Advanced Topic in Recommender Systems

Neighborhood Methods

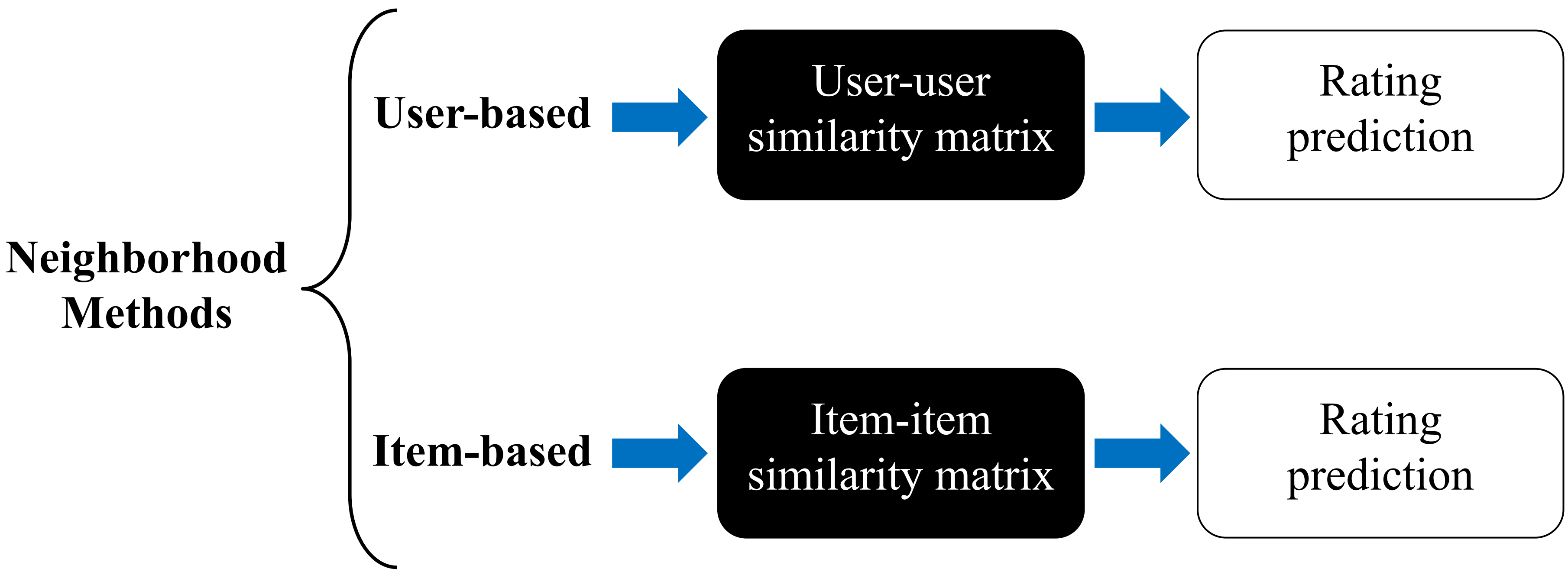
Masoud Mansoury
AMLab, University of Amsterdam
Discovery Lab, Elsevier

Neighborhood-based CF

- **User-based Collaborative Filtering**
  - The opinions of similar users on a target item is used to predict the rating for a target user.

- **Item-based Collaborative Filtering**
  - The rating on a target item is predicted based on its similarity to the items that a target user previously rated.

Neighborhood-based CF



Today …

- We review more **advanced algorithms** that are designed to improve the item-based neighborhood models.
- For this, we focus on **ranking problem**.

Item-based Method

- **Step 1: Similarity computation between all pair of items**
  - The similarity value between all pairs of the items are computed
- **Step 2: Neighbor selection**
  - k items with the highest similarity values with the target item are selected.
- **Step 3: Rating prediction**

The set of k closest items to target item i, which have received rating from user u.

