

Virginia Math Bulletin

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Issue 7 Volume 1 Fall 2020

View from the Chair

John Imbrie
Professor of Mathematics/Chair

I write at the conclusion of an extraordinary year for our department. As 2020 began, we were gearing up to host the AMS sectional, which was to start on March 12. Early indications were that large gatherings were a major contributor to the spread of the pandemic, so we hastily cancelled the event, which was to bring 700 participants to Charlottesville. Soon, the university cancelled classes and students were abruptly sent home. Faculty members and graduate students were tasked with quickly finding a way to hold classes online. The learning curve was steep, but we came together to reinvent our modes of teaching. Now we reach the conclusion of a semester with the majority of

instruction still being delivered remotely. The transition was, and continues to be, challenging, as many of us have put forth extraordinary time and effort while facing increasing demands caring for children and family members. Still, we have managed to thrive in ways that seemed out of reach in the dark days of March. Read ahead for an article by Leo Petrov on the ways we have managed to adapt to the new normal for teaching, research, and for our final exercises.

This year we are welcoming two new faculty members, Peter Humphries and Christian Reidys. Peter will start in the spring as his arrival in Charlottesville was delayed by the closure of the US embassy in Australia. Christian is one of the key figures in the new Biocomplexity Institute at UVa. Below, you will find profiles of them along with six new postdocs. The expanded postdoc roster includes two that are part of our multiyear program for transforming calculus instruction, and two that are supported by the Research Training Grant (RTG) -- see last year's Bulletin for a profile of the RTG grant.



This past summer we initiated a major new program for research experiences for undergraduates (REUs), combining Ken Ono's long-running program in number theory with a new program in geometry and topology, supported by the RTG grant. Despite the challenges of working remotely, the program was very successful -- see the article below by Ken Ono and Tom Mark. Other highlights of this Virginia Math Bulletin include an article on our new bridge program, by David Sherman (our new Director of Diversity, Equity, and Inclusion). Our Bridge to the Doctorate currently supports three post-baccalaureate students as they prepare for entry into a Ph.D. program.

Sadly, it was impossible to conduct the Gordon Keller math majors dinner. We had planned to host UVa graduate and McShane Prize winner Andrew Booker, who was in the news for work leading to the long-sought integer solutions to $x^3 + y^3 + z^3 = 33$ and $x^3 + y^3 + z^3 = 42$. We anxiously await the return of our normal activities, so that we can host Andrew and our majors once again.

In the meantime, with seminars and colloquia now delivered remotely via Zoom, we find that it is possible to participate remotely in lectures all over the world. I hope that our community of graduates and former members of the department will take this opportunity to reconnect with us by taking part in our online events. We'd love to hear from you as we move forward with another year of teaching, exploration, and discovery.

A handwritten signature of John Imbrie.

Supporting Us

The Mathematics Department is grateful for the generous support of its alumni and friends. The Department welcomes gifts annually to address its most urgent needs, as well as to the endowment which provides funding in perpetuity. To learn about how you can make a difference by supporting the Mathematics Department, please contact Becky Balber at beckybalber@virginia.edu or (434) 243-4978. To make a gift online, please visit <http://giving.as.virginia.edu/mathematics>.

New Faculty Profiles



Peter Humphries
Assistant Professor
(2020-)

Mixing methods from analytic number theory, automorphic forms, and representation theory in his work, Peter Humphries is a pure mathematician working broadly in the area of number theory, which involves the study of prime numbers and their distribution. His research has applications towards quantum chaos, and he is particularly interested in problems involving equidistribution and L-functions.

Humphries has published 13 papers in research journals such as Geometric and Functional Analysis, Compositio Mathematica, Mathematische Annalen, and Communications in Mathematical Physics. As a master's student, Humphries was awarded the J. G. Crawford prize by the Australian National University, a university-wide award in recognition of an outstanding research thesis.

Hailing from Melbourne, Australia, Humphries received his bachelor's and master's degrees from the Australian National University. He earned his Ph.D. in 2017 from Princeton University. Before joining UVA, he was a research associate at University College London, where he was funded by a grant from the European Research Council.

Humphries will be teaching Calculus III next spring.



Christian Reidys
Professor
(2020-)

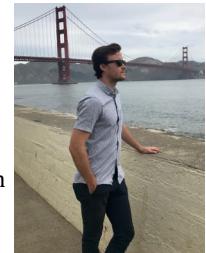
Christian Reidys has a joint appointment between the Mathematics Department and the Biocomplexity Institute at UVa. He is Director of the Mathematical Biocomplexity Division within the Biocomplexity Institute. His research is focused on discrete mathematics, computational biology, molecular evolution, RNA structure, DNA combinatorics, and topology of large data sets. This fall, he taught a graduate class in bio-mathematics. He has been advising postdocs Thomas Li, Ricky Chen, and Qijun He at the Biocomplexity Institute.

Reidis received his Ph.D. in 1995 from the University of Jena, Germany. He did postdoctoral work at the Los Alamos National Laboratory, and continued on staff member there until he became Professor at the Combinatorics Center at Nankei University, in 2006. He was Professor at the University of Southern Denmark from 2011-2015, after which he became Director and Professor at the Biocomplexity Institute at Virginia Tech. He is the author of two books, Combinatorial and Computational Biology of Pseudoknot RNA and An Introduction to Sequential Dynamical Systems.

Filippo Mazzoli
Whyburn Research Associate
(2020-)

Mazzoli's research interests lie in the broad realm of geometry and topology. More specifically, he works in hyperbolic and anti-de Sitter geometry, Teichmüller theory and low-dimensional topology. Mazzoli's main topic of research is the study of the deformation spaces of hyperbolic 3-manifolds, and their geometric properties. He is particularly interested in the relations between the geometry of the Teichmüller space (deformation space of conformal structures or hyperbolic metrics on a surface) and the notions of dual volume, W-volume and renormalized volume of hyperbolic 3-manifolds. He is also interested in the study of quantum invariants of finite volume hyperbolic 3-manifolds, and their connections with hyperbolic geometry. In his work, he uses techniques from differential geometry, geometric topology, and symplectic geometry.

Mazzoli received both his B.S. in mathematics (2014) and his M.S. in mathematics (2016) from the University of Pisa, Italy. He earned a Ph.D. in Mathematics from the University of Luxembourg (2020). In his teaching, he tries to always share with his students his enthusiasm for mathematics. This year, Mazzoli is teaching Calculus III and Applied Linear Algebra.



