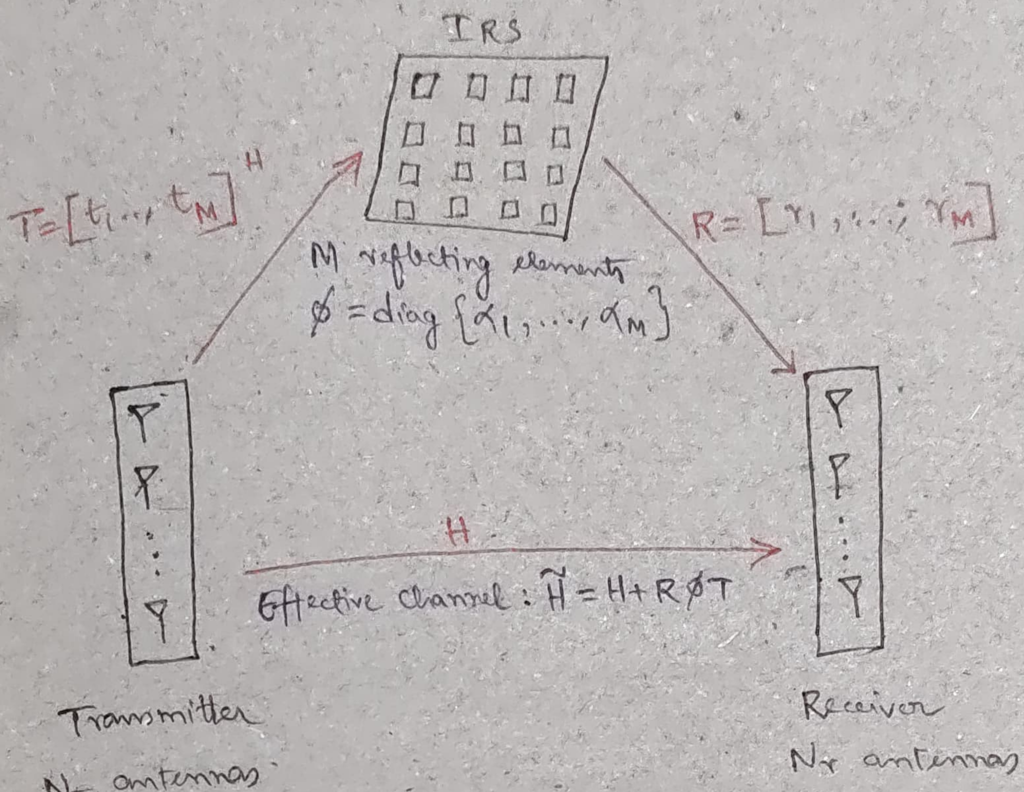
MIMO IRS Transceiver Design

IRS-aided MIMO

- Consider the multiple input multiple output (MIMO) System
  $N_r > 1$, $N_t > 1$



- Size of this MIMO system : $N_r \times N_t$
- Consider an IRS with M reflecting elements
  - Transmitter to Receiver direct channel
    $H \sim N_r \times N_t$
  - Transmitter to IRS channel
    $T \sim M \times N_t$
  - IRS to Receiver channel
    $R \sim N_r \times M$
- $\phi \sim M \times M$

- The complex reflecting matrix ϕ is

$$\phi = \operatorname{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_M\} = \begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_M \end{bmatrix}$$

where $\alpha_m \rightarrow$ IRS reflection coefficient for the m^{th} element

$$|\alpha_m| = 1$$

Phase of α_m adjusted in $[0, 2\pi)$

- The effective MIMO channel matrix from the transmitter to the receiver is

$$\tilde{H} = H + \underbrace{R\,\phi\,T}$$

↳ Channels through IRS (Artificial Multipath)

H: Source to Destination Channel

- Let $x \sim N_t \times 1$ denote the Transmitted signal vector

P_t denote the Maximum Transmit power

The transmit signal covariance matrix

$$Q \triangleq \mathbb{E}[\bar{x}\bar{x}^H] \sim N_t \times N_t$$

- Power constraint at the transmitter

$$\operatorname{Tr}(Q) = \mathbb{E}\{\bar{x}^H \bar{x}\} = \mathbb{E}\{\|\bar{x}\|^2\}$$

$$= \mathbb{E}\left\{\sum_{i=1}^{N_t} |x_i|^2\right\}$$

$$\leq P_t$$

(ie) Total transmit power $\leq P_t$

- MIMO Capacity

Note: Shanon's Capacity for 'Single Antenna'
$\log_2 (1 + SNR)$

MIMO channel Capacity (bits/s/Hz) is given by

$$C = \log_2 \det \left(I_{N_r} + \frac{1}{\sigma^2} \tilde{H} Q \tilde{H}^H \right)$$

where,
$Q \rightarrow$ Transmit Covariance Matrix
$\sigma^2 = N_0 =$ Noise Power

- The Alternating Optimization (AO) algorithm is employed to obtain the solution. It iteratively solves two smaller problems

$$\boxed{Q \quad \text{and} \quad \alpha_m.}$$

- Consider fixed reflection coefficients given $\{\alpha_m\}_{m=1}^{M}$

The Optimal transmit covariance matrix Q is given as follows:

