UNIT 1 INTRODUCTION TO STATISTICAL QUALITY CONTROL

Structure

1.1 INTRODUCTION

In our day-to-day life, we use different products and services from the time we get up until we get back to bed. For example, we use different types of products such as toothpaste, soap, detergent, clothes, food, gas stove, vehicle, phone, computer, TV, electric bulb, etc. and we also use different types of services such as water supply, electricity, transportation, health care, education, etc.

All of us wish that these products and services should fulfil certain specifications when we use them. If the product/service fulfils the specifications needed for its proper use, we say that it is of **good quality**. If it does not fulfil the specifications, it is said to be of **poor quality**.

In the face of ever-growing market competition, the main objective of the manufacturer or producer is to achieve quality assurance in manufacturing and service organisations so that his/her product/service can meet the existing competition in the market. In order to achieve this objective, different statistical tools have been developed, which are useful for controlling the quality of products vis-a-vis certain specifications or standard. The technique of

controlling product quality against the specifications using statistical tools is known as **Statistical Quality Control (SQC)**.

In this unit, you will learn about the concepts and various aspects of SQC. In Sec. 1.2, we define the term **quality** and discuss dimensions of quality, quality control and historical review of quality control. In Sec. 1.3, we describe various aspects of SQC, e.g., the elements and techniques of SQC, statistical process control and product control. We also discuss the causes of variation, which may be due to chance or could be assigned to some factors in the production process in Sec. 1.4. In Sec. 1.5, we introduce the control chart – a tool used in statistical quality control to indicate whether a process is under control or out-of-control. We explain the concept of 3σ limits, different patterns of the control chart and advantages and limitations of SQC in Secs. 1.5 to 1.8. In the next unit, you will study the control charts for variables.

Objectives

After studying this unit, you should be able to:

- explain the concepts of quality, quality control and statistical quality control (SQC);
- describe the need for statistical quality control;
- distinguish between causes of variation (chance causes and assignable causes) in the production process;
- describe the techniques of SQC;
- define process control and product control;
- explain the concept of control chart, the principle underlying 3σ limits and various control chart patterns; and
- discuss advantages and limitations of SQC.

1.2 QUALITY

In the introduction of the unit, we have just given you the flavour of the notion of quality. We now define the term **quality**. The dictionary meaning of quality is **Degree of Excellence**. It means that both products and services should be excellent. This definition of the term **quality** is rather subjective as it depends on the perception of **excellence** and varies from situation to situation or from person to person. If 20 different people are asked to define quality, most probably there will be 20 different answers. However, **the best** will be widely used. If we ask what quality they want, mostly people say they want the best. This indicates that the general public opinion about the term quality is **the best**. However, if we observe the buying habits of people, we find that most people buy an item at a discount price in a sale or a mid priced item rather than the most expensive one. This indicates that while buying something, people want the best that they can afford. Further, apart from price, there are many other things to be considered for quality such as cost, size, performance, warranty, appearance, etc.

The concept of quality as goodness or the best or luxury is not adequate for professionals working in quality control because **the best** depends on an individual's perception. For example, a person may think that a car is a good vehicle for Delhi roads, but a comparatively poor vehicle for the forests of

Himachal Pradesh, Madhya Pradesh, etc. because the best vehicle for these places would be one with four-wheel drive. Similarly, a good walking shoe is good for walking, but unsuitable for running. Therefore, the **intended use** of the product or service needs to be included in the concept of quality. So we need to define the term **quality**.

1.2.1 Defining Quality

There are several ways of defining quality:

1. Initially, quality was defined as **conforming to specifications**. This means that any product should be manufactured according to given specifications. If a product fulfils its specifications, it is considered to be a **quality** product.

   There are some problems with this definition. In some cases, it is found that even though a product conforms to all specifications, its utility is not up to the mark. For example, suppose a person wishes to purchase a touch screen mobile phone having clear sound. The shopkeeper shows him/her such a mobile set. But the person may feel that the size of the mobile phone is big. So he/she may not purchase it because it is not fit for his/her use. Therefore, from the point of view of customers, such products are not useful. Thus, there is a need to redefine the term quality.

2. The definition of quality was modified to include **fitness for use**. This means that the manufactured product should be such that it conforms to its specifications and is fit for use. However, it was found that in some cases, a product conformed to all specifications and was fit for use but could not be sold as it did not appeal to the customer. Therefore, to sell any product it is necessary to incorporate the **customer's viewpoint**.

3. According to the third definition, quality was **customer satisfaction**. This means that a product, which brings satisfaction to the customer could be termed as a quality product. Then it was found that every customer would have plenty of demands in respect of each product that he/she wished to buy. However, customers seldom express all of their expectations. For example, suppose a person goes to a restaurant and orders a pizza. If the pizza given to him is not hot, that person would definitely be unhappy. But it is also true that the customer would never mention that he/she wanted a hot pizza! There are many situations where the customers are unhappy, even when all their expressed or stated needs are fulfilled. Hence, while defining quality, there is a need for considering unexpressed or unstated needs of customers.

4. The fourth definition given for quality was **delighting the customer**. Delight is one step ahead of satisfaction. When a product fulfils both the expressed and unexpressed needs of the customer, he/she is delighted. However, this definition was also improved upon.

5. The fifth definition given for quality was **enchanting the customers**. According to this definition, the manufacturer plays a dual role: firstly, he/she should know the need of the customers. Secondly, he/she should make customers aware of this fact and also make them feel that these are the products they want. For example, an electric bulb manufacturer needs to educate customers about LEDs and make them want to buy LEDs as these are low on consumption of electricity even though if these cost slightly more. Now-a-days, every manufacturer is expected to follow this definition.

From the above discussion, we may conclude that in the manufacturing and services sector, the following aspects have to be incorporated in the definition of quality:

- conforming to specifications,
- fitness for use,
- customer satisfaction,
- delighting the customer, and
- enchanting the customers.

Having explained the concept of quality in industry, and defined it we now describe various dimensions of quality.

1.2.2 Dimensions of Quality

David A. Garvin is the C. Roland Christensen Professor of Business Administration at the Harvard Business School, Massachussetes USA.

