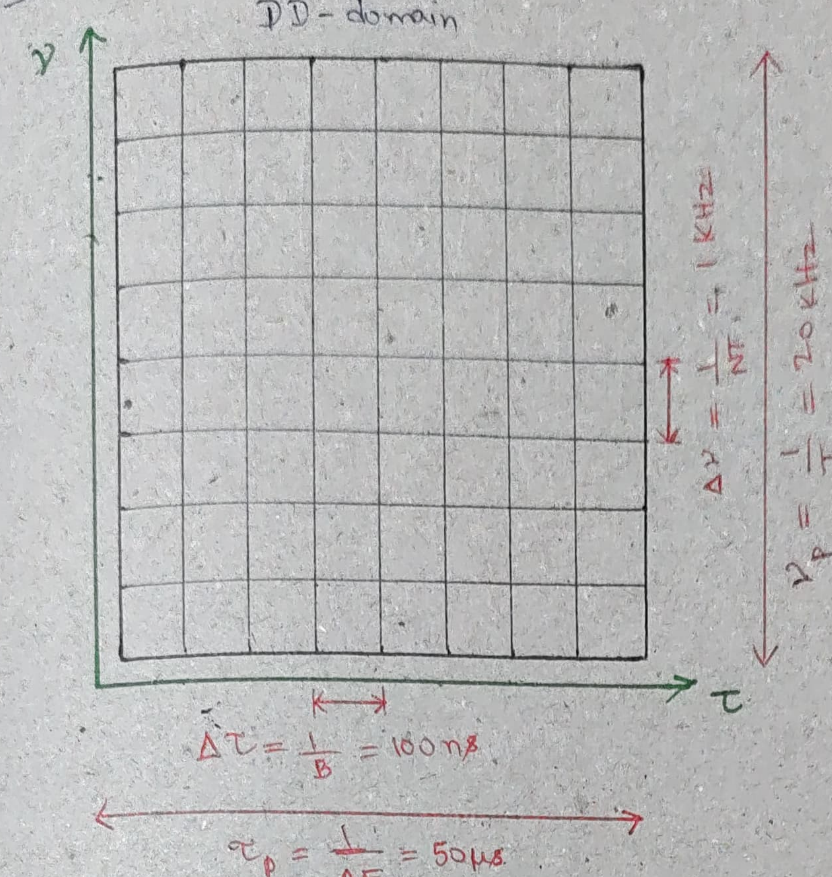
OTFS Signal Modulation and Generation

DD-domain Grid

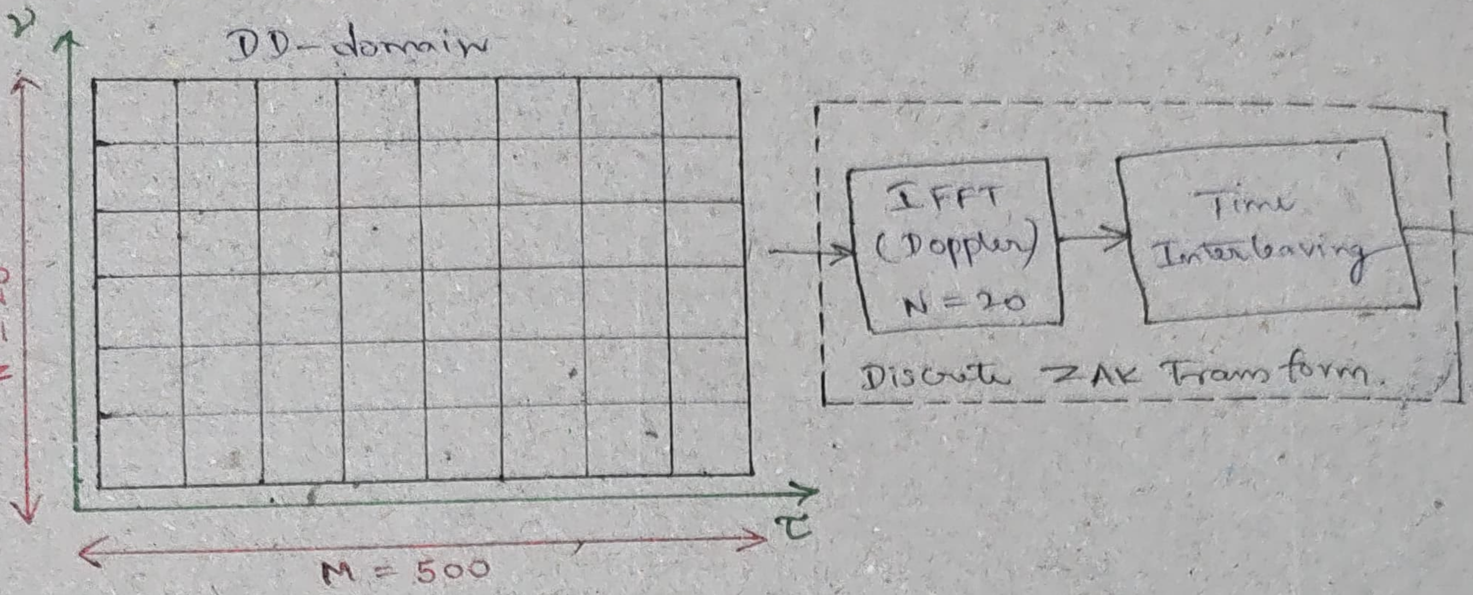


- No. of symbols along delay-axis, $M = \frac{\tau_p}{\Delta\tau} = 500$
- No. of symbols along doppler-axis, $N = \frac{\nu_p}{\Delta\nu} = 20$
- Total No. of symbols, $M \times N = 500 \times 20 = 10{,}000$
- These are spread over

