

$$\text{sum} :: [\text{Int}] \rightarrow \text{Int}$$

$$\text{sum} [] = 0$$

$$\text{sum} (x:xs) = x + \text{sum} xs$$

$$\begin{aligned} &:: \text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow \text{Bool} \\ \text{elem} &:: \text{Int} \rightarrow [\text{Int}] \rightarrow \text{Bool} \end{aligned}$$

$$\text{elem } y [] = \text{False}$$

$$\text{elem } y (x:xs) = x == y \quad || \quad \text{elem } y xs$$

$$\text{reverse} :: [a] \rightarrow [a]$$

$$\text{reverse} [] = []$$

$$\text{reverse} (x:xs) = \text{reverse } xs \mathrel{++} [x]$$

1 $\textcircled{Q} + \checkmark$

$$\begin{aligned} \text{sum } [1, 2, 3, 4] &= 1 + \text{sum } [2, 3, 4] \\ &= 1 + 2 + \text{sum } [3, 4] \\ &= 1 + 2 + 3 + \text{sum } [4] \\ &= \\ &= a + \text{sum } xs \end{aligned}$$

WANT / INVARIANT
Goal

$$\boxed{\text{sum}' \text{ ACC } xs = \text{ACC} + \text{sum } x}$$

$$\text{SUM ACC } [] = \text{ACC}$$

$$\text{SUM ACC } (x:xs) = \text{SUM } (\text{ACC} + x) \text{ } xs$$

$$\text{SUM } xs = \text{SUM } 0 \text{ } xs$$

$$\text{REVERSE } [1,2,3] = \text{REVERSE } [2,3] ++ [1]$$

$$= \text{REVERSE } [3] ++ [2] ++ [1]$$

$$= \text{REVERSE } [] ++ [3] ++ [2] ++ [1]$$

INVARIANT

$$\text{REV ACC } xs = \text{REVERSE } xs ++ \text{ACC}$$

$$\text{REV ACC } [] = \text{ACC}$$

$$\text{REV ACC } (x:xs) = \text{REV } (\text{ACC} + x) \text{ } xs$$

$$\text{REVERSE } xs = \text{REV } [] \text{ } xs$$

$$\text{Fizz Buzz } (m, n) [] = ([], [], [])$$

$$\text{Fizz Buzz } (m, n) (x:xs) =$$

$$\text{CASE } (x \text{ 'mod' } m == 0$$

$$x \text{ 'mod' } n == 0$$

) or

$$(True, _) \rightarrow (x:ms, ns, os)$$

$$(False, True) \rightarrow (ms, x:ns, os)$$

$$(False, False) \rightarrow (ms, ns, x:os)$$

WHERE

$$(ms, ns, os) = \text{Fizz Buzz } (m, n) xs$$