Week 8 : Session 1

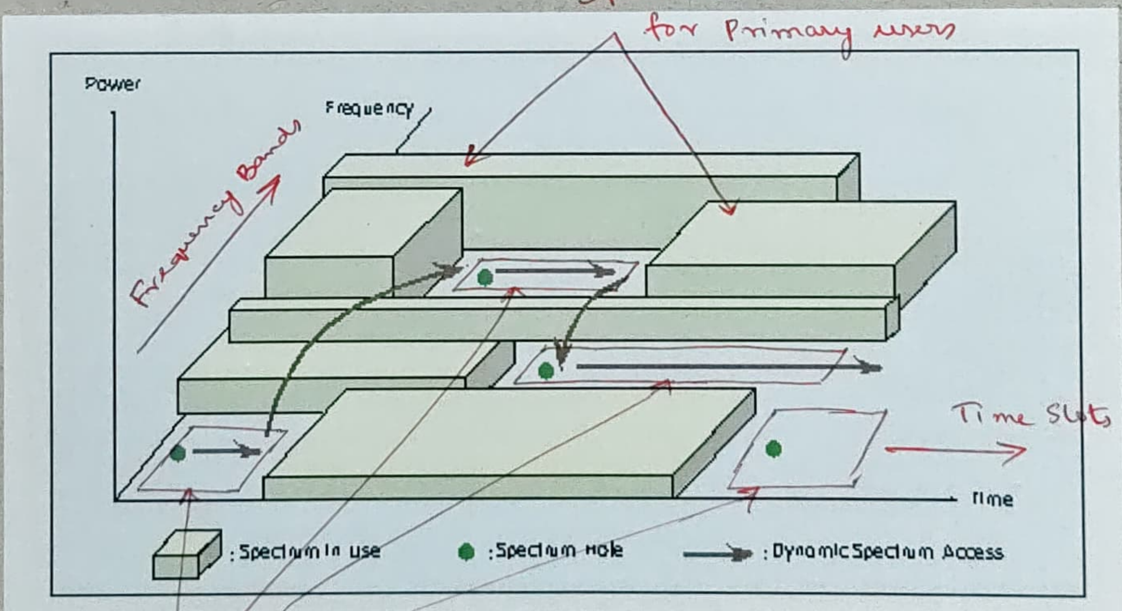
Cognitive Radio – Spectrum Sensing.

In this module, let us start looking at the application of principles of detection / Detection theory, that we've learnt so far. The Cognitive Radio (5G/6G Technology) more specifically use the detection, to sense the spectrum, whether it is currently occupied or vacant.

Traditional spectrum allocation is STATIC and INFLEXIBLE (i.e. the spectrum is allocated to a certain set of users called Licensed users, irrespective of whether they are using it or not). So, whenever they are using it, the spectrum is occupied, otherwise the spectrum is vacant. This leads to SPECTRAL HOLES.

The static spectrum allocation leads to Spectral Holes. This is because, the Radio spectrum allocated to <u>Licensed users/ Primary users</u> cannot be utilized by <u>Unlicensed users/ Secondary users</u>, even when it is Underutilized or Vacant!

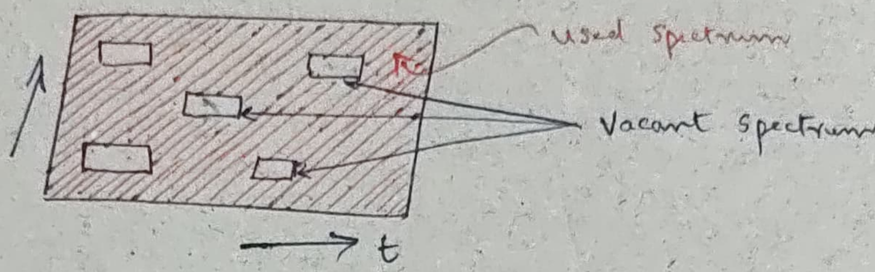
Spectrum in use
for Primary users



vacant (Spectral Holes)