$$B = \frac{1}{\Delta\tau} = 10\,MHz$$

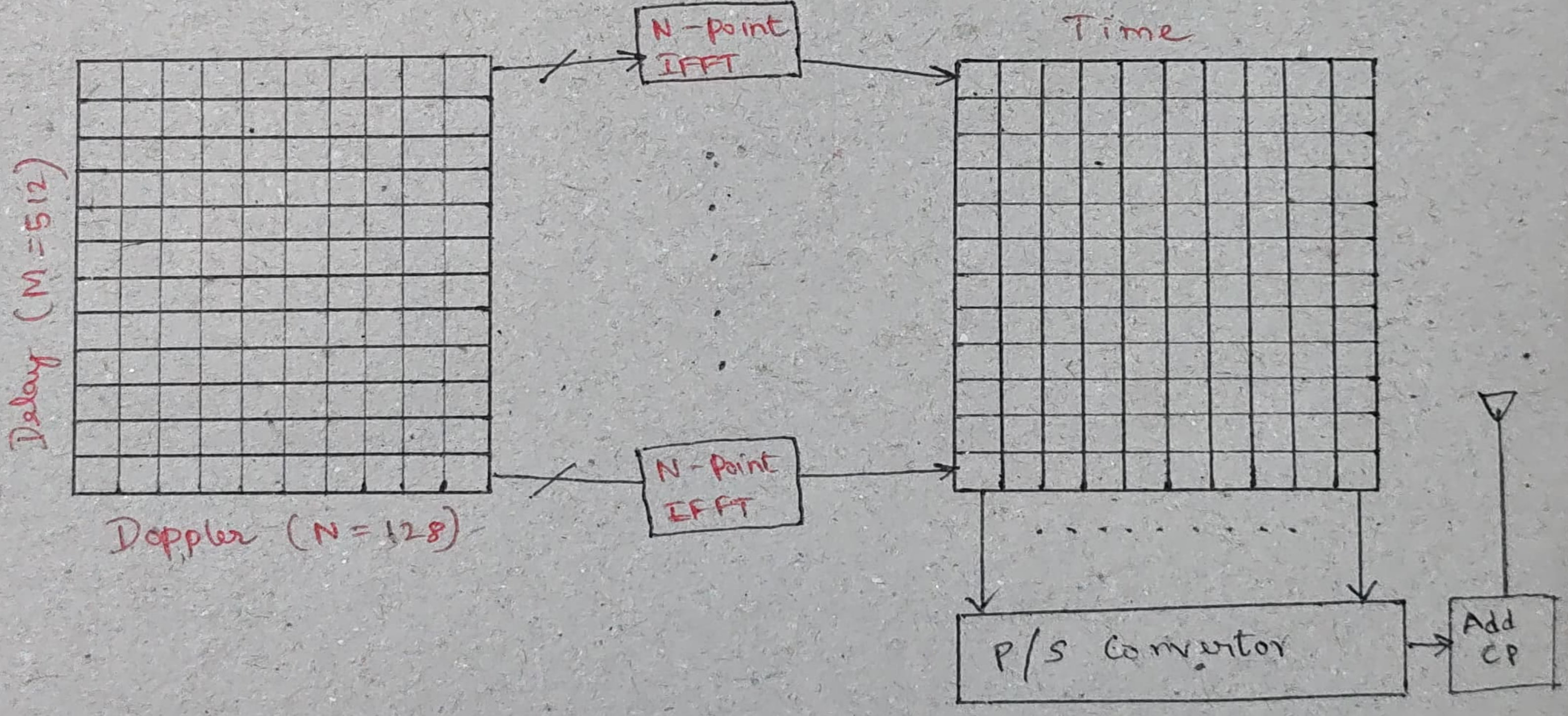
$$T_f = \frac{1}{\Delta\nu} = 1\,ms$$

OTFS Modulation procedure



- Perform IFFT along ν axis (size $N = 20$)
  This is followed by <u>Time Interleaving</u>
  (i.e) Transmit each row along τ axis.
- Therefore, IFFT of size $N = 20$ is repeated $M = 500$ times for OTFS.

OTFS Modulation procedure



Here, Total No. of Symbols, $M \times N = 512 \times 128$

$$= 2^9 \times 2^7$$

$$= 2^{16}$$

$$= 65536$$

Perform 128 point IFFT along Doppler, Time interleave and transmit.

OTFS Using OFDM

- X_{DD} is the $M \times N$ information symbol matrix in DD-domain
