Nonlinear Hydrodynamics of the Atmosphere and Ocean, with Examples from Sea Straits

Emin Özsoy[1], Adil Sözer[1], Özgür Gürses[1], Murat Gündüz[1], Gianmaria Sannino[2]

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(1) Institute Of Marine Sciences,
Middle East Technical University
Erdemli, Mersin, Turkey
(2) Italian National Agency For New Technologies,
Energy And Sustainable Economic Development
(ENEA-UTMEA), Rome, Italy

THE FUNDAMENTAL SOURCES OF NONLINEAR BEHAVIOR LEADING TO "DISORDER" IN THE OCEAN AND THE ATMOSPHERE:

Hydrodynamics:
-the (Navier-Stokes) equations of fluid dynamics are nonlinear
  advection (~material derivative) , mixing, dissipation
  nonlinear surface, bottom and lateral boundary conditions
-inherent instabilities of motion, eddying and meandering
  (wave instability,shear instability, static instability, diffusive-convective instabilities, barotropic – baroclinic instability etc.)
-transition to turbulence, fully developed turbulence = chaotic flow
-nonlinear equation of state – ocean or atmosphere:
  density=f(temperature, salinity) or density=(pressure, temperature)

Ecosystem dynamics:
-coupled nonlinear or "reactor" equations
  e.g. Lotka – Volterra predator-prey equations
-systems with memory and delayed response
-built on underlying physics, advection, diffusion, migration, swimming, clustering
-turbulence interaction
  e.g. orientation of zooplankton

PARTICULAR COMPLEXITY ISSUES AND CONSEQUENCES IN THE OCEAN AND THE ATMOSPHERE:

Hydrodynamics:

-"simple-looking" Navier-Stokes equations !

existence and uniqueness (smoothness) of solutions have not been proved!

(CLAY Mathematics Institute offers 1 million dollars for proof!)

- same eqns for ocean / atmosphere, with minor changes in physics, scales

-wide span of exact solutions under simplified cases (full solution only numerical)

-novel effects at geophysical scales due to earth's rotation and density stratification

-instabilities inherent

- leads to waves (surface, internal, topographic etc.) with unique propagation properties
- leads to gyres, eddies and meanders, at basin and sub-basin scales, internal / external
- leads to turbulence on local scales or geophysical turbulence at large scales

-regular and inverse (from small to large scales) nonlinear cascading of energy

-coherent structures and fronts emerge at local and geophysical scales

-consequences on mixing at local and large scales, wind stirring, convection, upwelling

-nonlinear equations of state – phase changes, ice physics, caballing etc. in the ocean

-parameterization in models replaces physics at eddy-resolving scales

-consequences on predictability

- typically 5 days in the ocean / atmosphere
- data assimilation in models
- ensemble modeling

Ecosystem dynamics: diversity, fragility, resilience, biogeochemical cycles built on physics

NONLINEAR PROCESSES, DISORDER, PREDICTABILITY:

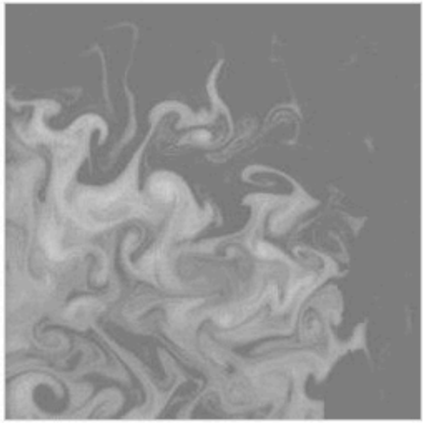
Instability is a fundamental property of fluids that cannot be ignored or escaped !
-waves, eddies, fronts, turbulence are essential components
-nonlinear systems – sensitivity to initial conditions, divergence of solutions
the 'butterfly effect' !
-significant loss of predictability in nonlinear coupled systems of high degrees of freedom
-forecasting success limited to no more than ~ 5 days in atmosphere / ocean
-'open systems': global versus limited area model forecasts
need external forcing or global closed budgets
-with appropriate parameterizations in climate models it is possible to make longer term, climatic predictions, but with less confidence
-partial solution with data assimilation
use of observations to update model forecasts
the Galilean 'scientific method': using observations along with models based on physics
importance of experiments:
http://mentalfloss.com/article/22913/hammer-and-feather-drop-moon
-ensemble forecasting (ensemble of models, forecasts with varying ic and bc)
-Bayesian statistics and Bayesian inference
update the probability estimate for a hypothesis as additional evidence is acquired

Clay Mathematics Institute 1 million $ prize !
for proof of uniqueness of solutions of the Navierf Stokes equations

CMI

ABOUT PROGRAMS MILLENNIUM PROBLEMS PEOPLE PUBLICATIONS EVENTS EUCLID

Navier–Stokes Equation



Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

This problem is: Unsolved

Rules:

Rules for the Millennium Prizes

Related Documents:

Official Problem Description

Related Links:

Lecture by Luis Cafarelli

The Generic Scalar Transport Equation

The universality of the three mechanisms discussed above makes it possible for us to construct a general differential equation that describes the conservation principle of a quantity. Note that the placement of the terms comes from the fundamental principles of deriving the conservation equations, i.e. transient and convective terms should balance diffusive and source terms. Therefore, some signs in the source terms may need to be modified as the source terms is always placed on the right hand side of the equation. Here it is

$$\underbrace{\frac{\partial \rho\phi}{\partial t}}_{\text{Accumulation}} + \underbrace{\nabla\cdot\left(\rho\mathbf{u}\phi\right)}_{\text{Convection}} = \underbrace{\nabla\cdot\left(\Gamma\nabla\phi\right)}_{\text{Diffusion}} + \underbrace{S_{\phi}}_{\text{Source}}$$

(Eq. 4)

Voila!

Eq. 4 can now be specialized to any process in the realm of heat, mass, and momentum transfer. For example, to obtain the continuity equation (for compressible flows) set phi = 1. Since diffusion is not present and in the absence of sources set those to zero to obtain

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\mathbf{u}\right) = 0$$

(Eq. 5)

To obtain the energy equation for an incompressible fluid, and in terms of the temperature, simply set phi = T to obtain

To obtain the energy equation for an incompressible fluid, and in terms of the temperature, simply set phi = T to obtain

$$\frac{\partial \rho T}{\partial t} + \nabla \cdot (\rho \mathbf{u} T) = \nabla \cdot \left(\frac{k}{c_p} \nabla T \right)$$

(Eq. 6)

Note that you can divide by the density of course. But I kept the above form so that it matches the generic equation.

Now for the Navier-Stokes equations, we replace phi by one velocity component at a time (remember, that the velocity components are also quantities and in this case their momentum is conserved). Let us consider a cartesian coordinate system and start with the axial velocity component. By replacing phi with u, and setting the appropriate form for the source term, we get

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho \mathbf{u} u) = \nabla \cdot (\mu \nabla u) - \frac{\partial p}{\partial x} + \rho g_x$$

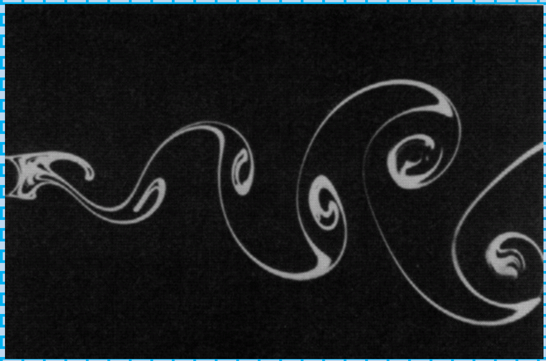
(Eq. 7)

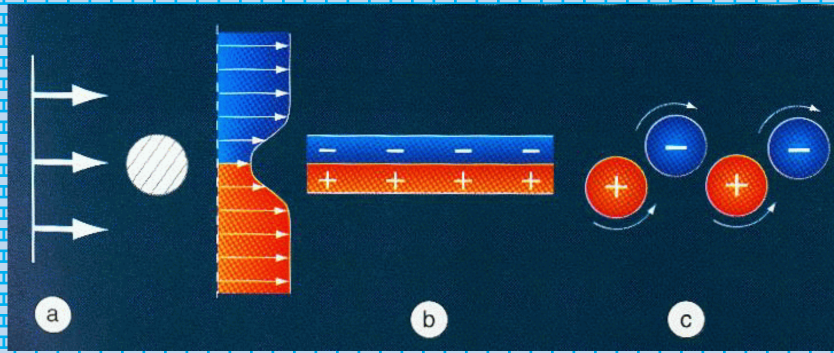
similarly, for the other coordinate directions, replace *u* by *v* and *w* respectively. The most subtle part is of course figuring out the source term. So if you just remember that typically the body forces in a fluid problem are those arising from the pressure and gravity, then the rest should be easy.

