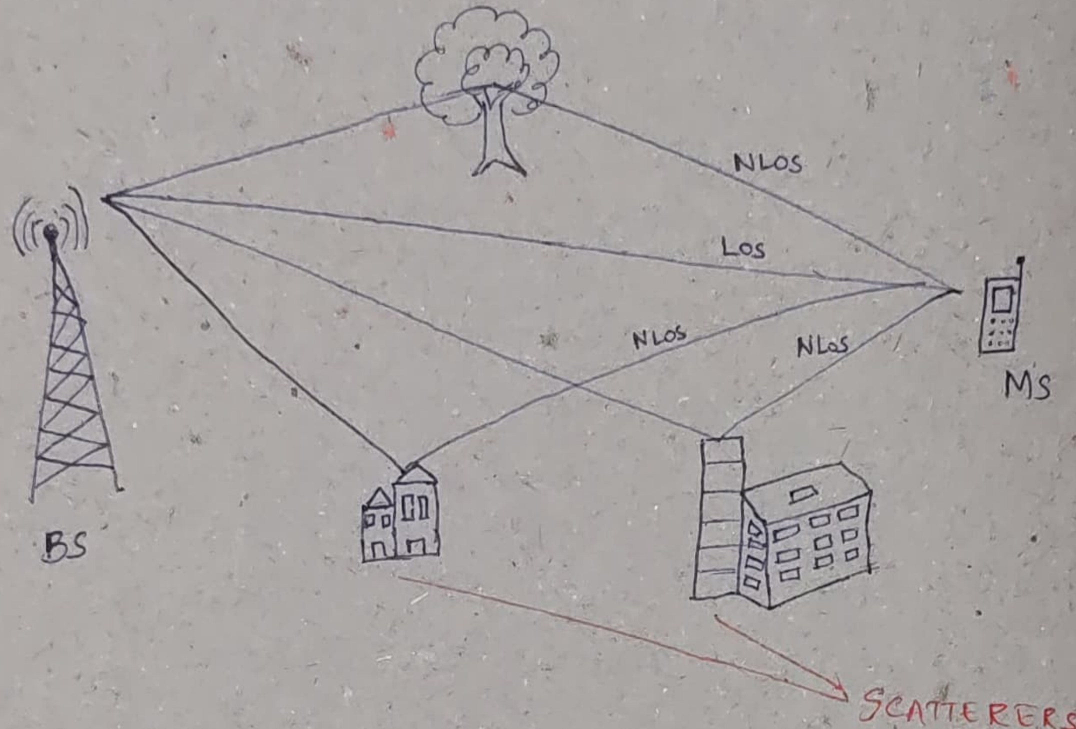
WIRELESS CHANNEL



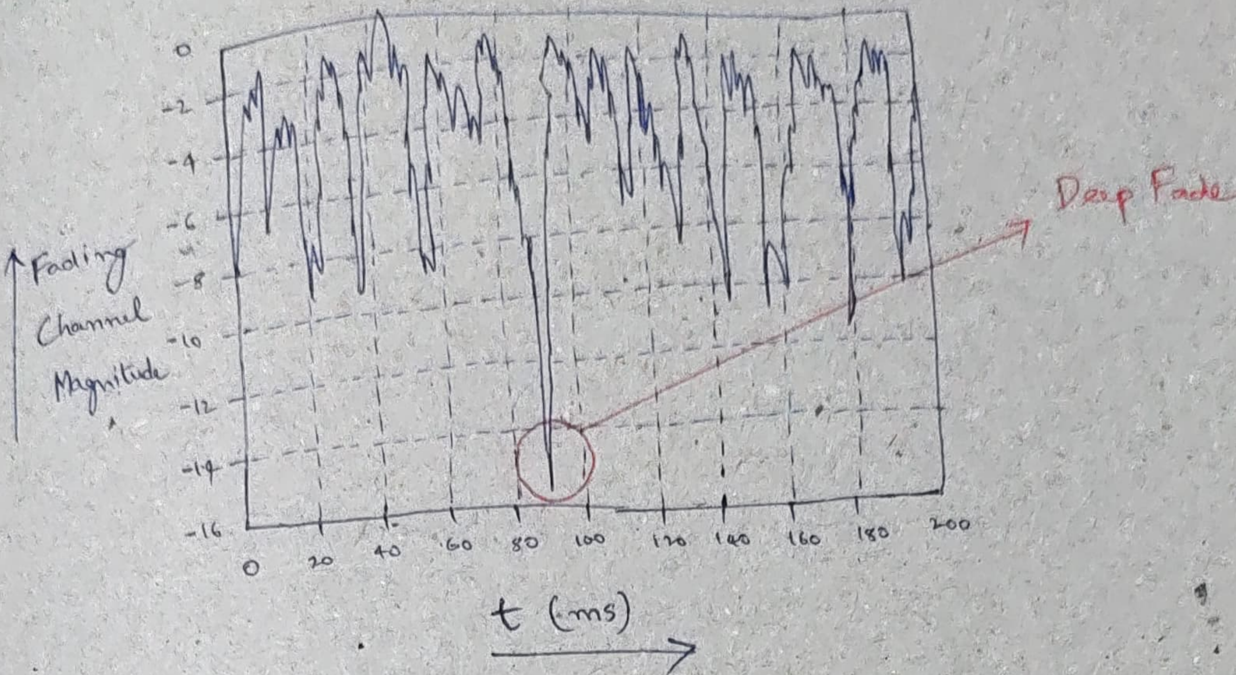
→ Fundamental difference b/w WIRED and WIRELESS Channel is that, there are multiple propagation paths. This is termed as Multipath propagation.

→ Multipath arises due to larger objects such as Trees, Buildings, etc.; These are termed as Scatterers.

→ This leads to multiple copies of the signal superpose at the receiver, resulting in Interference.

→ The interference can be constructive or destructive. Because of this, the SNR varies or fluctuates.

→ As the received signal power varies/fluctuates/fades... the wireless channel is also called as FADING CHANNEL



→ The received signal power where it <u>dips significantly</u> is termed as <u>DEEP FADE</u>

<u>WIRELESS CHANNEL MODEL</u>

The wireless channel model is given as

$$y = \underline{h}\,x + n$$

Fading channel coefficient

> **Note:** $\underbrace{y = x + w}_{\text{WIRELINE}}$

→ Signal output $= hx$

Output power $= P_0 = |h|^2 \cdot P$

$$P_0 = E\{|hx|^2\} = E\{|x|^2\} \cdot |h|^2$$

Note that, 'h' <u>determines the output power</u>.

- When $|h|$ is large, output power is large (<u>Constructive Interference</u>)
- When $|h|$ is small, output power is small (<u>Destructive Interference</u>)

→ The fading channel coefficient is random in nature. It is modelled as

$$\boxed{h = u + jv}$$

where, u, v are independent Gaussian RVs with mean $= 0$ $(E\{u\} = E\{v\} = 0)$ and Variance $= \frac{1}{2}$. $\left(E\{u^2\} = E\{v^2\} = \frac{1}{2}\right)$.

→ Therefore, 'h' is a complex Gaussian RV with

mean, $E\{h\} = E\{u\} + jE\{v\} = 0$ and

Variance, $E\{|h|^2\} = E\{u^2 + v^2\} = E\{u^2\} + E\{v^2\}$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1.$$

→ Another representation of h is

$$\boxed{h = a e^{j\phi}}$$

where, $a \rightarrow$ amplitude, $\Rightarrow a = |h| = \sqrt{u^2 + v^2}$

$\phi \rightarrow$ phase, $\Rightarrow \phi = \angle h$.

Note: Here, the phase 'ϕ' does not affect the output power P_o. In contrast, the magnitude/amplitude 'a' which very much affect the output power P_o.