In the previous section, we have explained the concept of quality and defined it. In 1988, David A. Garvin summarised eight basic elements of quality, which are known as the **dimensions of quality**. We describe them briefly.

i) Performance

The first dimension of quality is **performance**. It refers to the primary operating characteristics of a product. Consumers judge the quality of any product based on its performance after comparing it with the competitor's products or the prevailing market standard. For example, a mobile phone can be judged by its clarity of sound, weight, size, functions, etc. Similarly, a motorcycle can be judged by its pick-up, fuel efficiency, etc.

ii) Features

Features constitute the second dimension of quality. These refer to additional characteristics available in products along with the primary operating characteristics. For example, complimentary drinks and snacks in a flight or hotel, Bluetooth and FM in mobile, etc. A customers choice is also influenced by this dimension of quality.

iii) Reliability

Reliability refers to the probability of a product's failure within a specified time period. If a product fails frequently, we say that it is an unreliable product. For example, if a TV of a particular company requires frequent repair, we say that it is unreliable. There are many products such as laptops, TVs, automobiles, etc. in which customer's view about quality is influenced by the reliability of the product.

iv) Conformance (Agreement)

Conformance means meeting specifications. Customers obviously want that the product should meet its specifications. For example, when we purchase a motorcycle, we check whether the sitting space, weight, size, pick-up, fuel efficiency, etc. conform to the specifications mentioned by the company. This dimension also influences customers choice.

v) Durability

Durability refers to the measure of product life. It can also be understood as the operational life of the product, i.e., how long the product can be used without replacement. The life of an electric bulb is an example of durability:

when its filament burns up, the bulb needs to be replaced because at this stage, repair is not possible. Durability also changes customers view.

vi) Serviceability

Serviceability is the sixth dimension of the quality. Consumers are concerned not only about a product break-down, but also about the time taken before the product is serviced restored. Serviceability is concerned with how readily a product can be serviced back into operational mode. For example, suppose a customer wishes to buy a washing machine. While deciding on the company, he/she may consider how long the company takes to service or repair it in case it breaks down.

vii) Aesthetics

The seventh dimension of quality is **aesthetics**, which means how a product looks, sounds, feels, etc. The aesthetic value of a product is purely subjective. For example, some people may find a particular car aesthetically appealing while others may not.

viii) Reputation

Reputation is related to the past performance of the company. In many cases, customers also check out the quality of products made by the company in the past. For instance, if a company launches a new car, it is usually assumed by the customer that the new model would be successful because the past performance of other cars of that company was good.

In order to design and manufacture products of high quality, it is necessary to incorporate all **eight dimensions of quality**.

You may like to pause here and check your understanding about the definition of quality and its dimensions by answering the following exercise.

E1) Choose the correct option from the following:

i) Quality means
   a) fitness for use
   b) degree of excellence
   c) conformance to requirement or specifications
   d) all of the above

ii) Primary operating characteristic of a product is known as
   a) aesthetics
   b) reliability
   c) performance
   d) features

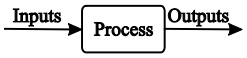
iii) Durability refers to a measure of
   a) product life
   b) specifications
   c) past performance of the product
   d) the probability of a product's failure within a specified time period

So far you have studied the definition of quality and its dimensions. We now introduce the concept of quality control.

1.2.3 Quality Control

We first understand what we mean by control.

A process is a series of actions or operations that transforms input to output.



The process/procedure/method that is applied to meet the specifications or standard is known as **control**. Process control works on feedback and comprises the following steps:

1. Choose the **parameter** to control, i.e., we first choose the characteristic that we intend to control such as length, height, weight, defects, etc.
2. Choose the **unit of measurement**, e.g., centimeter (cm), millimeter (mm), gram (g), etc.
3. Set the **standard** or goal for the parameter to control, e.g., 5 cm, 10 g, etc.
4. Select a sensing device, which can measure the parameter to control in terms of the unit of measure, e.g., scale, a weighing balance, etc.
5. Measure the actual performance.
6. Compare the actual performance with the standard.
7. Take necessary action when there is a difference between actual performance and the standard.

Thus, **quality control** can be defined as:

"The process by which we measure the quality characteristics of the product, compare them with the specifications or standard and take suitable actions whenever there is a difference between actual quality and the specifications or standard".

Now-a-days, quality is controlled by using statistical tools. This technique is known as **Statistical Quality Control**, which we describe in the next section. However, before you study further, you may like to know how the concept of quality control evolved historically.

1.2.4 Historical Review of Quality Control

The notion of quality control is not new. We may say that it began when human beings lived in forests and ate raw plants and animals. In those times, they used natural materials and may have faced problems such as which plants were suitable for eating and which ones were poisonous for health. With the evolution of early technology, human beings started processing natural materials. They made products and examined themselves whether these worked properly for their purpose or not. For example, they produced rough hewn stone tools. They may have checked the point of the tool to see whether it was sharp enough for their purpose. In those times, human beings were both manufacturers and users. This situation changed with time.

As civilisations arose, jobs were divided and different people got specialised in production of different goods and services. For example, farmers cultivated crops, potters made pots; weavers wove the cloth required for everyone. This specialisation created a separation between the producer and the user. Thus, the producer/manufacturer and the user were no longer the same individual. However, the goods were not so complicated and the user had long familiarity

with the products through prior use. Therefore, the user could assure that the products were fit for use. The situation changed drastically with the advent of industrialisation and the notion of quality control underwent a sea change.

For the sake of interest, we describe below the important stages in the historical evolution of quality control:

1. **Operator or craftsman quality control**

   From early civilisation until the industrial revolution, quality control was ensured/observed either by a single worker (operator or craftsman) or a very small number of workers who totally controlled the quality of their work. This situation continued until the nineteenth century.

2. **Foreman or supervisor control**

   Large factories were established after the industrial revolution to meet the increasing demand of consumers. Therefore, the number of workers increased in the factories. This gave rise to the need of supervisors who guided, controlled and administered other workers. Generally, in those times, the supervisor was picked up from among the workers of the factory who had thorough knowledge of the work. Checking the quality of a job done by workers was also the responsibility of the supervisor functions. This situation continued until the First World War.

3. **Inspector of quality control**

   The manufacturing system became more complex due to interchangeable components on a mass scale and it became essential to critically examine each component with the help of measuring instruments. Obviously, this job could not be performed by the supervisor because he/she had expertise only in one component of the product. Thus, a full time inspector came on the scene to inspect the finished goods and quality control was separated from production.

4. **Statistical quality control**

   Initially, quality control inspectors inspected all products. However, this was costly and time consuming. In 1924, **Walter A. Shewhart**, a researcher of Bell Telephone Laboratories (a Research and Development Unit of the American Telephone and Telegraph Company) developed the technique of **statistical control charts** for the control of product variability. This is usually considered as the beginning of statistical quality control (SQC).

   In the same decade, Harold F. Dodge and Harry G. Roming, researchers of Bell Telephone Laboratories, developed statistically based acceptance sampling as an alternative of 100% inspection.

   The techniques of control chart and acceptance sampling were used in Bell system, but neither of these were widely adopted outside it. However, this changed during the Second World War. The ordinance department of the U.S. Army was facing the problem of how to get large quantities of arms and ammunitions from multiple suppliers at acceptable levels of quality. In 1942, a quality control section was established in the War Department and acceptance sampling came to be used widely.

   After the Second World War, Japan was trying to revive from the devastations of world war. Japanese industries were almost destroyed and its leaders knew that rebuilding the industry was essential for the survival

Walter Andrew Shewhart
(1891-1967)

An American physicist, engineer and statistician, sometimes known as the father of **statistical quality control**.

