UNIT II: Statistical Control Charts

Detailed Study Notes

Duration: 20 Hours
Focus: Construction and statistical basis of control charts, variables and attributes, process capability, chart patterns.

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1. Statistical Basis of Control Charts

1.1 Definition and Purpose

Control Charts are graphical tools for monitoring process stability and detecting out-of-control (assignable causes) situations. They display sample statistics over time with:

- **Center Line (CL)**: Represents the in-control process mean
- **Upper Control Limit (UCL)**: Typically set at +3 standard deviations from CL
- **Lower Control Limit (LCL)**: Typically set at -3 standard deviations from CL

1.2 Three-Sigma (3σ) Basis

- **Statistical Basis**: Three-sigma (3σ) control limits are standard practice. For a normal process, about 99.73% of data points fall within ±3σ bounds.
- **Probability of False Alarm**: With 3σ limits, the probability of a single point falling outside limits when the process is truly in control is only 0.0027 (27 out of 10,000 points).
- **Type I Error (α)**: Risk of incorrectly signaling an out-of-control condition when none exists (~0.0027 per sample).
- **Type II Error (β)**: Risk of failing to detect a true out-of-control condition.

1.3 Advantages of Control Charts

✓ **Visual Clarity**: Easily visualizes process trends, shifts, and patterns over time
✓ **Prevention Philosophy**: Helps prevent defects and avoid unnecessary process adjustments
✓ **Diagnostic Value**: Enables process improvement through pattern analysis and root-cause identification
✓ **Early Detection**: Detects process changes quickly before significant defects occur
✓ **Process Capability**: Provides data to estimate process capability and set realistic targets
✓ **Reduced Costs**: Minimizes rework, scrap, and warranty claims through prevention

1.4 Disadvantages of Control Charts

✗ **Assumptions**: Assumes independence/normality of data; violations degrade performance
✗ **Sensitivity to Design**: Poorly chosen limits (too tight/too wide) cause either frequent false alarms or missed shifts
✗ **Sample Strategy Dependency**: Results heavily depend on rational subgrouping and

sampling frequency
✗ **Operator Confusion**: Multiple limits and pattern rules can confuse shop-floor personnel
✗ **False Alarms**: May trigger unnecessary investigations and process adjustments (tampering)
✗ **Autocorrelation Issues**: Chemical/process industries often have correlated data, reducing chart effectiveness

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2. Control Charts for Variables (X-bar, R-chart, S-chart)

2.1 Introduction to Variables Charts

Used when the quality characteristic is measured on a continuous scale. Variables charts provide rich quantitative information and high sensitivity to process changes.

Sample Size: Typically n = 3 to 5 units per subgroup, taken at regular intervals (e.g., every hour).

2.2 X-bar Chart (Average Chart)

Purpose: Monitors the process mean (central tendency) over time.

Construction Steps:

1. **Collect Data**: Take k preliminary samples (typically 20-25) each of size n
2. **Calculate Statistics**:
    - Sample mean: $\bar{x}_i = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$
    - Overall average: $\bar{\bar{x}} = \frac{1}{k}\sum_{i=1}^{k} \bar{x}_i$
3. **Set Control Limits**:
    - Center Line: $CL = \bar{\bar{x}}$
    - Upper Control Limit: $UCL = \bar{\bar{x}} + A_2\bar{R}$
    - Lower Control Limit: $LCL = \bar{\bar{x}} - A_2\bar{R}$

   (where A_2 is a constant depending on sample size n, found in statistical tables)

Interpretation: Points beyond control limits or nonrandom patterns indicate out-of-control conditions.

2.3 R-Chart (Range Chart)

Purpose: Tracks within-sample range, giving information on short-term process variability.

Why Control Variability First?: The X-bar control limits depend on process variability. If R is out of control, the X-bar limits are unreliable.

Construction Steps:

1. **Calculate Range**: $R_i = X_{max,i} - X_{min,i}$ for each sample
2. **Average Range**: $\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i$
3. **Set Control Limits**:
    - Center Line: $CL = \bar{R}$
    - Upper Control Limit: $UCL_R = D_4\bar{R}$
    - Lower Control Limit: $LCL_R = D_3\bar{R}$

(where D_3 and D_4 are constants from tables)

Advantage: Simple to compute; range is intuitive for operators.

2.4 S-Chart (Standard Deviation Chart)

Purpose: Uses sample standard deviation to estimate variability; more statistically efficient than range for large sample sizes.

When to Use: Preferred when n ≥ 10 (range becomes less efficient for larger samples).