- $X_{DD}(l, k)$ is the symbol at delay index l and doppler index k.
- The transmitter maps the DD-domain symbols to the TF-domain using the Inverse Symplectic Finite Fourier Transform (ISFFT)

$$X_{TF}(m, n) = \frac{1}{\sqrt{MN}} \sum_{l=0}^{M-1} \sum_{k=0}^{N-1} X_{DD}(l, k)\, e^{j2\pi\left(\frac{nk}{N} - \frac{ml}{M}\right)}$$

> Note: ISFFT denotes FFT w.r.t delay variable, and IFFT w.r.t doppler variable.

- This can be mathematically modeled as

$$X_{TF} = F_M X_{DD} F_N^H$$

DFT along Delay axis

N-dimensional IDFT w.r.t Doppler

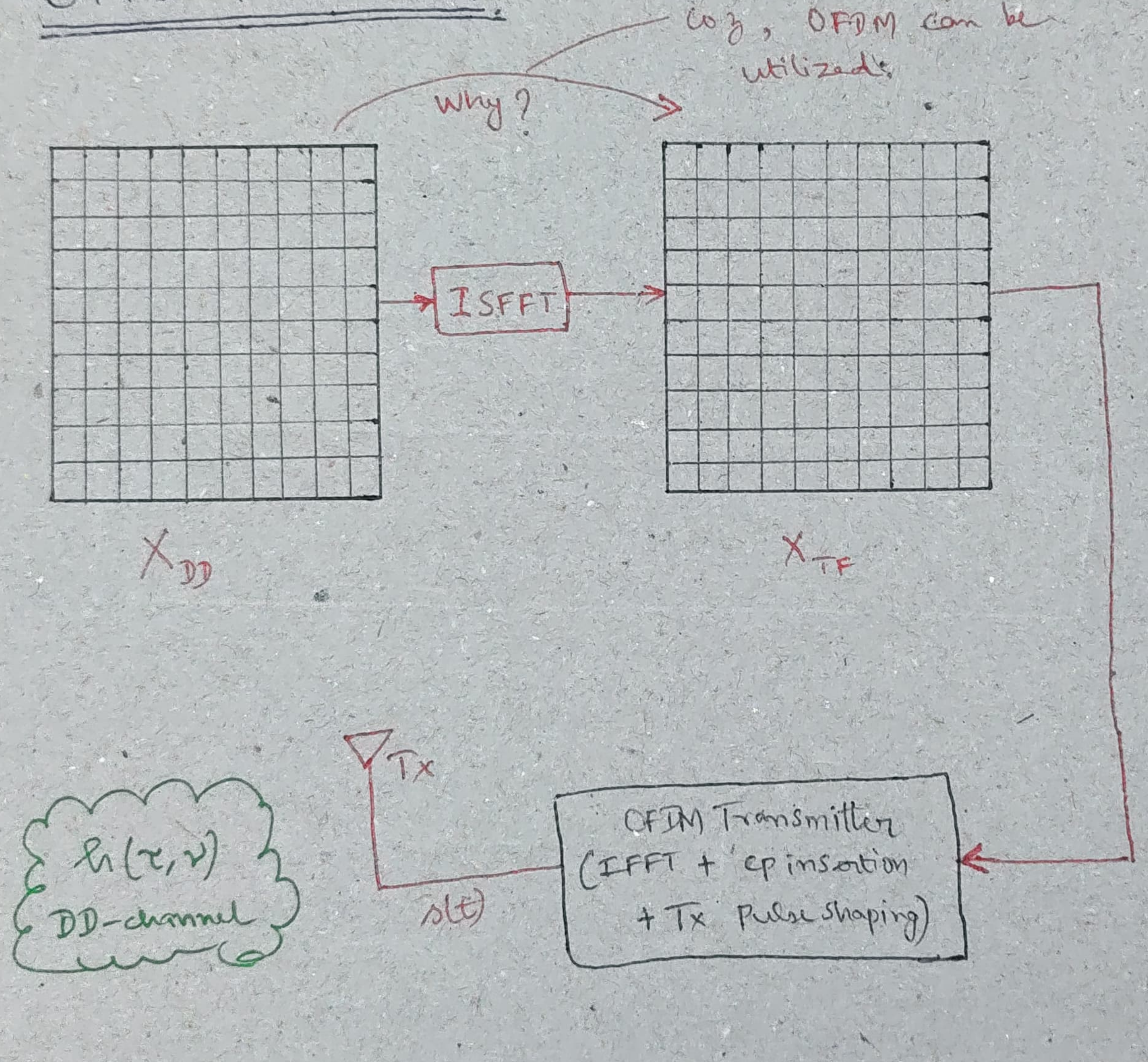
- F_M and F_N are the normalized-DFT matrices

