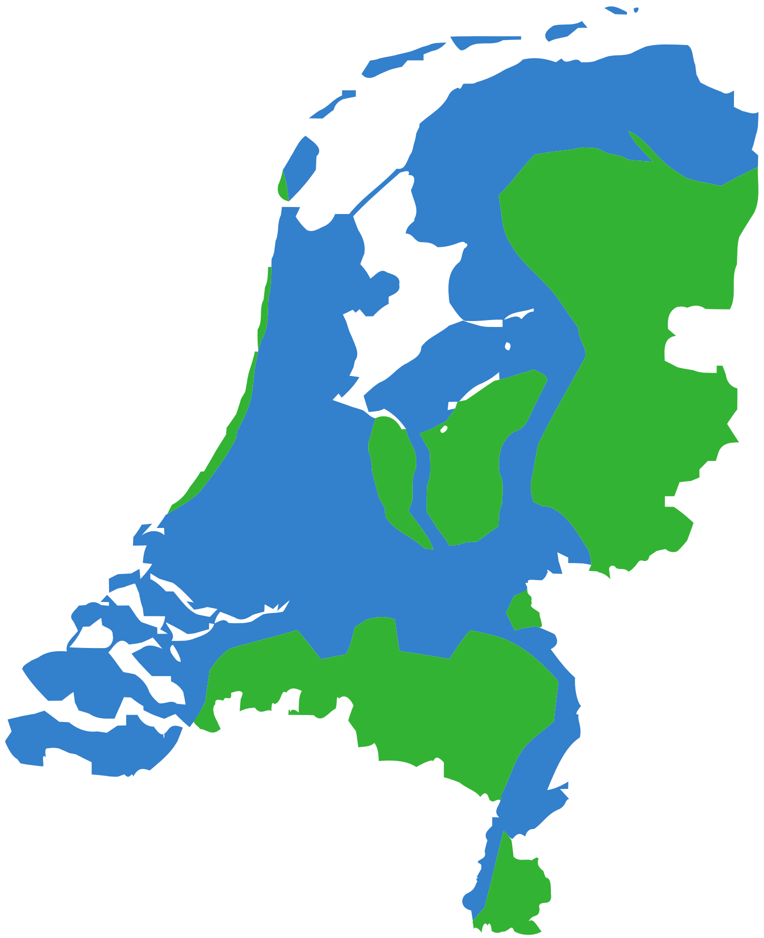
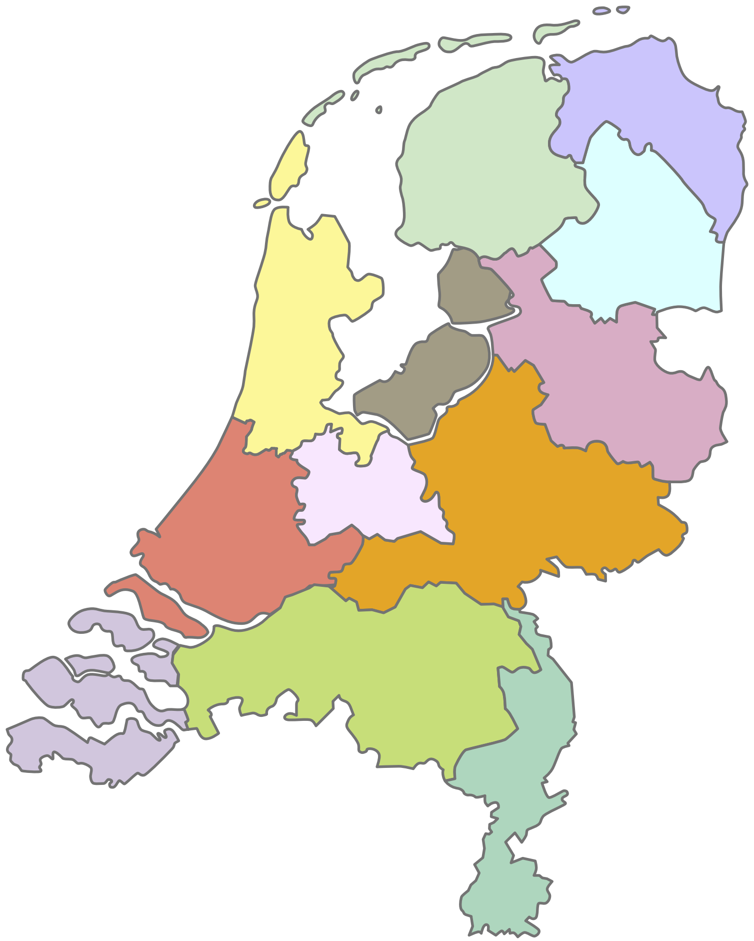
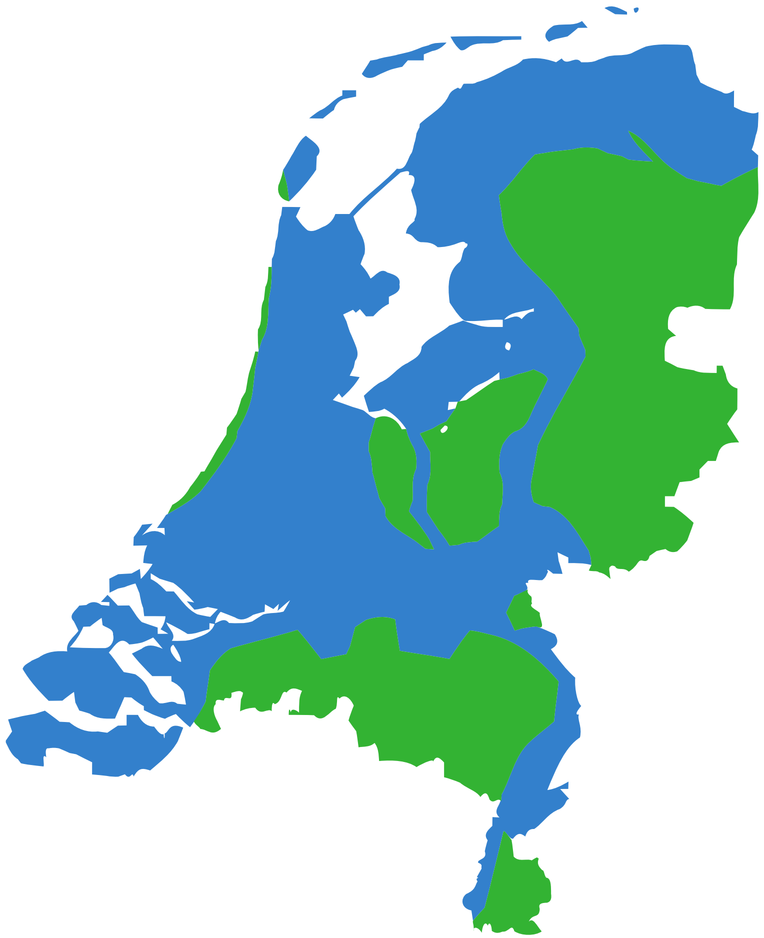
Introduction

Introduction



Introduction





What percentage of Utrecht is in danger of floods?

