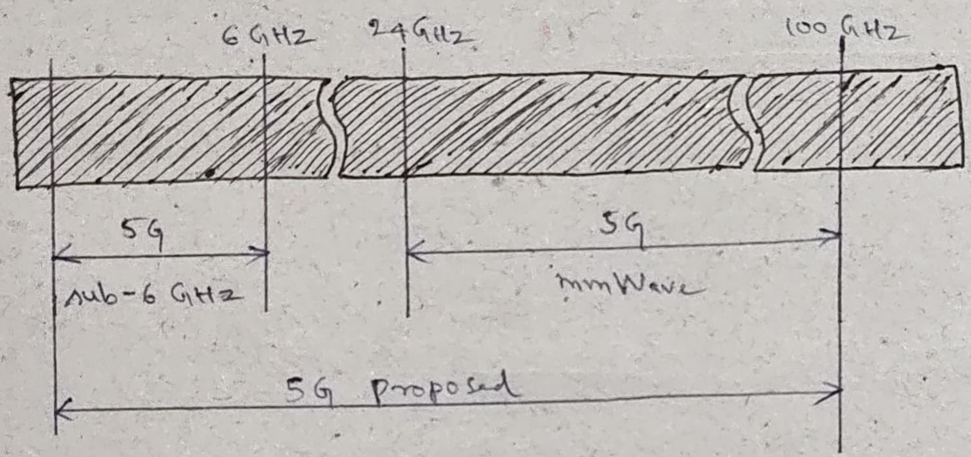
Projects 9, 10

mmWave MIMO technology and architecture

mmWave Communication

- mmWave communication operates in the 30 GHz to 300 GHz band, where large spectral bands are available.
- Large bandwidths (~ 2GHz) supports ultra-high data rates.
- Game changer for 5G !



Eg. 1

Evaluate the carrier frequencies for wavelengths 1mm and 10mm.

$$f_c = \frac{c}{\lambda} = \frac{3 \times 10^8}{1 \times 10^{-3}} = 3 \times 10^{11} = 300\ \text{GHz}$$

$$f_c = \frac{c}{\lambda} = \frac{3 \times 10^8}{10 \times 10^{-3}} = 3 \times 10^{10} = 30\ \text{GHz}$$

mm Wave propagation :

Frii's Law :

Transmit power P_t and receive power P_r, are related as

$$P_r = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \cdot P_t$$

where,

$d \rightarrow$ Tx - Rx distance

$\lambda \rightarrow$ Wavelength

$G_t, G_r \rightarrow$ Transmit, Receive antenna gains

$$P_r = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 P_t \propto \lambda^2$$

- Thus, conventional thinking says :

  Received power decreases as λ^2

- However, this fails to account for the gains.
- The gains G_t, G_r can increase as $\frac{1}{\lambda^2}$, since more antennas can be embedded into the same device.

$$\Rightarrow P_r = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 P_t \propto \frac{1}{\lambda^2} \times \frac{1}{\lambda^2} \times \lambda^2 = \frac{1}{\lambda^2}.$$

Thus, Received power can actually increase as $\frac{1}{\lambda^2}$, using large antenna arrays !

Eg.2

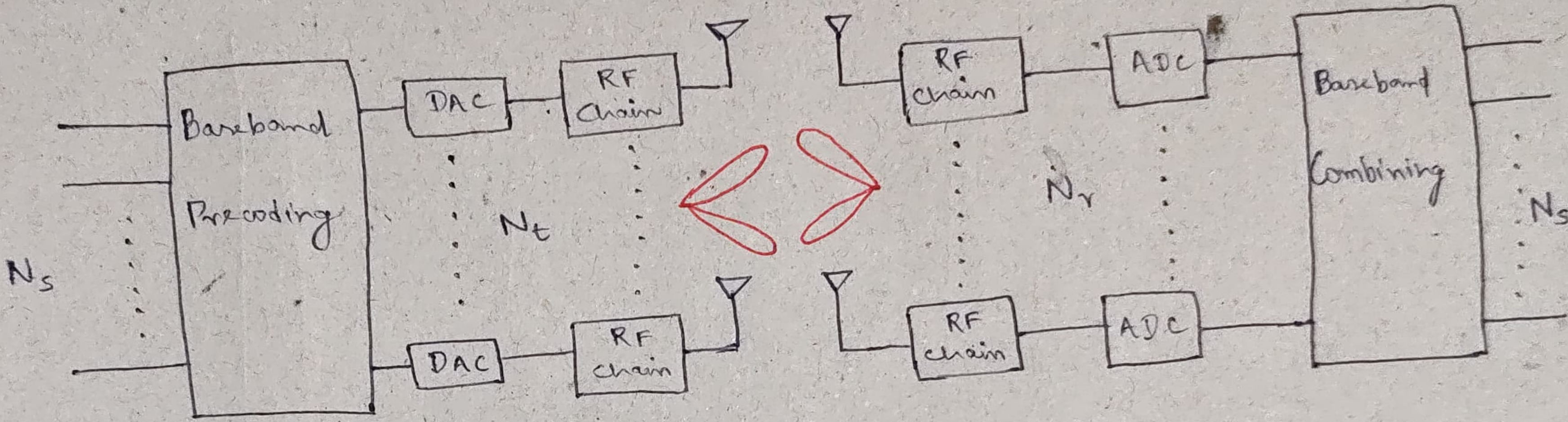
Let the carrier frequency f_c be increased by a factor of 10,

