Implementing Geometric Algorithms

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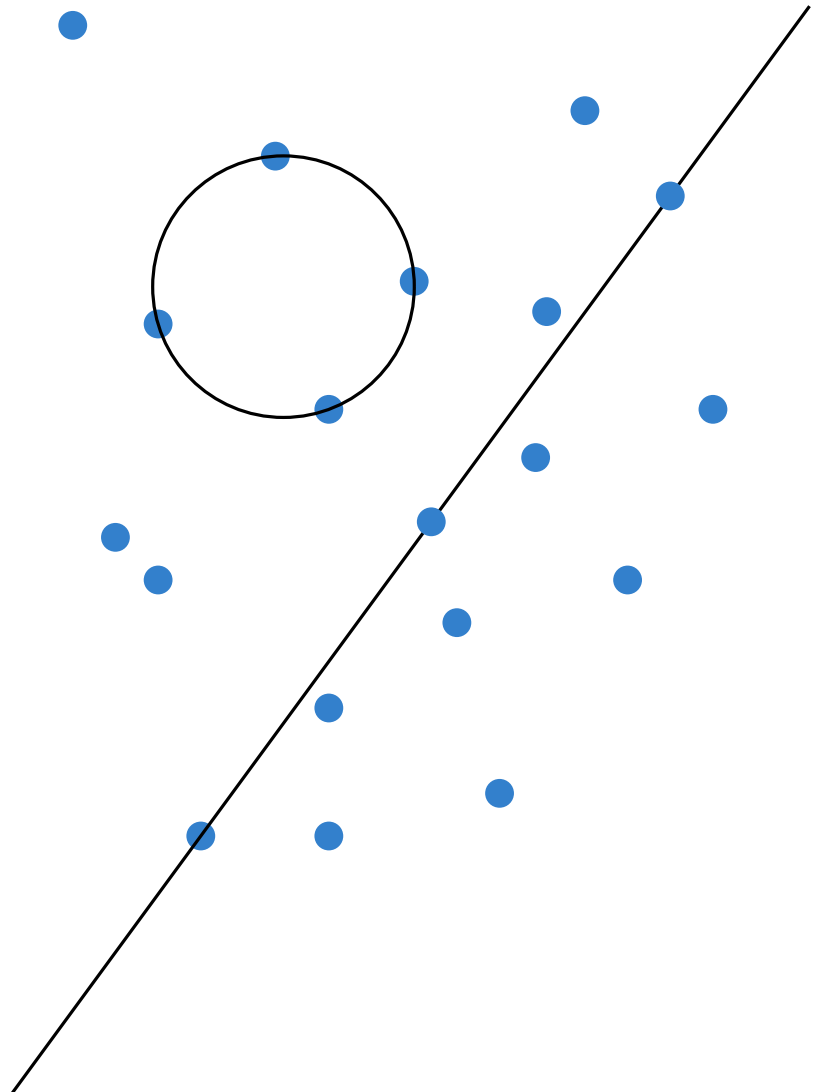
Difficulties:

- Dealing with Degenerate Input



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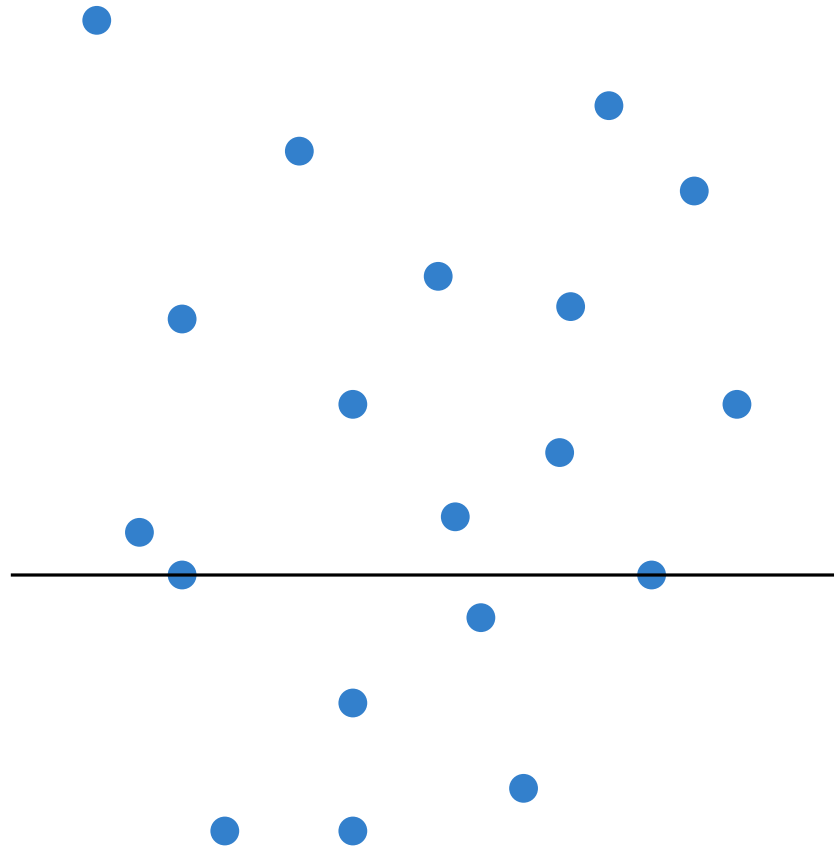

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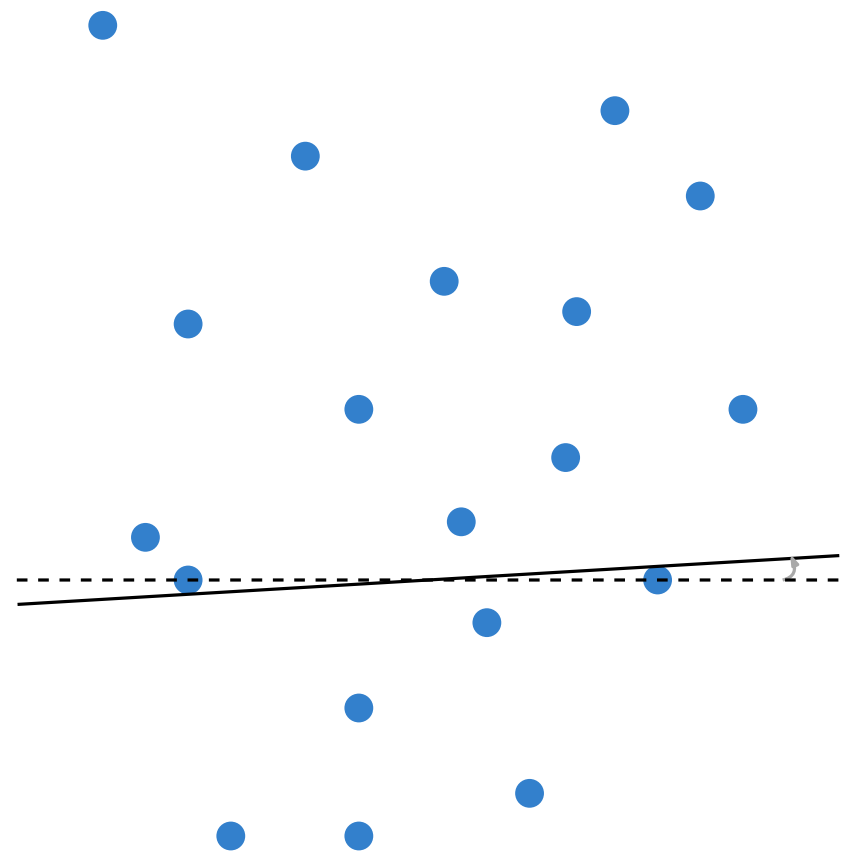