William Edwards Deming
(1900-1993)
An American engineer, statistician, professor, author, lecturer and

of the nation. Japanese industrialists decided to engage an American statistician as their consultant. The man they chose was W. Edwards Deming, who had studied with Walter Shewhart. Deming developed theories on how statistics could be used to improve industrial quality. He gave a series of lectures in Japan on statistical quality improvement methods. Japan adopted these methods and within a few decades, became one of the most successful industrial nations on the earth.

So far, we have given you an overview of the concepts of quality and quality control. We now discuss statistical quality control.

1.3 STATISTICAL QUALITY CONTROL (SQC)

In the highly competitive market today, the main objective of manufacturers or producers is to achieve quality assurance in manufacturing and service organisations. In order to achieve this objective, different statistical tools have been developed, which are useful for controlling the quality of products vis-a-vis the specifications or standard. The technique of using statistical tools for controlling product quality vis-a-vis the specifications is known as **Statistical Quality Control (SQC)**.

Statistical quality control is defined as the technique of applying statistical methods based on the theory of probability and sampling to establish quality standard and to maintain it in the most economical manner.

Let us now outline the elements that constitute SQC.

1.3.1 Elements of SQC

The following are the main elements of SQC:

a) Sample Inspection

We know that 100% inspection needs huge expenditure of time, money, labour and resources. Further, if the nature of the product is such that it is completely destroyed during the process of inspection, e.g., a bulb, candle, ammunition, food, etc., 100% inspection is not practicable. Therefore, SQC is based on **sampling inspection**. In sampling inspection method, some items or units (called sample) are randomly selected from the process and then each and every unit of the sample is inspected.

b) Use of Statistical Methods

Some commonly used statistical tools such as random sampling, mean, range, standard deviation, mean deviation, standard error and concepts such as probability, binomial distribution, Poisson distribution, normal distribution, etc., are used in SQC. Since, quality control method involves extensive use of statistics, it is termed as **Statistical Quality Control**.

c) Fundamental Objective

The fundamental objective of SQC is to decide whether the unit produced is according to its specifications or not. If the unit produced is not according to its specifications and there is a variation in quality, it becomes necessary to trace the causes of variation and eliminate them if possible.

d) Decision Making

With the help of SQC, we **decide** whether the quality of the product or the process of manufacturing/producing goods is under control or not.

e) Specifications, Production and Inspection

SQC method helps in deciding about the specifications, production and inspection of an product.

We now describe the techniques of statistical quality control.

1.3.2 Techniques of Statistical Quality Control

The important techniques used for statistical quality control can be broadly classified into two categories:

- Statistical Process Control (SPC) or simply Process Control, and
- Product Control.

These techniques are further classified into different categories as shown in Fig.1.1.

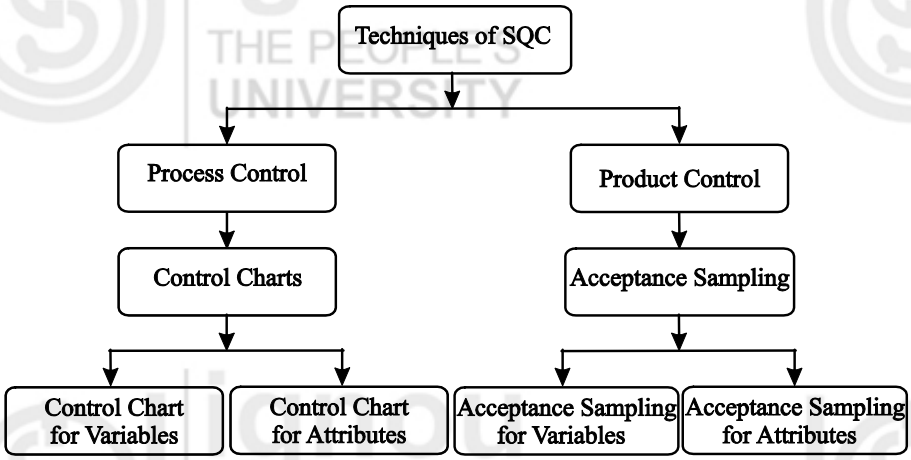


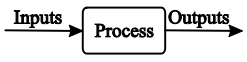
Fig. 1.1: Classification of SQC techniques.

Let us discuss both categories of SQC techniques in some detail.

1.3.3 Statistical Process Control (SPC)

Statistical Process Control (SPC) or simply process control (PC) is the first part of SQC. For understanding SPC, first of all, we should understand the concept of **process** in quality control.

A process is a series of operations or actions that transforms input to output. It is said to be stable or repeatable if the resulting output product is of the given specifications or standard quality. But sometimes, due to certain causes such as poor quality of raw material, change in the machine settings, use of unskilled work force, improper machine, etc., the stable process is disturbed. In such situations, we require a tool or technique through which we can control the process. This technique is known as **statistical process control (SPC)**.



Statistical process control is a technique used for understanding and monitoring the process by collecting data on quality characteristics periodically from the process, analysing them and taking suitable actions whenever there is a difference between actual quality and the specifications or standard.

Statistical process control technique is widely used in almost all manufacturing processes for achieving process stability and making continuous improvements in product quality. Its major tools are:

1. Histogram
2. Check sheet
3. Pareto chart
4. Cause and effect diagram
5. Process flow diagram
6. Scatter diagram
7. Control chart

Of these seven tools we shall describe only the technique of **"control chart"** given by W.A. Shewhart in 1924 because it is the most preferred technique today. In fact, it is probably an outstanding technique for controlling and improving quality. We introduce the basics of control chart in Sec. 1.5. In Units 2, 3 and 4 of this course, we discuss various types of control charts.

1.3.4 Product Control

In many situations, the product is so complex that a manufacturer cannot make all components or parts of the product. Therefore, one or more component (s) of a product are purchased from outside agents or suppliers and the manufacturer does not have a direct control over the quality of such component (s). In such situations, the manufacturer faces the problem of controlling the quality of component (s) sourced from elsewhere. In addition, the manufacturer needs to **control the quality of the final product**. So he/she also faces the problem of ensuring that the final product meets its specifications and various lots of the product do not contain an excessively large number of defective items. Such types of problems come under the category of **product control**.

Product control means to control the products in such a way that these are free from defects and conform to their specifications.

Initially, product control was done by 100% inspection, which means that each and every unit produced or received from the outside suppliers was inspected. This type of inspection has the advantage of giving complete assurance that all defective units have been eliminated from the inspected lot. However, it is time consuming and costly. Also, if a product is destroyed under inspection, e.g., light bulb, crackers, ammunition, picture tube of the TV, etc., 100% inspection is not possible.

Sampling inspection or acceptance sampling was developed as an alternative of 100% inspection.

Acceptance sampling is a technique in which a small part or fraction of items/units is selected randomly from a lot and the selected items/units are inspected to decide whether the lot should be accepted or rejected on the basis of the information obtained from sample inspection. This is how product control is achieved through acceptance sampling.