Using large antenna arrays, received power increases by a factor of 100.

mm Wave MIMO architectures

Sub-6 GHz MIMO

Carrier frequencies < 6 GHz.



- RF chain = Filter + Mixer + Amplifier + ADC/DAC
- For Passband to Baseband / Baseband to Passband conversion, RF chain is required.
- In Sub-6 GHz, all the signal processing is in baseband. One can employ only digital signal processing (DSP).
- This requires a separate RF chain and ADC for each antenna

mm MIMO architecture

In mmWave, not possible to have separate RF chain for each antenna. Why?

- Large number of antennas
  $\Rightarrow$ Large number of RF components + ADCs
- High bandwidth
  $\Rightarrow$ High sampling rate
  $\Rightarrow$ Very high power consumption!

Eg. 3 Consider a signal with maximum frequency $f_m = 500$ MHz. What is the minimum sampling rate required to avoid spectral distortion?

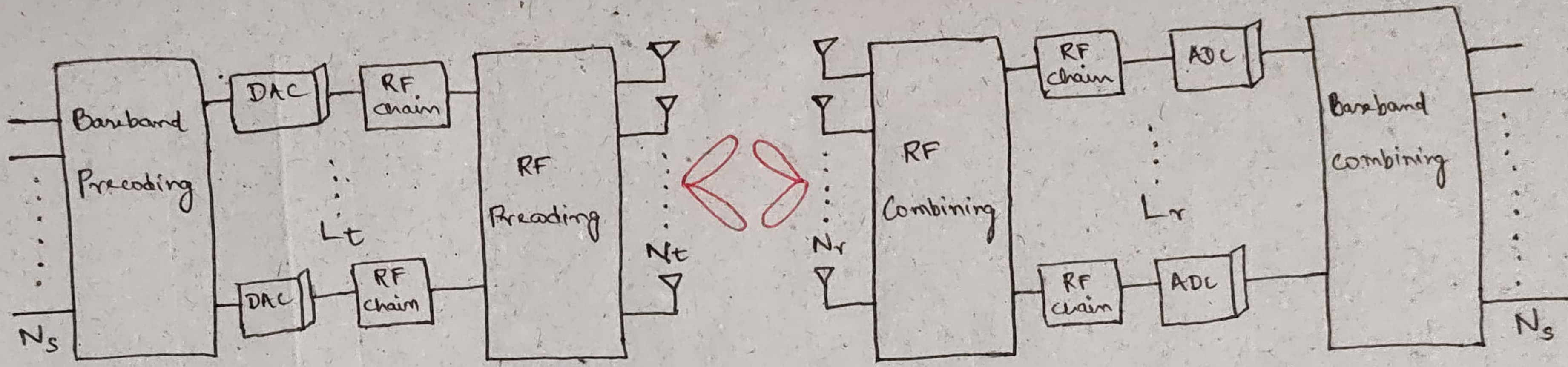
$$f_s = 2 f_m = 2 \times 500 \text{ MHz}$$
$$= 1 \text{ GHz}.$$

- Therefore, new architecture is required for mmWave MIMO.
- Signal processing is done in a mix of analog and digital domains.

