

Tugas Aljabar Linear Matrix

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1. Menentukan matrix A dan B

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix}$$

2. Menentukan OKE atau OBE 3 buah

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}_{H_{13}(A)} \quad A = \begin{bmatrix} 2 & 3 & 4 \\ -4 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix}_{K_{41(-3)}(B)} \quad B = \begin{bmatrix} -3 & 15 & 9 & 3 \\ -2 & 16 & -8 & 16 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & 20 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}_{H_{2(1)3(2)}(A)} \quad A = \begin{bmatrix} -1 & -2 & -3 \\ 0 & 6 & 9 \\ 2 & 3 & 4 \end{bmatrix}$$

3. Menentukan equivalen antara matrix A dengan C dan B dengan D

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -2 & -3 \\ 2 & 3 & 4 \\ -2 & 4 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}_{H_{23}(A)} \quad A = \begin{bmatrix} -1 & -2 & -3 \\ 2 & 3 & 4 \\ -4 & 0 & 1 \end{bmatrix}_{H_{3(1)1(-1)}} \quad A = \begin{bmatrix} -1 & -2 & -3 \\ 2 & 3 & 4 \\ -2 & 4 & 7 \end{bmatrix}$$

Jadi $A \sim C$

$$B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix} \quad D = \begin{bmatrix} -6 & 15 & 33 & -2 \\ 10 & 16 & 0 & -2 \\ 5 & 2 & -8 & 0 \\ -1 & 1 & 17 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix} K_{14}(B) \quad B = \begin{bmatrix} -6 & 15 & 9 & -2 \\ 10 & 16 & -8 & -2 \\ 5 & 2 & -5 & 0 \\ -1 & 1 & 8 & -7 \end{bmatrix} K_{3(2)2(1)}(B) \quad B = \begin{bmatrix} -6 & 15 & 33 & -2 \\ 10 & 16 & 0 & -2 \\ 5 & 2 & -8 & 0 \\ -1 & 1 & 17 & -7 \end{bmatrix}$$

Jadi, $B \sim D$

4. Menentukan determinan matriks A menggunakan metode sarrus dan B menggunakan metode kofaktor.

a). Menentukan determinan matriks A:

$$\begin{vmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$$

Menggunakan metode sarrus

$$\begin{vmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix} \begin{matrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{matrix} \begin{matrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{matrix}$$

$$(1 \times 0 \times 4) + (-2 \times 1 \times 2) + (-3 \times -4 \times 3) - (2 \times 0 \times (-3)) - (3 \times 1 \times (-1)) - (4 \times (-4) \times (-2)) = 0 + 4 + 36 - 0 + 3 - 32 = 11$$

Jadi, $\det(A) = 11$

b). Menggunakan metode kofaktor

$$B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix}$$

$$b_{11}K_{11} + b_{12}K_{12} + b_{13}K_{13} + b_{14}K_{14} =$$

$$-3 \begin{vmatrix} 16 & -8 & 10 \\ 2 & -5 & 5 \\ 1 & 8 & -1 \end{vmatrix} - 15 \begin{vmatrix} -2 & -8 & 10 \\ 0 & -5 & 5 \\ -7 & 8 & -1 \end{vmatrix} + 9 \begin{vmatrix} -2 & 16 & 10 \\ 0 & 2 & 5 \\ -7 & 1 & -1 \end{vmatrix} - (-6) \begin{vmatrix} -2 & 16 & -8 \\ 0 & 2 & -5 \\ -7 & 1 & 8 \end{vmatrix}$$

$$\begin{vmatrix} 16 & -8 & 10 \\ 2 & -5 & 5 \\ 1 & 8 & -1 \end{vmatrix} = 16(-35) - (-8)(-7) + 10(21) = -406$$

$$\begin{vmatrix} -2 & -8 & 10 \\ 0 & -5 & 5 \\ -7 & 8 & -1 \end{vmatrix} = (-2)(-35) - (-8)(35) + 10(-35) = 0$$

$$\begin{vmatrix} -2 & 16 & 10 \\ 0 & 2 & 5 \\ -7 & 1 & -1 \end{vmatrix} = (-2)(-7) - 16(35) + 10(14) = -406$$

$$\begin{vmatrix} -2 & 16 & -8 \\ 0 & 2 & -5 \\ -7 & 1 & 8 \end{vmatrix} = (-2)(21) - 16(-35) + (-8)(14) = 406$$

$$(-3)(-406) - 15(0) + 9(-406) - (-6)(406) = 406(3 - 9 + 6) = 406 \cdot 0 = 0$$

Jadi, $\det(B) = 0$

5. Menentukan Rank Matriks

$$A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \text{ karna tidak bisa dicari lagi, maka Rank (A) = 3}$$

$$B = \begin{bmatrix} -3 & 15 & 9 & -6 \\ -2 & 16 & -8 & 10 \\ 0 & 2 & -5 & 5 \\ -7 & 1 & 8 & -1 \end{bmatrix} \mathbf{B}_{3(1)4(1)} \quad B = \begin{bmatrix} -3 & 15 & 3 & -6 \\ -2 & 16 & 2 & 10 \\ 0 & 2 & 0 & 5 \\ -7 & 1 & 7 & -1 \end{bmatrix} \mathbf{B}_{1(1)3(1)} \quad B = \begin{bmatrix} 0 & 15 & 3 & -6 \\ 0 & 16 & 2 & 10 \\ 0 & 2 & 0 & 5 \\ 0 & 1 & 7 & -1 \end{bmatrix}$$

Maka, $\text{Rank}(B) = 3$