

1 Overture SYNTAX AND SEMANTICS

$v \in \mathbb{F}_p$, $w \in \text{String}$, $\iota \in \text{Clients} \subset \mathbb{N}$

$\varepsilon ::= r[w] \mid s[w] \mid m[w] \mid p[w] \mid v \mid \varepsilon - \varepsilon \mid \varepsilon + \varepsilon \mid \varepsilon * \varepsilon$ *expressions*

$x ::= r[w]@_\iota \mid s[w]@_\iota \mid m[w]@_\iota \mid p[w] \mid \text{out}_\iota$ *variables*

$\pi ::= m[w]@_\iota := \varepsilon@_\iota \mid p[w] := e@_\iota \mid \text{out}_\iota := \varepsilon@_\iota \mid \pi; \pi$ *protocols*

$$\begin{aligned} \llbracket \sigma, v \rrbracket_\iota &= v \\ \llbracket \sigma, \varepsilon_1 + \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota + \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 - \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota - \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 * \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota * \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, r[w] \rrbracket_\iota &= \sigma(r[w]@_\iota) \\ \llbracket \sigma, s[w] \rrbracket_\iota &= \sigma(s[w]@_\iota) \\ \llbracket \sigma, m[w] \rrbracket_\iota &= \sigma(m[w]@_\iota) \\ \llbracket \sigma, p[w] \rrbracket_\iota &= \sigma(p[w]) \end{aligned}$$

$$\frac{(\sigma, x := \varepsilon@_\iota) \Rightarrow \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} \quad \frac{(\sigma_1, \pi_1) \Rightarrow \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow \sigma_3}{(\sigma_1, \pi_1; \pi_2) \Rightarrow \sigma_3}}{(\sigma_1, \pi_1; \pi_2) \Rightarrow \sigma_3}$$

2 Overture ADVERSARIAL SEMANTICS

$(\sigma, x := \varepsilon@_\iota) \Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\}$ $\iota \in H$

$(\sigma, x := \varepsilon@_\iota) \Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \text{rewrite}_{\mathcal{A}}(\sigma_C, \varepsilon) \rrbracket_\iota\}$ $\iota \in C$

$(\sigma, \text{assert}(\varepsilon_1 = \varepsilon_2)@_\iota) \Rightarrow_{\mathcal{A}} \sigma$ if $\llbracket \sigma, \varepsilon_1 \rrbracket_\iota = \llbracket \sigma, \varepsilon_2 \rrbracket_\iota$, or $\iota \in C$

$(\sigma, \text{assert}(\phi(\varepsilon))@_\iota) \Rightarrow_{\mathcal{A}} \perp$ if $\neg\phi(\sigma, \llbracket \sigma, \varepsilon \rrbracket_\iota)$

$$\frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow_{\mathcal{A}} \sigma_3}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \sigma_3} \quad \frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \perp}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \perp}$$

$$\frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow_{\mathcal{A}} \perp}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \perp}$$

3 Overture CONSTRAINT TYPING

$\phi ::= x \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi$

$E ::= \phi \equiv \phi \mid E \wedge E$

We write $E_1 \models E_2$ iff every model of E_1 is a model of E_2 . Note that this relation is reflexive and transitive.

$$\begin{aligned} \lfloor x \rfloor &= x & \lfloor \varepsilon_1 + \varepsilon_2 @_\iota \rfloor &= \lfloor \varepsilon_2 @_\iota \rfloor + \lfloor \varepsilon_1 @_\iota \rfloor & \lfloor \varepsilon_1 - \varepsilon_2 @_\iota \rfloor &= \lfloor \varepsilon_2 @_\iota \rfloor - \lfloor \varepsilon_1 @_\iota \rfloor \\ \lfloor \varepsilon_1 * \varepsilon_2 @_\iota \rfloor &= \lfloor \varepsilon_2 @_\iota \rfloor * \lfloor \varepsilon_1 @_\iota \rfloor \end{aligned}$$

$$\lfloor \text{OT}(\varepsilon_1 @_{\iota_1}, \varepsilon_2, \varepsilon_3) @_{\iota_2} \rfloor = (\lfloor \varepsilon_1 @_{\iota_1} \rfloor \wedge \lfloor \varepsilon_3 @_{\iota_2} \rfloor) \vee (\neg \lfloor \varepsilon_1 @_{\iota_1} \rfloor \wedge \lfloor \varepsilon_2 @_{\iota_2} \rfloor)$$

$$\lfloor x := \varepsilon @_\iota \rfloor = x \equiv \lfloor \varepsilon @_\iota \rfloor \quad \lfloor \pi_1; \pi_2 \rfloor = \lfloor \pi_1 \rfloor \wedge \lfloor \pi_2 \rfloor$$

The motivating idea is that we can interpret any protocol π as a set of equality constraints $\lfloor \pi \rfloor$ and use an SMT solver to verify properties relevant to correctness, confidentiality, and integrity. Further, we can leverage entailment relation is critical for efficiency– we can use annotations to obtain a weakened precondition for relevant properties. That is, given π , program annotations or other cues can be used to find a minimal E with $\lfloor \pi \rfloor \models E$ for verifying correctness and security.

