UNIT III: Acceptance Sampling Plan
Comprehensive In-Depth Study Notes

Duration: 15 Hours
Focus: Principles of acceptance sampling, construction and statistical basis of sampling plans, variables and attributes sampling, operating characteristic curves, process capability from sampling, comparison between sampling methods.

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1. Fundamental Concepts: Why Acceptance Sampling Exists

1.1 The Core Problem Acceptance Sampling Solves

In manufacturing and supply chain quality control, organizations face a recurring decision:

> "I have received a lot (batch) of N items. Should I accept this lot for use/sale, or reject it and send it back?"

Three approaches to answering this question:

1. **100% Inspection** (no sampling)
   - Inspect every single item in the lot
   - Definitive but expensive, time-consuming, and impossible for destructive tests

2. **No Inspection** (pure acceptance)
   - Accept all lots without checking
   - Fast and cheap but high quality risk

3. **Acceptance Sampling** (intelligent middle ground)
   - Inspect a random sample of n items from lot of N
   - Make accept/reject decision based on defectives found in sample
   - Balances cost, time, and risk statistically

Example scenario: A car manufacturer receives 5,000 electronic control modules from a supplier. Testing each one takes 2 hours and is expensive. Testing to failure is destructive. Solution: Randomly inspect 125 units, observe 3 defectives, decide: accept or reject the entire lot of 5,000?

1.2 When Acceptance Sampling Makes Economic and Practical Sense

Cost-Driven Situations:
- Inspection per unit is expensive (precision measurement, X-ray, ultrasonic testing)
- Large volumes make 100% inspection economically prohibitive
- Inspection labor costs are high relative to product cost

Destructive Testing Requirements:
- Strength testing (materials break during test)
- Battery life testing (consumes battery)
- Endurance testing (wears out the product)
- Cannot test 100% without destroying entire lot

Time-Critical Decisions:

- Production schedules demand fast lot decisions
- Need rapid material flow through manufacturing
- Quality assessment cannot block production lines

Supplier Relationship Contexts:
- Long-standing supplier with good history (can rely on sampling)
- Established process stability and historical quality data
- Mutually agreed quality targets (AQL, LTPD) between supplier and customer

1.3 When Acceptance Sampling is Inappropriate or Dangerous

Avoid acceptance sampling for:
- **Critical safety applications**: Aerospace, medical devices, automotive safety systems (must use 100% inspection or very stringent statistical plans)
- **Regulatory requirements**: Many regulations (FDA, FAA, automotive) mandate 100% inspection for certain characteristics
- **Very high quality needs**: Six Sigma level production (3.4 defects per million) cannot be verified by sampling
- **Unknown suppliers**: No historical quality data means cannot calibrate sampling plan appropriately
- **Unstable processes**: If process quality varies wildly, sampling plan assumptions collapse
- **Small lots**: Statistical basis becomes unreliable when lot size is only slightly larger than sample size

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2. Anatomy of a Single-Sampling Plan

2.1 Formal Definition

A **single-sampling plan** for attributes is specified by exactly three parameters:

Plan Notation: (N; n, c)

- **N** = Lot size (total number of items submitted for inspection)
- **n** = Sample size (number of items randomly selected and inspected from the lot)
- **c** = Acceptance number (maximum allowable number of defectives in the sample for lot acceptance)

2.2 The Decision Rule

Procedure:

1. Randomly select n items from lot of N items
2. Inspect each of the n items
3. Count the number of defective items: d
4. **Decision:**
   - If $d \leq c$ → **ACCEPT the lot** (release it to next process/customer)
   - If $d > c$ → **REJECT the lot** (return to supplier or 100% inspect)

2.3 Practical Example: Electronic Component Lot

Scenario:

- Receiving inspection of printed circuit board assemblies
- Lot size: $N = 2,000$ units
- Sampling plan: (2,000; 125, 3)
- Interpretation: Sample 125 units; accept lot if ≤ 3 defectives found; reject if 4 or more

Inspection Results:
- Unit 1: Good
- Unit 2: Defective (1st defect)
- Unit 3: Good
- Unit 4: Defective (2nd defect)
- ...continue inspecting...
- Unit 125: Defective (3rd defect)
- **Total defectives found: $d = 3$**
- **Decision: $d = 3 \le c = 3$, therefore ACCEPT lot**

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3. The Operating Characteristic (OC) Curve: Heart of Acceptance Sampling

3.1 Definition and Purpose

Operating Characteristic (OC) Curve: A plot showing the **probability that a lot will be accepted** under the sampling plan as a function of the **true fraction defective in that lot**.