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Difficulties:

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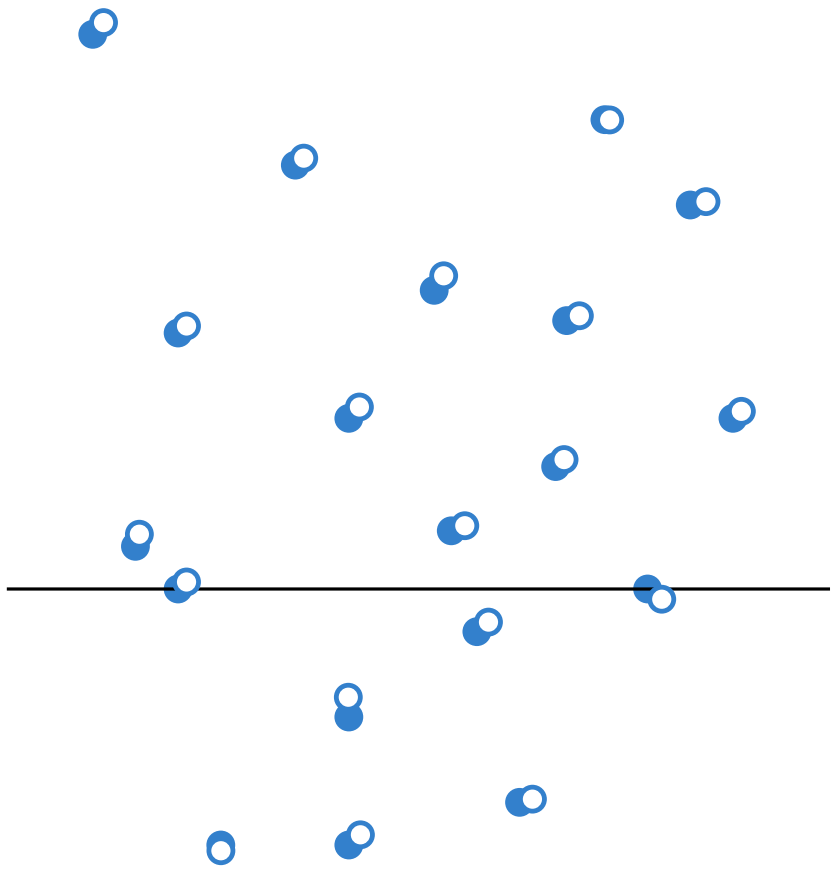
  - Symbolic pertubation



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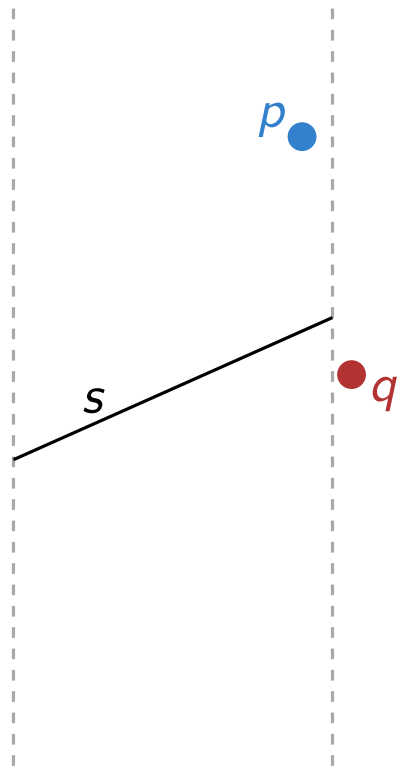