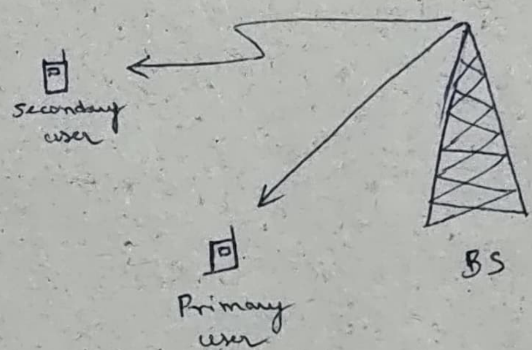
Secondary users can dynamically access the spectrum Holes.

The main goal of Cognitive Radio is to enable Dynamic spectrum Access, by a few selected Unlicensed / secondary users. This significantly improves the efficiency of the Spectrum utilization



$$\text{Efficiency } (\eta) = \frac{\text{Shaded area}}{\text{Total area}}$$

Therefore, SPECTRUM SENSING is a key aspect of Cognitive Radio (CR), to determine the presence / absence of primary users.



The secondary user senses the spectrum, to determine the presence / absence of Primary user. (ie) The secondary user listens / senses the spectrum to determine whether there is ongoing transmission between the BS and Primary user. If there is no ongoing transmission, then the secondary users can access the Spectrum. Hence, SPECTRUM SENSING is Key to enable Dynamic Spectrum Access, because if the spectrum is not awakened (ie if the spectral Hole is not detected), the spectrum cannot be dynamically accessed.

Only if the Spectral Hole is detected successfully, the secondary users can access the Spectrum. Hence this is a classical Detection Problem. So, the principles of detection / Detection theory plays a very important role in Cognitive Radio (Spectrum sensing).

Now, let us consider the Spectrum sensing problem, where $y(1), y(2), \ldots, y(N)$ denote the Output symbols. The corresponding input-output model is

$$\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} s(1) \\ s(2) \\ \vdots \\ s(N) \end{bmatrix} + \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(N) \end{bmatrix}$$

Random Signal

$$\Rightarrow \quad \bar{y} = \bar{s} + \bar{v}$$

- Let $s(i)$ be Random Complex Gaussian Signal $\left(\mathcal{CN}(0, \sigma_s^2)\right)$

$$s(i) = s_I(i) + j\, s_Q(i)$$

$$s_I(i) \sim \mathcal{N}(0, \sigma_s^2/2) \longrightarrow \text{Inphase component}$$

$$s_Q(i) \sim \mathcal{N}(0, \sigma_s^2/2) \longrightarrow \text{Quadrature component}$$

where, $s_I(i)$ and $s_Q(i)$ are independent identically distributed (iid). Thus, $s(i) = s_I(i) + j\, s_Q(i)$ is Circularly symmetric complex Gaussian.

$$E\{s(i)\} = 0, \quad \underbrace{E\{|s(i)|^2\}}_{\text{Signal Power}} = \sigma_s^2$$

- Similarly, Let $v(i)$ be complex Gaussian Noise $\left(\mathcal{CN}(0, \sigma^2)\right)$

$$v(i) = v_I(i) + j\, v_Q(i)$$

$$v_I(i) \sim \mathcal{N}(0, \sigma^2/2) \longrightarrow \text{Inphase Noise component}$$

$$v_Q(i) \sim \mathcal{N}(0, \sigma^2/2) \longrightarrow \text{Quadrature Noise component}$$

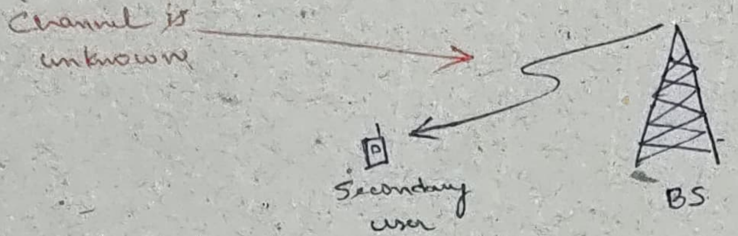
where, $v_I(i)$ and $v_Q(i)$ are iid zero mean Gaussian. Thus, $v(i) = v_I(i) + j\, v_Q(i)$ is Zero mean Circularly symmetric complex Gaussian.

