OTFS Signal Transform

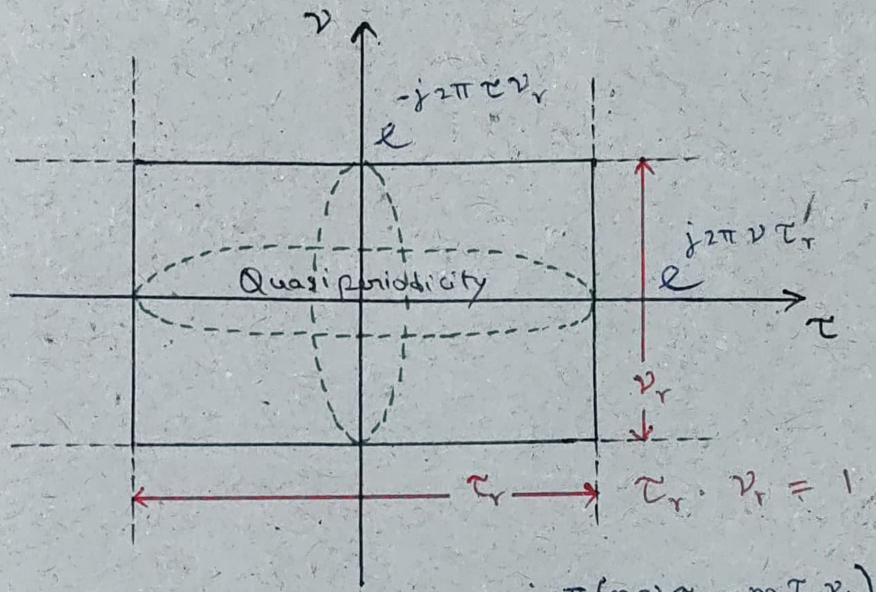
Signal Representations

- Any signal has <u>Time domain</u> and <u>Frequency domain</u> representations.
- The above two representations are <u>One dimensional</u>, interchangeable through Fourier Transform.

Fourier Transform: $$X(f) = \int_{-\infty}^{\infty} e^{-j2\pi ft} \, dt$$

Inverse Fourier Transform:
$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft} \, df$$

- DD signal representation $\phi(\tau, \nu)$ is <u>Quasiperiodic</u>.
  (i.e.) Periodic upto a <u>multiplicative phase</u>.



$$\phi(\tau + n\tau_p, \nu + m\nu_p) = \underbrace{e^{j2\pi(n\nu\tau_p - m\tau\nu_p)}}_{\text{Phase shift}} \phi(\tau, \nu)$$

where,

$\tau_p \rightarrow$ Delay period

$\nu_p \rightarrow$ Doppler period

⊙ $\phi(\tau,\nu)$ can be converted to <u>Time domain</u> using

<u>Z_t ZAK transform.</u>

$$Z_t(\phi) = \int_0^{\nu_p} \phi(t,\nu)\, e^{-j2\pi t\nu}\, d\nu$$

> Problem 1 :
>
> DTFT : $X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$
>
> Inverse DTFT : $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\cdot e^{j\Omega n}\, d\Omega$

⊙ $\phi(\tau,\nu)$ can be converted to <u>Frequency domain</u> using

<u>Z_f ZAK transform.</u>

$$Z_f(\phi) = \int_0^{\tau_p} \phi(\tau,f)\, e^{j2\pi f\tau}\, d\tau.$$

> Problem 2 :
>
> Evaluate $\int_0^{\nu_p} \delta(\nu-\nu_0)\, e^{-j2\pi t\nu_0}\, d\nu$
>
> $= e^{-j2\pi t\nu_0}$

