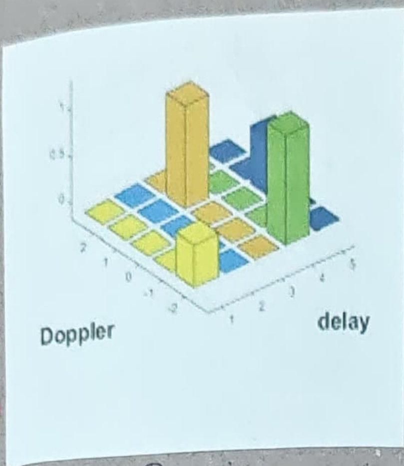
OTFS Signal Demodulation and End-to-End Model

DD-Domain Channel Model



DD - Domain

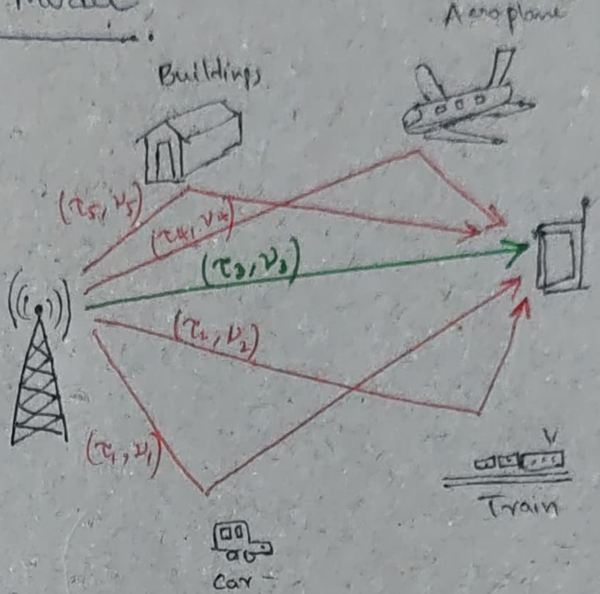


Fig. 5 Multipath components

- DD-domain wireless Channel model, $h(\tau, \nu)$

Delay

Doppler

$$h(\tau, \nu) = \sum_{i=1}^{L_p=5} h_i \, \delta(\tau - \tau_i) \, \delta(\nu - \nu_i)$$

where,

τ_i = Delay of i^{th} reflector

ν_i = Doppler of i^{th} reflector

h_i = Complex path-gain of i^{th} reflector

L_p = Number of multipath components

Received Signal

- Let $h(\tau, \nu)$ be the DD-domain impulse response of the channel.
- Given input signal $s(t)$, the received signal $r(t)$ is given as

$$r(t) = \iint h(\tau, \nu)\, s(t-\tau)\, e^{j2\pi\nu(t-\tau)}\, d\tau\, d\nu$$

Substituting the model for $h(\tau, \nu)$ in $r(t)$ yields

$$r(t) = \iint \underbrace{\left(\sum_{i=1}^{L_p} h_i\, \delta(\tau-\tau_i)\, \delta(\nu-\nu_i)\right)}_{h(\tau-\nu)} s(t-\tau)\, e^{j2\pi\nu(t-\nu)}\, d\tau\, d\nu$$

$$= \sum_{i=1}^{L_p} h_i\, s(t-\tau_i)\, e^{j2\pi\nu_i(t-\tau_i)} + w(t)$$

- The received signal is sampled with sampling duration

$$\frac{T}{M} = \frac{M/B}{M} = \frac{1}{B}$$

where,

$T = \frac{M}{B}$ = OFDM symbol duration

M = No. of subcarriers

- Assume τ_i, ν_i are integer multiples of Delay and Doppler resolutions respectively.

$\tau_i = \frac{l_i}{M\Delta f}$ → integer multiple of delay-resolution $\Delta\tau$

$\nu_i = \frac{k_i}{NT}$ → integer multiple of doppler-resolution $\Delta\nu$.