This is termed as

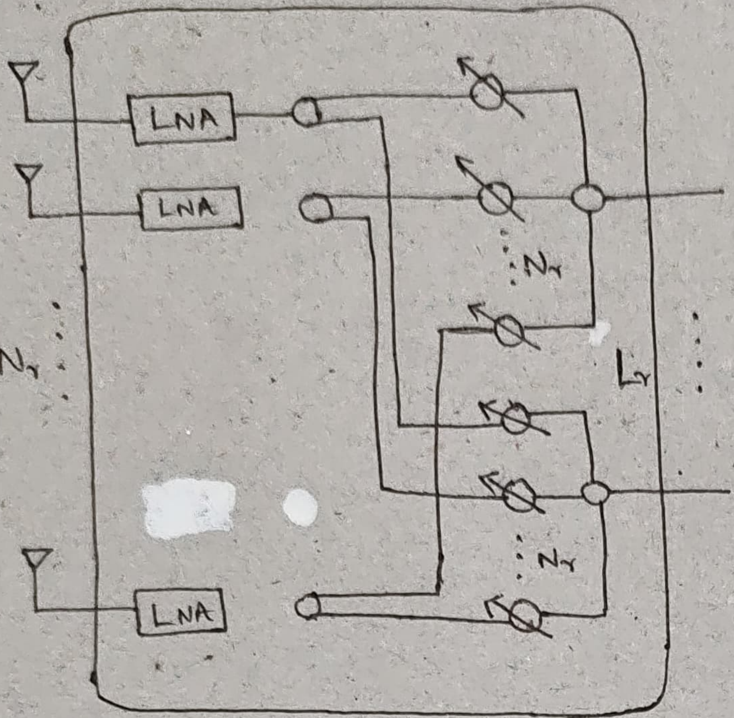
"Hybrid Analog-Digital Signal Processing".

Hybrid Processing



- Signal processing is divided between RF (analog) and Digital (baseband) domains. Hence this is hybrid analog - digital processing (hybrid RF - BB).

Hybrid architecture



- Each Antenna can connect to each RF Chain, through a phase shifter ($e^{j\phi}$) in the fully connected architecture.

Eg. 4

Given a 256 × 256 mmMIMO system with hybrid beamforming and 64 RF chains.

No. of phase shifters required in the fully connected architecture is

$$2 \times \underbrace{256 \times 64}_{} = 32,768$$

256 × 64 in the Tx side

256 × 64 in the Rx side

Hybrid Transceiver Design for mmWave MIMO

MIMO channel :

Consider the conventional MIMO system

$$\bar{x} = F\bar{s}$$

where, $F \rightarrow$ Precoder Matrix.

$$\tilde{y} = H\bar{x} + \bar{n}$$
$$= HF\bar{s} + \bar{n}$$

where,

$\bar{s} \rightarrow$ Symbol vector.

$\bar{x} \rightarrow$ Transmit vector

What is the best precoder matrix F ?

MIMO SVD

The SVD of the MIMO channel H is

$$H = U\Sigma V^H.$$

Let N_s symbols be transmitted.

Partition V as

$$V_{N_T \times N_T} = \left[\bar{V}_{N_T \times N_s} \quad \tilde{V}_{N_T \times (N_T - N_s)} \right]$$

where, $N_T \rightarrow$ No. of Transmit Antennas

$N_s \rightarrow$ No. of Symbols

Optimal Precoder

For the conventional MIMO system, the optimal precoder is

$$F = \bar{V}$$

This is also termed the <u>ideal fully digital precoder.</u>

<u>Eg. 5</u> Consider $N_T = 32$, $N_s = 4$

What is the size of V?

What is the size of the ideal fully digital precoder?

