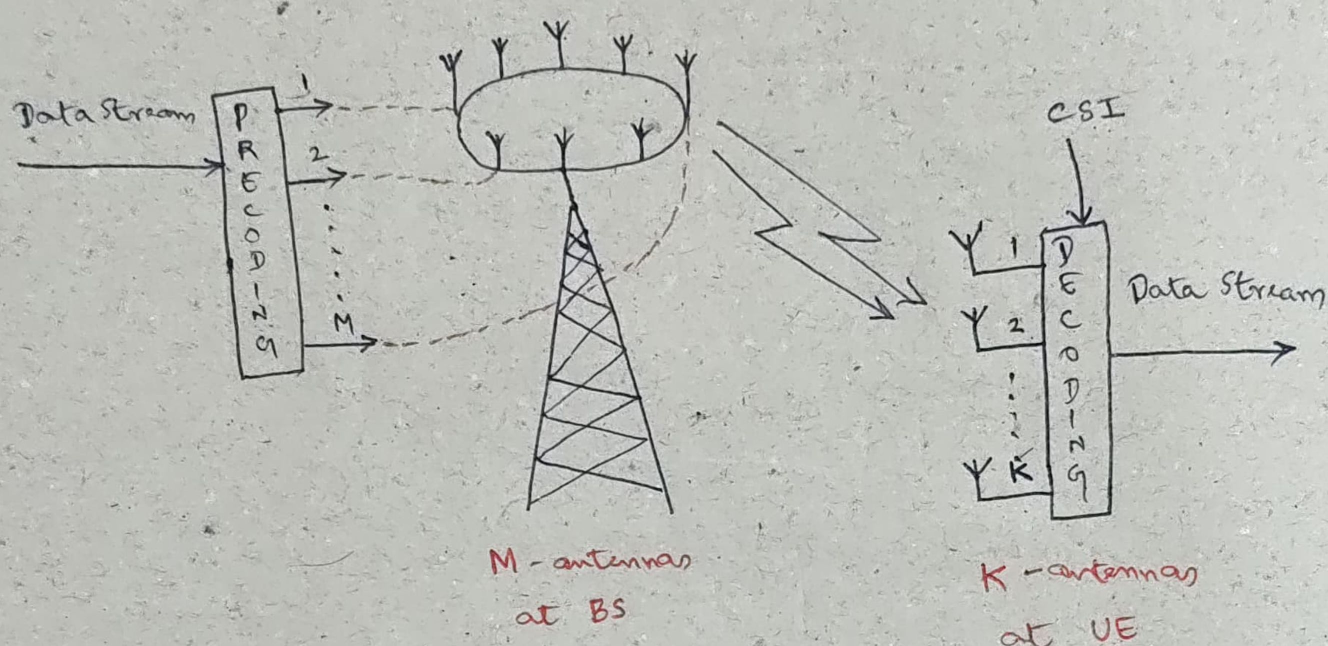
Projects 7, 8

Evolution of MIMO

(i) Point-to-point MIMO
(ii) Multi-user (MU) MIMO
(iii) Massive MIMO

① Point-to-point MIMO



⊙ Concentrated array of M-antennas transmits data to a UE / BS that has a concentrated array of K-antennas.

⊙ The Spectral efficiency of the link is

$$C \propto \min(M, K) \log_2 (\rho_d)$$

where ρ_d is the Signal-to-Noise power ratio (SNR)

Eg. 1 Consider an 8 x 4 MIMO system. It follows that

$$C \propto \min(8,4) \log_2(\rho_d)$$

(ie) $C \propto 4 \log_2(\rho_d)$.

Challenges

① Point - to - point MIMO faces several challenges. Absence of multipath results in increased correlation. (As the antennas are placed close to each other). This leads to lower capacity.

② Multiple antennas at user

⇒ Several RF chains

⇒ Equipment is costly

③ Achieving capacity requires complex signal processing, which requires SVD.