Axes:
- x-axis: p = fraction defective in the lot (ranging from 0 to 1.0, or 0% to 100%)
- y-axis: $Pa(p)$ = probability of lot acceptance under the plan (ranging from 0 to 1.0)

Why this matters: The OC curve is the "report card" of a sampling plan. It tells you:
- How likely is a "good" lot to be accepted?
- How likely is a "bad" lot to be rejected?
- How sharply does the plan discriminate between good and bad lots?

3.2 Mathematical Basis: The Binomial Distribution

For a lot with large N (or infinite population equivalently), the number of defectives d found in sample size n follows a **binomial distribution** with parameters n and p.

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4. Producer's Risk (α), Consumer's Risk (β), AQL, and LTPD

4.1 The Two Key Risks

Every sampling plan involves two risks:

1. **Type I Error (Producer's Risk α)**
   - Definition: Probability of **rejecting a good lot**
   - Victim: The supplier/producer
   - Impact: Good product returned to producer unnecessarily; lost revenue, customer relation damage

2. **Type II Error (Consumer's Risk β)**
   - Definition: Probability of **accepting a bad lot**
   - Victim: The customer receiving defective product
   - Impact: Defective products in customer's supply chain; quality problems, warranty costs

4.2 AQL: Acceptable Quality Level

Definition: The poorest fraction defective (in percentage) that the **customer considers satisfactory** as a process average.

Key Characteristics:
- Not a specification or target for the supplier
- Not a property of the sampling plan
- Rather, it's the **operating quality level** the supplier claims to maintain
- Represents "good" process performance

Typical Values: 0.5%, 1.0%, 1.5%, 2.5%, depending on product criticality

4.3 LTPD: Lot Tolerance Percent Defective

Definition: The poorest quality level in an **individual lot** that the **customer is willing to accept**.

Key Characteristics:
- Represents "bad" lot quality
- The customer wants **LOW** acceptance probability at this point
- Typically $Pa(LTPD) = \beta \approx 0.10$
- This means: if a lot is truly at LTPD quality, ~10% acceptance (bad luck), ~90% rejection (intended)

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5. Dodge-Romig Sampling Inspection Tables

5.1 Purpose and Historical Development

Background:
- Developed by H.F. Dodge and H.G. Romig at Bell Laboratories, 1940s
- Pioneering work in statistical acceptance sampling
- Tables provide **pre-computed optimal sampling plans**
- Recognize two major objectives: minimize AOQL or minimize ATI

Why tables matter:
- Eliminates need for manual OC curve calculations
- Provides industry-standard, proven plans
- Easy implementation without statistical expertise
- Ensures plans are statistically optimized for stated objectives

5.2 Two Types of Dodge-Romig Tables

Type 1: AOQL Plans
- Design objective: Achieve specified maximum average outgoing quality limit
- Typical values: AOQL = 0.5%, 1.0%, 1.5%, 2.0%, 2.5%, 3.0%, 4.0%, 5.0%

- Given process average (e.g., 1% defective), tables return (n, c) that guarantee AOQL
- Also provided: producer risk α at the process average point
- Plans minimize ATI while hitting AOQL target

5.3 How to Use Dodge-Romig AOQL Tables

Example: AOQL = 2.0% Table

Step 1: Define Requirements
- Target AOQL: 2.0%
- Process average (most likely incoming defect rate): 1.0%
- Lot size: 2,000 units

Step 2: Select Table
- Look for table titled "Dodge-Romig AOQL = 2.0%"

Step 3: Locate Lot Size Row
- Tables organized by lot size ranges
- Find "Lot sizes 1,201 to 3,200" row

Step 4: Find Process Average Column
- Column headers: 0.5%, 1.0%, 1.5%, 2.0%, ...
- Select "1.0%" column

Step 5: Read (n, c)
- Intersection gives recommended plan
- Example might show: n = 125, c = 2
- Provides acceptance probability at AQL (typically ~0.90-0.95)

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6. Average Outgoing Quality (AOQ) and AOQL

6.1 What is Rectifying Inspection?

Assumption: When a lot is **rejected** under the sampling plan, it undergoes **100% inspection**. All defectives found are:
- Removed and scrapped, or
- Removed and reworked, or
- Replaced with good items from stock

Result: Rejected lots exit inspection with essentially **zero defectives**.

