Projects : 3, 4, 5, 6

MIMO receiver design and optimization

Introduction to MIMO

- MIMO (Multiple Input Multiple Output)
- Key technology in 4G/5G.

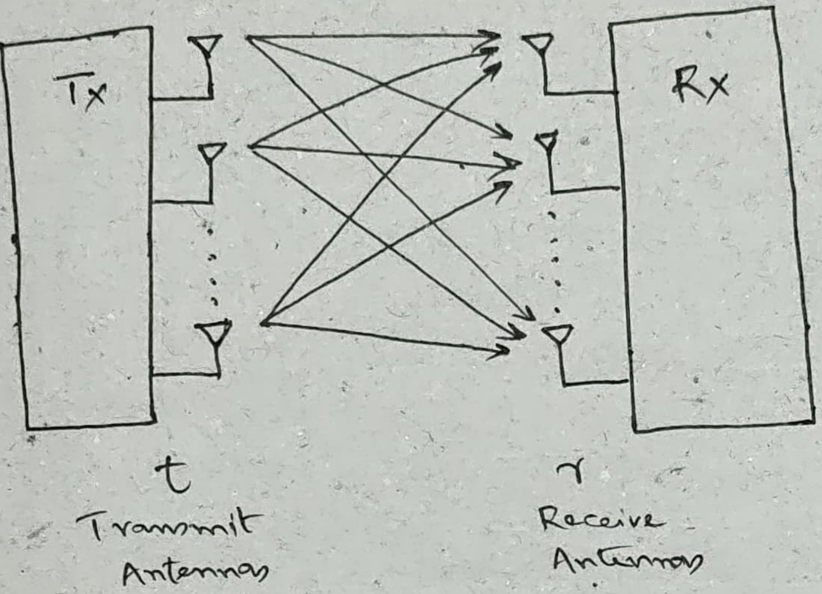

Figure : MIMO System

MIMO advantages

⊙ Multiple Antennas → Beamforming → Increased Reliability

⊙ MIMO can also be used to increase data rates.

- By transmitting multiple information streams in parallel.
- This is termed as Spatial Multiplexing.

MIMO system model

MIMO system model can be represented as

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1t} \\ h_{21} & h_{22} & \cdots & h_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{r1} & h_{r2} & \cdots & h_{rt} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{bmatrix}}_{\bar{n}}$$

$$\bar{y} = H \bar{x} + \bar{n}$$

MIMO Receivers

(i) ZF receiver. (Suppresses the interference b/w different symbols)

- The ZF receiver minimizes the error.

$$\min \|\bar{y} - H\bar{x}\|^2$$

- The estimate of the symbol vector is

$$\hat{x} = (H^H H)^{-1} H^H \bar{y}$$

- The quantity $(H^H H)^{-1} H^H$ is termed as the pseudo-inverse of H. (ie) $(H^H H)^{-1} H^H \cdot H = I$.

- Let $L = r - t + 1$.

What is the probability that bit received is Error?

$$BER \approx \frac{1}{2^L} \, {}^{2L-1}C_L \left(\frac{1}{SNR}\right)^L$$

$$= \frac{1}{2^L} \times \frac{(2L-1)!}{L! \times (L-1)!} \times \left(\frac{1}{SNR}\right)^L$$

(ii) LMMSE receiver. (Suppresses the interference b/w different symbols and NOISE)

- Minimum mean of the squared error

$$\min E\left\{ \| \hat{x} - \bar{x} \|^2 \right\}$$

- The LMMSE receiver for the MIMO system is given as

$$\hat{x} = \left(\frac{N_0}{E_s} I + H^H H \right)^{-1} H^H \bar{y}$$

$$= \left(\frac{1}{SNR} I + H^H H \right)^{-1} H^H \bar{y}$$

- At high SNR, LMMSE $\rightarrow$ ZF.

SVD and MIMO optimization

SVD (Singular value Decomposition)

- The SVD of a $r \times t$ matrix H is given as

$$H = U \Sigma V^H$$

- U (size $r \times r$), V (size $t \times t$) are Unitary matrices.

$$U^H U = U U^H = I$$

$$V^H V = V V^H = I$$

The matrix Σ is a diagonal matrix,

- contains singular values $\sigma_1, \sigma_2, \ldots, \sigma_t$ on the main diagonal.

$$\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_t \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_t \geq 0.$$

Note:

* SVD exists for any Matrix.
* EVD exists for SQUARE matrix.

MIMO beamforming

* Start with the MIMO model

$$\bar{y} = H\bar{x} + \bar{n}$$

$$= U \Sigma V^H \bar{x} + \bar{n}$$

* At the transmitter, we precode the transmit symbol vector $\bar{x} = V \tilde{x}$

$$\Rightarrow \bar{y} = U \Sigma V^H (V \tilde{x}) + \bar{n}$$

$$\Rightarrow \bar{y} = U \Sigma \tilde{x} + \bar{n}$$

* At the receiver, we use U^H as the Combiner.

$$\Rightarrow \tilde{y} = U^H \bar{y}$$

$$\Rightarrow \tilde{y} = U^H (U \Sigma \tilde{x} + \bar{n})$$

$$\Rightarrow \tilde{y} = \Sigma \tilde{x} + \tilde{n}.$$

* Effective MIMO model is given as

$$\underbrace{\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_t \end{bmatrix}}_{\tilde{y}} = \underbrace{\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_t \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_t \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_t \end{bmatrix}}_{\tilde{n}}$$

Optimal MIMO power.

If the symbols are of power $E\{|\tilde{x}_i|^2\} = P_i$;

the optimal value of P_i is

$$\underbrace{P_i = \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_i^2}\right)^+}_{\text{Water - filling power Allocation}}$$

The parameter λ can be found from the constraint

$$\sum_{i=1}^{t} \left(\frac{1}{\lambda} - \frac{N_0}{\sigma_i^2}\right)^+ = P_0$$