$$F_N(i,j) = \frac{1}{\sqrt{N}} e^{-\frac{j2\pi ij}{N}}, \quad 0 \le i,j \le N-1$$

$$F_M(i,j) = \frac{1}{\sqrt{M}} e^{-\frac{j2\pi ij}{M}}, \quad 0 \le i,j \le M-1$$

- X_{TF} is the $M \times N$ information symbol matrix in TF-domain
- $X_{TF}(m,n)$ is the symbol over subcarrier m and symbol time n.
- Note that, the frame duration is $T_f = NT$.

OTFS Transmission



- Perform multiple M-point IFFTs, one for each of the N columns of X_{TF}, followed by CP insertion and transmission.

- Each column of X_{TF} is of size M.

$$M = \frac{B}{\Delta F} = \text{\# subcarriers}$$

- The time-domain transmit signal matrix S of size $M \times N$ is

$$S = P_{tx} F_M^H X_{TF}$$

Pulse shaping filter

- Substituting $X_{TF} = F_M X_{DD} F_N^H$ yields

IFFT matrix

FFT matrix

$$S = P_{tx} \cancel{F_M^H} \cancel{F_M} X_{DD} F_N^H$$

$$= P_{tx} X_{DD} F_N^H$$

- Form N columnwise blocks of size M, transmit the MN samples <u>serially.</u>

- The <u>transmit vector</u> $\bar{s}$ of size MN is

$$\bar{s} = \underbrace{vec(S)} = vec(P_{tx} X_{DD} F_N^H)$$

MN x 1

$$= (F_N^H \otimes P_{tx}) \bar{x}_{DD}$$

stacking of the columns of S
i.e. vectorizing.

where

$$\bar{x}_{DD} = vec(X_{DD})$$

- For the special case of a rectangular pulse, $P_{tx} = I_M$.

$$\bar{s} = \text{vec}(S) = \text{vec}(\underbrace{P_{tx}}_{M \times M} \underbrace{X_{DD}}_{M \times N} \underbrace{F_N^H}_{N \times N})$$

$$= \text{vec}(X_{DD} F_N^H)$$

$$= \underbrace{(F_N^H \otimes I_M)}_{MN \times MN} \underbrace{\bar{x}_{DD}}_{MN \times 1}$$

where,

$$\bar{x}_{DD} = \text{vec}(X_{DD})$$

- Prior to transmission, similar to OFDM, a cyclic Prefix (CP) of length L is appended to $\bar{s}$. This removes the inter-frame interference at the receiver.

Problem 5: Determine $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

Kronecker product is defined as

$$\begin{bmatrix} a_{11} & a_{12} & \square \\ a_{21} & \ddots & \square \\ \square & \square & \square \end{bmatrix} \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \square \\ a_{21}B & \ddots & \square \\ \square & \square & \square \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 & -4 \\ 2 & 1 & 4 & 2 \\ -2 & 4 & 1 & -2 \\ -4 & -2 & 2 & 1 \end{bmatrix}$$

Problem 6: What is size of $F_N^H \otimes I_M$?

$MN \times MN$