<u>OTFS Example</u>

⊙ consider a <u>quasiperiodic 2D pulse train</u> in DD domain

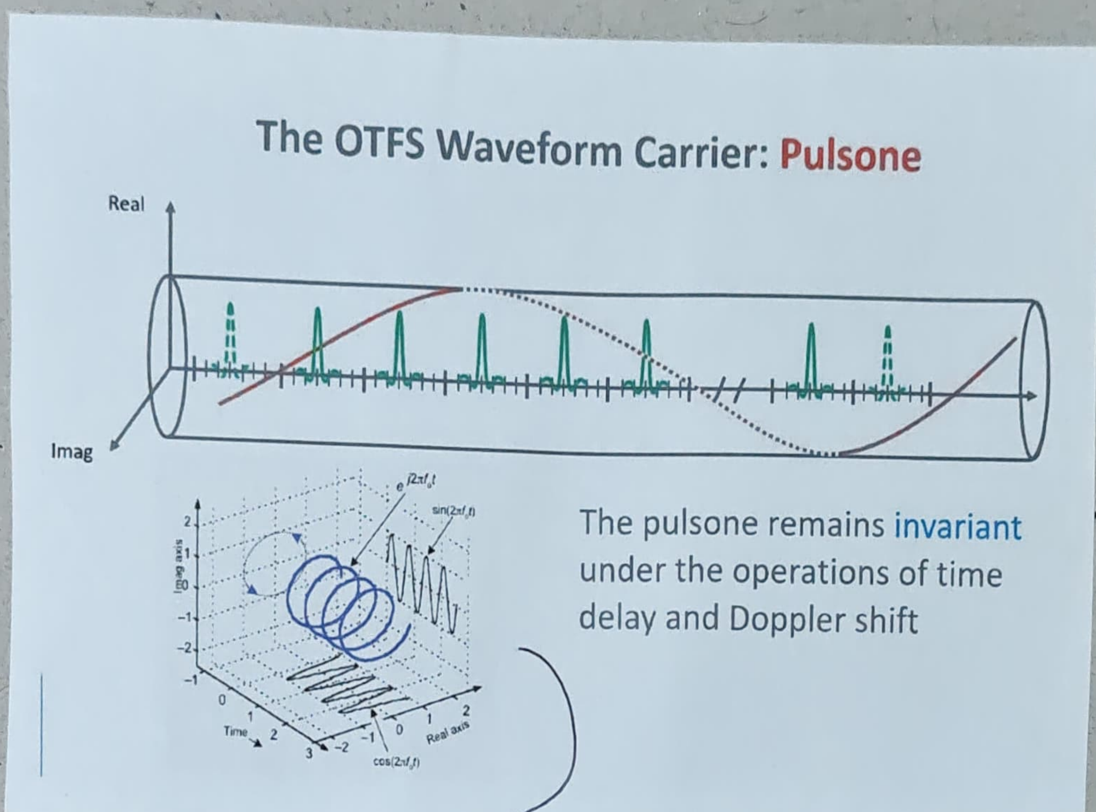
$$\phi(\tau,\nu) = \underbrace{\left[\sum_{m=-\infty}^{\infty} \delta(\tau-\tau_0-m\tau_p)\right]}_{\text{pulse train in delay}} \underbrace{\left[\sum_{n=-\infty}^{\infty} \delta(\nu-\nu_0-n\nu_p)\right]}_{\text{pulse train in doppler}}$$

Taking Z_t yields the <u>pulsone</u>.

(i) replace $\tau \rightarrow t$, integrate along <u>Doppler</u>.

$$\Rightarrow z_t(\phi) = \left[\sum_{m=-\infty}^{\infty} \delta(t-\tau_0-m\tau_p)\right] \times \underbrace{\int_0^{\nu_p} \left[\sum_{n=-\infty}^{\infty} \delta(\nu-\nu_0-n\nu_p)\right] e^{-j2\pi t\nu}\, d\nu}_{\int_0^{\nu_p} \delta(\nu-\nu_0)\, e^{-j2\pi t\nu}\, d\nu}$$

$$= \left[\sum_{m=-\infty}^{\infty} \delta(t-\tau_0-m\tau_p)\right] \times e^{-j2\pi t\nu_0}$$