Eg. 2

Consider correlation coefficient between channel coefficients $\rho = 1$. What is the rank of 4 x 4 MIMO channel ?

The Rank of 4x4 MIMO channel reduces to 1, as all coefficients are same!

② Multi-user MIMO (MU-MIMO)

MU-MIMO separates the

single K antenna user

into

K single antenna users.

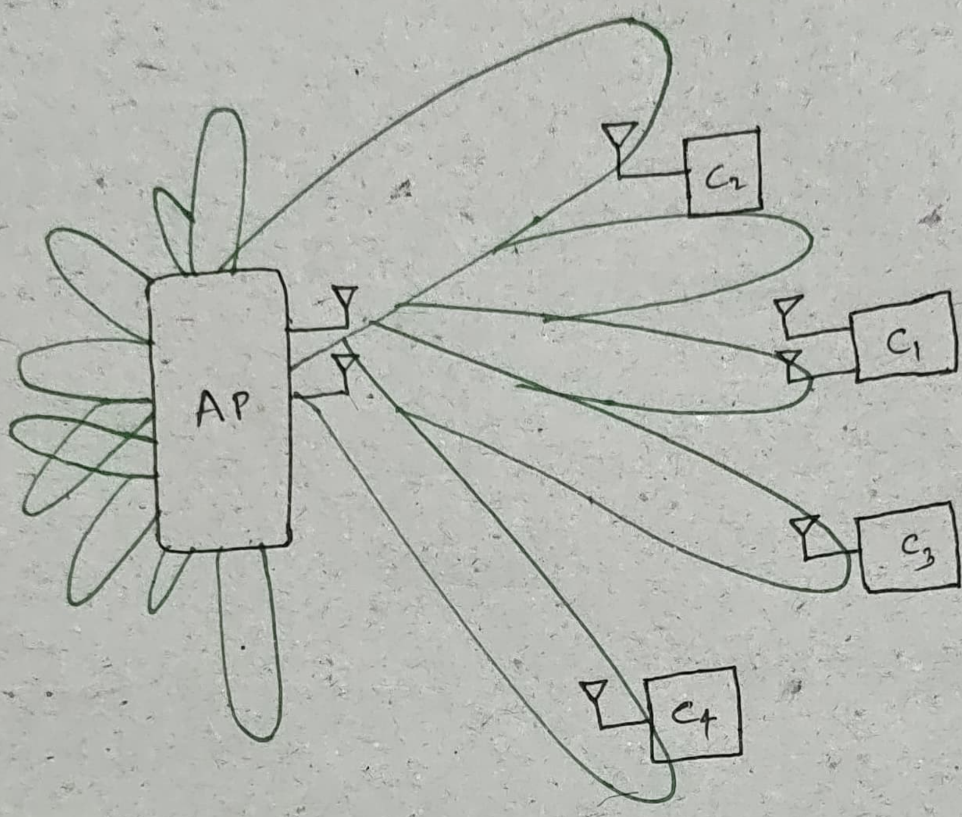


Figure: Multi-user MIMO

Advantages.

① Larger user separation

⇒ Channels are uncorrelated

⇒ higher capacity

② Single antenna user terminals are required.

⇒ No. of RF chains is lower

⇒ lower cost !

Challenges.

Achieving Capacity has higher complexity.

(ii) increased complexity of MU precoding and decoding.

③ Massive MIMO (A revolutionary technology)

Features.

In mMIMO,

No. of BS antennas (M)

is several times the

No. of users (K).

M ~ 250 − 300 antennas

K ~ 40 − 50 users

mMIMO Processing.

