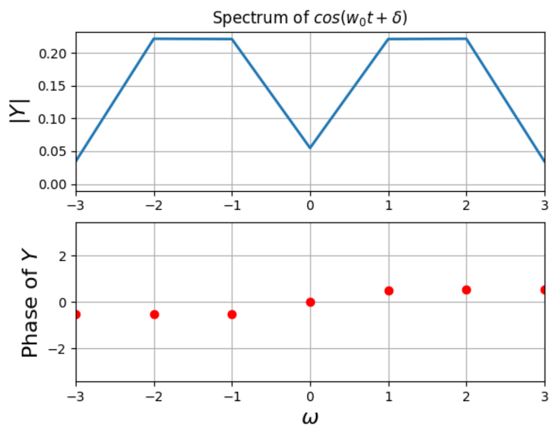
We need to estimate ω and δ for a signal $\cos((\omega t + \delta)$ for 128 samples between $[-\pi, \pi)$. We estimate omega using a weighted average. We have to extract the digital spectrum of the signal and find two peaks at $\pm\omega_o$, and estimate ω and δ.



We estimate omega by performing a Mean average of ω over the magnitude of $|Y(j\omega)|$. For delta we consider a widow on each half of ω (split into two positive and negative values) and extract their mean slope. The intuition behind this is that, a circular shift in the time domain of a sequence results in the linear phase of the spectra.

2.4 Question 4: White Gaussian Noise

We repeat the same exact process on question 3 but with noise added to the original signal.
For true value of ω and $\delta = 1.5$ and 0.5 respectively I got:



$\omega = 1.51631796485824 12$
$\delta = 0.506776265719626$ (No noisy case)
$\omega = 1.9850993495694094$
$\delta = 0.529289485221217$ (Noisy case)

2.5 Question 5: DFT of Chirped signal

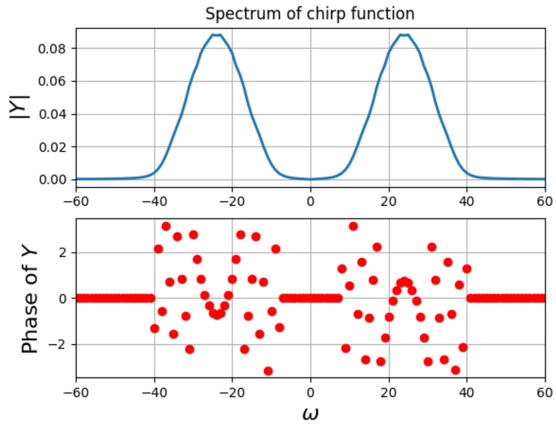
In this question we analyze a chirp signal which is an FM signal where frequency is directly proportional to time. A chirp signal we shall consider is given by

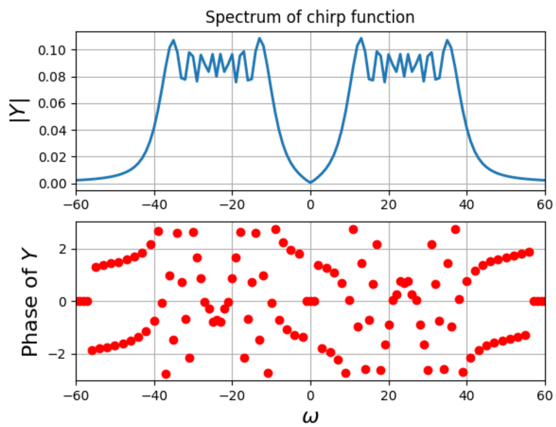
$$f(t) = cos(16t(1.5 + \frac{t}{2\pi}))$$

The FFT of the chirp is given by: We note that the frequency response is spread between 5-50 rad/sec. A large section of this range appears due to Gibbs phenomenon. On windowing, only frequencies between 16 and 32 rad/sec remain.

2.6 Question 6: Time-frequency plot

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide. Extract, the DFT of each and store as a column in a 2D array. Then plot the array as a surface plot to show how the frequency of the signal varies with time. This is new. So far we worked either in time or in frequency. But this is a time- frequency" plot, where we get localized DFTs and show how the spectrum evolves in time. We do this for both phase and magnitude.





Let us explore the surface plots:

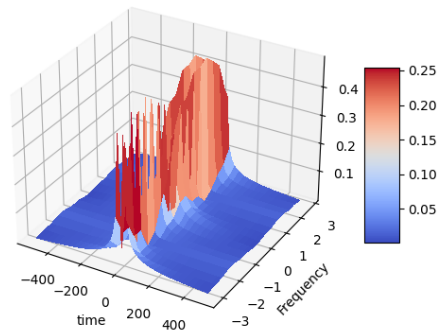


Figure 1: Windowed Chopped Chirp function: "Fourier Transform"

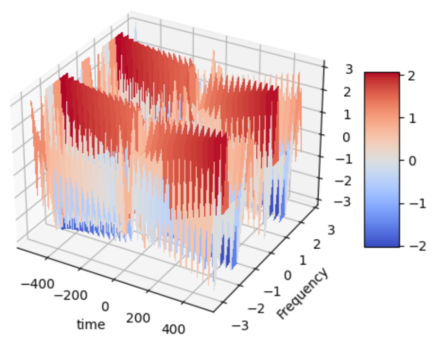


Figure 2: Windowed Chopped Chirp function: "Phase of Fourier Transform"

3 Conclusion

In this assignment, we have covered the requirement of windowing in the case of non-periodic series in DFt's. In particular this is to mitigate the effect of Gibbs phenomena owing to the discontinuous nature of the series $\tilde{x}[n]$ realised by a discrete fourier transform. The last question addresses the time varying spectra for a chirped signal, where we plot fourier spectra for different time slices of a signal. We noted the case of sparse number of slices and hence took more closely spaced slices. The general properties of a fourier spectra for a chirped signal are observable in the time varying plots i.e., existence of two peaks (slow growth), vanishing of chirp effects in case of a windowed transform, and a phase plot that periodically varies with reduced phase near maximum values.