William Olsen
RTG Research Associate
(2020-)

William Olsen works in geometric topology, with particular interest in smooth 4-dimensional manifolds. His doctoral dissertation provided the first concrete relationship between the powerful invariants of smooth 4-manifolds defined by Ozsváth and Szabó on the one hand, and the theory of trisections of 4-manifolds on the other. Ozsváth-Szabó theory provides a means to study subtle properties of smooth 4-manifolds, for example distinguishing between manifolds that are homeomorphic but not diffeomorphic, while the theory of trisections is a rapidly developing field that gives an essentially combinatorial method to describe and manipulate 4-manifolds. By demonstrating a connection between these fields, Bill's thesis is pointing the way to further developments in both areas.

Bill finished his Ph.D. in 2020 at the University of Georgia, where his dissertation advisor was David Gay; he spent the year 2019-20 as a visitor at the Max Planck Institute in Bonn, Germany. Bill joins UVa with support from the department's RTG grant from the National Science Foundation, and besides teaching regular classes will assist in the department's summer research programs for undergraduates.

New Faculty Profiles

Thomas Polstra Research Associate (2020-)

Thomas Polstra's research interests lie in Commutative Algebra. More specifically, his research specialties include prime characteristic singularities, uniformity in commutative algebra, the behavior of numerical measurements of singularities, and the theory of surface singularities. At UVA he is working with Craig Huneke. Prior to joining the mathematics department at the University of Virginia, Thomas was a National Science Foundation postdoc at the University of Utah after receiving his PhD at the University of Missouri. Thomas has served the mathematics community by organizing high school math circles, organizing conferences, designing a REU course, and mentoring undergraduate research projects. This year, Polstra is teaching Calculus II.



Walker Stern RTG Research Associate (2020-)

Walker Stern works in higher category theory, with applications to homotopy theory, algebra, and mathematical physics. His current research interests include 2-Segal spaces, topological field theories, and $(\infty, 2)$ categories.



Walker received his BA and MA at Brandeis University and spent a year teaching at Wentworth Institute of Technology before moving to Germany for graduate school. He received his doctorate in mathematics from Universität Bonn under the direction of Toby Dyckerhoff. Before coming to UVA, he was a postdoc for one year at Universität Hamburg. This year, Stern is teaching Calculus III and Introduction to Geometry.

Wei-Lun Tsai Research Associate (2020-)

Wei-Lun Tsai hails from Taiwan, where he earned his undergraduate degree from National Taiwan University in 2010, and a Master's degree in 2012. He earned his Ph.D. from Texas A&M University in 2020, where he was advised by Riad Masri.



Wei-Lun Tsai's research interests are in arithmetic geometry and algebraic and analytic number theory. His research specialties include the theory of class groups for number fields, rational elliptic curves, harmonic Maass forms, and L-functions. He will be working in the Algebra Group, and he will be

mentored by Ken Ono. Tsai and Ono began their collaboration in May 2020, and they have already written a few joint papers on elliptic curves and modular forms. Furthermore, Tsai served as a research mentor in the 2020 UVA REU.

Before joining the UVA Department of Mathematics, Tsai mentored undergraduate students for two semesters through the Texas A&M University Directed Reading Program (DRP). Also, for three summers he served as a Teaching Assistant for the REU at Texas A&M University. He worked in the number theory research group, mentoring a total of seven students over the past three summers. He co-authored three papers with these students; one has been published in Mathematical Research Letters, one has been published in Journal of Number Theory, and one is submitted for publication. One of these students won a Goldwater Scholarship and the 2019 Joint Mathematics Meetings--Student Poster Session Award.

Tsai has taught various mathematics courses, from college level to graduate level, including Mathematical Concepts, Calculus (and pre-calculus), and Complex Analysis. He believes that teaching and mentoring are extremely important parts of the profession of mathematics. He is now passionate about both teaching and research, and he excited to develop his skills in these areas at UVA. This year, Tsai is teaching Calculus III and Survey of Algebra.

Gennady Uraltsev Whyburn Research Associate (2020-)

Gennady received his Ph.D. in mathematics from Bonn University and prior to joining UVA he was a H.C. Wang postdoc at the Cornell Math Department. Gena's research interests are in real-variable harmonic analysis and the theory of singular integral operators. His main area of work is in time-frequency analysis, an area initiated by Carleson with his celebrated result on the pointwise convergence of Fourier series for square integrable functions.

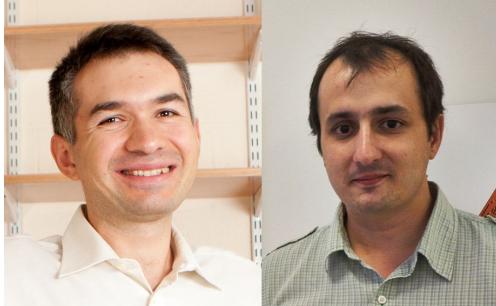


Gena's PhD thesis (2016) was concerned with further developing Do-Thiele's Lebesgue space theory over outer measures. After his thesis, Gena has written papers about vector valued and weighted bounds for classical operators in time frequency analysis, such as the variation-norm Carleson operator and the bilinear Hilbert transform. He is looking for other applications of the new methods to study important open problems in harmonic analysis, such as the question about uniform estimates for the bilinear Hilbert transform. This year, Uraltsev is teaching Calculus III and Basic Real Analysis.

Faculty Awards and Honors

Leonid Petrov Receives the 2020 Bernoulli Prize for a Joint Paper

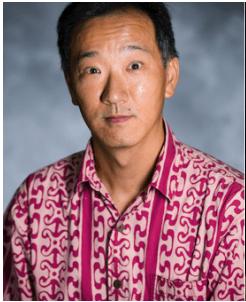
The Bernoulli Prize for an Outstanding Survey Article is to recognize authors of an influential survey publication in probability. The 2020 prize is given to Alexei Borodin (MIT) and Leonid Petrov (UVA) for the article Integrable probability: From representation theory to Macdonald processes. Probability Surveys, v.11:1–58, 2014.



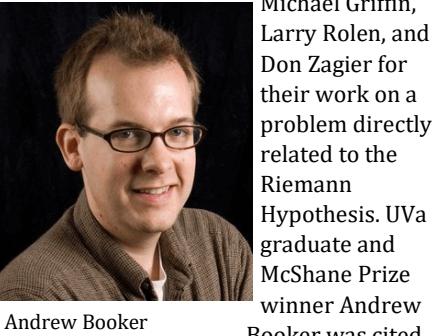
Alexei Borodin and Leonid Petrov

Popular Mechanics Honors Ken Ono and Andrew Booker for the Top Two Math Breakthroughs in 2019

In a recent Popular mechanics article UVa faculty member Ken Ono was honored along with co-authors



Ken Ono and Andrew Booker



Michael Griffin, Larry Rolen, and Don Zagier for their work on a problem directly related to the Riemann Hypothesis. UVa graduate and McShane Prize winner Andrew Booker was cited

along with Andrew Sutherland for work leading to the long-sought integer solutions to $x^3+y^3+z^3=33$ and $x^3+y^3+z^3=42$.

