

MATEMATIKA I

INTEGRAL

-2. Integral Beberapa Fungsi Transenden-

Fungsi Transenden

- Fungsi aljabar: polinom, rasional, akar, mutlak
- Fungsi transenden: yang bukan merupakan fungsi aljabar

contoh: invers fungsi, fungsi logaritma asli, fungsi eksponen asli dll

2.1 Integral Fungsi-fungsi Eksponen dan Ln

Ingat kembali turunan fungsi-fungsi eksponen dan ln!

✓ $\int e^x dx = e^x + C$

✓ $\int a^x dx = \frac{1}{\ln a} a^x + C$

✓ $\int \ln x dx = x \ln x - x + C, x > 0$

✓ $\int \frac{1}{x} dx = \ln|x| + C, x \neq 0$

✓ Rumus-rumus diatas dapat dikombinasikan dengan teknik substitusi yang sudah dipelajari



Logaritma Asli

Definisi:

Fungsi Logaritma Asli dinyatakan oleh \ln , didefinisikan sebagai

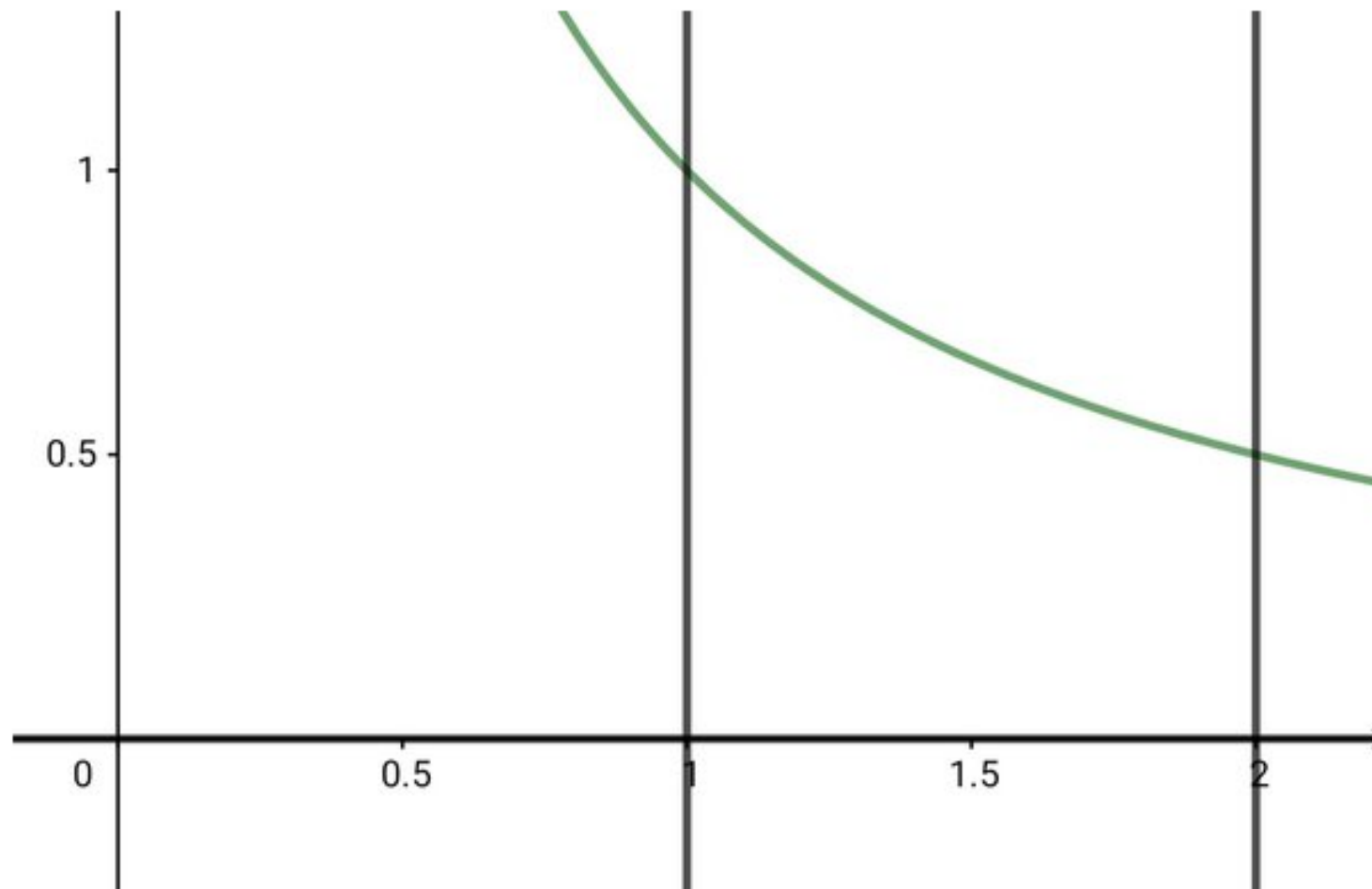
$$f(x) = \ln x = \int_1^x \frac{1}{t} dt, \quad x > 0$$

$$f(x) = \int x^r dx = \frac{1}{r+1} x^{r+1} + C, \quad r \neq -1$$

$$\text{jika } r = -1, \int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$