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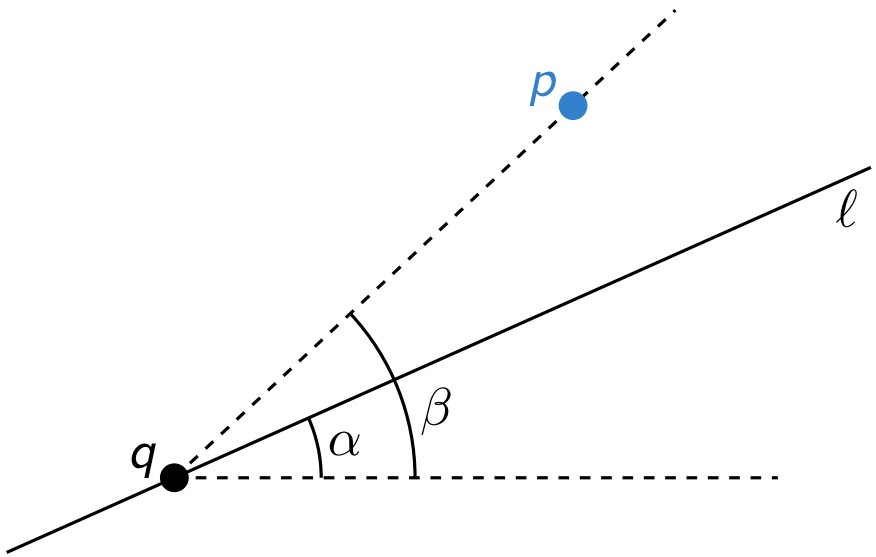
  - Use better predicates?

isLeftOfLine(p, ℓ) :



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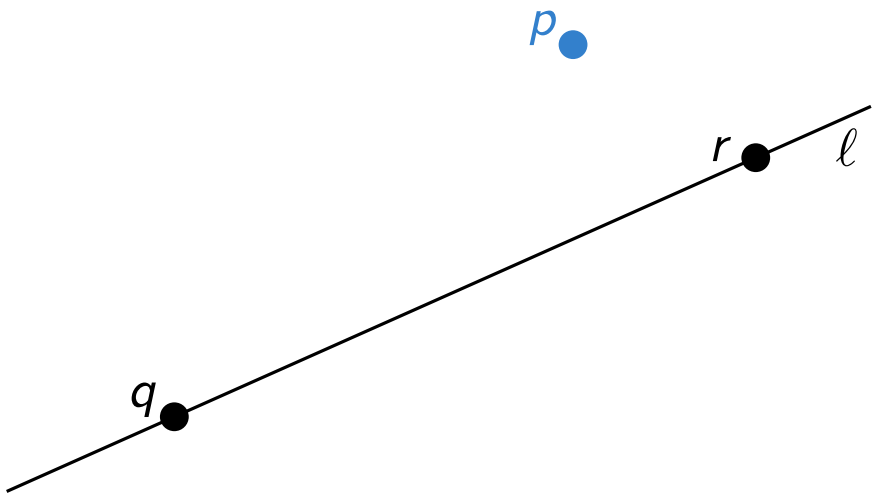

$$\begin{vmatrix} q_x & q_y & 1 \\ r_x & r_y & 1 \\ p_x & p_y & 1 \end{vmatrix} < 0$$

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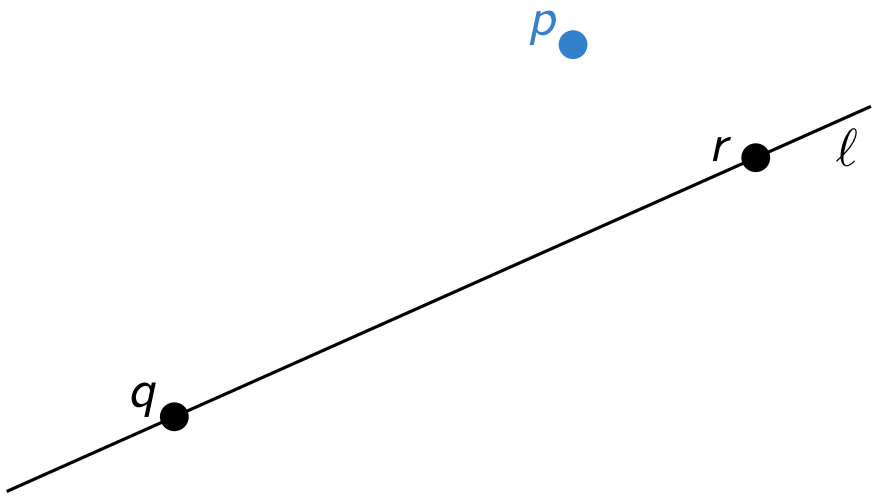

