

## Class 3: Well-Ordering Principle

### Schedule

This week, you should read MCS Chapter 2 and MCS Chapter 3 (at least through the end of Section 3.4).

**Problem Set 1** is due **Friday at 6:29pm**.

Office hours started Monday afternoon. See the course calendar for the full office hours schedule.

### Notes and Questions

What properties must a sensible ordering function have?

**Definition.** A set is *ordered* with respect to an ordering relation (e.g.,  $<$ ), if two things hold. First: every pair of unequal elements  $a, b$  in  $A$  either satisfy  $a < b$  or  $b < a$ . By the well-ordering principle, there must be a smallest element  $m \in C$ . 4. Reach a contradiction (this is the creative part!). One way to reach a contradiction would be to show  $P(m)$ . Another way is to show there must be an element  $m' \in C$  where  $m' < m$ . 5. Conclude that  $C$  must be empty, hence there are no counter-examples and  $P(n)$  always holds.

**Example: Betable Numbers.** A number is *betable* if it can be produced using some combination of \$2 and \$5 chips. Prove that all integer values greater than \$3 are betable.

**Example: Division Property.** Given integer  $a$  and positive integer  $b$ , there exist integers  $q$  and  $r$  such that:  $a = qb + r$  and  $0 \leq r < b$ .

*The whole problem with the world is that fools and fanatics are always so certain of themselves, and wiser people so full of doubts.* Bertrand Russell