Size of $V = N_T \times N_T = 32 \times 32$

Size of $\tilde{V} = N_T \times (N_T - N_s)$

$= 32 \times (32 - 4) = 32 \times 28$

Size of ideal fully digital precoder $= N_T \times N_s$

$= 32 \times 4$

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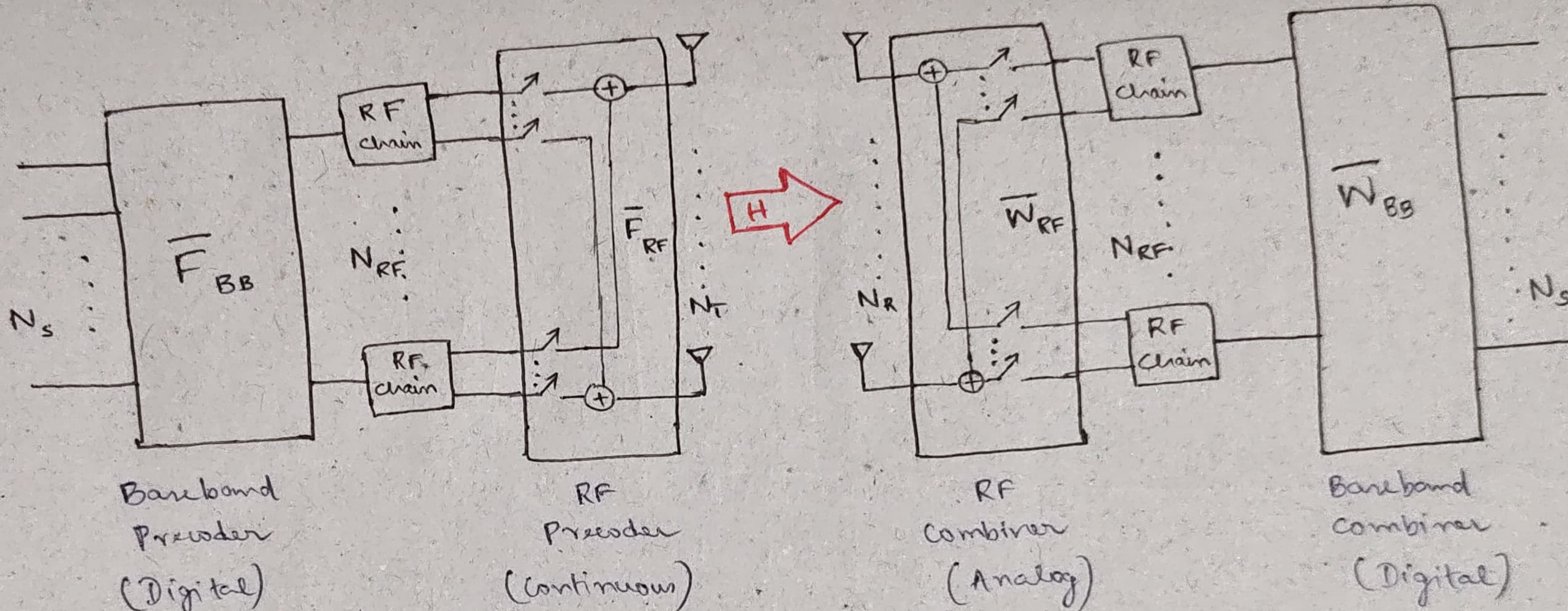
mmWave MIMO - Precoding / Combining



Note: No. of RF chains < No. of Tx / Rx antennas.

- mmWave MIMO precoding model

$$\tilde{y} = H\bar{x} + \bar{n}$$
$$= H F_{RF} F_{BB} \bar{s} + \bar{n},$$
$$= H F \bar{s} + \bar{n}$$

The sizes of the precoding matrices are

- RF Precoder Matrix,

$$F_{RF} \sim N_T \times N_{RF}$$

- Baseband Precoder Matrix,

$$F_{BB} \sim N_{RF} \times N_S$$

Eg. 6

Consider $N_T = 32$, $N_{RF} = 8$, $N_S = 4$

What are the sizes of the precoding matrices?

$$F_{RF} = N_T \times N_{RF} = 32 \times 8 = 256$$
$$F_{BB} = N_{RF} \times N_S = 8 \times 4 = 32$$

