

## Session 3

Sometimes the actual decision problem was not very well articulated. In the decision problems, there is always a cost or benefit and your goal is to find a tradeoff between them. So, when you are articulating the decision problem you are going to find what is the cost or benefit in the decision problem and what is the state variable space. The decision problems mean the set of options that someone has to make. For example, there is a fixed budget for food and you are going to purchase nutrition and calories in order to maximize the quantify nutrition health benefits.

### Formulization for statistical decision model:

Action set  $\Rightarrow A = a_0, a_1, \dots, a_m$  or  $A \subseteq R^M$

Example: you have a fixed budget and you got two assets including safe and risky assets. You are going to decide what proportion of your portfolio to put at the risky asset and how much put at the safe asset.

Ex 1: Portfolio allocation, 2 assets including safe and risky

Action: the fraction of the portfolio in a risky asset

$$0 \leq a \leq 1 \quad A = [0, 1] \equiv \Delta^1$$

Ex 2: Portfolio allocation, 3 assets including cash, bonds, and stocks

Let  $a_i$  fraction of portfolio allocated to asset  $i, i=\{1,2,3\}$

We must have  $0 \leq a_i \leq 1, \sum_{i=1}^3 a_i = 1$

$$A = \{ \langle a_1, a_2, a_3 \rangle \mid 0 \leq a_i \leq 1 \text{ for } i = 1, 2, 3, \sum_{i=1}^3 a_i = 1 \}$$

$\equiv \Delta^2$  The unit simplex in  $R^3$

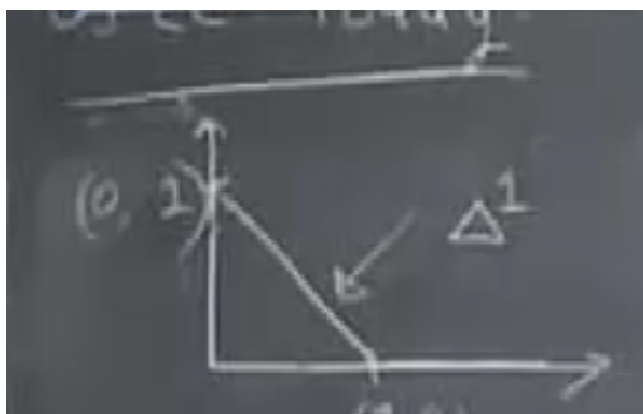


Figure 1: 1

There are a set of points, each of them is non-negative and their summation is equal to the one.

The following is the two-dimensional object in the three-dimensional space

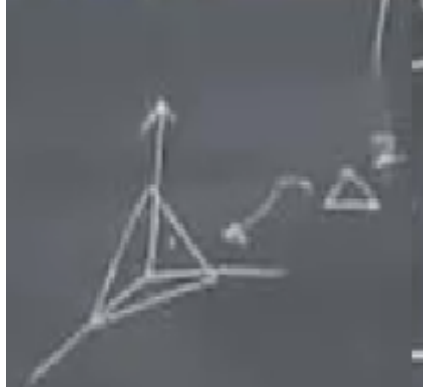


Figure 2: 2

Problems where you got to allocate some scarce resource according to some kind of finding constraints, the action set often ends up looking something like the above figures.

States:  $x \in X$

$x$  refers to the observable data about the state of the worlds

$X$ : state space or sample space

In some applications we are going to have a dataset, we are going to analyze then and we will come up with some estimated parameters and some actions based on that. In other cases, we may not yet know the values of our data but we may anticipate.

Example: Medical test, the previous example related to whether or not to perform the test to gain new information.

You have to decide based on the trade of associate with that test before you get the information

Therefore,  $x$  in some application is the observable data which is not yet observed.

When  $x$  is uncertain, we represent its value as a random variable,  $X$ .

EX: Medical test,  $X = \{\text{test is negative}, \text{test is positive}\} = \{0, 1\}$

Let  $X$  random variable corresponding to the test result.

We write  $X \sim \beta(\theta)$

$X$  is distributed as a Bernoulli the parameter  $\theta$

$$Pr\{X = 1\} = \theta$$

$$Pr\{X = 0\} = 1 - \theta$$

Ex: X is next Thursday temperature at Charlottesville airport

$$X \sim N(\theta, \sigma^2)$$

Based on the weather you may decide about shutting down schools. It will cost you if you decide to shut down the school while the weather is good. Also, it will cost you if you do not shut down the school when the weather is not good.

Suppose that  $X = \{1, 2, \dots, N\}$ , there are N states

Ex: X is a random variable

$$\text{Let } \theta_i = Pr\{X = i\}, i = 1, 2, \dots, n$$

$$\text{Then } \Theta = \langle \theta_1, \theta_2, \dots, \theta_N \rangle$$

defines a probability distribution on X

$$\text{Here } 0 \leq \theta_i \leq 1, \sum \theta_i = 1$$

The set of all possible  $\theta = \text{simplex}^{n-1}$

What is corresponding to the good information? Having high probabilities for some outcomes and low probabilities for the rest. Ideally, having the probability equal to one for one of the outcomes and zero for the rest.

What is corresponding to the bad or no information? Having equal probabilities, uniform distribution.

If  $N=2$ ,

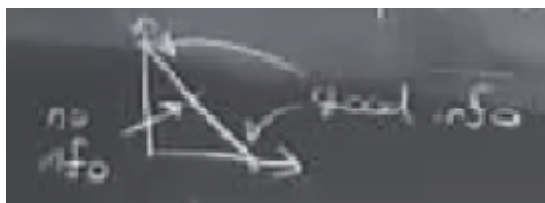


Figure 3: 3