Paul Bourdon and Sara Maloni Win 2020 All-University Teaching Awards

We are happy to announce that Sara Maloni and Paul Bourdon are receiving University of Virginia All-University Teaching Awards. They are among this year's group of ten recipients.



Paul Bourdon and Sara Maloni

Ken Ono Elected Chair of the Mathematics Section of AAAS

Ken Ono was elected Chair of the Mathematics Section of the American Association for the Advancement of Science.

Jennifer Morse Elected to Fellows of the AMS

Congratulations to Professor Jennifer Morse for being elected to the 2021 Class of Fellows of the AMS, "for contributions to algebraic combinatorics and representation theory and service to the mathematical community".



Jennifer Morse

Ken Ono Named Among the Most Influential Mathematicians

Ken Ono, Thomas Jefferson Professor of Mathematics, has been named the 15th most influential mathematician by Academic Influence.

Karen Parshall Elected Fellow of AAAS

Congratulations to Commonwealth Professor of Mathematics and History Karen Parshall who was elected as a Fellow of the American Association for the Advancement of Science, "for outstanding contributions to the history of mathematics, combined with extraordinary service to the mathematical and historical sciences."



Karen Parshall

Ken Ono Awarded Honorary Professorship at IIT Guwahati

We are pleased to announce that Professor Ken Ono was awarded Honorary Professorship at Indian Institute of Technology, Guwahati.

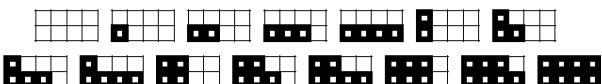
Faculty Research

Wrapping Polynomial Computations in Combinatorics

By Jennifer Morse

Remember when you were taught to expand $(x + y)^2$, multiplying out term by term? It wasn't particularly hard, but I doubt your teacher would have gotten a shiny, red apple for asking you to expand out $(x + y)^6$ on a quiz. Were you thrown for a loop when discovering that the same expansion allows you to compute some nasty trig integrals and to prove the power rule in Calculus? Did you again turn to multiplying polynomials for intuition when finally learning the definition of an algebra (while being encouraged by your 12 year old sister who was sailing through her own algebra homework)?

Polynomial computation doesn't go away, but would you believe that it can be beautifully wrapped in a game of counting? Finding the coefficient of x^2y^4 in $(x + y)^6$ turns into a matter of counting the ways to stack boxes against the wall of a 2×4 rectangle:



Such combinatorial solutions are not only elegant and fun, but they may simply be the only way to go. Computing with symmetric polynomials, multivariate polynomials with the property of being invariant under any permutation of the variables:

$$-17x_1x_2 - 17x_1x_3 - 17x_2x_3 \text{ but not } -17x_1x_2 - 15x_1x_3 - 17x_2x_3,$$

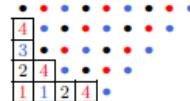
is accomplished by a rule using a 100-year old combinatorial structure called Young tableaux (fill the above boxes with positive numbers so that they are increasing along rows and up columns). What's more, the same rule for computing products of symmetric polynomials captures the structure of the cohomology of the variety of r -dimensional subspaces of C^n and the decomposition of Young modules into irreducible representations.

The challenge of finding a combinatorial structure to naturally facilitate computation is a central theme in algebraic combinatorics. My interest is to find creative ways to describe computational problems which have applications to enumerative geometry and representation theory. A broad class of problems connects with a distinguished family of multivariate polynomials called Macdonald polynomials. These are symmetric functions which arise in the wave equations of certain systems of quantum relativistic particles on a circle, and as the Frobenius images of bigraded Garsia-Haiman modules. Key information is stored in products of



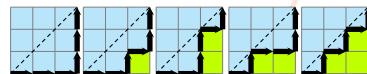
the distinguished families or in their coefficients when decomposed into more classical symmetric function bases. However, the computations are now extremely intricate and efficient rules are notoriously hard to come by.

This was my focus in the late 1990's when Alain Lascoux, Luc Lapointe, and I uncovered the k -Schur functions (Duke Math. J. 2003). The k -Schur functions are a basis for the span of a subset of Macdonald polynomials determined by a positive integer k . These symmetric functions led Lapointe and me to a modernization of Young tableaux which is compatible with the Bruhat order on the type-A affine Weyl group; a periodic coloring of the numbers comes into play:



Our investigations led to the further discovery that parameterless k -Schur functions can be used to reflect the structure of quantum cohomology (products of k -Schur functions generate certain three-point Gromov-Witten invariants). Thomas Lam went on to prove a conjecture of Mark Shimozono that parameterless k -Schur functions represent Schubert classes for the homology of the affine Grassmannian of SL_{k+1} (J. Amer. Math. Soc. 2008).

These surprising connections strengthened my resolve to fully develop the combinatorics necessary for computing with k -Schur functions. Just recently, my work with Blasiak, Pun, and Summers (J. Amer. Math. Soc. 2019) settled problems posed with the inception of k -Schur functions 20 years prior. A key idea was that realizing k -Schur functions as a subclass of the larger family of Catalan functions offers powerful inductive techniques not available in a confined setting. Catalan functions are graded Euler characteristics of certain vector bundles on the flag variety, studied in greatest generality by Chen-Haiman (Berkeley Thesis 2010) and Panyushev (Selecta Math. 2010). The related combinatorial mechanism is given by Dyck paths, prescribing boxes stacked beneath a staircase:



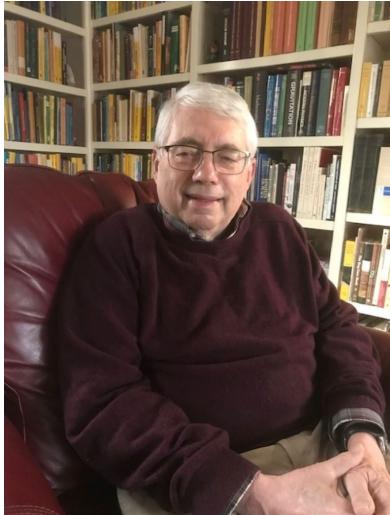
The enumeration of Dyck paths gives the Catalan numbers, sequences which have been around since the 1800's. By forging a relationship between the affine tableaux and Dyck paths, we are finally able to deduce natural combinatorial rules for computing k -Schur functions.

