

## Class 7: Sets

### Schedule

**Problem Set 3** is due **Friday at 6:29pm**.

I will have “make-up” office hours tomorrow (Wednesday) 3:30-4:30pm (in addition to my usual Wednesday 1-2pm office hours). There have been some adjustments to the office hours schedule, please check the calendar.

### Notes and Questions

What is a *data type*? What are the differences between a *mathematical data type* and a data type in your favorite programming language?

A **set** is an unordered collection of objects. A set is defined by its membership operation:  $x \in S$  is true if  $x$  is in the set  $S$ .

### Set Operations

Subset:  $\subseteq$  (note that this does not mean *strict subset*)

$$A \subseteq B \iff \forall x \in A. \_\_\_ \in \_\_\_.$$

Set Equality: =

$$A = B \iff A \_\_\_ B \wedge B \_\_\_ A.$$

Set Union:  $\cup$

$$\forall x. x \in A \cup B \iff x \in A \_\_\_ x \in B.$$

Set Intersection:  $\cap$

$$\forall x. x \in A \cap B \iff x \in A \_\_\_ x \in B.$$

Set Difference:  $-$

$$\forall x. x \in A - B \iff x \in A \wedge x \notin B.$$

Set Complement:  $\bar{S}$

$$\forall x. x \in D. x \in \bar{A} \iff x \notin A.$$

( $D$  is the “domain of discourse”, the universe of all objects under discussion.)

**Russell's Paradox**

$S_R =$  the set of all sets that are not members of themselves

Is  $S_R \in S_R$ ?

**Set Practice**

Here are some practice problems involving sets. We won't go through these in class, but you should ask questions about any are unclear. (At least a few of these will be on Exam 1.)

1. Define  $A \subset B$  (strict subset).
2. Prove  $A \cup B \equiv B \cup A$ .
3. Prove  $A - B = \emptyset \iff A \subseteq B$ .
4. Prove  $A = B \iff (\forall a \in A. a \in B) \wedge (\forall b \in B. b \in A)$ .