Acceptance sampling is also of different types. We shall discuss it in detail in Unit 5.

You may like to pause now and check your understanding of various aspects of SQC discussed in this section. Answer the following exercises.

E2) Choose the correct option from the following:

i) Statistical quality control (SQC) is a technique of
   a) process control
   b) product control
   c) both (a) and (b)

ii) The statistical techniques used in statistical quality control are
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

iii) Process control is achieved through
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

iv) Product control is achieved through
   a) control charts
   b) acceptance sampling plans
   c) both (a) and (b)

You have learnt that SQC techniques are used for process control and product control so that the products of desired quality may be manufactured. We now explain the causes of variation in the process.

1.4 CAUSES OF VARIATION

Variation in manufactured products is inevitable; it is a fact of nature and industrial life. Even when a production process is well designed or carefully maintained, no two products are identical. The difference between any two products could be very large, moderate, very small or even undetectable depending on the sources of variation. For example, the weight of a particular model of automobile varies from unit to unit, the weight of packets of milk may differ very slightly from each other, the length of refills of ball pens, the diameter of cricket balls may also be different and so on. The existence of variation in products affects quality. So the aim of SQC is to trace the sources of such variation and try to eliminate them as far as possible. The causes of variation are broadly classified into two categories:

1. Chance causes, and
2. Assignable causes.

1.4.1 Chance Causes

Chance causes are also known as **random** or **natural** or **common causes**. Even in a well designed or carefully maintained production process, variability exists in the product due to some natural/random causes. Even if the process is operated under the same conditions, that is, the quality of raw materials used is same and there is no change in the machine settings, operators or the environment, there is a specific pattern of variability in the product. For

example, the diameter of ball bearings varies slightly, there is a slight variation in the weight of cricket balls, the fuel efficiency of a particular model of automobile is varies slightly and so on. Such variability is due to different common or chance causes, may which affect the process output in minor ways. Such causes are known as **chance causes of variation**. These may arise due to, inflexibility of aged machines, variability in purchased material, poor lighting, extent of worker training or other non obvious reasons. These may or may not be present at the same time, but when taken together produce random results. If the quality of the output varies too much due to chance causes, the process must be redesigned or modified to eliminate one or more of these causes. Since process redesigning or modification is the responsibility of the management, the elimination of common/chance causes of variation is usually the responsibility of the management and not that of workers. It may not be possible to eliminate all chance causes in a process. However, even **if variation due to chance causes is present in the production process, it is still said to be under statistical control.**

1.4.2 Assignable Causes

Another kind of variability may be present occasionally in the output of a process. The causes of such type of variability are **not** due to the process design, but take place because of changes in raw material, machine, operator, environment or any other component of the process. These causes are called **assignable causes** and are also known as **special** or **non-random** or **unnatural causes**. Accidental improper setting of the machine, a worker falling ill and still continuing to work, change of operators or shift, breakages, misreading of scales, batch of defective raw material, etc. are examples of assignable causes.

Since the effect of assignable causes is localised within a process, these may be eliminated by workers or their immediate supervisor. The variability due to assignable causes is generally larger than the variability due to chance causes and it usually represents an unacceptable level of process performance. A process that is operating in the presence of assignable causes is said to be an **out-of-control process**.

You can now check your understanding of the causes of variation by answering the following exercise.

E3) A company manufactures cricket balls. The statistical quality controller of the company finds that there is a variation in the weight of cricket balls. Answers the following:

i) The variation in the weight of cricket balls may be due to

   a) assignable causes

   b) chance causes

   c) both (a) and (b)

ii) If the variation in the weight is due to chance causes, it is

   a) controllable

   b) not controllable

   c) both (a) and (b)

iii) If the variation in the weight is due to assignable causes, it is

   a) controllable

b) not controllable
c) both (a) and (b)

iv) The variation due to chance causes
a) is tolerable
b) does not affecting the quality of a product
c) is uncontrollable
d) all of the above

v) The variation due to assignable causes
a) can be removed
b) cannot be removed
c) can be removed sometimes
d) can be removed most of the times

Having explained the causes of variations in the production process, we now introduce the technique of control chart. It is the main technique used for process control.

1.5 CONTROL CHARTS

You will agree that graphical representation is one of the most sensitive statistical instruments. So we can represent the quality characteristic of the output product such as weight, length, diameter, defects, etc. graphically to understand, describe or monitor process variation. The idea of representing quality characteristics graphically was first given by Walter A. Shewhart. He invented **control charts** for the industrial processes to distinguish acceptable (chance) variation from the assignable variation. He observed that with the help of control charts, the occurrence of assignable causes of variation could be detected quickly and corrective action could be taken to eliminate them.

A **control chart** is a two-dimensional graphical display of a quality characteristic that has been measured or computed in terms of mean or other statistic from samples and plotted against the sample number or time at which the sample is taken from the process.

The concept of control chart is based on the theory of sampling and probability. In a control chart, a sample statistic of a quality characteristic such as mean, range, proportion of defective units, etc. is taken along the Y-axis and the sample number or time is taken along the X-axis. A control chart consists of three horizontal lines, which are described below:

1. **Centre Line (CL) –** The centre line of a control chart represents the value which can have three different interpretations depending on the available data. First, it can be the **average value of the quality characteristic** or the **average of the plotted points**. Second, it can be a **standard** or **reference value**, based on representative prior data or an **aimed (targeted) value** based on specifications. Third, it can be the **population parameter** if that value is known. The **centre line is usually represented by a solid line**.

2. **Upper Control Line –** The upper control line represents the **upper value of the variation in the quality characteristic**. So this line is called **upper conrol limit (UCL)**. Usually, the **UCL is shown by a dotted line**.
3. **Lower Control Line –** The lower control line represents the **lower value of the variation in the quality characteristic**. So this line is called **lower control limit (LCL)**. Usually, the **LCL is shown by a dotted line**.

The UCL and LCL also have three interpretations depending on the available data same as the centre line. These limits are obtained using the concept of **3σ (three sigma) limits**, which we shall describe in Sec. 1.6.

For construction of control charts, we select samples of few units/items at regular intervals from the process. Then we measure the quality characteristic, e.g., length of foil, diameter of ball bearing, weight of cricket ball, etc. for each unit of the selected samples or count the number of defects, number of defective units, etc. for each selected sample. After that, we calculate the statistic such as mean, standard deviation, range, proportion of defective unit, etc. for each sample. Then the values of the calculated statistic are plotted against the sample number or time. The sample points on the graph may be joined by a line. Joining the sample points by line segments is not compulsory but by doing so, we can easily understand their sequence (pattern) over time.

If all sample points lie on or in between the upper and lower control limits, the control chart indicates that the **process is under statistical control**. That is, only chance causes are present in the process. **No assignable cause is present** in the process. However, if one or more sample points lie outside the control limits, the control chart alarms (indicates) that the process is **not under statistical control**. Some **assignable causes are present** in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

However, a control chart cannot tell us what is wrong it. In fact, it is the responsibility of the supervisor or quality control manager to find out what has gone wrong.