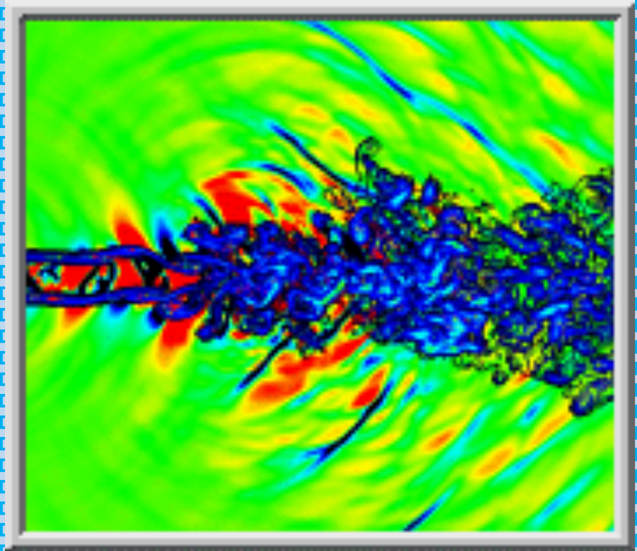
REFERENCES

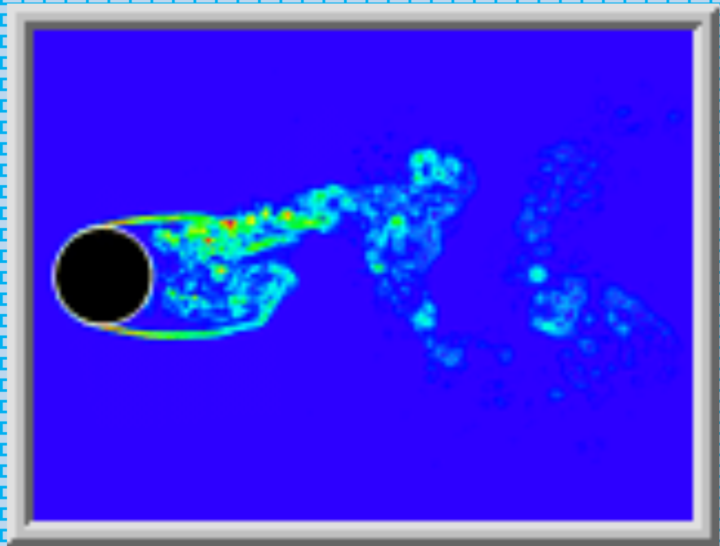
[1] Patankar, S. V., Numerical Heat Transfer and Fluid Flow, Hemisphere, Washington, D.C., 1980.

