Problem Set 2

Deliverable: Submit your responses as a single PDF file on the collab site before **6:29pm** on **Friday, 9 September**. The PDF you submit can be a scanned handwritten file (please check the scan is readable), or a typeset PDF file (e.g., generated by LaTeX or Word).

Collaboration Policy - Read Carefully

For this assignment, you should work in groups of *one* to *three* students, so long as the *total number of siblings the members of your group have is divisible by three.* (Note that this means that someone with, say, one sibling, should not work alone, but either needs to find a teammate with two (or 5 or 8, etc.) siblings, or two teammates whose sibling counts sum to the right number. If you have half-siblings, you can decide to count them as either 0, 1/2, or 1.)

If you work with teammates, you should submit one assignment that represents your collective best work with all of your names and UVA ids on it. Everyone on a team should understand everything you turn in for the assignment well enough to be able to produce it completely on your own. You are encouraged to use the `#teaming` channel to find suitable teammates.

You may discuss the assignment with anyone you want and get help from others, so long as it is help in the spirit of learning how to do things yourself not getting answers you don't understand.

Remember to follow the course pledge you read and signed at the beginning of the semester. For this assignment, you may consult any outside resources you want, including books, papers, web sites and people. You may consult an outside person (e.g., another friend who is a CS major but is not in this class) who is not a member of the course staff, but that person cannot type anything in for you and all work must remain your own and outside sources should never give you specific answers to problem set questions. If you use resources other than the class materials, lectures and course staff, you should document this clearly on your submission.

You are strongly encouraged to start early and take advantage of the scheduled office hours for this course.

Preparation

This problem set focuses on Chapter 2 and Chapter 3 (through 3.5) of the MCS book, and Class 3 and Class 4.

Directions

Solve all the problems on the next page. For full credit, your answers should be correct, clear, well-written, and convincing.

(Un)Well-Ordered Sets

For each of these question, answer if the given set and comparator is *well-ordered.* Support your answer with a brief, but clear and convincing, argument.

1. The empty set; $<$.
2. The set of integers less than 2102; $>$.
3. The set of positive rational numbers with lowest terms, $\frac{p}{q}$ where $q < 2102$; $<$.
4. The set of positive rational numbers; to compare two rational numbers, a, b, write them as fractions in lowest terms (which we know exist because of the well-ordering principle on the integers!), $a = \frac{p_a}{q_a}$, $b = \frac{p_b}{q_b}$. Then, $a \prec b$ iff $p_a q_b < p_b q_a$.

Well-Ordering Principle Proofs (and Non-Proofs)

5. MCS Problem 2.2 (explain clearly which proof step is invalid and why).
6. (Similar to Problem 2.6) The "Exponential Losses" Casino has chips with value \$1, \$2, \$4, \$8, \$16, ..., $\$2^k$, but has a rule that bettors may not use more than one of the same value of chip to make any bet. Prove that all integer bets from \$1 to $\$2^{k+1}$ can be made.
7. MCS Problem 2.17.

Logical Formulas

8. Write each of the following natural language statments as a logical formula. Your goal is to produce a simple and clear statement whose meaning matches what you believe is the intent of the natural language statements. If there are logical ambiguities in the statement, explain them. Be especially careful to notice when "and" means (logical) or, and "or" means (logical) xor.

a. *Excessive bail shall not be required, nor excessive fines imposed, nor cruel and unusual punishments inflicted.* (US Consitution, Eigth Amendment)

b. ??

c. (optional, for true linguistic logicians only!) "The University of Virginia does not discriminate on the basis of age, color, disability, gender identity, marital status, national or ethnic origin, political affiliation, race, religion, sex (including pregnancy), sexual orientation, veteran status, and family and genetic information, in its programs and activities as required by Title IX of the Education Amendments of 1972, the Americans with Disabilities Act of 1990, as amended, Section 504 of the Rehabilitation Act of 1973, Titles VI and VII of the Civil Rights Act of 1964, the Age Discrimination Act of 1975, the Governor's Executive Order Number One (2014), and other applicable statutes and University policies." (University of Virginia's "Notice of Non-Discrimination and Equal Opportunity")

9. Prove $\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q$.

10. Determine if the following statements are logically equivalent, and support your answer with a clear proof.

$$Q \implies (P \implies R) \qquad \text{and} \qquad (Q \implies P) \implies R$$

11. Given the following statements, prove S:

$$Q, \quad \neg P, \quad L \implies P, \quad Q \implies M, \quad \neg(\neg Q \vee P) \implies \neg R, \quad (\neg L \wedge M) \implies S \vee R$$

