UNIVERSITY of VIRGINIA
DEPARTMENT of MATHEMATICS

Schur dualities arising from quantum symmetric pairs

A symmetric pair $(\mathfrak{g}, \mathfrak{g}^\theta)$ consists of a Lie algebra $\mathfrak{g}$ and a fixed point subalgebra $\mathfrak{g}^\theta$, where θ is an involution on $\mathfrak{g}$. The corresponding quantum symmetric pair $(\mathbf{U}, \mathbf{U}^\imath)$, introduced by Letzter for finite type and Kolb for Kac-Moody type, is a q-deformation of $(\mathfrak{g}, \mathfrak{g}^\theta)$. Here $\mathbf{U}$ is the quantum group associated to $\mathfrak{g}$ and $\mathbf{U}^\imath$ is a coideal subalgebra of $\mathbf{U}$. The algebra $\mathbf{U}^\imath$, now referred as the **$\imath$quantum group**, can be viewed as natural generalizations of quantum groups.

The Schur-Jimbo duality is one of the most fundamental topics in representation theory, bridging the irreducible representations of a Hecke algebra with those of a Drinfeld-Jimbo quantum group. In [1], the authors establish an $\imath$Schur duality where the Hecke algebra of type B, and the $\imath$quantum group of quasi-split type AIII satisfy a double centralizer property. Furthermore, it was shown that the Kazhdan-Lusztig basis of type B coincides with the $\imath$canonical bases arising from tensor product modules of $\imath$quantum groups.

We study $\imath$quantum groups of AIII in a more general sense and establish an $\imath$Schur duality which can be viewed as a common generalization of Jimbo-Schur duality and Bao-Wang's quasi-split $\imath$Schur duality. The $\imath$canonical bases in this setting provides a common generalization of both type A and type B Kazhdan-Lusztig bases.

Theorem ([2]) Let $(\mathbf{U}, \mathbf{U}^\imath)$ be a quantum symmetric pair of type AIII. The $\imath$quantum group $\mathbf{U}^\imath$ and the Hecke algebra of type B form a double centralizer property on a tensor module of $\mathbf{U}$.

The exploration of $\imath$Schur dualities within the framework of quantum symmetric pairs provides a natural extension of classical Schur dualities. Specifically, the type AI $\imath$quantum group serves as a quantization of the special orthogonal Lie algebra, while the type AII $\imath$quantum group serves as a quantization of the spin Lie algebra. These classcial limits form Schur dualities with the Brauer algebra.

Theorem ([3]) Let $(\mathbf{U}, \mathbf{U}^\imath)$ be a quantum symmetric pair of super type AI or AII. The $\imath$quantum group $\mathbf{U}^\imath$ and the q-Brauer algbera form a double centralizer property on a tensor module of $\mathbf{U}$.

Motivated by the immense success of quantum symmetric pairs, we introduce quantum supersymmetric pairs associated with Lie superalgebras in [4] and establish their fundamental properties. Furthermore, we extend the aforementioned $\imath$Schur dualities to the super setting.

References

[1] H. Bao and W. Wang, *A new approach to Kazhdan-Lusztig theory of type B via quantum symmetric pairs*, Astérisque **402** (2018), vii+134 pp.

[2] Y. Shen and W. Wang, *$\imath$Schur duality and Kazhdan-Lusztig basis expanded*, Adv. Math. **427** (2023), Paper No. 109131.

[3] W. Cui and Y. Shen, *Canonical basis of q-Brauer algebras and $\imath$Schur dualities*, Math Res. Let. (to appear).

[4] Y. Shen, *Quantum supersymmetric pairs and $\imath$Schur duality of type AIII*, submitted.

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