Sampled Output

- Sampled received signal

$$r(p) = r(t)\Big|_{t=\frac{pT}{M}}$$

$$= \sum_{i=1}^{L_p} h_i\, s\left(\frac{pT}{M} - \tau_i\right) e^{j2\pi \nu_i\left(\frac{pT}{M} - \tau_i\right)} + w\left(\frac{pT}{M}\right)$$

- Substituting $\tau_i = \frac{l_i}{M\Delta f}$, $\nu_i = \frac{k_i}{NT}$.

And using $T\Delta f = 1$

$$r(p) = \sum_{i=1}^{L_p} h_i\, s(p - l_i)\, e^{j2\pi \frac{k_i(p-l_i)}{MN}} + w(p)$$

- Note that the term $s(p-l_i)\, e^{j2\pi \frac{k_i(p-l_i)}{MN}}$ implies the following.
  - Take the signal $s(p)$
  - Multiply it with the phase factor $e^{j2\pi \frac{k_i p}{MN}}$ arising due to doppler.
  - This results in the signal $s(p)\, e^{j2\pi \frac{k_i p}{MN}}$
  - Now shift this by l_i-units to obtain $s(p-l_i)\, e^{j2\pi \frac{k_i(p-l_i)}{MN}}$
  - Furthermore, due to CP addition in $s(p)$ and periodic nature of the complex exponential $e^{j2\pi \frac{k_i p}{MN}}$

$$s([p-l_i]_{MN})\, e^{j2\pi \frac{k_i([p-l_i]_{MN})}{MN}}$$

$$= s(p-l_i)\, e^{j2\pi \frac{k_i(p-l_i)}{MN}},$$

- In vector notation :
  - Take the vector $\bar{s}$
  - Multiply it with a diagonal matrix

$$\text{diag}\left\{ e^{j2\pi \frac{k_i p}{MN}} \right\}_{p=0}^{MN-1}$$

  - Then circularly shift its elements by l_i-units.
  - The l-circular shifts on a vector $\bar{s}$ can be obtained by multiplying it by Π^l, where Π is a standard permutation matrix of size $MN \times MN$.

- Hence, the signal received due to i^{th} multipath component, having complex-path gain h_i, delay l_i and Doppler k_i can be obtained as

$$s \rightarrow \bar{s} = \underbrace{\Delta^{k_i}}_{\text{Due to Doppler}} \bar{s} \rightarrow \underbrace{\Pi^{l_i}}_{\text{Due to delay}} \bar{s} = \Pi^{l_i} \Delta^{k_i} \bar{s} \rightarrow \underbrace{h_i}_{\text{Path Gain}} \Pi^{l_i} \Delta^{k_i} \bar{s}$$

- The sampled output vector $\bar{r}$ of size $MN \times 1$ can be expressed as

$$\underbrace{\bar{r}}_{\text{After removing CP}} = \begin{bmatrix} r(0) \\ r(1) \\ \vdots \\ r(MN-1) \end{bmatrix} = \left(\sum_{i=1}^{L_p} h_i \Pi^{l_i} \Delta^{k_i} \right) \underbrace{\bar{s}}_{MN \times 1} + w$$

$$= H \bar{s} + \bar{w}.$$

where, $L_p \rightarrow$ No. of multipath components

$h_i \rightarrow$ Complex Gain

$l_i \rightarrow$ delay index

$k_i \rightarrow$ doppler index

- Δ is the diagonal matrix of phase factors.

$$\Delta = \text{diag}\left\{ e^{j 2\pi \frac{p}{MN}} \right\}_{p=0}^{MN-1}$$

$$\Delta^{ki} = \begin{bmatrix} 1 & & & \\ & e^{j2\pi \frac{ki}{MN}} & & \\ & & e^{j2\pi \frac{2ki}{MN}} & \\ & & & \ddots & \\ & & & & e^{j2\pi (MN-1)\frac{ki}{MN}} \end{bmatrix}$$

- The permutation matrix Π is defined as

$$\Pi = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & 1 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 1 & 0 \end{bmatrix}_{MN \times MN}$$

(In Identity Matrix, shift the columns circularly to get Permutation Matrix)

- Define $\Pi^0 = I_{MN}$

$$\Pi^1 = \Pi$$

$$\Pi^2 = \Pi \times \Pi.$$

$$\Pi^2 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & & & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\Pi^l = \underbrace{\Pi \times \Pi \times \cdots \times \Pi}_{l\text{-times}}$$

(ii) l times circularly shifted

$\Rightarrow$ Delay of l samples.