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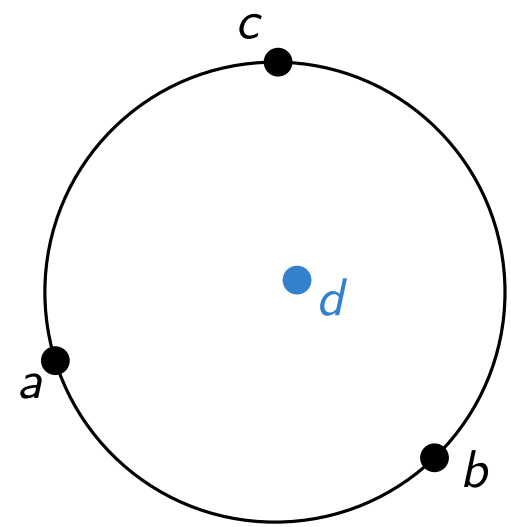

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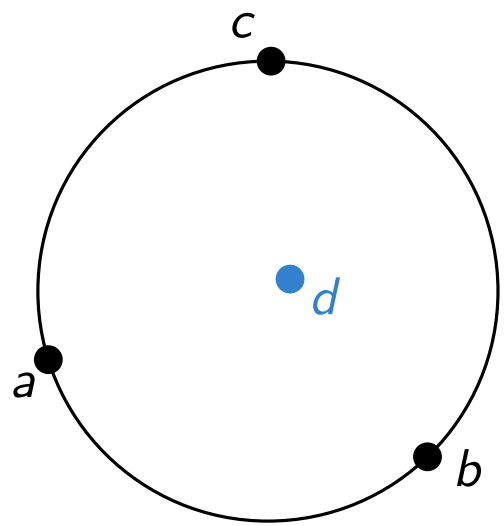

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  - Use fixed precision numbers
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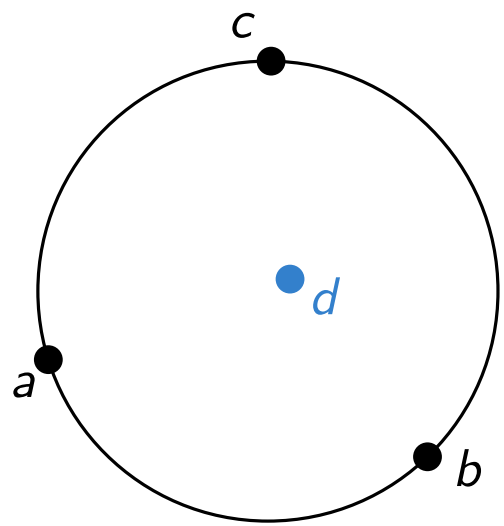

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$\Longrightarrow$ Use CGAL, LEDA, etc.

