Projects 11, 12, 13, 14

Non-Orthogonal Multiple Access (NOMA)

Conventional Orthogonal Multiple Access (OMA) techniques include

- Time Division Multiple Access (TDMA) → 2G
- Code Division Multiple Access (CDMA) → 3G
- Orthogonal Frequency Division Multiple Access (OFDMA) → 4G & 5G

What are the orthogonal resources in TDMA, CDMA and OFDMA?

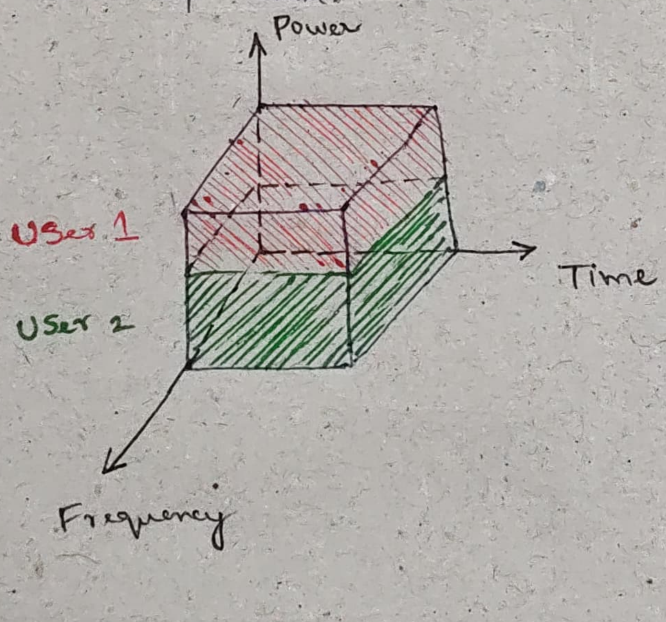
TDMA → Time

CDMA → Codes

OFDMA → Sub carriers

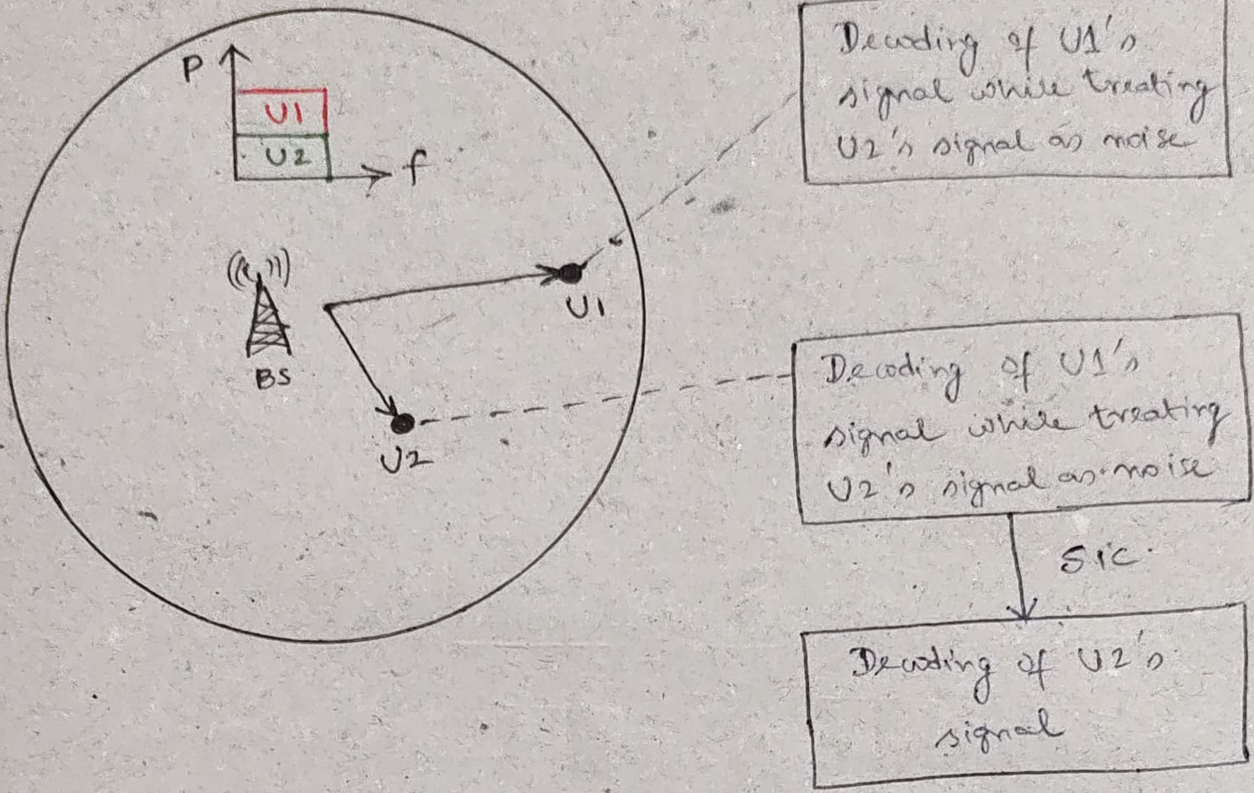
NOMA

In NOMA, several users are multiplexed over the same frequency / time resources. As a result, throughput of NOMA can be significantly larger than that of OMA.



NOMA interference Cancellation

NOMA downlink (DL)



Eg. 1 Describe the decoding process for 3-users, U_1, U_2 and U_3.

(i) U_1 decodes its symbol with interference from U_2, U_3.

(ii) U_2 decodes U_1 with interference from U_2, U_3.
U_2 then cancels symbol of U_1.
Next, it decodes its own symbol with interference from U_3.

(iii) U_3 decodes U_1 with interference from U_2, U_3.
U_3 then cancels symbol of U_1.
Next, it decodes symbol of U_2 with interference from U_3.
It then cancels symbol of U_2 and decodes its symbol.

Successive Interference Cancellation (SIC).

- BS creates a <u>superposed</u> signal containing data of both users.

