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Combination Theorems for Convergence Groups

First studied by Poincaré and Klein in 1883, *Kleinian groups* are discrete subgroups of $\mathrm{PSL}(2,\mathbf{C})$, the orientation preserving isometries of the real 3-dimensional hyperbolic space, $\mathbb{H}^3_{\mathbf{R}}$. These groups have deep connections to 3-manifolds, and Teichmuüller theory. In particular, *geometrically finite* Kleinian groups have been of great interest. A Kleinian group G being geometrically finite roughly means the quotient $\mathbb{H}^3_{\mathbf{R}}/G$ decomposes into a compact piece containing all of the topology and finitely many 'ends.'

The classical combination theorems of Maskit give sufficient combinatorial conditions stating when two geometrically finite Kleinian groups G_1 and G_2 generate a new geometrically finite Kleinian group G isomorphic to either an HNN extension or amalgamated free product of the original groups.

In joint work with Theodore Weisman, we prove generalizations of these classical theorems for the setting of *discrete convergence groups*. These are groups G acting on a compact space M with dynamics mimicking that of Kleinian groups acting on the Riemann sphere $\partial\mathbb{H}^3_{\mathbf{R}} \cong \mathbf{C} \cup \{\infty\}$. Our dynamical assumptions closely resemble the *ping-ping lemma* of geometric group theory, and the below picture shows the adapted ping-pong dynamics which arise from our assumptions. These dynamics are analyzed carefully in our proof.

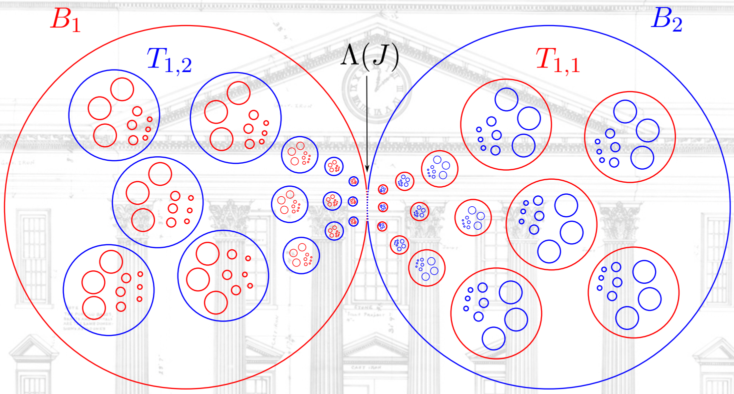

Figure 1: A depiction of the ping-pong dynamics arising from our hypotheses.

We end with a possible application of these theorems for studying different types of convergence for sequences of representations which act as convergence groups. Specifically, we study when the *algebraic limit* and *geometric limit* of a sequence do not coincide. Such examples exist for Kleinian groups using methods which do not generalize well to other matrix groups, and these combination theorems give a possibility for generalization to other settings.

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