3.1 Confidentiality Types

$$\text{DEFTY} \quad \frac{\emptyset, E \vdash \phi : \text{vars}(\phi) \quad \text{ENCODE} \quad \frac{E \models \phi \equiv \phi' \oplus r[w]@_l \quad \oplus \in \{+, -\} \quad R, E \vdash \phi' : T}{R; \{r[w]@_l\}, E \vdash \phi : \{c(r[w]@_l, T)\}}}{R; \{r[w]@_l\}, E \vdash \phi : \{c(r[w]@_l, T)\}}$$

$$\text{SEND} \quad \frac{R, E \vdash \lfloor \varepsilon@_l \rfloor : T}{R, E \vdash x := \varepsilon@_l : (x : T)} \quad \text{SEQ} \quad \frac{R_1, E \vdash \pi_1 : \Gamma_1 \quad R_2, E \vdash \pi_2 : \Gamma_2}{R_1; R_2, E \vdash \pi_1; \pi_2 : \Gamma_1; \Gamma_2}$$

Definition 3.1. $R, E \vdash \pi : \Gamma$ is *valid* iff it is derivable and $\lfloor \pi \rfloor \models E$.

$$\frac{\iota \in C}{\Gamma, C \vdash \Gamma(m[w]@_l)} \quad \frac{\Gamma, C \vdash T_1 \cup T_2}{\Gamma, C \vdash T_1} \quad \frac{\Gamma, C \vdash \{m[w]@_l\}}{\Gamma, C \vdash \Gamma(m[w]@_l)}$$

$$\frac{\Gamma, C \vdash \{r[w]@_l\} \quad \Gamma, C \vdash \{c(r[w]@_l, T)\}}{\Gamma, C \vdash T}$$

THEOREM 3.2. *If $R, E \vdash \pi : \Gamma$ is valid and for all H, C it is not the case that $\Gamma, C \vdash \{s[w]@_l\}$ for $\iota \in H$, then π satisfies gradual release.*

3.1.1 Example.

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m[x]@1 := s2(s[x], -r[x], r[x])@2
// m[x]@1 == s[x]@2 + -r[x]@2
// m[x]@1 : { c(r[x]@2, { s[x]@2 }) }

m[y]@1 := 0T(s[y]@1, -r[y], r[y])@2
// m[y]@1 == s[y]@1 + -r[y]@2
// m[y]@1 : { c(r[y]@2, { s[y]@1 }) }

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3.2 Compositional Type Verification in *Prelude*

(*Need to fix the following to allow reduction of x . – Chris*)

$$\begin{array}{c}
 \text{MSG} \\
 \frac{e_1 \Rightarrow \varepsilon \quad e_2 \Rightarrow \iota \quad R_1, E \Vdash \lfloor \varepsilon @ \iota \rfloor : (R_2, T)}{R_1, E \vdash x := e_1 @ e_2 : (x : T, R_1; R_2, E \wedge x \equiv \lfloor \varepsilon @ \iota \rfloor)} \\
 \\
 \text{ENCODE} \\
 \frac{e_1 \Rightarrow \varepsilon \quad e_2 \Rightarrow \iota \quad e_3 \Rightarrow \phi \quad E \models \lfloor \varepsilon @ \iota \rfloor \equiv \phi \quad R_1, E \Vdash \phi : (R_2, T)}{R_1, E \vdash x := e_1 @ e_2 \text{ as } e_3 : (x : T, R_1; R_2, E \wedge x \equiv \phi)} \\
 \\
 \text{APP} \\
 \frac{\text{sig}(f) = \{E_1\} x_1, \dots, x_n \{\Gamma, R, E_2\} \quad e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad \rho = [v_1/x_1] \cdots [v_n/x_n] \quad E \models \rho(E_1)}{R_1, E \vdash f(e_1, \dots, e_n) : (\rho(\Gamma), R_1; \rho(R), E \wedge \rho(E_2))} \\
 \\
 \text{SEQ} \\
 \frac{R_1, E_1 \vdash \pi_1 : (\Gamma_2, R_2, E_2) \quad R_2, E_2 \vdash \pi_2 : (\Gamma_3, R_3, E_3)}{R_1, E_1 \vdash \pi_1; \pi_2 : (\Gamma_2; \Gamma_3, R_3, E_3)} \\
 \\
 \text{SIG} \\
 \frac{C(f) = x_1, \dots, x_n, \mathbf{c} \quad \rho = [v_1/x_1] \cdots [v_n/x_n] \quad \emptyset, \rho(E_1) \vdash \mathbf{c} : (\rho(\Gamma), \rho(R), \rho(E_2))}{f : \{E_1\} x_1, \dots, x_n \{\Gamma, R, E_2\}}
 \end{array}$$