$$x_s = \sqrt{a_1 \rho_s}\, x_1 + \sqrt{a_2 \rho_s}\, x_2$$

where,
$a_1 \rightarrow$ Power allocated to U1
$a_2 \rightarrow$ Power allocated to U2
$\rho_s \rightarrow$ Total transmit SNR.

$$\rho_s = \frac{P_s}{\sigma^2}$$

where, $P_s \rightarrow$ Total transmit Power

$$a_1 + a_2 = 1$$

- At <u>User 1</u>, the <u>received signal</u> is

$$y_1 = h_1 \left(\sqrt{a_1 \rho_s}\, x_1 + \sqrt{a_2 \rho_s}\, x_2 \right) + n_1$$

$$= h_1 \sqrt{a_1 \rho_s}\, x_1 + h_1 \sqrt{a_2 \rho_s}\, x_2 + n_1$$

SINR for decoding x_1 at User 1 is

$$\gamma_{u1}^{x_1} = \frac{a_1 \rho_s |h_1|^2}{a_2 \rho_s |h_1|^2 + 1}$$

$$\text{SINR} = \frac{\text{Signal Power}}{\text{Interference + Noise Power}}$$

- At <u>User 2</u>,

$$y_2 = h_2 \left(\sqrt{a_1 \rho_s}\, x_1 + \sqrt{a_2 \rho_s}\, x_2 \right) + n_2$$

$$= h_2 \sqrt{a_1 \rho_s}\, x_1 + h_2 \sqrt{a_2 \rho_s}\, x_2 + n_2$$

SINR for decoding x_1 at User 2, is.

$$\gamma_{u2}^{x_1} = \frac{a_1 \rho_s |h_2|^2}{a_2 \rho_s |h_2|^2 + 1}$$

First U2 has to decode U1's signal

After decoding 1st User and Cancelling interference (successfully);

$$y_2 = h_2 \sqrt{a_2 \rho_s}\, x_2 + n_2$$

SNR for decoding x_2 at User 2 is

$$\gamma_{u_2}^{x_2} = a_2 \rho_s |h_2|^2$$

No interference

Eg. 2 For a 3-user scenario, what is the transmit signal and SINR for decoding?

