Lexical alignment: IBM models 1 and 2

MLE via EM for categorical distributions

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Translation data

Let's assume we are confronted with a new language
and luckily we managed to obtain some sentence-aligned data

the black dog	$\square$ $\circledast$
the nice dog	$\square$ $\cup$
the black cat	$\boxdot$ $\circledast$
a dog chasing a cat	$\boxdot$ $\triangleleft$ $\square$

Translation data

Let's assume we are confronted with a new language
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Is there anything we could say about this language?

Translation by analogy

the black dog	$\square \circledast$
the nice dog	$\square \cup$
the black cat	$\boxdot \circledast$
a dog chasing a cat	$\boxdot \triangleleft \square$

A few hypotheses:

Translation by analogy

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A few hypotheses:

- $\square \iff$ dog

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- $\boxdot \iff$ cat
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- nouns seem to preceed adjectives

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A few hypotheses:

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- nouns seem to preceed adjectives
- determines are probably not expressed
- *chasing* may be expressed by $\triangleleft$
  and perhaps this language is OVS

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A few hypotheses:

- $\square \iff$ dog
- $\boxdot \iff$ cat
- $\circledast \iff$ black
- nouns seem to preceed adjectives
- determines are probably not expressed
- *chasing* may be expressed by $\lhd$
  and perhaps this language is OVS
- or perhaps *chasing* is realised by a verb with swapped arguments

Probabilistic lexical alignment models

This lecture is about operationalising this intuition

- through a probabilistic learning algorithm
- for a non-probabilistic approach see for example [?]

Content

Lexical alignment

Mixture models

IBM model 1

IBM model 2

Remarks

Word-to-word alignments

Imagine you are given a text

the black dog	el perro negro
the nice dog	el perro bonito
the black cat	el gato negro
a dog chasing a cat	un perro presiguiendo a un gato

Word-to-word alignments

Now imagine the French words were replaced by placeholders

the black dog	F_1 F_2 F_3
the nice dog	F_1 F_2 F_3
the black cat	F_1 F_2 F_3
a dog chasing a cat	F_1 F_2 F_3 F_4 F_5

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and suppose our task is to have a model explain the original data

Word-to-word alignments

Now imagine the French words were replaced by placeholders

the black dog	$F_1\ F_2\ F_3$
the nice dog	$F_1\ F_2\ F_3$
the black cat	$F_1\ F_2\ F_3$
a dog chasing a cat	$F_1\ F_2\ F_3\ F_4\ F_5$

and suppose our task is to have a model explain the original data
by generating each French word from exactly one English word

Generative story

For each sentence pair independently,

1. observe an English sentence $e_1, \cdots, e_m$
   and a French sentence length n
2. for each French word position j from 1 to n
   2.1 select an English position a_j
   2.2 conditioned on the English word e_{a_j}, generate f_j

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We have introduced an alignment
which is not directly visible in the data

Data augmentation

Observations:

the black dog | el perro negro

Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \rightarrow f_j$

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Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \to f_j$

the black dog | $(A_1, E_{A_1} \to F_1)$ $(A_2, E_{A_2} \to F_2)$ $(A_3, E_{A_3} \to F_3)$

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Observations:

$$\text{the black dog} \mid \text{el perro negro}$$

Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \to f_j$

$$\text{the black dog} \mid (1, E_{A_1} \to F_1)\ (A_2, E_{A_2} \to F_2)\ (A_3, E_{A_3} \to F_3)$$

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$$\text{the black dog} \mid (1, \text{the} \to \text{el})\ (A_2, E_{A_2} \to F_2)\ (A_3, E_{A_3} \to F_3)$$

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Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \to f_j$

$$\text{the black dog} \mid (1, \text{the} \to \text{el})\ (3, E_{A_2} \to F_2)\ (A_3, E_{A_3} \to F_3)$$

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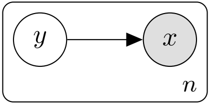
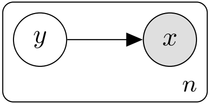
Content

Mixture models: generative story



- c mixture components
- each defines a distribution over the same data space $\mathcal{X}$
- plus a distribution over components themselves

Mixture models: generative story



- c mixture components
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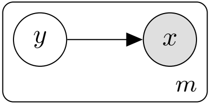
Generative story

1. select a mixture component $y \sim p(y)$
2. generate an observation from it $x \sim p(x|y)$

Mixture models: likelihood



Incomplete-data likelihood

$$p(x_1^m) = \prod_{i=1}^{m} p(x_i) \tag{1}$$

$$= \prod_{i=1}^{m} \sum_{y=1}^{c} \underbrace{p(x_i, y)}_{\text{complete-data likelihood}} \tag{2}$$

$$= \prod_{i=1}^{m} \sum_{y=1}^{c} p(z)p(x_i|y) \tag{3}$$

Interpretation

Missing data

- Let y take one of c mixture components
- Assume data consists of pairs (x, y)
- x is always observed
- y is always missing