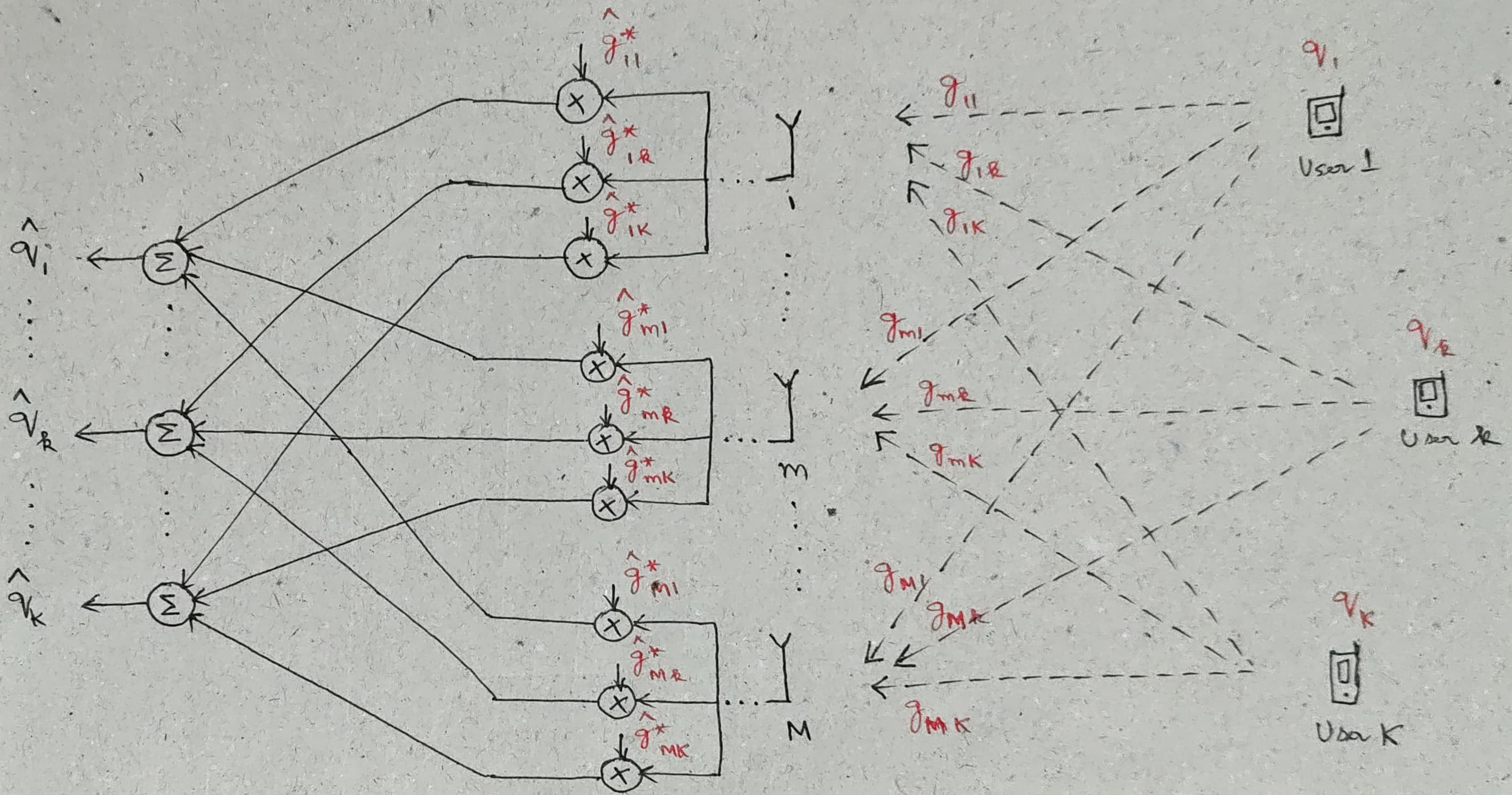
⊙ Simple Matched Filter (MF) can be employed for user separation on the UL.

⊙ Conjugate beamforming (CB) for user multiplexing on the DL.

$$\boxed{\begin{array}{l}\text{Note :}\\ \quad -\text{ZF/LMMSE receiver not required}\\ \quad -\text{No Matrix inverse operations}\end{array}}$$

⊙ In Massive MIMO, linear precoding and decoding suppress interference!

