## **Topics: Normal distribution, Functions of Random Variables**

- 1. The time required for servicing transmissions is normally distributed with  $\mu$  = 45 minutes and  $\sigma$  = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
  - A. 0.3875
  - B. 0.2676
  - C. 0.5
  - D. 0.6987

ANS:- Mean=45 & St. Deviation=8

Work will commence after 10mins so mean would be 45+10=55

Car will be ready within 1hr=60

 $S_{0,60-55/8=0.62}$ , from the z value table=0.723

Probability of service manager can't meet his commitment would be 1.

1-0.723= 0.267

- 2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean  $\mu$  = 38 and Standard deviation  $\sigma$  =6. For each statement below, please specify True/False. If false, briefly explain why.
  - A. More employees at the processing center are older than 44 than between 38 and 44.

ANS:- Mean=38 & St. Deviation=6

Probability of employees older then 44

(44,38,6)=1-0.841=0.158

Probability of employees b/w 38&44

(38,38,6)=0.5

0.841 - 0.5 = 0.341

As we can see probability of employees b/w 38&44 is more, so therefore this statement is false.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS:-Mean=38 St. Deviation=6,x=30

Probability of employees under the age 30

(30,38,6) = 0.091

Total no of employees 400\*0.091 = 36.4, so therefore this statement is true.

3. If  $X_1 \sim N(\mu, \sigma^2)$  and  $X_2 \sim N(\mu, \sigma^2)$  are *iid*normal random variables, then what is the difference between 2  $X_1$  and  $X_1 + X_2$ ? Discuss both their distributions and parameters.

- 4. Let  $X \sim N(100, 20^2)$ . Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
  - A. 90.5, 105.9
  - B. 80.2, 119.8
  - C. 22, 78
  - D. 48.5, 151.5
  - E. 90.1, 109.9

ANS:- Mean=100 & std.deviation=20

Probability of random variable taking a value b/w them is 0.99

(0.99) = 2.32

2.32\*20+100=146.4

-2.32\*20+100=53.48

Hence D option would be the nearest value.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions  $Profit_1 \sim N(5, 3^2)$  and  $Profit_2 \sim N(7, 4^2)$  respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

ANS:- annual profit=proft1 + profit2

Mean=profit1 (mean) +profit2(mean) = (5+7)=12

St. Deviation=sqrt of 3&4

$$=(9+16) = 25 = 5.$$

12\*45= 540- Mean

5\*45=225-Std.deviation

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS:- Range= 95%

$$(0.95,540,225) = 99.00,980.99.$$

B. Specify the 5<sup>th</sup> percentile of profit (in Rupees) for the company.

ANS:- 5<sup>th</sup> percentile, for that we need to find the z score value

Z score value of 90% = 1.645

Mean+(-1.645)\*St. Deviation

540-1.645\*225 = 169.87

C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:- Probability of making loss 1st =(0,5,3) = 0.0477

Probability of making loss  $2^{nd} = (0,7,4) = 0.040$ 

1<sup>st</sup> division made more loss compare to 2<sup>nd</sup> division.