The similarity degree of item j and target item i

Rating by user u on item j

$$\hat{r}_{ui} = \frac{\sum_{j \in Q_i(u)} Sim(i,j) \times r_{uj}}{\sum_{j \in Q_i(u)} |Sim(i,j)|}$$

Ranking task

- **How Item-based method can be used in ranking task?**

Ranking task

- **How Item-based method can be used in ranking task?**

- **In ranking problem, we do not necessarily need to know the predicted rating value**
  - For example, the predicted rating value does not need to be in the range of 5-star ratings
  - We want to know between items A and B, which one is more relevant to the user's preferences

Ranking task

- **Therefore, the equation for predicting the rating value of an item for a user can be simiplified to compute the degree to which the item is relevant to user.**

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Ranking task

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$$\hat{r}_{ui} = \sum_{j \in Q_i(u)} Sim(i,j) \times r_{uj}$$

- r_{uj} **is the rating value if available, otherwise if the rating is missed, it is set to 0.**

Ranking task

- **Therefore, the equation for predicting the rating value of an item for a user can be simiplified to compute the degree to which the item is relevant to user.**

$$\hat{r}_{ui} = \sum_{j \in \cancel{Q_i(u)}} Sim(i, j) \times r_{uj}$$

- **r_{uj} is the rating value if available, otherwise if the rating is missed, it is set to 0.**
- **Neighbor selection step can be ignored and instead, the prediction can be performed over all items**
  - Among all items, the one that user liked would have effect on the prediction
  - The rest of the items will not affect the prediction as $r_{uj} = 0$.

Ranking task

- **Therefore, the equation for predicting the rating value of an item for a user can be simiplified to compute the degree to which the item is relevant to user.**

$$\hat{r}_{ui} = \sum_{j \in \mathbb{I}\mathrm{tems}} Sim(i, j) \times r_{uj}$$

- **r_{uj} is the rating value if available, otherwise if the rating is missed, it is set to 0.**

Ranking task

- **Therefore, the equation for predicting the rating value of an item for a user can be simiplified to compute the degree to which the item is relevant to user.**

$$\hat{r}_{ui} = \sum_{j \in \mathrm{Items}} Sim(i, j) \times r_{uj}$$

- r_{uj} **is the rating value if available, otherwise if the rating is missed, it is set to 0.**

- *The term "score prediction" is also used instead of "rating prediction".*

Example

- **User's profile**

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \boxed{0} & \boxed{5} & \boxed{0} & \boxed{0} & \boxed{4} \end{array}$$

- **Item-item similarity matrix**

$Sim =$

	i_1	i_2	i_3	i_4	i_5
i_1	1	0.3	0.4	0.1	0.7
i_2	0.3	1	0.5	0.6	0.2
i_3	0.4	0.5	1	0.8	0.9
i_4	0.1	0.6	0.8	1	0.5
i_5	0.7	0.2	0.9	0.5	1

Example

- **User's profile**

$$\begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline R_{u1} = & 0 & 5 & 0 & 0 & 4 \\ \hline \end{array}$$

- **Item-item similarity matrix**

$$Sim = \begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 1 & 0.3 & 0.4 & 0.1 & 0.7 \\ \hline i_2 & 0.3 & 1 & 0.5 & 0.6 & 0.2 \\ \hline i_3 & 0.4 & 0.5 & 1 & 0.8 & 0.9 \\ \hline i_4 & 0.1 & 0.6 & 0.8 & 1 & 0.5 \\ \hline i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 1 \\ \hline \end{array}$$

- **To avoid self-recommendation, diagonal entries are set to zero.**
  - Target item has the highest similarity to itself and would lead to recommending itself.