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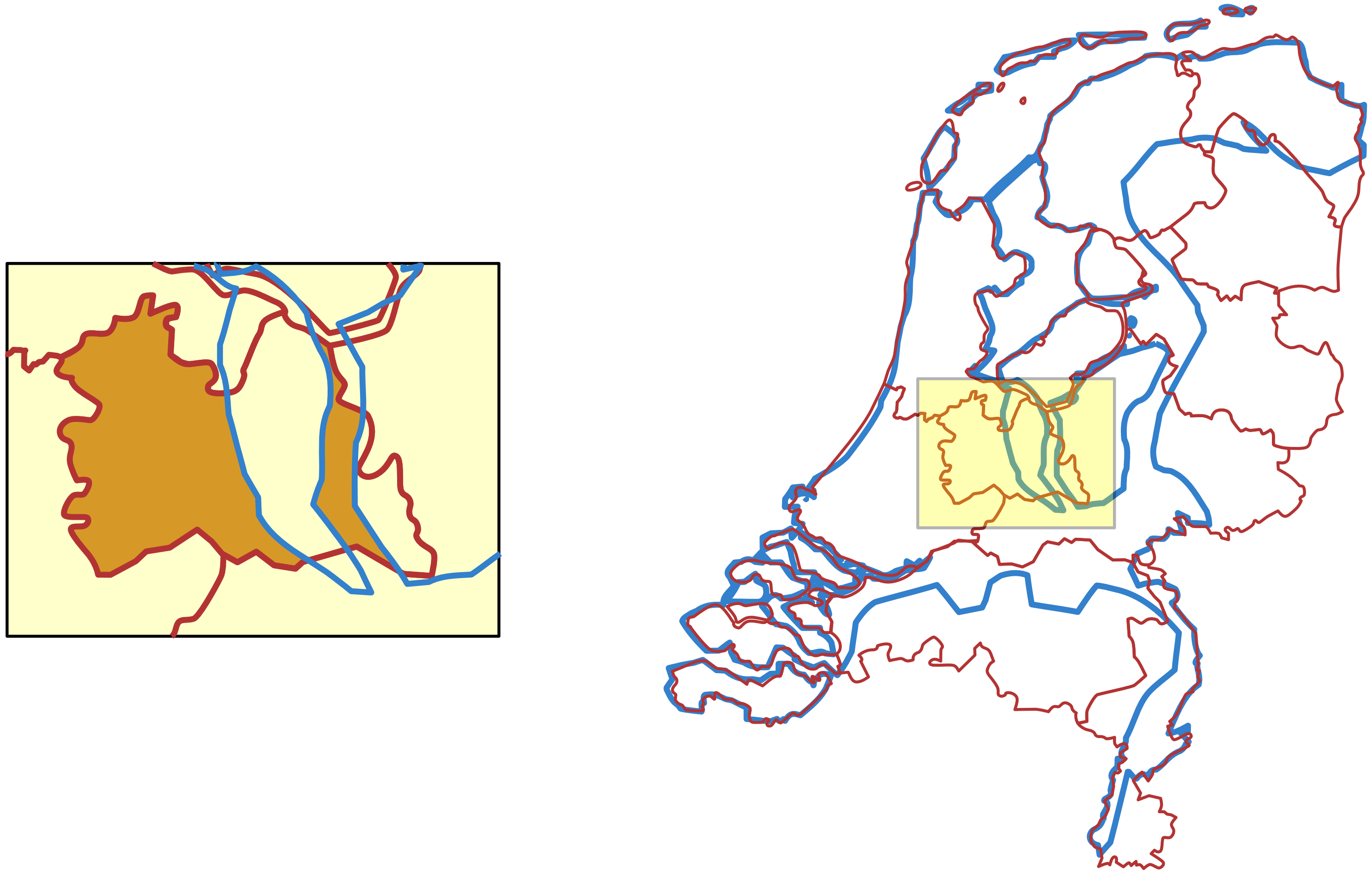
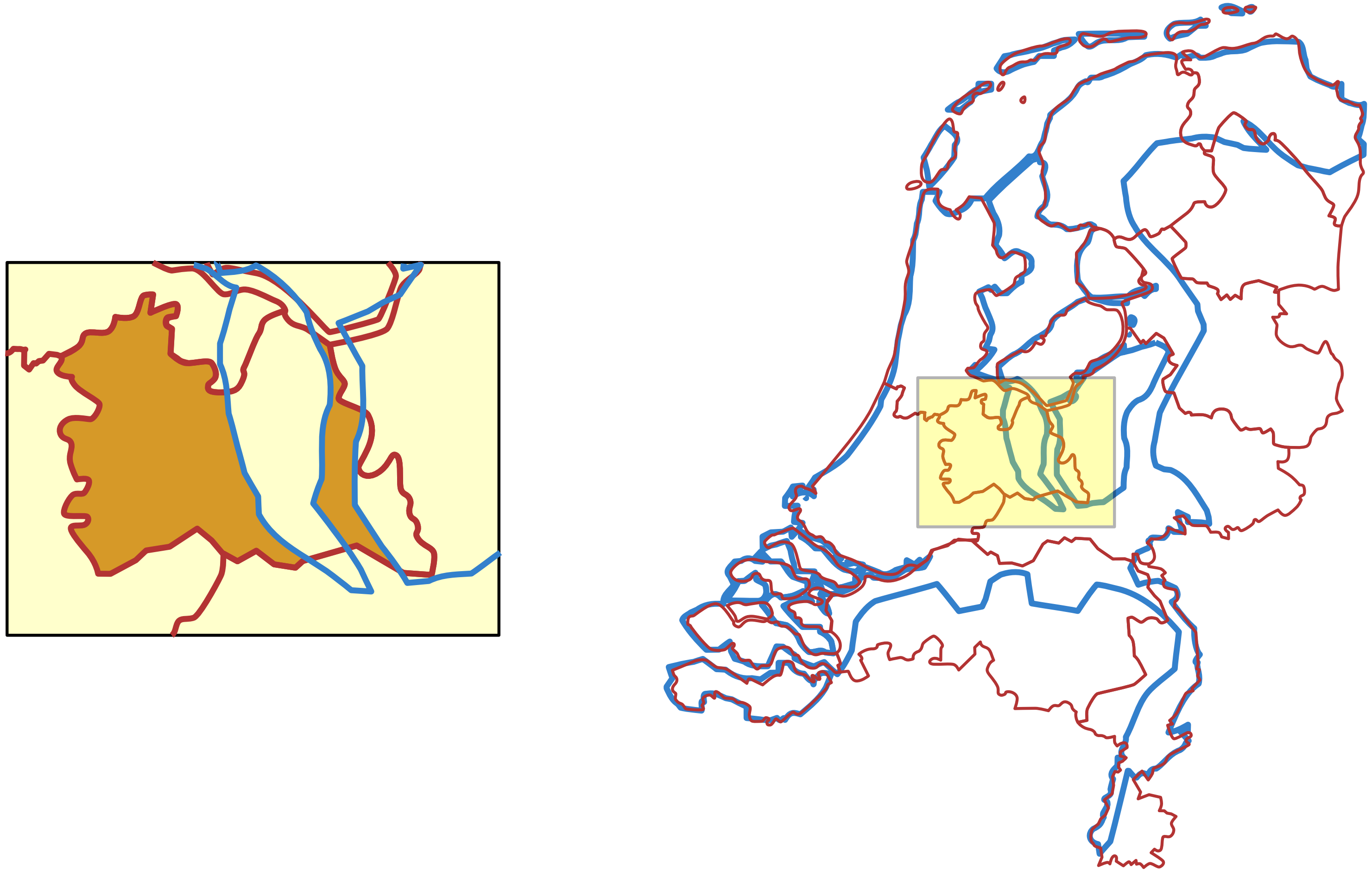


Map Overlay

Problem:

Given two maps M_1 and M_2, compute the overlay of M_1 and M_2.



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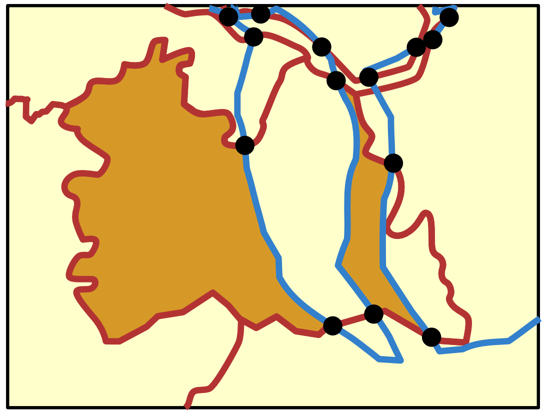

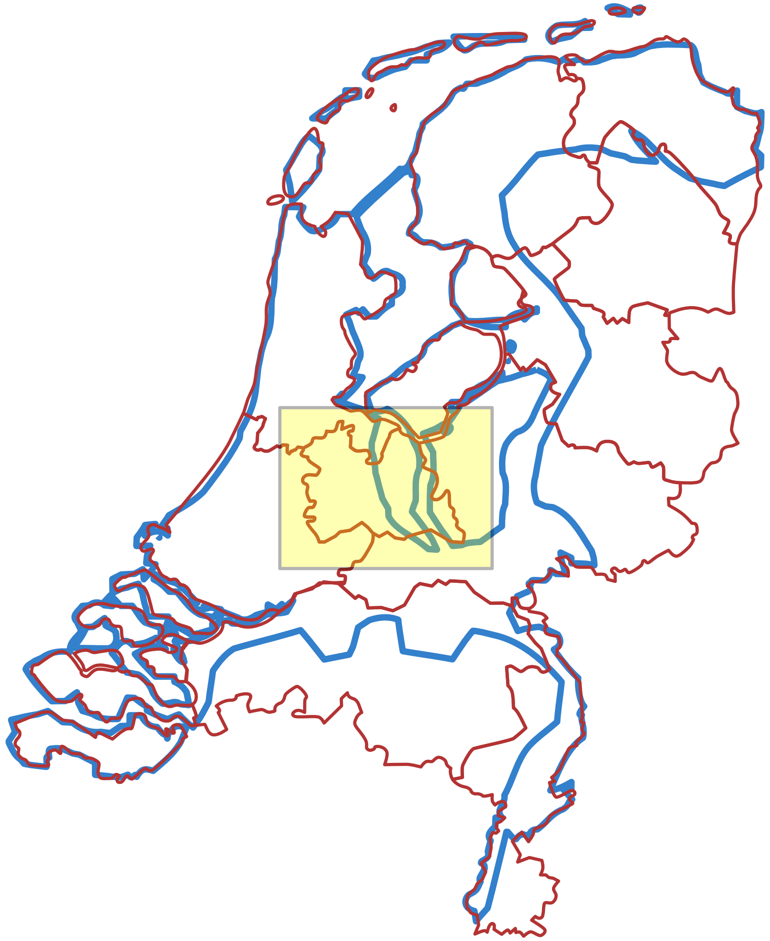

- How to compute all intersection points?

Map Overlay

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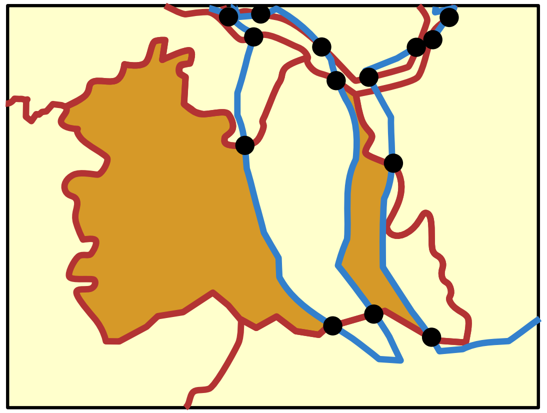
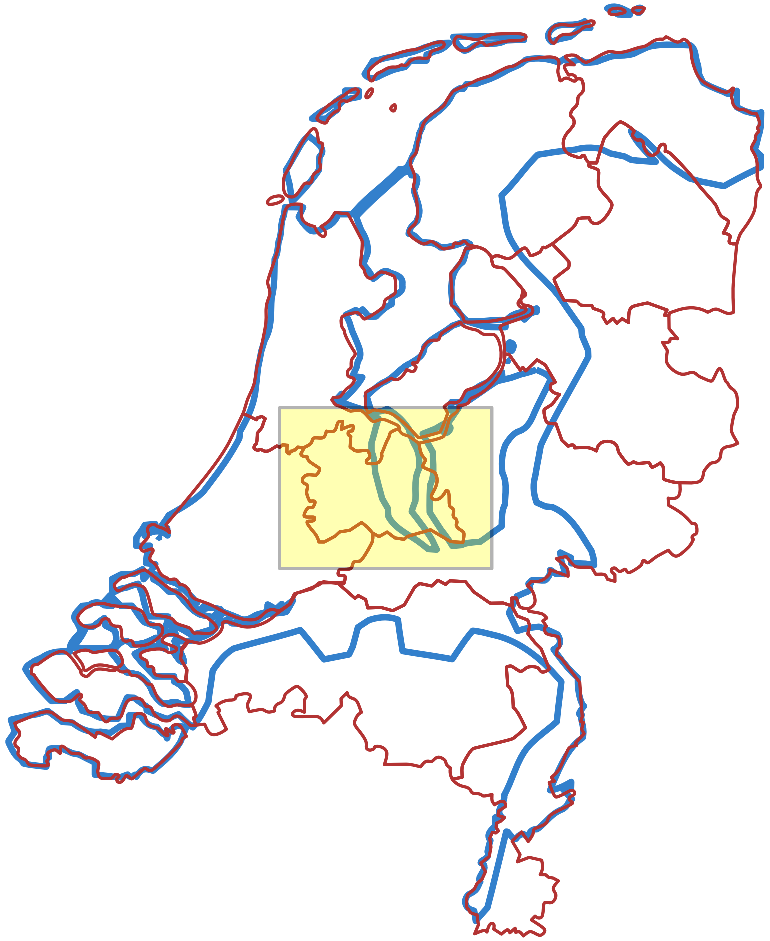
Given two maps M_1 and M_2, compute the overlay of M_1 and M_2.