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mmWave Precoder

Ideally, we desire

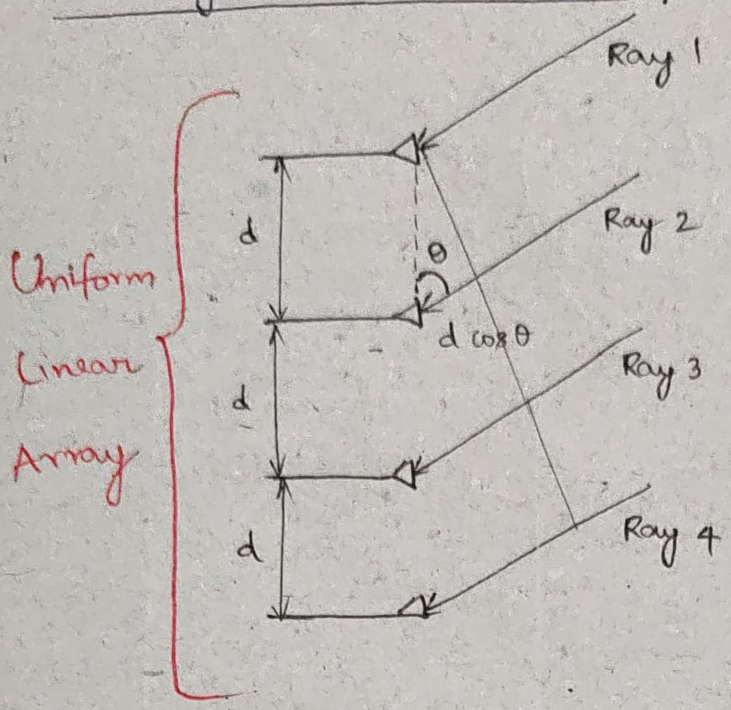
$$\bar{V} = F_{RF} F_{BB}$$

$\bar{V} \rightarrow$ Ideal Precoder

Also, entries of F_{RF} have to be unit-magnitude phase-shifters!

$$e^{j\phi}$$

Array response vector



- Consider signal at antenna 1 to be $e^{j2\pi f_c t}$

- Signal at antenna 2 is

$$e^{j2\pi f_c \left(t - \underbrace{\frac{d\cos\theta}{c}}_{\text{delay}}\right)} = e^{j2\pi f_c t} \cdot e^{-j\frac{2\pi}{\lambda} d\cos\theta}$$

- Signal at antenna 3 is

$$e^{j2\pi f_c \left(t - \underbrace{\frac{2d\cos\theta}{c}}_{\text{delay}}\right)} = e^{j2\pi f_c t} \cdot e^{-j\frac{4\pi}{\lambda} d\cos\theta}$$

- Let

$\theta_r \rightarrow$ Angle of Arrival @ Receiver

$\theta_t \rightarrow$ Angle of Departure @ Transmitter

⊙ Response of array to ray arriving / departure at θ

Array Manifold vectors/
Array Signature vectors/
Array Response vectors.

At Receiver, $\bar{a}_R(\theta^r) = \frac{1}{\sqrt{N_R}} \begin{bmatrix} 1 \\ e^{-j\frac{2\pi}{\lambda} d \cos\theta^r} \\ e^{-j\frac{4\pi}{\lambda} d \cos\theta^r} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}(N_R-1) d \cos\theta^r} \end{bmatrix}$

At Transmitter, $\bar{a}_T(\theta^t) = \frac{1}{\sqrt{N_T}} \begin{bmatrix} 1 \\ e^{-j\frac{2\pi}{\lambda} d \cos\theta^t} \\ e^{-j\frac{4\pi}{\lambda} d \cos\theta^t} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}(N_T-1) d \cos\theta^t} \end{bmatrix}$

where, $\frac{1}{\sqrt{N_R}}$, $\frac{1}{\sqrt{N_T}}$ are Normalizing factor.

Precoder Optimization

- Ideally, $\bar{a}_T(\theta^t)$ can be made the columns of RF precoder, because its entries are Unit magnitude.

- Since AoD's are unknown, one can use the dictionary matrix.

$$A_T = \underbrace{\left[\bar{a}_T(\theta_1) \quad \bar{a}_T(\theta_2) \quad \cdots \quad \bar{a}_T(\theta_G)\right]}_{\text{All possible angles of departure (or) Code book}}$$

- Further, one can set

$$\bar{V} = F_{RF} F_{BB} = A_T \bar{F}$$

where,

$\bar{V} \rightarrow$ Ideal Precoder

$A_T \rightarrow$ Dictionary Matrix / codebook.

- F_{RF} Contains only N_{RF} columns
- $\bar{F}$ can only contain N_{RF} non-zero rows.

(ie) Max. No. of non-zero rows $= N_{RF}$

- What kind of matrix is $\bar{F}$?

It is simultaneous Sparse Matrix!

- The optimization problem for precoder design is

$$\arg\min \left\| \bar{V} - A_T \tilde{F}_{BB} \right\|_F^2$$

$$\text{s.t.}, \left\| \text{diag}\left(\tilde{F}_{BB} \tilde{F}_{BB}^H \right) \right\|_0 = N_{RF}$$

This can be solved via <u>Simultaneous Orthogonal Matching Pursuit</u> (SOMP).

