ALAMOUTI CODE

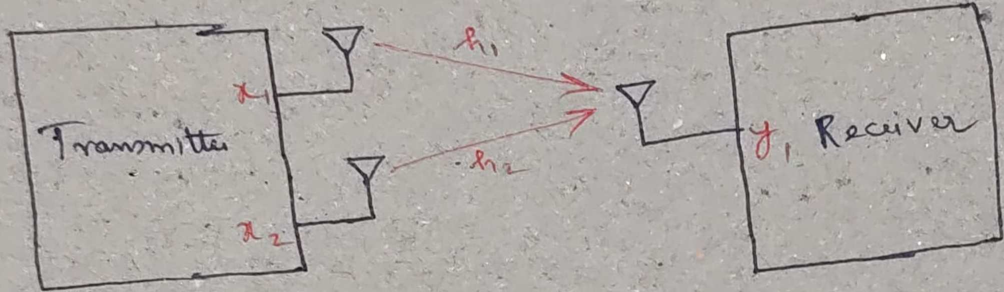
→ It is an Orthogonal Space Time Block Code

→ Alamouti Code is for a System with

- 2 transmit antennas ($t = 2$)
- 1 receive antennas ($r = 1$)

1×2 SYSTEM

Also known as MISO System



The MISO channel is given as

$$\bar{h}^T = [h_1 \quad h_2]$$

where, h_1 → channel coefficient b/w Rx Antenna and Tx Antenna 1.

h_2 → Channel coeffient b/w Rx Antenna and Tx Antenna 2.

⊙ In the first time instant, the transmit vector is given as $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

where, x_1 is transmitted from Tx. Antenna 1

x_2 is transmitted from Tx. Antenna 2

Therefore, the output in time instant 1 is given as

$$y_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

$$\boxed{y_1 = h_1 x_1 + h_2 x_2 + n_1}$$

⊙ In the second time instant, the transmit vector is given as $\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix}$

where, $-x_2^*$ is transmitted from Tx. Antenna 1

x_1^* is transmitted from Tx. Antenna 2.

Therefore, the output at time instant 2 is given as

$$y_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2$$

$$\boxed{y_2 = -h_1 x_2^* + h_2 x_1^* + n_2}$$

⊙ Consider now y_2^*

$$y_2^* = \begin{bmatrix} h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2^*$$

$$\boxed{y_2^* = h_2^* x_1 + (-h_1^*) x_2 + n_2^*}$$

⊙ The net system model is given as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

→ Noise Vector

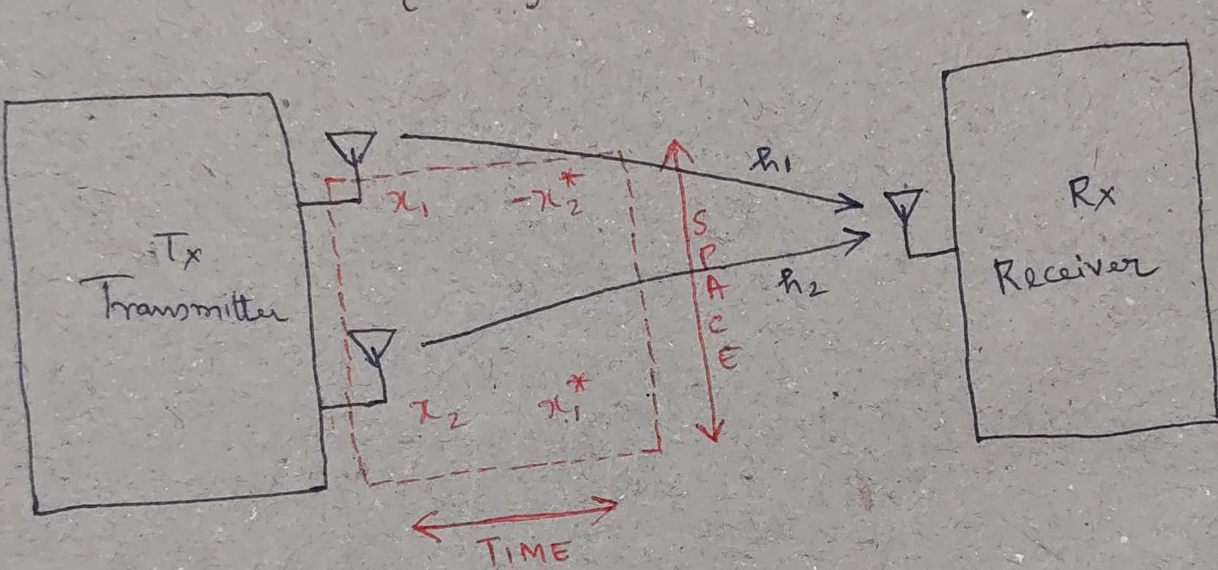
→ Symbol vector

→ 2 × 2 Alamouti Matrix

→ Output vector.

The Total Power (P) is split equally between x_1 and x_2.

(ie) $E\{|x_1^2|\} = E\{|x_2^2|\} = \frac{P}{2}$



TIME ⇒ 2 Transmit time instants.