Note: Sometimes the sample points may be inside the control limits but may behave in a systematic or non-random manner. This is also an indication that the process is out-of-control. For example, suppose 22 of the last 25 sample points lie below the centre line but above the lower control line and only 3 of these points lie above the centre line but below the upper control line. This pattern indicates that the process is **not under statistical control** because it is not random in appearance. We shall explore this aspect in more detail in Sec 1.7.

You may like try the following exercise before studying further.

E4) Choose the correct option from the following:

i) Control chart is a
   a) one-dimensional chart.
   b) two-dimensional chart.
   c) three-dimensional chart.
   d) none of the above.

ii) Control chart consists of
   a) one control line.

b) two control lines.

c) three control lines.

d) four control lines.

iii) If one or more sample points lie outside the control limits, the control chart indicates that

a) there is no assignable cause in the process and the process is under statistical control.

b) there is at least one assignable cause in the process and the process is under statistical control.

c) there is at least one assignable cause in the process and the process is out of statistical control.

d) there is no assignable cause in the process and the process is out of statistical control.

iv) Control charts in statistical quality control are used for

a) describing the pattern of variation.

b) checking whether the variability in the product is within the tolerance limits or not.

c) both (a) and (b).

In Sec. 1.5, you have learnt about the control chart, which contains a centre line (CL), lower control limit (LCL) and upper control limit (UCL). We now discuss how to obtain the centre line and control limits for a control chart. The UCL and LCL are also called 3σ limits.

1.6 3σ LIMITS

The quality characteristic can be described by a probability distribution or a frequency distribution. In most situations, a quality characteristic follows a **normal distribution** or can be approximated by a normal distribution. You have studied the normal distribution in Unit 14 of MST-003 entitled Probability Theory and you know the probability that a normally distributed random variable (X) lies between $\mu - 3\sigma$ and $\mu + 3\sigma$ is 0.9973 where μ and σ are the mean and the standard deviation of the random variable (X). Thus,

$$P[\mu - 3\sigma \leq X \leq \mu + 3\sigma] = 0.9973$$

So the probability that the random variable X lies outside the limits $\mu \pm 3\sigma$ is 1– 0.9973 = 0.0027, which is very small. It means that if we consider 100 samples, most probably 0.27 of these may fall outside the $\mu \pm 3\sigma$ limits. So if an observation falls outside the 3σ limits in 100 observations, it is logical to suspect that something might have gone wrong. Therefore, the control limits on a control chart are set up by using 3σ limits. The **UCL and LCL of a control chart are called 3σ limits** of the chart. The question is: How do we calculate 3σ limits?

Suppose M is a sample statistic (e.g., mean, range, proportion of defectives, etc.) that measures some quality characteristic of interest. Further suppose that

μ_M and σ_M are the mean and standard error (standard deviation) of the sample statistic M, respectively. Then the centre line and control limits for controlling the quality characteristic are given by:

$$\text{Centre line (CL)} = \mu_M \quad \text{... (1)}$$

$$\text{Upper control limit (UCL)} = \mu_M + 3\sigma_M \quad \text{... (2)}$$

$$\text{Lower control limit (LCL)} = \mu_M - 3\sigma_M \quad \text{... (3)}$$

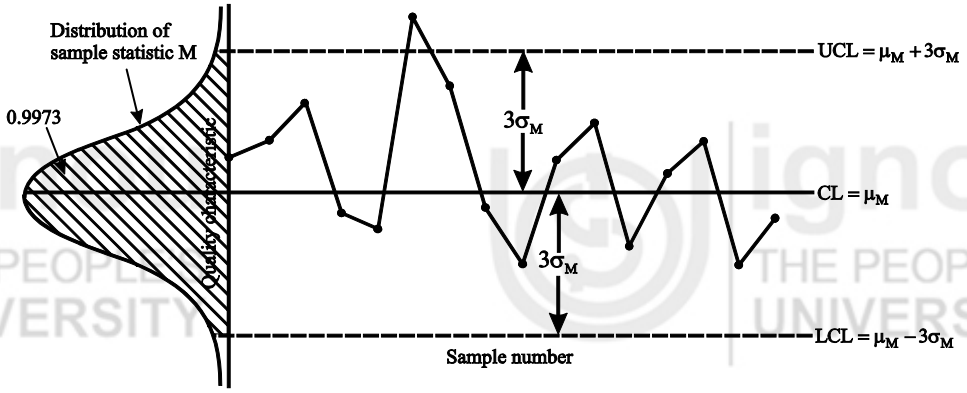


Fig. 1.2: 3σ limits.

The chart in Fig. 1.2 shows the centre line (CL), lower control limit (LCL) and upper control limit (UCL). The UCL and LCL are set at the distance $\pm 3\sigma_M$ from the centre line (μ_M). Note from Fig. 1.2 that the area covered between the $\text{UCL}\left(=\mu_M + 3\sigma_M\right)$ and $\text{LCL}\left(=\mu_M + 3\sigma_M\right)$ is 0.9973 (99.73%). So the probability that an observation falls outside these limits is 0.0027.

If the sample points fall between the control lines, the process is said to be **under statistical control**. But, if one or more points lie outside the control limits, control chart alarms (indicates) that the **process is not under statistical control**. Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them and then continue the production process.

You may now like to check your understanding of 3σ limits by answering the following exercise.

E5) If μ and σ represent the mean and standard deviation of the process, the lower and upper three sigma control limits for a control chart are given by:

a) $\mu - 3\sigma^2$ and $\mu + 3\sigma^2$

b) $\mu - 3\sigma$ and $\mu + 3\sigma$

c) $\mu^2 - 3\sigma^2$ and $\mu^2 + 3\sigma^2$

d) $\mu^2 - 3\sigma$ and $\mu^2 + 3\sigma$

In Sec. 1.6., you have learnt that the points on the control chart describe a pattern. If one or more sample points fall outside the control limits, the process

is said to be out-of-control and if all sample points are inside or on the control limits, the process is said to be under statistical control. But in many cases, the sample points may lie within the control limits and yet show an unnatural or specific pattern. This is also an indication of assignable causes. So to decide whether the process is under statistical control or not, it is also important to analyse the **pattern of the sample points**. This is what we discuss in Sec. 1.7.

1.7 CONTROL CHART PATTERNS

The **patterns of the control chart** are broadly classified into two categories:

1. Natural patterns of variation, and
2. Unnatural patterns of variation.

We first discuss the natural patterns of variation.