Thus, computational complexity is significantly low!



<u>Figure</u> : Matched Filter (MF) for mMIMO UL

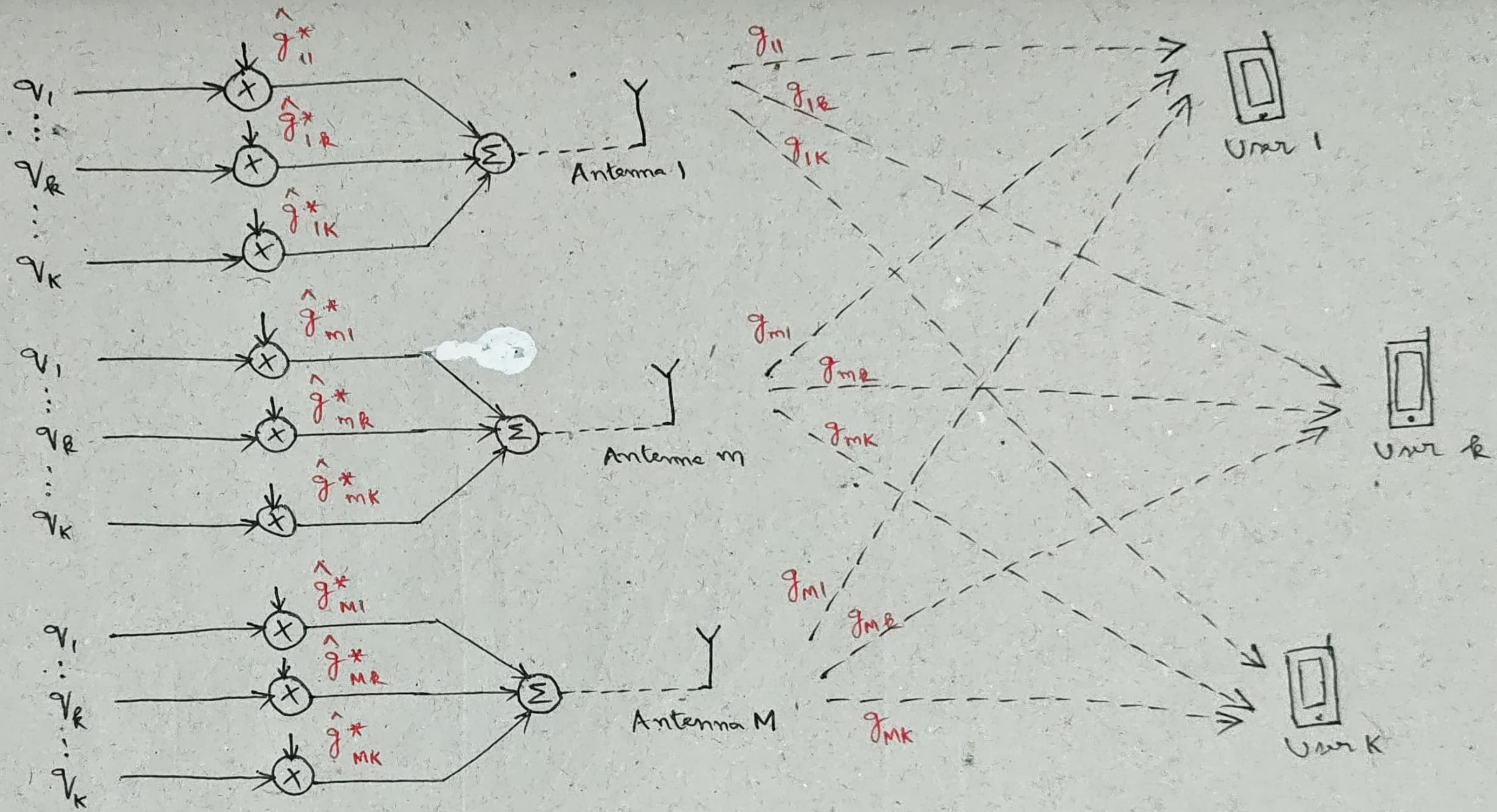


Figure: Conjugate beamforming (CB) for mMIMO DL

Eg. 3

What are the equations for the Massive MIMO System.