$$x_s = \sqrt{a_1 \rho_s}\, x_1 + \sqrt{a_2 \rho_s}\, x_2 + \sqrt{a_3 \rho_s}\, x_3$$

$$y_3 = h_3 \sqrt{a_2 \rho_s}\, x_2 + h_3 \sqrt{a_3 \rho_s}\, x_3 + n_2$$

$$\gamma_{u_3}^{x_2} = \frac{a_2 \rho_s |h_3|^2}{a_3 \rho_s |h_3|^2 + 1}$$

$$\gamma_{u_3}^{x_1} = \frac{a_1 \rho_s |h_3|^2}{a_2 \rho_s |h_3|^2 + a_3 \rho_s |h_3|^2 + 1}$$

NOMA Outage

- Outage occurs when data rate of user is lower than the desired rate

- Let $\tilde{R}_1, \tilde{R}_2$ denote the desired rates of User 1 and User 2.

Probability of Outage - U1

The condition of outage for user 1 is

$$\underbrace{C_{u_1}^{x_1} = \log_2\left(1 + \gamma_{u_1}^{x_1}\right)}_{\text{Maximum Rate for successful decoding}} < \underbrace{\tilde{R}_1}_{\text{Expected Data Rate}}$$

$$\Rightarrow 1 + \gamma_{u_1}^{x_1} < 2^{\tilde{R}_1}$$

$$\Rightarrow \gamma_{u_1}^{x_1} < 2^{\tilde{R}_1} - 1$$

$$\Rightarrow \frac{a_1 \rho_s |h_1|^2}{1 + a_2 \rho_s |h_1|^2} < R_1$$

$$\Rightarrow |h_1|^2 < \frac{R_1}{(a_1 - a_2 R_1)\rho_s}$$

Let b_1 be defined as

$$b_1 = |h_1|^2$$

$h_1 \rightarrow$ Rayleigh RV
$|h_1|^2 \rightarrow$ Exponential RV

Let $E\{b_1\} = \beta_1$.

PDF of b_1 is

$$\underbrace{f_{B_1}(b_1) = \frac{1}{\beta_1} e^{-\frac{b_1}{\beta_1}}, \quad b_1 \geq 0}_{\text{Exponential random variable}}$$

The Probability of outage for U1 is

$$\Pr\left\{ b_1 < \frac{R_1}{(a_1 - a_2 R_1)\rho_s} \right\}$$

$$= \int_0^{\frac{R_1}{(a_1-a_2R_1)\rho_s}} f_{B_1}(b_1)\, db_1$$

$$P_o = 1 - \exp\left(- \frac{R_1}{\rho_s B_1 (a_1 - a_2 R_1)} \right)$$

⊙ Probability of Outage - U2.

User 2 decoding fails if either decoding of x_1 or x_2 fails.

Probability of outage at user 2 is

$$\Pr\{ \underbrace{C_{u2}^{x_1}}_{\text{Maximum possible rate to transmit } x_1} < \tilde{R}_1 \cup C_{u2}^{x_2} < \tilde{R}_2 \}$$

$$C_{u_2}^{x_1} < \tilde{R}_1$$

$$\Rightarrow \underbrace{\log_2\left(1 + \gamma_{u_2}^{x_1}\right)}_{\text{Max. possible Rate to transmit}} < \tilde{R}_1$$

$$\Rightarrow \frac{a_1 \rho_s |h_2|^2}{a_2 \rho_s |h_2|^2 + 1} < 2^{\tilde{R}_1} - 1 = R_1$$

$$\Rightarrow |h_2|^2 < \frac{R_1}{(a_1 - a_2 R_1)\rho_s}$$

Decoding of x_1 fails.

$$C_{u2}^{x_2} < \tilde{R}_2$$

$$\Rightarrow \log_2\left(1+\gamma_{u2}^{x_2}\right) < \tilde{R}_2$$

$$\Rightarrow a_2 \rho_s |h_2|^2 < 2^{\tilde{R}_2} - 1 = R_2$$

$$\Rightarrow |h_2|^2 < \frac{R_2}{a_2 \rho_s}$$ | Decoding of x_2 fails.