1.7.1 Natural Patterns of Variation

In Unit 1 of MST-002 entitled Analysis of Quantitative Data, you have studied that the central tendency is the characteristic of the distribution. Most observations tend to concentrate near the centre (mean) of the distribution and very few points lie near the tails. Since normal distribution is symmetrical about its mean (μ), i.e., the centre line is at μ (CL = μ), we expect that half the points will lie above the centre line and half below it. We also know that for the normal distribution

$$P[\mu - 3\sigma \le X \le \mu + 3\sigma] = 0.9973$$

This means that 99.73% observations lie between the 3σ limits. So of a total of 100 observations, 99.73 observations will lie inside the 3σ limits and only 0.27 observations may lie outside the 3σ limits.

We may conclude that a control chart having a natural pattern of variation has the following three characteristics:

i) Most points lie near the centre line of the chart.

ii) Very few points lie near the control limits.

iii) None of the points fall outside the control limits.

A typical control chart with 3σ limits is shown in Fig.1.3.

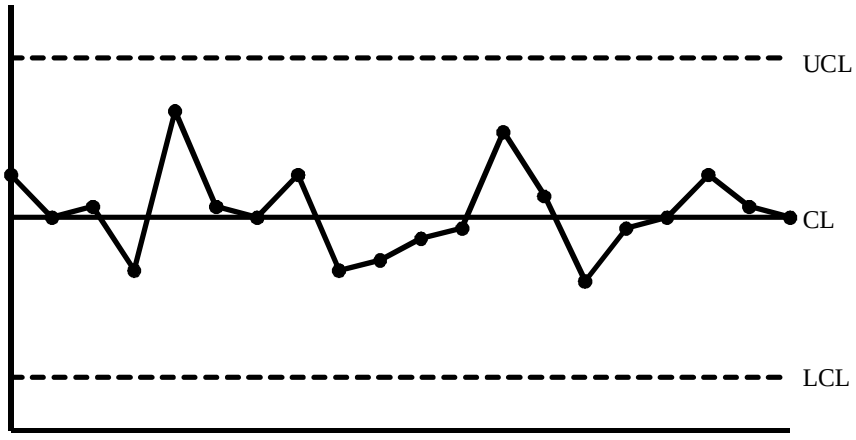


Fig. 1.3: Typical control chart.

1.7.2 Unnatural Patterns of Variation

The Western Electric Company engineers have identified 15 unnatural typical patterns in control charts. In this section, we discuss the most important types of unnatural patterns:

1. Extreme Variation

If one or more samples point are significantly different from the other points and lie outside the control limits of the control chart as shown in Fig.1.4, we say that there is an extreme variation in the chart. Some assignable causes are present in the process and corrective action is necessary to bring the process under control.

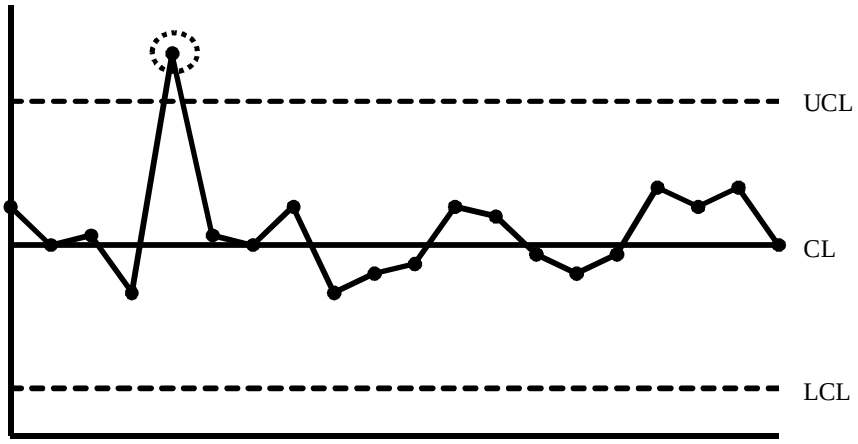


Fig. 1.4: Extreme variation.

Causes of extreme variation are:

i) Error in measurement, recording and calculations,

ii) Wrong setting and defective machine tools or erroneous use,

iii) Power failures for short time,

iv) Use of a new tool, failure of the component at the time of test, etc.

A sample point falling outside the control limits is a clear indication of the presence of assignable causes. There are other situations where in the pattern of sample points on the chart indicates the presence of assignable causes, although all points may lie within the control limits. Such situations are discussed below.

2. Trend

If consecutive points on the control chart tend to move upward or downward as shown in Fig. 1.5, it can be assumed that the process indicates a trend. If proper care or corrective action is not taken, the process may go out-of-control.

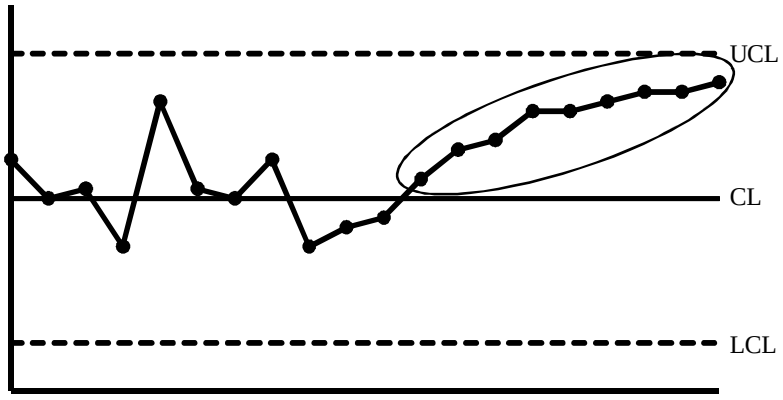


Fig. 1.5: Trend pattern.

Causes of trend pattern are:

i) Tool or die wear,

ii) Gradual change in temperature or humidity,

iii) Gradual wearing of operating machine parts,

iv) Gradual deterioration of equipment, etc.

3. Cycles

When consecutive points exhibit a cyclic pattern (as shown in Fig. 1.6), it is also an indication that assignable causes are present in the process which affect it periodically.

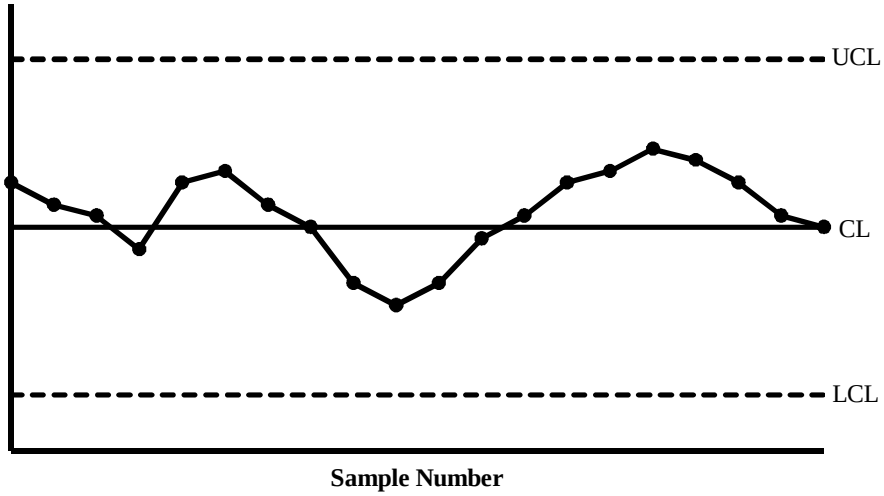


Fig. 1.6: Cyclic pattern.