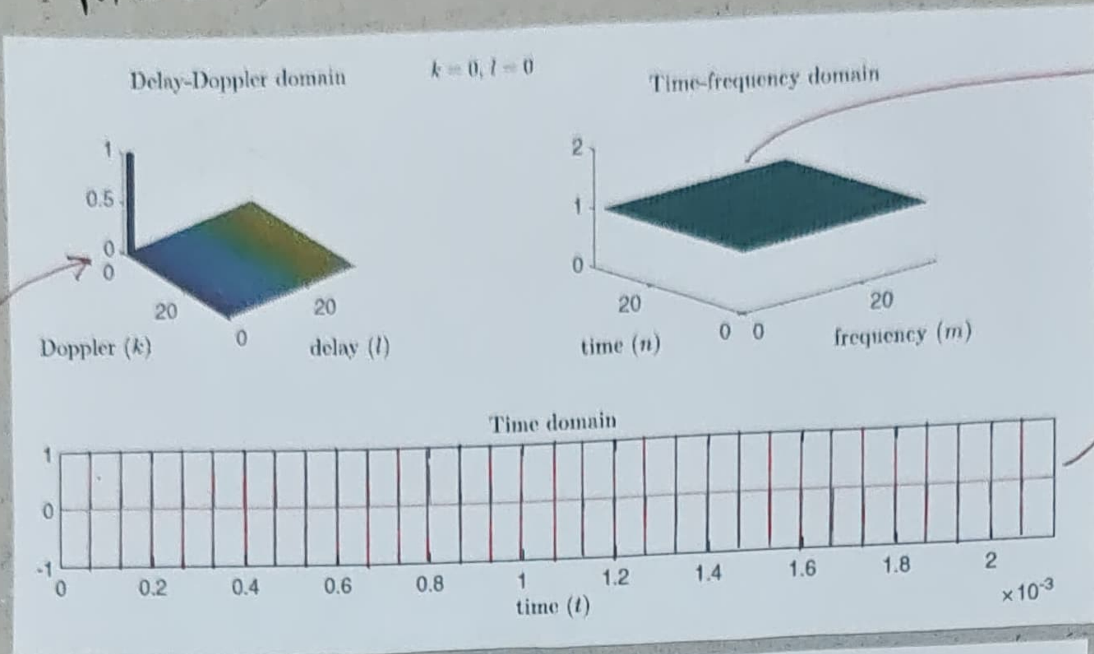
- Locally, it looks like impulse
- Globally, it looks like tone
- spreads in the TF domain