Most exciting is the wealth of new problems springing from this approach; there is enough fodder to feed the hungriest of combinatorial monsters.

Brian Parshall, Gordon Whyburn Professor of Mathematics Emeritus

Brian Parshall received his doctoral degree in mathematics from Yale in 1971. After completing two years of service in the United States Army, he joined the UVA Department of Mathematics in 1972 as an Assistant Professor. He became a full professor in 1983 and in 1996 he became the Gordon Whyburn Professor of Mathematics. As Chair of the department from 1993-1999, he shaped the department in that decade and beyond. Always an advocate of the highest academic standards in the faculty, he was also generous with service to the department. He returned as chair from 2010-2012.

Brian's research is in representation theory and cohomology of algebraic and quantum groups, as well as of finite dimensional Lie and associative algebras. It remains highly influential. He published over 100 articles and mentored 9 Ph.D. students, and helped found in 2009 the NSF-funded annual workshop series called Southeast Lie Theory.



Brian Parshall

Numerous practicing mathematicians today are grateful for his mentorship and intellectual leadership. Especially noteworthy are Brian's research collaborations. One famous collaboration, known as CPS, for Edward Cline, Brian Parshall, and Leonard Scott, created its first joint research publication at UVA in 1972-3, and twenty-eight in all over the ensuing 40 years, until Ed Cline's death in 2012. The trio CPS is believed to be the longest running three person research team known in all of mathematics. Brian had other high profile collaborations, including seven publications with Jianpan Wang at East China Normal University, who later became president of that university, and thirteen with Eric Friedlander of Southern California University, who later served a term as President of the (national flagship) American Mathematical Society. Brian, himself, was named a Fellow of the American Mathematical Society in 2012.

Brian retires from active service to the University in 2020 but will continue his scholarship and mentorship for many years to come.

Bridge to the Doctorate Program

By David Sherman

In Fall 2020 the math department was one of five UVa departments to welcome its first cohort in a new graduate school initiative, the Bridge to the Doctorate. This is a post-baccalaureate program for students from groups that are underrepresented in their disciplines and who have not had sufficient training and research experiences to prepare them for admission to doctoral programs. Participants are fully funded for two years.

The inaugural group of bridge students is Adrian Avalos, Alex Jenny, and Reina Kirkendall. We are excited about their talent and potential! Unfortunately the pandemic has constrained opportunities to include them in department life, but they are making the best of it, taking classes, meeting with faculty mentors, and participating in online conferences and other opportunities for professional development.

Asked to provide short comments for this article, all three

students expressed appreciation for faculty mentoring. Alex stated, "By being supported and encouraged by mentor professors to learn and grow as a mathematician, the bridge program has given me the opportunity to learn topics I would otherwise struggle with alone." Reina wrote, "I have the opportunity to take classes that weren't offered at my undergraduate institution and I'm excited to learn which math topics excite me." Adrian "suspect(s) that by the end of the bridge program, I will be equipped with the tools necessary to achieve a PhD."

The bridge program is administered within the math department by a committee of seven faculty: myself and Julie Bergner, Evangelia Gazaki, Ben Hayes, Thomas Koberda, Tom Mark, and Tai Melcher. We welcome applicants for Fall 2021, so please pass the word on to any students or communities you think may be interested! The deadline for applications is March 1, and more information can be found at <https://math.virginia.edu/graduate/bridge/>, including a link to the application page in the graduate school. Inquiries may be directed to David Sherman (dsherman@virginia.edu).

Virginia Mathematics Lecture Series

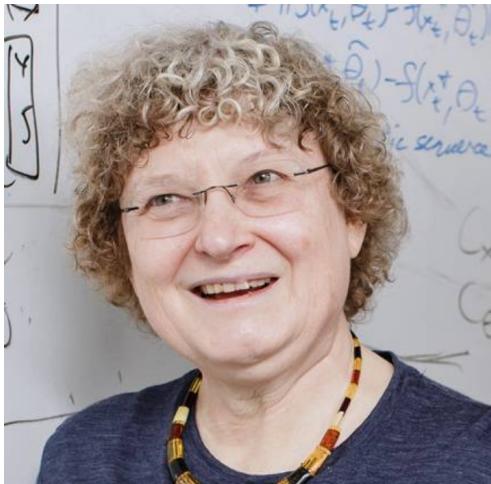
The Virginia Mathematics Lectures are a Distinguished Lecture Series that IMS established in 2014

Spotlight: Ingrid Daubechies

The Department of Mathematics welcomed Ingrid Daubechies for a virtual visit on September 14 and 15, 2020. Her visit featured both a department colloquium and a public lecture, as well as a lunch with our local AWM chapter.

Ingrid Daubechies is the James Duke Professor of Mathematics and Computer Engineering at Duke University. She is recognized for her study of the mathematical methods that enhance image-compression technology. Professor Daubechies is a member of the National Academy of Engineering, the National Academy of Sciences (NAS), and the American Academy of Arts and Sciences. Her research accomplishments have garnered her a MacArthur Fellowship, NAS Mathematics Prize, Steele Prize, and Nemmers Prize, to name a few.

Professor Daubechies' colloquium discussed diffusion methods



that can be used to identify low-dimensional manifolds underlying high-dimensional datasets, and illustrated that by pinpointing additional mathematical structure, improved results can be obtained. Much of the talk drew on a case study from her collaboration with biological morphologists.

Her public lecture highlighted ways that mathematics can help art historians and art conservators to study and understand art works, their manufacture process, and their state of conservation. Her talk featured several instances of such collaborations, explaining the role of mathematics and illustrating the approach with extensive documentation of the art.

Professor Daubechies also participated in one of the UVa AWM chapter's lunchtime Q&A's, which was a great opportunity for members of our community to hear more informally about her mathematics and her distinguished career.

Spotlight: Greg Lawler

The 12th Virginia Mathematics Lectures were given at UVa by Greg Lawler February 12-14, 2020.

Greg Lawler is the George Wells Beadle Distinguished Service Professor of Mathematics and Statistics at the University of Chicago. Professor Lawler has made many contributions to probability theory and is particularly known for his work in developing the theory of the Schramm-Loewner evolution. He is a fellow of the American Mathematical Society, and a recipient of the George Pólya Prize and the Wolf Prize in Mathematics.

His first lecture "Random walks: Simple and self-avoiding" began with a review of the most common model for random behavior,



the "drunkard's walk", where at each time an individual chooses their step from some probability distribution. He discussed what happens when one puts some constraints on the walker to try to avoid places already visited, and he explored the relationship between the "fractal dimension" of a random path and the ambient dimension in which it lives.