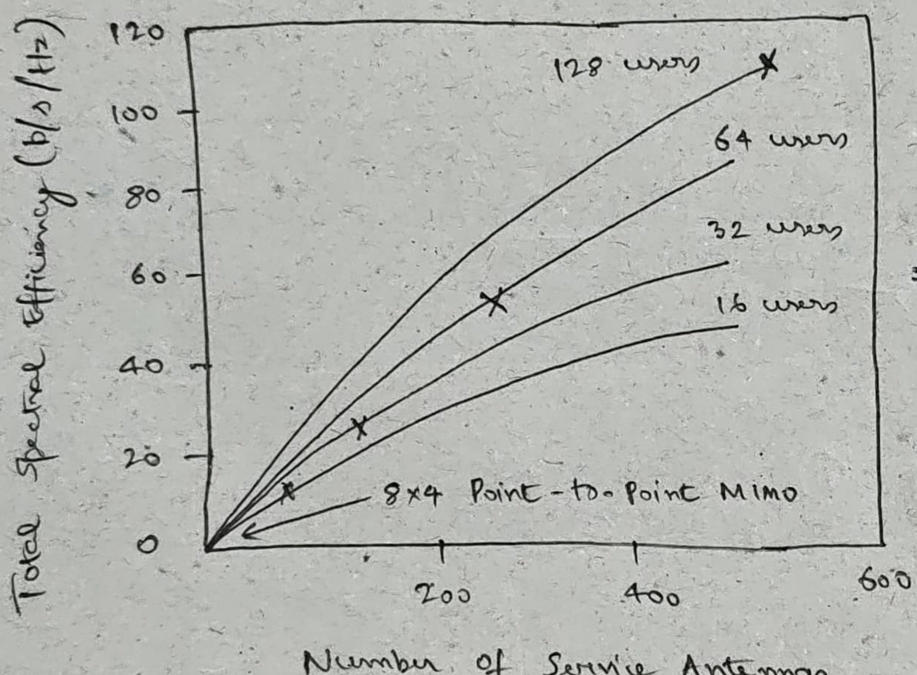
- For UL symbol estimation,

$$\hat{q}_1 = \sum_{m=1}^{M} g_{m1}^{*} \, y_m$$

- For DL symbol precoding,

$$x_1 = \sum_{k=1}^{K} g_{1k}^{*} \, q_k$$

mMIMO performance



M = 250 Antennas

K = 64 users

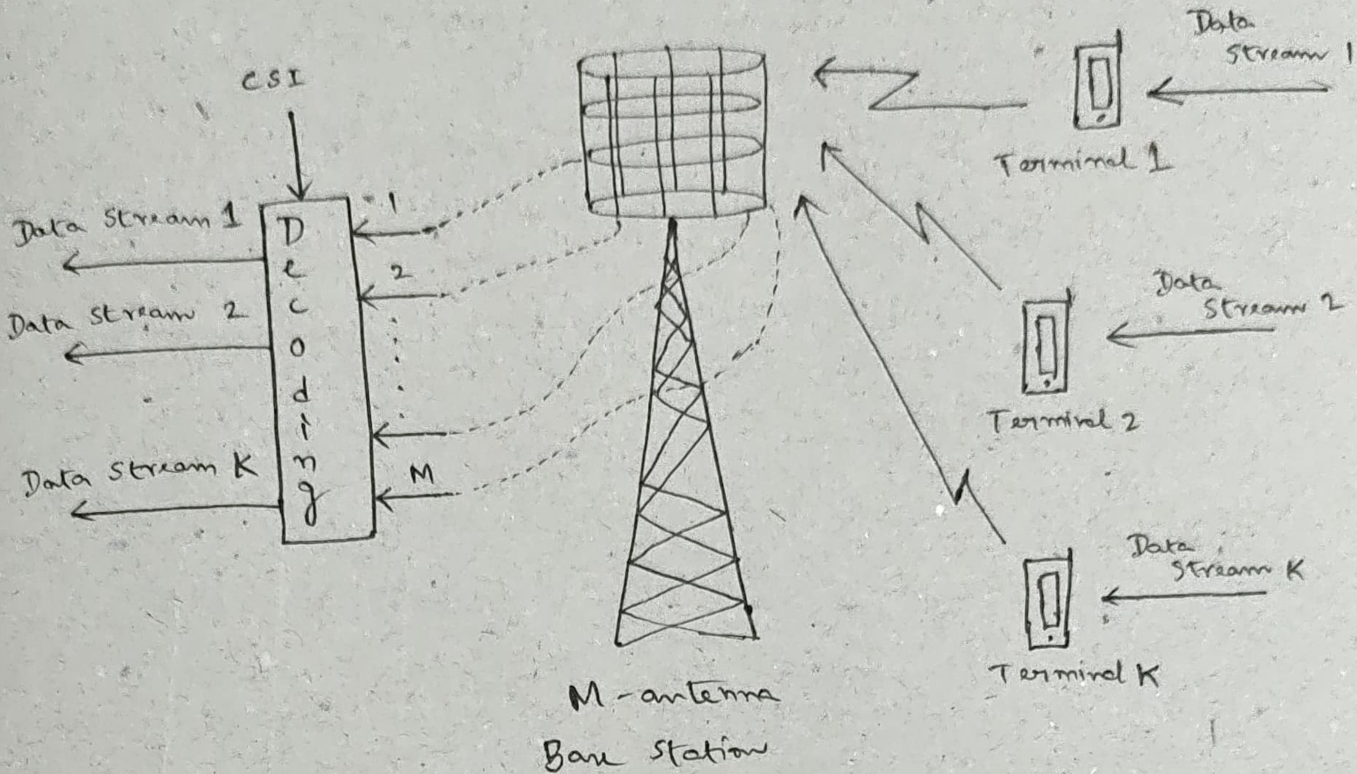
⇒ Spectral efficiency ~ 50 bits/s/Hz.

Plot: Total Spectral efficiency with variation in No. of BS antennas for particular K.

For 8×4 point to point MIMO,

Spectral efficiency = 1.3 bits/s/Hz.

mMIMO receiver design and analysis



Massive MIMO model

- The system has one BS equipped with M antennas.
- The cell has K single-antenna users
- Let $\bar{g}_k$ denote the $M \times 1$ channel vector between BS and user k.

$$\bar{g}_k = \begin{bmatrix} g_{1k} \\ g_{2k} \\ \vdots \\ g_{mk} \\ \vdots \\ g_{Mk} \end{bmatrix}$$

- g_{mk} is the channel between the m^{th} antenna of BS and k^{th} user.

$$E\{|g_{mk}|^2\} = \underline{\underline{\beta_k}}$$

- β_k models the geometric attenuation (path loss $\propto \frac{1}{d^2}$) and shadow fading. It is the <u>large scale fading factor</u>.

- The $M \times 1$ received vector y at the BS is

$$\bar{y} = \sqrt{P_u} \underbrace{\begin{bmatrix} \bar{g}_1 & \bar{g}_2 & \cdots & \bar{g}_K \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}}_{\bar{n}}$$

$$\Rightarrow \boxed{\bar{y} = \sqrt{P_u}\; G \bar{x} + \bar{n}}$$

- $\bar{n}$ is a vector of iid zero-mean complex Gaussian noise samples.

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}, \quad E\{|n_i|^2\} = 1$$

$$E\{n_i\} = 0$$

Noise variance is set to $\underline{1}$, without loss of generality.