Causes of cyclic pattern are:

i) Rotation of operators,

ii) Periodic changes in temperature and humidity,

iii) Periodicity in the mechanical or chemical properties of the material,

iv) Seasonal variation of incoming component, etc.

4. Shifts

When a series of consecutive points falls above or below the centre line of the chart, it can be assumed that a shift in the process has taken place. This indicates the presence of some assignable causes. Such a pattern is shown in Fig. 1.7.

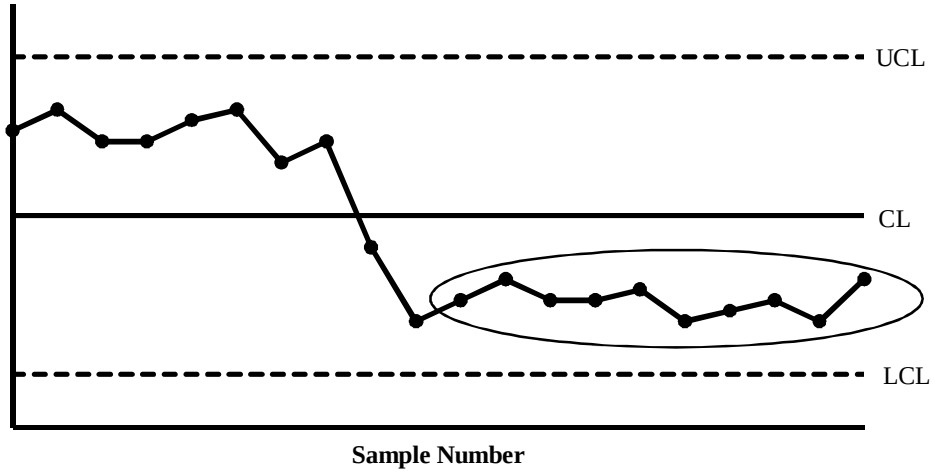


Fig. 1.7: Shift pattern.

Generally, it is assumed that when 7 consecutive points lie above or below the centre line, a shift has occurred.

Causes of shift pattern are:

i) Change in material,

ii) Change in machine setting,

iii) Change in operator, inspector, inspection equipment, etc.

5. Erratic Fluctuations

When the sample points of control chart tend to fall near or slightly outside the control limits with relatively few points near the centre line as shown in Fig. 1.8, it can be assumed that erratic fluctuation has taken place. This indicates the presence of some assignable causes. The causes for erratic fluctuations are slightly difficult to identify. These may be due to different causes acting at different times in the process.

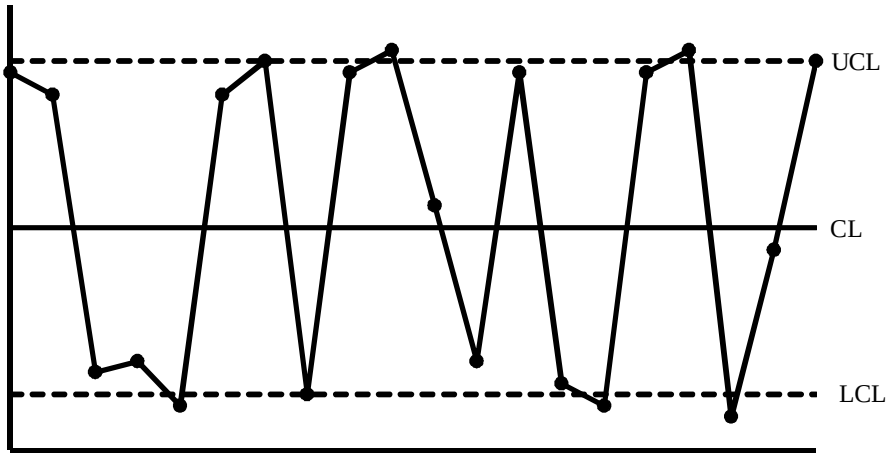


Fig. 1.8: Erratic fluctuations.

Causes of erratic fluctuations are:

i) Different types of materials being processed,

ii) Change in operator, machine, inspection, equipment,

iii) Frequent adjustment of machine, etc.

You may now like to review what you have studied about control charts patterns.

E6) Choose the correct option from the following:

i) If the points on the chart have continuous movements upward/downward, the pattern is called

a) freak pattern b) shift pattern

c) trend pattern d) cyclic pattern

ii) If all points on the control chart are within the control limits and the pattern of the points show trend, the process is said to be

a) under control b) out-of-control

c) both (a) and (b)

iii) To check whether the process is under control or out-of-control, we see

a) the pattern of the sample points on the chart

b) the position of the sample points on the control chart

c) both (a) and (b)

1.8 ADVANTAGES AND LIMITATIONS OF SQC

When a lot of items/units is manufactured, the manufacturer has two methods to check the quality of the lot: firstly, he/she could check each and every item and decide about the quality of the product, i.e., 100% inspection. Secondly, he/she could use the statistical quality control methods, i.e., inspect a small number of items and decide about the quality of the entire lot of the produced product. SQC has many advantages over 100% inspections, which are listed below:

A lot is the collection of units or items.

1. Ease of Application

An excellent feature of statistical quality control is that it is easy to apply. While developing the statistical methods for quality control, skilled and intelligent persons are required. However, even those persons who have not had extensive specialised training can apply statistical methods easily.

2. Reduction in Costs

The cost of inspection is reduced. In SQC as only a part or fraction of a lot is taken and inspected.

3. Greater Efficiency

Inspection of every item is bound to reduce the efficiency of a quality control inspectors because of dullness. Inspectors are more alert while using.SQC as only a part is inspected.

4. Early Detection of Faulty Units

SQC consists of continuous checking of the quality of the product. When a sample point falls outside the control limits it gives the signal that the process is not under statistical control. If some assignable causes are present in the process, necessary corrective action can be taken. Therefore, SQC ensures an early detection of faults and results in minimum wastage of items.

5. Helpful in Specification

Using SQC, we can find out whether the produced item is under control or not, that is, whether the item meets the specifications within the tolerance limits or not. If the variation is beyond the tolerance limits, SQC gives a danger signal and necessary corrective action can be taken. So, as long as statistical control continues, specifications can be accurately predicted for the future, which cannot be guaranteed by 100% inspection.

6. Ensures Overall Coordination

SQC methods ensure coordination between managers managing specifications, production and inspection. It provides a basis to resolve the differences arising among the various interests in an organisation.

7. Determination of the Effect of Change in the Process

With the help of control charts, we can easily detect whether or not a change in the production process results in a significant change in the quality.