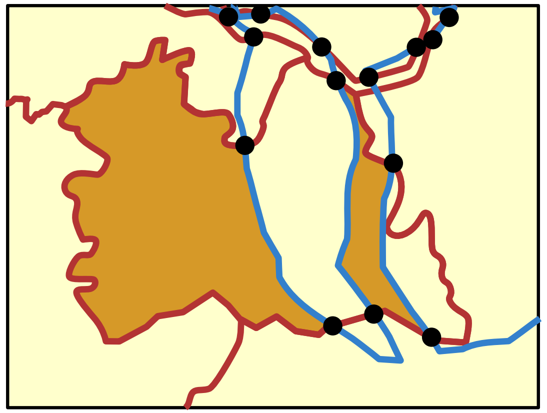


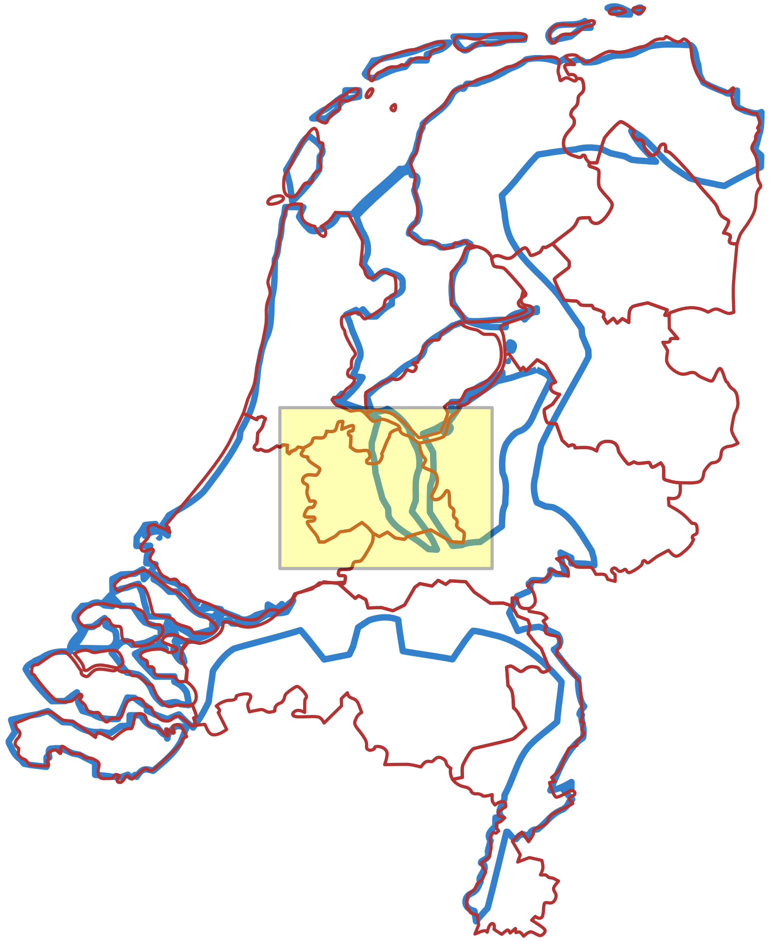
- How to compute all intersection points?
- How to represent (the overlay of) a map?

Map Overlay

Problem:

Given two maps M_1 and M_2, compute the overlay of M_1 and M_2.





- How to compute all intersection points?

- How to represent a planar subdivision?

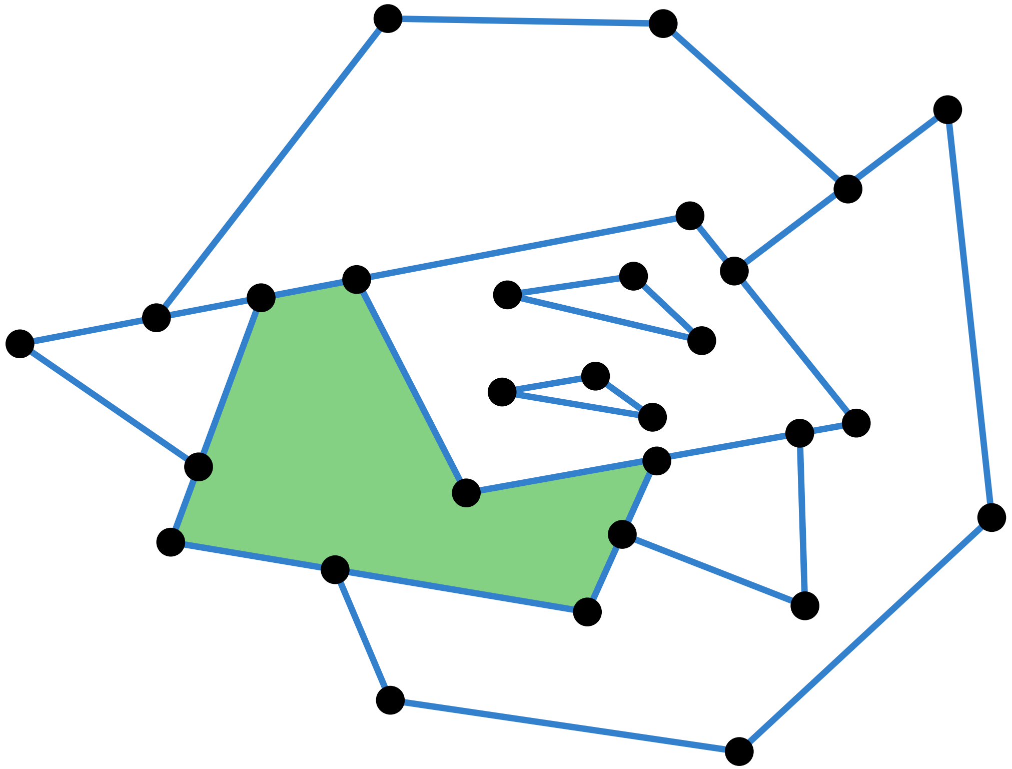
Representing a planar subdivision

A planar subdivision is an embedded planar graph, consisting of:

Vertices

Edges

Faces



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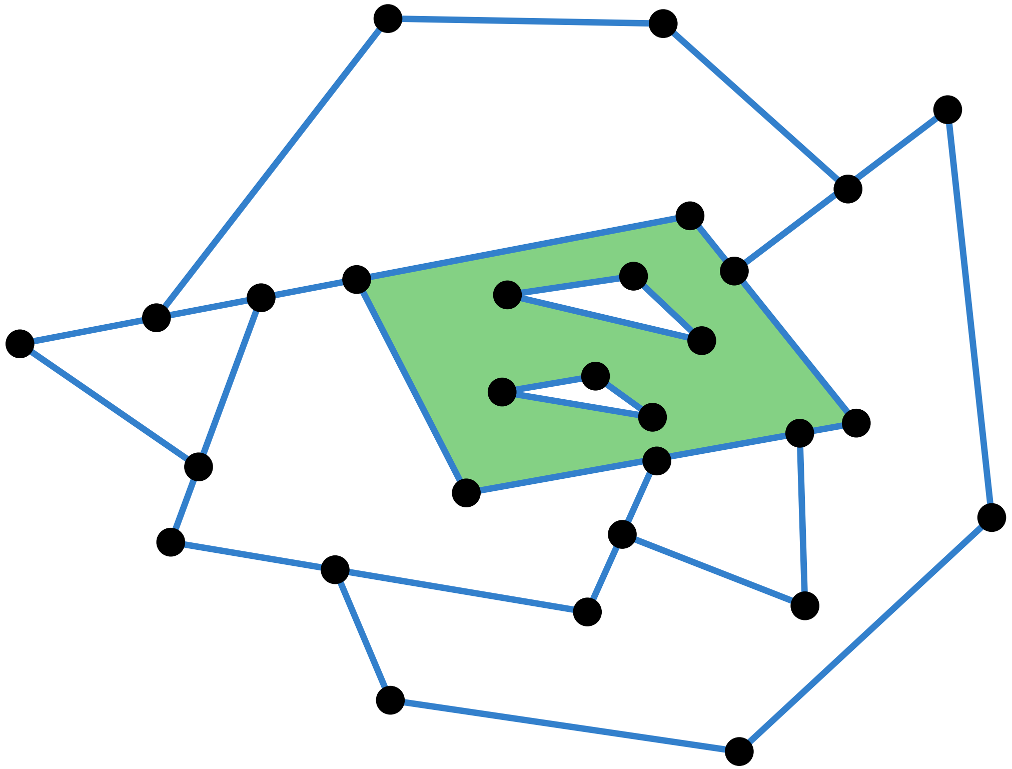
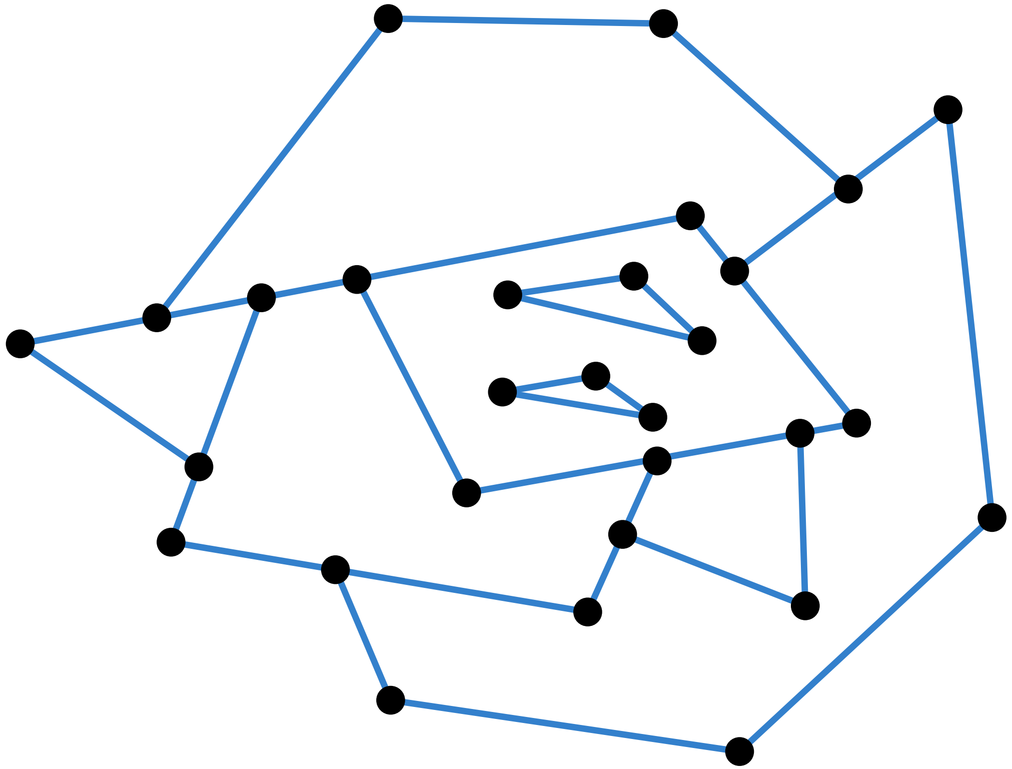