The second lecture "Conformal invariance and the two-dimensional critical phenomenon" focused on the prediction by theoretical physicists that lattice models from equilibrium statistical physics "at criticality" in two dimensions have limits that are conformally invariant. Professor Lawler surveyed some of the incredible work of the last twenty years making these ideas precise and rigorous, the starting point of which was the development of the Schramm-Loewner evolution.

The final lecture "Loop measures and the loop-erased random walk" described some relatively recent work on two related models, loop measures and the loop-erased random walk.

Undergraduate Accomplishments

Distinguished Major Program

In its third year, our revamped Distinguished Major Program continues to pick up pace. This year four students — Sebastian Haney, Yifei Yang, Trent Lucas, and Brandon Ward — successfully completed the distinguished mathematics major program. This involves completing a research project under the supervision of the faculty member, assembling their findings in a thesis and presenting these at a public defense.

Sebastian Haney

(**Advisor:** Tom Mark)

Title: Cylindrical contact homology of Brieskorn 3-manifolds
Abstract: Cylindrical contact homology (CCH) is a Floer-theoretic invariant of contact manifolds proposed in 2000 by Eliashberg, Givental, and Hofer as part of the symplectic field theory package. Intuitively cylindrical contact homology is an attempt to define the Morse homology of a certain action functional induced by a contact form. Defining CCH in full generality is still a work in progress, but in 2019 Hutchings and Nelson showed that it is a well-defined contact invariant for a subclass of contact 3-manifolds satisfying additional geometric constraints. I will describe joint work with Tom Mark in which we calculate cylindrical contact homology for the Brieskorn 3-manifolds which are Seifert fibrations over hyperbolic orbifolds with their standard contact structures, which I will also review.

Trent Lucas

(**Advisor:** Sara Maloni)

Title: Hyperbolic Structures on Surfaces and their representation space

Abstract: Given a surface, viewed as a topological object, we can endow it with a geometry. In fact, we can equip most surfaces with a hyperbolic structure, which is a geometry that locally resembles the hyperbolic plane. Our goal in this talk is to understand the relationship between different hyperbolic structures on the same surface. This is encoded by a space called Teichmüller space, where "points" are hyperbolic structures and similar structures are "close". We will see how the Teichmüller space behaves under a natural action of the symmetry group of a surface (i.e. its mapping class group), and how this generalizes to the representation space of the surface's fundamental group. In particular, we will discuss a theorem of Marche' and Wolff relating a Conjecture of Goldman to a question asked by Bowditch.

Brandon Ward

(**Advisor:** Julie Bergner)

Title: Topology, Combinatorics, and Categories: from Δ to Θ_2
Abstract: Finite ordinals arise as we try to model topological spaces by combinatorial information, that is, sets of discrete points and subsets thereof. The collection of all finite ordinals and the order preserving maps between them constitute the category Δ , which we use to define a successful combinatorial model for topological spaces, called simplicial sets. Any map in Δ can be written as the composite of certain generating maps of Δ ; we can explicitly describe simplicial sets by means of these generating maps and the relations amongst them. Our goal in this talk is two-fold: first, we consider how Δ arises in our attempt to model a topological space combinatorially; second, we consider a generalization of Δ , the category Θ_2 , and try to understand the combinatorics of this category by presenting a set of generating maps and relations amongst them.

Yifei Yang

(**Advisor:** Malek Abdesselam)

Title: Diagrammatic Methods in Classical Invariant Theory

Abstract: Classical invariant theory studies the properties of polynomials. In particular, we wish to learn when polynomial functions will stay "invariant" or "unchanged" under the actions of linear groups. We will provide a basic overview of invariants and covariants under the context of binary forms and the special linear group SL_2 . Furthermore, we would like to introduce an alternative approach to investigate invariant theory: the diagrammatic methods. Developed from 19th century invariant theories, diagrammatic methods construct graphical representations to describe algebraic objects such as matrices, tensors, binary forms, and covariants. We hope to offer an expository view about how the diagrammatic methods work and serve as an intrinsic way to analyze invariant theory.

2019-2020 Mathematics Department Prize Winners

E.J. McShane Prize

Sebastian Haney and Trent Lucas

The E.J. McShane Prize is given annually to graduating math majors of outstanding achievement in mathematics.



Sebastian Haney



Trent Lucas

Edwin E. Floyd Prize

Nathan Li and Zach Baugher

The Edwin E. Floyd Prize is given annually to second- or third-year majors who show exceptional promise in mathematics.



Zachary Baugher

Undergraduate Activities

Undergraduate research: mathematical fluid dynamics

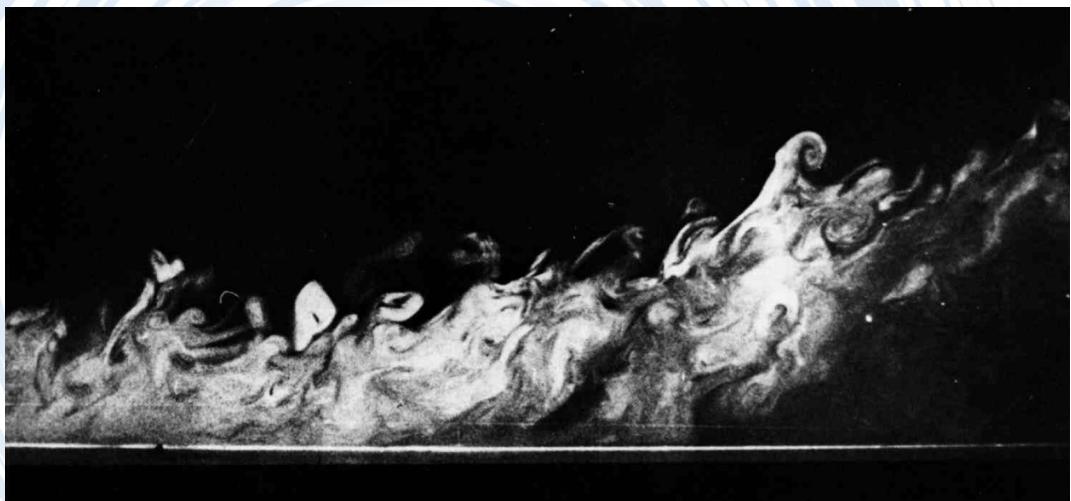
By Zoran Grujic

Mathematical fluid dynamics is an area of applied analysis tasked with providing a mathematical framework for the scientific study of fluid flows, and in particular with infusing mathematical rigor into turbulence phenomenology. One of the key principles of turbulence is the energy cascade: the energy is transferred from larger to smaller scales (a nonlinear process), and the cascade continues until a 'dissipation scale' is reached—at this point, the diffusion (a linear process) takes over and dissipates the energy in the form of heat. One of the big challenges has been relating the geometry of the flow with the analytical theory.

