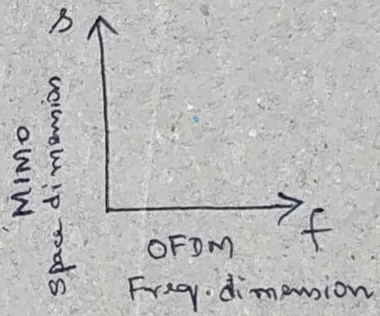
OFDM TECHNOLOGY

(ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING)

→ One of the most extensively used wireless technology.

→ OFDM is widely employed in most of the modern cellular (4G-LTE, 5G-NR) and Wi-Fi (802.11n, 802.11ac, 802.11ax, ...) systems.

→ OFDM + MIMO



- Enables transmission over very large Bandwidth.
- Enables Ultra-High Data Rates
- SPACE + FREQUENCY Multiplexing implies Extra ordinarily high data rates!!!

SINGLE CARRIER MODULATION

- Consider Bandwidth B and a single carrier.
- Symbol time/symbol duration is given by $\frac{1}{B}$.
- Symbol Rate (i.e.) No. of Symbols/sec is given by B.

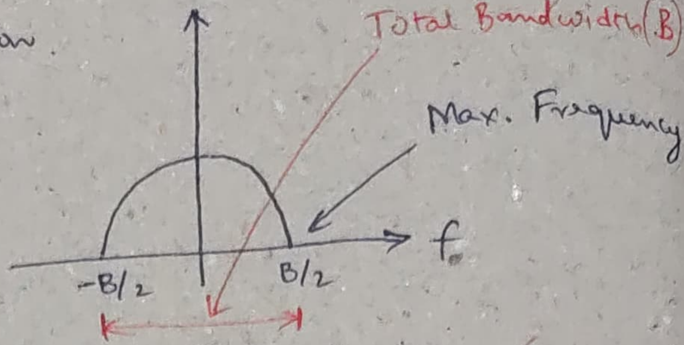
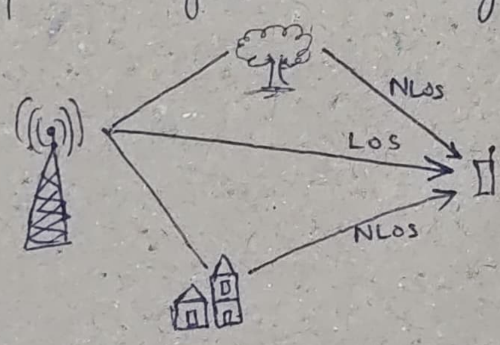


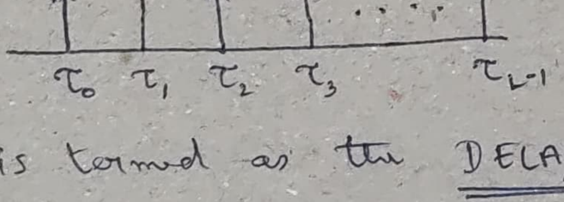
Fig. Single Carrier

Bandwidth (B) ↑	Symbol duration ($1/B$) ↓
1 kHz	1 ms
10 kHz	0.1 ms
1 MHz	1 μs
10 MHz	0.1 μs

- Recall, due to multipath propagation of wireless channel, different copies of the signal arriving with different delays!



- As a result, the multipath components are spread over time.



- This is termed as the DELAY SPREAD of wireless channel.

Typically $\simeq$ 2 - 3 μs

- At Larger Bandwith, eventually the
  Delay spread > Symbol Time !!!
- The signal copies with different delays superpose at the receiver

This leads to ISI. (Inter Symbol Interference)

which implies,

* Significant distortion!
* BER is very high
* INFORMATION is lost.

These are the Problems as we go larger Bandwidths.

Is it possible to ELIMINATE ISI ?

Yes. This is what OFDM achieves precisely.

How ?