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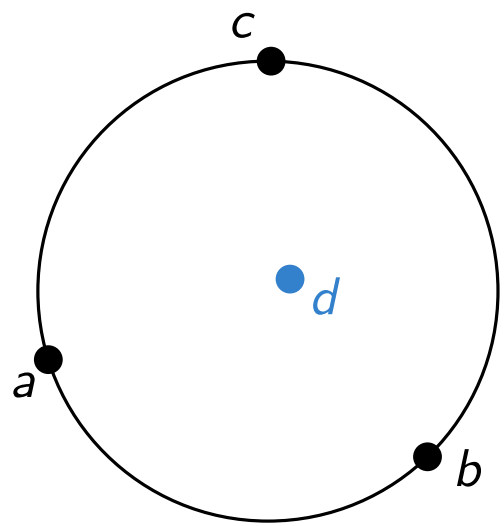

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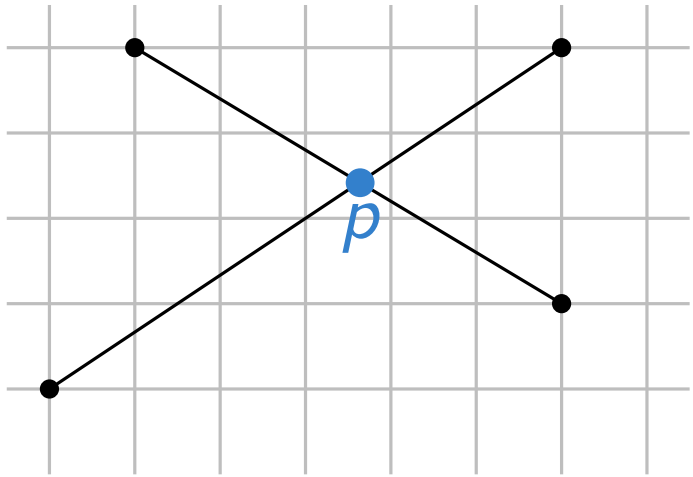

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  - Theory: primitives in $O(1)$ time
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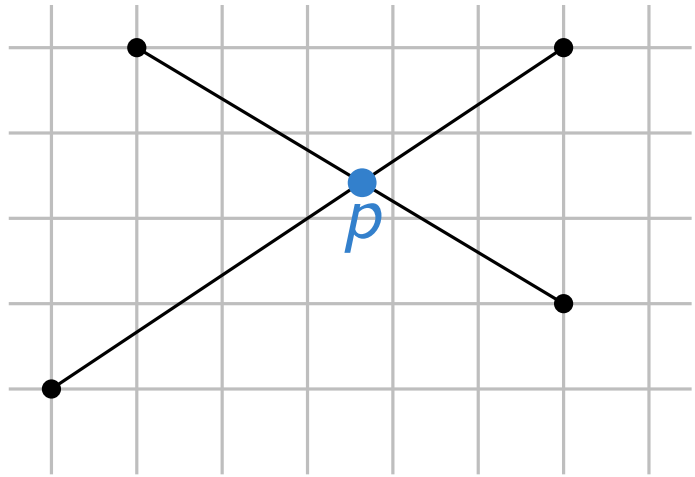

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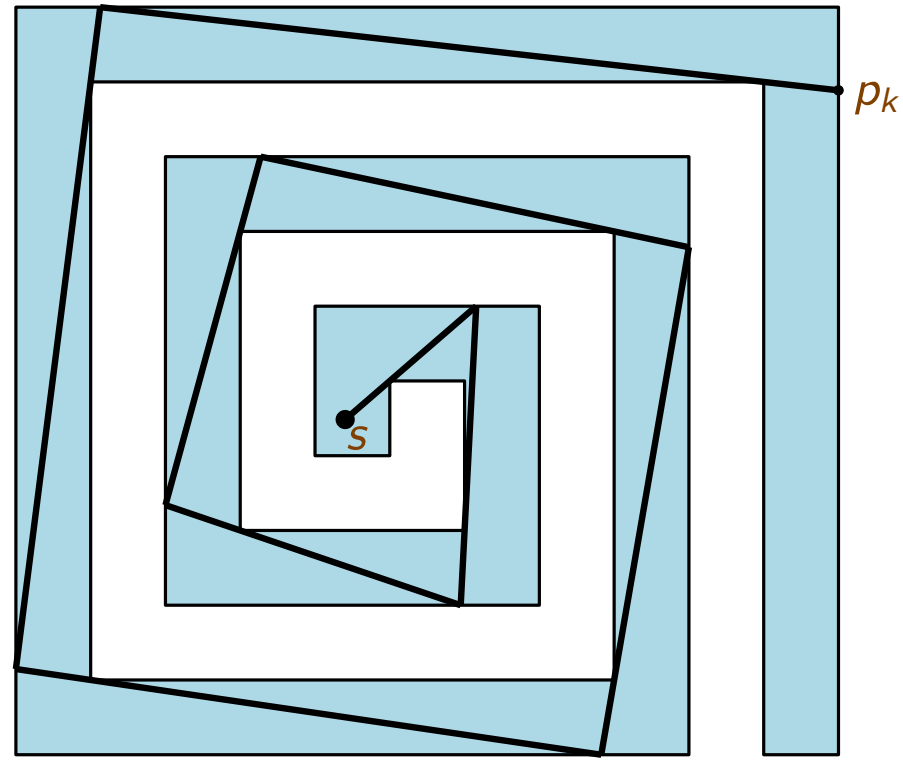