Therefore, the outage occurs if

$$|h_2|^2 < \max\left\{\frac{R_1}{(a_1 - a_2 R_1)\rho_s}, \frac{R_2}{a_2 \rho_s}\right\}.$$

Let b_2 be defined as

$$b_2 = |h_2|^2$$

Let $E\{b_2\} = E\{|h_2|^2\} = \beta_2$

PDF of $b_2 = |h_2|^2$ is given as

$$f_{B_2}(b_2) = \frac{1}{\beta_2} e^{-\frac{b_2}{\beta_2}}$$

The probability of outage for decoding at user 2 is evaluated as

$$\int_0^{\max\left\{\frac{R_1}{(a_1-a_2R_1)\rho_s}, \frac{R_2}{a_2\rho_s}\right\}} f_{B_2}(b_2)\, db_2$$

$$= 1 - \exp\left(-\frac{1}{\beta_2}\max\left\{\frac{R_1}{(a_1-a_2R_1)\rho_s}, \frac{R_2}{a_2\rho_s}\right\}\right).$$

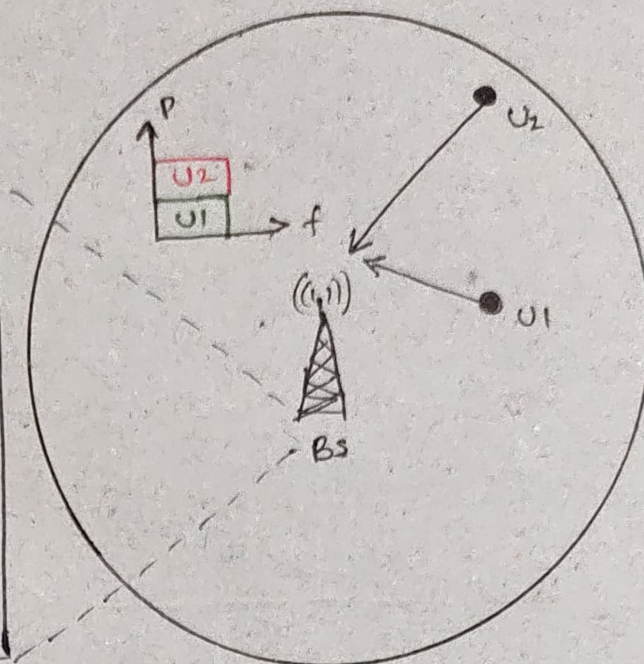
NOMA Uplink (UL)

Signal Decoding at BS

Decoding of U1's signal while treating U2's signal as Noise

$\downarrow$ SIC

Decoding of U2's signal



- BS receives the <u>superposed</u> signal comprising of symbols of both users

$$y_{BS} = h_1\sqrt{a_1\rho_s}\,x_1 + h_2\sqrt{a_2\rho_s}\,x_2 + n_{BS}.$$

- BS decodes x_1 followed by x_2 using SIC.

$$\gamma_{BS}^{x_1} = \frac{a_1\rho_s|h_1|^2}{1+a_2\rho_s|h_2|^2} \qquad \Big| \quad \text{SINR} = \frac{\text{Signal Power}}{\text{Noise} + \text{Interference}}$$

What are the received uplink signal and $\gamma_{BS}^{x_2}$ for a 3 user scenario?

$$y_{BS} = h_1\sqrt{a_1\rho_s}\,x_1 + h_2\sqrt{a_2\rho_s}\,x_2 + h_3\sqrt{a_3\rho_3}\,x_3 + n_{BS}$$

$$\gamma_{BS}^{x_2} = \frac{a_2\rho_s|h_2|^2}{1+a_3\rho_s|h_3|^2}$$

⊙ Probability of Outage - U1

The outage condition for user 1 can be simplified as

$$\log_2\left(1+\gamma_{BS}^{x_1}\right) < \tilde{R}_1$$

$$\Rightarrow \quad 1+\gamma_{BS}^{x_1} < 2^{\tilde{R}_1}$$

$$\Rightarrow \quad \gamma_{BS}^{x_1} < 2^{\tilde{R}_1} - 1$$

$$\Rightarrow \quad \frac{a_1 \rho_s |h_1|^2}{1 + a_2 \rho_s |h_2|^2} < R_1$$

depends on h_2

$$\Rightarrow \quad |h_1|^2 < \frac{R_1\left(1 + a_2 \rho_s |h_2|^2\right)}{a_1 \rho_s}$$

