What is this module about?

→ To introduce the cutting-edge Wireless technologies

- ⊙ Multiple-antenna system
- ⊙ MIMO (Multiple Input Multiple Output) technology
- ⊙ OFDM (Orthogonal Frequency Division Multiplexing) System

These form the basis of modern Wireless Systems.

- Cellular (LTE, 5G-NR)
- Wi-Fi (802.11ac, 802.11ax)

→ To understand the basic principles and models of wireless systems.

→ To use a lot of mathematical tools and analysis

BASIC DIGITAL COMMUNICATION

Why is conventional wireline communication different from wireless?

Reason: Channel is FIXED!!

(ie) Wired medium
Propagation channel
Propagation path
Propagation distance
Nature of the medium
Delay
Gain of the medium
Attenuation of the medium

All these properties of the wireline channel are fixed.

Whereas, in wireless communication, the channel is DYNAMIC. (ie) the channel changes w.r.t time.

To understand the implications of Channel being FIXED, we have to model the wireline digital system.

Communication system has 4 components.

1. Received signal y
2. Transmit signal x
3. Noise n
4. Channel h ✗ Coz, Channel is fixed for Wireline Communication System

$\boxed{\text{SNR - Signal to Noise Power Ratio}}$

SNR - Signal to Noise Power Ratio
- Very very IMPORTANT quantity in communication which will be referred to frequently.

A simple model for communication system,

$$y = x + \underbrace{n}_{}$$

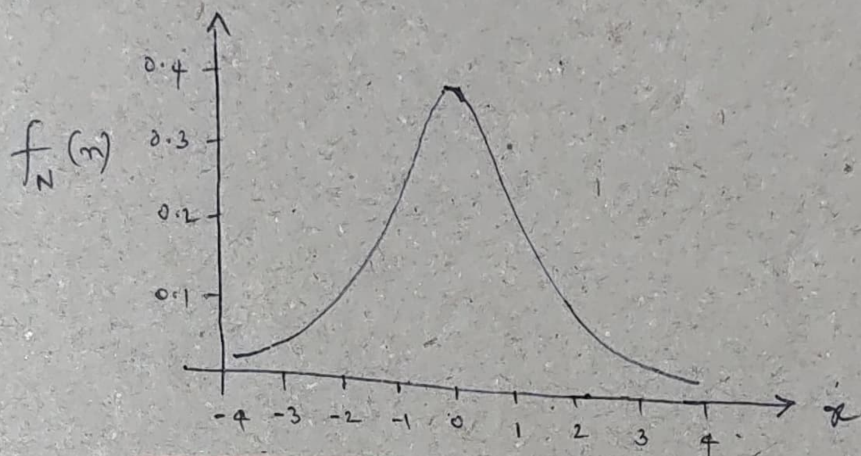
← Additive Noise

SIGNAL

- Signal Power is defined as $E\{|x|^2\} = P$
  where $E\{\cdot\}$ denotes the expected value or average
- NOT any signal is permitted.
- Communication signals has to have a specific structure, to convey maximum information (bits)
- This is termed as MODULATION.

NOISE

- The noise n is termed as AWGN. (Additive White Gaussian Noise)
- Noise is additive
- Noise is Gaussian (Probability density function, PDF of noise is Gaussian in nature)



$$\boxed{f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\mu)^2}{2\sigma^2}}}$$

where, $\mu = \text{Mean} = E\{N\}$

$\sigma^2 = \text{Variance} = E\{(N-\mu)^2\}$

Variance tells, how far the spread would range from the Mean.
Larger the variance, Larger would be the noise power

Typically, $\mu = 0$ for Noise.

$$\Rightarrow f_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

where, $\sigma^2 = \text{Variance} = E\{|N|^2\}$

⊙ Noise is White (Power Spectral Density, PSD of noise is flat or constant across the frequency spectrum)

$$S_{nn}(\Omega) = \frac{N_o}{2} = \text{Constant}$$

⊙ similar to white light!

⊙ contains all frequencies.

⊙ How is PSD defined?

→ Fourier transform of auto-correlation

$$R_{nn}(l) = E\{n(k)\, n(k+l)\} = \underbrace{\frac{N_o}{2}\,\delta(l)}_{\text{Impulse}}$$

2 Noise Samples

l - Lag

Noise samples are distinct. Hence they're mostly uncorrelated.

