Transit Route Choice Decision-making Using an Integrated Bayesian Statistical Inference

Project proposal for University of Virginia SYS 6014 Decision Analysis Spring 2020

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INTRODUCTION

Smart card automated fare collection systems are being used more and more by public transit agencies. While their main purpose is to collect revenue, they also produce large quantities of very detailed data on onboard transactions. These data can be very useful to transit planners, from the day-to-day operation of the transit system to the strategic long-term planning of the network. Since travel information provided by smart card is on individual level, it brings an opportunity of understanding individual travel behavior and improve the accuracy of existing transit route assignment model.

According to this data, we have a chance to make a decision-making model on the individual level. Since we have all the public transit data in Incheon, we can use those data to build a predictive model and generate the probability of each transit route in a route choice set. Here the route choice set is a set including all alternative routes for a certain OD (origin to destination) pair which also implies the set of options for decision-maker. Our decision-maker is the passenger. And then apply the decision-making analysis which in this study is based on transit utility function. In this study, we choose the origin of Bupyeongu Office and the destination of Onsu for the OD pair as our research object.

MODEL OF THE DECCISION PROBLEM

Problem Initialization

To specify the stated problem, assume a pair of origin and destination as showed in figure 1. The information provided by smart card data only includes tap-in and tap-out time, but cannot match to specific routes. Since travel time can be inferred from the smart card data, the main objective of this study is to find out if through travel time, the individual route choices or the probability of it could be derived.

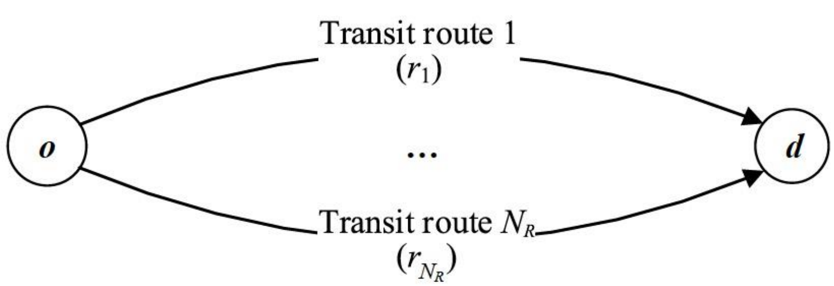


Figure 1 Illustration of Origin-Destination and Route Alternatives

The smart data comes from a Korea transit operation company named T-money. A sample of data is showed below, which include information of origin, destination, travel time, route number, and cost for each single card use.

First_stop	Last_stop	Unlinked_Trip	Length	Time	Ttime	Chain	mode1	route1	route1_obs	vehicle1
2755	2801376	2	8198	23	29	Y	203	20000000		.
2801394	2755	2	7336	23	25	Y	480	28206004	1	128717257
2755	2801417	2	7788	26	30	Y	203	20000000		.
2801394	2755	2	7336	26	29	Y	480	28206004	1	128717254
2755	2756	1	1000	16	16	Y	203	20000000		.
2801636	2801384	1	4083	16	16	Y	480	28217002	1	128717066
2801394	2755	2	7336	27	33	Y	480	28206004	1	128717253
2756	2801631	2	6724	26	34	Y	203	20000000		.
2801394	2755	2	7336	21	27	Y	480	28206004	1	128717252
2755	2801417	2	7788	33	41	Y	203	20000000		.
2755	2801376	2	8198	27	32	Y	203	20000000		.
2801394	2755	2	7336	21	24	Y	480	28206004	1	128717251
2755	2756	1	1000	4	4	Y	203	20000000		.
2756	2801384	2	7119	31	38	Y	203	20000000		.
2801394	2755	2	7336	20	23	Y	480	28206004	1	128717252
2801379	2756	2	7176	21	25	Y	476	28030014	1	128721340
2756	2801376	2	7198	21	26	Y	203	20000000		.
1816	1806	1	4400	10	10	Y	202	20000000		.

Figure 2 A Sample of Smart Card Data

To achieve better benefit of this study, the proper origin and destination selected for this study should have route alternatives that compete with each other instead of having one option is obviously better than the rest. With this consideration, the origin of Bupyeongu Office and the destination of Onsu in Incheon, Korea were selected, as showed in Figure 3. There are two route options as showed, 1) the orange one has longer distance but without transfer, while 2) the yellow one has shorter distance but will need to transfer. A week of data is available and will be used in the model analysis in the following sections.