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- **Item-item similarity matrix**

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- **To avoid self-recommendation, diagonal entries are set to zero.**
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Example

- **User's profile**

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline \end{array}$$

i_1	i_2	i_3	i_4	i_5
0	5	0	0	4

- **Item-item similarity matrix**

$Sim =$

	i_1	i_2	i_3	i_4	i_5
i_1	0	0.3	0.4	0.1	0.7
i_2	0.3	0	0.5	0.6	0.2
i_3	0.4	0.5	0	0.8	0.9
i_4	0.1	0.6	0.8	0	0.5
i_5	0.7	0.2	0.9	0.5	0

$\hat{r}_{u1i1} = ?$

Example

- User's profile

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline 0 & 5 & 0 & 0 & 4 \end{array}$$

- Item-item similarity matrix

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\ i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\ i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\ i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\ i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \end{array}$$

$\hat{r}_{u1i1} = ?$

Example

- **User's profile**

$$\begin{array}{cccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \boldsymbol{R_{u1}} = & 0 & 5 & 0 & 0 & 4 \end{array}$$

- **Item-item similarity matrix**

$$\boldsymbol{Sim} = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\ i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\ i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\ i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\ i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \end{array}$$

$$\boldsymbol{\hat{r}_{u1i1} = 0 \times 0 + 5 \times 0.3 + 0 \times 0.4 + 0 \times 0.1 + 4 \times 0.7 = 4.3}$$

Example

- **User's profile**

$$
\begin{array}{c|c|c|c|c|c|}
 & i_1 & i_2 & i_3 & i_4 & i_5 \\
\hline
R_{u1} = & 0 & 5 & 0 & 0 & 4 \\
\hline
\end{array}
$$

- **Item-item similarity matrix**

$$
Sim = \begin{array}{c|c|c|c|c|c|}
 & i_1 & i_2 & i_3 & i_4 & i_5 \\
\hline
i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\
\hline
i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\
\hline
i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\
\hline
i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\
\hline
i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \\
\hline
\end{array}
$$

$$\hat{r}_{u1i1} = 0 \times 0 + 5 \times 0.3 + 0 \times 0.4 + 0 \times 0.1 + 4 \times 0.7 = 4.3$$

$$\hat{r}_{u1i3} = ?$$

Example

- **User's profile**

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline 0 & 5 & 0 & 0 & 4 \end{array}$$

- **Item-item similarity matrix**

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\ i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\ i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\ i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\ i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \end{array}$$

$$\hat{r}_{u1i1} = 0\times0 + 5\times0.3 + 0\times0.4 + 0\times0.1 + 4\times0.7 = 4.3$$

$$\hat{r}_{u1i3} = 0\times0.4 + 5\times0.5 + 0\times0 + 0\times0.8 + 4\times0.9 = 6.1$$

Example

- User's profile

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline 0 & 5 & 0 & 0 & 4 \end{array}$$

- Item-item similarity matrix

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\ i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\ i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\ i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\ i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \end{array}$$

$$\hat{r}_{u1i1} = 0\times0 + 5\times0.3 + 0\times0.4 + 0\times0.1 + 4\times0.7 = 4.3$$

$$\hat{r}_{u1i3} = 0\times0.4 + 5\times0.5 + 0\times0 + 0\times0.8 + 4\times0.9 = 6.1$$

$$\hat{r}_{u1i4} = ?$$

Example

- **User's profile**

$$R_{u1} = \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline 0 & 5 & 0 & 0 & 4 \end{array}$$

- **Item-item similarity matrix**

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & 0.3 & 0.4 & 0.1 & 0.7 \\ i_2 & 0.3 & 0 & 0.5 & 0.6 & 0.2 \\ i_3 & 0.4 & 0.5 & 0 & 0.8 & 0.9 \\ i_4 & 0.1 & 0.6 & 0.8 & 0 & 0.5 \\ i_5 & 0.7 & 0.2 & 0.9 & 0.5 & 0 \end{array}$$

$$\hat{r}_{u1i1} = 0\times0 + 5\times0.3 + 0\times0.4 + 0\times0.1 + 4\times0.7 = 4.3$$

$$\hat{r}_{u1i3} = 0\times0.4 + 5\times0.5 + 0\times0 + 0\times0.8 + 4\times0.9 = 6.1$$

$$\hat{r}_{u1i4} = 0\times0.1 + 5\times0.6 + 0\times0.8 + 0\times0 + 4\times0.5 = 5$$