Properties of RVs

- Let $X_1, X_2, \ldots, X_N$ be iid. (independent identically distributed) RVs with mean μ.

  Law of Large numbers.

$$\frac{X_1 + X_2 + \ldots + X_N}{N} \longrightarrow \mu$$

- We have,

$$\frac{\bar{g}_k^H \bar{g}_k}{M} = \frac{\|\bar{g}_k\|^2}{M} = \frac{|g_{1k}|^2 + |g_{2k}|^2 + \ldots + |g_{Mk}|^2}{M}$$

$$= \beta_k$$

This is termed as Channel hardening.

Beam-forming gain.

- Consider beamforming vector $\bar{\omega}$

$$E\left\{|\bar{\omega}^H \bar{g}_k|^2\right\} = \beta_k \|\bar{\omega}\|^2$$

This is the beamforming gain.

- Consider a unit-norm beamformer

$$\tilde{\omega} = \frac{\bar{\omega}}{\|\bar{\omega}\|}$$

$$E\left\{|\tilde{\omega}^H \bar{g}_i|^2\right\} = \beta_i \|\tilde{\omega}\|^2 = \beta_i$$

mMIMO receiver

- Consider user 1 as the desired user.

The received signal can be split into desired signal and interference.

$$\bar{y} = \underbrace{\sqrt{p_u}\,\bar{g}_1 x_1}_{\text{desired}} + \underbrace{\sqrt{p_u}\sum_{i=2}^{K} \bar{g}_i x_i + \bar{n}}_{\text{interference}}$$

- The matched filter for user 1 is

$$r_1 = \frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{y}$$

$$= \frac{\bar{g}_1^H}{\|\bar{g}_1\|}\left(\sqrt{p_u}\,\bar{g}_1 x_1 + \sqrt{p_u}\sum_{i=2}^{K}\bar{g}_i x_i + \bar{n}\right)$$

$$= \underbrace{\sqrt{p_u}\,\|\bar{g}_1\| x_1} + \underbrace{\sqrt{p_u}\sum_{i=2}^{K}\frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{g}_i x_i + \frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{n}}$$

- mMIMO SINR

$$\text{SINR} = \frac{p_u\|\bar{g}_1\|^2}{p_u\sum_{i=2}^{K} E\left\{\left|\frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{g}_i\right|^2\right\} + E\left\{\left|\frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{n}\right|^2\right\}}$$

where,

- Beamforming gain property

$$E\left\{\left|\frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{g}_i\right|^2\right\} = \beta_i$$

- As noise samples have unit variance,

$$E\left\{\left|\frac{\bar{g}_1^H}{\|\bar{g}_1\|}\bar{n}\right|^2\right\} = 1$$

Therefore, mMIMO SINR can be obtained as

$$SINR = \frac{p_u \|\bar{g}_1\|^2}{p_u \sum_{i=2}^{K} \beta_i + 1}$$

Power scaling.

We now derive the power scaling property of mMIMO. consider now

$$p_u = \frac{E_u}{M}$$

(ie) power of each user is decreased inversely as number of antennas.

As $M \to \infty$, the SINR scales as

$$SINR = \frac{p_u \|\bar{g}_1\|^2}{p_u \sum_{i=2}^{K} \beta_i + 1} = \frac{\frac{E_u}{M}\|\bar{g}_1\|^2}{\frac{E_u}{M}\sum_{i=2}^{K} \beta_i + 1}$$

$$= \frac{E_u \overbrace{\frac{\|\bar{g}_1\|^2}{M}}^{\beta_1}}{E_u \underbrace{\left(\frac{1}{M}\sum_{i=2}^{K}\beta_i\right)}_{0} + 1} \to E_u \beta_1$$

User Orthogonality

Therefore, mMIMO is able to suppress the MUI using only the MF which has a very low complexity!

Power Scaling

Power of users can decrease as $\frac{1}{M}$ while maintaining constant SINR!

This is one of the major advantages of mMIMO.