Definition 3.3. sig is verified iff $f : \text{sig}(f)$ is valid for all $f \in \text{dom}(\text{sig})$.

The following theorem holds for protocols with default preprocessing.

THEOREM 3.4. *If sig is verified and $\emptyset, \emptyset \vdash e : (\Gamma, R, E)$ then $e \Rightarrow \pi$ and $R, E \vdash \pi : \Gamma$ is valid.*

3.3 Integrity Types

$$\begin{array}{c}
 \text{VALUE} \quad \text{SECRET} \quad \text{RANDO} \\
 \Gamma, \emptyset, E \vdash_i v : \emptyset \cdot \text{High} \quad \Gamma, \emptyset, E \vdash_i s[w] : \{s[w]@i\} \cdot \mathcal{L}(\iota) \quad \Gamma, \emptyset, E \vdash_i r[w] : \{r[w]@i\} \cdot \mathcal{L}(\iota) \\
 \\
 \text{MSG} \quad \text{PUBM} \quad \text{INTEGRITYWEAKEN} \\
 \Gamma, \emptyset, E \vdash_i m[w] : \Gamma(m[w]@i) \quad \Gamma, \emptyset, E \vdash_i p[w] : \Gamma(p[w]) \quad \frac{\Gamma, R, E \vdash_i \varepsilon : T \cdot \zeta_1 \quad \zeta_1 \leq \zeta_2}{\Gamma, R, E \vdash_i \varepsilon : T \cdot \zeta_2} \\
 \\
 \text{ENCODE} \\
 \frac{\Gamma, \emptyset, E \vdash_i \varepsilon : T \cdot \zeta \quad E \models \lfloor \varepsilon @ \iota \rfloor = \phi \oplus r[w]@i' \quad \oplus \in \{+, -\}}{\Gamma, r[w]@i, E \vdash_i \varepsilon : \{c(r[w]@i', \Gamma(\phi))\} \cdot \zeta} \\
 \\
 \text{BINOP} \\
 \frac{\Gamma, R_1, E \vdash_i \varepsilon_1 : T_1 \cdot \zeta \quad \Gamma, R_2, E \vdash_i \varepsilon_2 : T_2 \cdot \zeta \quad \oplus \in \{+, -, *\}}{\Gamma, R_1; R_2, E \vdash_i \varepsilon_1 \oplus \varepsilon_2 : T_1 \cup T_2 \cdot \zeta}
 \end{array}$$

$$\begin{array}{c}
\text{SEND} \\
\frac{\Gamma, R, E \vdash_t \varepsilon : T \cdot \mathcal{L}(t) \quad E' \models E \wedge x = \lfloor \varepsilon @ t \rfloor}{\Gamma, R, E \vdash x := \varepsilon @ t : \Gamma; x : T \cdot \mathcal{L}(t), E'} \\
\\
\text{ASSERT} \\
\frac{E \models \lfloor \varepsilon_1 @ t \rfloor = \lfloor \varepsilon_2 @ t \rfloor}{\Gamma, R, E \vdash \text{assert}(\varepsilon_1 = \varepsilon_2) @ t : \Gamma, E} \\
\\
\text{SEQ} \\
\frac{\Gamma_1, R_1, E_1 \vdash \pi_1 : \Gamma_2, E_2 \quad \Gamma_2, R_2, E_2 \vdash \pi_2 : \Gamma_3, E_3}{\Gamma_1, R_1; R_2, E_1 \vdash \pi_1; \pi_2 : \Gamma_3, E_3} \\
\\
\text{CONSTRAINT} \\
\frac{\Gamma_1, R, E_1 \vdash \pi : \Gamma_2, E_2 \quad E'_1 \models E'_1 \quad E_2 \models E'_2}{\Gamma_1, R, E'_1 \vdash \pi : \Gamma_2, E'_2} \\
\\
\text{MAC} \\
\frac{E \models m[\text{wm}] @ t = m[\text{wk}] @ t + (m[\text{delta}] @ t * m[\text{ws}] @ t) \quad \Gamma(m[\text{ws}] @ t) = T \cdot \zeta}{\Gamma, R, E \vdash \text{assert}(m[\text{wm}] = m[\text{wk}] + (m[\text{delta}] * m[\text{ws}])) @ t : \Gamma; m[\text{ws}] @ t : T \cdot \text{High}, E}
\end{array}$$