→ FT of an Impulse is flat across the entire frequency spectrum.

SNR

⊙ SNR is the ratio of Signal Power to Noise Power.

$$y = x + n$$

$$SNR = \frac{E\{|x|^2\}}{E\{|n|^2\}} = \frac{P}{N_o/2} = \frac{2P}{N_o}$$

→ SNR of the wireline channel is approximately constant, because the channel is FIXED. Therefore, no variation or fluctuations in SNR.

→ Whereas, SNR of the wireless channel is not constant. It is infact Random, as the channel is Random. Therefore, there exists variations or fluctuations in SNR.

Performance of Communication Systems

- In order to characterize the performance of a communication system, **BER** is used.
- What is BER (Bit-Error Rate) ?

  Probability that a single bit is in error.

  Eg BER $= 10^{-2} \Rightarrow \frac{1}{100} \Rightarrow 1\%$

  (ie) Approx. 1% of bits are in error

  1 bit in every 100 bits
(or)
10 bits in every 1000 bits } on an average are in error.

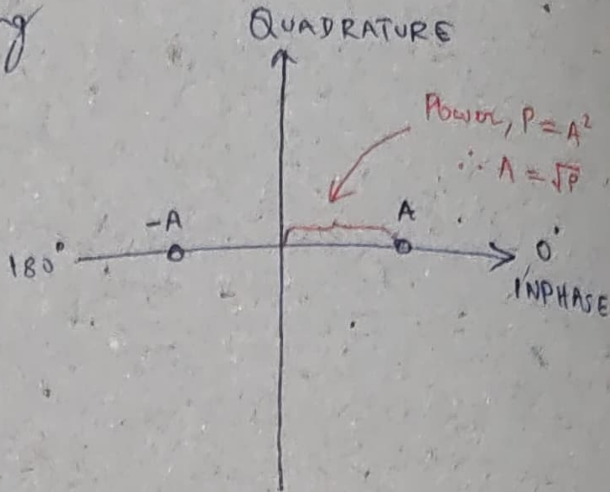
DIGITAL MODULATION

- Mapping of information bits to signals that can be transmitted over the channel
- Various formats of Digital Modulation
  - BPSK, QPSK, QAM

BPSK

- Binary Phase Shift Keying
- $x \in \{+A, -A\}$
- Two phases : 0° and 180°
- Signal constellation :

  $\{+A, -A\}$



- Bit Mapping :

  $0 \longrightarrow +A$

  $1 \longrightarrow -A$

$M = 2$ Symbols

No. of bits/symbol $= \log_2 M$

$= \log_2 2$

$= 1$ bit/Sym

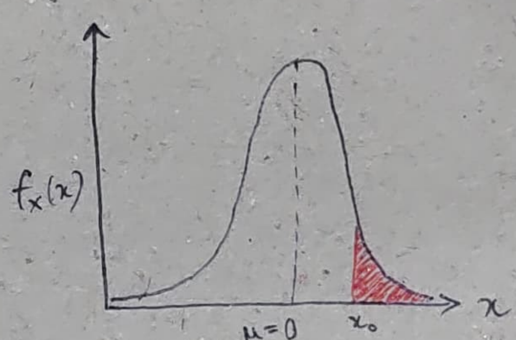
- $SNR = \dfrac{P}{N_0/2} = \dfrac{2P}{N_0}$
- $BER = Q(\sqrt{SNR}) = Q\left(\sqrt{\dfrac{2P}{N_0}}\right)$

Gaussian Q - function

→ $Q(\cdot)$ is the Complementary Cumulative Distribution Function (CCDF) of the Standard Gaussian RV.

→ Standard Gaussian RV : (Mean, $\mu = 0$; Variance, $\sigma^2 = 1$)

$$\Rightarrow f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



→ WKT.

