MIMO - OFDM

- Combines the benefits of both
  MIMO (Spatial Multiplexing)
  +
  OFDM (Frequency division Multiplexing)
- This leads to Ultra High data rates !!
  4G LTE ~ 100 Mbps
  5G NR ~ 1 Gbps

MIMO - OFDM Channel Model

- Recall in MIMO,
  $r \rightarrow$ No. of Receive Antennas
  $t \rightarrow$ No. of Transmit Antennas

The channel between the i^{th} Receive Antenna and j^{th} Transmit Antenna is FREQUENCY SELECTIVE, as the Bandwidth is large (WIDE BAND). Thus, the total number of frequency selective channels between each Rx. Antenna and Tx. Antenna pair is $r \times t$.

Also, there are L channel Taps between the i^{th} Receive Antenna and j^{th} Transmit Antenna which can be represented as

$$h_{ij}(0),\ h_{ij}(1),\ \ldots\ldots,\ h_{ij}(L-1)$$

Therefore, total number of Channel Taps for the Frequency Selective MIMO channel is

$$r \times t \times L.$$

⊙ MIMO - OFDM Transmission

On each Transmit antenna j, perform IFFT. (i.e) Load the subcarriers.

N Symbols loaded on Subcarriers for each transmit antenna j are

$$X_j(0), X_j(1), \ldots, X_j(N-1)$$

$\Rightarrow X_j(k) \rightarrow$ Symbol loaded on subcarrier k @ Transmit antenna j

$$\left.\begin{array}{l}\text{Total number of symbols loaded on}\\ \text{the subcarriers of all Tx. Antennas}\end{array}\right\} = \underbrace{N}_{\text{Frequency dimension}}\ \underbrace{t}_{\text{Space dimension}}$$

IFFT on each Tx. Antenna can be performed as shown below.

$$\boxed{X_j(0), X_j(1), \ldots, X_j(N-1)} \text{ Freq. Domain}$$

$$\downarrow$$

$$\boxed{\text{IFFT}}$$

$$\downarrow$$

$$\boxed{x_j(0), x_j(1), \ldots, x_j(N-1)} \text{ Time Domain}$$

Total No. of IFFT blocks on all Tx. Antennas

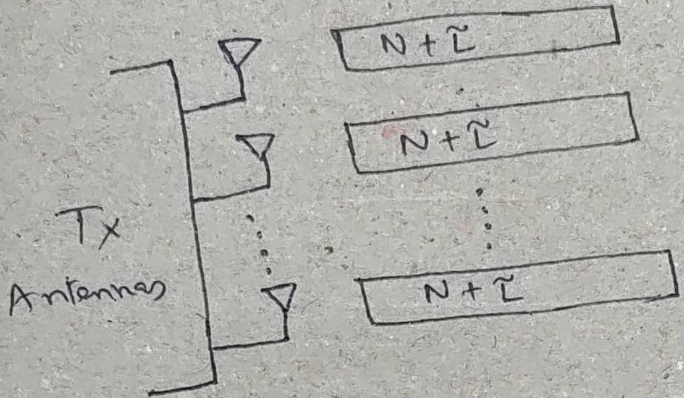
$\Rightarrow t$ IFFTs

(One IFFT @ Each Tx. Antenna).

⊙ Add CP on each Tx. Antenna as shown below.

$$\underbrace{\underbrace{x_j(N-\tilde{L}), \ldots, x_j(N-2), x_j(N-1)}_{\text{CP samples } (\tilde{L}) \text{ or } (N_{cp})}, \underbrace{x_j(0), x_j(1), \ldots, x_j(N-1)}_{\text{Original samples } (N)}}_{N+\tilde{L} \text{ samples on each Tx. Antenna}}$$

⊙ Size of the Total transmission block



Total No. of samples considering all Tx. Antennas $\Bigg\} = (N+\tilde{L})\,t$ (or)

$(N+N_{cp})\,t$

⊙ After removal of CP, Linear Convolution becomes Circular convolution.

$$y_i(k) = \sum_{j=1}^{t} h_{ij} \circledast x_j + w_i(k)$$

x_j → Samples on Tx. Antenna j

h_{ij} → Channel b/w Rx. Antenna i and Tx. Antenna j

MIMO - OFDM Receiver

On each Receive antenna i, perform FFT. (ie) Unload the subcarriers.