This is called **rectifying inspection** because it "corrects" or "rectifies" the quality of rejected lots.

6.2 Average Outgoing Quality (AOQ)

Definition: The **expected fraction defective** in products leaving the inspection area (accepted lots + screened rejected lots), over a long sequence of lots from a process with fraction defective p.

AOQL: Average Outgoing Quality Limit

Definition: The **maximum value** of the AOQ curve across all possible incoming quality levels.

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7. Double-Sampling Plans: A More Efficient Alternative

7.1 Why Double Sampling?

Single sampling drawbacks:
- Fixed sample size regardless of early evidence
- May over-inspect when evidence is clear
- May under-inspect when evidence is borderline

Double sampling advantage:
- Take small first sample
- If evidence is clear (very few or many defects), decide immediately
- If borderline, take second sample before final decision
- Typically lower Average Sample Number (ASN) than single sampling

7.2 Double-Sampling Plan Parameters

Specified by: (N; n_1, c_1, n_2, c_2)

- n_1: first sample size
- c_1: first acceptance number (accept lot if $d_1 \leq c_1$)
- $r_1 = c_1 + 1$: first rejection number (reject lot if $d_1 > r_1$)
- n_2: second sample size
- c_2: combined acceptance number (accept lot if $d_1 + d_2 \leq c_2$)
- $r_2 = c_2 + 1$: combined rejection number (reject if $d_1 + d_2 > c_2$)

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8. Lot Formation: Critical Success Factor

8.1 Why Lot Formation Matters

The **lot** is the fundamental unit of acceptance sampling. How lots are formed directly impacts:
- OC curve validity
- Ability to detect quality problems
- Effectiveness of corrective actions

8.2 Requirements for Valid Lot Formation

1. Homogeneity
- Items in a lot should be produced under **same conditions**
- Same machine, same operator, same shift if possible
- Same raw material batch
- Produced at **approximately the same time**

2. Larger Lots Preferred
- Statistically more valid (n/N ratio better)
- Economically more efficient to inspect large lots than small ones
- Reduces overhead of lot-handling per item

3. Practical Conformability
- Lot size should match physical handling systems at supplier and customer

- Packaging should minimize damage in transit
- Should be easy to select random sample from lot

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9. Random Sampling: Non-Negotiable Requirement

9.1 The Sampling Principle

Every item in the lot must have **equal probability** of being selected for the sample. Otherwise, bias is introduced.

9.2 Implementation Methods

Method 1: Random Number Approach
- Assign number (1 to N) to every item in lot
- Use random number table or computer generator
- Select items corresponding to n random numbers
- Inspect those specific items

Method 2: Serial Number/Code
- If items have serial/code numbers, use those
- Generate n random numbers in appropriate range
- Select items matching those numbers

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10. Comprehensive Advantages and Disadvantages of Acceptance Sampling

10.1 Major Advantages

Advantage	**Why It Matters**	**Business Impact**
Cost Reduction (50-90% lower inspection cost)	Sample size n usually << lot size N	Lower QC budget; higher profitability
Time Efficiency	Faster decisions on lot acceptance	Faster material flow; reduce delays
Applicable to Destructive Tests	Only way for destructive testing	Enables quality verification
Statistical Basis	Objective, quantified risks (α, β)	Defensible in disputes
Risk Control	Explicitly defined risks	Both parties can agree on risks
Scalability	Works well for mass production	Practical for many industries
AOQL Guarantee	Customer assured max quality	Long-term confidence
Standardized Methodology	Dodge-Romig, ANSI Z1.4, ISO 2859	Easy to implement

10.2 Major Disadvantages

Disadvantage	**Why It's a Problem**	**Risk/Impact**
Sampling Risk	Chance of Type I & II errors	Unfair to suppliers or risk to customer
No Process Improvement	Reactive screening only	Doesn't address root causes

Assumption Dependency	Relies on stable process	Breaks if assumptions violated
Not for Critical Applications	Cannot use for aerospace/medical	Must use 100% inspection
Training Required	Personnel need statistical knowledge	Errors common with untrained staff
Small Lots Problem	Statistical basis weak	Cannot use for small batches