OTFS Demodulation

- Construct now the output matrix $R \sim M \times N$ by taking elements columnwise from $\bar{r}$.

$$R = \text{vec}^{-1}(\bar{r})$$

- Received signal $r(t)$ is processed by a filter, matched to the receiver pulse $p_{rx}(t)$ of duration T, repeated N-times.

$$y(f,t) = \int p^*_{rx}(t'-t)\, r(t')\, e^{-j2\pi f(t'-t)}\, dt'$$

- This is followed by sampling in TF-domain to yield

$$Y_{TF}(m,n) = y(f,t)\Big|_{f=m\Delta f,\ t=nT}$$

$$= \int p^*_{rx}(t'-nT)\, r(t')\, e^{-j2\pi m\Delta f(t'-nT)}\, dt'$$

- $Y_{TF} \in \mathbb{C}^{M \times N}$: TF - demodulated symbol matrix.

Its $(m,n)^{th}$ element denotes the demodulated symbol on m^{th} subcarrier in the n^{th} symbol duration.

- Perform receive pulse shaping and FFT, to obtain $Y_{TF} \sim M \times N$:

$$Y_{TF} = F_M \underbrace{P_{rx}}_{M \times M} \underbrace{R}_{M \times N}$$

- The demodulated DD-domain OTFS signal $Y_{DD} \sim M \times N$ is obtained via symplectic finite fourier Transform (SFFT) of Y_{TF} as

$$Y_{DD}(l,k) = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} Y_{TF}(m,n)\, e^{-j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)}$$

> Note:
> The SFFT performs IFFT along frequency variable and FFT along time variable

- Equivalently;

$$Y_{DD} = F_M^H \, Y_{TF} \, F_N$$

$$= F_M^H \left(F_M P_{rx} R\right) F_N$$

$$= P_{rx} R F_N$$

Furthermore, vectorizing the above result, we have

$$\bar{y}_{DD} = \mathrm{vec}(Y_{DD}) = (F_N \otimes P_{rx})\,\bar{r}$$

- Substituting the expression for $\bar{r}$ and $\bar{s} = (F_N^H \otimes P_{tx})\bar{x}_{DD}$, one obtains the end-to-end relationship in the DD-domain as

$$\underbrace{\bar{y}_{DD}}_{MN\times 1} = H_{DD}\underbrace{\bar{x}_{DD}}_{MN\times 1} + \bar{v}_{DD}$$