The Binary Hypothesis Testing problem for Spectrum sensing is

Under NULL Hypothesis $H_0 : \bar{y} = \bar{v}$

Under ALTERNATIVE Hypothesis $H_1 : \bar{y} = \bar{s} + \bar{v}$

$$\left.\begin{array}{l} \bar{s} \rightarrow \text{Signal vector} \\ \bar{v} \rightarrow \text{Noise vector} \end{array}\right\} \text{ } \bar{s} \text{ and } \bar{v} \text{ are independent complex Gaussian.}$$



Since the channel is <u>Unknown</u>, one can use the <u>Energy Detector (ED)</u>. Thus, the test statistic is $\|\bar{y}\|^2$.

$$\|\bar{y}\|^2 = \underbrace{|y(1)|^2 + |y(2)|^2 + \dots + |y(N)|^2}_{\text{Energy of Signal}}$$

Now, compare the Test statistic $\|\bar{y}\|^2$ with a suitable threshold γ, to get the Energy Detector (ED).

(i) Choose $H_1 : \|\bar{y}\|^2 > \gamma$.

Choose $H_0 : \|\bar{y}\|^2 \leq \gamma$

⊙ The Probability of False Alarm (P_{FA}) is.
(Signal is Absent, but decision is Signal Present)

$$P_{FA} = P_r\left(\|\bar{y}\|^2 > \gamma \ ; H_0\right)$$

$$y(i) = y_I(i) + j\, y_Q(i)$$
$$= v_I(i) + j\, v_Q(i)$$

$$\underbrace{y_I(i),\ y_Q(i)}_{\text{iid}} \sim \mathcal{N}(0, \sigma^2/2)$$

$$y_I(i) = v_I(i) \sim \mathcal{N}(0, \sigma^2/2)$$

$$y_Q(i) = v_Q(i) \sim \mathcal{N}(0, \sigma^2/2)$$

Thus, $\|\bar{y}\|^2 > \gamma$

$$\Rightarrow \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

$$\Rightarrow \underbrace{\sum_{i=1}^{N} \left|\frac{y_I(i)}{\sigma/\sqrt{2}}\right|^2 + \left|\frac{y_Q(i)}{\sigma/\sqrt{2}}\right|^2}_{\text{Sum of squares of 2N iid Zero mean Standard Gaussian RVs.}} > \frac{\gamma}{\sigma^2/2}$$

This is called as Central chi-squared RV with 2N degrees of freedom. $\left(\chi^2_{2N}\right)$.

$$\Rightarrow P_{FA} = P_r\left(\|\bar{y}\|^2 > \gamma \ ; H_0\right)$$

$$\Rightarrow P_{FA} = P_r\left(\frac{\|\bar{y}\|^2}{\sigma^2/2} > \frac{\gamma}{\sigma^2/2} \ ; H_0\right)$$

$$\Rightarrow \boxed{P_{FA} = Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)}$$

CCDF of chi-squared RV with 2N degrees of freedom.