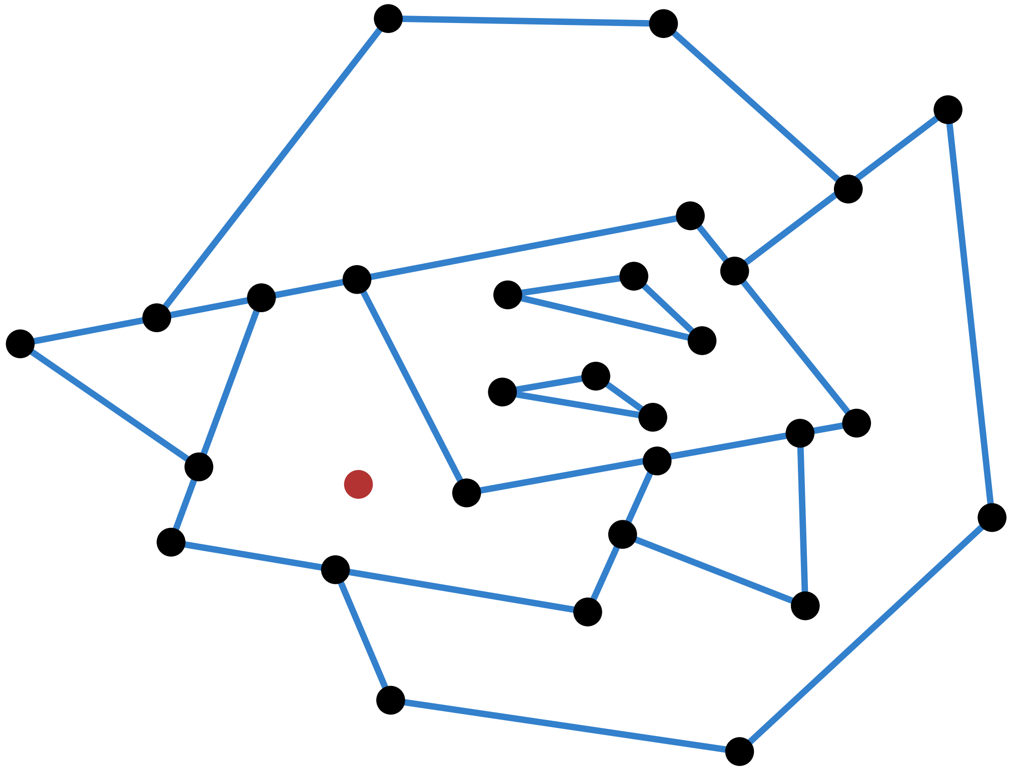
Representing a planar subdivision

What operations should are representation support?



Representing a planar subdivision

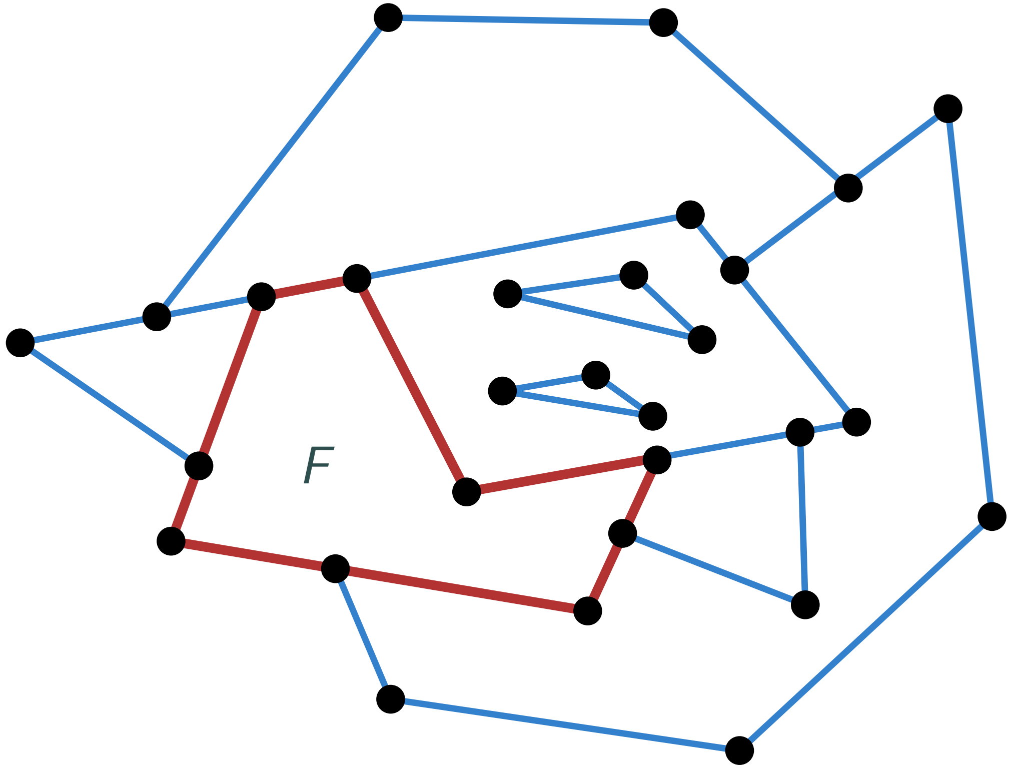
What operations should are representation support?



Representing a planar subdivision

What operations should are representation support?

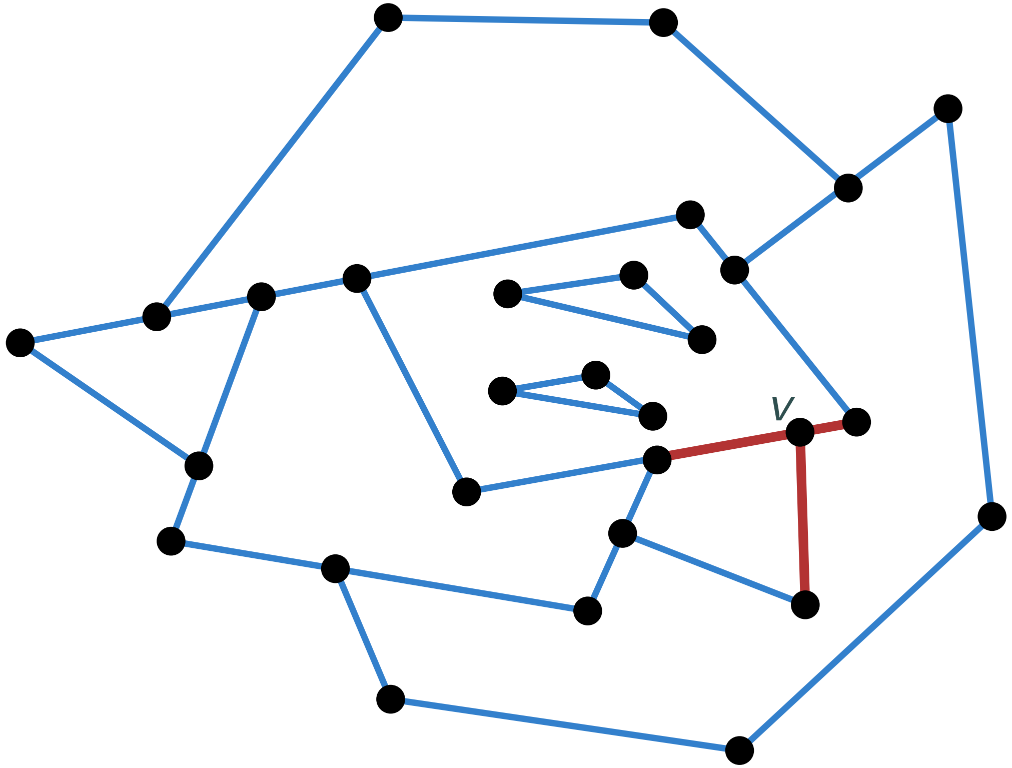
- List edges or vertices bounding a face F



Representing a planar subdivision

What operations should are representation support?

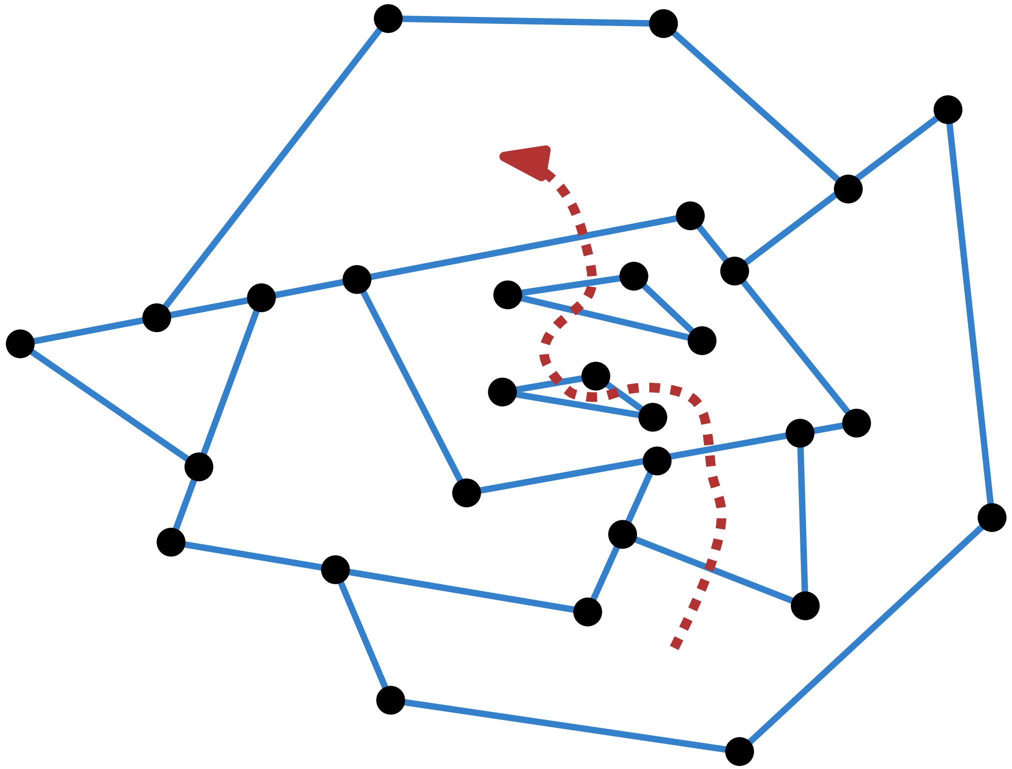
- List edges or vertices bounding a face F
- List edges incident to a vertex v



Representing a planar subdivision

What operations should are representation support?