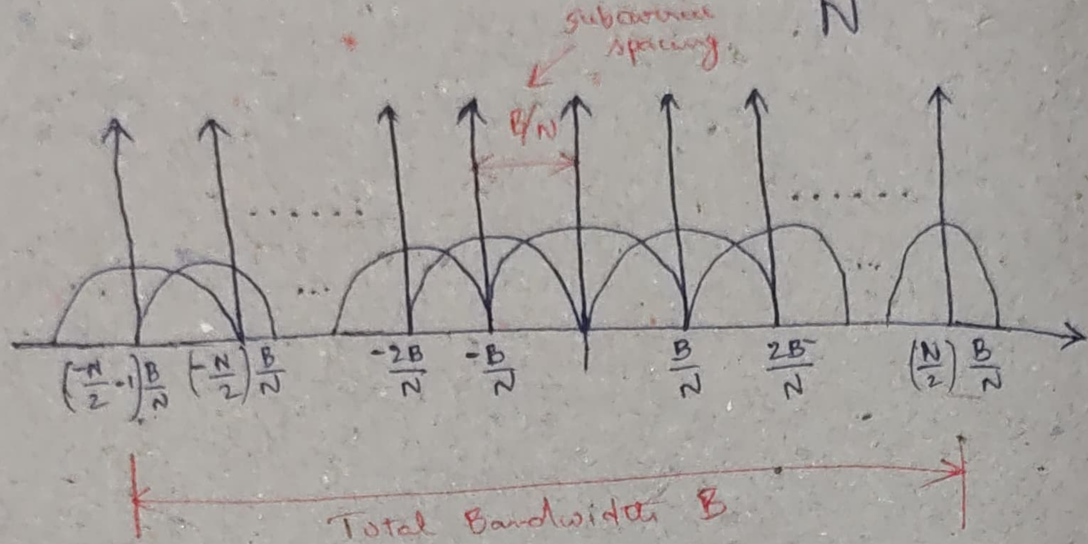
Instead of using single carrier, use N subcarriers, which is called Multicarrier Modulation.

MULTI CARRIER MODULATION

- Consider Bandwidth B and N sub-carriers.

(i) Divide the Bandwidth B into N SUB BANDS

Bandwidth of each SUBBAND $= \frac{B}{N}$.



Example

$B = 10\,\text{MHz}$, $N = 1000$ subcarriers

Bandwidth of each Subcarrier (or) Subcarrier spacing $= \frac{B}{N} = \frac{10 \times 10^6\,\text{Hz}}{1000} = 10\,\text{KHz}$

- Subcarriers are placed at integer multiples of $\frac{B}{N}$.

$$\ldots, \frac{-2B}{N}, \frac{-B}{N}, 0, \frac{B}{N}, \frac{2B}{N}, \frac{3B}{N}, \ldots$$

- Let $\boxed{\frac{B}{N} = f_0}$

$\Rightarrow$ Subcarriers are placed at integer multiples of f_0.

$$\ldots, -2f_0, -f_0, 0, f_0, 2f_0, 3f_0, \ldots$$

- Carrier frequency f_c can be represented as

$$\boxed{e^{j2\pi f_c t}}$$

Therefore, the k^{th} subcarrier (subcarrier placed at kf_0) can be represented as

$$\boxed{e^{j2\pi k f_0 t}}$$

- On each subcarrier, modulate the symbol X_k.

$$\boxed{X_k \cdot e^{j2\pi k f_0 t}}$$

- The transmit signal is given as the sum of all the symbols across all the subcarriers.

$$\boxed{x(t) = \sum_k X_k \, e^{j2\pi \cdot k f_0 t}}$$

Symbol LOADED on subcarrier k

Subcarrier Index

- The received signal (considering noiseless scenario) is given as

$$y(t) = x(t) + n(t)$$
$$= \sum_k X_k \, e^{j2\pi k f_0 t} + 0$$

(Ignoring noise for simplicity!)

⊙ Note that $y(t) = x(t) = \sum_k X_k \cdot e^{j2\pi k f_0 t}$

is the FOURIER SERIES expansion of $x(t)$.

⊙ The coefficient X_l can be extracted as follows.

$$X_l = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} y(t) \cdot e^{-j2\pi l f_0 t} \, dt$$

Integrating over one period (T) $\} = \frac{1}{f_0}$

$$= \frac{1}{T} \int_{-T/2}^{T/2} y(t) \, e^{-j2\pi l f_0 t} \, dt.$$

⊙ Recall, $f_0 = \frac{B}{N}$

$$\Rightarrow \boxed{T = \frac{1}{f_0} = \frac{N}{B}}$$

where $T \rightarrow$ OFDM Symbol duration.