① The Probability of Detection (P_D) is (Signal is present, And the decision is H_1)

$$P_D = P_r\left(\|\bar{y}\|^2 > \gamma \ ; H_1\right)$$

$$y(i) = y_I(i) + j\, y_Q(i)$$
$$= s_I(i) + j\, s_Q(i) + v_I(i) + j v_Q(i)$$
$$= \left(s_I(i) + v_I(i)\right) + j\left(s_Q(i) + v_Q(i)\right)$$

independent $\begin{cases} s_I(i) \sim \mathcal{N}(0, \sigma_s^2/2) \\ v_I(i) \sim \mathcal{N}(0, \sigma^2/2) \end{cases}$

$$s_I(i) + v_I(i) \sim \mathcal{N}\left(0, \frac{\sigma^2 + \sigma_s^2}{2}\right)$$

Similarly, $s_Q(i) + v_Q(i) \sim \mathcal{N}\left(0, \frac{\sigma^2 + \sigma_s^2}{2}\right)$

Thus, $\underbrace{y_I(i), y_Q(i)}_{\text{Inphase and Quadrature components of signal}} \sim \mathcal{N}\left(0, \frac{\sigma^2 + \sigma_s^2}{2}\right)$

Thus, $\|\bar{y}\|^2 > \gamma$

$$\Rightarrow \sum_{i=1}^{N} |y_I(i)|^2 + |y_Q(i)|^2 > \gamma$$

$$\Rightarrow \underbrace{\sum_{i=1}^{N} \left|\frac{y_I(i) - 0}{\sqrt{\frac{\sigma^2+\sigma_0^2}{2}}}\right|^2 + \left|\frac{y_Q(i) - 0}{\sqrt{\frac{\sigma^2+\sigma_0^2}{2}}}\right|^2}_{\text{Sum of squares of 2N iid zero mean Standard Gaussian RVs}} > \frac{\gamma}{(\sigma^2+\sigma_0^2)/2}$$

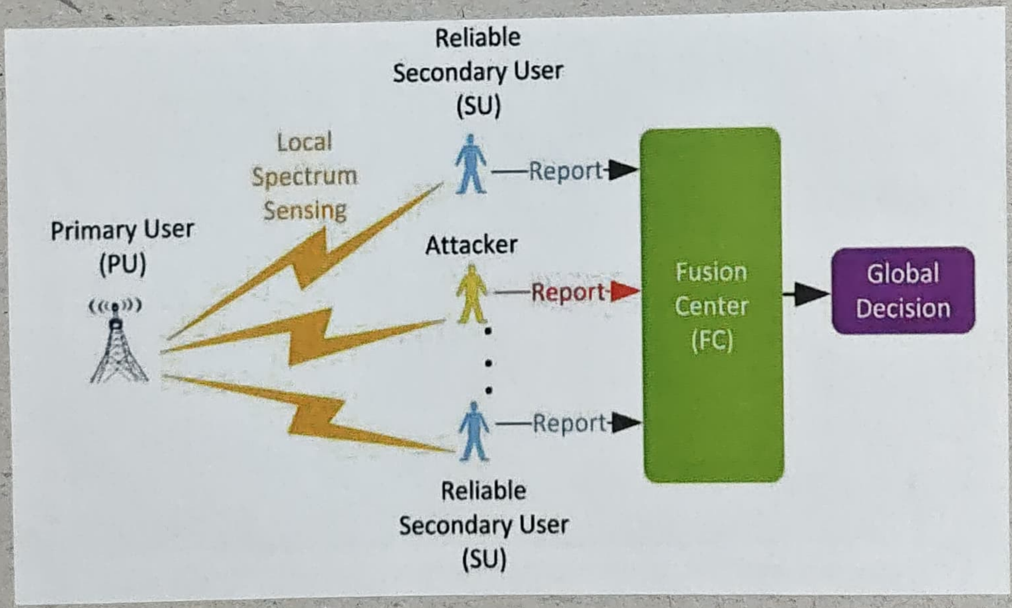
This is called as central chi-squared RV with 2N degrees of freedom $\left(\chi^2_{2N}\right)$

$$\Rightarrow P_D = P_r\left(\frac{\|\bar{y}\|^2}{(\sigma^2+\sigma_0^2)/2} > \frac{\gamma}{(\sigma^2+\sigma_0^2)/2} ; H_1\right)$$

$$\Rightarrow \boxed{P_D = Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma^2+\sigma_0^2)/2}\right)}$$

Week 8 : Session 2

Cooperative Spectrum sensing:



In general, there exists multiple secondary users. Each secondary user senses the spectrum, make a decision and report to the Fusion Center (FC). The FC makes decision based on all the decisions.

