Class 7: Sets

Schedule

Problem Set 3 is due **Friday at 6:29pm**.

Notes and Questions

What is a *data type*? What are the differences between a *mathematical data type* and a data type in your favorite programming language?

A **set** is an unordered colection of objects. A set is defined by its membership operation: $x \in S$ is true if x is in the set S.

Set Operations

Subset: $\subseteq$ (note that this does not mean *strict subset*)

$$A \subseteq B \iff \forall x \in A.\_\_\_\_ \in \_\_\_\_.$$

Set Equality: $=$

$$A = B \iff A\_\_\_\_B \wedge B\_\_\_\_A.$$

Set Union: $\cup$

$$\forall x.x \in A \cup B \iff x \in A\_\_\_\_x \in B.$$

Set Intersection: $\cap$

$$\forall x.x \in A \cap B \iff x \in A\_\_\_\_x \in B.$$

Set Difference: $-$

$$\forall x.x \in A - B \iff x \in A \wedge x \notin B.$$

Set Complement: $\overline{S}$

$$\forall x.x \in D.x \in \overline{A} \iff x \notin A.$$

(D is the "domain of discourse", the universe of all objects under discussion.)

Russell's Paradox

$$S_R = \text{ the set of all sets that are not members of themselves}$$

Is $S_R \in S_R$?

Set Practice

Here are some practice problems involving sets. We won't go through these in class, but you should ask questions about any are unclear. (At least a few of these will be on Exam 1.)

1. Define $A \subset B$ (strict subset).

2. Prove $A \cup B \equiv B \cup A$.

3. Prove $A - B = \emptyset \iff A \subseteq B$.

4. Prove $A = B \iff (\forall a \in A.\, a \in B) \wedge (\forall b \in B.\, b \in A)$.

