Lecture 5. Data types and type classes

Functional Programming 2017/18

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Goals

- Define your own data types
  - Simple, parametric and recursive
- Define your own type classes and instances

Chapter 8 (until 8.6) from Hutton's book

In the previous lectures...

... we have only used built-in types!

- Basic data types
  - `Int`, `Bool`, `Char`...
- Compound types parametrized by others
  - Some with a definite amount of elements, like tuples
  - Some with an unbound number of them, like lists

It's about time to define our own!

Direction

```
data Direction = North
               | South
               | East
               | West
```

- `data` declares a new **data type**
- The name of the type must start with **U**ppercase
- Then we have a number of *constructors* separated by |
  - Each of them also starting by uppercase
  - The same constructor cannot be used for different types
- Such a simple data type is called an *enumeration*

Building a list of directions

Each constructor defines a *value* of the data type

```
> :t North
North :: Direction
```

You can use `Direction` in the same way as `Bool` or `Int`

```
> :t [North, West]
[North, West] :: [Direction]
> :t (North, True)
(North, True) :: (Direction, Bool)
```

Pattern matching over directions

To define a function, you proceed as usual:

1. Define the type

   ```
   directionName :: Direction -> String
   ```

2. Enumerate the cases
   - The cases are each of the constructors

   ```
   directionName North = _
   directionName South = _
   directionName East  = _
   directionName West  = _
   ```

Pattern matching over directions

3. Define each of the cases

```
directionName North = "N"
directionName South = "S"
directionName East  = "E"
directionName West  = "W"
```

```
> map directionName [North, West]
["N","W"]
```

Built-in types are just data types

- `Bool` is a simple enumeration

```
data Bool = False | True
```

- `Int` and `Char` can be thought as very long enumerations

```
data Int  = ... | -1 | 0 | 1 | 2 | ...
data Char = ... | 'A' | 'B' | ...
```

  - The compiler treats these in a special way

Points

Data types may store information within them

```
data Point = Point Float Float
```

- The name of the constructor is followed by the list of types of each argument
- Constructor and type names may overlap

Using points

- To create a point, we use the name of the constructor followed by the value of each argument

```
> :t Point 2.0 3.0
Point 2.0 3.0 :: Point
```

- To pattern match, we use the name of the constructor and further matchs over the arguments

```
norm :: Point -> Float
norm (Point x y) = sqrt (x*x + y*y)
```

  - Do not forget the parentheses!

```
> norm Point x y = x * x + y * y
<interactive>:2:6: error:
• The constructor ‘Point’ should have 2 arguments,
  but has been given none
```

Constructors are functions

Each constructor in a data type is a function which build a value of that type given enough arguments

```
> :t North
North :: Direction  -- No arguments
> :t Point
Point :: Float -> Float -> Point  -- 2 arguments
```

They can be arguments or results of higher-order functions

```
zipPoint :: [Float] -> [Float] -> [Point]
zipPoint xs ys = map Point (zip xs ys)
```

Shapes

A data type may have zero or more *constructors*, each of them holding zero or more *arguments*

```
data Shape = Rectangle Point Float Float
           | Circle    Point Float
           | Triangle  Point Point Point
```

We call these **algebraic data types**, or **ADTs**

Pattern matching over shapes

Each case starts with a constructor – in uppercase – and matches the arguments

```
area :: Shape -> Float
area (Rectangle _ w h) = w * h
area (Circle    _ r)   = pi * r * r
area (Triangle  x y z) = sqrt (s*(s-a)*(s-b)*(s-c))
                         -- Heron's formula
  where a = distance x y
        b = distance y z
        c = distance x z
        s = (a + b + c) / 2

distance (Point u1 u2) (Point v1 v2)
  = sqrt ((u1-v1)*(u1-v1)+(u2-v2)*(u2-v2))
```

ADTs versus object-oriented classes

```
abstract class Shape {
   abstract float area();
}
class Rectangle : Shape {
  public Point corner;
  public float width, height;
  public float area() { return width * height; }
}
// More for Circle and Triangle
```

- There is no *inheritance* involved in ADTs
- Constructors in an ADT are *closed*, but you can always add *new subclasses* in a OO setting
- Classes bundle *methods*, functions for ADTs are defined *outside* the data type

Lists and trees of numbers

Data types may refer to themselves

- They are called **recursive** data types

```
data ListOfNumbers
  = EmptyList | OneMore Int ListOfNumbers

data TreeOfNumbers
  = EmptyTree | Node Int TreeOfNumbers TreeOfNumbers
```

Cooking `elemList`

1. Define the type

```
elemList :: Int -> ListOfNumbers -> Bool
```

2. Enumerate the cases
   - One equation per constructor

```
elemList x EmptyList      = _
elemList x (OneMore y ys) = _
```

3. Define the cases

```
elemList x EmptyList = False
elemList x (OneMore y ys)
  | x == y    = True
  | otherwise = elemList x ys
```