OTFS Basis Function

$M = 32$, $N = 32$, $\Delta f = 15$ kHz

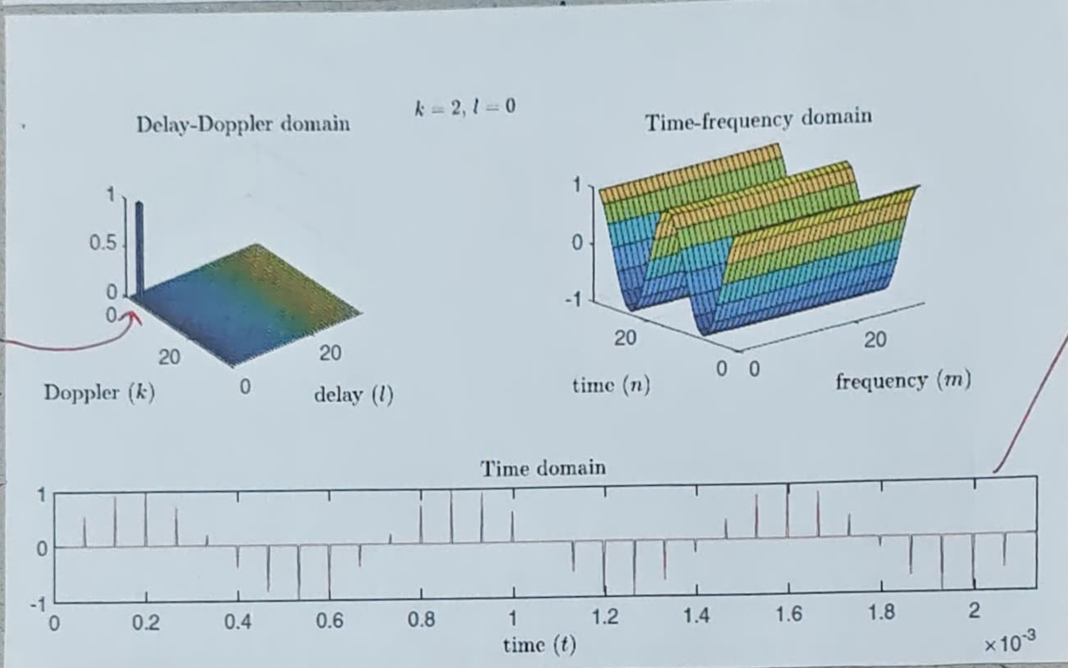
Spread in TF domain

(0,0)