Construction:

1. **Calculate Standard Deviation**: $s_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \bar{x}_i)^2}$
2. **Average Standard Deviation**: $\bar{s} = \frac{1}{k}\sum_{i=1}^{k} s_i$
3. **Set Control Limits**:
    - Center Line: $CL = \bar{s}$
    - UCL: $UCL_s = B_4\bar{s}$
    - LCL: $LCL_s = B_3\bar{s}$

2.5 Rational Sub-grouping

Definition: The practice of forming subgroups so that variation *within* a subgroup is due only to common (chance) causes, while variation *between* subgroups can capture assignable causes.

Two Approaches:

1. **Snapshot Method** (Consecutive Units):
    - Each sample consists of units produced at the same time
    - Minimizes within-sample variability
    - Best for detecting process mean shifts
    - Example: Take 5 consecutive units every hour
2. **Random Sample Method** (Stratified):
    - Each sample is a random selection from all units produced since last sample
    - Captures all types of process changes
    - Better for acceptance decisions
    - Risk: If process drifts within sampling interval, ranges become artificially large

Advantages of Good Subgrouping: Maximizes sensitivity to real shifts; yields realistic variability estimates.

Disadvantages of Poor Subgrouping: Completely masks problems; can hide critical patterns.

2.6 Revised and Modified Control Limits

Process:

1. Plot preliminary data against trial limits
2. Identify out-of-control points and investigate for assignable causes
3. If assignable cause found, remove that point
4. Recalculate control limits with remaining points

5. Recheck all points against new limits
6. Repeat until all points plot in control

Purpose: Ensures control limits reflect a truly in-control process for future monitoring.

2.7 Advantages of Variables Charts

✓ **High Sensitivity**: Detect relatively small shifts in process mean (≈1-2σ shifts)
✓ **Rich Information**: Provide estimates of both mean and variability
✓ **Capability Estimation**: Enable calculation of Cp and Cpk indices
✓ **Small Sample Sizes**: n=3-5 sufficient for effective control
✓ **Industry Standard**: Widely used and understood in manufacturing
✓ **Diagnostic Power**: Patterns reveal root causes of variability

2.8 Disadvantages of Variables Charts

✗ **Measurement Required**: Need precise, continuous measurement equipment
✗ **Complexity**: More formulas and constants; requires statistical knowledge
✗ **Subgrouping Critical**: Poor rational subgrouping severely compromises effectiveness
✗ **Autocorrelation Sensitive**: Correlated data (common in chemical processes) violates independence assumption
✗ **Non-Normality**: Performance degrades if data aren't approximately normal
✗ **Single Observations**: Ineffective when only one measurement available per time period

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3. Control Charts for Attributes (np-chart, p-chart, c-chart, u-chart)

3.1 When to Use Attribute Charts

- Quality characteristic is a count or classification (pass/fail, defective/non-defective)
- Measurement is impractical or expensive
- Interest is in proportion/number of defects, not precise measurements
- Data follows binomial (defectives) or Poisson (defects) distribution

3.2 np-Chart (Number of Defectives)

Purpose: Monitors the number of defective items in a fixed-size sample.

Requirements: Constant sample size n for all subgroups.

Construction:

- Plot np (actual number of defectives) for each sample
- $CL = n\bar{p} = \frac{1}{k}\sum_{i=1}^{k}(np)_i$
- $UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$
- $LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

where $\bar{p} = n\bar{p}/n$

Advantage: Easy for operators (whole number counts).

3.3 p-Chart (Proportion Defective)

Purpose: Monitors the fraction/proportion of defective items; useful when sample size varies.

Construction:

- Plot p = (number defective)/n for each sample
- $CL = \bar{p} = \frac{\sum \text{(number of defectives)}}{\sum \text{(sample size)}}$
- Control limits depend on sample size: $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$

Advantage: Accommodates variable sample sizes; normalized to proportion.

3.4 c-Chart (Number of Defects per Unit)

Purpose: Tracks number of defects per inspection unit (e.g., defects per electronic board, per meter of fabric).

Requirements: Fixed inspection area/unit size.

Construction (Poisson distribution):

- Plot c (actual number of defects) for each unit
- $CL = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$
- $UCL = \bar{c} + 3\sqrt{\bar{c}}$
- $LCL = \bar{c} - 3\sqrt{\bar{c}}$ (or 0 if negative)

Use Case: Multiple defects possible; each unit inspected uniformly.

3.5 u-Chart (Defects per Unit, Variable Unit Size)

Purpose: Tracks defects per unit when inspection unit size varies.

