MISO IRS Transceiver Design

⊙ MISO IRS system model

- Consider a multiple input single output (MISO) System.

$$N_r = 1 , \quad N_t > 1$$

- Consider an IRS with N reflecting elements

⊙ BS to User channel

$$\bar{h}^H_{sd} = \begin{bmatrix} h^*_{sd,1} & h^*_{sd,2} & \cdots & h^*_{sd,N_t} \end{bmatrix}_{1 \times N_s}$$

⊙ BS to IRS channel

$$H_{sr} \sim N \times N_t$$

⊙ IRS to User channel

$$\bar{h}^H_{rd} = \begin{bmatrix} h^*_{rd,1} & h^*_{rd,2} & \cdots & h^*_{rd,N} \end{bmatrix}_{1 \times N}$$

- Recall, the reflecting matrix $\Theta \sim N \times N$ is

$$\Theta = \begin{bmatrix} \beta_1 e^{j\theta_1} & & \\ & \ddots & \\ & & \beta_N e^{j\theta_N} \end{bmatrix}$$

where $\left.\begin{matrix} \beta_n = \text{Amplitude} \\ \theta_n = \text{Phase shift} \end{matrix}\right\}$ of the n^{th} element

- Transmission model
  - Let x is the information symbol.
  - The transmit beamformer is $\bar{w} \sim N_t \times 1$, — To improve SNR

    satisfying $\|\bar{w}\|^2 \leq P_t$ → Transmit power
  - The transmit vector is $\bar{w}x$.

- IRS received signal model
  - The received signal y at the user is — Beamformer

$$y = \Big(\underbrace{\bar{h}_{rd}^{H}\, \Theta\, H_{sr}}_{\text{Component through the IRS}} + \underbrace{\bar{h}_{sd}^{H}}_{\text{Direct form}}\Big)\bar{w}x + z$$

$$E\{|x|^2\} = 1 \quad \text{and} \quad E\{|z|^2\} = 1$$

- Output Power
  - The output power is given as

$$\left|\bar{h}_{rd}^{H}\, \Theta\, H_{sr}\, \bar{w} + \bar{h}_{sd}^{H}\, \bar{w}\right|^2$$

$$= \left|\bar{w}\left(\bar{h}_{rd}^{H}\, \Theta\, H_{sr} + \bar{h}_{sd}^{H}\right)\right|^2$$

$$= \left|\underbrace{\bar{h}_{eff}^{H}\, \bar{w}}\right|^2$$

Dot product between 2 vectors (Matched Filter)

$$\Rightarrow \bar{h}_{eff}^{H} \propto \bar{w}$$

- MISO IRS optimization

- We employ the principle of <u>alternating Optimization</u>:

<u>Problem 2</u>: What is the beam-former $\bar{\omega}$ that achieves $\max \left| \bar{h}_{eff}^{H} \bar{\omega} \right|^2$

$$\text{s.t. } \|\bar{\omega}\|^2 \leq P_t$$

The beamformer $\bar{\omega}$ is

$$\bar{\omega} = \frac{\bar{h}_{eff}}{\|\bar{h}_{eff}\|} \sqrt{P_t}$$

- <u>Step 1</u>: Maximal Ratio Transmission (MRT)

For a given Θ, beamformer $\bar{\omega}$ is determined as

$$\bar{\omega} = \sqrt{P_t} \frac{\left(\bar{h}_{rd}^{H} \Theta H_{sr} + \bar{h}_{sd}^{H}\right)^H}{\left\|\bar{h}_{rd}^{H} \Theta H_{sr} + \bar{h}_{sd}^{H}\right\|}$$

$$= \sqrt{P_t} \frac{H_{sr}^{H} \Theta^{H} \bar{h}_{rd} + \bar{h}_{sd}}{\left\|H_{sr}^{H} \Theta^{H} \bar{h}_{rd} + \bar{h}_{sd}\right\|}$$

- <u>Step 2</u>: Optimal IRS

- For a given $\bar{\omega}$, the <u>optimal phases</u> can be determined as below.
- Employing the <u>triangle inequality</u>, we have

$$\left|\bar{h}_{rd}^{H} \Theta H_{sr} \bar{\omega} + \bar{h}_{sd}^{H} \bar{\omega}\right| \leq \underbrace{\left|\bar{h}_{rd}^{H} \Theta H_{sr} \bar{\omega}\right|} + \underbrace{\left|\bar{h}_{sd}^{H} \bar{\omega}\right|}$$

Same phase

(i.e.) Magnitude of Sum $\leq$ Sum of the Magnitudes

Problem 3 : Let $\angle(\bar{h}_{sd}^H \bar{w}) = \phi_0$.

How to choose θ to maximize

$$\left| \bar{h}_{rd}^H \Theta H_{sr} \bar{w} + \bar{h}_{sd}^H \bar{w} \right|$$

Solution : Phase of each term in $\bar{h}_{rd}^{H} \Theta H_{sr} \bar{w}$ can be matched to ϕ_0.

- Equality holds when

$$\angle(\bar{h}_{rd}^H \Theta H_{sr} \bar{w}) = \angle(\bar{h}_{sd}^H \bar{w}) = \phi_0$$

- Phase of the signal through the BS-IRS-User link is aligned with that of the BS-User direct link.
- Set $\beta_n = 1$
- Note that $\bar{h}_{rd}^H \Theta H_{sr} \bar{w}$

$$= [e^{j\theta_1}, e^{j\theta_2}, \dots, e^{j\theta_N}] \, \mathrm{diag}(\bar{h}_{rd}^H) H_{sr} \bar{w}$$

$$= \sum_{n=1}^{N} e^{j\theta_n} h_{rd,n}^{*} (\underbrace{\bar{h}_{sr,n}^H}_{n^{th} \text{ row of IRS elements}} \cdot \bar{w}).$$

- To maximize Output SNR

due to complex conjugate

$$\underbrace{\theta_n - \angle h_{rd,n} + \angle(\bar{h}_{sr,n}^H \bar{w})}_{\text{Sum of the phases}} = \phi_0$$

$$\Rightarrow \theta_n = \phi_0 + \angle h_{rd,n} - \angle(\bar{h}_{sr,n}^H \bar{w})$$

$$\Rightarrow \theta_n = \mathrm{mod}\left(\phi_0 + \angle h_{rd,n} - \angle(\bar{h}_{sr,n}^H \bar{w}), 2\pi\right)$$

- The phase optimization and beamformer optimization steps are performed alternatively until convergence.