$CDF = F_X(x) = P(X \le x)$

$CCDF = \bar{F}_X(x) = 1 - CDF$

$= P(X > x)$

$PDF = f_X(x) = \dfrac{d}{dx} F_X(x)$

$\Rightarrow CDF = F_X(x) = \int f_X(x)$

$PDF = \dfrac{d}{dx} CDF$

$CDF = \int PDF$

$$\boxed{\therefore Q(x) = P(X \ge x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt}$$

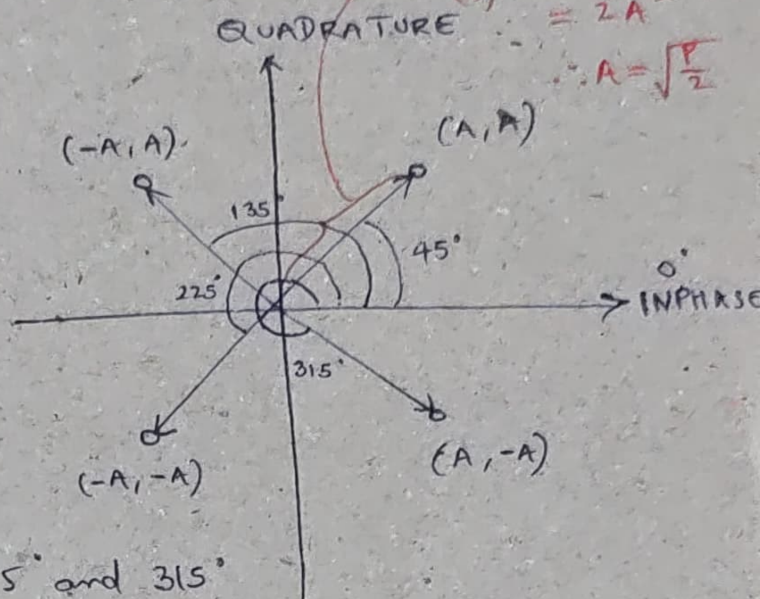
QPSK

- Quadrature Phase Shift Keying.
- $x_I \in \{+A, -A\}$

  $x_Q \in \{+A, -A\}$

radius $= \sqrt{A^2+A^2} = \sqrt{2A^2}$
$= \sqrt{2}\,A$

Power (P) $= (\sqrt{2}\,A)^2$
$= 2A^2$
$\therefore A = \sqrt{\frac{P}{2}}$



- Four phases : 45°, 135°, 225° and 315°
- Signal constellation :
  $\{A+jA,\ A-jA,\ -A+jA,\ -A-jA\}$
- Bit Mapping :
  - 00 → (A, A)
  - 01 → (A, -A)
  - 11 → (-A, -A)
  - 10 → (-A, A)

M = 4 Symbols
No. of bits/sym $= \log_2 M$
$= \log_2 4$
$= 2$ bits/sym

- $\text{SNR} = \dfrac{P}{\left(\frac{N_0}{2} + \frac{N_0}{2}\right)} = \dfrac{P}{N_0}$
- $\text{BER} = Q\left(\sqrt{\text{SNR}}\right) = Q\left(\sqrt{\frac{P}{N_0}}\right)$
- QPSK symbol is in error when either of the bits is in error.

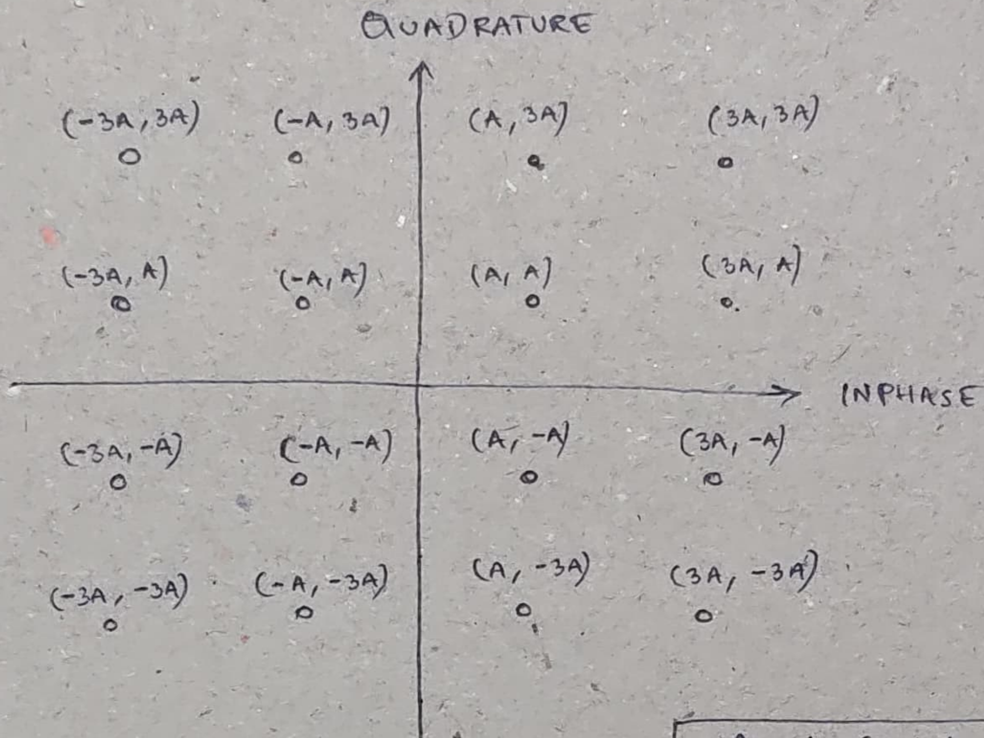
  Symbol Error Rate, $\text{SER} = 1 - (1-\text{BER})^2$
  $\approx 2 \times \text{BER}$
  $= 2\,Q\left(\sqrt{\text{SNR}}\right)$
  $= 2\,Q\left(\sqrt{\frac{P}{N_0}}\right)$