AMPLITUDE

→ Amplitude 'a' follows the Rayleigh PDF.

$$f_A(a) = \begin{cases} 2a\,e^{-a^2}, & a \geq 0 \\ 0, & a < 0 \end{cases}$$

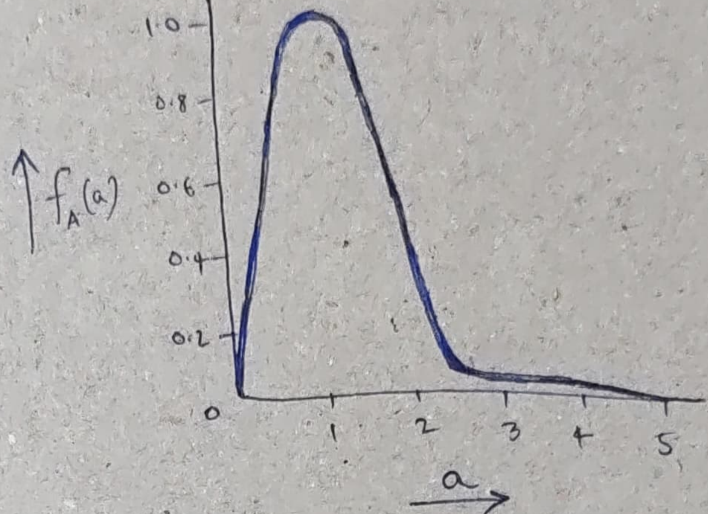


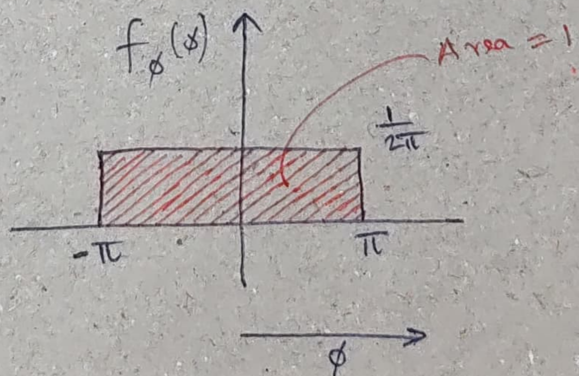
Fig. Rayligh PDF

→ Such a channel is called as Rayligh Fading channel.

PHASE

→ Phase 'ϕ' is Uniformly distributed in $[-\pi, \pi]$.

$$f_\phi(\phi) = \frac{1}{2\pi}, \quad -\pi < \phi < \pi$$



WIRELESS CHANNEL : SYMBOL DETECTION

WKT $y = hx + n$

- At the receiver, EQUALIZATION is carried out first.

Invert the effect of Channel co-efficient 'h'.

(ii) $z = \frac{1}{h} \times y = \frac{1}{h}(hx + n) = \underbrace{x}_{\text{UNDISTORTED SIGNAL}} + \frac{n}{h}$

- How the receiver knows the value of 'h' ?
  - → Using Channel Estimation
  - → Estimate the channel co-efficient 'h' using the pilot symbols (known).

- Recall ; BPSK signal constellation

$x \in \{+A, -A\}$.

Therefore, detection can be carried out as follows:

→ If $z \geq 0$, then it is more likely that $\hat{x} = +A$

→ If $z < 0$, then it is more likely that $\hat{x} = -A$

where, $\hat{x}$ → Estimate of x

This is termed as Threshold Detector (or) Maximum Likelihood (ML) detector.

OUTPUT SNR : SNR at the receiver

- Recall, the output power, $P_o = |h|^2 \cdot P$
$$= a^2 \cdot P$$

- The output SNR is defined as
$$SNR_o = \frac{O/P \text{ Power}}{\text{Noise Power}} = \frac{|h|^2 \cdot P}{N_o/2}$$
$$= |h|^2 \cdot \frac{2P}{N_o}$$
$$= a^2 \cdot SNR$$

WIRELESS BER

- BER for BPSK, $Q(\sqrt{SNR_o}) = Q(\sqrt{a^2 \cdot SNR})$

where $a \rightarrow$ random quantity.

Hence, to calculate the actual BER, One has to average w.r.t PDF of a.