SPACE ⇒ 2 Antennas.

⊙ Coding across SPACE and TIME. Hence termed as STBC.
(Space Time Block Code)

⊙ Alamouti Code is an STBC.

2 x 2 ALAMOUTI MATRIX

$$H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

$\bar{C}_1 \quad \bar{C}_2$

- The columns of the ALAMOUTI MATRIX are ORTHOGONAL $\Rightarrow$ Inner product of column vectors $= 0$

Let $\bar{C}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}$ and $\bar{C}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$

$$\bar{C}_1^H \cdot \bar{C}_2 = \begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

$$= \cancel{h_1^* h_2} - \cancel{h_2 h_1^*}$$

$$= 0.$$

The Orthogonality between $\bar{C}_1$ and $\bar{C}_2$ makes decoding very Easy!

Hence, Alamouti Code is termed as O STBC. (orthogonal Space Time Block Code).

- Since matrix is Orthogonal, decoding can be simply performed by multiplying by inverse

$$H^T = \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}.$$

$$H^H = (H^T)^* = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}$$

$$\boxed{H^{-1} = \frac{1}{\|\bar{h}\|^2} H^H = \frac{1}{\|\bar{h}\|^2} \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}}$$

$$\text{Where } \|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$$

Decoder operation,

$$\boxed{H^{-1}\bar{y} = \frac{1}{\|\bar{h}\|^2} H^H \bar{y}}$$

EXAMPLE

- Consider the MISO channel

$$\bar{h}^T = [h_1 \quad h_2] = [1-2j \quad -2+j]$$

$$\|\bar{h}\|^2 = |h_1|^2 + |h_2|^2$$
$$= 1+4+4+1$$
$$= 10.$$

where, $h_1 \rightarrow$ Channel coefficient b/w Rx. Antenna and Tx. Antenna 1.

$h_2 \rightarrow$ Channel coefficient b/w Rx. Antenna and Tx. Antenna 2.

- In the <u>first time instant</u>, consider the transmit vector

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1-j \\ 1+j \end{bmatrix}$$

where, x_1 is transmitted from Tx. Antenna 1

x_2 is transmitted from Tx. Antenna 2

- In the second time instant, consider the transmit vector

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} = \begin{bmatrix} -1+j \\ -1+j \end{bmatrix}$$

where, $-x_2^*$ is transmitted from Tx. Antenna 1

x_1^* is transmitted from Tx. Antenna 2

- The Alamouti Matrix is given as

$$H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} 1-2j & -2+j \\ -2-j & -1-2j \end{bmatrix}$$

$$\qquad\qquad\qquad\qquad \bar{c}_1 \qquad \bar{c}_2$$

Let $\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix} = \begin{bmatrix} 1-2j \\ -2-j \end{bmatrix}$ and

$\bar{c}_1^H = \begin{bmatrix} 1+2j & -2+j \end{bmatrix}$

$\bar{c}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix} = \begin{bmatrix} -2+j \\ -1-2j \end{bmatrix}$

$\bar{c}_1$ and $\bar{c}_2$ are ORTHOGONAL, because their Inner Product $\bar{c}_1^H \bar{c}_2 = 0$

$$\Rightarrow \begin{bmatrix} 1+2j & -2+j \end{bmatrix} \begin{bmatrix} -2+j \\ -1-2j \end{bmatrix} = (1+2j)(-2+j) + (-2+j)(-1-2j)$$

$$= \cancel{(1+2j)(-2+j)} - \cancel{(-2+j)(1+2j)}$$

$$= 0$$

Hence, Alamouti Code is termed as <u>OSTBC</u> (Orthogonal Space Time Block Code).

- Decoding can be simply performed by multiplying by inverse.

$$H^{-1} = \frac{1}{\|\bar{h}\|^2} H^H = \frac{1}{10} \begin{bmatrix} 1+2j & -2+j \\ -2-j & -1+2j \end{bmatrix}$$

ALAMOUTI BER

- The Alamouti System Output SNR for each stream is given as

$$SNR_o = \|\bar{h}\|^2 \frac{P/2}{N_o} = \frac{1}{2}\|\bar{h}\|^2 \frac{P}{N_o}$$

- The Alamouti System Output BER is given as

$$\boxed{BER = \frac{3}{SNR^2}}$$

Recall

QPSK $SNR = \frac{P}{N_o}$

BPSK $SNR = \frac{2P}{N_o}$

BER is decreasing as $\frac{1}{SNR^2}$

$\Rightarrow$ Diversity order = 2.

ADVANTAGE

- Recall, In the Time instant 1, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are being transmitted from Tx. Antenna 1 and Tx. Antenna 2 respectively.
- Also, In the Time instant 2, $\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix}$ are being transmitted from Tx. Antenna 1 and Tx. Antenna 2 respectively.
- Hence, Alamouti Code does not need knowledge of channel (CHANNEL STATE INFORMATION) at Transmitter for Beamforming.

EXAMPLE

Evaluate Alamouti BER for SNR = 20 dB = 100

$$BER = \frac{3}{SNR^2} = \frac{3}{(100)^2} = 3 \times 10^{-4}$$