Session 4

A= actions $a \in A$

X= states $x \in X$

Payoffs, based on your model, depending on the realized state of the world (X) or the value of the unobserved parameter.

In the case of a medical test, the realized payoff depends on whether or not the patient has a bacterial or viral infection.

There is an unobserved parameter θ which describes the likelihood of that patient having a bacterial or viral infection.

The realized payoff is not depending on θ but it depends on the state. Then, in this case, it makes sense to model your payoff based on states, not the unobserved parameter. However, the expected value depends on probabilities.

In other applications, the payoff may depend on an unobserved parameter. As an example, given you are working for a company and you are marketing for a new drug and you are trying to decide the estimate the benefits or the returns to the company. It depends on the market share that the drug will get. Therefore, it may make sense to consider the market share as a parameter. What really matters here is you are correctly specifying what is the thing that actually determines payoff.

Payoff: Depending on situation,

might model as function of $(a, x) or 4(a, \theta)$

Suppose payoff depends on (a,x)

Let u(a,x), realized payoff as a function of realized x.

Before the value of x is known, model it as a random variable X

Let p(x)=pr(X=x), Assuming X is discrete

Then $E[u(a, x)] = -\sum p(x)u(a, x)$

If the goal is to max E[u(a,x)], then the optimal action is a^* is the one that maximize E[u(a,x)]:

$a^* = argmax E[u(a, x)]$

If the goal is to avoid the bad outcome at all cost then a^* is

$a^* =$

Think of each action as a lottery

$a_1 \longmapsto < u(a_1, x_1), u(a_1, x_2), ..., u(a_1, x_N) >$

$p(x_1), p(x_2), ..., p(x_N)$

$a_2 \longmapsto < ... >) >$

.

.

.

X is the temperature,

Each action is whether or not wear the jacket

The way you choose the action is selecting the action with the highest expected payoff.

If you have two identical jackets, wearing each of them is one action and their payoffs will be equal. Therefore, they are the same action and you can consider them as one choice.

If your goal is to avoid that outcome that has a cost, for each of the actions, we identify the worse possible outcome that is associated with that outcome. We do not care about the probabilities.

$\underline{u}(a_1) = \min_{x \in X} u(a_1, x)$

The above equation shows the minimum possible worse case pay off if we choose action one. we have the same relationships for other actions.

$\underline{u}(a_2) = \min_{x \in X} u(a_2, x)$ . . .

In this problem we want to avoid worse outcomes. Therefore, choose a^* as follows:

$a^* = argmax\underline{u}(a_i)$

To choose this way is called following a "Max-Min rule".

Payoffs in terms of unobserved parameters

Suppose we model $u(a, \theta)$: payoff is function of action and parameter θ

Suppose that we want to choose an action to max $u(a, \theta)$

Assume that we have some opportunities to do some statistical sampling from some data which tells us something about θ and it helps us to sharpen our beliefs and based on that we can choose the right action.

Suppose we do not know much about θ, but we can collect data.

Ex: $x_1, \ldots, x_n \sim N(\mu, \sigma)$ and it helps us to estimate θ

First, collect data: $x_1, ..., x_n$

Second, estimate $N(\mu, \sigma)$

Third, choose optimal action given new information about $N(\mu, \sigma)$

How to do the above procedure? Frequentist: $\hat{\mu} = \frac{\Sigma_{i=1}^{n} \lambda_i}{n}$

Ex: You have a flight when you will leave your house? You will estimate the time it will take to arrive there. There is a cost to get the airport early and there is a cost to get there late. Why do not get the estimated value to get there at the right time? The payoff is not symmetric; if you get there a little bit early, there is a small cost. However, if you get there a little bit late, the cost will be huge. Therefore, considering the estimated μ, and choosing the action that maximizes the payoff is not, in fact, the optimal decision-making procedure. What you need is a rigorous way of deciding how much error there is and then dealing with the payoff and the optimal action given this error. This is what statistical decision theory will do for us. In the airport example, you will not consider the estimated arrival time (μ). In fact, you know there is an error and you will take into account that error.