PDF of a $\rightarrow$ $f_A(a) = 2a \cdot e^{-a^2}$

Average $\rightarrow$ $\int_0^\infty \underbrace{g(a)}_{\text{Function of a}} \cdot \underbrace{f_A(a)}_{\text{PDF of a}} \, da$

Therefore, BER for BPSK modulation for the wireless channel is given as

$$BER = \int_{-\infty}^{\infty} Q\left(\sqrt{a^2 \times SNR}\right) f_A(a)\, da$$
$$= \int_{-\infty}^{\infty} Q\left(\sqrt{a^2 \times SNR}\right) 2a e^{-a^2}\, da$$
$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$
$$= \frac{1}{2}\left(1 - \sqrt{\frac{2P/N_o}{2+2P/N_o}}\right)$$

EXAMPLE

① Evaluate BER_s of Wireline and Wireless Channels with BPSK transmission and SNR = 12 dB.

$SNR = 12\,dB = 15.85$

$10 \log_{10} SNR = SNR\_dB$

$10 \log_{10} SNR = 12$

$\log_{10} SNR = 1.2$

$SNR = 10^{1.2}$

$SNR = 15.85$

• $$BER_{WIRELINE} = Q\left(\sqrt{SNR}\right)$$

$$= Q\left(\sqrt{15.85}\right)$$

$$= \underline{\underline{3.44 \times 10^{-5}}} = \frac{3.44}{10^5}$$

Means, if 10^5 bits are transmitted, then 3.44 bits are in error.

(ie) In every 10^7 transmitted bits, 344 bits are error.

• $$BER_{WIRELESS} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{15.85}{2+15.85}}\right)$$

$$= 0.0288$$

$$= \underline{\underline{2.88 \times 10^{-2}}} = \frac{2.88}{10^2}$$

Means, if 10^2 bits are transmitted, then 2.88 bits are in error.

(ie) In every 10000 bits, 288 bits are in error.

• Inference:

BER OF WIRELESS IS SIGNIFICANTLY HIGHER THAN BER OF WIRELINE !!

$$\boxed{BER_{WIRELESS} >> BER_{WIRELINE}}$$

② Evaluate BERs of Wireline and Wireless Channels with BPSK transmission and SNR = 20 dB.

SNR = 20 dB = 100

$10 \log_{10} SNR = SNR\_dB$

$10 \log_{10} SNR = 20$

$\log_{10} SNR = 2$

$SNR = 10^2$

$SNR = 100$

⊙ $BER_{WIRELINE} = Q(\sqrt{SNR})$

$= Q(\sqrt{100})$

$= \underline{\underline{7.62 \times 10^{-24}}} = \frac{7.62}{10^{24}}$

⊙ $BER_{WIRELESS} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$

$= \frac{1}{2}\left(1 - \sqrt{\frac{100}{102}}\right)$

$= \underline{\underline{5 \times 10^{-3}}} = \frac{5}{10^3}$

⊙ Inference:

BER of WIRELESS is 10^{21} times HIGHER THAN BER OF WIRELINE !!

⊙ Why is the BER of WIRELESS is so high?

— PTO.

WIRELINE BER SIMPLIFIED

- WKT, Gaussian Q-function

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

- The simple bound for Q-function

$$Q(x) \leq \frac{1}{2} e^{-\frac{1}{2}x^2}$$

- The wireline BER can be simplified as

$$Q(\sqrt{SNR}) \leq \frac{1}{2} e^{-\frac{1}{2} SNR}$$

$$\boxed{BER_{WIRELINE} \approx e^{-\frac{1}{2} SNR}}$$

- Therefore, $BER_{WIRELINE}$ decreases exponentially w.r.t SNR.