Cooking `elemTree`

1. Define the type

   ```
   elemTree :: Int -> TreeOfNumbers -> Bool
   ```

2. Enumerate the cases

   - Each constructor needs to come with as many variables as arguments in its definition

   ```
   elemList x EmptyTree       = _
   elemList x (Node y rs ls) = _
   ```

3. Define the simple (base) cases

   ```
   elemList x EmptyTree = False
   ```

Cooking `elemTree`

4. Define the other (recursive) cases
   - Each recursive appearance of the data type as an argument usually leads to a recursive call in the function

```
elemList x (Node y rs ls)
  | x == y     = True
  | otherwise = elemList x rs || elemList x ls

-- Or simpler
elemList x (Node y rs ls)
  = x == y || elemList x rs || elemList x ls
```

Cooking `treeToList`

1. Define the type

```
treeToList :: TreeOfNumbers -> ListOfNumbers
```

2. Enumerate the cases

```
treeToList EmptyTree        = _
treeToList (Node x ls rs) = _
```

3. Define the simple (base) cases

```
treeToList EmptyTree        = EmptyList
```

Cooking `treeToList`

4. Define the other (recursive) cases

```
treeToList (Node x ls rs)
  = OneMore x (concatList ls' rs')
    where ls' = treeToList ls
          rs' = treeToList rs

-- Left as an exercise to the audience
concatList :: ListOfNumbers -> ListOfNumbers
           -> ListOfNumbers
concatList xs = _
```

Polymorphic data types

We have seen examples of types which are parametric

- Lists like `[Int]`, `[Bool]`, `[TreeOfNumbers]`...
- Tuples `(A, B)`, `(A, B, C)` and so on

Functions over these data types can be polymorphic

- They work regardless of the parameter of the type

```
(++) :: [a] -> [a] -> [a]
zip  :: [a] -> [b] -> [(a, b)]
```

Optional values

`Maybe T` represents a value of type `T` which might be absent

```
data Maybe a = Nothing
             | Just a
```

- In the declaration of a polymorphic data type, the name `Maybe` is followed by one or more type variables
  - Type *variables* start with a lowercase letter
- The constructors may refer to the type variables in their arguments
  - In this case, `Just` holds a value of type `a`

Optional values

```
> :t Just True
Maybe Bool
> :t Nothing
Maybe a
```

Note that `Nothing` has a polymorphic type, since there is no information to fix what `a` is

Cooking `find`

`find p xs` finds the first element in `xs` which satisfies `p`

- Such an element may not exist
  - Think of `find even [1,3]`, or `find even []`
- Other languages resort to `null` or magic `-1` values
- Haskell always marks a possible absence using `Maybe`

1. Define the type

```
find :: (a -> Bool) -> [a] -> Maybe a
```

2. Enumerate the cases

```
find p []     = _
find p (x:xs) = _
```

Cooking `find`

3. Define the simple (base) cases

```
find _ [] = Nothing
```

4. Define the other (recursive) cases

```
find p (x:xs) | p x       = Just x
              | otherwise = find p xs
```

elem in terms of find

Let me define a small utility function

```
isJust :: Maybe a -> Bool
isJust Nothing  = False
isJust (Just _) = True
```

Then we can define elem as a composition of other functions

```
elem :: Eq a => a -> [a] -> Bool
elem x = isJust . find (== x)
```

Trees for any type

We can generalize our `TreeOfNumbers` data type

- This is a polymorphic and recursive data type
- Mind the parentheses around the arguments

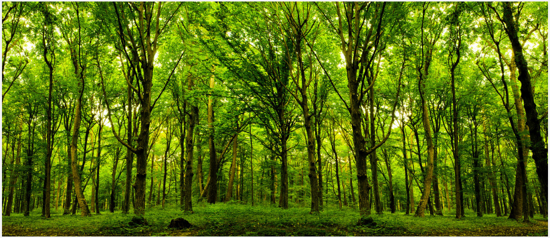
```
data Tree a = Leaf
            | Node a (Tree a) (Tree a)
```

More recipes with trees

Next lecture

Many more operations over trees!

- Including *search* trees



Nominal versus structural typing

```
data Point  = Point  Float Float
data Vector = Vector Float Float
```

- These types are *structurally* equal
  - They have the same number of constructors with the same number and type of arguments
- But for the Haskell compiler, they are unrelated
  - You cannot use one in place of the other
  - This is called *nominal* typing

```
> :t norm
norm :: Vector -> Float
> norm (Point 2.0 3.0)
Couldn't match ‘Vector’ with ‘Point’
```

Type classes

Oveloaded types

From previous lectures…

Some functions work uniformly for all types

```
reverse :: [a] -> [a]
```

But others require the type to satisfy a constraint

```
elem :: Eq a  => a -> [a] -> Bool
(+)  :: Num a => a ->  a  -> a
```

- `Eq` and `Num` are called **(type) classes**
- Each type which satisfies the constraint is an **instance**
  - `Int` is an instance of class `Eq`
- **Warning!** Terminology conflict with other languages