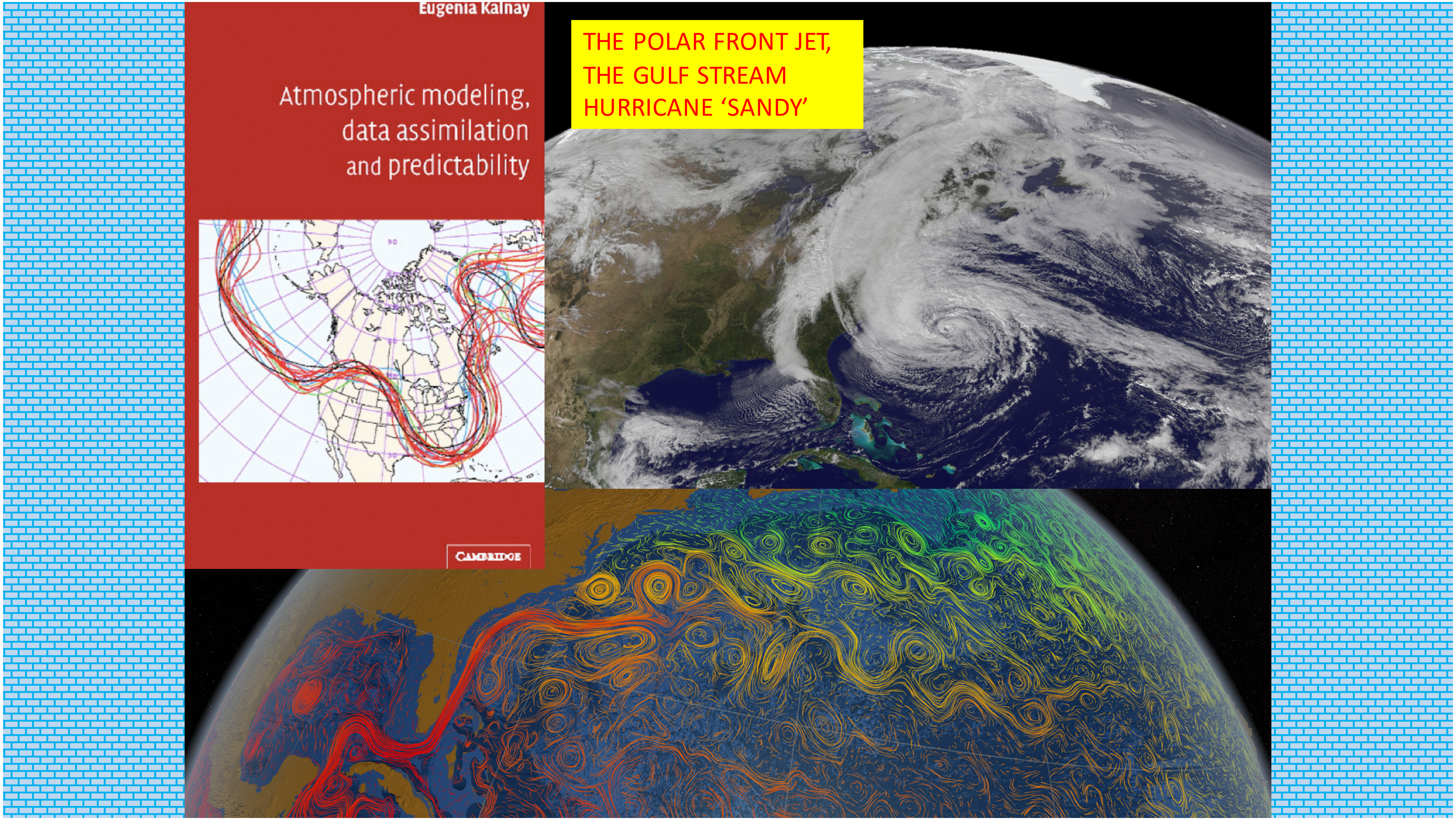
LABORATORY SCALE EDDIES AND JETS



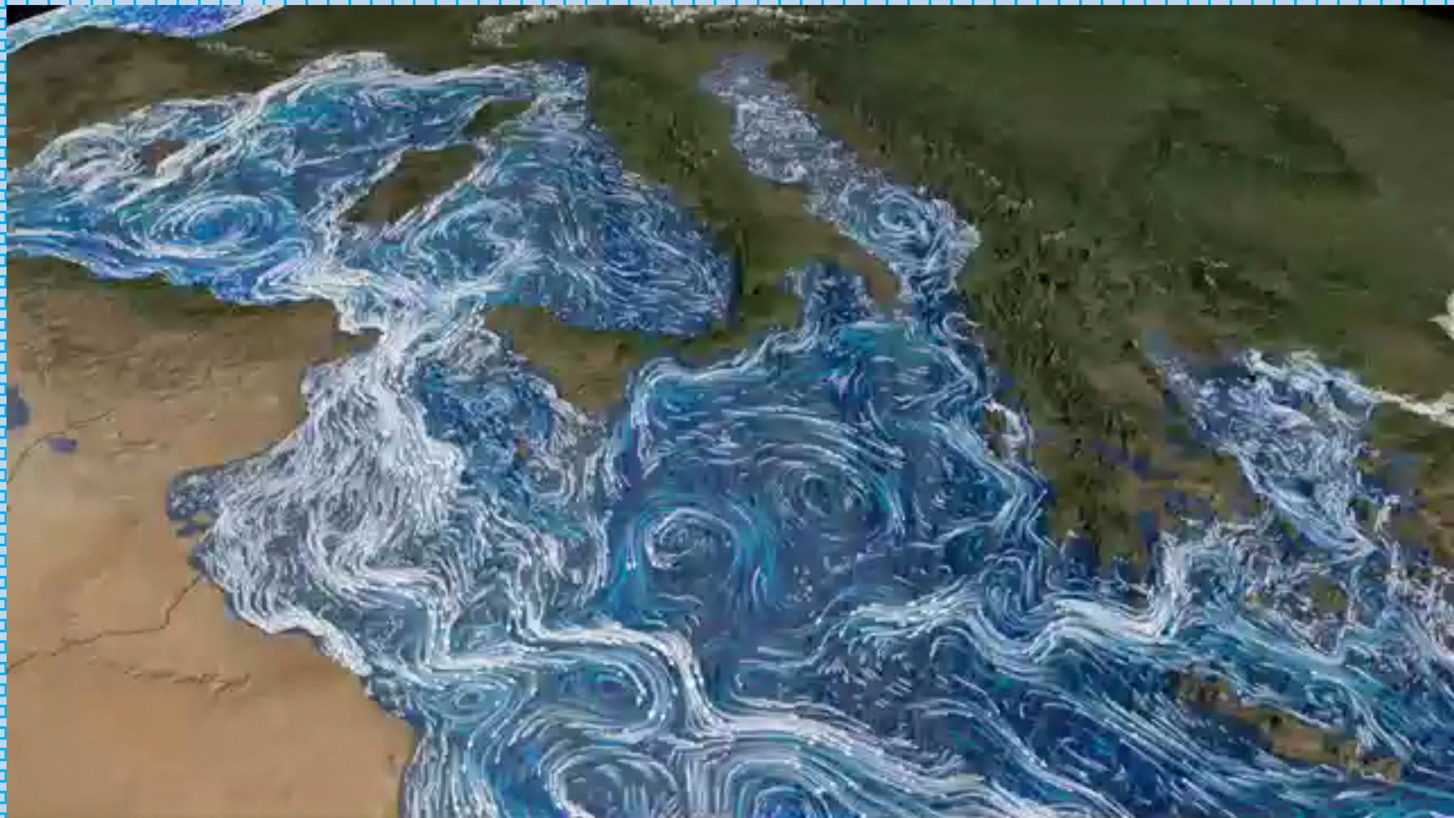




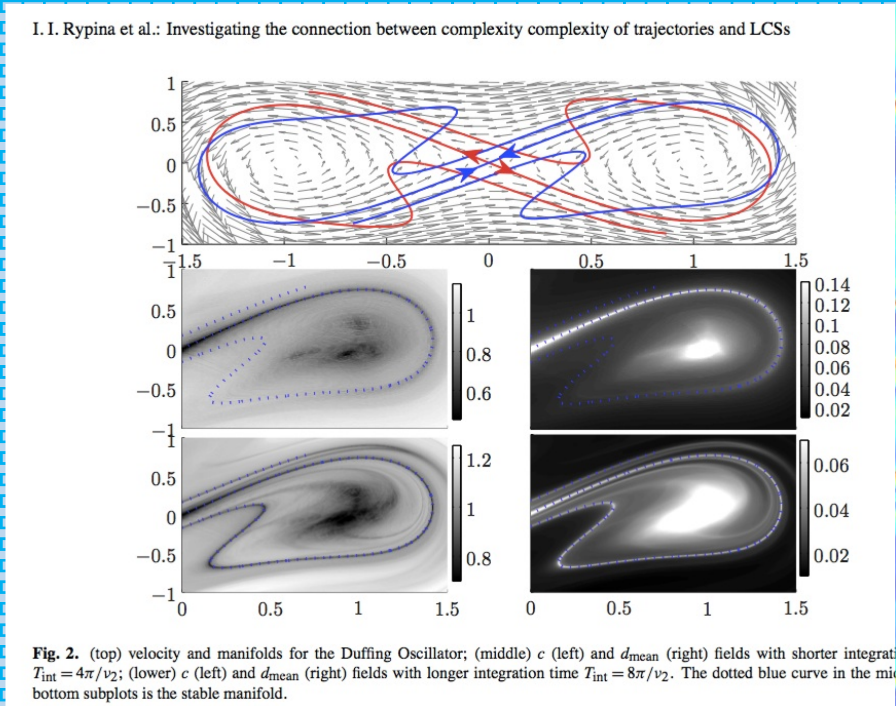


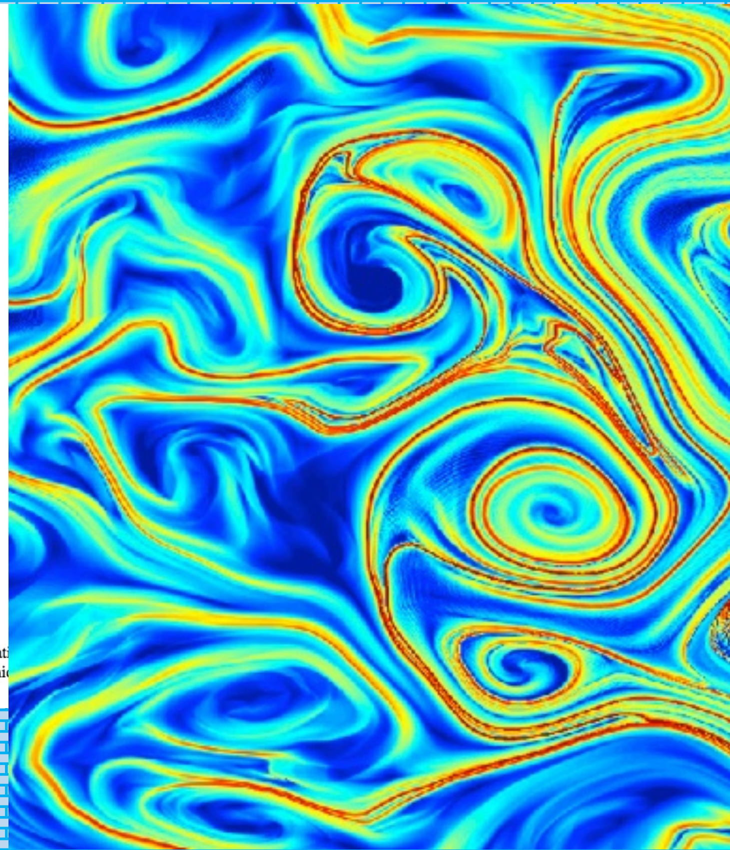

Eugenia Kalnay
Atmospheric modeling, data assimilation and predictability
CAMBRIDGE
THE POLAR FRONT JET,
THE GULF STREAM
HURRICANE 'SANDY'

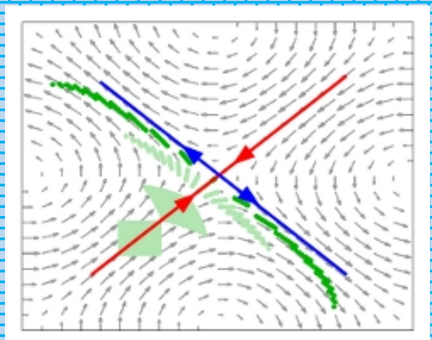
MEDITERRANEAN EDDIES AND JETS



LAGRANGIAN COHERENT STRUCTURES (LCS)