QAM / 2^{2n} QAM / M-QAM

- Quadrature Amplitude Modulation

$n=1 \rightarrow$ 4 QAM $\rightarrow$ 2 bits/sym (QPSK)
$n=2 \rightarrow$ 16 QAM $\rightarrow$ 4 bits/sym
$n=3 \rightarrow$ 64 QAM $\rightarrow$ 6 bits/sym
$n=4 \rightarrow$ 256 QAM $\rightarrow$ 8 bits/sym
$n=5 \rightarrow$ 1024 QAM $\rightarrow$ 10 bits/sym.

- 16-QAM



$M = 16$ Symbols
No. of bits/sym $= \log_2 M$
$= \log_2 16$
$= 4$ bits/sym

- $x_I \in \{-3A, -A, A, 3A\}$
  $x_Q \in \{-3A, -A, A, 3A\}$

- Signal constellation :

$\{-3A+j3A,\ -3A+jA,\ -3A-jA,\ -3A-j3A,$
$-A+j3A,\ -A+jA,\ -A-jA,\ -A-j3A,$
$A+j3A,\ A+jA,\ A-jA,\ A-j3A,$
$3A+j3A,\ 3A+jA,\ 3A-jA,\ 3A-j3A\}$

⊙ Bit Mapping :

0000 → (−3A, 3A)
0001 → (−3A, A)
0011 → (−3A, −A)
0010 → (−3A, −3A)

0100 → (−A, 3A)
0101 → (−A, A)
0111 → (−A, −A)
0110 → (−A, −3A)

1100 → (A, 3A)
1101 → (A, A)
1111 → (A, −A)
1110 → (A, −3A)

1000 → (3A, 3A)
1001 → (3A, A)
1011 → (3A, −A)
1010 → (3A, −3A)

Symbol Error Rate (SER) for M−QAM

$$SER \approx 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3P}{N_o(M-1)}}\right)$$

$M=4$, $SER = 4\left(1-\frac{1}{\sqrt{4}}\right) Q\left(\sqrt{\frac{3P}{N_o(4-1)}}\right) = 2\,Q\left(\sqrt{\frac{P}{N_o}}\right)$

$M=16$, $SER = 4\left(1-\frac{1}{\sqrt{16}}\right) Q\left(\sqrt{\frac{3P}{N_o(16-1)}}\right) = 3\,Q\left(\sqrt{\frac{P}{5N_o}}\right)$

$M=64$, $SER = 4\left(1-\frac{1}{\sqrt{64}}\right) Q\left(\sqrt{\frac{3P}{N_o(64-1)}}\right) = \frac{7}{2}\,Q\left(\sqrt{\frac{P}{21\,N_o}}\right)$

$M=256$, $SER = 4\left(1-\frac{1}{\sqrt{256}}\right) Q\left(\sqrt{\frac{3P}{N_o(256-1)}}\right) = \frac{15}{4}\,Q\left(\sqrt{\frac{P}{85\,N_o}}\right)$

$M=1024$, $SER = 4\left(1-\frac{1}{\sqrt{1024}}\right) Q\left(\sqrt{\frac{3P}{N_o(1024-1)}}\right) = \frac{31}{8}\,Q\left(\sqrt{\frac{P}{341\,N_o}}\right)$