Interpretation

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Inference: posterior distribution over possible y for each x

$$p(y|x) = \frac{p(y,x)}{\sum_{y'=1}^{c} p(y',x)} \tag{4}$$

$$= \frac{p(y)p(x|y)}{\sum_{y'=1}^{c} p(y')p(x|y')} \tag{5}$$

Non-identifiability

Different parameter settings, same distribution

Suppose $\mathcal{X} = \{a, b\}$ and $c = 2$
and let $p(y = 1) = p(y = 2) = 0.5$

y	$x = a$	$x = b$
1	0.2	0.8
2	0.7	0.3
$p(x)$	0.45	0.55

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Problem for parameter estimation by hillclimbing

Maximum likelihood estimation

Suppose a dataset $\mathcal{D} = \{x^{(1)}, x^{(2)}, \cdots, x^{(m)}\}$

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the score function is

$$l(\theta) = \sum_{i=1}^{m} \log p_\theta(x^{(i)})$$

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Likelihood of iid observations

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the score function is

$$l(\theta) = \sum_{i=1}^{m} \log p_\theta(x^{(i)})$$

then we choose

$$\theta^\star = \arg\max_\theta l(\theta)$$

MLE for categorical: estimation from fully observed data

Suppose we have **complete data**

- $\mathcal{D}_{\text{complete}} = \{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$

MLE for categorical: estimation from fully observed data

Suppose we have **complete data**

- $\mathcal{D}_{\text{complete}} = \{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$

Then, for a **categorical distribution**

$$p(x|y) = \theta_{y,x}$$

and $n(y, x|\mathcal{D}_{\text{complete}}) =$ *count of* (y, x) *in* $\mathcal{D}_{\text{complete}}$

MLE solution:

$$\theta_{y,x} = \frac{n(y, x|\mathcal{D}_{\text{complete}})}{\sum_{x'} n(y, x'|\mathcal{D}_{\text{complete}})}$$

MLE for categorical: estimation from incomplete data

Expectation-Maximisation algorithm [?]

E-step:

- for every observation x, imagine that every possible latent assignment y happened with probability $p_\theta(y|x)$

$$\mathcal{D}_{\text{completed}} = \{(x, y = 1), \ldots, (x, y = c) : x \in \mathcal{D}\}$$

MLE for categorical: estimation from incomplete data

Expectation-Maximisation algorithm [?]

M-step:

- reestimate θ as to climb the likelihood surface
- for categorical distributions $p(x|y) = \theta_{y,x}$
  y and x are categorical
  $0 \leq \theta_{y,x} \leq 1$ and $\sum_{x \in X} \theta_{y,x} = 1$

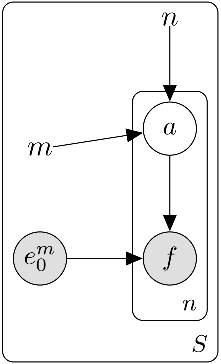
$$\theta_{y,x} = \frac{\mathbb{E}[n(y \to x|\mathcal{D}_{\text{completed}})]}{\sum_{x'} \mathbb{E}[n(y \to x'|\mathcal{D}_{\text{completed}})]} \tag{6}$$

$$= \frac{\sum_{i=1}^{m} \sum_{y'} p(y'|x^{(i)}) \mathbb{1}_y(y') \mathbb{1}_x(x^{(i)})}{\sum_{i=1}^{m} \sum_{x'} \sum_{y'} p(y'|x^{(i)}) \mathbb{1}_y(y') \mathbb{1}_{x'}(x^{(i)})} \tag{7}$$

$$= \frac{\sum_{i=1}^{m} p(y|x^{(i)}) \mathbb{1}_x(x^{(i)})}{\sum_{i=1}^{m} \sum_{x'} p(y|x^{(i)}) \mathbb{1}_{x'}(x^{(i)})} \tag{8}$$

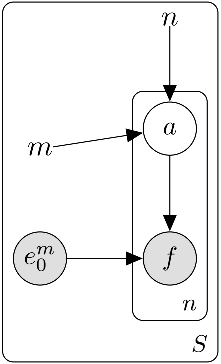
Content

IBM1: a constrained mixture model



Constrained mixture model

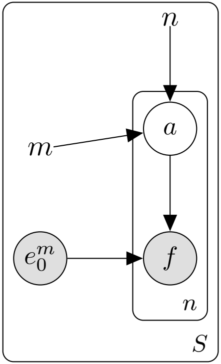
IBM1: a constrained mixture model



Constrained mixture model

- mixture components are English words

IBM1: a constrained mixture model



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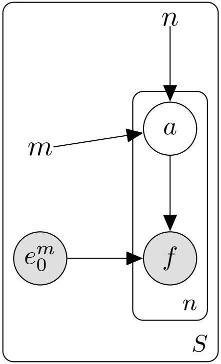