Construction:

- Calculate u = c/n (defects per unit)
- $CL = \bar{u}$
- $UCL = \bar{u} + 3\sqrt{\bar{u}/n_i}$
- $LCL = \bar{u} - 3\sqrt{\bar{u}/n_i}$

Advantage: Flexible; accommodates varying inspection areas.

3.6 Advantages of Attribute Charts

✓ **Simplicity**: Only counting required; no measurement equipment
✓ **Operator-Friendly**: Easy to understand pass/fail concepts
✓ **Practical**: Applicable to many service and manufacturing contexts
✓ **Cost-Effective**: Low inspection cost per unit
✓ **Multiple Defects**: c and u charts capture multiple defect types

3.7 Disadvantages of Attribute Charts

✗ **Lower Sensitivity**: Less ability to detect small process shifts
✗ **Larger Samples**: Need bigger samples than variables charts for same detection power
✗ **Less Information**: Count provides less detail than measurement
✗ **Approximation Errors**: Binomial/Poisson approximations can fail with very high or very low defect rates
✗ **Binary Data**: No information about how bad a defect is, only presence/absence

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4. Comparison: Variable vs Attribute Charts

Aspect	Variables Charts (X-bar, R, S)	Attributes Charts (p, np, c, u)
Data Type	Continuous measurements (length, weight, time)	Discrete counts (pass/fail, defects)
Information Content	Rich—provides mean and variability	Limited—only counts or proportions
Sensitivity	High—detects small shifts (~1σ)	Lower—better for large shifts
Sample Size	Small (n=3-5)	Usually larger (n=50-200+)
Measurement	Requires precise measurement tools	Only counting required
Complexity	More statistical knowledge needed	Simpler, operator-friendly
Best Use	Precision manufacturing, tight specs	Final inspection, pass/fail criteria
Cost	Higher per measurement	Lower per inspection
Capability Info	Direct Cp/Cpk calculation	Indirect; less detailed

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5. Analysis of Patterns on Control Charts

5.1 Non-Random Patterns

Even if all points fall within control limits, non-random patterns indicate assignable causes:

Pattern Types:

1. **Runs**: Sequence of points consistently above or below center line
    - Rule: 8+ consecutive points on one side signals an assignable cause
2. **Trends**: Consistent increase or decrease in values
    - Rule: 6+ points in a row consistently increasing/decreasing
3. **Cycles**: Repeating pattern of highs and lows
    - Suggests cyclic assignable cause (e.g., operator fatigue, material batches)
4. **Clustering**: Points hugging the center line
    - May indicate measurement system issues or subgroup sampling problems
5. **Stratification**: Two separate parallel lines
    - Often indicates two machines, two shift conditions, or two measurement systems

5.2 Western Electric Control Chart Rules

Standard sensitivity rules (beyond single 3σ point):

Rule	Signal	Meaning
1	1 point beyond ±3σ	Out of control
2	2 of 3 consecutive points beyond ±2σ (same side)	Likely shift
3	4 of 5 consecutive points beyond ±1σ (same side)	Drift detected
4	8 consecutive points on one side of CL	Process shift
5	6+ points trending up or down	Trend detected

Benefit: Rules increase sensitivity to detect small changes faster.

Risk: May increase false alarm rate; requires clear communication with operators.

5.3 Advantages of Pattern Analysis

✓ Detects gradual shifts missed by single-point limits
✓ Identifies systematic causes (cycles, trends)
✓ Enables faster corrective action
✓ Improved process understanding

5.4 Disadvantages

✗ Complex rules can confuse operators
✗ Increased false alarm risk
✗ Requires trained personnel to interpret patterns
✗ May waste resources investigating false signals

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6. Estimation of Process Capability

6.1 Process Capability Definition

Process Capability: The ability of a process operating in statistical control to produce output within specified customer tolerance limits.

6.2 Capability Indices from Control Chart Data

Step 1: Verify Control

- X-bar and R/S charts must show only common causes (statistical control)
- Remove points with identified assignable causes and recalculate limits

Step 2: Estimate Process Parameters

From X-bar/R chart:

- Process mean: $\hat{\mu} = \bar{\bar{x}}$
- Process standard deviation: $\hat{\sigma} = \frac{\bar{R}}{d_2}$ (where d_2 is a constant from tables)

From X-bar/S chart:

- Process mean: $\hat{\mu} = \bar{\bar{x}}$
- Process standard deviation: $\hat{\sigma} = \frac{\bar{s}}{c_4}$ (where c_4 is a constant)

6.3 Capability Index Formulas

Cp (Potential Capability Index):

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

- Assumes process is centered at midpoint of specification limits
- **Interpretation**:
    - Cp > 1.33: Capable process (recommended minimum)
    - 1.0 < Cp < 1.33: Marginally capable; improvement recommended
    - Cp < 1.0: Incapable; improvements essential