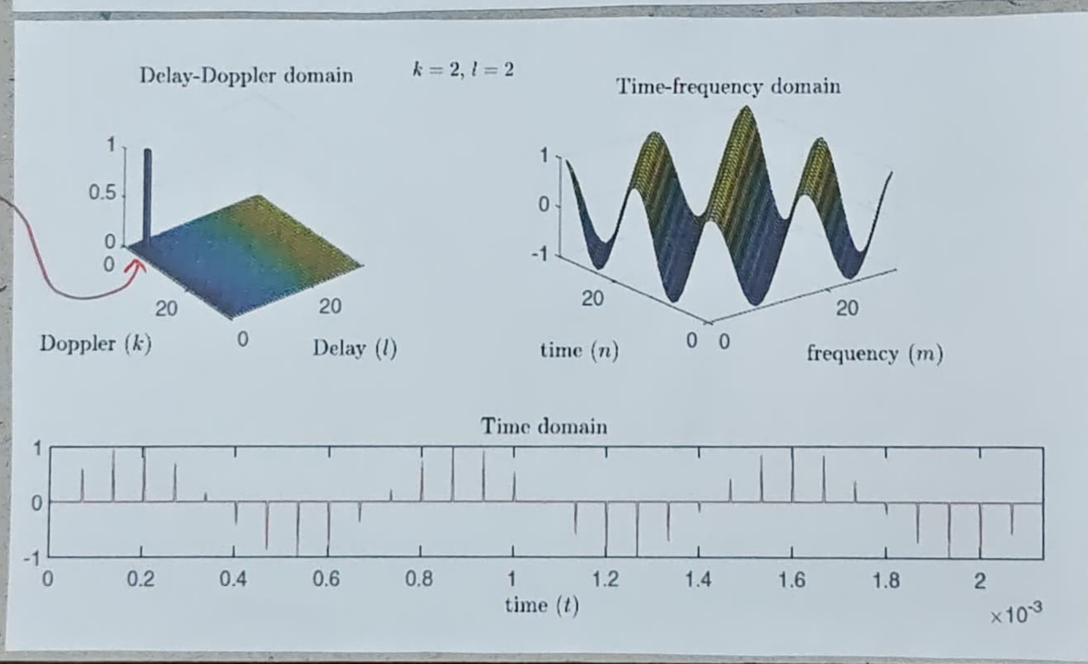
Magnitude is constant

Doppler is shifted



Locally Pulse
Globally Tone

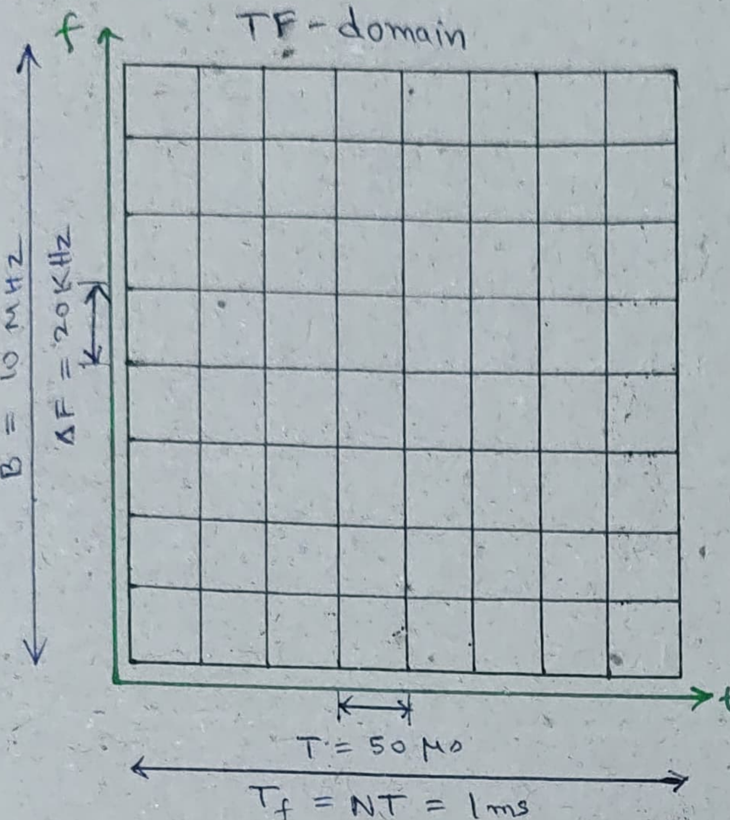
Doppler & Delay shifted



OTFS Modulation

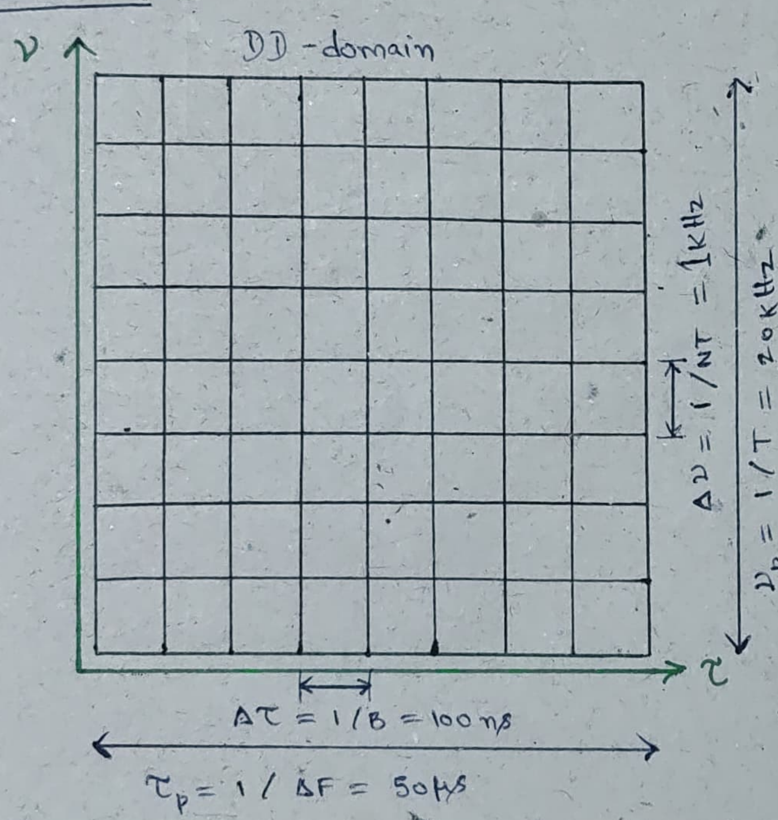
- Place a 2D grid in the DD-domain
- Position a QAM symbol over each grid-point
- Transmit it using ZAK transform Z_t