WIRELESS BER SIMPLIFIED

- The wireless BER can be simplified as

$$\frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right) = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{SNR\left(\frac{2}{SNR}+1\right)}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{1}{\frac{2}{SNR}+1}}\right)$$

$$= \frac{1}{2}\left(1 - \left(\frac{2}{SNR}+1\right)^{-1/2}\right)$$

$$\approx \frac{1}{2}\left(1 - \left(1 - \frac{1}{2}\cdot\frac{2}{SNR}\right)\right)$$

$$\approx \frac{1}{2}\left(\frac{1}{SNR}\right)$$

First order Taylor Series Approximation.

$$(1+x)^{-1/2} \approx 1 - \frac{1}{2}x$$

$$\boxed{BER_{WIRELESS} \approx \frac{1}{SNR}}$$

- Therefore, $BER_{WIRELESS}$ decreases gradually w.r.t SNR.

EXAMPLE

(3) Calculate the SNR, required to achieve $BER = 10^{-6}$ in both Wireline and Wireless channels.

WIRELINE

$$Q\left(\sqrt{SNR}\right) = BER_{WIRELINE}$$

$$\sqrt{SNR} = Q^{-1}\left(10^{-6}\right) \approx 4.75$$

$$SNR = 4.75^2 = 22.56$$

$$\boxed{SNR_{WIRELINE} = 13.53 \text{ dB}}$$

WIRELESS

$$\frac{1}{2}\left(\frac{1}{SNR}\right) = BER_{WIRELESS}$$

$$SNR = \frac{1}{2}\left(\frac{1}{10^{-6}}\right) = 5 \times 10^5$$

$$\boxed{SNR_{WIRELESS} = 57 \text{ dB}}$$

Ratio of SNRs

$$\frac{SNR_{WIRELESS}}{SNR_{WIRELINE}} = \frac{57 \text{ dB}}{13.53 \text{ dB}} = \frac{5 \times 10^5}{22.56} \approx 22{,}000$$

Inference

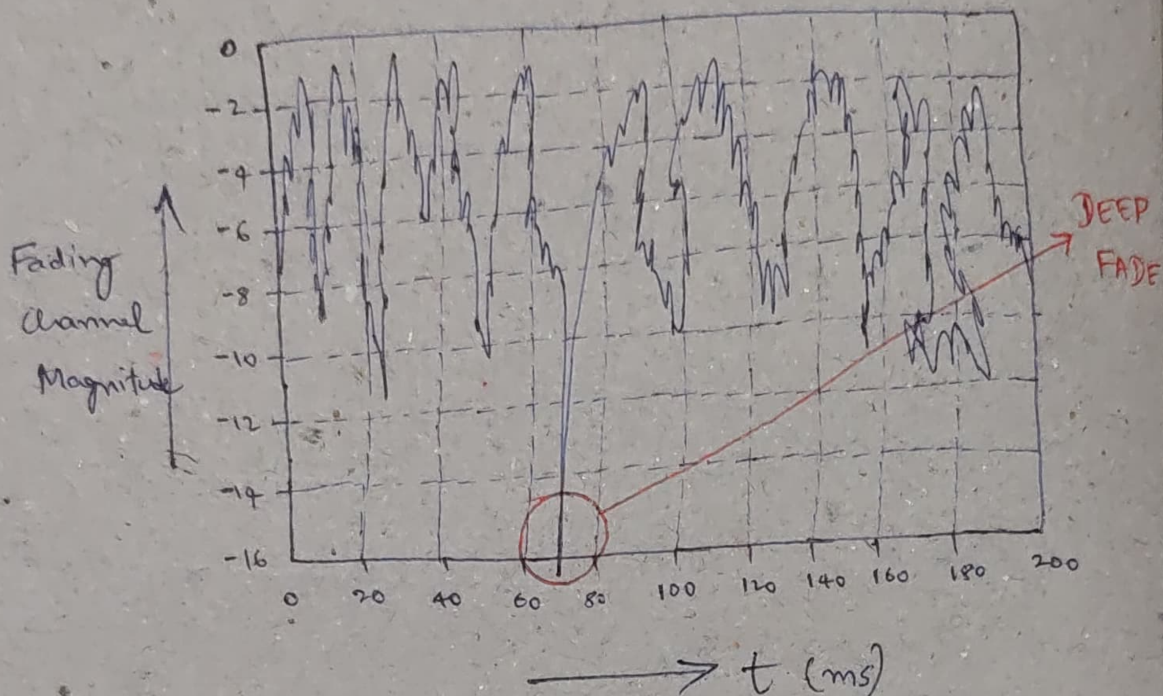
This implies that, Wireless system requires 22,000 times more transmit power than Wireline System, to achieve the $BER = 10^{-6}$.