In general …

$$Sim = \begin{array}{c|c|c|c|c|c|} & \boldsymbol{i_1} & \boldsymbol{i_2} & \boldsymbol{i_3} & \boldsymbol{i_4} & \boldsymbol{i_5} \\ \hline \boldsymbol{i_1} & 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ \hline \boldsymbol{i_2} & s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ \hline \boldsymbol{i_3} & s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ \hline \boldsymbol{i_4} & s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ \hline \boldsymbol{i_5} & s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \\ \hline \end{array}$$

$$R = \begin{array}{c|c|c|c|c|c|} & \boldsymbol{i_1} & \boldsymbol{i_2} & \boldsymbol{i_3} & \boldsymbol{i_4} & \boldsymbol{i_5} \\ \hline \boldsymbol{u_1} & r_{u1i1} & r_{u1i2} & r_{u1i3} & r_{u1i4} & r_{u1i5} \\ \hline \boldsymbol{u_2} & r_{u2i1} & r_{u2i2} & r_{u2i3} & r_{u2i4} & r_{u2i5} \\ \hline \boldsymbol{u_3} & r_{u3i1} & r_{u3i2} & r_{u3i3} & r_{u3i4} & r_{u3i5} \\ \hline \boldsymbol{u_4} & r_{u4i1} & r_{u4i2} & r_{u4i3} & r_{u4i4} & r_{u4i5} \\ \hline \end{array}$$

In general …

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ i_2 & s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ i_3 & s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ i_4 & s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ i_5 & s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \end{array}$$

$$R = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & r_{u1i1} & r_{u1i2} & r_{u1i3} & r_{u1i4} & r_{u1i5} \\ u_2 & r_{u2i1} & r_{u2i2} & r_{u2i3} & r_{u2i4} & r_{u2i5} \\ u_3 & r_{u3i1} & r_{u3i2} & r_{u3i3} & r_{u3i4} & r_{u3i5} \\ u_4 & r_{u4i1} & r_{u4i2} & r_{u4i3} & r_{u4i4} & r_{u4i5} \end{array}$$

$$\widehat{R} = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & ? & ? & ? & ? & ? \\ u_2 & ? & ? & ? & ? & ? \\ u_3 & ? & ? & ? & ? & ? \\ u_4 & ? & ? & ? & ? & ? \end{array}$$

In general …

$$Sim = \begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ \hline i_2 & s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ \hline i_3 & s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ \hline i_4 & s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ \hline i_5 & s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \\ \hline \end{array}$$

$$R = \begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & r_{u1i1} & r_{u1i2} & r_{u1i3} & r_{u1i4} & r_{u1i5} \\ \hline u_2 & r_{u2i1} & r_{u2i2} & r_{u2i3} & r_{u2i4} & r_{u2i5} \\ \hline u_3 & r_{u3i1} & r_{u3i2} & r_{u3i3} & r_{u3i4} & r_{u3i5} \\ \hline u_4 & r_{u4i1} & r_{u4i2} & r_{u4i3} & r_{u4i4} & r_{u4i5} \\ \hline \end{array}$$

$$\hat{R} = \begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & \hat{r}_{u1i1} & ? & ? & ? & ? \\ \hline u_2 & ? & ? & ? & ? & ? \\ \hline u_3 & ? & ? & ? & ? & ? \\ \hline u_4 & ? & ? & ? & ? & ? \\ \hline \end{array}$$

In general …

$Sim =$

	i_1	i_2	i_3	i_4	i_5
i_1	0	s_{i1i2}	s_{i1i3}	s_{i1i4}	s_{i1i5}
i_2	s_{i2i1}	0	s_{i2i3}	s_{i2i4}	s_{i2i5}
i_3	s_{i3i1}	s_{i3i2}	0	s_{i3i4}	s_{i3i5}
i_4	s_{i4i1}	s_{i4i2}	s_{i4i3}	0	s_{i4i5}
i_5	s_{i5i1}	s_{i5i2}	s_{i5i3}	s_{i5i4}	0

