Class 5: Satisfiability

Schedule

Problem Set 2 is due **Friday at 6:29pm**.

Notes and Questions

Definition: satisfiable. A logical formula is *satisfiable* if there is *some* way to make it **true**. That is, there is at least one assignment of truth value to its variables that makes the forumla true.

Definition: conjunctive normal form (CNF). A logical formula that is written as a conjunction of *clauses*, where each clause is a disjunction of *literals*, and each literal is either a variable or a negation of a variable, is in *conjunctive normal form*. If each clause has excatly three literals, it is called *three conjunctive normal form* (3CNF).

$$(a_1 \vee a_2 \vee \neg a_3) \wedge (a_1 \vee \neg a_2 \vee a_3) \wedge (\neg a_1 \vee a_2 \vee \neg a_3) \wedge (\neg a_1 \vee a_2 \vee a_3)$$

Show that every logical formula can be written in 3-conjunctive normal form.

What is the maximum number of (different) clauses in a 3CNF formula involving 5 variables?

What is the maximum number of (different) clauses in a *satisfiable* 3CNF formula involving 5 variables?
What is the maximum number of (different) clauses in a *valid* 3CNF formula involving 5 variables?

Logical Quantifiers

Proofs can certify that a computing system will always behave correctly, something that no amount of testing can do.

$\forall x \in S.P(x)$ is equivalent to: $\neg(\exists x \in S.$ $\quad\quad\quad\quad$ $)$