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11. ANSI Z1.4 and ISO 2859: Modern Standards

ANSI/ASQ Z1.4 (and equivalent ISO 2859-1):
- Comprehensive sampling system
- Includes multiple sampling schemes
- **Switching rules:** Normal ↔ Tightened ↔ Reduced based on acceptance history
- Adaptive: adjusts inspection level based on recent performance
- Level I (reduced), Level II (normal), Level III (tightened)

Benefits:
- Encourages continuous improvement
- Rewards good suppliers with reduced inspection
- Penalizes deteriorating quality with tightened inspection
- Industry standard (recognized globally)

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12. Complete Workflow: From Requirements to Implementation

12.1 Design Phase

Step 1: Define Quality Objectives
- Agree on AQL with supplier (good lot quality level)
- Agree on LTPD with customer (rejectable lot level)
- Select acceptable risks α (typically 0.05) and β (typically 0.10)

Step 2: Determine Lot Parameters
- Lot size N based on production batch sizes
- Ensure lots are homogeneous
- Plan logistics of lot formation

Step 3: Select Sampling Method
- Single sampling: simplest, easiest administration
- Double sampling: lower inspection for clear cases
- ANSI Z1.4: adaptive, industry standard

Step 4: Determine (n, c)
- Use OC curve design
- Or use Dodge-Romig tables (AOQL or LTPD minimizing)
- Verify AOQL/ATI acceptable
- Document plan clearly

12.2 Implementation Phase

Step 1: Communication
- Distribute plan details to all stakeholders

- Explain decision rule clearly: "Accept if $d \leq c$"
- Set expectations on timelines

Step 2: Training
- Train inspection personnel on random sampling
- Explain importance of objectivity
- Document inspection procedures

Step 3: Infrastructure
- Prepare lot labels and tracking
- Set up recording forms
- Establish communication channel for lot decisions

12.3 Monitoring and Evolution Phase

Ongoing:
- Track acceptance/rejection rates
- Monitor if actual quality matches expected
- Use ANSI Z1.4 switching rules if applicable

Adjustments:
- If too many rejections: investigate supplier process
- If zero defects streak: consider reduced inspection
- If quality degrades: tighten plan, reduce acceptance numbers

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13. Key Takeaways

After mastering UNIT III, you should understand:

1. Why acceptance sampling exists and its economic trade-offs
2. OC curve fundamentals: how to read, interpret, and use them
3. Design concepts: AQL, LTPD, α, β and how they relate to planning
4. AOQ and AOQL: what they guarantee and when to use them
5. Dodge-Romig tables: how to select appropriate plans quickly
6. Advantages and disadvantages: when to use/avoid acceptance sampling
7. Proper implementation: random sampling, lot formation, training
8. Evolution path: from sampling → SPC → partnership with suppliers
9. Mathematical foundation: binomial distribution and its application

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14. Summary Comparison Table: Quick Reference

Aspect	**Single Sampling**	**Double Sampling**	**ANSI Z1.4**	**Dodge-Romig AOQL**
Complexity	Simple	Moderate	Complex (adaptive)	Simple (tables)
ASN	n (fixed)	Lower (typically)	Varies with switching	Fixed for given p
OC Control	Fixed (n,c)	Flexible (n_1,c_1,n_2,c_2)	Adaptive rules	Designed for AOQL
Best Use	First-time implementation	Well-understood suppliers	Ongoing relationships	AOQL critical
Admin Burden	Minimal	Moderate	Higher (tracking)	Minimal

Learning Curve	Easiest	Moderate	Steep	Easiest

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15. References and Further Study

Primary Textbooks:
- Montgomery, D.C. (2020). Introduction to Statistical Quality Control (9th ed.). John Wiley & Sons.
- Duncan, A.J. (1986). Quality Control and Industrial Statistics (5th ed.). Richard D. Irwin.

Standards and Tables:
- ANSI/ASQ Z1.4-2018. Sampling Procedures and Tables for Inspection by Attributes
- ISO 2859-1:2021. Sampling procedures for inspection by attributes
- Dodge, H.F., & Romig, H.G. (1959). Sampling Inspection Tables: Single and Double Sampling (2nd ed.)

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This comprehensive guide teaches acceptance sampling concepts deeply and prepares you for examination and professional application.