After CP removal, the samples on each Receive Antenna i are

$$y_i(0),\ y_i(1),\ \ldots,\ y_i(N-1)$$ Time domain

$$\downarrow$$

$$\boxed{\text{FFT}}$$

$$\downarrow$$

$$Y_i(0),\ Y_i(1),\ \ldots,\ Y_i(N-1)$$ Freq. domain

$\Rightarrow Y_i(k) \rightarrow$ Symbol Unloaded from Subcarrier k @ Receive antenna i

Total number symbols unloaded from the subcarriers of all Rx. Antennas $\Bigg\} = N\, r$

Frequency dimension

Space dimension

Total No. of FFT blocks on all Rx. Antennas

$\Rightarrow r$ FFTs

(One FFT @ Each Rx. Antenna)

MIMO - OFDM SYSTEM MODEL

The net MIMO-OFDM system model for subcarrier k is given below.

$$\underbrace{\begin{bmatrix} Y_1(k) \\ Y_2(k) \\ \vdots \\ Y_r(k) \end{bmatrix}}_{Y(k)} = \underbrace{\begin{bmatrix} H_{11}(k) & H_{12}(k) & \cdots & H_{1t}(k) \\ H_{21}(k) & H_{22}(k) & \cdots & H_{2t}(k) \\ \vdots & \vdots & \ddots & \vdots \\ H_{r1}(k) & H_{r2}(k) & & H_{rt}(k) \end{bmatrix}}_{H(k)} \underbrace{\begin{bmatrix} X_1(k) \\ X_2(k) \\ \vdots \\ X_t(k) \end{bmatrix}}_{X(k)} + \underbrace{\begin{bmatrix} W_1(k) \\ W_2(k) \\ \vdots \\ W_r(k) \end{bmatrix}}_{W(k)}$$

$$\boxed{Y(k) = H(k)\,X(k) + W(k)}$$

$k = 0, 1, \ldots, N-1$ (N PARALLEL MIMO CHANNELS)

(FREQUENCY DOMAIN MODEL)

where,

$Y(k) \rightarrow r \times 1$ Output vector for all Rx. Antennas on subcarrier k.

$H(k) \rightarrow r \times t$ MIMO (FLAT) channel Matrix for subcarrier k

$X(k) \rightarrow t \times 1$ Transmit vector of Symbols loaded on subcarrier k for all Tx. Antennas

$W(k) \rightarrow r \times 1$ Noise vector for subcarrier k

$H_{ij}(k) \rightarrow$ MIMO-OFDM channel coefficients b/w i^{th} Rx. Antenna and j^{th} Tx. Antenna for Subcarrier k.

Recall, $\left(h_{ij}(0), h_{ij}(1), \ldots, h_{ij}(L-1), \underbrace{0, 0, 0, \ldots 0}_{N-L \text{ zeros}} \right)$

$\downarrow$

N-POINT FFT

$\downarrow$

$\left(H_{ij}(0), H_{ij}(1), \ldots, H_{ij}(N-1) \right)$

⊙ How many such parallel MIMO systems are there?

ONE for each SUBCARRIER.

$\Rightarrow$ N PARALLEL MIMO CHANNELS.

$$\left.\begin{aligned} \text{(i.e.)}\ Y(0) &= H(0)\, X(0) + W(0) \\ Y(1) &= H(1)\, X(1) + W(1) \\ &\ \vdots \\ Y(N-1) &= H(N-1)\, X(N-1) + W(N-1) \end{aligned}\right\} \text{N PARALLEL MIMO CHANNELS}$$

⊙ How to recover $X(k)$?

MIMO receiver can be used for each subcarrier k.

(i) Zero - Forcing Receiver

$$\hat{X}(k) = \underbrace{\left(H^H(k)\, H(k)\right)^{-1} H^H(k)}_{\text{Pseudo-Inverse of } H(k)} \cdot Y(k)$$

(ii) Linear Minimum Mean Square Error Receiver

$$\hat{X}(k) = \left(H^H(k)\, H(k) + \frac{1}{SNR} I\right)^{-1} H^H(k) \cdot Y(k).$$

$$\boxed{\begin{aligned} &\text{As } SNR \to \infty, \\ &\text{LMMSE Receiver} \longrightarrow \text{ZF Receiver.} \end{aligned}}$$