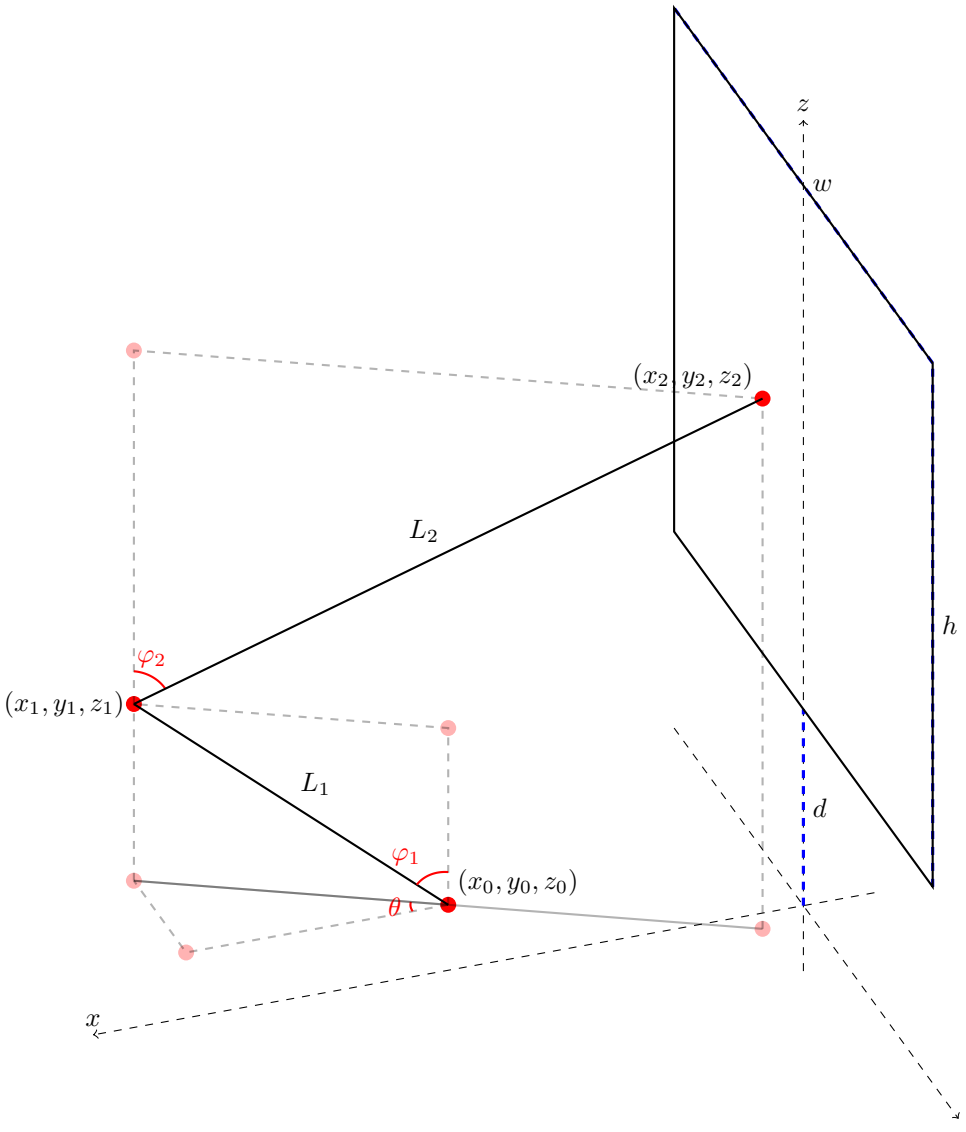
MAT 4070 (314S3) - Project

April 9, 2025

1 Three Axis Robotic Arm



Let (x_0, y_0, z_0) denote the base point of the arm, and L_1 and L_2 be the length of the links. Further, let φ_i denote the angle between y-axes and the link L_i $(i = 1, 2)$, and θ denote the angle between z-axes and the xz-projection of L_1.

Question 1. *Compute the coordinates of the points* (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *in terms of* $x_0, y_0, z_0, \theta, \varphi_1, L_1, \varphi_2, L_2$.

$$\begin{cases} x_1 = x_0 + L_1 \sin(\varphi_1)\cos(\theta) \\ y_1 = y_0 - L_1 \sin(\varphi_1)\sin(\theta) \\ z_1 = z_0 + L_1 \cos(\varphi_1) \end{cases}$$
$$\begin{cases} x_2 = x_1 - L_2 \sin(\varphi_2)\cos(\theta) \\ y_2 = y_1 + L_2 \sin(\varphi_2)\sin(\theta) \\ z_2 = z_1 + L_2 \cos(\varphi_2). \end{cases}$$