(i) In scenarios with multiple secondary users, each can communicate / Report the sensing decision to the Fusion Center. The FC can subsequently make a final decision based on an appropriate FUSION RULE.

AND Fusion Rule

Fusion center (FC) decides H_1 : only if all sensors report H_1

H_0 : otherwise.

(i) Decision of FC = 1 , only if all sensors report 1.

0 , if one or more sensors report 0.

(i) FC performs 'AND' of all decisions.

This is known as optimistic rule.

Let No. of sensors = K.

Let P_D and P_{FA} denote the probabilities of detection and false alarm for each sensor.

- P_{FA} at Fusion center $\left(P_{FA}^{FC}\right)$
  - All sensors falsely report the presence of Primary User
    (i) All sensors report 1.

$$P_{FA}^{FC} = \underbrace{P_{FA} \times P_{FA} \times \ldots \times P_{FA}}_{K \text{ Times}}$$

$$\Rightarrow \boxed{P_{FA}^{FC} = \left(P_{FA}\right)^K = \left(Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)\right)^K}$$

- P_D at Fusion center $\left(P_D^{FC}\right)$
  - Each sensor reports the presence of Primary user

$$P_D^{FC} = \underbrace{P_D \times P_D \times \ldots \times P_D}_{K \text{ Times}}$$

$$\Rightarrow \boxed{P_D^{FC} = \left(P_D\right)^K = \left(Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma_s^2+\sigma^2)/2}\right)\right)^K}$$

OR Fusion Rule

Fusion Center (FC) decides H_1 : if atleast one of the sensors report H_0.

H_0 : otherwise.

(i) Decision of FC $= 1$, if any sensor report 1
$= 0$, if all sensors report 0 :

(ii) FC performs 'OR' of all sensor decision.
This is known as Conservative Rule.

- P_{FA} at Fusion center (P_{FA}^{FC}).

P_{FA}^{FC} = Probability atleast one sensor falsely detects
= 1 − Probability None falsely detects

$$\boxed{P_{FA}^{FC} = 1 - (1 - P_{FA})^K = 1 - \left(1 - Q_{\chi^2_{2N}}\left(\frac{\gamma}{\sigma^2/2}\right)\right)^K}$$

- P_D at Fusion center (P_D^{FC})

P_D^{FC} = Probability atleast one sensor detects
= 1 − Probability None detects

$$\boxed{P_D^{FC} = 1 - (1 - P_D)^K = 1 - \left(1 - Q_{\chi^2_{2N}}\left(\frac{\gamma}{(\sigma^2+\sigma_0^2)/2}\right)\right)^K}$$

Generalized Fusion Rule

Fusion center (FC) decides H_1 : Only if atleast L sensors or more report H_1

H_0 : Otherwise

- The False Alarm probability is given as

$$\boxed{P_{FA}^{FC} = \sum_{k=L}^{K} {}^K C_k \, (P_{FA})^k \, (1 - P_{FA})^{K-k}}$$

$\frac{K!}{k!\,(K-k)!}$

(k out of K sensors can be chosen as ${}^K C_k$)

Any set of k sensors report False alarm.

- The Probability of detection is given as

$$\boxed{P_D^{FC} = \sum_{k=L}^{K} {}^{K}C_{k} \, (P_D)^{k} \, (1 - P_D)^{K-k}}$$

Number of combinations of k sensors from total of K sensors

k sensors correctly detect
$K-k$ sensors fail to detect

Binomial Probability Distribution