In Summer 2018, math major Jiayi Wang explored a possibility of a power-law dependence between a 'typical small scale' associated with the coherent vortex structures in a 3D turbulent channel, a geometric scale, and an analytic dissipation scale. One should note that the rigorous theory here is fundamentally incomplete. In this project, Jiayi made use of a dataset made publicly available as a part of the Johns Hopkins Turbulence Database (JHTDB). Once the data was harvested, she visualized the coherent vortex structures in ParaView, extracted the information on the 'scale of sparseness' of the regions of intense vorticity, and performed the data analysis. The investigation revealed that there is indeed a statistically significant evidence of the power-law dependence, providing a boost for the future theoretical efforts.

In summer 2019, Jiayi engaged in her second REU-type research project under the guidance of mathematics professor Zoran Grujic. This time, the topic was the study of possible formation of singularities in the 3D fractional Navier-Stokes (NS) equations, describing the motion of a 3D incompressible, viscous, Newtonian fluid with a reduced diffusion described by the fractional Laplacian operator. The fractional NS equations serve as a bridge between the 3D Navier-Stokes equations (full diffusion, described by the classical Laplace operator) and the 3D Euler equations (inviscid flow, zero diffusion). The question of whether a singularity can form in an initially smooth/regular flow is still open for the full range of the models, from Euler to NS, and is one of the major open

problems in the mathematical physics. A physically relevant task is to formulate a condition—consistent with the geometry of turbulent flows observed in the highly-resolved computational simulations—that would prevent a singularity formation. There is a nontrivial amount of work in this direction in the case of both Euler and NS equations, but very little in the case of the fractional NS. One of the prominent morphological signatures of turbulent flows is the local coherence of the vorticity direction, i.e., locally, the vortex lines are nearly-aligned. This is especially visible in the regions of intense vorticity. In her work, Jiayi established that a scaling-invariant, dynamical balance between the coherence of the vorticity direction and the vorticity magnitude, in the averaged sense, suffices to prevent a formation of the singularity—the first result of this type for the fractional NS. The proof utilized an array of sophisticated tools from functional and harmonic analysis, and is on the level of a Ph.D. student actively working in this area.



coherent vortex structures in turbulent boundary layer, from An Album of Fluid Motion by Milton Van Dyke

As a matter of fact, this work resulted in her solo paper, Balance of the vorticity direction and the vorticity magnitude in 3D fractional Navier-Stokes equations, Applied Mathematics & Optimization (a highly ranked journal in applied analysis), <https://doi.org/10.1007/s00245-020-09684-1>. In addition, Jiayi presented her findings at MAA Undergraduate Poster Session at the Joint Mathematics Meeting (JMM) in Denver, Colorado, where she received an "Outstanding Poster" award. She is currently applying for graduate programs in mathematics and applied mathematics.



Jiayi Wang

Undergraduate Activities

2020 UVA REU Program

The UVa Department of Mathematics organized a summer research experience for undergraduate students in June and July 2020. The program was supported by grants from the National Science Foundation (i.e. RTG grant and another REU grant), the National Security Agency, the Templeton World Charity Foundation, as well as some private donors. The program entitled "Number Theory, Representation Theory and Topology" was headed by a management team consisting of Professors Julie Bergner, Thomas Koberda, Thomas Mark, and Ken Ono.

The diverse group of 28 participants included 13 men and 15 women from colleges and universities across the country, including the Lafayette College, University of Maryland, Wake Forest University, and Marshall University in the mid-Atlantic region as well as many further-off schools such as Harvard, Harvey Mudd, MIT, Stanford, University of Chicago, University of Michigan and the University of California.

Although the pandemic necessitated significant changes to the planned program, the participants enjoyed 6 weeks of virtual research activities. These activities included a lecture series, various mini-courses, panel discussions, and in-depth guided research with UVa faculty mentors, supported by UVa graduate students and postdocs.

The invited speakers were: Henri Darmon (McGill), Sang-hyun Kim (KIAS), James Maynard (Oxford University), Lillian Pierce (Duke), Ken Ribet (UC Berkeley), and Maryna Viazovska (Ecole Polytechnique, Lausanne), as well as UVa mathematics faculty members Benjamin Hayes, Slava Krushkal, Nick Kuhn, Sara Maloni, and Ken Ono.

The mini-courses were:

Essentials of topology and geometry of manifolds
(Instructors: Julie Bergner, Thomas Koberda, Tom Mark)

Beyond the Mordell-Weil Theorem
(Instructor: Charlotte Ure)

Complex multiplication: Constructing Hilbert class fields (Instructor: Jinbo Ren)

Representation Theory and Symmetric Groups (Instructor: You Qi)

Brauer Groups: What are they and what are they good for?
(Instructor: Evangelia Gazaki)



As well as receiving guidance from the lead faculty members and guest mentors Jesse Thorner (University of Illinois, Urbana Champaign) and Wei-Lun Tsai,

participants worked closely with graduate student mentors from the UVa math department: Ross Akhmechet, Will Craig, Peter Johnson, Christopher Lloyd, Eleanor McSpirit, Badri Pandey, Ian Runnels, Shunyu Wan and Jiajun Yan.

The participants completed 8 papers that have been submitted for publication in peer reviewed mathematics journals. These papers are:

M. Amir and L. Hong, On L-functions of modular elliptic curves and certain K3 surfaces, Ramanujan Journal, accepted for publication.

T. Blum, C. Choi, A. Hoey, J. Iskander, K. Lakein, and T. Martinez, On class numbers, torsion subgroups, and quadratic twists of elliptic curves, submitted for publication.

S. Dembner and V. Jain, Hyperelliptic curves and newform coefficients, submitted for publication.

S. Dembner and V. Jain, A note on congruences for weakly holomorphic modular forms, submitted for publication.

M. Hanada and R. Madhukara, Fourier coefficients of level 1 Hecke eigenforms, submitted for publication.

L. Hong and S. Zhang, Towards Heim and Neuhauser's unimodality conjecture on the Nekrasov-Okounkov polynomials, Research in Number Theory, accepted for publication.

T. Khale, C. O'Kuhn, A. Panidapu, A. Sun, and S. Zhang, A Bombieri-Vinogradov Theorem for primes in short intervals and small sectors, submitted for publication.