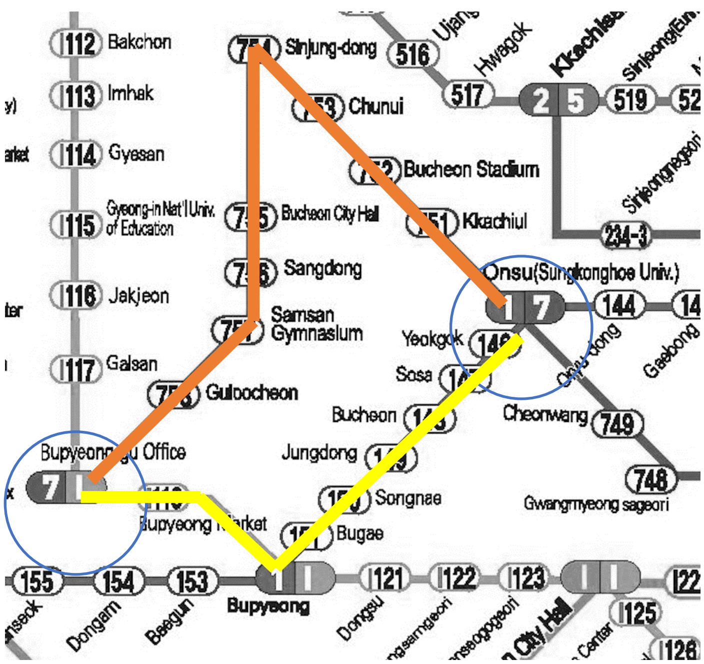


Figure 3 Origin and Destination Locations and Route Alternatives

Decision Analysis

Back to this transportation scenario, we have the information about the average time consumption for each route, the transit fare for each route and the posterior probability of an individual passenger choosing a specific route, conditional on the observation of the passenger's journey time. The cost set here is including time cost and money cost. Our utility function is generally defined as follows:

$$EU(r_i) = -\Pr\left(choice_{qr} \middle| t_q\right) \cdot (t_q + fare_q)$$

Our utility value is estimated in terms of the time consumption and the ticket cost. For the decision-maker, which is the passenger here, is definitely going to make his/her decision and choose to go through a cheaper and faster path for his/her trip.

PREDICTIVE MODEL

Bayesian inference method applied to the question mentioned above was first introduced by Richardson and Green (1). Rephrasing our objective in Bayesian language is that what is the probability of an individual passenger choosing a specific route, conditional on the observation of the passenger's journey time. In conformity with Bayes' rule, the translated equation including such relationship is showed below:

$$\Pr\left(choice_{qr} \middle| t_q\right) = \frac{Pr\left(choice_{qr}\right) * \Pr\left(t_q \middle| choice_{qr}\right)}{Pr\left(t_q\right)}$$

Where,

$\Pr\left(\mathrm{t_q} \middle| \mathrm{choice_{qr}}\right)$ represents a likelihood that the observed travel time would be $\mathrm{t_q}$ given an evidence of route r being actually chosen by the passenger;

$\Pr\left(\mathrm{choice_{qr}}\right)$ represents the prior probability, which reflects intrinsically the possibility that, among the population Q, route r itself could potentially be used relative to other alternatives.

Additionally, based on the law of total probability, $Pr\left(t_q\right)$ is included for normalization, thus the above equation could be simplifying to the following:

$$\Pr\left(choice_{qr} \middle| t_q\right) \propto Pr\left(choice_{qr}\right) * \Pr\left(t_q \middle| choice_{qr}\right)$$

In such way, the solution of our problem focuses on two terms: one, the prior probability

$\Pr\left(\mathrm{choice_{qr}}\right)$; and two, the likelihood function $\Pr\left(\mathrm{t_q} \middle| \mathrm{choice_{qr}}\right)$. The following sections will discuss the method of solving these two terms in details.

Mathematical linkage between the problem and the method(s)

Assuming the travel time for a given route between a pair of origin and destination follows a Gaussian distribution, the proposed model is a weighted sum of all possible alternatives by mixing probabilities. Let m(t) donate the proposed mixture Gaussian distribution, and t is the travel time between a pair of origin and destination. The number of route alternatives is N, and the travel time distribution of each route is $c_i(t;\theta_i)$. The mixture Gaussian is formulated as followed:

$$m(t) = \sum_{i \in R} \omega_i * c_i(t;\theta_i);$$

$$\sum_{i \in R} \omega_i = 1$$

With sample smart card data of observations of individual travel time, an empirical mixture distribution can be gained. However, little knowledge is known about the component distributions. If given more information to each transit service, their average travel time is supposed to be distinguishable. With properly identified component distributions, for any given observation, the posterior probability could be given by:

$$\Pr\left(choice_{qr} \middle| t_q\right) = \frac{\omega_i * c_i(t;\theta_i)}{\sum_{j \in R} \omega_j * c_j\left(t;\theta_j\right)}$$

