# Lawrence's Laking List of Labeled Luxurious Equations[1]. [MK III]

### $3DCalculus$

$F(r) = -\nabla V(r)$
Force is MINUS gradient of potential
(Equilibrium) Stationary Point:
$f_x(a,b) = f_y(a,b) = 0$
(Stable) Minimum:
$f_{xx} > 0, f_{yy} > 0, f_{xx}f_{yy} > (f_{xy})^2$
(Unstable) Maximum:
$f_{xx} < 0, f_{yy} < 0, f_{xx}f_{yy} > (f_{xy})^2$
(Unstable) Saddle:
$f_{xx}f_{yy} < (f_{xy})^2$
Chain Rule:
$\frac{d}{dt}[f(a,b,c...)] = \frac{\delta f}{\delta a}\frac{da}{dt} + \frac{\delta f}{\delta b}\frac{db}{dt} + \frac{\delta f}{\delta c}\frac{dc}{dt}...$
Conservative Feild:
if( $\frac{\delta F_x}{\delta y} == \frac{\delta F_y}{\delta x}$ ){  }
$dxdy = \mid \frac{\delta(x,y)}{\delta(u,v)} \mid dudv$
$\frac{\delta(x,y)}{\delta(u,v)} = \frac{\delta x}{\delta u}\frac{\delta y}{\delta v} - \frac{\delta x}{\delta v}\frac{\delta y}{\delta u}$
Greens Theorem:
$\int_c Pdx + Qdy = \iint_s (\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y})dxdy$

### $LogRules$

$\log_b M * N = \log_b M + \log_b N$
$\log_b \frac{M}{N} = \log_b M - \log_b N$
$\log_b M^K = K \log_b M$
$\log_b 1 = 0$
$\log_b b = 1$
$b^{\log_b K} = K$
$\log_a b \frac{\log_c b}{\log_c a}$
$\frac{1}{\log_a b} = \log_b a$

### $WaveEquation$

$u_{tt} = c^2 u_{xx}$

### $FourierSeries$

$f(x) = \frac{1}{2}a_0 \sum_{n=1}^{\infty}[a_n \cos(\frac{2nx\pi}{a}) + b_n \sin(\frac{2nx\pi}{a})]$
$a_n = \frac{2}{a}\int_0^a f(x)\cos(\frac{2nx\pi}{a})dx$
$b_n = \frac{2}{a}\int_0^a f(x)\sin(\frac{2nx\pi}{a})dx$

### $FourierHalf(Sin)Series$

$f(x) = \sum_{n=1}^{\infty} \sin(\frac{nx\pi}{a})$
$c_n = \frac{2}{a}\int_0^a f(x)\sin(\frac{nx\pi}{a})dx$

### $HeatEquation$

$\frac{\delta U}{\delta t} = K\frac{\delta^2 U}{\delta x^2}$ Were U is the temperature T
$U(x,t) = \sum_{n=1}^{\infty} c_n e{-}K(\frac{n\pi}{a})^2 t \sin(\frac{nx\pi}{a})$
$c_n = \frac{2}{a}\int_0^a f(x)\sin(\frac{nx\pi}{a})dx$

### $CalculusEquations$

$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
$\frac{d}{dx}[u^n(x)] = n \times u^{n-1}(x) \times \frac{d}{dx}[u(x)]$
$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
$f_{xy} = f_{yx}$

### $GettinTriggyWithIt$

$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$

### $2DJacobian$

$\mid \frac{\delta(x,y)}{\delta(u,v)} \mid = \mid \begin{matrix} \frac{\delta x}{\delta u} & \frac{\delta y}{\delta u} \\ \frac{\delta x}{\delta v} & \frac{\delta y}{\delta v} \end{matrix} \mid = \mid \frac{\delta x}{\delta u}\frac{\delta y}{\delta v} - \frac{\delta x}{\delta v}\frac{\delta y}{\delta u} \mid$
Polar Jacobian = r