A. Panidapu and J. Thorner, The CM Sato-Tate distribution in short intervals, submitted for publication.

Number theory participant Vanshika Jain has been awarded

Honorable Mention for the 2021 Alice T. Schafer Prize by the Association for Women in Mathematics. The Schafer Prize recognizes the best undergraduate women in mathematics.



UVa Mathematics is running a similar program in 2021 (see <https://uva.theopenscholar.com/reu/program>). The deadline for 2021 applications for the 2021 program is February 15, 2021.

Please spread the word.

Recent PhDs

Thomas Sale

Advisor: Weiqiang Wang

Quantum Symmetric Pairs and Quantum Supergroups at Roots of 1



A quantum group, as conceived by Drinfeld and Jimbo, is the quantization of an enveloping algebra via the quantum parameter v . In analogue with the theory of algebraic groups in prime characteristic, Lusztig laid the foundations of a theory of quantum groups when v has been specialized to a root of 1. Among his fundamental results and constructions are a quantum Frobenius homomorphism, a Steinberg tensor product theorem and the small quantum group.

In this dissertation, we extend the aforementioned results to two related settings. A quantum symmetric pair is the quantization of a symmetric pair of a Lie algebra and its fixed point subalgebra under an involution; the corresponding subalgebra is called an \imath quantum group. In the first part of the dissertation, we show that Lusztig's Frobenius homomorphism restricts to a map of \imath quantum groups in finite type. We also formulate the small \imath quantum group and compute its dimension. A number of elements are shown to be central in the \imath quantum group at a root of 1. In ADE type, the action of the \imath quantum group at a root of 1 on the quantized adjoint module gives rise to a Lie algebra isomorphic to the symmetric pair subalgebra.

A quantum covering group is an algebra with parameters v and π , where $\pi^2 = 1$. When π is specialized to 1, it is a quantum group of anisotropic Kac-Moody type, and when π is specialized to -1 , it is a quantum supergroup. In the second part, we establish analogues of Lusztig's Frobenius homomorphism and Steinberg tensor product theorem for quantum covering groups. Moreover, we formulate the small quantum covering group; in finite type, we compute its dimension. The specialization of these constructions to $\pi = 1$ recovers those of Lusztig.



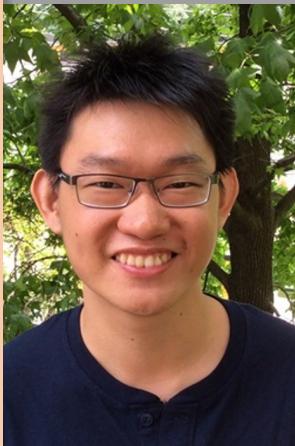
Huy Dang

Advisor: Andrew Obus

Hurwitz Trees and Deformations of Cyclic Covers

The main focus of this thesis is equal characteristics deformations of Artin-Schreier covers (of curves). We formulate some conditions on a combinatorial object called Hurwitz tree to determine the existence of certain types of deformations of given degree p cyclic branched covers. Furthermore, by applying these Hurwitz tree criteria, we show that the moduli space of Artin-Schreier curves of fixed genus g is connected when g is sufficiently large.

Recent PhDs



Christopher Chung

Advisor: Weiqiang Wang

Quantum Covering Groups and Quantum Symmetric Pairs

A quantum covering group \mathbf{U}_π is an algebra with parameters q and π subject to $\pi^2 = 1$ and it admits an integral form; it specializes to the usual quantum group at $\pi = 1$ and to a quantum supergroup of anisotropic type at $\pi = -1$. In this dissertation, we establish the Frobenius-Lusztig homomorphism and Lusztig-Steinberg tensor product theorem in the setting of quantum covering groups at roots of 1, recovering Lusztig's constructions for quantum groups at roots of 1 when we specialize at $\pi = 1$.

We develop a theory of quantum symmetric pair $(\mathbf{U}_\pi, \mathbf{U}_\pi^\sharp)$, where \mathbf{U}_π^\sharp is a coideal subalgebra of \mathbf{U}_π . When specializing at $\pi = 1$, the pair $(\mathbf{U}_\pi, \mathbf{U}_\pi^\sharp)$, reduces to a quantum symmetric pair of G. Letzter and its Kac-Moody generalization by Kolb. We give a Serre presentation for \mathbf{U}_π^\sharp of quantum symmetric pairs $(\mathbf{U}_\pi, \mathbf{U}_\pi^\sharp)$ for quantum covering groups, introducing the i^π -Serre relations and i^π -divided powers. We also develop a quasi K -matrix in this setting, which leads to a construction of ω canonical bases for the highest weight integrable \mathbf{U}_π -modules and their tensor products regarded as \mathbf{U}_π^\sharp -modules, as well as an ω canonical basis for the modified form of the ω quantum group \mathbf{U}_π^\sharp . Again, specializing at $\pi = 1$ we recover the Serre presentation of \mathbf{U}^\sharp by Chen-Lu-Wang and the canonical basis construction of Bao-Wang. The specialization at $\pi = -1$ leads to new constructions for quantum supergroups.

Andrew Kobin Advisor: Andrew Obus

Wild Ramification and Stacky Curves

The local structure of Deligne–Mumford stacks has been studied for decades, but most results require a tameness hypothesis that avoids certain phenomena in positive characteristic. We tackle this problem directly and classify stacky curves in characteristic $p > 0$ with cyclic stabilizers of order p using higher ramification data. Our approach replaces the local root stack structure of a tame stacky curve, similar to the local structure of a complex orbifold curve, with a more sensitive structure called an Artin–Schreier root stack, allowing us to incorporate the ramification data directly into the stack. A complete classification of the local structure of stacky curves, and more generally Deligne–Mumford stacks, will require a broader understanding of root structures, and we begin this program by introducing a higher-order version of the Artin–Schreier root stack. Finally, as an application, we compute dimensions of Riemann–Roch spaces for some examples of stacky curves in positive characteristic and suggest a program for computing spaces of modular forms using the theory of stacky modular curves.



Recent PhDs



Bogdan Krstic Advisor: Nicholas Kuhn

Bispans in Quasicategories

A Tambara functor is an algebraic system indexed by the subgroups of a fixed finite group, possessing additive and multiplicative inductions along subgroup inclusions as well as a twisted distributive law between the two inductions. These arise in equivariant homotopy theory and group representation theory, with examples coming from representation rings, generalized character rings, equivariant K -theory, Burnside rings, group cohomology, algebraic K -theory, and homotopy groups of equivariant E_∞ -ring spectra. Tambara functors are defined using bispan categories, which simultaneously encode the inductive system and distributive law. Many Tambara functors can be defined in a compatible manner for all groups at once, suggesting the notion of a global Tambara functor. Encoding the distributivity properties for global equivariant phenomena suggests passage to bispans in the bicategory of finite groupoids, but the complicated nature of bispan composition means that an axiomatic elaboration of global Tambara functors has yet to be provided, although work of Schwede suggests a relationship to global Mackey functors with power operations. In this thesis, we provide a quasicategorical bispan construction, initiating the development of the higher categorical framework needed to study global Tambara functors.