Explicitly, we have

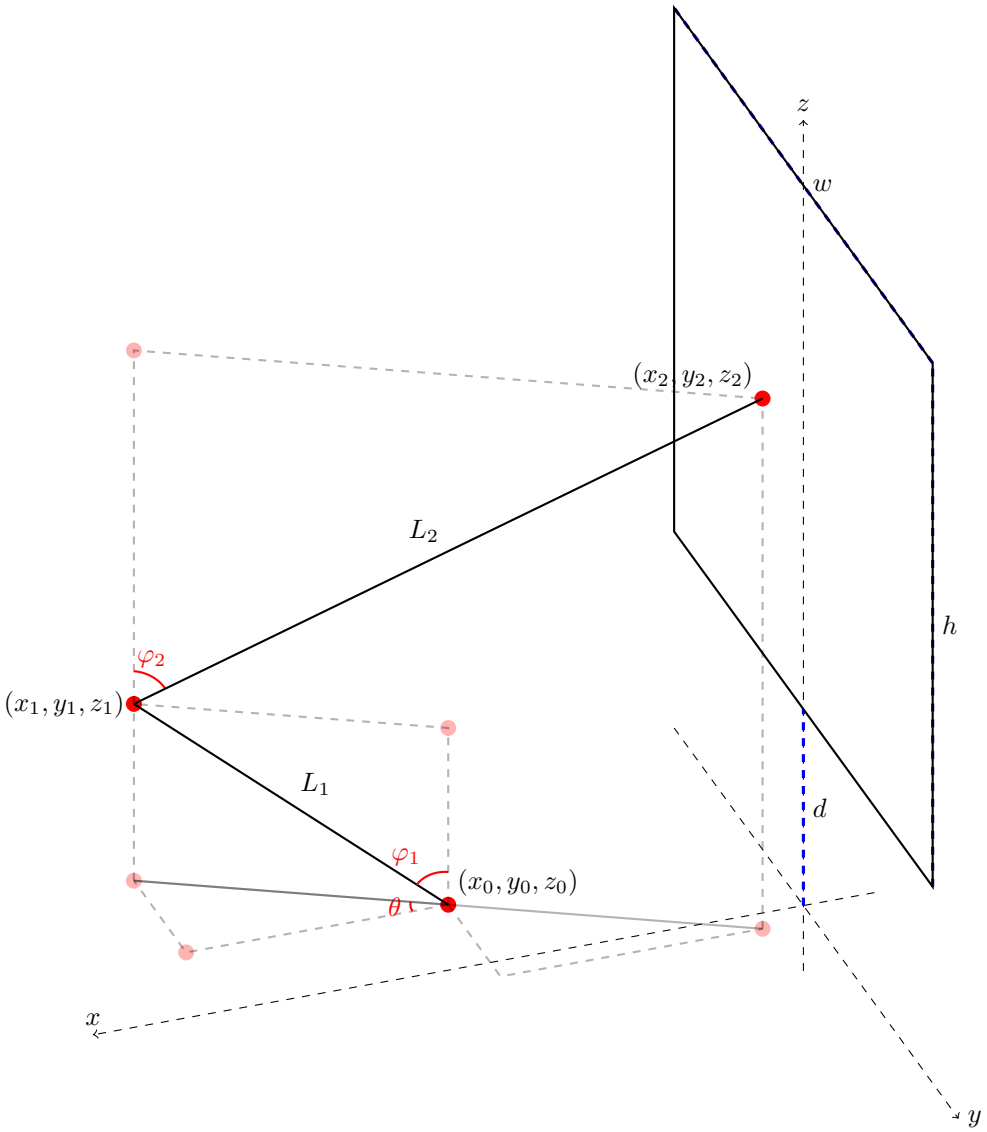
$$\begin{cases} x_2 = x_0 + \big[L_1 \sin(\varphi_1) - L_2 \sin(\varphi_2)\big]\cos(\theta) \\ y_2 = y_0 - \big[L_1 \sin(\varphi_1) - L_2 \sin(\varphi_2)\big]\sin(\theta) \\ z_2 = z_0 + L_1 \cos(\varphi_1) + L_2 \cos(\varphi_2). \end{cases}$$

Question 2. *Under which restrictions* (x_2, y_2, z_2) *belongs to the white board (i.e.,* $x_2 = 0$, $-\frac{w}{2} \le y_2 \le \frac{w}{2}$ *and* $d \le z_2 \le d + h$*)? Find ranges for* $\theta, \varphi_1, \varphi_2$.

$$\begin{cases} x_1 - x_0 = L_1 \sin(\varphi_1)\cos(\theta) \\ -(y_1 - y_0) = L_1 \sin(\varphi_1)\sin(\theta) \\ z_1 - z_0 = L_1 \cos(\varphi_1) \end{cases}$$
$$\begin{cases} -(x_2 - x_1) = L_2 \sin(\varphi_2)\cos(\theta) \\ y_2 - y_1 = L_2 \sin(\varphi_2)\sin(\theta) \\ z_2 - z_1 = L_2 \cos(\varphi_2). \end{cases}$$
$$\implies \begin{cases} \tan(\theta) = -\dfrac{y_1 - y_0}{x_1 - x_0} \\ \tan(\varphi_1) = \dfrac{x_1 - x_0}{z_1 - z_0}\sec(\theta) \end{cases}$$
$$\begin{cases} \tan(\theta) = -\dfrac{y_2 - y_1}{x_2 - x_1} \\ \tan(\varphi_2) = \dfrac{y_2 - y_1}{z_2 - z_1}\csc(\theta) \end{cases}$$
$$\implies \begin{cases} \theta = -\arctan\left(\dfrac{y_1 - y_0}{x_1 - x_0}\right) \\ \theta = -\arctan\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right) \\ \theta = -\arctan\left(\dfrac{y_2 - y_0}{x_2 - x_0}\right) \\ \varphi_1 = \arctan\left(\dfrac{x_1 - x_0}{z_1 - z_0}\sec(\theta)\right) \\ \varphi_2 = \arctan\left(\dfrac{y_2 - y_1}{z_2 - z_1}\csc(\theta)\right) \end{cases}$$