Lemma.
A MinLinkPath of length k between s and t in a simple polygon whose vertices, as well as s and t, have bit-complexity $\log n$, may contain vertices of bit-complexity $\Omega(k \log n)$.

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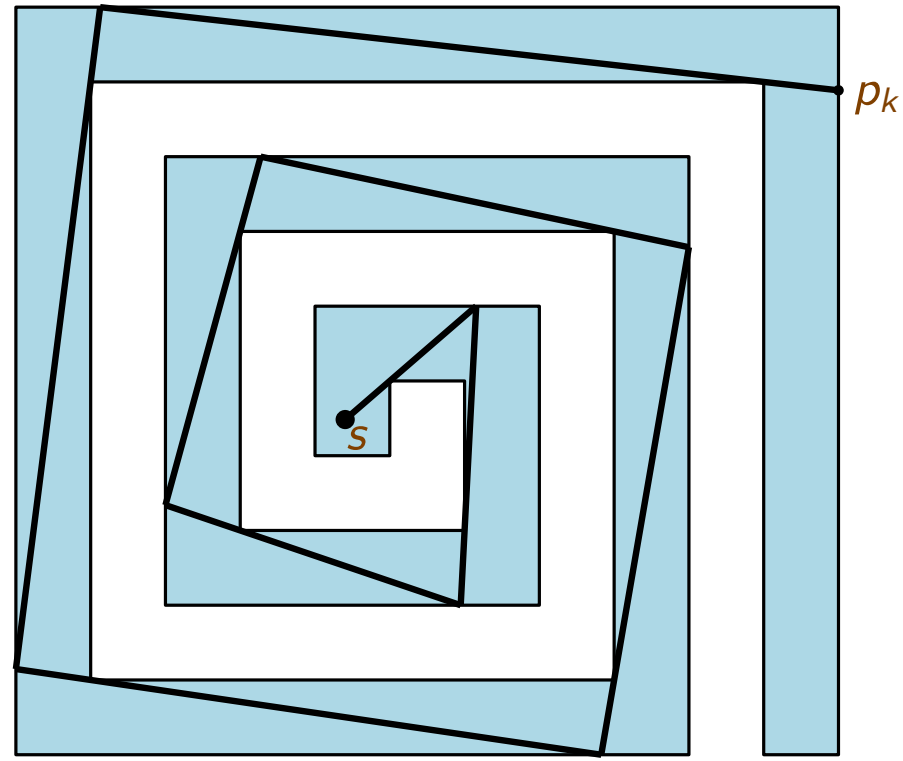

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$\Longrightarrow$ In Real RAM we can compute a MinLinkPath in $O(n \log n)$ time.
However, if we measure bit complexity it takes $O(n^2 \log n)$ time.

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