As $c_i(t;\theta_i)$ is not known, in this study, it is assumed two types of distribution, Gaussian and lognormal. Then the question shifted to find out the parameters of each distribution. On the basis of existing data of travel time observations, those parameters could be figured out if the route-specific travel time data can be completely identified from among pooled observations. In practice, this may not be the case. In this context, the Expectation-Maximization (EM) algorithm is widely acknowledged to be an effective approach to estimate the parametric model by fitting observation data. It is an iterative algorithm implemented by the following 4 steps:

i. Initialize parameters;

ii. For 'Expectation' (E-step), calculate $\Pr\left(choice_{qr} \middle| t_q\right)$ using intial parameters;

iii. For 'Maximization' (M-step), evaluate and update it to be new values;

iv. Repeat (ii) and (iii) to converge to an optimum solution.

For initializing the parameters, for instance, a set of well-qualified starting values could be derived from K-means clustering. It partitions all the actual observations into a specified number, 'K', of 'clusters' each having a centroid, i.e. a mean, where $K \geq 2$. In the context of passengers' route choices that being discussed in this paper, a cluster should be equivalent to a group of passengers who chose the same transit route of which the mean travel time is considered the centroid. That is, each observation of travel time would be assigned to one cluster, and the member observations are supposed to be tightly close to its mean. This could be achieved by minimizing the sum of all the mean squared errors between the members and their centroid, over all clusters.

TRADE-OFF

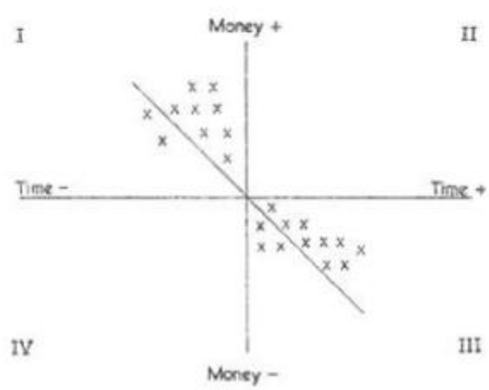
By far the most commonly used trade-off situation has been the choice of route for passengers. This situation has a number of advantages for the purpose of this estimation problem. Travelers are repeating the journey regularly and may therefore be fairly well informed concerning the alternatives. Genuine trade-off situations frequently occur. The journey to work in large cities is moderately unpleasant whatever mode or route is chosen, so that there may be no significant variation in other imponderables such as comfort. Moreover, the situation for which data is available is also the most frequently and important non-working time journey.

Imagine a set of individual passengers faced with alternative routes or public transport modes to choose for the journey to work. Here, Incheoners, for example, might use Korean railway service or bus. For each individual, compare the journey time and cost differences between the chosen service and that which is considered to be the next best alternative. A possible trade-off situation of this kind is shown in Table.

	Time (mins)	Cost (p)	Implicit time value if chosen
Mode A	15	12	At least 24 p per hour
Mode B	20	10	Less than 24p per hour

In this case Mode A is five minutes quicker but 2p dearer. If this mode is chosen then we may presume that the five minutes saved are worth at least 2p which the traveler has had to pay to obtain the saving. Assuming that the modes are similar in all other qualitative respects, it would not be rational for him to choose this mode otherwise. Conversely, if he chose the slower route, he would be implicitly valuing time at no more than 24p per hour. Again, his choice would not be rational if he valued time more highly than this. If we consider a set of passengers, some will show maximum and other minimum values of time by the actual choices made. For some, of course, the quickest route will also be the cheapest; such cases yield no useful information. Our problem is then to find the value of time which is most compatible with these choices (i.e. the rate which makes the smallest number of observed choices appear irrational). This can be obtained fairly rapidly by inspection.

The same procedure may be presented diagrammatically. Each individual can be plotted on a graph showing the advantage or disadvantage of the chosen mode with respect to time and money, as showing in the following.