We provide a construction of a quasicategory of bispans in a locally cartesian closed quasicategory which is compatible with the span quasicategory of Barwick. In particular, we obtain a quasicategory whose simplices consist of diagrams encoding higher composites of bispans. To this end, we develop and study quasicategorical analogues of exponential diagrams, which are the categorical construction governing composition in bispan categories.

We first generalize a theory of bispans in the category of finite sets appearing in the thesis of Cranch, creating a “decomposed” bispan quasicategory. The decomposed bispan diagrams of Cranch are sub-simplicial sets of the bispan diagrams used for the main construction, and we establish pleasant properties of these inclusions. With these results in hand, we prove that what is *a priori* a simplicial set of bispans is in fact trivially fibered over the decomposed bispan quasicategory, thus obtaining the desired quasicategory of bispans.

Recent PhDs

Carleson Embeddings into Weighted Outer Measure Spaces

Recent progress in harmonic analysis in obtaining L^p norm inequalities for modulation invariant operators has been in part due to the formalization of time-frequency analysis methods under an outer measure framework developed in [DT15]. The framework codifies the underlying nature of such an analysis and shifts the difficulty in proofs to obtaining L^p norm estimates on projection operators mapping from a classic L^p space into an outer measure L^p space; such maps are referred to as (generalized) Carleson embeddings. This dissertation seeks to extend known generalized Carleson embeddings in outer L^p theory from non-weighted settings to weighted settings. The highlight estimate of this work is a generalized Carleson embedding for the wave packet transform

$$f \mapsto \mathbb{F}_\phi(f)(y, \eta, t) := \int_{\mathbb{R}} f(x) \frac{1}{t} e^{i\eta(y-x)} \phi\left(\frac{y-x}{t}\right) dx, \quad (y, \eta, t) \in \mathbb{R} \times \mathbb{R} \times (0, \infty)$$

of a function $f \in L^p(\mathbb{R}, w)$ into a weighted outer L^p space situated in $\mathbb{R} \times \mathbb{R} \times (0, \infty)$ for exponents $p > 2$ and Muckenhoupt weights $w \in A_{p/2}$. The wave packet transform is a projection of modulation invariant operators into $\mathbb{R} \times \mathbb{R} \times (0, \infty)$ as mentioned in [DT15, DPO18, Ura16] and generalized Carleson embeddings of this transform are known in non-weighted settings. The proof utilizes weighted phase plane techniques adapted to a continuum along with new restriction L^2 estimates for the wave packet transform.

Mark Lewers

Advisor: Yen Do



Mark Schrecengost

Advisor: Peter Abramekno



Finite Generation of RGD Systems with Exceptional Links

Let $(G, (U_\alpha)_{\alpha \in \Phi}, T)$ be an RGD system. The most prominent examples are Kac-Moody groups, which are infinite dimensional analogs of semisimple Lie groups. These groups have an associated twin building Δ on which the group G will act strongly transitively. We say that the building Δ satisfies condition (co) if the collection of chambers opposite any chamber is gallery connected. It is known that if Δ satisfies (co), the subgroup U_+ of G is generated by some finite set of fundamental root groups, and thus is finitely generated if these root groups are finitely generated.

We will help close the gap in the literature relying on condition (co) by proving when the subgroup U_+ of an RGD system associated to rank-3 buildings is finitely generated. Most of the time, the group U_+ will not be finitely generated, and we will give sufficient conditions to guarantee the infinite generation of U_+ . We will then modify this approach to see that another group not covered by the conditions is also not finitely generated. Our main strategy will be to produce a large family of surjective homomorphisms from U_+ which send relatively few U_α to non-identity elements, implying that some of these U_α must be in any generating set.

Finally, we will show that there are two cases where U_+ remains finitely despite Δ not satisfying (co). We will use an approach which relies on defining a distance between root groups, and showing that most root groups can be expressed in terms of those closer to the fundamental chamber. This approach can also give another proof of the finite generation of U_+ with condition (co).

2019-2020 Outstanding Mathematics Graduate Teaching Assistant Award:

Andrew Kobil



2019-2020 All-University Graduate Teaching Award:

Sarasij Maitra



Online Life at the Math Department

University of Virginia transitioned to remote teaching in March 2020, and our Department is staying mostly online for the 2020-21 academic year.

The end of the Spring semester was unusual and dramatic. It felt like in-person social interactions in the Department stopped. We started weekly online "tea time" Zoom meetings at the usual department tea times on Thursdays at 3:15, to preserve communication at least in some form.

This has become our new social tradition in the online world.

In May, we gathered students and their families across the nation and the world for the 2020 online installment of the Department of Mathematics



Diploma Ceremony (shown in the picture).

The research part of our activities changed, too. Now anyone can attend any mathematical seminar or conference happening around the world and listen to the live or recorded broadcast of fresh mathematical ideas. However, it turned out that the online format cannot imitate another very important part of mathematical seminars and conferences - the cherished "coffee breaks" between talks, packed with exchange of ideas and sparks of new collaborations with colleagues. When it becomes possible to travel again, this aspect of mathematical research will be the main reason to resume in-person conferences and seminars.

Over the summer of 2020, the University took efforts to prepare for the upcoming online semester. I served as an "Instructional Partner" - a point of contact for my colleagues who worked hard to redesign their courses for the online teaching environment. Everyone had to recreate lectures, blackboard and chalk, quizzes, group discussions, homework, exams, and other components of effective teaching in a fully digital form. These changes inevitably lead to implementing some teaching innovations which

benefited student learning. For example, some of us "inverted" our classrooms: The instructor pre-records the lectures, students view them as a part of the homework,

while the class meeting times are dedicated to discussing problems and examples.

In November 2020 I was advising math majors and some first- and second-year students and asked them how they feel about studying in the new online format. I was surprised that most of the advisees said that this format worked fine for them, and they felt "mostly okay" taking classes online. We already know that this teaching format is continuing through Spring 2020. While we will do our best to educate the students in these circumstances, we cannot wait to see everyone back in classroom for a real blackboard and chalk experience.

What are you doing? We'd love to hear from you!

Email: math-newsletter@virginia.edu

News & Events: <http://math.virginia.edu/allnews/>

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