- Initialize F_{RF} as the NULL matrix

$$F_{RF}^{(0)} = [\,]$$

- Set <u>initial residue</u> = ideal precoder

$$F_{res}^{(0)} = \bar{V}$$

- In iteration k, find column of A_T that has <u>maximum correlation</u> with $F_{res}^{(k-1)}$

$$\Psi = \underbrace{A_T^H}_{\text{dictionary}} \cdot \underbrace{F_{res}^{(k-1)}}_{\text{residue}}$$

$$i(k) = \arg\max \left[\Psi \Psi^H \right]_{l,l}$$

- Augment the RF precoder with the $i(k)^{th}$ column of A_T.

$$F_{RF}^{(k)} = \left[F_{RF}^{(k-1)} \middle| \bar{a}_T\left(\theta_{i(k)}\right) \right]$$

- Find the best approximation to the ideal baseband precoder $F_{BB}^{(k)}$

$$F_{BB}^{(k)} = \underbrace{\left(\left(F_{RF}^{(k)}\right)^H F_{RF}^{(k)} \right)^{-1} \left(F_{RF}^{(k)}\right)^H}_{\text{Pseudo-inverse}} \bar{V}$$

- Update residue $F_{res}^{(k)}$ for iteration k as

$$F_{res}^{(k+1)} = \frac{\bar{V} - F_{RF}^{(k)} F_{BB}^{(k)}}{\left\| \bar{V} - F_{RF}^{(k)} F_{BB}^{(k)} \right\|_F}$$

- Choose N_{RF} columns.
- $F_{RF}^{(0)} = [\,]$, $F_{res}^{(0)} = \bar{V}$
- For $1 \leq k \leq N_{RF}$

$$\Psi = A_T^H F_{res}^{(k-1)}$$

$$i(k) = \arg\max \left[\Psi \Psi^H \right]_{l,l}$$

$$F_{RF}^{(k)} = \left[F_{RF}^{(k-1)} \,\middle|\, \bar{a}_T(\theta_{i(k)}) \right]$$

$$F_{BB}^{(k)} = \left(\left(F_{RF}^{(k)}\right)^H F_{RF}^{(k)} \right)^{-1} \left(F_{RF}^{(k)}\right)^H \bar{V}$$

$$F_{res}^{(k)} = \frac{\bar{V} - F_{RF}^{(k)} F_{BB}^{(k)}}{\left\| \bar{V} - F_{RF}^{(k)} F_{BB}^{(k)} \right\|_F}$$

end for

SOMP Algorithm

Combiner Optimization

- The optimal LMMSE combiner can be computed in a similar fashion.
- The optimal LMMSE combiner is

$$\overline{W}_M^H = F_{BB}^H F_{RF}^H H^H \times \left(H F_{RF} F_{BB} F_{BB}^H F_{RF}^H H^H + N_s \sigma^2 I\right)^{-1}$$

- It can be shown that the MSE minimization is equivalent to

$$\min \left\| R_{yy}^{1/2} \left(\overline{W}_M - W_{RF} W_{BB}\right)\right\|_F^2$$

$$R_{yy} = H F_{RF} F_{BB} F_{BB}^H F_{RF}^H H^H + N_s \sigma^2 I$$

$$= R_{yy}^{1/2} R_{yy}^{H/2}$$

- W_{RF} can be formed from the receive array response vectors $\overline{a}_R(\theta_\ell^r)$. However, these are unknown!
- Start with the dictionary matrix

$$A_R = \left[\overline{a}_R(\theta_1) \;\; \overline{a}_R(\theta_2) \;\; \cdots \;\; \overline{a}_R(\theta_G)\right]$$

- The optimization problem for receiver design can be formulated as

$$\min \left\| R_{yy}^{1/2} \left(\overline{W}_M - A_R \tilde{W}_{BB} \right) \right\|_F^2$$

- SOMP can once again be used to design $\tilde{W}_{BB}$
- $W_{RF}^{(0)} = [\,]$, $W_{res}^{(0)} = \overline{W}_M$
- For $1 \leq k \leq N_{RF}$

$$\Psi = A_R^H R_{yy} W_{res}^{(k-1)}$$

$$i(k) = \arg\max \left[\Psi \Psi^H \right]_{\ell,\ell}$$

$$W_{RF}^{(k)} = \left[W_{RF}^{(k-1)} \,\middle|\, \bar{a}_R(\theta_{i(k)}) \right]$$

$$W_{BB}^{(k)} = \left(\left(W_{RF}^{(k)} \right)^H R_{yy} W_{RF}^{(k)} \right)^{-1} \left(W_{RF}^{(k)} \right)^H R_{yy} \overline{W}_M$$

$$W_{res}^{(k)} = \frac{\overline{W}_M - W_{RF}^{(k)} W_{BB}^{(k)}}{\left\| \overline{W}_M - W_{RF}^{(k)} W_{BB}^{(k)} \right\|_F}$$

end for.