Total Bandwidth (B)	Bandwidth of each Subcarrier (B/N)	Symbol duration (N/B)
1 kHz	1 Hz	1 s
10 kHz	10 Hz	0.1 s
1 MHz	1 kHz	1 ms
10 MHz	10 kHz	0.1 ms

* Assume N = 1000

Typical DELAY SPREAD ≃ 2 - 3 μs

⊙ Even at larger Bandwidth, the
Symbol Time >> Delay spread.
$\Rightarrow$ No ISI.
Thus, OFDM avoids Inter Symbol Interference.

⊙ Extracting the l^{th} symbol (X_l)

$$f_0 \int_0^{1/f_0} y(t)\, e^{-j 2\pi l f_0 t}\, dt$$

$$= f_0 \int_0^{1/f_0} \left(\sum_k X_k e^{j 2\pi k f_0 t} \right) e^{-j 2\pi l f_0 t}\, dt$$

$$= \sum_k X_k \underbrace{\left(f_0 \int_0^{1/f_0} e^{j 2\pi (k-l) f_0 t}\, dt \right)}$$

$$= \sum_k X_k \underbrace{\delta(k-l)}_{\text{Discrete Delta function}} \quad \begin{cases} = 0, & \text{if } k \neq l \\ 1, & \text{if } k = l \end{cases}$$

$\because$ Subcarriers are ORTHOGONAL

INNER PRODUCT of 2 diff. subcarriers is Zero.

$$= X_l$$

Thus, able to recover l^{th} symbol (X_l).

SUMMARY

- $e^{j2\pi k f_0 t}$, $e^{j2\pi l f_0 t}$ are ORTHOGONAL.

- ORTHOGONAL $\Rightarrow$ Subcarriers are Orthogonal
- FREQUENCY DIVISION $\Rightarrow$ Dividing Bandwidth into multiple sub bands
- MULTIPLEXING $\Rightarrow$ Parallel transmission of multiple symbols over multiple subcarriers in same band/channel (Wide Bandwidth channel)

- Thus, OFDM enables Transmission over Large Bandwidth without ISI, with extremely high Data rates.

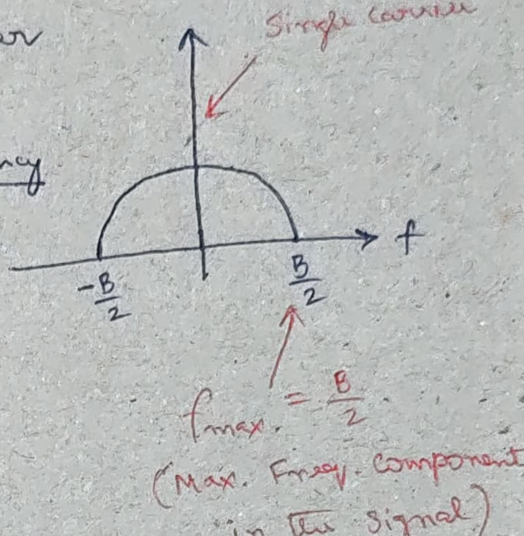
  Say, LTE ~ 100 Mbps
  LTE-A ~ 500 Mbps. } due to OFDM + MIMO

OFDM GENERATION

$\sum_k X_k e^{j 2\pi k f_0 t}$ implies 1000's of Subcarriers, which is difficult to generate.

The simple technique to generate the OFDM signal is "SAMPLING".

→ Consider a single carrier



The Minimum Sampling frequency required to sampling the signal, which is band limited to $\frac{B}{2}$, without loosing any information is given by

$$f_s = 2 f_{max} = 2 \times \frac{B}{2} = B.$$

This is termed as NYQUIST criterion.