Class definition

```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
```

- The name of the type class starts with **U**ppercase
- We declare a type variable – `a` in this case – to stand for the overloaded type in the rest of the declaration
- Each type class defines one or more **methods** which must be implemented for each instance
  - We do *not* write the constraint in the methods

Missing instances

```
> Point 2.0 3.0 == Point 2.0 3.0
<interactive>:2:1: error:
    • No instance for (Eq Point)
      arising from a use of ‘==’
```

- You have to give the instance declaration for your own data types, even for built-in type classes
  - In some cases, the compiler can write them for you

Instance declarations

```
instance Eq Point where
  Point x y == Point u v = x == u && y == v
  Point x y /= Point u v = x /= u || y /= v
```

- Almost like the class declaration, except that
  - The type variable is substituted by a real type
  - Instead of method types, you give the implementation

```
> Point 2.0 3.0 == Point 2.0 3.0
True
```

Instance signatures

It is useful to write the specialized type for the instance in the declaration

```
instance Eq Point where
  (==) :: Point -> Point -> Bool
  Point x y == Point u v = x == u && y == v
  (/=) :: Point -> Point -> Bool
  Point x y /= Point u v = x /= u || y /= v
```

The Haskell standard does not allow this

- But you can do this if you write at the top of the file

  ```
  {-# language InstanceSigs #-}
  ```

Recursive instances

Type class instances for polymorphic types may depend on their parameters

- For example, equality of lists, tuples, and trees
- These requisites are listed in front of the declaration

```
instance Eq a => Eq [a] where
  []     == []     = True
  []     == _      = False
  _      == []     = False
  (x:xs) == (y:ys) = x == y && xs == ys

instance (Eq a, Eq b) => Eq (a, b) where
  (x, y) == (u, v) = x == u && y == v
```

Overlapping instances

Imagine that I want tuples of `Int`s to work slightly different

```
instance Eq (Int, Int) where
  (x, y) == (u, v) = x * v == y * u
```

You *cannot* do this! This instance **overlaps** with the other one given for generic tuples

Superclasses

A class might demand that other class is implemented

- We say that such a class has a **superclass**
- For example, any class with an ordering – `Ord` – has to implement equality – `Eq`

```
class Eq a => Ord a where
  (<), (>), (<=), (>=) :: a -> a -> Bool
  min, max             :: a -> a -> a

instance (Ord a, Ord b) => Ord (a, b) where
  (x, y) < (u, v) | x == u    = y < v
                  | otherwise = x < u
```

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The meanings of =>

- In a type, it constraints a polymorphic function

```
elem :: Eq a => a -> [a] -> Bool
```

- In a class declaration, it introduces a superclass

```
class Eq a => Ord a where ...
```

  - All instances of `Ord` must be instances of `Eq`

- In an instance declaration, it defines a requisite

```
instance Eq a => Eq [a] where ...
```

  - A list `[T]` supports equality only if `T` supports it

Before => you write an *assumption* or *precondition*

Default definitions

We could also write the following instance `Eq Point`

```
instance Eq Point where
  Point ... == Point ... = _   -- as before
  p /= q = not (p == q)
```

In fact, this definition of `(/=)` works for *any* type

- You can include a *default* definition in `Eq`
- If an instance does not have a explicit definition for that method, the default one is used

```
class Eq a where
  (==), (/=) :: a -> a -> Bool
  x /= y = not (x == y)
```

Default definitions

- You could have also defined `(/=)` *outside* of the class

```
(/=) :: Eq a => a -> a -> Bool
x /= y = not (x == y)
```

  - This definition cannot be overriden in each instance
- Why do we prefer `(/=)` to live in the class?
  - Performance! For some data types it is cheaper to check for disequality than for equality

Automatic derivation

- Writing equality checks is boring
  - Go around all constructors and arguments
- Writing order checks is even more boring
- Turning something into a string is also boring

Let the compiler work for you!

```
data Point = Point Float Float
             deriving (Eq, Ord, Show)
```

Historical note: many of the advances in automatic derivation of type classes where done here at UU

Define your own data types!

Data types in Haskell are simple and cheap to define

- Introduce one per concept in your program

```haskell
-- the following definition
data Status  = Stopped | Running
data Process = Process ... Status ...
-- is better than
data Process = Process ... Bool ...
-- what does 'True' represent here?
```

- Use type classes to share commonalities

Overloaded syntax

Numeric constants' weird type

What is going on?

```
> :t 3
3 :: Num t => t
```

Numeric constants can be turned into any `Num` type

```
> 3 :: Integer
3
> 3 :: Float
3.0
> 3 :: Rational   -- Type of fractions
3 % 1             -- Numerator % Denominator
```

Range syntax

The range syntax `[n .. m]` is a shorthand for

```
enumFromTo n m
```

`enumFromTo` lives in the class `Enum`, which can be automatically derived for enumerations

```
data Direction = North | South | East | West
                 deriving (Eq, Ord, Show, Enum)
```

```
> [South .. West]
[South, East, West]
```