$R =$

	i_1	i_2	i_3	i_4	i_5
u_1	r_{u1i1}	r_{u1i2}	r_{u1i3}	r_{u1i4}	r_{u1i5}
u_2	r_{u2i1}	r_{u2i2}	r_{u2i3}	r_{u2i4}	r_{u2i5}
u_3	r_{u3i1}	r_{u3i2}	r_{u3i3}	r_{u3i4}	r_{u3i5}
u_4	r_{u4i1}	r_{u4i2}	r_{u4i3}	r_{u4i4}	r_{u4i5}

$\hat{R} =$

	i_1	i_2	i_3	i_4	i_5
u_1	$\hat{r}_{u1i1}$	$\hat{r}_{u1i2}$	?	?	?
u_2	?	?	?	?	?
u_3	?	?	?	?	?
u_4	?	?	?	?	?

In general …

$$
\boldsymbol{Sim} = \begin{array}{c} \\ \boldsymbol{i_1} \\ \boldsymbol{i_2} \\ \boldsymbol{i_3} \\ \boldsymbol{i_4} \\ \boldsymbol{i_5} \end{array}
\begin{array}{c}
\begin{array}{ccccc} \boldsymbol{i_1} & \boldsymbol{i_2} & \boldsymbol{i_3} & \boldsymbol{i_4} & \boldsymbol{i_5} \end{array} \\
\begin{array}{|c|c|c|c|c|}\hline
0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ \hline
s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ \hline
s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ \hline
s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ \hline
s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \\ \hline
\end{array}
\end{array}
$$

$$
\boldsymbol{R} = \begin{array}{c} \\ \boldsymbol{u_1} \\ \boldsymbol{u_2} \\ \boldsymbol{u_3} \\ \boldsymbol{u_4} \end{array}
\begin{array}{c}
\begin{array}{ccccc} \boldsymbol{i_1} & \boldsymbol{i_2} & \boldsymbol{i_3} & \boldsymbol{i_4} & \boldsymbol{i_5} \end{array} \\
\begin{array}{|c|c|c|c|c|}\hline
r_{u1i1} & r_{u1i2} & r_{u1i3} & r_{u1i4} & r_{u1i5} \\ \hline
r_{u2i1} & r_{u2i2} & r_{u2i3} & r_{u2i4} & r_{u2i5} \\ \hline
r_{u3i1} & r_{u3i2} & r_{u3i3} & r_{u3i4} & r_{u3i5} \\ \hline
r_{u4i1} & r_{u4i2} & r_{u4i3} & r_{u4i4} & r_{u4i5} \\ \hline
\end{array}
\end{array}
$$

$$
\boldsymbol{\hat{R}} = \begin{array}{c} \\ \boldsymbol{u_1} \\ \boldsymbol{u_2} \\ \boldsymbol{u_3} \\ \boldsymbol{u_4} \end{array}
\begin{array}{c}
\begin{array}{ccccc} \boldsymbol{i_1} & \boldsymbol{i_2} & \boldsymbol{i_3} & \boldsymbol{i_4} & \boldsymbol{i_5} \end{array} \\
\begin{array}{|c|c|c|c|c|}\hline
\hat{r}_{u1i1} & \hat{r}_{u1i2} & \hat{r}_{u1i3} & ? & ? \\ \hline
? & ? & ? & ? & ? \\ \hline
? & ? & ? & ? & ? \\ \hline
? & ? & ? & ? & ? \\ \hline
\end{array}
\end{array}
$$

In general …

$$Sim = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ i_2 & s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ i_3 & s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ i_4 & s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ i_5 & s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \end{array}$$

$$R = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & r_{u1i1} & r_{u1i2} & r_{u1i3} & r_{u1i4} & r_{u1i5} \\ u_2 & r_{u2i1} & r_{u2i2} & r_{u2i3} & r_{u2i4} & r_{u2i5} \\ u_3 & r_{u3i1} & r_{u3i2} & r_{u3i3} & r_{u3i4} & r_{u3i5} \\ u_4 & r_{u4i1} & r_{u4i2} & r_{u4i3} & r_{u4i4} & r_{u4i5} \end{array}$$

$$\hat{R} = \begin{array}{c|ccccc} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline u_1 & \hat{r}_{u1i1} & \hat{r}_{u1i2} & \hat{r}_{u1i3} & \hat{r}_{u1i4} & ? \\ u_2 & ? & ? & ? & ? & ? \\ u_3 & ? & ? & ? & ? & ? \\ u_4 & ? & ? & ? & ? & ? \end{array}$$