What is the Reason?

PTO.

$$P_{WIRELESS} \gg P_{WIRELINE}$$

The reason behind the requirement of more transmit power in Wireless than Wireline System is due to the consequence of the DEEP FADE phenomenon.



- Recall, in a Wireless system, the SNR fades/fluctuates/varies
- DEEP FADE is a sharp drop in the SNR.
- This implies, the Signal is buried in Noise.

(i.e) SIGNAL IS LOST

$\Rightarrow$ Output Signal Power $<$ Noise power

$$|h|^2 P < N_0$$

$$a^2 P < N_0$$

$$a^2 < \frac{N_0}{P}$$

$$\boxed{a^2 < \frac{1}{SNR}}$$

This is the CONDITION FOR DEEP FADE.

The Probability of Deep Fade, P_{DF} can be evaluated as follows.

$$P_{DF} = P_r\left(a^2 < \frac{1}{SNR}\right)$$

$$= P_r\left(a < \frac{1}{\sqrt{SNR}}\right)$$

where, $a \rightarrow$ Rayleigh RV.

The Probability Density Function of 'a' $\rightarrow f_A(a)$

$$P_{DF} = \int_0^{\frac{1}{\sqrt{SNR}}} f_A(a)\, da = \int_0^{\frac{1}{\sqrt{SNR}}} 2a\, e^{-a^2}\, da$$

$$= 1 - e^{-\frac{1}{SNR}}$$

$$\approx 1 - \left(1 - \frac{1}{SNR}\right).$$

$$e^{-x} \approx 1 - x$$

$$\boxed{P_{DF} = \frac{1}{SNR}}$$

WKT $\quad BER_{WIRELESS} = \frac{1}{2}\left(\frac{1}{SNR}\right) = \frac{1}{2}\left(P_{DF}\right)$

This implies that,

$$\boxed{\text{BIT ERROR RATE} \propto \text{PROBABILITY OF DEEP FADE}}$$

Therefore, high BER of the wireless channel is a direct consequence of DEEP FADE..

- DEEP FADE causes a huge spike in the BER.
- CHALLENGE!

One has to overcome the Problem of Deep Fade without having to increase the Transmit Power.

BER and SER for QPSK and QAM

WKT $BER_{WIRELESS} = \frac{1}{2}\left(\frac{1}{SNR}\right)$

SNR for QPSK modulation $= \frac{P}{N_0}$

- BER for QPSK $= \frac{1}{2}\left(\frac{1}{P/N_0}\right) = \frac{1}{2}\left(\frac{N_0}{P}\right) = \frac{1}{2}\left(\frac{1}{SNR}\right)$
- SER for QPSK $= 2 \times BER = 2 \times \frac{1}{2}\left(\frac{1}{SNR}\right) = \frac{1}{SNR}$
- SER of M-QAM $= 4\left(1 - \frac{1}{\sqrt{M}}\right) \times \frac{1}{2} \times \frac{1}{\frac{3P}{N_0(M-1)}}$

$$= 4\left(1 - \frac{1}{\sqrt{M}}\right) \times \frac{1}{2} \times \frac{(M-1)}{3 \times \frac{P}{N_0}}$$

$$= 4\left(1 - \frac{1}{\sqrt{M}}\right) \times \frac{1}{2} \times \frac{(M-1)}{3 \times SNR}$$

Directly proportional to $\frac{1}{SNR}$

Thus, One cannot improve the performance by simply changing the modulation.

Then, how to improve wireless performance?

This can be achieved via DIVERSITY.

FUNDAMENTAL TO WIRELESS SYSTEM.