Circular Motion and Gravitation

Uniform Circular Motion

Important characteristics

- Firstly in uniform circular motion velocity is constant, and direction is continuously changing.
- Interestingly even though uniform circular motion has constant velocity there is a centripetal acceleration pointed to the center of the circle.
  - How is this possible? In linear motion $a = \frac{v}{t}$ for constant acceleration and $a = \frac{dv}{dt}v$ more generally. How can there be acceleration when the change in velocity is 0?
  - Well as stated before even though the velocity is constant the direction is always changing. This requires acceleration.
    - A good way to understand this is through the scenario where a bowling ball is moving at some velocity v and you try to move it in a circle with a broom stick. In order to do this u hit it with a broom stick directing a force to the center of the circular path you want.

Basic Formulas and Derivations

Basic Formulas

- $a = \frac{v^2}{r}$
  - Acceleration is proportional to v^2 and inversely proportional to r
- $w = \frac{v}{r}$
  - w is the angular acceleration and it is the change in angle over time. It is constant in uniform circular motion, and its units are usually radians/s.
- $f = \frac{1}{T}$
  - f is the frequency and T is the period. Period is the amount of time for one rotation, and frequency is the number of rotations per unit of time. The units for f are Hz, and for period are seconds.

Derivations for further formulas

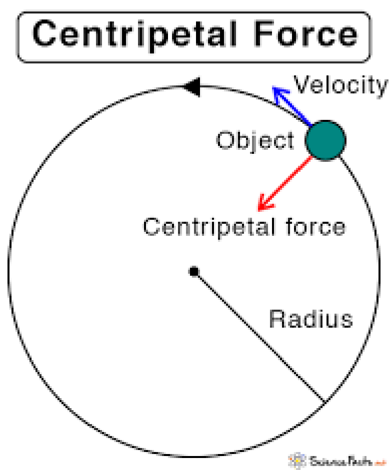
- $v = \frac{2\pi r}{T}$
  - The circumference of a circle is $2\pi r$ which is also the distance traveled in one rotation.
  - The amount of time it takes for one rotation is T
  - $v = \frac{d}{t}$ and $d = 2\pi r$ and $t = T$, therefore $v = \frac{2\pi r}{T}$
- $v = 2\pi rf$

- We can rearrange the previous equation to $v = 2\pi r \cdot \frac{1}{T}$
- We know that $\frac{1}{T} = f$ therefore $v = 2\pi r f$

Uniform Circular Motion With Forces

Centripetal Force

- Remember that Newtons Second Law is $\Sigma F = F_{net} = ma$
  - In circular motion we know that the acceleration is the centripetal acceleration pointed to the center
  - This centripetal acceleration is created by the centripetal force
- The centripetal force is the net force of an object in circular motion
  - It can be defined as $F_{net} = ma = m\frac{v^2}{r}$



Problem Tips

> Rather than going over every possible problem scenario I will offer some tips and steps to approaching these types of problems

1. First tip is to not forget everything you practiced in previous forces topics. Even though these problems have the added complexity of circular motion they are still largely similar
2. Draw your free body diagrams. This is super important to understand the problem, and also see understand all the forces in the problem
3. If needed split the problem into components
4. Remember that centripetal force is a NET FORCE which is the sum of all forces

Gravitation

> Lots of the topics of gravitation are framed and studied as circular motion problems, so don't forget everything covered.

Important Facts About Gravity

- Gravity is always an attractive force
- Gravity operates over all scales
- Gravity is much weaker than the other 3 forces; however, since it operates over all scales, at large scales it becomes the primary source.

Important Formulas

- $F_1 = F_2 = G\frac{m_1 m_2}{r^2}$
  - Force 1 is the force that object 1 applies on object 2, and force 2 is the force that object 2 applies on object 1.
  - G is the gravitational constant or $6.67 \cdot 10^{-11}$ as you can see it is very small
  - m_1 is the mass of the first object and m_2 is the mass of the second object
  - r is the distance between the two objects
- $g = G\frac{m}{r^2}$
  - g is the acceleration due to gravity from the object
  - G is the gravitational constant
  - r is the effective radius or distance from the center of the object
  - This formula is obtained from the previous formula
    - Remember that g is an acceleration and $F = ma$ so therefore $m_2 g = G\frac{m_1 m_2}{r^2}$. Then we can cancel out the m_2 and are left with $g = G\frac{m_1}{r^2}$ or $g = G\frac{m}{r^2}$
- $U_G = -G\frac{m_1 m_2}{r}$
  - U_G is the potential energy due to gravity
  - G is the gravitational constant
  - m_1,m_2 are the masses
  - r is the distance between the two objects

Density formulas

- Here the previous formulas that use density instead of masses. $
  - Mass is equal to volume times density
  - For spheres: $m = \frac{4}{3}\pi r^3 \rho$
- We can then plug this in for any ms to get the density based formulas for gravity
- $F_1 = F_2 = G\frac{(\frac{4}{3}\pi r^3 \rho_1)(\frac{4}{3}\pi r^3 \rho_2)}{r^2}$
- $g = G\frac{\frac{4}{3}\pi r^3 \rho}{r^2}$

Proportionality Problems

- In Gravitation many problems on the AP Exam are proportionality so here we review proportions.
- We can create proportionality statements by disregarding all variables that are kept constant. For example let's say we double the distance between 2

objects and keep the rest of the variables the same.

  - The proportionality equation for the gravitation between the 2 objects is $F \propto \frac{1}{r^2}$

- Now we can use this for proportionality problems. For example lets say our distance between the 2 objects doubles, now this new force is one forth of the original

Keplers Third Law

- $T^2 \propto r^3$
  - In planetary motion the force of gravity serves as the centripetal force
  - We can start with the formula for centripetal force: $F_{net} = \frac{mv^2}{r}$
  - We can substitute F_{net} with the formula for gravitational force, or $G\frac{m_1m_2}{r^2}$ -> $G\frac{m_1m_2}{r^2} = \frac{m_2v^2}{r}$
  - We can cancel the m_2 and plugin $\frac{2\pi r}{T}$ for v to get $G\frac{m_1}{r^2} = \frac{4\pi^2 r}{T^2}$
  - Finally after rearranging we get $T^2 = \frac{4\pi^2 r^3}{GM}$, which is how we get the proportionality statement