- List edges or vertices bounding a face F
- List edges incident to a vertex v
- Efficiently "walk" in the subdivision
- ...



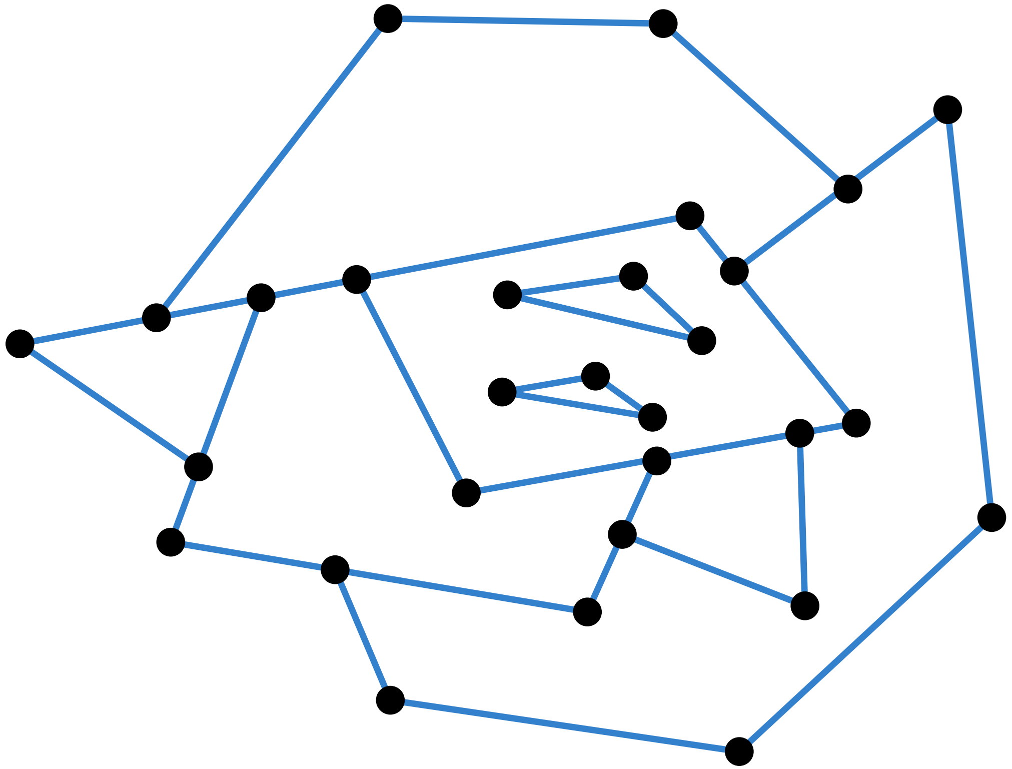
The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a doubly-connected edge list (DCEL)

Vertices

Edges

Faces



The Doubly-Connected Edge List (DCEL)

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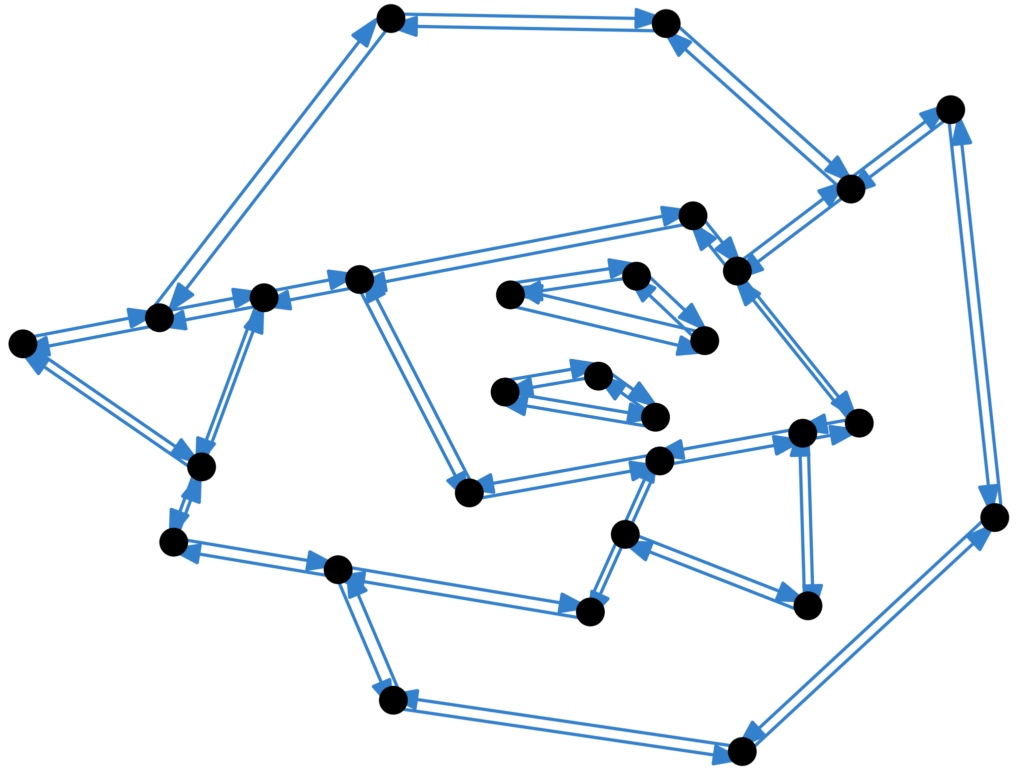
Vertices

Half-edges

Faces



The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a
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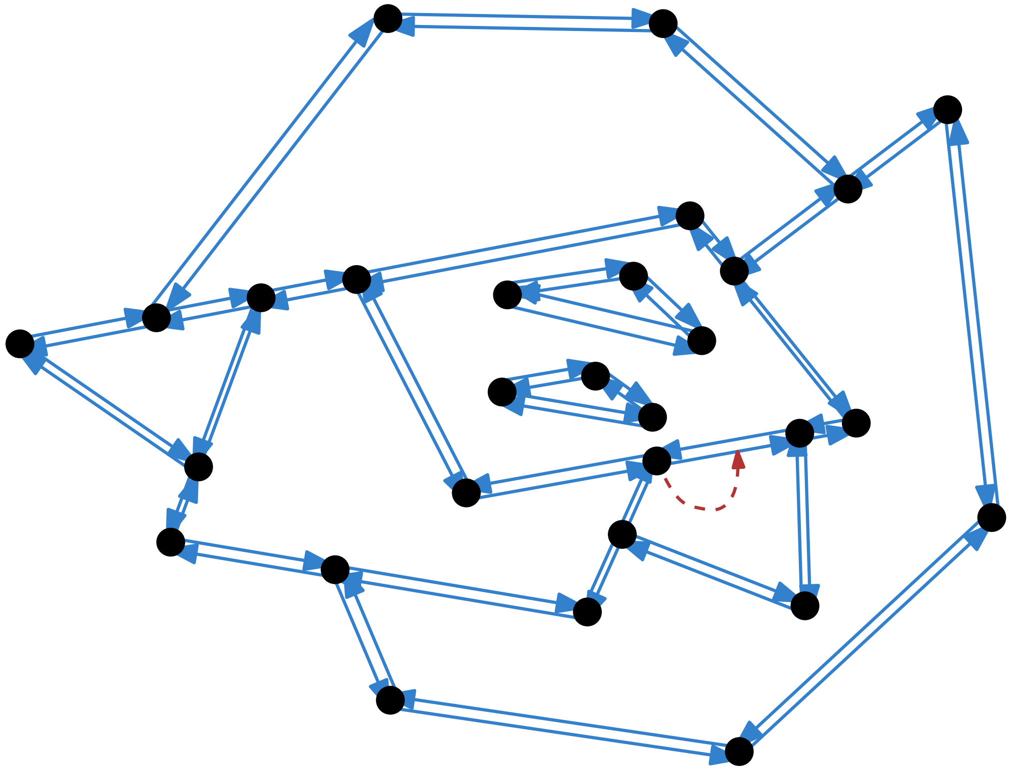
Vertices

Coordinates

An arbitrary outgoing half-edge



Half-edges

Faces

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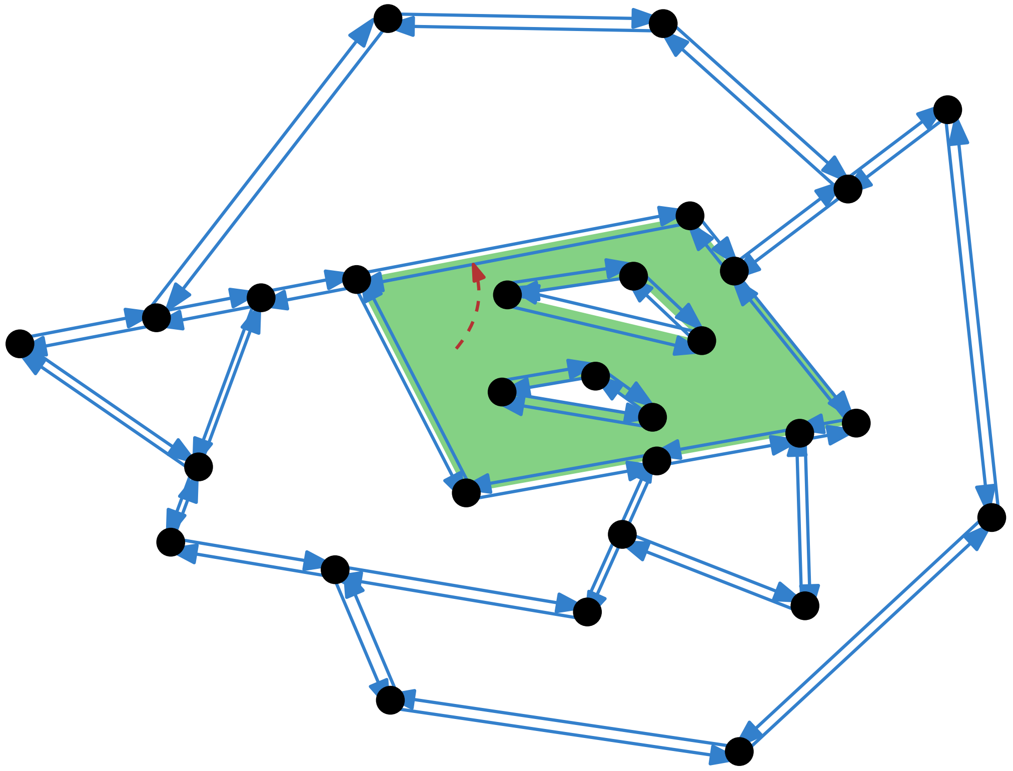