In general …

$$Sim = \begin{array}{c|c|c|c|c|c|} & i_1 & i_2 & i_3 & i_4 & i_5 \\ \hline i_1 & 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ \hline i_2 & s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ \hline i_3 & s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ \hline i_4 & s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ \hline i_5 & s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \\ \hline \end{array}$$

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$$\hat{R} = R \times Sim$$

Similarity matrix is the core of the algorithm

$$Sim = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{array} \begin{array}{c} \begin{array}{ccccc} i_1 & i_2 & i_3 & i_4 & i_5 \end{array} \\ \left[\begin{array}{ccccc} 0 & s_{i1i2} & s_{i1i3} & s_{i1i4} & s_{i1i5} \\ s_{i2i1} & 0 & s_{i2i3} & s_{i2i4} & s_{i2i5} \\ s_{i3i1} & s_{i3i2} & 0 & s_{i3i4} & s_{i3i5} \\ s_{i4i1} & s_{i4i2} & s_{i4i3} & 0 & s_{i4i5} \\ s_{i5i1} & s_{i5i2} & s_{i5i3} & s_{i5i4} & 0 \end{array}\right] \end{array}$$

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0					
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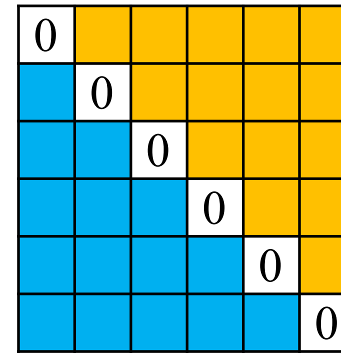

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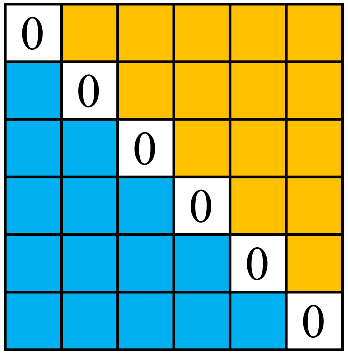

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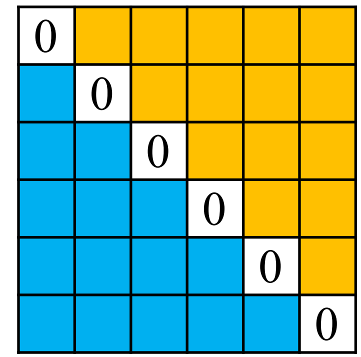
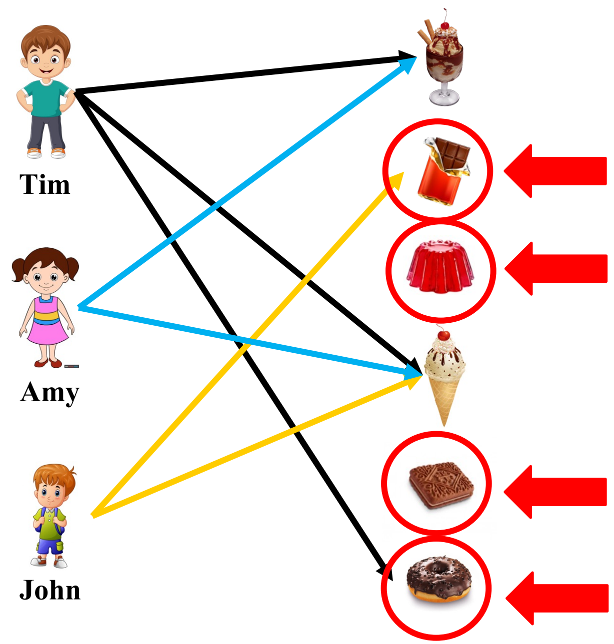


$$\frac{(n \times n) - n}{2}$$

  - For example, for 100,000 items, the similarity matrix requires 4,999,950,000 computations.