The Sampling duration / Sampling interval is given as

$$T_s = \frac{1}{f_s} = \frac{1}{B}.$$

→ Thus, the l^{th} sample will be at

$$t = l T_s = l \times \frac{1}{B} = \frac{l}{B}.$$

$$\therefore x(l) = \sum_{k} X_k \, e^{j2\pi k f_0 t}$$

$$= \sum_{k} X_k \, e^{j2\pi k \frac{B}{N} \cdot \frac{l}{B}}$$

Simple SCALING/ MULTIPLICATION factor

$$= \boxed{\frac{1}{N}} \underbrace{\sum_{k} X_k \, e^{j2\pi k \frac{l}{N}}}$$

IDFT, which can be efficiently implemented using IFFT.

Therefore, <u>Samples</u> of the OFDM signal can be generated very efficiently, without loosing any information (Thanks to the NYQUIST criterion) using the <u>IFFT</u> algorithm.

← Widely implemented in DSP chipsets.

← computational complexity of $N \log_2 N \sim N$

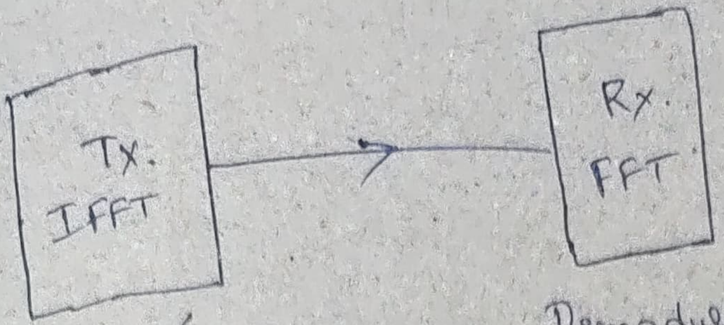
$X_0, X_1, \ldots, X_{N-1}$
(Frequency Domain SUBCARRIERS)

↓

IFFT

↓

$x(0), x(1), \ldots, x(N-1)$
(Time Domain SAMPLES)



Modulate/Load the sub carriers

Demodulate/UnLoad the sub carriers

ISI Channel Model

- WKT, the NO ISI (only Fading) channel model

$$y(k) = h\, x(k) + n(k)$$

- The ISI channel model is given by

current symbol

L-1 Previous symbols

L Channel Taps

$$y(k) = h(0)\,x(k) + h(1)\,x(k-1) + h(2)\,x(k-2) + \cdots + \cdots + h(L-1)\,x(k-(L-1)) + n(k).$$

$$\boxed{\Rightarrow y(k) = \sum_{l=0}^{L-1} h(l) \cdot x(k-l) + n(k)}$$

$$\Rightarrow y(k) = \underline{h * x} + n(k)$$

Thus, the ISI channel performs <u>LINEAR CONVOLUTION</u>.

CYCLIC PREFIX

Prior to transmission, we add CYCLIC PREFIX (CP) to an OFDM block.

$$\overbrace{\underbrace{x(N-\tilde{L}), \ldots, x(N-2), x(N-1)}_{\tilde{L}\text{ CP samples}}}^{\text{Prefix at Head}}, \underbrace{x(0), x(1), x(2), \ldots, \overbrace{\ldots, x(N-1)}^{\tilde{L}\text{ samples from Tail}}}_{\text{N original samples generated by IFFT}}$$

$$\left.\begin{array}{c}\text{Total Number}\\ \text{of Samples}\end{array}\right\} = \left.\begin{array}{c}\text{N IFFT samples}\\ +\\ \tilde{L}\text{ CP samples}\\ (10-15\%\text{ of N})\end{array}\right\} = N + \tilde{L}$$

The Output corresponding to $x(0)$ is

L Channel Taps

$$\begin{aligned} y(0) = {} & h(0)\,x(0) \quad \text{— Current Symbol} \\ & + h(1)\,x(N-1) \\ & + h(2)\,x(N-2) \\ & + \ldots \\ & + \ldots \\ & + h(L-1)\,x(N-(L-1)) \\ & + \ldots \\ & + \ldots \\ & + 0 \times x(2) \\ & + 0 \times x(1) \\ & + n(0). \end{aligned}$$

L-1 Previous Symbols.