### $Vectors$

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$
$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \bullet \vec{c}) - \vec{c}(\vec{a} \bullet \vec{b})$
$\mid \vec{v} \mid = \mid \begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid = \sqrt[2]{a^2 + b^2 + c^2}$
$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \mid \begin{bmatrix} i & j & k \\ a & b & c \\ d & e & f \end{bmatrix} \mid = i \mid \begin{bmatrix} b & c \\ e & f \end{bmatrix} \mid -j \mid \begin{bmatrix} a & c \\ d & f \end{bmatrix} \mid +k \mid \begin{bmatrix} a & b \\ d & e \end{bmatrix} \mid$
$\mid \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid = ad - bc$
$\frac{d}{dx}(\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}) = \begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix}$
$\nabla f = \begin{pmatrix} \frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y} \\ \frac{\delta f}{\delta z} \end{pmatrix}$
$D_{\vec{n}} f = \vec{n} \bullet \nabla f$

[1] Made in LaTeX

## $MatrixAndMatricies$

Fig 0. There is no spoon

An (n× k) matrix has n rows ans k columns.
Matrix (a× b) can be multiplied by (c× d) if b= c and becomes (a×d).
Anticlockwise Rotation through $\theta$ about O: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
Reflection in the line $y = (\tan(\theta))x$ : $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$
Transverse of a Matrix:
$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$
Determinant Facts:

- if two rows or columns are swapped det() docent change
- if a row or column is multiplied by k det() scales by a factor of k
- adding a multiple of one row (or collumn) to another docent change the det().
- det(A) = det(A$^T$)
- det(AB) = det(A) × det(B)

I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$...
A A$^{-1}$ = I
A matrix can transform co-ordinates: $\begin{pmatrix} x \\ y \end{pmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$ When used to transform an area: $det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \pm \frac{AreaOfS'}{AreaOfS}$

## $Eigen - ValuesandVectors$

Eigenvalues:
det $\begin{bmatrix} a-\lambda & b & c \\ d & e-\lambda & f \\ g & h & i-\lambda \end{bmatrix} = 0$ were $\lambda$ are the eigenvalues, for an (n × n) matrix there are n eigenvectors.
The Eigenvalues add up to "the sum of the leading diagonal" of the matrix ($\lambda_1 + \lambda_2 + \lambda_3$ = a + e + i).
The product of the Eigenvalues is the determinant of the matrix so if the determinant is 0 at least one eigenvector is 0, and vice versa.

Eigenvectors:
$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ where $\lambda$ are the already calculated eigenvalues and $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ is the associated eigenvector (one Eigenvalue for each Eigenvector)

## $SimplifiedTrig$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
$\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$
$\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$

## $Motivation$

f(x) = | x |
Stay Positive!

## $HandyDifferentials$

| f(x) | f′(x) |
|---|---|
| tan(x) | sec$^2$(x) |
| sec(x) | sec$(x)\tan(x)$ |
| cot(x) | -csc$^2$(x) |
| csc(x) | $-\csc(x)\cot(x)$ |
| ln sec(x) | $\tan(x)$ |
| ln sin(x) | $\cot(x)$ |
| tanh(x) | sech$^2(x)$ |
| arcsin(x) | $\frac{1}{\sqrt{1-x^2}}$ |
| arccos(x) | $-\frac{1}{\sqrt{1-x^2}}$ |
| arctan(x) | $\frac{1}{1+x^2}$ |
| arcsinh(x) | $\frac{1}{\sqrt{1+x^2}}$ |
| arcCosh(x) | $\frac{1}{\sqrt{x^2-1}}$ |
| arctanh(x) | $\frac{1}{1-x^2}$ |
| $-\frac{\cos^{n+1}(x)}{n+1}$ | $\sin(x)\cos^n(x)$ |
| $\frac{\sin^{n+1}(x)}{n+1}$ | $\sin^n(x)\cos(x)$ |

## $SpaceToDrawPumbaAfterFinishingTheExam$