Detection of
stable manifolds

Hyperbolic trajectories

FORECAST SKILL INCREASE OVER THE YEARS

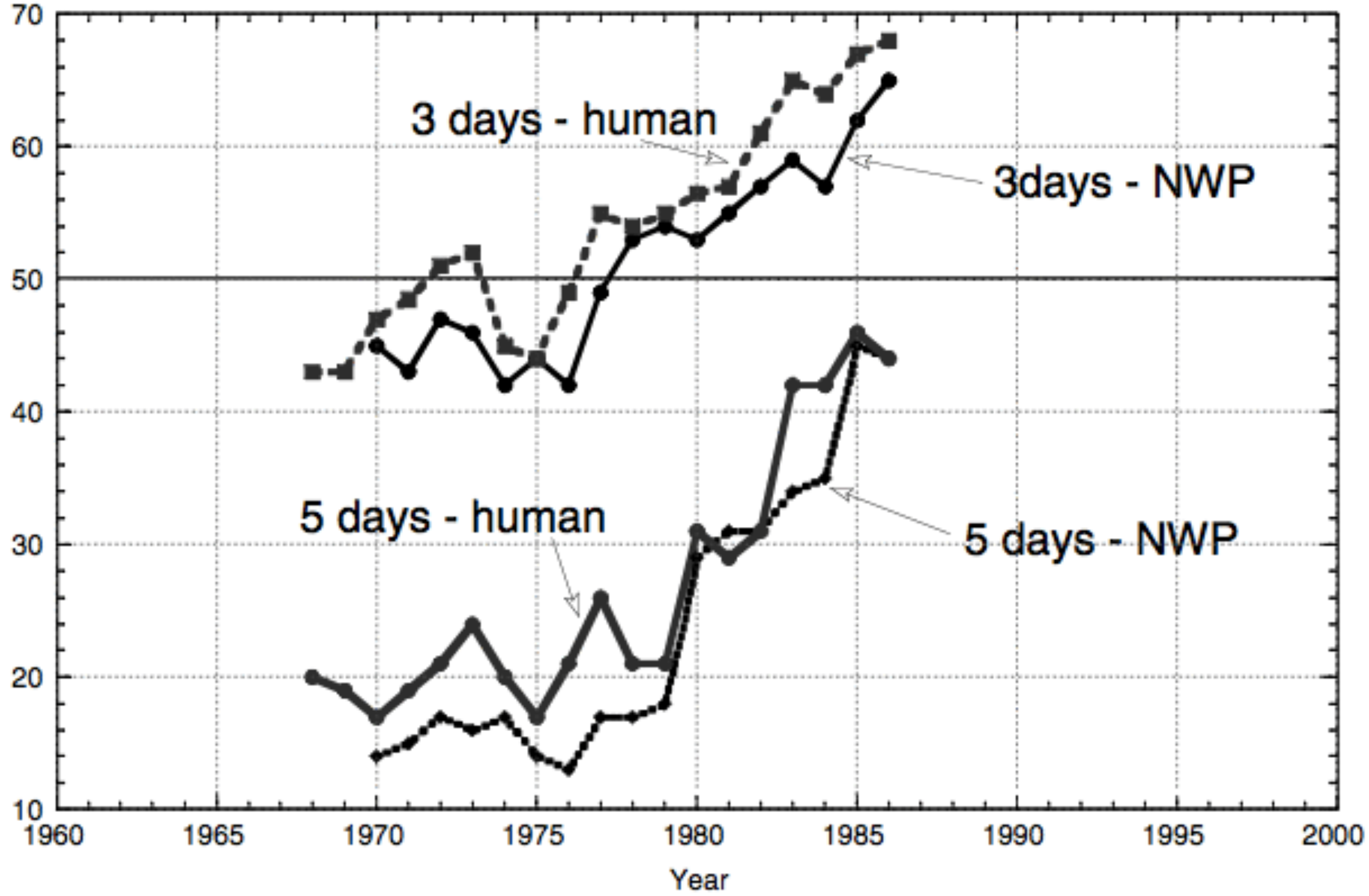


Figure 1.5.2: Hughes data: comparison of the forecast skill in the medium-range from NWP guidance and from human forecasters.

EARTH'S CLIMATE HISTORY

Passing an uncertain future to coming generations © Greg Wray

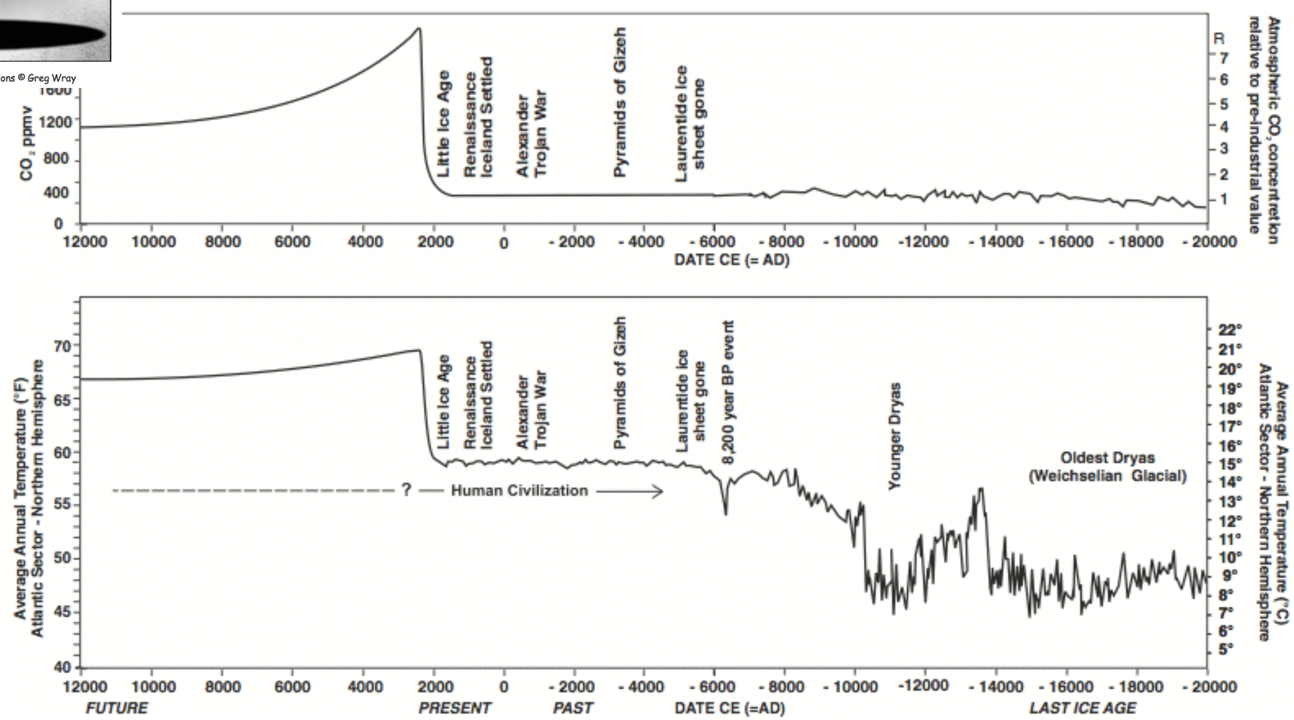


Fig. 28.10 Past and projected future atmospheric CO_2 levels and Earth surface temperatures. The temperature variation during the last glacial are based on the Greenland ice cores, and best reflect the Atlantic sector of the Northern Hemisphere

