Class 19: Reviewing Infinities

Schedule

Problem Set 8 is now due on **Friday, Nov 3**.

See PDF Version for Notes.

Comparing sets Recap:

Definition. Sets A and B have the same cardinality, denoted by $|A| = |B|$ if there is a bijection between A and B.

Definition. Cardinality of A is at least as big as cardinality of B, denoted by $|A| \geq |B|$ if and only if *either* of the following is true (they are equivalent).

1. There is a surjective function from A to B.
2. There is a total injective function from B to A.

Infinite Sets Recap

Definition. A set C is *infinite* if and only if *either* of the following happens (they are all equivalnet).

1. Dedekind-infinite: There is a bijection between C and a strict subset B of C.
2. There is *no* bijection between C and any $\mathbb{N}_k$ for any natural number $k \in \mathbb{N}$.
3. There exists a surjective function from C to $\mathbb{N}$.
4. There exists a total injective function from $\mathbb{N}$ to C.

Definition. A set C is *countable* if and only if there exists a surjective function from $\mathbb{N}$ to C. (That is, ≤ 1 arrow out from $\mathbb{N}$, ≥ 1 arrow in to C.)

Definition. A set C is *countably infinite* if and only if there exists a bijection between C and $\mathbb{N}$.

Cantor's Theorem

For **all** sets, S, $|pow(S)| > |S|$.

What does this mean for $|\mathbb{N}|$?

What is the cardinality of all the real numbers?

Show there is a bijection between $(0, 1)$ and $pow(\mathbb{N})$.

Show there is a bijection between $[0, 1]$ and the cube $[0, 1] \times [0, 1] \times [0, 1]$.