⊙ Let SVD of $\tilde{H}$ be given as

$$\tilde{H} = \tilde{U} \tilde{\Sigma} \tilde{V}^H = \tilde{U} \begin{bmatrix} \tilde{\sigma}_1 & & \\ & \ddots & \\ & & \tilde{\sigma}_D \end{bmatrix} \tilde{V}^H$$

where, $\alpha_i \rightarrow$ Singular values
$D \rightarrow \text{rank}(\tilde{H}) \leq \min(N_t, N_r)$

⊙ Optimal Q is given as

$$Q = \tilde{V} \underbrace{\begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_D \end{bmatrix}}_{\text{Power loading Matrix}} \tilde{V}^H$$

⊙ p_i is the optimal power allocated to the i^{th} data stream.

$$P_i = \left[\frac{1}{\lambda} - \frac{\sigma^2}{\tilde{\sigma}_i^2}\right]^+ \quad \text{(water-filling strategy)}$$

$$= \begin{cases} \frac{1}{\lambda} - \frac{\sigma^2}{\tilde{\sigma}_i^2} & ; \text{ when } \frac{1}{\lambda} \geq \frac{\sigma^2}{\tilde{\sigma}_i^2} \\ 0 & ; \text{ when } \frac{1}{\lambda} < \frac{\sigma^2}{\tilde{\sigma}_i^2} \end{cases}$$

where,

$N_o = \sigma^2 =$ Noise Power

$\lambda \rightarrow$ Lagrange multiplier satisfying

$$\underbrace{\sum_{i=1}^{D} p_i}_{\text{Sum of all the Transmit Power}} = \underbrace{P_t}_{\text{Total Transmit Power}}$$

(ii) $$\sum_{i=1}^{D} \left[\frac{1}{\lambda} - \frac{\sigma^2}{\tilde{\sigma}_i^2}\right]^+ = P_t$$

- IRS phases

- Optimization of α_m with given Q and $\{\alpha_i\}_{i=1}^{M}{}_{i \neq m}$,

- The Objective function w.r.t each α_m is rewritten as

$$f_m = \log_2 \det\left(A_m + \alpha_m' B_m + \alpha_m^* B_m^H\right)$$

(Refer Appendix A)

- Consider A_m and B_m defined in Appendix B.

The Optimal IRS phase α_m is given by

$$\alpha_m = \begin{cases} e^{-j \arg\{\lambda_m\}} & , \ \mathrm{Tr}\left(A_m^{-1} B_m\right) \neq 0 \\ 1 & , \ \mathrm{Tr}\left(A_m^{-1} B_m\right) = 0 \end{cases}$$

$\lambda_m \rightarrow$ Maximum magnitude eigen value of $A_m^{-1} B_m$

(Refer Appendix B)

Appendix A

- Effective IRS-aided MIMO channel can be re-written as

$$\tilde{H} = H + R\phi T$$

$$= H + \sum_{m=1}^{M} \alpha_m r_m t_m^H$$

where,

$r_m \in \mathbb{C}^{N_r \times 1}$ denotes the m^{th} column of R

$t_m^H \in \mathbb{C}^{1 \times N_t}$ denotes the m^{th} row of T

- Eigen value decomposition (EVD) of Q:

$$Q = U_Q \Sigma_Q U_Q^H$$

where, Q is a Positive Semi-definite (PSD)

(i.e.) diagonal elements of Σ_Q are non-negative

- Define:

$$H' = H U_Q \Sigma_Q^{1/2}$$

$$U_Q \Sigma_Q^{1/2} = Q^{1/2}$$

$$T' = T U_Q \Sigma_Q^{1/2} = [\bar{t}_1', \ldots, \bar{t}_M']^H$$

$$\bar{t}_m' = \Sigma_Q^{1/2} U_Q^H \bar{t}_m$$

⊙ capacity expression can be now written as :

$$C = \log_2 \det\left(I_{N_r} + \frac{1}{\sigma^2} \tilde{H} U_Q \Sigma_Q U_Q^H \tilde{H}^H\right)$$

$$= \log_2 \det\left(I_{N_r} + \frac{1}{\sigma^2} \tilde{H} U_Q \Sigma_Q^{1/2} \left(\tilde{H} U_Q \Sigma_Q^{1/2}\right)^H\right)$$

$$= \log_2 \det\left(I_{N_r} + \frac{1}{\sigma^2} (H' + R\phi T')(H' + R\phi T')^H\right)$$