Gampangannya, $\ln a$ adalah luas daerah yang dibatasi oleh grafik $y = \frac{1}{x}$, $y = 0$, $x = 1$, dan $x = a$.



$$y = {}^{10}\log a$$

$$y = {}^e\ln a$$

$$e \approx 2,718...(\text{bilangan Euler})$$

Sifat-sifat fungsi logaritma asli:

Jika $a > 0, b > 0$, dan r bilangan rasional, maka

i) $\ln 1 = 0$

ii) $\ln(ab) = \ln a + \ln b$

iii) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

iv) $\ln(a^r) = r \ln a$

Selesaikan

$$1. \int \frac{1}{2x+1} dx =$$

$$2. \int \frac{1}{1-x} dx =$$

$$3. \int \frac{5}{2x+7} dx =$$

$$4. \int \frac{2x}{x^2-5} dx =$$

$$5. \int \frac{x}{10-x^2} dx =$$

Fungsi Eksponen Asli

Invers dari fungsi logaritma asli. Notasi : $\exp(x) = e^x$

$$x = {}^e \ln y \Leftrightarrow y = e^x$$

Sifat-sifat $\exp(x)$

$$e^0 = 1$$

$$e^1 = e \approx 2,718...$$

$${}^e \ln e = 1 \rightarrow e^1 = e$$

$$e^a \cdot e^b = e^{a+b}$$

$$\frac{e^a}{e^b} = e^{a-b}$$

$$(e^a)^b = e^{a \cdot b}$$



$$1. \int e^{-4x} dx$$

$$2. \int e^{\frac{x}{2}} dx$$

$$3. \int e^{2x-1} dx$$

$$4. \int 2x^2 e^{x^3} dx$$

$$5. \int \frac{e^x}{e^x - 1} dx$$

2.1 Integral Fungsi-fungsi Eksponen dan Ln

$$1. \int \frac{e^x + e^{2x} - 1}{e^x} dx$$

$$2. \int e^{2x+1} dx$$

$$3. \int 2^{(3x+2)} dx$$

$$4. \int (t+1)e^{(t^2+2t-1)} dt$$

$$5. \int x \ln(x^2 + 1) dx$$

$$6. \int \frac{e^x}{(e^x + 2)} dx$$

$$7. \int e^x (e^x + 4)^5 dx$$

$$8. \int \frac{\ln \sqrt{x^2 + 2}}{\sqrt{4x^4 + 8x^2}} dx =$$

2.2 Integral Fungsi Invers Trigonometri dan Hiperbolik

$$✓ \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$✓ \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$✓ \int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$$

$$✓ \int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1-x^2} + C$$

$$✓ \int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln|1+x^2| + C$$

$$✓ \int \sinh x dx = \cosh x + C$$

$$✓ \int \cosh x dx = \sinh x + C$$

$$✓ \int \tanh x dx = \ln(\cosh x) + C$$



2.2 Integral Fungsi Invers Trigonometri dan Hiperbolik

$$\int u \, dv = u \cdot v - \int v \, du \text{ (integral parsial pertemuan ke-4)}$$

$$\int \tan^{-1} x \, dx = \dots$$

$$\rightarrow u = \tan^{-1} x ; dv = dx$$

$$\rightarrow du = \frac{1}{1+x^2} dx ; v = x$$

$$\begin{aligned} \int \tan^{-1} x \, dx &= x \cdot \tan^{-1} x - \int x \cdot \frac{1}{1+x^2} dx \\ &= x \cdot \tan^{-1} x - \frac{1}{2} \int 2x \cdot \frac{1}{1+x^2} dx \\ &= x \cdot \tan^{-1} x - \frac{1}{2} \int \frac{d(1+x^2)}{1+x^2} dx \\ &= x \cdot \tan^{-1} x - \frac{1}{2} \cdot \ln|1+x^2| + C \end{aligned}$$

2.2 Integral Fungsi Invers Trigonometri dan Hiperbolik

Latihan soal. Selesaikan!

1. $\int \left(\frac{1}{\sqrt{1-x^2}} - \frac{2}{1+x^2} \right) dx$

2. $\int (\sin^{-1} 5x + 2 \tan^{-1} 2x) dx$

3. $\int \cosh x \sinh^3 x dx$

4. $\int (\sinh^2 x - \cosh^2 x) dx$

Hint: $\cosh^2 x - \sinh^2 x = 1$

