Problem Set 4

Deliverable: Submit your responses as a single PDF file on the collab site before **6:29pm** on **Friday, 23 September**. The PDF you submit can be a scanned handwritten file (please check the scan is readable), or a typeset PDF file (e.g., generated by LaTeX or Word).

Collaboration Policy - Read Carefully

For this assignment, you should work in groups of *one* to *four* students of your choice with no restrictions. The rest of the collaboration policy is identical to what it was on PS3, and is not repeated here.

Preparation

This problem set focuses on Chapter 4 (up to Section 4.4) of the MCS book, and Class 7 and Class 8.

Directions

Solve all **TODO:** ?? problems. For maximum credit, your answers should be correct, clear, well-written, and convincing. The problems marked with ($\star$) are believed to be challenging enough that it is not necessary to solve them well to get a "green-star level" grade on this assignment (although we certainly hope you will try and some will succeed!)

Sets

1. For each set S defined below, indicate whether or not it is equivalent to A, where A and B are any sets. Support your answer with a brief explanation.

   a. $S = A \cup \emptyset$.

   b. $S ::= \{x | x \in A \wedge x \in \overline{B}\}$

   c. $S ::= \{x | x \in A \wedge x \notin \overline{A}\}$

   d. $S ::= A \cap (B \cup A)$.

   e. $S ::= A - (B \cap \overline{B})$.

2. Use the definitions of the set operations to prove that for all sets A and B,

   $$A = (A \cap B) \cup (A - B).$$

3. In Class 7, we defined set difference as:

$$\forall x.x \in A - B \iff x \in A \wedge x \notin B.$$

Provide an alternate (but equivalent in meaning) definition of set different using only the other defined set operations (you may use any of the union ($\cup$), intersection ($\cap$), and complement ($\overline{S}$) operations in your definition, but no other operations or qualifiers). A good answer will include a proof that shows your definition is equivalent to the original set difference definition.

4. Solve MCS Problem 4.14.

Functions and Relations

5. For each function described below, identify a *domain* and *codomain* that make the function *total*. For example, for $f(x) ::= 1/x$ you could correctly answer that the domain is $\mathbb{R} - \{0\}$ and codomain is $\mathbb{R}$.

a. $f(x) ::= x + 1$

b. $f(x) ::= \frac{x}{7}(x - 1)$

c. $f(S) ::=$ $\text{minimum}_<(S \cap \mathbb{N})$ where $\text{minimum}_<$ is defined as:

$$\text{minimum}_<(A) = x \in A where \forall a \in A - \{x\}.\, x < a.$$

