Session 6

Big picture: How to make optimal choices under statistical uncertainty?

- A decision-maker (a.k.a. *actor*) chooses one option from a menu of possible *actions*.
- Payoffs from that choice depending on the action chosen, and on the *true* value of the state of nature. Depending on the way we are modeling, the true state of nature could either be unobservable realized random variable x or could be the value of the unobserved parameters but in either way, we call it the state of nature.
  - Payoffs may equivalently be represented as *losses* — negative payoffs relative to some baseline.
- The true value of the state of nature is *uncertain.* Hence the decision-maker confronts a problem of *decision-making under uncertainty.*
  - Each possible action thus maps to a *lottery* over uncertain payoffs.

Uncertainty

- Typically, the decision-maker will possess some information about the likelihood of different states of nature.
- This information can be represented formally by treating the state of nature as a random variable drawn from a known distribution. The tricks here are trying to represent those distributions formally, so we have a quantify handle on what we know and what we do not know. Then, those quantify representation of the uncertainty in an optimized way.

Optimal choice

- Given a complete enumeration of
  - the menu of possible actions, (a)
  - the set of possible states of nature (θ),
  - a probability distribution over the set of states of nature, representing the likelihood of states, and – a *payoff function* that gives the payoffs (or losses) from each possible combination of action $\times$ state, in some units of measure.

  then each possible action maps to a *lottery* over payoffs (losses).

Optimization: Defining objectives

- Finally, given a lottery over payoffs, the actor chooses an *optimal* action guided by a *decision-making principle*:
  - *Example:* Maximize expected payoffs.

  - *Example:* Maximize the expected utility of payoffs.
  - *Example:* 'Minimax': Choose the action to minimize possible loss, irrespective of probabilities. (an extreme form of risk aversion)

- The decision-making principle encodes the actor's attitude towards risk – the willingness to accept losses in some uncertain states of the world, in exchange for achieving gains in other states of the world.
  - The study of attitudes towards risk and loss is a huge topic in economics, finance, and psychology. We will cover it for only glancingingly.
  - For your applications, just at minimum be aware that the actor's optimum choices may not be driven by the goal to maximize expected gains (= minimize expected losses).

Defining the objective function is not trivial. Depending on your application you may have a very clear objective or you may not have a clear set of objectives.

After modeling your problem, you will have a set of actions, their payoffs, and corresponding probabilities. Now, what you should do? Which option would be best?

You have to have some principles for converting lotteries over payoffs into an optimal choice. There are different ways to do this, one of the ways is maximization expectation payoffs. Have it in your mind that when you have uncertainty, you should not collapse the uncertainty, and then consider expected value.

Ex (Flight): We were consulting with the atmospheric scientists, and they had planes that could fly around and collect data. Part of the challenge was the amount of the flight and the number of the days in a season was restricted. They did not just have one decision about where to go. They had three possible options. There was a menu of choices instead of yes/no. There were including 1) Flight in zone one, 2) Flight in zone two, 3) Flight in zone three, or do not fly. The trick was what they care about it at the end of the season was what portfolio of the data did they have. What they wanted was to have more data is better but they also wanted more balanced data. They wanted data with nearly the same portion from all three zones. So, we had to extract from them what was the utility function. How much would they care about balancing the portfolio? How much more data would they give up to have more balance data? The atmospheric scientists did not know what is the utility function. We had to go find their preferences in this process to get enough information about how to make this trade-off. Then, we used the utility function as an objective function. In those cases, the goal is not just to maximize as much data as possible but they also want a balanced portfolio. In this application, we need to think about the shape of the utility function.

Integrating predictive and inferential tools into the decision calculus

In general, your predictive model serves to *reduce uncertainty* over future values of the states of nature.

They *sharpen the probability distribution* over states of nature.

(In terms of probability theory: they involve a *change of measure*.)

The actor can now choose an optimal action based not on her *prior* (or *naive*) beliefs, but on her *posterior* beliefs, conditioned on the current data.

Payoff functions: Define in terms of x (state) or θ (parameter value)?

Depending on your application, it may make sense to model payoffs (or losses) as either a function of observed realized state $x \in \mathbb{X}$, e.g., x denotes realized temperature:

$$L_0 = L_0(a; x)$$

or in terms of the value of an unobserved parameter, e.g., θ denotes mean temperature:

$$L_1 = L_1(a; \theta)$$

Ex: Suppose you are going to think about how much you are spending on energy or fuel tomorrow. For example, you want to decide how to set the thermostat and your losses are based on your expenditure and your comfort. Depending on how your decision-making problem is set up, it makes sense to represent that explicitly in terms of your on realized observable temperature, or it could make sense to think about it in terms of unobserved parameters of the model.

In the example flight x was for each day in the field season, there is a realization either the condition is good for data collection or not. The action, in that case, is either fly on that day or not. So, θ will describe the likelihood of the good or bad conditions, but at the end of the field season what the decision-maker cared about is not the likelihood, they cared about how much data they got. Based on these, the loss is explicitly based on the realized outcomes.

Parameter-dependent payoffs as a reduced form of state-dependent payoffs

In many cases, you can represent the θ formulation as the *reduced form* of the x formulation, e.g.

$$L_1(a; \theta) = E[L_0(a; X)|\theta]$$

Ex: If X is a random variable and the distribution over X governs by θ. So, a realization is a draw from the distribution and the parameters.

Ex: You are going to do fraud detection. You have financial transactions that come to the system. The computer has to very quickly identify the likelihood if the transaction is fraud or not. You have a lot of data and a small percentage of them are likelihood probability and the action is whether or not to flag that transaction and give it more screening by a human analyst.

Building and Running a decision rule:

1- Building and calibrating the tools

2- Put into the production

There is raw data and preprocessing will be done on them. The output will be the structured data, which is data in terms which you can actually use. You use the structured data to calibrate your model. What it allows you to do is to get new input data and map your prior beliefs to the posterior believes about the data. Then, given your posterior beliefs, the output of the optimization model will be the optimal choice.

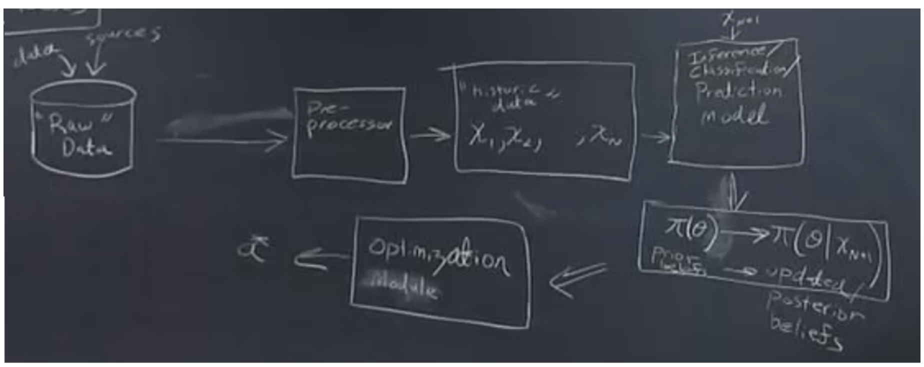

Figure 1: 1

Intertemporal optimization: It is for a set of approaches for dealing cases, in which we have a set of copies of decisions and they have an impact on each other.