where $H_{DD} \sim MN \times MN$, $\bar{v}_{DD}$ are defined as

$$H_{DD} = (F_N \otimes P_{rx})\, H\, (F_N^H \otimes P_{tx})$$

$$\bar{v}_{DD} = (F_N \otimes P_{rx})\,\bar{w}$$

- For the standard rectangular pulse $(P_{rx} = P_{tx} = I_M)$

$$H_{DD} = (F_N \otimes I_M)\, H\, (F_N^H \otimes I_M)$$

$$\bar{v}_{DD} = (F_N \otimes I_M)\,\bar{w}$$

Note: Size of Matrices

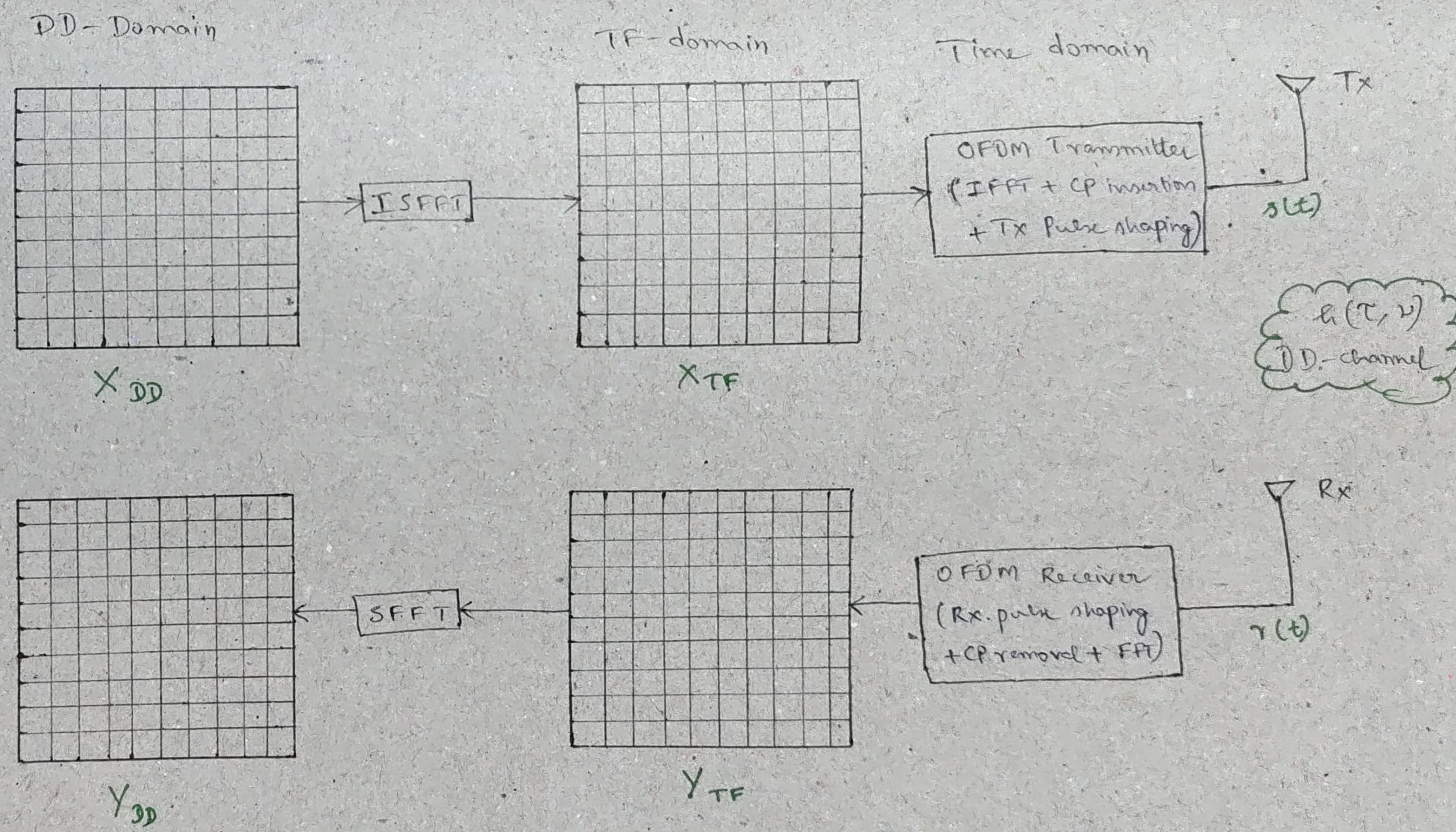
$(F_N \otimes P_{rx}) \longrightarrow MN \times MN$ (F_N: $N\times N$, P_{rx}: $M\times M$)

$H \longrightarrow MN \times MN$

$(F_N^H \otimes P_{tx}) \longrightarrow MN \times MN$

$$(\underset{m\times n}{A} \otimes \underset{p\times q}{B}) = [\ \ ]_{mp\times nq}$$

- In Summary...


DD - Domain
TF - domain
Time domain
ISFFT
OFDM Transmitter
(IFFT + CP insertion
+ Tx Pulse shaping)
Tx
s(t)
h(τ, ν)
DD - channel
Rx
r(t)
OFDM Receiver
(Rx. pulse shaping
+ CP removal + FFT)
SFFT
X DD
X TF
Y TF
Y DD