

A petri dish containing a bacterial culture, showing numerous small, rod-shaped bacteria. The dish is set against a light yellow background.

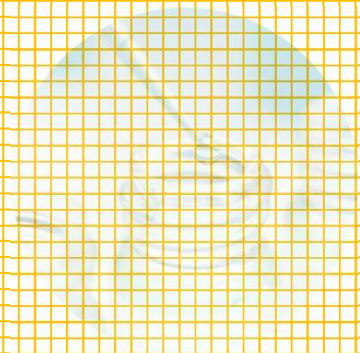
DERET TAYLOR

A small, solid blue horizontal line.

MATEMATIKA 2

Mengapa Perlu belajar deret kuasa / deret fungsi, khususnya deret Taylor

- Sebagai nilai pendekatan dari suatu fungsi
- Dapat digunakan untuk menganalisis eror suatu pengukuran
- Penyajian deret dapat mempermudah komputasi



Perhatikan deret Taylor untuk fungsi satu variable berikut

Pendekatan sampai **order satu** dan **order dua**

$$P_1(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$P_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2$$

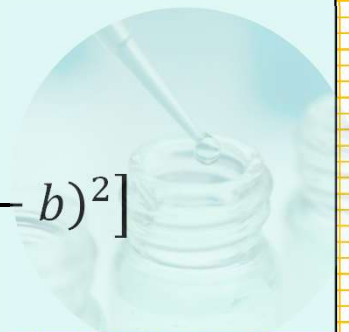
Misalkan $f: R^2 \rightarrow R$ fungsi dua variable, maka deret Taylor di titik (a, b)

1) Pendekatan sampai **order satu**

$$P_1(a, b) = f(a, b) + [f_x(a, b)(x - a) + f_y(a, b)(y - b)]$$

2) Pendekatan sampai **order dua**

$$P_2(a, b) = f(a, b) + [f_x(a, b)(x - a) + f_y(a, b)(y - b)] \\ + \frac{1}{2} [f_{xx}(a, b)(x - a)^2 + 2f_{xy}(a, b)(x - a)(y - b) + f_{yy}(a, b)(y - b)^2]$$



Contoh 1

Deretkan fungsi $f(x, y) = 1 - e^{-x^2-3y^2}$ di titik $(0,0)$ sampai order satu dan dua.

$$f_x(x, y) = 2xe^{-x^2-2y^2}$$

$$f_y(x, y) = 4ye^{-x^2-2y^2}$$

$$f_{xx}(x, y) = (2 - 4x^2)e^{-x^2-2y^2}$$

$$f_{yy}(x, y) = (4 - 16y^2)e^{-x^2-2y^2}$$

$$f_{xy}(x, y) = -8xye^{-x^2-2y^2}$$

$$\begin{aligned} P_1(x, y) &= f(0, 0) + [f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0)] \\ &= (1 - e^0) + (0x + 0y) = 0 \end{aligned}$$

$$\begin{aligned} P_2(x, y) &= f(0, 0) + [f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0)] \\ &\quad + \frac{1}{2}[f_{xx}(0, 0)(x - 0)^2 + 2f_{xy}(0, 0)(x - 0)(y - 0) + f_{yy}(0, 0)(y - 0)^2] \\ &= (1 - e^0) + (0x + 0y) + \frac{1}{2}[2x^2 + 2 \cdot 0xy + 4y^2] \\ &= x^2 + 2y^2 \end{aligned}$$

Contoh 2

Deretkan fungsi $f(x, y) = \cos(x) \cos(y)$ di titik $(0,0)$ sampai order dua.

		$x = 0, y = 0$
$f(x, y)$	$\cos x \cos y,$	1
f_x	$-\sin x \cos y,$	0
f_y	$-\cos x \sin y,$	0
f_{xx}	$-\cos x \cos y,$	-1
f_{xy}	$\sin x \sin y,$	0
f_{yy}	$-\cos x \cos y,$	-1

$$f(x, y) = f(0, 0) + x f_x(0, 0) + y f_y(0, 0) + \frac{1}{2!} [x^2 f_{xx}^2(0, 0) + 2xy f_{xy}(0, 0) + y^2 f_{yy}(0, 0)]$$

$$f(x, y) \cong 1 + x(0) + y(0) + \frac{1}{2!} [x^2(-1) + 2xy(0) + y^2(-1)]$$

$$f(x, y) \cong 1 - x^2 - y^2$$