Constrained mixture model

- mixture components are English words
- but only English words that appear in the English sentence can be assigned
- a_j acts as an indicator for the mixture component that generates French word f_j
- e_0 is occupied by a special NULL component

Parameterisation

Alignment distribution: uniform

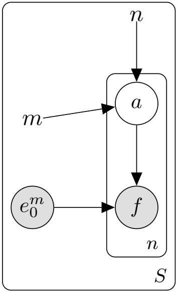
$$p(a|m,n) = \frac{1}{m+1} \tag{9}$$

Lexical distribution: categorical

$$p(f|e) = \mathrm{Cat}(f|\theta_e) \tag{10}$$

- where $\theta_e \in \mathbb{R}^{v_F}$
- $0 \leq \theta_{e,f} \leq 1$
- $\sum_f \theta_{e,f} = 1$

IBM1: incomplete-data likelihood



Incomplete-data likelihood

$$
\begin{aligned}
p(f_1^n|e_0^m) &= \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} p(f_1^n, a_1^n|e_{a_j}) && (11)\\
&= \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \prod_{j=1}^{n} p(a_j|m,n)p(f_j|e_{a_j}) && (12)\\
&= \prod_{j=1}^{n} \sum_{a_j=0}^{m} p(a_j|m,n)p(f_j|e_{a_j}) && (13)
\end{aligned}
$$

IBM1: posterior

Posterior

$$p(a_1^n|f_1^n, e_0^m) = \frac{p(f_1^n, a_1^n|e_0^m)}{p(f_1^n|e_0^m)} \tag{14}$$

Factorised

$$p(a_j|f_1^n, e_0^m) = \frac{p(a_j|m,n)p(f_j|e_{a_j})}{\sum_{i=0}^{m} p(i|m,n)p(f_j|e_i)} \tag{15}$$

MLE via EM

E-step:

$$\mathbb{E}[n(\mathsf{e} \to \mathsf{f}|a_1^n)] = \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} p(a_1^n|f_1^n, e_0^m) n(\mathsf{e} \to \mathsf{f}|A_1^n) \tag{16}$$

$$= \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \prod_{j=1}^{n} p(a_j|f_1^n, e_0^m) \mathbb{1}_{\mathsf{e}}(e_{a_j}) \mathbb{1}_{\mathsf{f}}(f_j) \tag{17}$$

$$= \prod_{j=1}^{n} \sum_{i=0}^{m} p(a_j = i|f_1^n, e_0^m) \mathbb{1}_{\mathsf{e}}(e_i) \mathbb{1}_{\mathsf{f}}(f_j) \tag{18}$$

M-step:

$$\theta_{e,f} = \frac{\mathbb{E}[n(e \to f|a_1^n)]}{\sum_{f'} \mathbb{E}[n(e \to f'|a_1^n)]} \tag{19}$$

EM algorithm

Repeat until convergence to a local optimum

1. For each sentence pair
   1.1 compute posterior per alignment link
   1.2 accumulate fractional counts
2. Normalise counts for each English word

Content

Alignment distribution

Positional distribution

$p(a_j|m,n) = \mathrm{Cat}(a|\lambda_{j,m,n})$

- one distribution for each tuple (j, m, n)
- support must include length of longest English sentence
- extremely over-parameterised!

Alignment distribution

Positional distribution
$p(a_j|m, n) = \mathrm{Cat}(a|\lambda_{j,m,n})$

- one distribution for each tuple (j, m, n)
- support must include length of longest English sentence
- extremely over-parameterised!

Jump distribution **[?]**

- define a jump function $\delta(a_j, j, m, n) = a_j - \left\lfloor j\frac{m}{n} \right\rfloor$
- $p(a_j|m, n) = \mathrm{Cat}(\Delta|\lambda)$
- Δ takes values from $-$longest to $+$longest

Content

Note on terminology: source/target vs French/English

From an alignment model perspective all that matters is

- we condition on one language and generate the other
- in IBM models terminology, we condition on *English* and generate *French*

From a noisy channel perspective, where we want to translate a *source* sentence f_1^n into some *target* sentence e_1^m

- Bayes rule decomposes $p(e_1^m|f_1^n) \propto p(f_1^n|e_1^m)p(e_1^m)$
- train $p(e_1^m)$ and $p(f_1^n|e_1^m)$ independently
- **language model:** $p(e_1^m)$
- **alignment model:** $p(f_1^n|e_1^m)$
- note that the alignment model conditions on the target sentence (English) and generates the source sentence (French)

Limitations of IBM1-2

- too strong independence assumptions
- categorical parameterisation suffers from data sparsity
- EM suffers from local optima

Extensions

Fertility, distortion, and concepts [**?**]

Dirichlet priors and posterior inference [**?**]

- + no NULL words [**?**]
- + HMM and efficient sampler [**?**]

Log-linear distortion parameters and variational Bayes [**?**]

First-order dependency (HMM) [**?**]

- E-step requires dynamic programming [**?**]

References I