$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + h(L-1)x(N-(L-1))$$
$$+ \dots + 0 \times x(2) + 0 \times x(1) + n(0)$$

$$y(1) = h(0)x(1) + h(1)x(0) + h(2)x(N-1) + \dots + h(L-1)x(N-(L-2))$$
$$+ \dots + 0 \times x(3) + 0 \times x(2) + n(1)$$

$$y(2) = h(0)x(2) + h(1)x(1) + h(2)x(0) + h(3)x(N-1) + \dots + h(L-1)x(N-(L-3))$$
$$+ \dots + 0 \times x(4) + 0 \times x(3) + n(2).$$

The OFDM samples are shifting in a circular fashion! This is termed as

CIRCULAR CONVOLUTION:

Because of the addition of CYCLIC PREFIX, LINEAR convolution becomes CIRCULAR convolution.

(ie) $y(k) = h \circledast x + v(k)$

What happens when we take FFT at the receiver?

CIRCULAR CONVOLUTION

$$\boxed{\underbrace{y(l)}_{\text{OUTPUT}} = \underset{\text{CHANNEL}}{h} \circledast \underset{\text{OFDM SIGNAL}}{x} + \underbrace{v(l)}_{\text{NOISE}}}$$

$$\downarrow$$

$$\boxed{\text{FFT}}$$

$$\downarrow$$

PRODUCT IN FREQUENCY DOMAIN

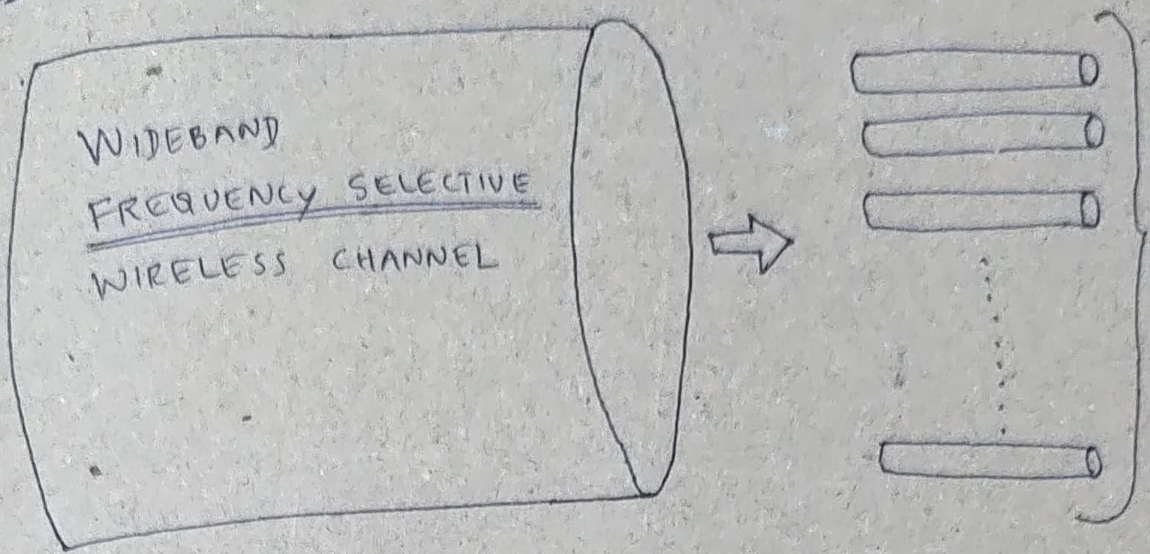
$$\boxed{\underbrace{Y_k}_{\text{OUTPUT FOR SUBCARRIER 'k'}} = \underbrace{H_k}_{\text{CHANNEL COEFFICIENT FOR SUBCARRIER 'k'}} \times \underbrace{X_k}_{\text{SYMBOL ON SUBCARRIER 'k'}} + \underbrace{V_k}_{\text{NOISE FOR SUBCARRIER 'k'}}}$$

Therefore, we have

$$\left.\begin{aligned} Y_0 &= H_0 \times X_0 + V_0 \\ Y_1 &= H_1 \times X_1 + V_1 \\ &\vdots \\ Y_{N-1} &= H_{N-1} \times X_{N-1} + V_{N-1} \end{aligned}\right\} \text{N parallel Channels, One for each Subcarrier.}$$

- $k = 0, 1, 2, \ldots N-1 \rightarrow$ Subcarrier Indices
- NO ISI on any subcarrier !!
- Thus, OFDM completely avoids the ISI.

