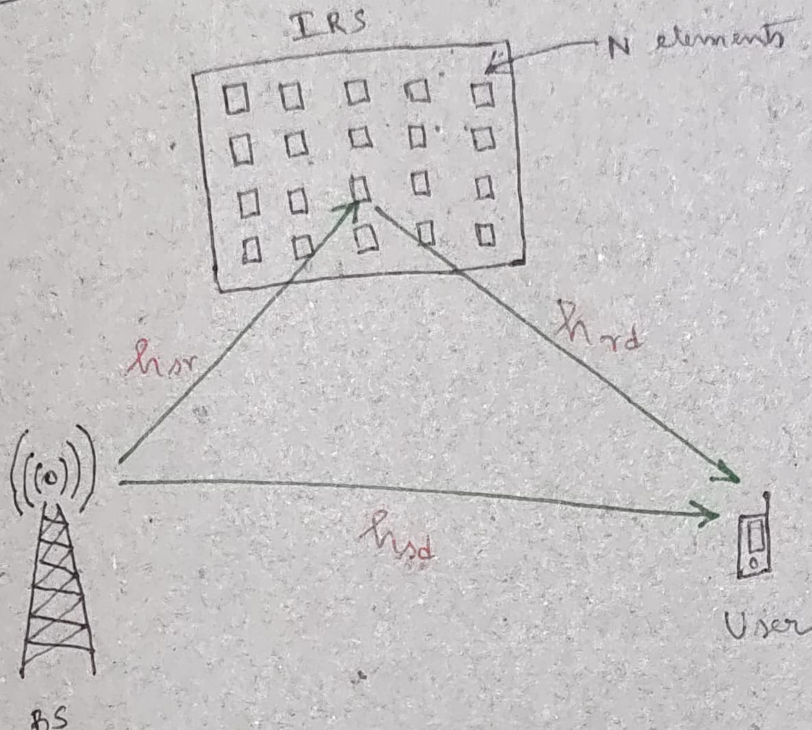
SISO IRS Model and Optimization

IRS-aided SISO model:



- Consider an IRS with N reflecting elements.
  - BS (source) to User (destination) channel

$$h^*_{sd}$$

  - BS (source) to IRS channel

$$\bar{h}_{sr} = \begin{bmatrix} h_{sr,1} \\ h_{sr,2} \\ \vdots \\ h_{sr,N} \end{bmatrix} \leftarrow \text{N channel coefficients}$$

  - IRS to User (destination) channel

$$\bar{h}^H_{rd} = \begin{bmatrix} h^*_{rd,1} & h^*_{rd,2} & \cdots & h^*_{rd,N} \end{bmatrix}$$

IRS model

- The diagonal complex reflecting matrix $\Theta \sim N \times N$ is given as

$$\Theta = \begin{bmatrix} \beta_1 e^{j\theta_1} & & \\ & \ddots & \\ & & \beta_N e^{j\theta_N} \end{bmatrix}$$

where, $\beta_n \rightarrow$ Amplitude / Gain $\left.\right\}$ for the n^{th} element
$\theta_n \rightarrow$ Phase shift

Why reflecting matrix is diagonal?   Each element reflects only the particular signal which is incident on the same.

- The received signal y at the user is given as

$$y = \left(\sum_{n=1}^{N} \beta_n e^{j\theta_n} \underbrace{h^*_{rd,n}}_{n^{th}\text{ reflector to destination}} \underbrace{h_{sr,n}}_{n^{th}\text{ source to reflector}} + h^*_{sd} \right) \underbrace{\sqrt{P_t}}_{\text{Transmit signal Power}} x + \underbrace{Z}_{\text{Noise}}$$

$$= \left(\bar{h}^H_{rd} \, \Theta \, \bar{h}_{sr} + h^*_{sd} \right) \sqrt{P_t} \, x + Z$$

- $E\{|x|^2\} = 1$, $E\{|n|^2\} = 1$

The receive SNR at the user is given as

$$\gamma = \frac{P_t \left| \bar{h}^H_{rd} \, \Theta \, \bar{h}_{sr} + h^*_{sd} \right|^2}{\sigma^2}$$

$$= \frac{P_t \left| \sum_{n=1}^{N} \beta_n e^{j\theta_n} \overbrace{h^*_{rd,n} h_{sr,n}}^{\text{Fixed channel coefficients}} + h^*_{sd} \right|^2}{\sigma^2}$$

SISO IRS Optimization

- How to maximize the output SNR ?

$$\max_{\theta,\beta} \left| \sum_{n=1}^{N} \beta_n e^{j\theta_n} h_{rd,n}^{*} h_{sr,n} + h_{sd}^{*} \right|^2$$

$$0 \le \theta_n \le 2\pi$$

$$0 \le \beta_n \le 1$$

- The SNR objective can be written as

$$\left| \sum_{n=1}^{N} \beta_n e^{j\theta_n} h_{rd,n}^{*} h_{sr,n} + h_{sd}^{*} \right|^2$$

$$= \left| \sum_{n=1}^{N} \beta_n \alpha_{sr,n} \alpha_{rd,n} e^{j(\theta_n + \phi_{sr,n} - \phi_{rd,n})} + \alpha_{sd} e^{-j\phi_{sd}} \right|^2$$

where,

$$h_{sr,n} = \alpha_{sr,n} e^{j\phi_{sr,n}}$$

$$h_{rd,n} = \alpha_{rd,n} e^{j\phi_{rd,n}}$$

$$h_{sd} = \alpha_{sd} e^{j\phi_{sd}}$$

- For coherent signal construction

$$\theta_n + \phi_{sr,n} - \phi_{rd,n} = -\phi_{sd}$$

$$\Rightarrow \theta_n = \mathrm{mod}\left[-\phi_{sd} - \phi_{sr,n} + \phi_{rd,n} , 2\pi \right]$$

⊙ The SNR objective reduces to

$$\left| \sum_{n=1}^{N} \beta_n e^{j\theta_n} h^*_{rd,n} h_{sr,n} + h^*_{sd} \right|^2$$

$$= \left| \sum_{n=1}^{N} \beta_n \alpha_{sr,n} \alpha_{rd,n} + \alpha_{sd} \right|^2$$

Note that $0 \le \beta_n \le 1$

To maximize the above objective, set $\beta_n = 1$

Problem 1 : What is the maximum value of quantity below ?

$$\max_{\theta_1, \theta_2} \left| a_1 e^{j\theta_1} + a_2 e^{j\theta_2} \right|^2$$

Maximum value is $|a_1 + a_2|^2$, which occurs when $\theta_1 = \theta_2$.