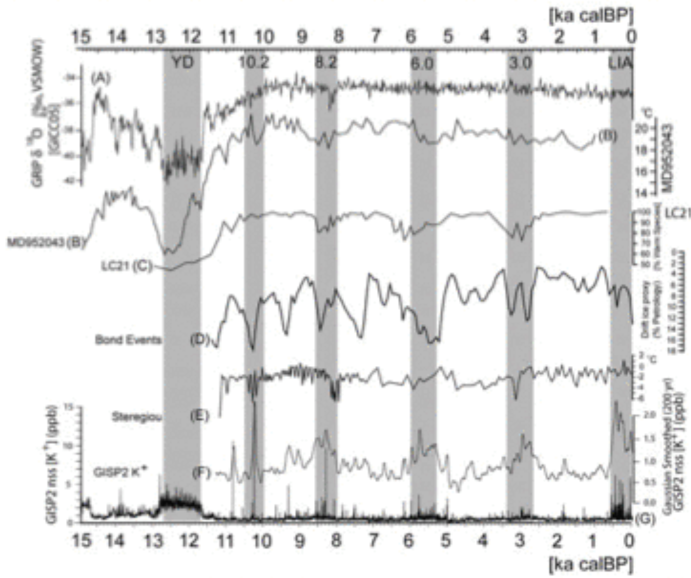


Fig. 1. Northern Hemisphere Palaeoclimate Records showing Holocene Rapid Climate Change (RCC) (site map cf. *Fig. 2), (A) Greenland GISP2 ice-core $\delta^{18}O$ (*Grootes et al. 1993*), (B) Western Mediterranean (Iberian Margin) core MD95–2043, sea surface temperature (SST) C37 alkenones (*Cacho et al. 2001; Fletcher et al. 2008*), (C) Eastern Mediterranean core LC21 (SST) fauna (*Rohling et al. 2002*), (D) North Atlantic Bond-Events, stacked petrologic tracers of drift ice from cores MC52–V29191+MC21–GGC22 (*Bond et al. 2001*), (E) Romania (Steregoiu), Mean Annual Temperature of the Coldest Month (MTC, °C) (*Feurdean et al. 2008*), (F) Gaussian smoothed (200 yr) GISP2 potassium (non-sea salt [K⁺]; ppb) ion proxy for the Siberian High (*Mayewski et al. 1997; Meeker and Mayewski 2002*), (G) High-Resolution GISP2 potassium (non-sea salt [K⁺]; ppb) ion proxy for the Siberian High (*Mayewski et al. 1997; Meeker and Mayewski 2002*).*

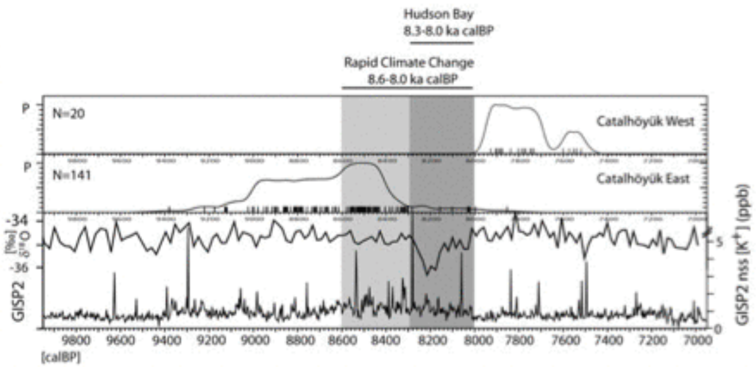


*Fig. 18. Radiocarbon Dates from Çatalhöyük (Central Anatolia) in comparison to selected climate records. Upper: ¹⁴C-Data from Çatalhöyük West (N = 20), ¹⁴C-Data from Çatalhöyük East (N = 141) (*cf. Appendix I, Radiocarbon Database*). Lower: Greenland GISP2 ice-core $\delta^{18}O$ (*Grootes et al. 1993*); GISP2 potassium (non-sea salt [K⁺]; ppb) ion proxy for the Siberian High (*Mayewski et al. 1997; Meeker and Mayewski 2002*).*

RAPID CLIMATE CHANGE (RCC) IN THE EASTERN MEDITERRANEAN (Weninger et al., 2009)

ÇATALHÖYÜK

RAPID CLIMATE CHANGE (RCC) IN THE EASTERN MEDITERRANEAN (Weninger et al., 2009)

BOSPHORUS FREEZING EVENTS

LITTLE ICE AGE
15^{th} – 16^{th} centuries AD

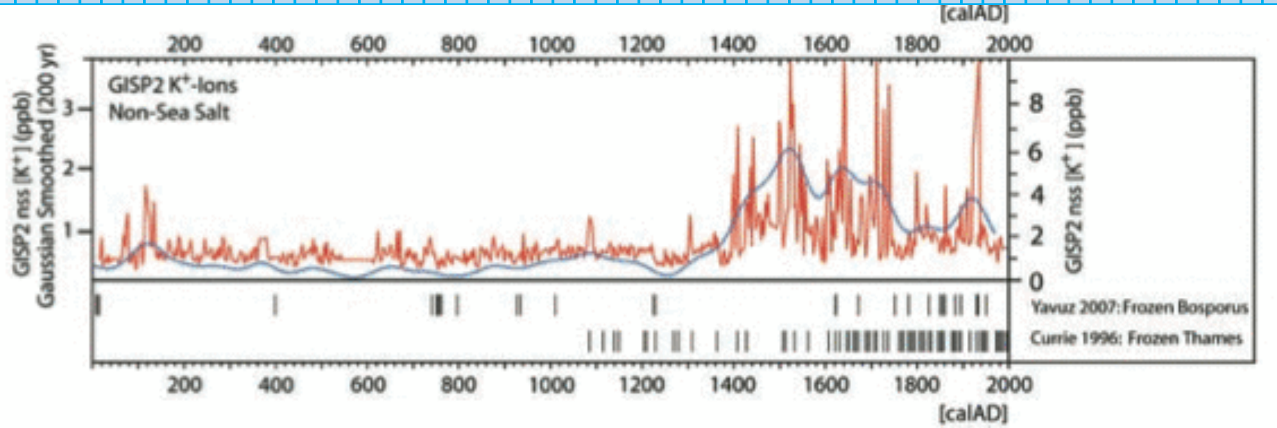
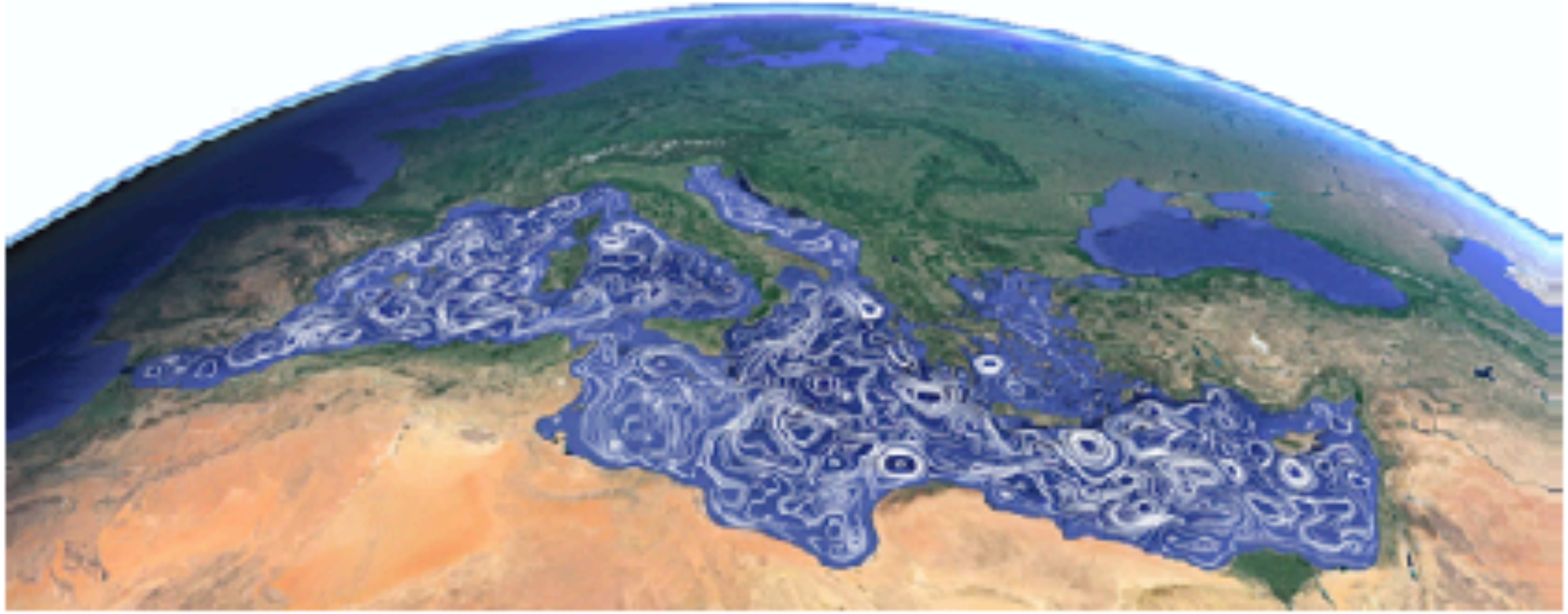