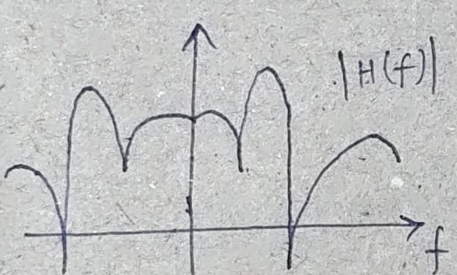
OFDM



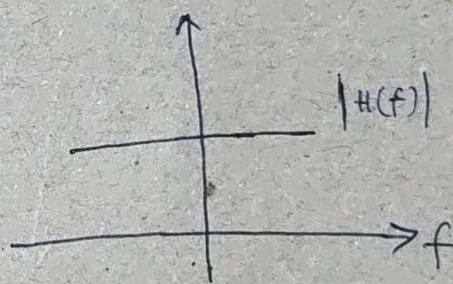
N PARALLEL
NARROWBAND
FLAT FADING
OFDM SUBCHANNELS

What is OFDM achieving?

OFDM is converting a $\overbrace{\text{TIME DOMAIN ISI CHANNEL}}^{\text{BAD}}$ into $\underbrace{\text{N PARALLEL ISI FREE FREQUENCY DOMAIN SUBCARRIER}}_{\text{GOOD}}$ channels.



Frequency Selective



Flat Fading

- Channel coefficient H_k are given as

$$\boxed{\underbrace{h(0), h(1), \ldots h(L-1)}_{L}, \underbrace{0, 0, 0, \ldots 0}_{N-L \text{ Zeros}}}$$

$$\Big\downarrow \text{ N-Point zero-Padded FFT}$$

$$\boxed{H_0, H_1, H_2, \ldots, H_{N-1}}$$

Subcarrier channel coefficients.

The coefficient for k^{th} subcarrier is given as

$$H_k = \sum_{l=0}^{L-1} h(l)\, e^{-j 2\pi \frac{kl}{N}}$$

- The Outputs Y_k are given as

$$\boxed{\begin{array}{c} y(0), y(1), \ldots, y(N-1) \\ \text{(TIME DOMAIN)} \end{array}}$$

$$\Big\downarrow \text{ FFT}$$

$$\boxed{\begin{array}{c} Y_0, Y_1, \ldots, Y_{N-1} \\ \text{(FREQUENCY DOMAIN)} \end{array}}$$

(Freq. Domain Subcarrier Outputs)

(i) CP removal

$$\underbrace{x(N-\tilde{L}), \ldots, x(N-2), x(N-1)}_{\tilde{L} \text{ CP samples}}, \underbrace{x(0), x(1), x(2), \ldots, x(N-1)}_{N \text{ original samples}}$$

At the receiver, we remove the outputs corresponding to the CP. That is, we dont need $y(N-1), y(N-2), \ldots$. We only consider outputs starting from $y(0)$.

Hence, CP removal is performed.

(ii) Remove the outputs corresponding to CP samples.

OFDM SYSTEM EXAMPLE

- Consider an OFDM system with Bandwidth $B = 10$ MHz and Number of Subcarriers $N = 1024$.

Note: The No. of Subcarriers is always Power of 2. (2^n) because, the radix of FFT is always power of 2.

- Bandwidth of each subcarrier $= \frac{B}{N} = \frac{10 \times 10^6}{1024} = 9.77$ kHz

- What is the size of IFFT at transmitter?

  IFFT is used for LOADING the subcarriers.

  Hence, the size of IFFT = No. of Subcarriers
  $= N = 1024$.

- OFDM sample duration $= \frac{1}{B} = \frac{1}{10 \times 10^6} = 0.1\ \mu s.$

- OFDM symbol duration (without CP) $= N \times \frac{1}{B}$

  $= 1024 \times \frac{1}{10 \times 10^6}$

  $= 102.4\ \mu s.$

- Consider No. of CP samples $N_{cp} = 80$ samples.

  CP samples duration $= N_{cp} \times \frac{1}{B}$

  $= 80 \times \frac{1}{10 \times 10^6}$

  $= 8\ \mu s.$

- How many samples after addition of CP ?

