Class 8: Relations

Schedule

Problem Set 3 is due **Friday at 6:29pm**.

Functions

A **function** is a mathematical datatype that associates elements from one set, called the *domain*, with elements from another set, called a *codomain*.

$$f : domain \rightarrow codomain$$

If the function is *total*, every element of the domain has one associated codomain element; if the function is *partial*, there may be elements of the domain that do not have an associated codomain element.

Defining a function. To define a function, we need to describe how elements in the domain are associated with elements in the codomain.

Set Products. A *Cartesian product* of sets $S_1, S_2, \cdots, S_n$ is a set consisting of all possible sequences of n elements where the i^{th} element is chosen from S_i.

$$S_1 \times S_2 \times \cdots \times S_n = \{(s_1, s_2, \cdots, s_n) | s_i \in S_i\}$$

How many elements are in $A \times B$?

What are the (sensible) domains and codomains of each function below:

$$f(n) ::= |n| \qquad f(x) ::= x^2 \qquad f(n) ::= n+1 \qquad f(a,b) ::= a/b$$

$$f(x) ::= \sqrt{x} \qquad f(S) ::= \text{minimum}_{<}(S)$$

For which of them is the function *total*?

How are functions in your favorite programming languages like and unlike mathematical functions?

Relations

A **binary relation**, R, consists of a domain set, A, and a codomain set, B, and a subset of $A \times B$ called the *graph* of R.

For each statement below, give the name and at least one example.

A binary relation where no element of A has more than one outgoing edge:

A binary relation where every element of A has exactly one outgoing edge:

A binary relation where every element of B has exactly one incoming edge:

A binary relation where every element of A has exactly one outgoing edge and every element of B has exactly one incoming edge:

If there exists a *bijective* relation between S and T defined by the graph G which of these *must* be true:

a. there exists some *injective* relation between S and T.
b. there exists some *bijective* relation between T and S.
c. there exists a *total* function, $f : S \to T$.
d. $S - T = \emptyset$.
e. the number of elements of S is equal to the number of elements of T.
f. $G - (S \times T) = \emptyset$.
g. $(S \times T) - G = \emptyset$.
h. $(S \times T) - G \neq \emptyset$.