*Fig. 3. Freezing Events during the last 2000 years in the Bosporus, the southern Black Sea and Marmara region, derived from historical documents (*Yavuz et al. 2007*), compared to the GISP2 nss [K^+] ion record (*Mayewski et al. 1997; 2004*). Also shown is the historical record (1000–2000 AD) of the Thames freezing (*Currie et al. 1996*).*



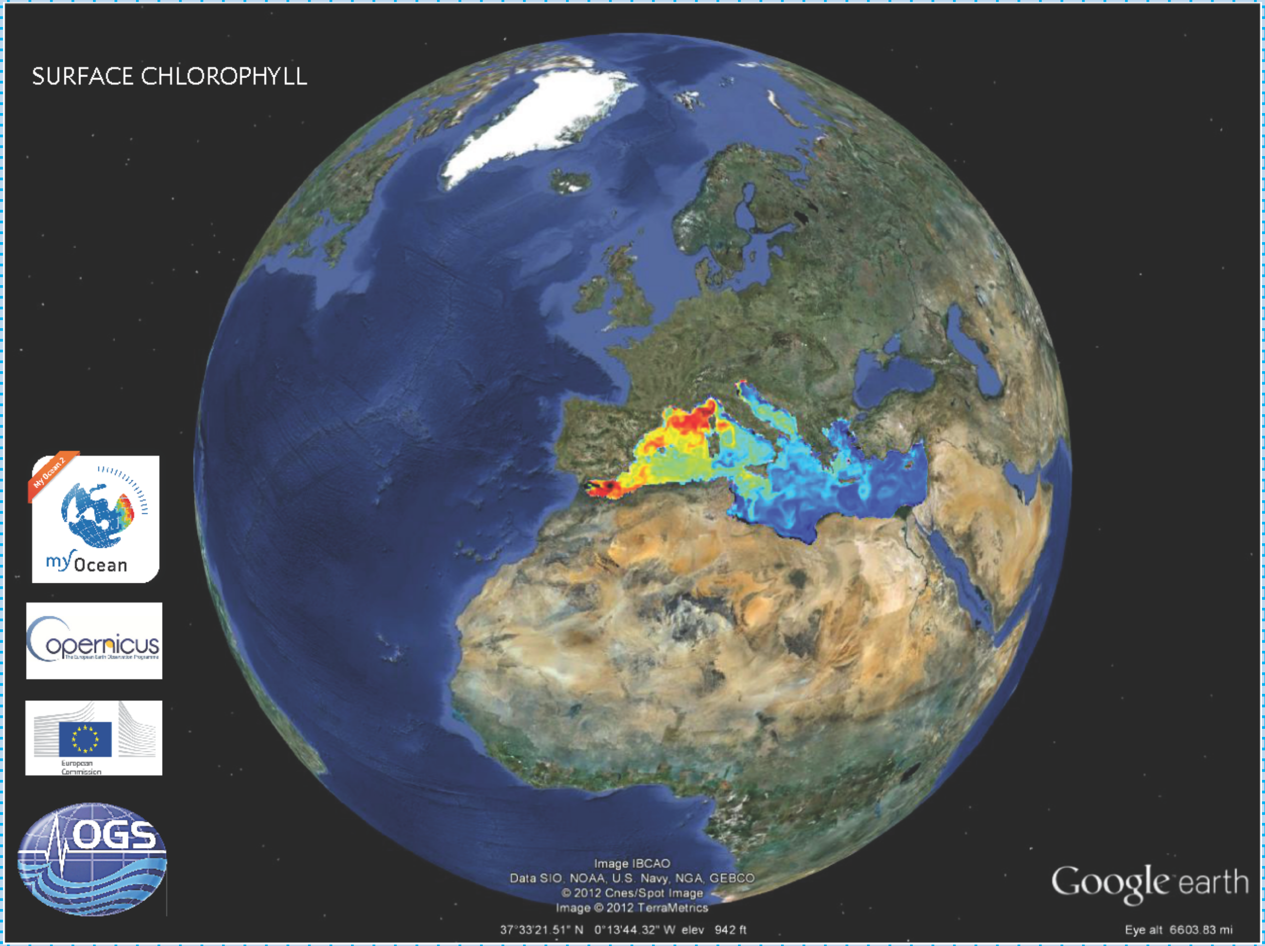

SURFACE CHLOROPHYLL
myOcean
Copernicus
European Commission
OGS
Image IBCAO
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
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Image © 2012 TerraMetrics
37°33'21.51" N 0°13'44.32" W elev 942 ft
Google earth
Eye alt 6603.83 mi

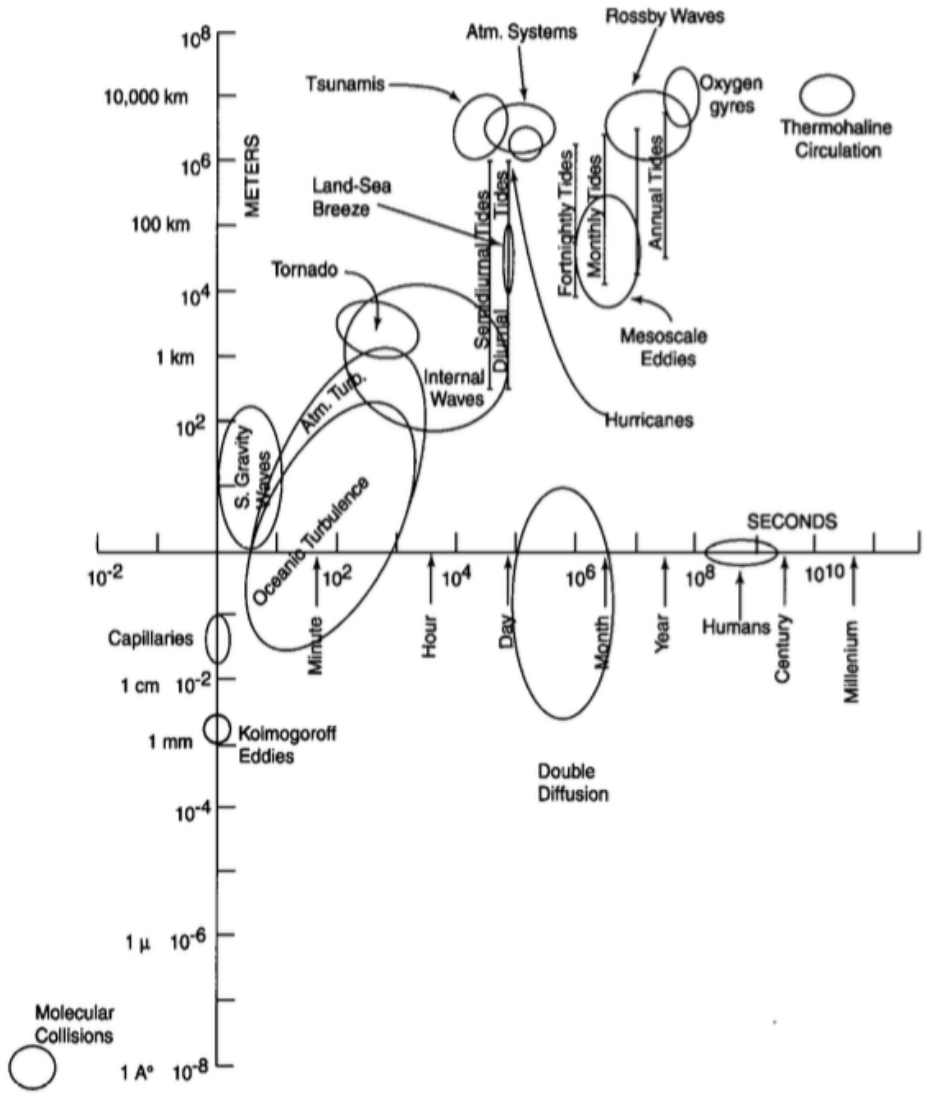

Figure 1.5.1 The range of spatial and temporal scales of motions in the atmosphere and the oceans. The motions span over a 10-decade range in both space and time.