Vertices
- Coordinates
- An arbitrary outgoing half-edge

Half-edges

Faces
- An arbitrary half-edge on its outer boundary

The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a
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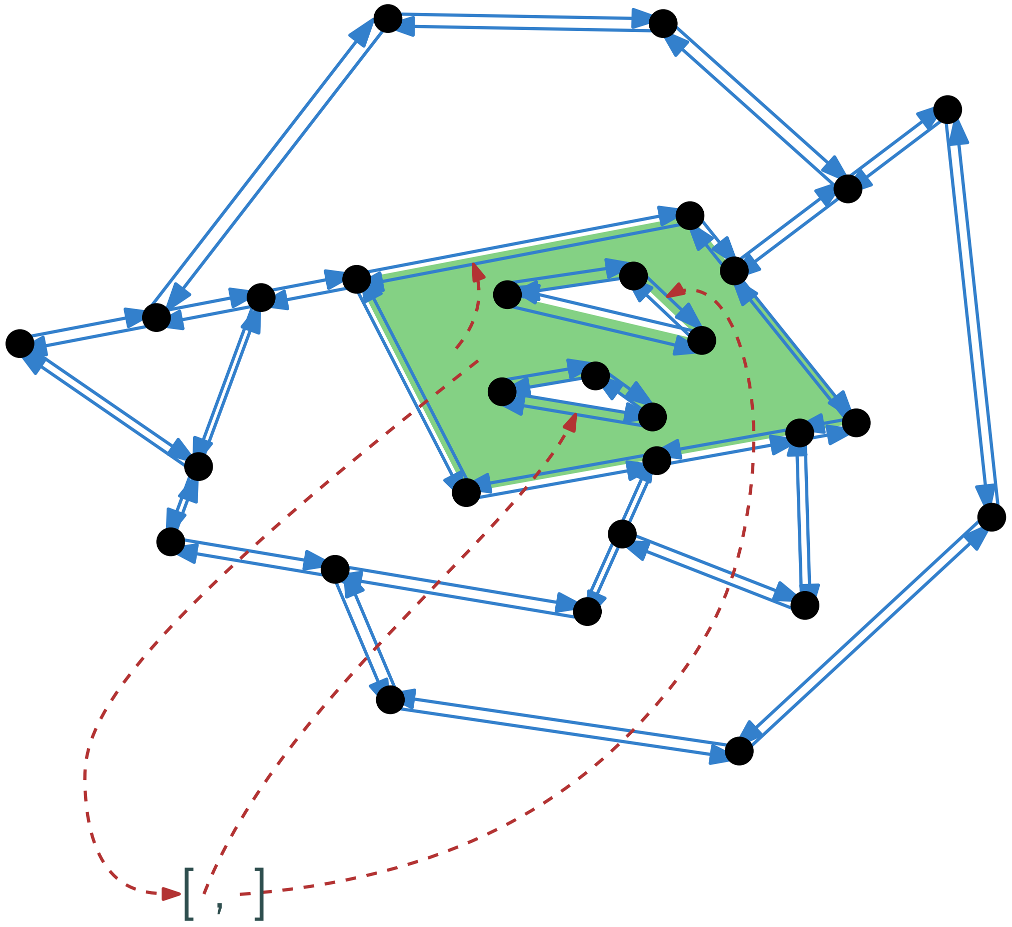


Vertices

Coordinates

An arbitrary outgoing half-edge

Half-edges

Faces

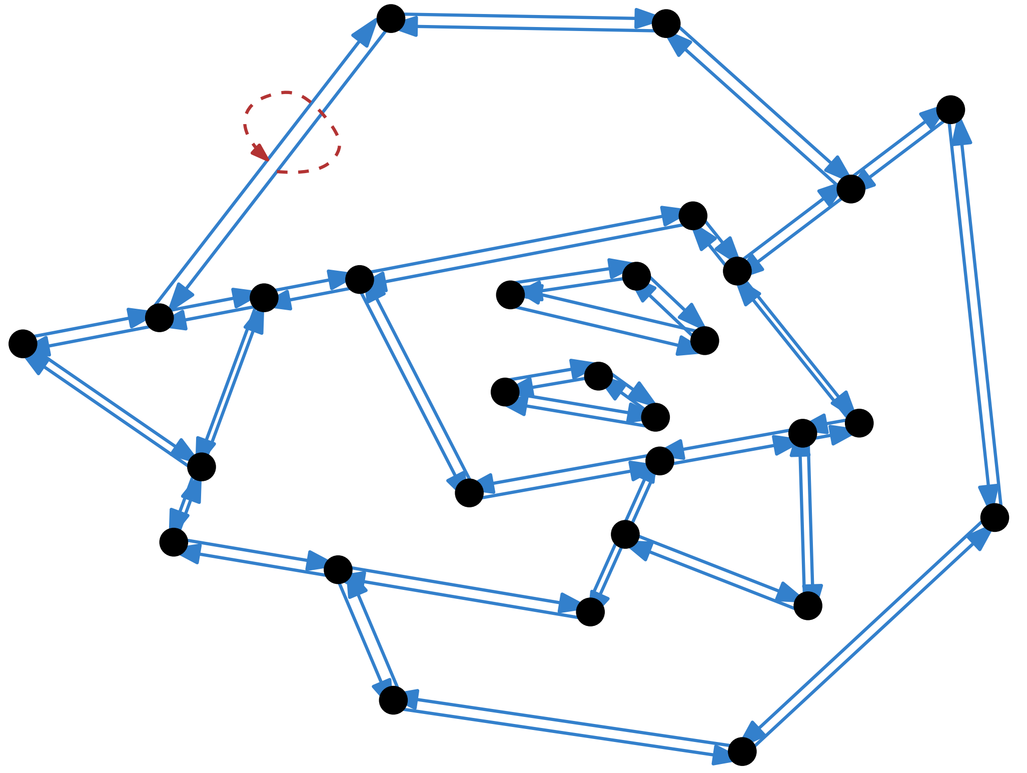
An arbitrary half-edge on its outer boundary

A list of holes in the face;

for each hole an arbitrary half-edge.

The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a doubly-connected edge list (DCEL)



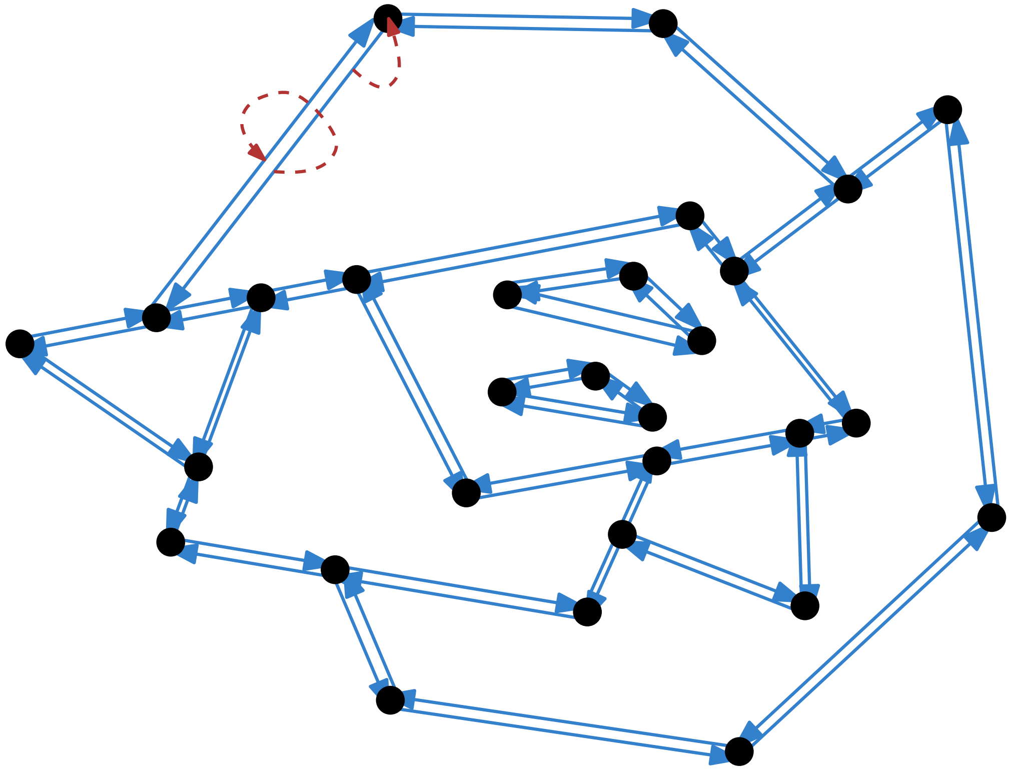
Vertices
- Coordinates
- An arbitrary outgoing half-edge

Half-edges
- Twin

Faces
- An arbitrary half-edge on its outer boundary
- A list of holes in the face;
- for each hole an arbitrary half-edge.