8. Equilibrium in Consumer's and Producer's Risk

Methods such as quality control and acceptance sampling help in maintaining equilibrium between the consumer's risk and producer's risk.

9. Wider Applications

It is not only useful in the examination of items produced in small numbers, but also when bulk production has to be done.

10. Unique Method

Statistical quality control is helpful for those items that get destroyed on being examined for a given quality characteristic, for example, the intensity of match sticks, average life of compact fluorescent lamp (CFL), strength of glass, etc., In such cases, 100% inspection will spoil the entire lot and create a huge loss.

However, SQC has some **limitations**, which are described below:

1. When a sample of the items drawn from the lot is not a true representative of the entire lot, does not have the same characteristics as the lot from which it is drawn. Then a good lot may be rejected and a bad one may be accepted. This is the **main limitation** of SQC.
2. SQC cannot be used mechanically for any production process without studying the process and without adequate knowledge about it.
3. SQC applied on a production process provides only the information that the process is under control or out-of-control. It cannot take any action for improvement.

We end this unit by giving a summary of its contents.

1.9 SUMMARY

1. **Quality** means:

- conforming to specifications,
- fitness for use,
- customer satisfaction,
- delighting the customer, and
- enchanting the customers.

2. Quality has eight **dimensions**:

   i) Performance ii) Features iii) Reliability iv) Conformance

   v) Durability vi) Serviceability vii) Aesthetics

   viii) Reputation

3. Quality control is defined as the process by which we measure the quality characteristics of the product, compare them with the specifications or standard and take suitable actions whenever there is a difference between actual quality and the specifications or standard.
4. The technique of controlling product quality vis-a-vis the specifications using statistical tools is known as **Statistical Quality Control (SQC)**.
5. Statistical quality control can be broadly classified into two categories:
   - Statistical Process Control (SPC) or simply Process Control, and
   - Product Control or Acceptance Sampling.
6. S**tatistical process control** is a technique used for understanding and monitoring the process by collecting the data on quality characteristic periodically from the process, analysing them and take suitable actions whenever there is a difference between actual quality and the specifications.
7. **Product control** means to control the producta in such a way that these are free from defects and conform to their specifications.
8. The causes of variation are broadly classified into two categories:
   - Chance or random or natural or common causes, and
   - Assignable or non-random or unnatural or special causes.
9. The control chart is a two-dimensional graphical display of a quality characteristic that has been measured or computed in terms of means or other statistics from a sample and plotted against the sample number or time at which the sample is taken from the process.
10. A control chart consists of three horizontal lines:

    Centre Line (CL) – The centre line of a control chart represents the value which can have three different interpretations depending on the available data. First, it can be the **average value of the quality characteristic** or the **average of the plotted points**. Second, it can be a **standard** or **reference value**, based on representative prior data or an **aimed (targeted) value** based on specifications. Third, it can be the **population parameter** if that value is known. The **centre line is usually represented by a solid line**.

Upper Control Line – The upper control line represents the **upper value of the variation in the quality characteristic**. So this line is called **upper conrol limit (UCL)**. Usually, the **UCL is shown by a dotted line**.

Lower Control Line – The lower control line represents the **lower value of the variation in the quality characteristic**. So this line is called **lower control limit (LCL)**. Usually, the **LCL is shown by a dotted line**.

11. If all sample points **lie on or in between the upper and lower control limits**, the control chart indicates that the process is **under statistical control**. However, if **one or more points lie outside the control limits**, the control chart alarms (indicates) that the process is **not under statistical control**. Some assignable causes are present in the process.
12. There are two types of patterns of sample points on the control chart:
    - Natural patterns of variation, and
    - Unnatural patterns of variation

1.10 SOLUTIONS/ANSWERS

E1) i) Option (d) is the correct option because we know that quality has to incorporate the following:
- conforming to specifications,
- fitness for use,
- customer satisfaction,
- delighting the customer, and
- enchanting the customers.

ii) Option (c) is the correct option because we know that the **performance** is the primary operating characteristics of a product. **Features** are the additional characteristics available in product along with the primary operating characteristics. **Reliability** refers to the probability of a product's failure within a specified time period. **Aesthetics** means how a product looks, sounds, feels, etc.

iii) Option (a) is the correct option because we know that **durability** means a measure of product life. **Conformance** means meeting the specifications or standard. **Reputation** is related to the past performance of the company. **Reliability** refers to the probability of a product's failure within a specified time period.

E2) i) Option (c) is the correct option because we know that statistical quality control is a technique of both process control and product control.

ii) Option (c) is the correct option because we know that statistical quality control is a technique of both process control and product control. The control charts are used for process control and acceptance sampling plans are used for product control.

iii) Option (a) is the correct option because we know that process control is achieved through control charts, whereas product control is achieved through acceptance sampling plans.

iv) Option (b) is the correct option because we know that product control is achieved through acceptance sampling plans, whereas process control is achieved through control charts.

E3) i) Option (c) is the correct option because we know that the variation in quality characteristic may be due to assignable causes and chance causes.

ii) Option (b) is the correct option because we know that the variation due to chance causes is not controllable whereas the variation due to assignable causes is controllable.

iii) Option (a) is the correct option because we know that the variation due to assignable causes is controllable whereas the variation due to chance causes is not controllable.

iv) Option (d) is the correct option because we know that chance causes affect the process output in minor ways.

v) Option (a) is the correct option because we know that the variation due to assignable causes can be removed always.

E4) i) Option (b) is the correct option because we know that the control chart is a two-dimensional graphical display of a quality characteristic.

ii) Option (c) is the correct option because we know that the control chart consists of the centre line, upper control line, and lower control line.

iii) Option (c) is the correct option because we know that if one or more points lie outside the control limits, the process is not under statistical control. Some assignable causes are present in the process.

iv) Option (c) is the correct option because we know that control charts in statistical quality control are used for describing the pattern of variation and checking whether the variability in the product is within the tolerance limits or not.

E5) Option (b) is the correct option because we know that the lower and upper three sigma control limits for a control chart are $\mu \pm 3\sigma$ where μ and σ are the mean and the standard deviation of a random variable.

E6) i) Option (c) is the correct option because we know that if some consecutive points on the control chart tend to move upward or downward, it is called a **trend** pattern. If one sample point is significantly different from the other points and lies outside the control limits of the control chart, we say that there is an **extreme variation (freak)** in the chart. If a series of consecutive points falls above or below the centre line of the chart, it can be assumed that a **shift** in the process has taken place. If points on the chart have peaks and troughs which repeat themselves, we say that there is a **cyclic** pattern. These patterns are also an indication of the assignable causes.

ii) Option (b) is the correct option because we know that if the sample points show trend, it is also an indication of assignable causes and the process is out-of-control.

iii) Option (c) is the correct option because for checking whether the process is under control or out-of-control, we see the pattern of the sample points on the chart as well as the position of the sample points on the control chart.