Table 1.5.1 Time and Spatial Scales of Various Oceanic and Atmospheric Phenomena

Process	Length scale	Timescale	Rossby number
Dissipative scales	1–2 mm	~1 s	$\sim 10^4$
Vertical mixing (ocean)	1–100 m	several minutes	$\sim 10^2$
Vertical mixing (atm)	1–1000 m	several minutes	$\sim 10^2$–10^3
Surface waves	1–100	several seconds	~ 10–10^2
Internal waves	1–10 km	mins–hrs	$\sim 10^{-1}$–1
Double diffusion	1 cm–10 m	days–weeks	~10
Coastal upwelling	10–20 km	days	$\sim 10^{-1}$–1
Mesoscale eddies	10–400 km	weeks–months	$\sim 10^{-1}$
Atm. weather patterns	100–5000 km	days–weeks	$\sim 10^{-1}$–1
Ocean fronts	5–50 km	weeks	$\sim 10^{-1}$
Boundary currents	50–100 km	months	$\sim 10^{-1}$
Basin gyres	2000–15000 km	years	$\sim 10^{-2}$
Ocean tides	100–1000 km	1/2, 1 day, ...	$\sim 10^{-4}$–10^{-2}
Tornadoes	few km	<1 hr	$\sim 10^3$
Hurricanes	500–2000 km	days	~10
Rossby waves	1000 km	months–years	$\sim 10^{-3}$
Tsunamis	100 km	day	$\sim 10^2$
Land–sea breeze	10–50 km	1/2 day	~1

Note that the mean free path from kinetic theory gives a length scale of about $\sim 10^{-7}$ m at sea level for the atmosphere, and the corresponding timescale associated with molecular collisions is $\sim 10^{-4}$ s.

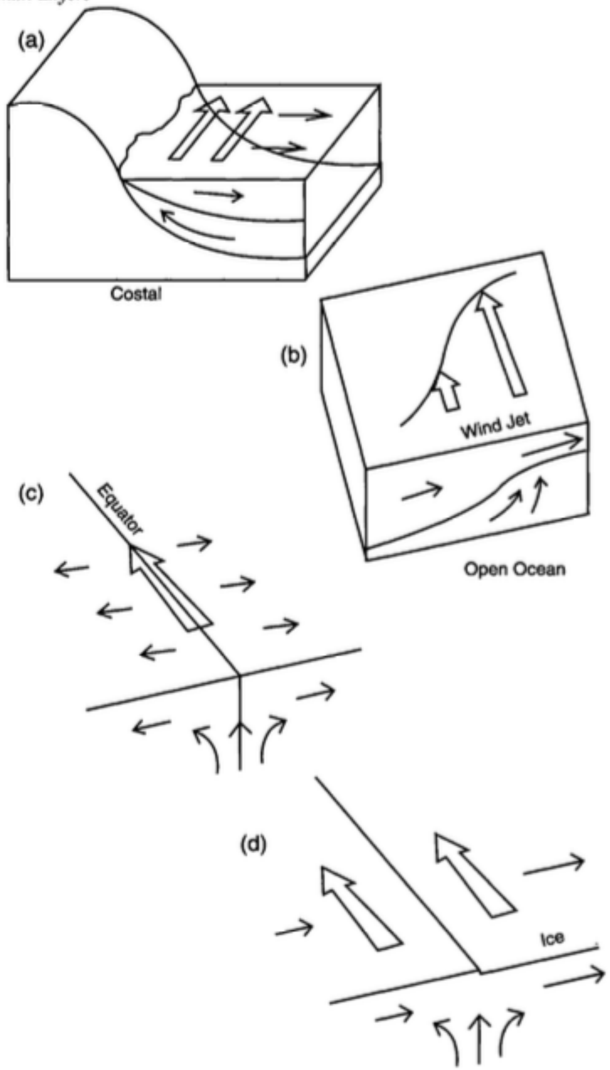

Figure 1.8.2 Upwelling situations in the ocean: (a) coastal upwelling induced by a wind stress parallel to the coast with the coast to its left (in the northern hemisphere), (b) upwelling induced by the curl of the wind stress in the open ocean (for example, off the Arabian coast during summer monsoons), (c) equatorial upwelling due to easterlies, and (d) upwelling along an ice edge (in the northern hemisphere).

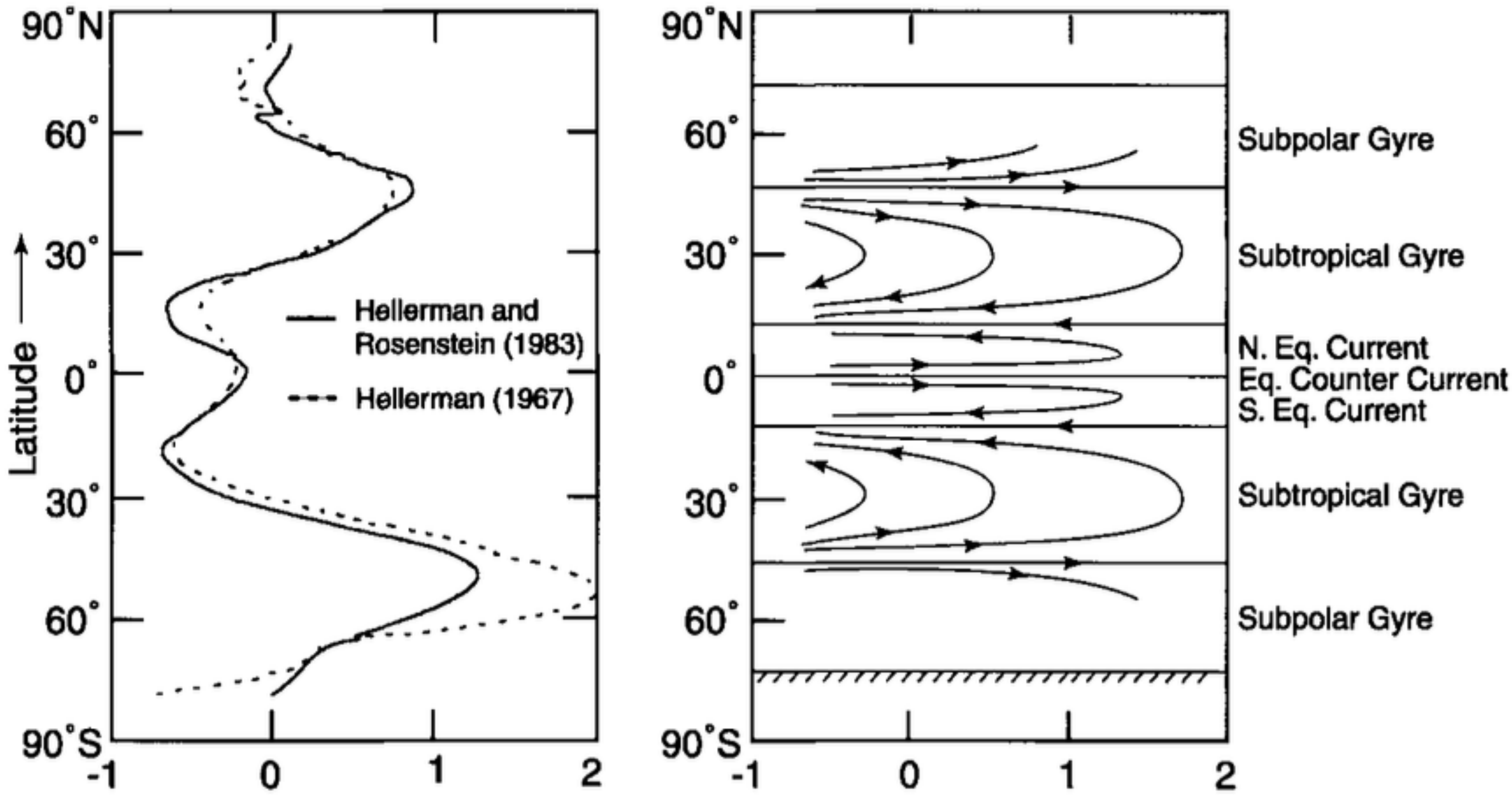

Figure 1.9.2 The meridional distribution of zonal long term mean winds (left) and the corresponding Sverdrup circulation (right). From Hellerman and Rosenstein (1983) and adapted from Pedlosky (1966).