TF Grid



- Subcarrier BW
  $\Delta F = 20\ \text{kHz}$
- No. of subcarriers
  $M = 500$
- Bandwidth
  $B = M \cdot \Delta F$
  $= 500 \times 20\ \text{kHz}$
  $= 10\ \text{MHz}$
- OFDM symbol duration
  $T = \frac{1}{\Delta F} = \frac{1}{20\ \text{kHz}} = 50\ \mu s$
- No. of OFDM symbols, $N = 20$
- Frame duration
  $T_f = NT = 20 \times 50\ \mu s = 1\ \text{ms}.$

DD Grid



⊙ Delay resolution (Sampling Interval)

$$\Delta\tau = \frac{1}{B} = \frac{1}{M\,\Delta F} = \frac{1}{500 \times 20\,KHz} = 100\,ns.$$

⊙ Delay period (OFDM symbol duration)

$$\tau_p = \frac{1}{\Delta F} = M\,\Delta\tau = 500 \times 100\,ns = 50\,\mu s$$

⊙ Doppler resolution

$$\Delta\nu = \frac{1}{NT} = \frac{1}{T_f} = \frac{1}{1\,ms} = 1\,KHz$$

⊙ Doppler period

$$\nu_p = \frac{1}{T} = N\,\Delta\nu = 20 \times 1\,KHz = 20\,KHz$$

Inference:

From TF and DD-domain Grids, it is observed that, $\tau_p \times \nu_p = T \times \Delta F = 50\,\mu s \times 20\,KHz = \underline{1}$

Problem 3: Given Subcarrier BW, $\Delta F = 25\,KHz$
No. of OFDM symbols, $N = 50$
No. of subcarriers, $M = 200$

Bandwidth,
$B = M\Delta F$
$= 200 \times 25\,KHz$
$= 5\,MHz$

(a) OFDM Symbol duration, $T = \frac{1}{\Delta F} = \frac{1}{25\,KHz} = 40\,\mu s$

(b) Frame duration, $T_f = N \times T = 50 \times 40\,\mu s = 2\,ms$

(c) Delay resolution (Sampling Interval), $\Delta\tau = \frac{1}{B} = \frac{1}{5\,MHz} = 200\,ns$

(d) Delay period (OFDM symbol duration), $\tau_p = \frac{1}{\Delta F} = M\Delta\tau = 200 \times 200\,ns = 40\,\mu s$

(e) Doppler resolution, $\Delta\nu = \frac{1}{NT} = \frac{1}{T_f} = \frac{1}{2\,ms} = 500\,Hz$

(f) Doppler period, $\nu_p = \frac{1}{T} = N\,\Delta\nu = 50 \times 500\,Hz = 25\,KHz.$

Problem 4 : Given $\tau_p = 40\ \mu s$, $\Delta\tau = 200\ ns$

$\Delta\nu = 500\ Hz$, $\nu_p = 25\ KHz$

(a) Compute M, N.

$$M = \frac{\tau_p}{\Delta\tau} = \frac{40\mu s}{200\ ns} = 200$$

$$N = \frac{\nu_p}{\Delta\nu} = \frac{25\ KHz}{500\ Hz} = 50$$

(b) Compute B, T_f.

$$B = \frac{1}{\Delta\tau} = 5\ MHz$$

$$T_f = \frac{1}{\Delta\nu} = 2\ ms$$

(c) What is the total no. of symbols in the DD-Grid ?

$$M \times N = 200 \times 50 = 10,000.$$

(i) 10,000 symbols are spread over $B = 5\ MHz$, $T_f = 2\ ms$.