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compute $\quad P_r\left(|h_1|^2 = b_1 < \frac{R_1\left(1+a_2\rho_s |h_2|^2\right)}{a_1 \rho_s}\right)$

$$P_r\left(b_1 < \frac{R_1(1+a_2\rho_s b_2)}{a_1\rho_s}\right)$$

$$= \int_0^{\frac{R_1(1+a_2\rho_s b_2)}{a_1\rho_s}} \frac{1}{\beta_1} e^{-\frac{b_1}{\beta_1}} \, db_1$$

$$= 1 - e^{-\frac{R_1(1+a_2\rho_s b_2)}{\beta_1 a_1 \rho_s}}$$

$$= F_{B1}\left(\frac{R_1(1+a_2\rho_s b_2)}{a_1\rho_s}\right)$$

The Probability of outage for user 1 can be evaluated as follows :

Note that,

$$F_{B1}(x) = P_r(B1 \leq x)$$

$$= \int_0^x f_{B1}(b1)\, db_1 = \int_0^x \frac{1}{\beta_1} e^{-\frac{b_1}{\beta_1}}\, db_1$$

$$= 1 - e^{-\frac{x}{\beta_1}}$$

Probability of outage for user 1 is

$$\int_0^\infty F_{B1}\left(\frac{R_1(1+a_2 \rho_s b_2)}{a_1 \rho_s}\right) f_{B_2}(b_2)\, db_2$$

$$= \int_0^\infty \left(1 - e^{-\frac{R_1(1+a_2\rho_s b_2)}{a_1 \rho_s \beta_1}}\right) \frac{1}{\beta_2} e^{-\frac{b_2}{\beta_2}}\, db_2$$

On further simplification,

$$= 1 - \frac{e^{-\frac{R_1}{a_1 \rho_s \beta_1}}}{1 + \frac{R_1 a_2 \beta_2}{a_1 \beta_1}}$$

⊙ Probability of Outage - U_2

Resulting signal after decoding and removing signal of x_1 is

$$y_2 = h_2 \sqrt{a_2 \rho_s}\, x_2 + n_{BS}$$

SNR for user 2 is

$$\gamma_{BS}^{x_2} = a_2 \rho_s |h_2|^2$$

Outage does not occur for user 2 when

$$\underbrace{\log_2\left(1+\gamma_{BS}^{x_1}\right)}_{\text{Max. Rate of error free decoding of } x_1} \geq \tilde{R}_1 , \quad \underbrace{\log_2\left(1+\gamma_{BS}^{x_2}\right)}_{\text{Max. Rate of error free decoding of } x_2} \geq \tilde{R}_2$$

$$\Rightarrow \frac{a_1 \rho_s |h_1|^2}{1+a_2 \rho_s |h_2|^2} \geq R_1 , \quad a_2 \rho_s |h_2|^2 \geq R_2$$

$$\Rightarrow |h_1|^2 \geq \frac{R_1\left(1+a_2 \rho_s |h_2|^2\right)}{a_1 \rho_s} , \quad |h_2|^2 \geq \frac{R_2}{a_2 \rho_s}$$

Therefore, probability of outage for user 2 is

$$P'_{out} = 1 - P_r\left\{ b_1 \geq \frac{R_1(1+a_2\rho_s b_2)}{a_1 \rho_s} , \; b_2 \geq \frac{R_2}{a_2 \rho_s} \right\}$$

$$= 1 - \int_{\frac{R_2}{a_2\rho_s}}^{\infty} \bar{F}_{B1}\left(\frac{R_1(1+a_2\rho_s b_2)}{a_1\rho_s}\right) f_{B2}(b_2)\, db_2$$

This can be further simplified as

$$1 - \int_{\frac{R_2}{a_2\rho_s}}^{\infty} e^{-\frac{R_1(1+a_2\rho_s b_2)}{a_1\rho_s \beta_1}} f_{B_2}(b_2)\, db_2$$

$$= 1 - \frac{e^{-\left(\frac{R_1}{a_1\rho_s\beta_1} + \frac{R_2}{a_2\rho_s\beta_1} + \frac{R_1 R_2}{a_1\rho_s\beta_1}\right)}}{1 + \frac{R_1 a_2 \beta_2}{a_1\beta_1}}$$