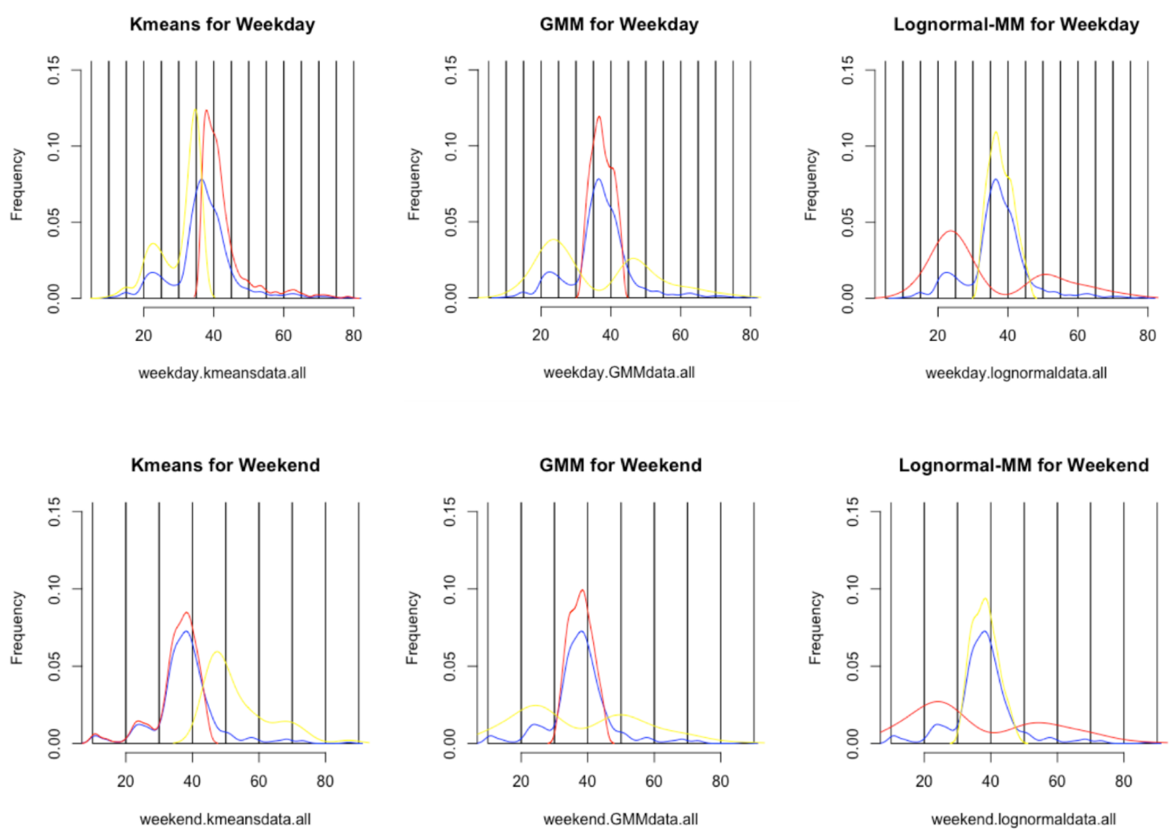


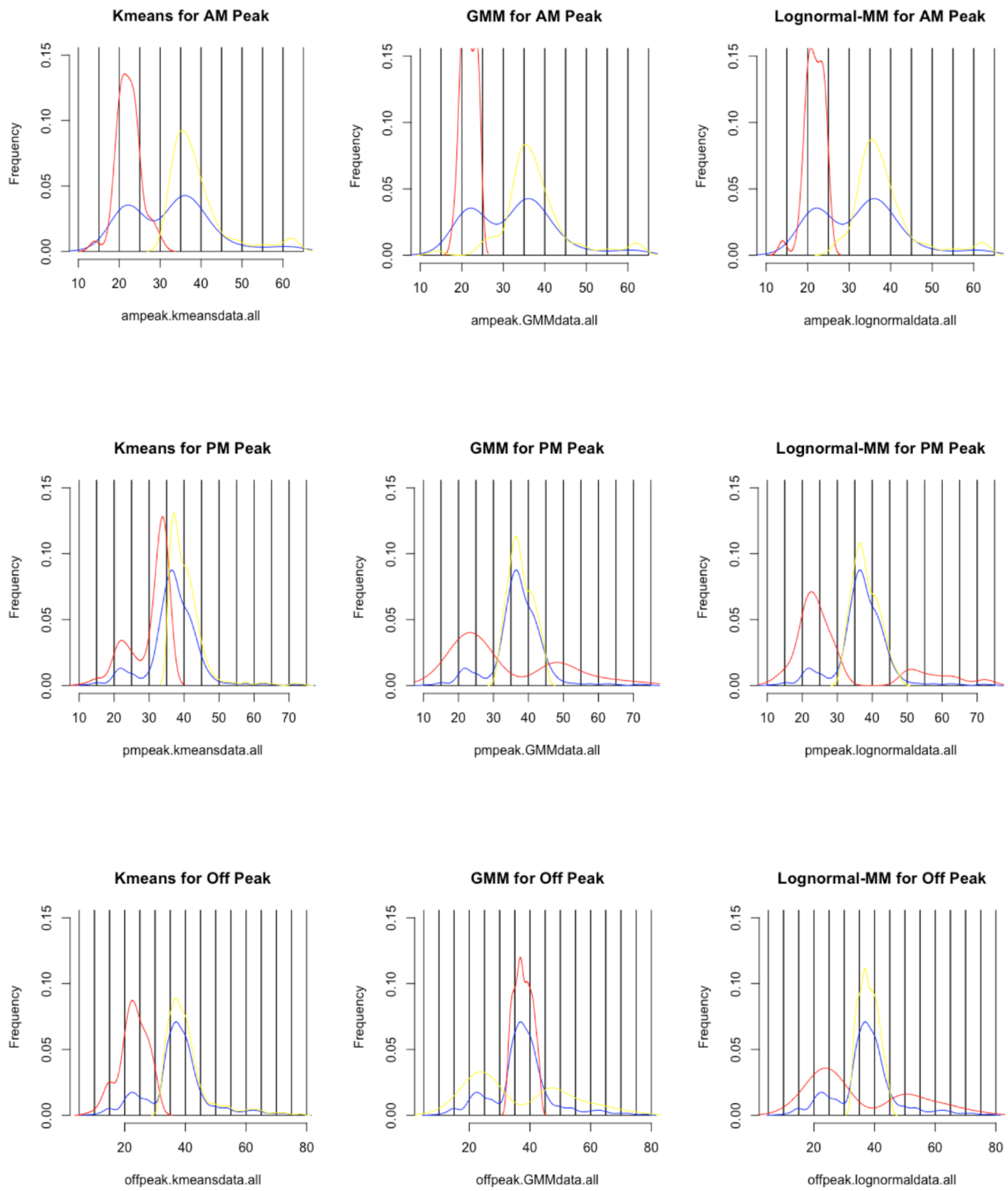
Times saved is shown on the horizontal axis, money saved on the vertical axis.

The inspection for the best estimate of the value of time is easily undertaken on the graph. For, as we have time on one axis and money on the other, the slope of a straight line through the origin in quadrants I and III gives a trade-off rate or value of time. To be explained as rational, individuals need to be to the right of the line in both quadrants I and III. This is because points to the right of the line in quadrant I have a trade-off rate of money for time greater than that shown by the line, whilst those to the right in quadrant III have a trade-off rate less than the actual rate. These are the rationality conditions previously described. Thus, to find the best estimate of the value of time, we pit a straight line through the origin and rotate it until it reaches the position at which the greatest possible number of individuals lie to the right of it. The slope of the line then gives the trade-off rate or value of time.

RESULTS & CONCLUSION

Both Gaussian and Lognormal mixture model are adopted to fit the observed travel time, and the results are showed in the figures below. To better understand the data, we divided them into different scenarios: weekday, weekend, AM/PM peak hours, and non-peak hours. In these figure, blue line stands for the pooled data; red line stands for route choice 1, and yellow line stands for route choice 2. Overall, travelers are more likely to choose choice 1 because although it has longer distance, without transfer, the variation of travel time is smaller than route 2, except for PM peak hours. The results indicate that people prefer a route that has more reliable travel time over relative travel distance. Moreover, this result could potentially point out the lack of frequency of route 1 during PM peak hours since travelers are less likely to choose route 1 over route 2.





It should be noted that, in this study, the alternative routes were predefined. Besides, it only assumed a fixed set of routes available for passengers travelling between the given O-D and each individual was considered to make a choice from an identical choice set. Nevertheless, in reality, all passengers that are demanding to travel between the same O-D might have their own different route choice sets. This refers to two aspects. On the one hand, the choice sets may differ among those passengers, because different people might take into account different alternative routes and carry out different choice tasks. On the other hand, all alternatives encompassed in a choice set may not be equally perceived by an individual in terms of route attributes. Such attributes involve a variety of factors influencing passengers' travel decisions, including systematic variables (e.g. service frequency, walking distance for interchange), individual perceptions to over-crowding and seat availability, provision of real-time information, and other uncertainties, etc. Then a challenge will be on how to explicitly specify or identify each individual's perceived choice set, and will be considered in future model improvements.

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