The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a doubly-connected edge list (DCEL)



Vertices

Coordinates

An arbitrary outgoing half-edge

Half-edges

Twin

Origin

Faces

An arbitrary half-edge on its outer boundary

A list of holes in the face;

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The Doubly-Connected Edge List (DCEL)

We store the planar subdivision in a
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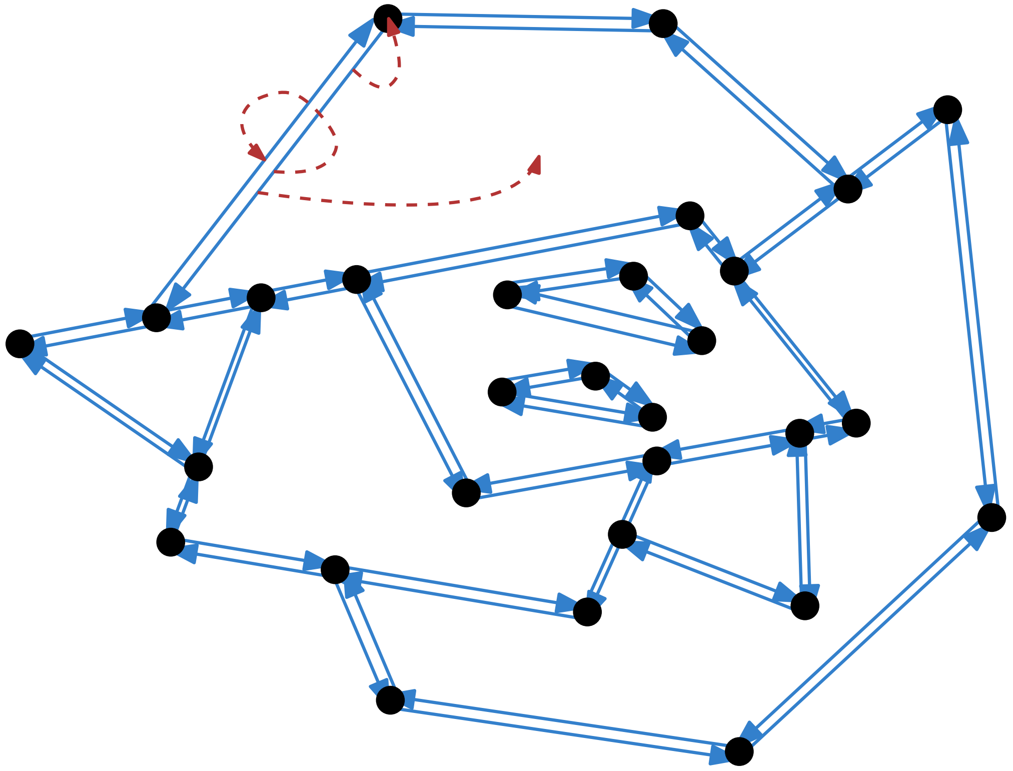

Vertices
- Coordinates
- An arbitrary outgoing half-edge

Half-edges
- Twin
- Origin
- Incident face

Faces
- An arbitrary half-edge on its outer boundary
- A list of holes in the face;
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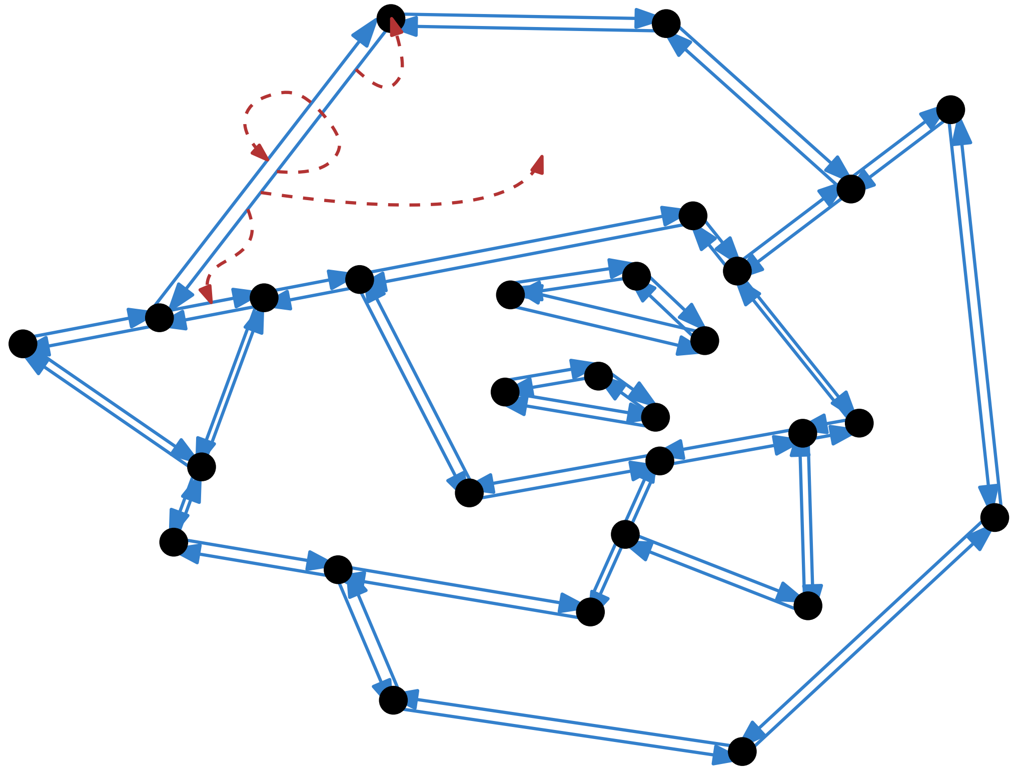
We store the planar subdivision in a doubly-connected edge list (DCEL)



Vertices

Coordinates

An arbitrary outgoing half-edge

Half-edges

Twin

Origin

Incident face

Next

Faces

An arbitrary half-edge on its outer boundary

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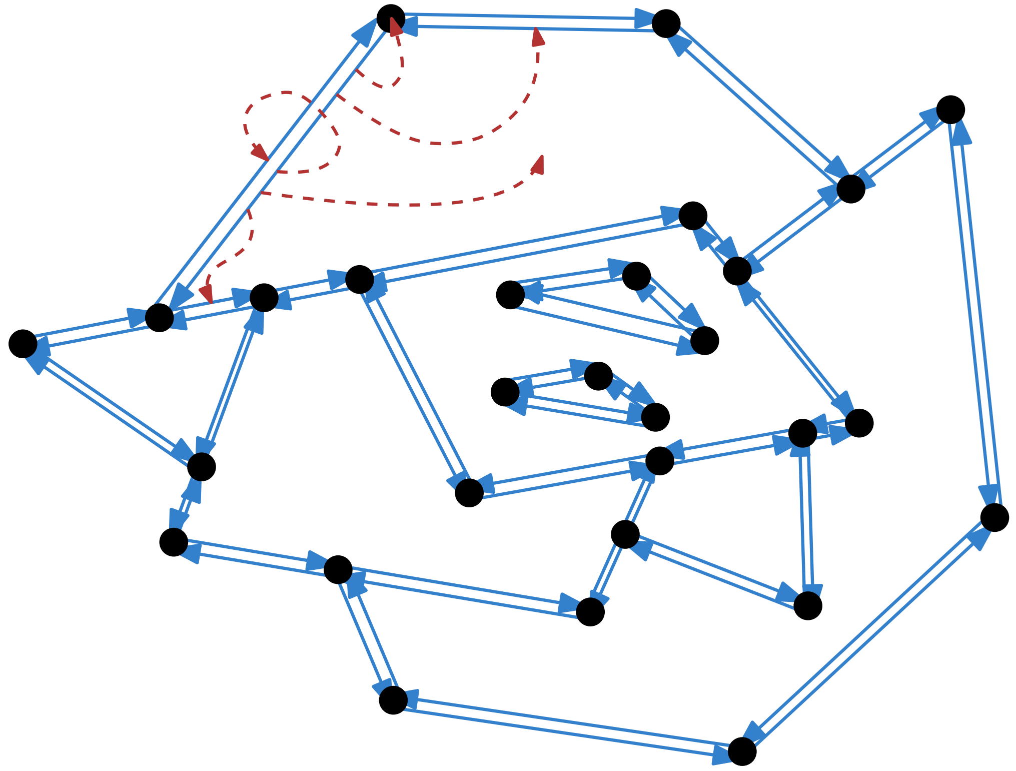
We store the planar subdivision in a doubly-connected edge list (DCEL)



Vertices
- Coordinates
- An arbitrary outgoing half-edge

Half-edges
- Twin
- Origin
- Incident face
- Next
- Prev

Faces
- An arbitrary half-edge on its outer boundary
- A list of holes in the face;
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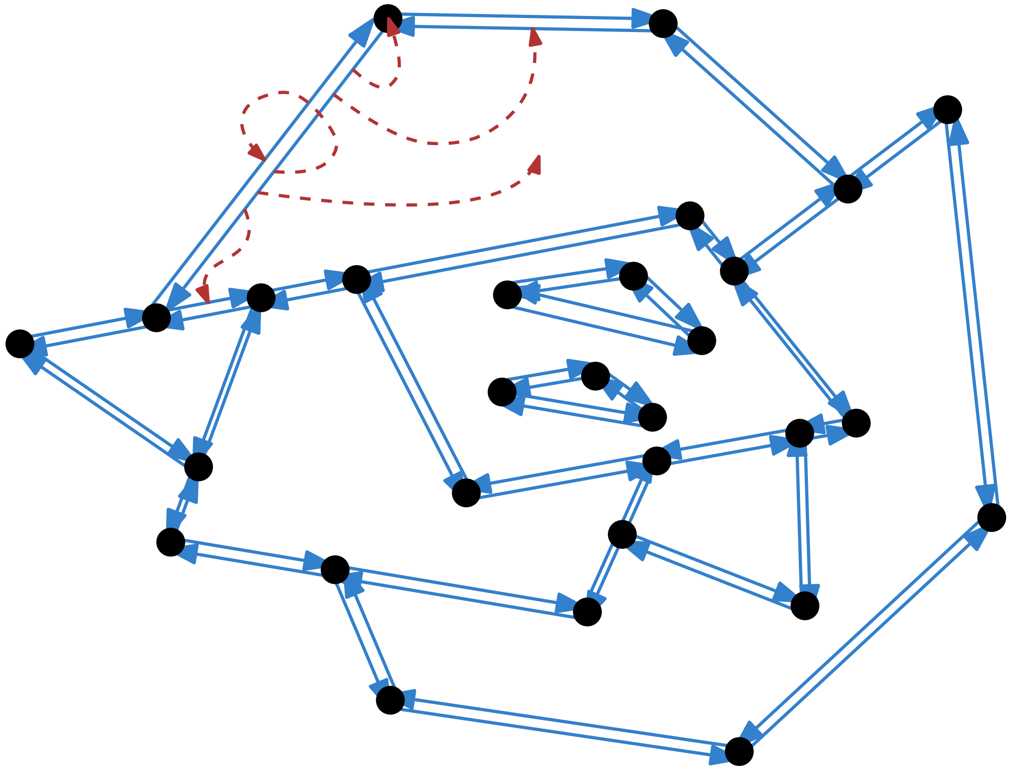