Question 3. *For the point* $(0, y, z)$ *belonging to the white board, compute* $\theta, \varphi_1, \varphi_2$.

$$L_3^2 = (x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2$$
$$L_1^2 + 2L_2L_3\cos(\alpha) = L_2^2 + L_3^2$$
$$L_2^2 + 2L_1L_3\cos(\beta) = L_1^2 + L_3^2$$



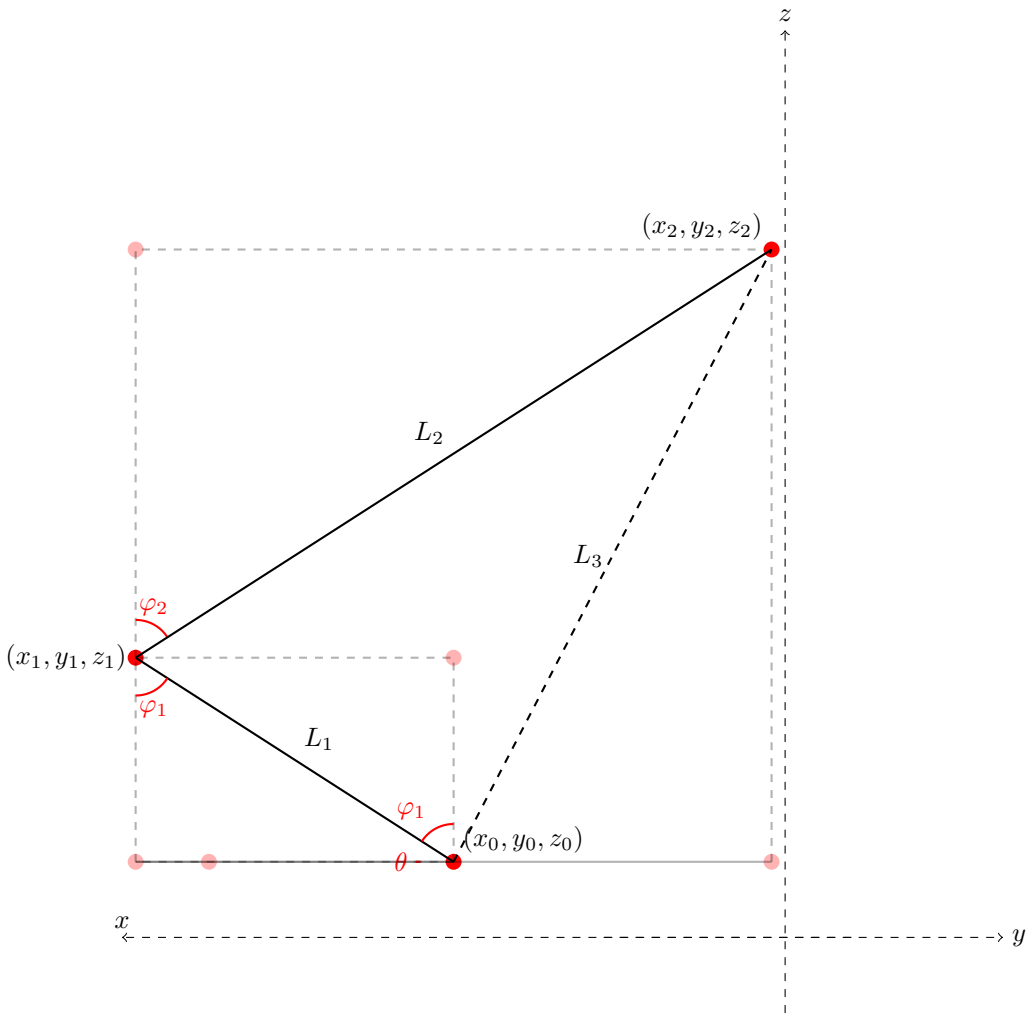
$$\alpha = \arccos\left(\frac{L_2^2 + L_3^2 - L_1^2}{2L_2L_3}\right)$$

$$\beta = \arccos\left(\frac{L_1^2 + L_3^2 - L_2^2}{2L_1L_3}\right)$$

$$\delta = \arccos\left(\frac{|z_2 - z_0|}{L_3}\right)$$

$$\varphi_2 = \alpha + \delta$$

$$\varphi_1 = \beta - \delta$$


z
(x_2, y_2, z_2)
L_2
L_3
φ_2
(x_1, y_1, z_1)
φ_1
L_1
φ_1
(x_0, y_0, z_0)
θ
x
y