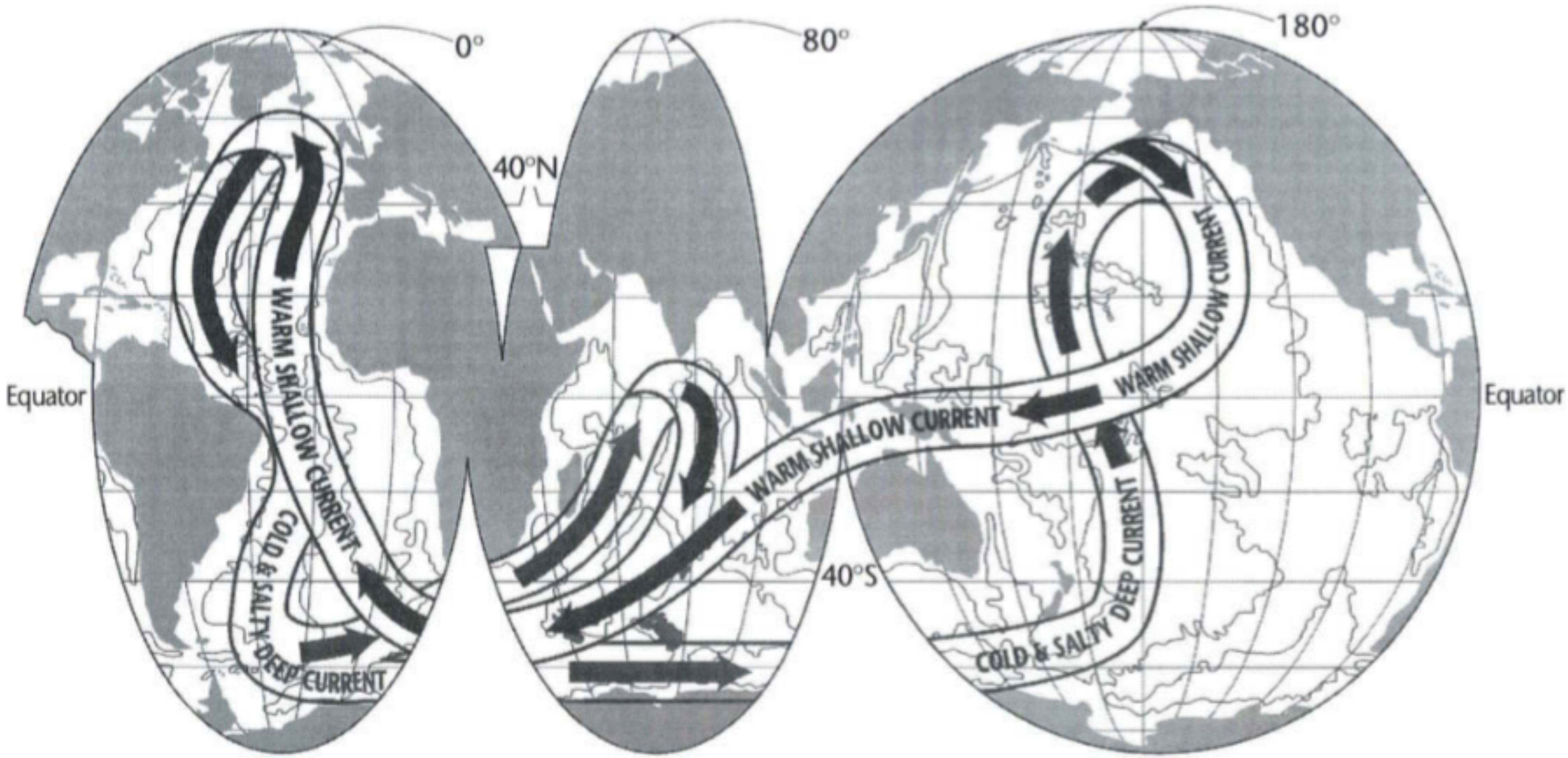

Figure 1.16.1 Meridional thermohaline circulation (the conveyor belt) in the global oceans, the two-layer version suggested by Broecker. From Schmitz (1995).

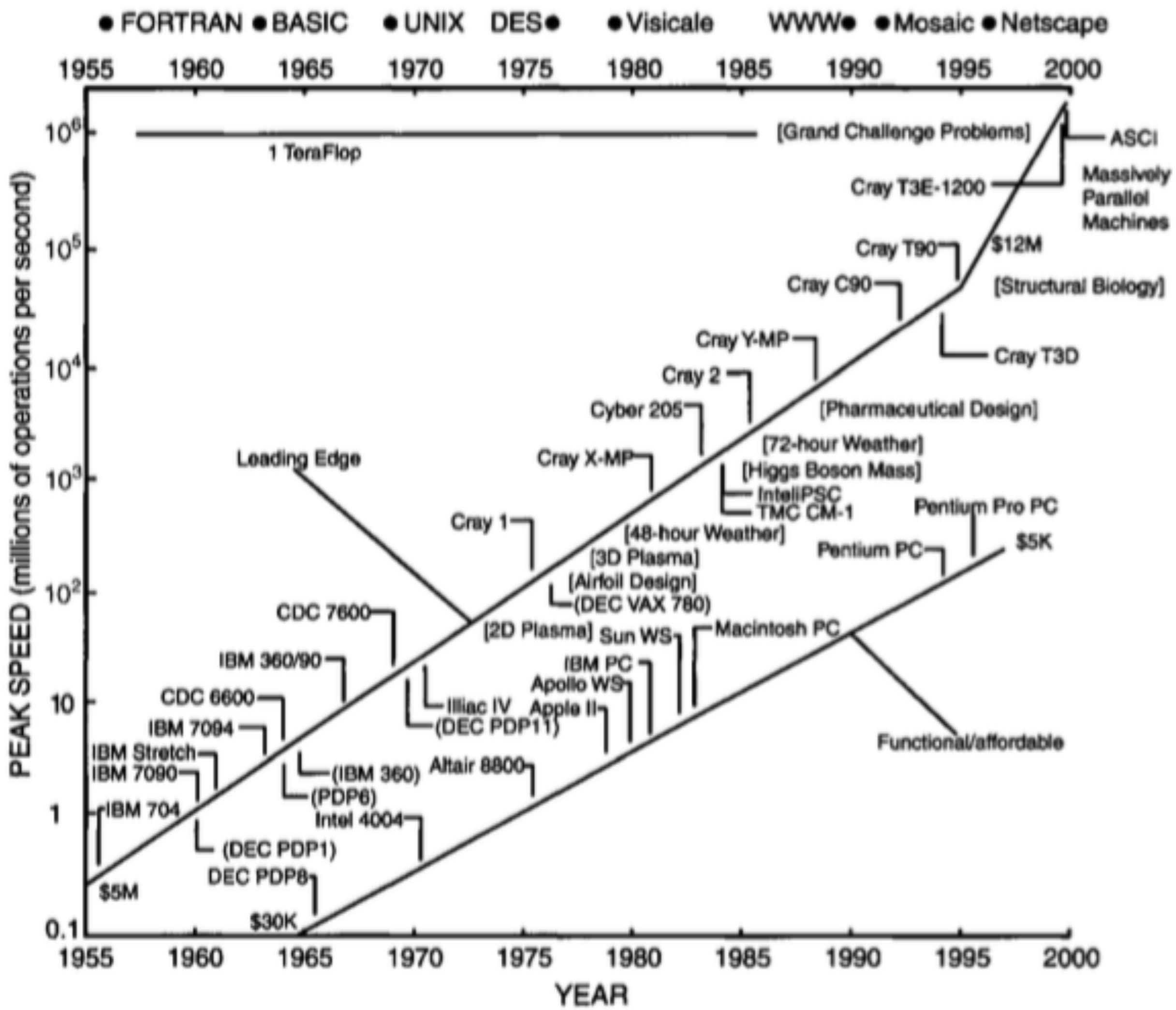

Figure 2.1.1 Evolution of computing since 1955. Both mainframes and workstations are shown. Adapted from Brenner (1996).

SEA STRAITS

BOSPHORUS AND DARDANELLES STRAITS IN PARTICULAR

Sea straits constitute complex, high-energy physical environments with rapid currents, hydraulic transitions, stratification and turbulence, and are controlled strongly by geometric constraints, often creating complex multi-scale interactions influencing the states of the interconnected basins. A wide range of coupled motions at scales extending from short-term to climatic co-exist and interact in sea straits and their adjacent basins.