Vertices
- Coordinates
- An arbitrary outgoing half-edge
- Label

Half-edges
- Twin
- Origin
- Incident face
- Next
- Prev
- Label

Faces
- An arbitrary half-edge on its outer boundary
- A list of holes in the face;
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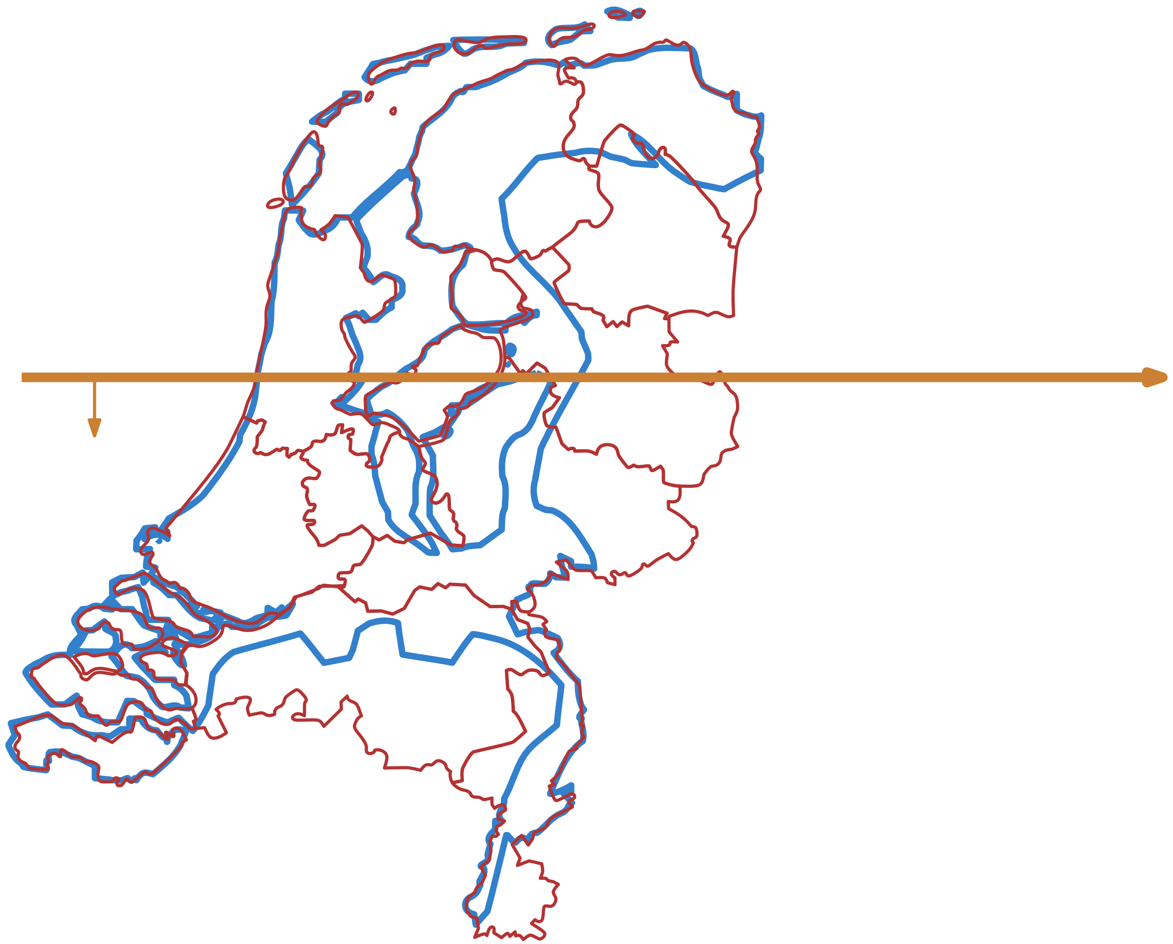
- Label

Map Overlay

Compute the overlay of M_1 and M_2:

Use a sweep line algorithm to compute the intersection points and maintain the vertex and half-edge information above the sweep line.

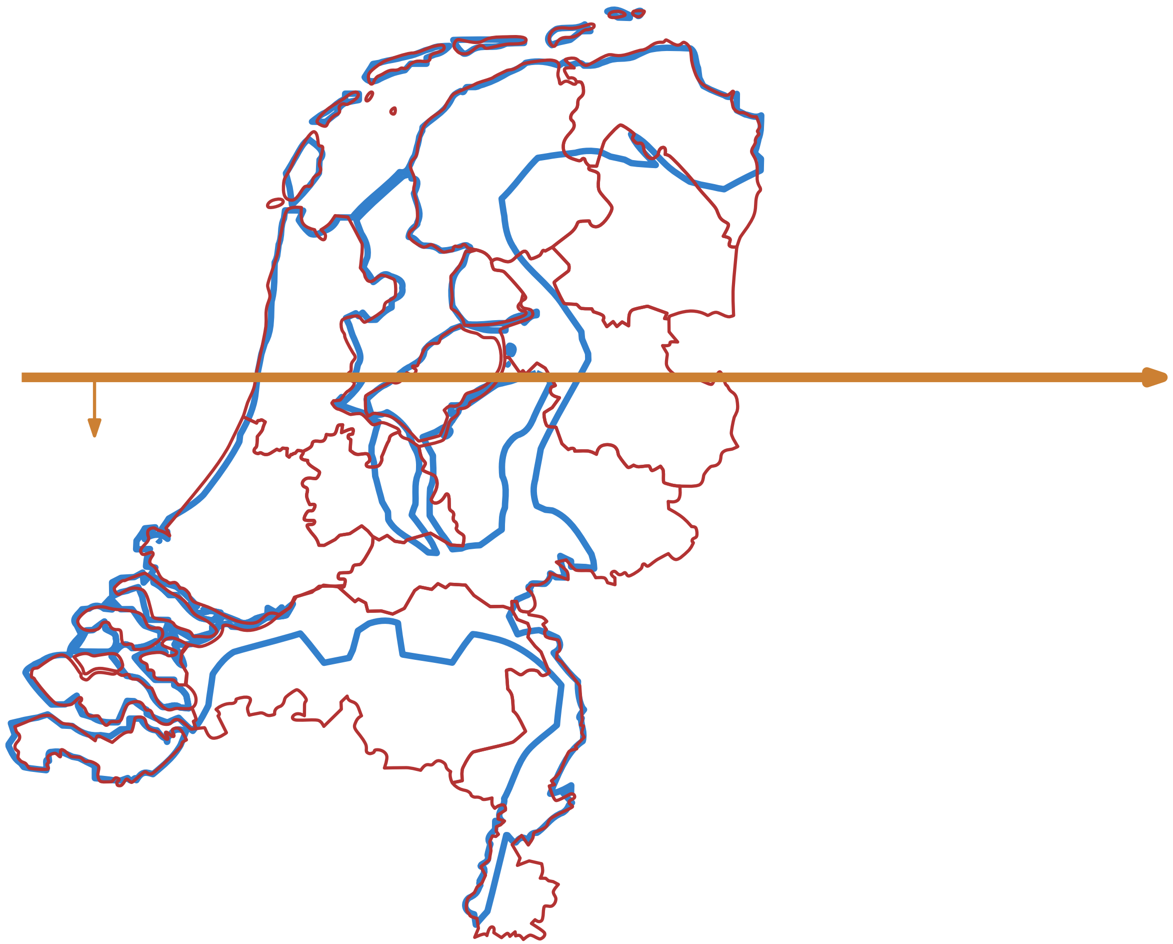


Map Overlay

Compute the overlay of M_1 and M_2:

Use a sweep line algorithm to compute the intersection points and maintain the vertex and half-edge information above the sweep line.

Use a second sweep to compute the face information.



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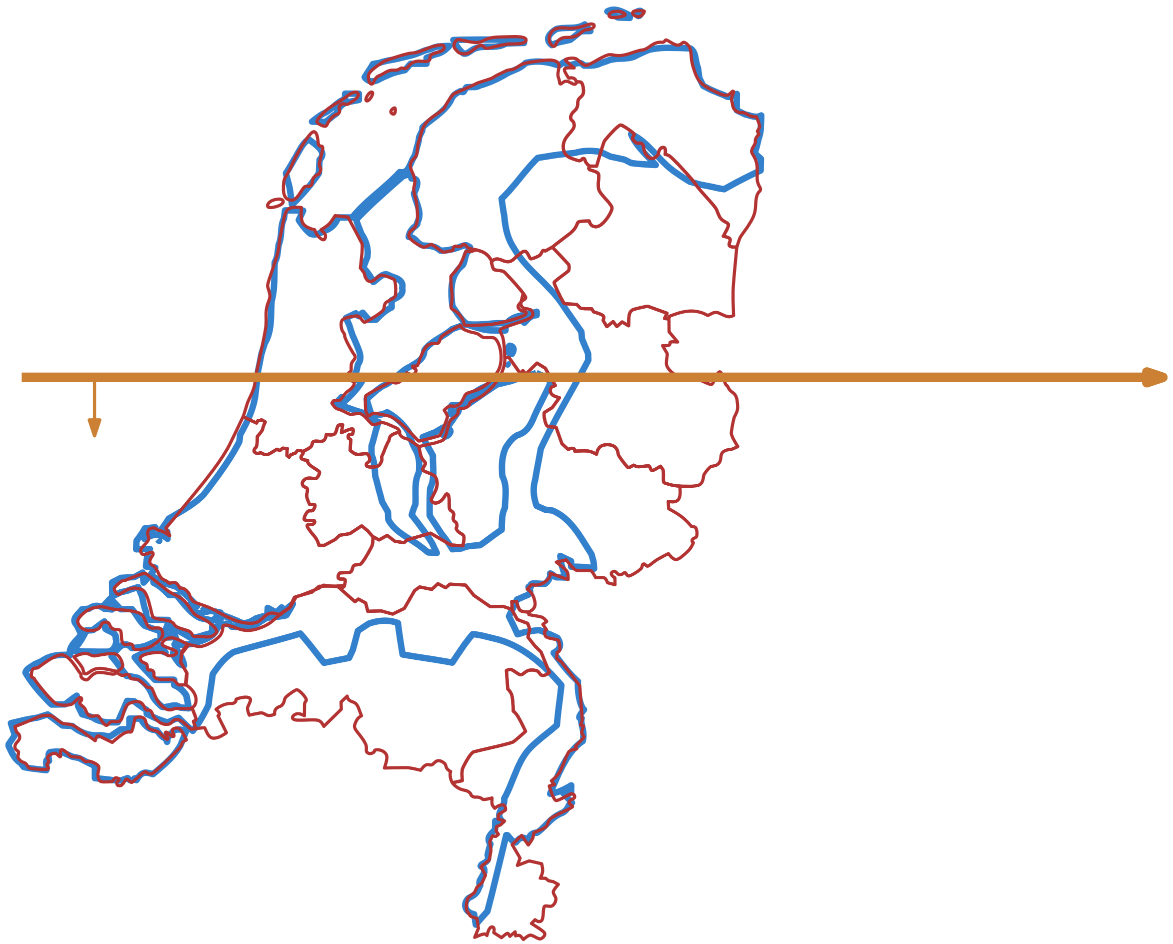

Theorem. We can compute the overlay of M_1 and M_2 in $O((n+k)\log n)$ time, where n is the total number of line segments in M_1 and M_2, and k is the number of line segments in the output.