$$= \log_2 \det\left(I_{N_r} + \frac{1}{\sigma^2} H'' H''^H\right)$$

where,

$$H'' = H' + R\Phi T' = H' + \sum_{i=1}^{M} \alpha_i \bar{r}_i \bar{t}_i'^H$$

$$C = \log_2 \det\left(A_m + \alpha_m B_m + \alpha_m^* B_m^H\right)$$

$$A_m = I_{N_r} + \left(H' + \sum_{i=1, i\neq m}^{M} \alpha_i \bar{r}_i \bar{t}_i'^H\right)\left(H' + \sum_{i=1, i\neq m}^{M} \alpha_i \bar{r}_i \bar{t}_i'^H\right)^H$$

$$+ \frac{1}{\sigma^2} \bar{r}_m \bar{t}_m'^H \bar{t}_m' \bar{r}_m^H$$

$$B_m = \frac{1}{\sigma^2} \bar{r}_m \bar{t}_m'^H \left(H'^H + \sum_{i=1, i\neq m}^{M} \alpha_i^* \bar{t}_i' \bar{r}_i^H\right)$$

Appendix B

- Let A_m^{-1} exists
- $\mathrm{rank}(B_m) \leq 1$
- Capacity expression:

$$C = \underbrace{\log_2 \det\left(I_{N_r} + \alpha_m A_m^{-1} B_m + \alpha_m^* A_m^{-1} B_m^H\right)}_{f_m}$$

$$+ \underbrace{\log_2 \det(A_m)}_{\text{independent of } \alpha_m}$$

- $\mathrm{rank}(A_m^{-1} B_m) \leq \mathrm{rank}(B_m) \leq 1$

* If $\mathrm{rank}(A_m^{-1} B_m) = 0$, $A_m^{-1} B_m = 0$

  $\Rightarrow$ any α_m with $|\alpha_m| = 1$ is optimal

* If $\mathrm{rank}(A_m^{-1} B_m) = 1$,

  $\Rightarrow A_m^{-1} B_m$ is either diagnozable or non-diagnozable

Case 1: $A_m^{-1} B_m$ is diagnozable

EVD: $A_m^{-1} B_m = U_m \Lambda_m U_m^{-1}$

$$\Sigma_m = \mathrm{diag}\{\lambda_m, 0, \ldots, 0\} \in \mathbb{C}^{N_r \times N_r}$$

$$f_m = \log_2 \det\left(I_{N_r} + \alpha_m A_m^{-1} B_m + \alpha_m^* A_m^{-1} B_m^H\right)$$

$$= \log_2 \det\left(I_{N_r} + \alpha_m U_m \Lambda_m U_m^{-1} + \alpha_m^* A_m^{-1} U_m^{-1H} \Lambda_m^H U_m^H A_m\right)$$

$$= \log_2 \det\left(I_{N_r} + \alpha_m \Lambda_m + \alpha_m^* \underbrace{U_m^{-1} A_m^{-1} U_m^{-1H}}_{V_m^{-1}} \Lambda_m^H \underbrace{U_m^H A_m U_m}_{V_m}\right)$$

$$= \log_2 \det\left(I_{Nr} + \alpha_m \Lambda_m + \alpha_m^* \bar{v}_m \lambda_m^* v_m'^T\right)$$

$$= \log_2\left(\left(1 + |\lambda_m|^2 (1 - v_{m1}' v_{m1}\right) + 2\,\mathrm{Re}\{\alpha_m \lambda_m\}\right)$$

Optimal solution to (P1-m): $\alpha_m^* = e^{-j \arg\{\lambda_m\}}$

Case 2 : $A_m^{-1} B_m$ is non-diagnolizable

$$A_m^{-1} B_m = \bar{u}_m \bar{v}_m^H$$

$A_m^{-1} B_m$ is non-diagnolizable if $\bar{u}_m^H \bar{v}_m = Tr(A_m^{-1} B_m) = 0$

$$f_m = \log_2 \det \left(I_{Nr} + \alpha_m A_m^{-1} B_m + \alpha_m^* A_m^{-1} B_m^H \right)$$

$$= \log_2 \det \left(I_{Nr} + \alpha_m \bar{U}_m \bar{v}_m^H + \alpha_m^* A_m^{-1} \bar{U}_m \bar{v}_m^H A_m \right)$$

Since $\bar{u}_m^H \bar{v}_m = 0$

$$f_m = \log_2 \det \left(I_{Nr} - A_m^{-1} \bar{v}_m \bar{u}_m^H \left(I_{Nr} - \alpha_m^* \bar{v}_m \bar{u}_m^H \right) A_m \bar{U}_m \bar{v}_m^H \right)$$

$$= \log_2 \det \left(I_{Nr} - A_m^{-1} \bar{v}_m \bar{u}_m^H A_m \bar{U}_m \bar{v}_m^H \right)$$

f_m is independent of α_m

$\Rightarrow$ any α_m with $|\alpha_m| = 1$ is optimal.