$\underbrace{x(N-\tilde{L}), \ldots, x(N-2), x(N-1)}_{80 \text{ CP samples}}, \underbrace{x(0), x(1), x(2), \ldots, x(N-1)}_{1024 \text{ Original samples}}$

Total No. of samples with CP $= N + N_{cp}$

$= 1024 + 80$

$= \underline{\underline{1104 \text{ samples.}}}$

$\Rightarrow \underbrace{x(943), \ldots, x(1022), x(1023)}_{80 \text{ CP samples}}, \underbrace{x(0), x(1), \ldots, x(1023)}_{1024 \text{ Original samples}}$

- Total duration (with CP) $= 102.4\,\mu s + 8\,\mu s$

$= \underline{\underline{110.4\,\mu s}}$

- Let us say, each subcarrier is loaded with QPSK symbols (2 bits/symbol). What is the effective bit rate ?

$$\frac{2 \frac{\text{bits}}{\text{Sub carrier}} \times 1024 \text{ subcarriers}}{\text{Total duration (with CP)}}$$

$$= \frac{2 \times 1024}{110.4 \times 10^{-6}}$$

$$= \underline{\underline{18.55 \text{ Mbps.}}}$$

- % Loss in Spectral efficiency $= \dfrac{N_{cp}}{N + N_{cp}} \times 100$

$$= \frac{80}{80+1024} \times 100$$

$$= \underline{\underline{7.24\,\%}}$$

- As the CP addition is redundant, it causes loss in the spectral efficiency.
- Also, it is evident that the CP absorbs the ISI / distortion / Inter Block Interference, which makes the channel reliable.
- Hence, CP addition is a Trade-off between the loss in spectral efficiency (VS) reliability.

OFDM TRANSMISSION EXAMPLE

⊙ consider an $N=4$ subcarrier system

$$\left.\begin{aligned} X_0 &= 1+j \\ X_1 &= 1-j \\ X_2 &= 1+2j \\ X_3 &= 2-j \end{aligned}\right\} \text{4 SYMBOLS LOADED ON SUBCARRIERS.}$$

What are the Time domain samples?

The TD samples $x(0)$, $x(1)$, $x(2)$ and $x(3)$ are given by the IFFT.

$$\boxed{x(l) = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{j2\pi \frac{kl}{N}}}$$

Here, $N=4$,

$$x(l) = \frac{1}{4}\sum_{k=0}^{3} X_k \, e^{j\frac{\pi}{2}kl}$$

$$x(0) = \frac{1}{4}\left(X_0 + X_1 + X_2 + X_3\right)$$

$$= \frac{1}{4}\left(1+j+1-j+1+2j+2-j\right)$$

$$= \frac{5}{4} + \frac{1}{4}j$$

$$x(1) = \frac{1}{4}\left(X_0 + jX_1 - X_2 - jX_3\right)$$

$$= \frac{1}{4}\left(1+j+j(1-j) - (1+2j) - j(2-j)\right)$$

$$= -\frac{1}{2}j$$

$$x(2) = -\frac{1}{4} + \frac{5}{4}j$$

$$x(3) = 0.$$

Therefore, the Time domain transmit samples before CP addition are

$$\boxed{x(0), x(1), x(2), x(3) = \frac{5}{4} + \frac{1}{4}j,\ \frac{-1}{2}j,\ \frac{-1}{4} + \frac{5}{4}j,\ 0}$$

- Let cyclic Prefix (CP) be of duration "1 sample. ($N_{cp} = 1$)
  The transmit frame is given by

$$x(3), x(0), x(1), x(2), x(3)$$

$\Rightarrow$ The transmitted OFDM block with CP is

$$\boxed{0,\ \frac{5}{4} + \frac{1}{4}j,\ \frac{-1}{2}j,\ \frac{-1}{4} + \frac{5}{4}j,\ 0}$$

OFDM RECEPTION EXAMPLE

(i) Consider the Output samples (TIME DOMAIN)

$$\left.\begin{matrix} y(0) = 1, & y(1) = \frac{1}{2} \\ y(2) = \frac{1}{2}, & y(3) = 1 \end{matrix}\right\} \text{AFTER CP REMOVAL.}$$