Cpk (Process Capability Index—accounts for centering):

$$C_{pk} = \min\left(\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right)$$

- More realistic; reflects both variability and process centering
- **Interpretation**: Same as Cp, but more stringent when process is off-center

- Industry standard: Cpk ≥ 1.33 for new processes; ≥ 1.67 for critical characteristics

6.4 Practical Use of Capability Analysis

- **Design Phase**: Assess whether process can meet specifications before full production
- **Contract Negotiations**: Justify quality claims to customers
- **Improvement Priority**: Identify processes needing the most improvement
- **Make-or-Buy Decisions**: Evaluate outsourcing vs in-house production

6.5 Advantages of Capability Analysis

✓ Quantifies ability to meet customer specs in simple metric
✓ Identifies which processes need improvement
✓ Supports business decisions (pricing, contracts, investment)
✓ Tracks improvement over time
✓ Differentiates between variability and centering issues (Cp vs Cpk)

6.6 Disadvantages and Limitations

✗ Meaningful only if process is in statistical control
✗ Assumes approximate normality; non-normal data gives misleading results
✗ Estimates only from past data; doesn't guarantee future performance
✗ Doesn't address special causes; only common-cause capability
✗ Can be misused to justify poor performance if out-of-control data included

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7. Comprehensive Advantages & Disadvantages Summary

7.1 Advantages of Control Charts

Advantage	Benefit
Early Detection	Catches problems before major defects occur
Prevention Philosophy	Builds quality in rather than inspecting for defects
Reduce Waste	Eliminates scrap and rework through prevention
Lower Costs	Fewer defects = reduced warranty, returns, liability
Process Improvement	Identifies specific assignable causes for targeted action
Data-Driven	Objective statistical basis for decisions
Predictability	Enables forecasting and resource planning
Operator Empowerment	Clear decision rules enable shop-floor action
Capability Assessment	Quantifies process potential
Continuous Improvement	Provides framework for ongoing optimization

7.2 Disadvantages of Control Charts

Disadvantage	Challenge
Statistical Assumptions	Independence/normality assumptions often violated
Sample Strategy Dependent	Poor subgrouping completely compromises effectiveness
Sensitive Design	Wrong limits cause false alarms or missed shifts
Complexity	Multiple rules and limits confuse operators
False Alarms	Can trigger unnecessary investigations (waste)
Autocorrelation Issues	Common in chemical/process industries; requires advanced charts
Non-Normal Data	Manufacturing processes often produce skewed/non-normal output
Investment Required	Equipment, training, analysis infrastructure needed
Initial Setup Effort	Significant time to establish stable baseline
Over-Control Risk	Tampering (excessive adjustment) worsens variability

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8. Key Takeaways

1. **Foundation**: 3σ limits provide ~99.73% confidence; about 0.27% false alarm risk
2. **Variable Charts Superiority**: X-bar/R/S charts more sensitive than attribute charts; superior for precision manufacturing
3. **Rational Subgrouping Critical**: Success depends absolutely on intelligent subgroup selection
4. **Pattern Analysis Important**: Western Electric rules increase sensitivity but require clear protocols
5. **Capability Estimation**: Cp/Cpk indices quantify ability to meet specifications; require in-control data
6. **Integration with Process**: Charts are a means to improvement, not the goal itself
7. **Operator Training Essential**: Personnel must understand purpose, rules, and out-of-control action plans

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9. Summary Table: Control Chart Selection Guide

Situation	Best Chart	Reason
Continuous measurements, small samples	X-bar/R or X-bar/S	High sensitivity; rich info
Pass/fail inspection	p-chart or np-chart	Simple; direct proportions
Defects per unit, fixed area	c-chart	Poisson model ideal
Defects per unit, variable area	u-chart	Adjusts for area variation
Very high-volume process	X-bar/R with rational grouping	Cost-effective; early detection
Service/transaction data	Attribute charts	Counting easier than measuring
Chemical process with drifting mean	Special/advanced charts (CUSUM, EWMA)	Standard Shewhart ineffective
Low defect rate	Modified attribute charts	Standard binomial limits break down

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10. References

Montgomery, D.C. (2020). *Introduction to Statistical Quality Control* (9th ed.). John Wiley & Sons.

Deming, W.E. (2000). *Out of the Crisis*. MIT Press.

Western Electric Company. (1956). *Statistical Quality Control Handbook*. Delco Electronics.

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This document provides comprehensive coverage of UNIT II topics with practical examples, mathematical foundations, and comparative analysis of advantages and disadvantages for all control chart types.