Limitations

- **Similarity computation is sometimes impossible due to *sparsity* issue.**
  - Sparsity refers to the percentage of *missing values* in rating matrix.
  - When the rating matrix is highly sparse, sometimes no neighbor can be found for a target item.
  - Therefore, the similarity computation and rating prediction is impossible.



SLIM algorithm

- **Sparse LInear Method**

- **SLIM improves item-based neighborhood model in creating item-item similarity matrix by addressing the aforementioned limitations.**

- **Instead of computing similarity value between each pair of items, it learns the similarity matrix through an optimization process.**

The idea behind SLIM algorithm

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Ideal, but not achievable. Therefore, $\boldsymbol{R - R \times Sim \approx 0}$

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$$\underset{\boldsymbol{Sim}}{\text{minimize}} \quad (\boldsymbol{R} - \boldsymbol{R}\times\boldsymbol{Sim})^2 + \boldsymbol{regularization}$$

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4) **Update matrix Sim based on observed *error***

Why is SLIM more effective?

- **Less computation is needed for building the similarity matrix compared to item-based neighborhood method**
  - At most, the number of computation is the number of available ratings
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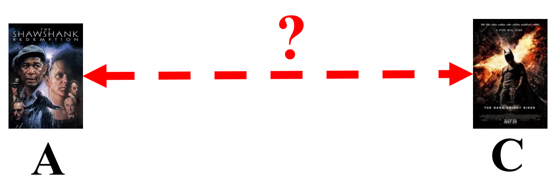

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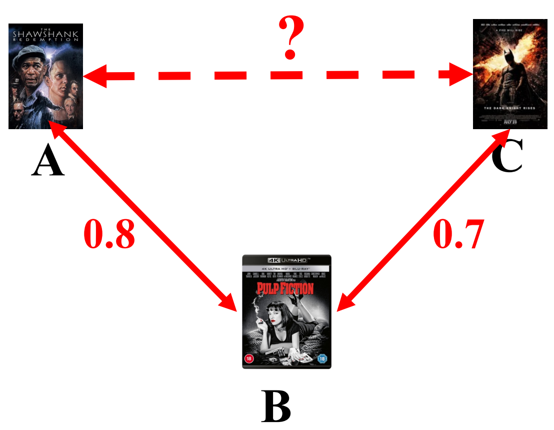

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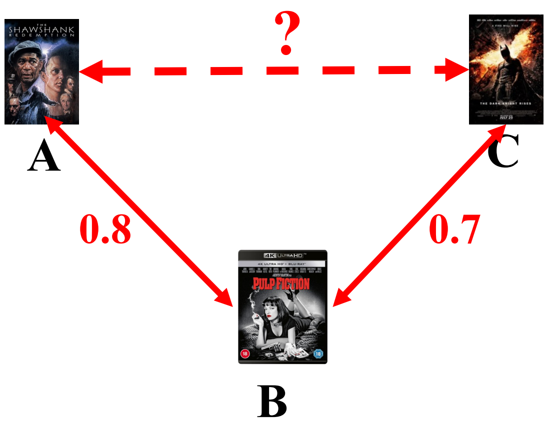
    - Then, can we approximately estimate the similarity between A and B?



Summary

- **Item-based neighborhood method showed superior performance.**

- **SLIM algorithm improves item-based method in two aspects**
  - It computes similarity matrix with less computations
  - It does not require neighbors for computing similarity values and can estimate the similarity value between all pair of items.

Advanced Topic in Recommender Systems

Neighborhood Methods

Masoud Mansoury
AMLab, University of Amsterdam
Discovery Lab, Elsevier