The subcarrier outputs (FREQUENCY DOMAIN) are given by FFT.

$$\boxed{Y_k = \sum_{l=0}^{N-1} y(l) \cdot e^{-j 2\pi \frac{kl}{N}}}$$

Here $N = 4$,

$$Y_k = \sum_{l=0}^{3} y(l) \cdot e^{-j \frac{\pi}{2} kl}$$

$$Y_0 = y(0) + y(1) + y(2) + y(3)$$
$$= 1 + \frac{1}{2} + \frac{1}{2} + 1 = 3$$

$$Y_1 = y(0) - j\,y(1) - y(2) + j\,y(3)$$
$$= 1 - j\frac{1}{2} - \frac{1}{2} + j(1) = \frac{1}{2} + \frac{1}{2}j$$

$$Y_2 = 0$$
$$Y_3 = \frac{1}{2} - \frac{1}{2}j$$

Therefore, the frequency domain subcarrier outputs after CP removal are

$$\boxed{Y_0, Y_1, Y_2, Y_3 = 3, \frac{1}{2} + \frac{1}{2}j, 0, \frac{1}{2} - \frac{1}{2}j}$$

OFDM SUBCARRIER CHANNEL COEFFICIENTS EXAMPLE

⊙ Consider the $L=2$ Channel Taps given as

$$h(0) = 1, \quad h(1) = \frac{1}{2}$$

What are the subcarrier channel coefficients?
($N = 4$ subcarriers). (H_0, H_1, H_2, H_3).

The subcarrier channel coefficients (H_0, H_1, H_2, H_3) are given by the Zero-Padded FFT of the Channel Taps $\left(1, \frac{1}{2}, 0, 0\right)$.

due to

$$\text{(ii)} \quad \boxed{H_k = \sum_{l=0}^{L-1} h(l) \cdot e^{-j 2\pi \frac{kl}{N}}}$$

$N=4, L=2$

$$H_k = \sum_{l=0}^{1} h(l) \cdot e^{-j \frac{\pi}{2} k l}$$

$$= h(0) + h(1)\, e^{-j \frac{\pi}{2} k}.$$

$$H_0 = h(0) + h(1)$$
$$= 1 + \frac{1}{2} = \frac{3}{2}$$

$$H_1 = h(0) - j\, h(1)$$
$$= 1 - \frac{1}{2} j$$

$$H_2 = \frac{1}{2}$$

$$H_3 = 1 + \frac{1}{2} j$$

Therefore, the subcarrier channel coefficients are

$$\boxed{H_0, H_1, H_2, H_3 = \frac{3}{2}, 1 - \frac{1}{2} j, \frac{1}{2}, 1 + \frac{1}{2} j}$$

BER OF OFDM

- Consider the channel taps $h(0), h(1), \ldots, h(L-1)$.
  Assume they are Rayleigh fading with Unit Power

$$E\{|h(l)|^2\} = 1$$

Noise samples $v(l)$ are iid with power N_0.

$$E\{|v(l)|^2\} = N_0$$

Symbols loaded on subcarriers have power P.

- The effective SNR for QPSK is given by

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

where, $L \rightarrow$ No. of channel Taps
$N \rightarrow$ No. of sub carriers

- The BER of OFDM for QPSK is given by

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{\rho_{eff}}{2+\rho_{eff}}}\right) \approx \frac{1}{2}\left(\frac{1}{\rho_{eff}}\right)$$

$$\Rightarrow \boxed{BER = \frac{1}{2} \times \frac{N}{L \times SNR}}$$

BER OF OFDM EXAMPLE

What is the BER of OFDM for QPSK with

$SNR = 30\ dB = 10^3$

$L = 8$ channel taps and

$N = 64$ subcarriers.

$$\rho_{eff} = \frac{L}{N} \times \frac{P}{N_0} = \frac{L}{N} \times SNR$$

$$= \frac{8}{64} \times 10^3$$

$$BER = \frac{1}{2} \times \frac{1}{\rho_{eff}} = \frac{1}{2} \times \frac{64}{8} \times 10^{-3}$$

$$= 4 \times 10^{-3}$$