

Final Exam Geometric Algorithms, April 17, 2014, 09.00–12.00

Read every question carefully, make sure you understand it, and be sure to answer the question. Read the question again after answering it, as a check whether you really answered the question. Answer the easier questions first, and then the harder ones. Answer questions in sufficient but not too much detail. You may not use the textbook during the exam, and you may not use or refer to any algorithms or results from the book.

Be sure to put your name on every piece of paper you hand in. Also write down your studentnummer. If you write readable, unambiguous, and technically correct, you get one point for free (in particular, do not write “line” if you mean “line segment” and do not write “Step 1 takes $n \log n$ time” when you mean “Step 1 takes $O(n \log n)$ time”). The other nine points can be earned by answering the questions correctly. Good luck!

1. (1 point) Consider the set of points $P = \{(0, 0), (1, 7), (2, 2), (3, 4), (6, 3)\}$. Let L be the set of lines corresponding to P in dual space. Explain your answers to the following questions:
 - (a) How many vertices does the Voronoi diagram of P have?
 - (b) How many bounded faces does the arrangement $\mathcal{A}(L)$ have?
2. (1 point) Let P be a set of n points in the plane.
 - (a) What is the worst-case running time for computing the convex hull of P ?
 - (b) What is the construction time for building a range-tree (without fractional cascading) T on P ?
 - (c) What is the storage requirement for T ?
 - (d) What is the query time for a range-reporting query on T , assuming there are k points in the requested range?
3. (1 point) In the *linear programming* problem, we are interested in computing the lowest point that lies in the common intersection of a set of halfplanes. Formulate the dual version of this problem.
4. (1 point) Let S be a planar subdivision in doubly-connected edge list representation (DCEL). Let s be a line segment that properly intersects exactly one given edge e of S (you are given a pointer to one of the half-edges of e), and does not touch any other part of S . Describe an algorithm to insert s into S , and analyze its running time.
5. (1 point) The *Art Gallery Theorem* states that for a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient to have every point of the polygon visible from at least one camera. Prove this theorem.
6. (2 points) Let P be a set of points in the plane. The *minimum spanning tree* (MST) of P is the shortest graph $G = (P, E)$ that connects all points of P , where the length of a graph is defined as the total length of all its edges.
 - (a) Argue that the edges in the minimum spanning tree are a subset of the edges of the Delaunay Triangulation of P .
 - (b) Sketch an algorithm to compute the MST, and analyze its running time.
7. (2 points) Let D be a collection of n disks in the plane. We are interested in the smallest disk that contains all disks of D . Describe a randomized incremental construction algorithm to compute this, and analyze its running time.