4 Prelude SYNTAX AND SEMANTICS

$$\begin{array}{l}
\ell \in \text{Field}, y \in \text{EVar}, f \in \text{FName} \\
e ::= v \mid r[e] \mid s[e] \mid m[e] \mid p[e] \mid e \text{ binop } e \mid \text{let } y = e \text{ in } e \mid \\
\quad f(e, \dots, e) \mid \{\ell = e; \dots; \ell = e\} \mid e.\ell \\
c ::= m[e] @ e := e @ e \mid p[e] := e @ e \mid \text{out} @ e := e @ e \mid \text{assert}(e = e) @ e \mid \\
\quad f(e, \dots, e) \mid c; c \mid \text{pre}(E) \mid \text{post}(E) \\
\text{binop} ::= + \mid - \mid * \mid ++ \\
v ::= w \mid t \mid \varepsilon \mid \{\ell = v; \dots; \ell = v\} \\
fn ::= f(y, \dots, y)\{e\} \mid f(y, \dots, y)\{c\} \\
\phi ::= r[e] @ e \mid s[e] @ e \mid m[e] @ e \mid p[e] \mid \text{out} @ e \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi \\
E ::= \phi \equiv \phi \mid E \wedge E
\end{array}$$

$$\frac{e[v/y] \Rightarrow v'}{\text{let } y = v \text{ in } e \Rightarrow v'}$$

$$\frac{C(f) = y_1, \dots, y_n, e \quad e_1 \Rightarrow v_1 \dots e_n \Rightarrow v_n \quad e[v_1/y_1] \dots [v_n/y_n] \Rightarrow v}{f(e_1, \dots, e_n) \Rightarrow v}$$

$$\frac{e_1 \Rightarrow v_1 \dots e_n \Rightarrow v_n}{\{\ell_1 = e_1; \dots; \ell_n = e_n\} \Rightarrow \{\ell_1 = v_1; \dots; \ell_n = v_n\}} \quad \frac{e \Rightarrow \{\dots; \ell = v; \dots\}}{e.\ell \Rightarrow v} \quad \frac{e_1 \Rightarrow w_1 \quad e_2 \Rightarrow w_2}{e_1 ++ e_2 \Rightarrow w_1 w_2}$$

$$\begin{array}{c}
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{on}, \text{assert}(e_1 = e_2)@e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2)@_{\iota}, (E_1, E_2 \wedge \lfloor \varepsilon_1 @_{\iota} \rfloor = \lfloor \varepsilon_2 @_{\iota} \rfloor), \text{on})} \\
\\
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{off}, \text{assert}(e_1 = e_2)@e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2)@_{\iota}, (E_1, E_2, \text{off}))} \\
\\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{on}, m[e_1]@_{e_2} := e_3@_{e_4}) \Rightarrow (\pi; m[w]@_{\iota_1} := \varepsilon@_{\iota_2}, (E_1 \wedge m[w]@_{\iota_1} = \lfloor \varepsilon@_{\iota_2} \rfloor, E_2), \text{on})} \\
\\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{off}, m[e_1]@_{e_2} := e_3@_{e_4}) \Rightarrow (\pi; m[w]@_{\iota_1} := \varepsilon@_{\iota_2}, (E_1, E_1), \text{off})} \\
\\
(\pi, (E_1, E_2), \text{on}, \text{pre}(E)) \Rightarrow (\pi, E_1, E_2 \wedge E, \text{off}) \\
\\
(\pi, (E_1, E_2), \text{off}, \text{post}(E)) \Rightarrow (\pi, (E_1 \wedge E, E_2), \text{on}) \\
\\
\frac{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}_1) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2) \quad (\pi_2, (E_{21}, E_{22}), \text{sw}_2, \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), \text{sw}_3)}{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}_1; \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), \text{sw}_3)} \\
\\
\frac{e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad C(f) = y_1, \dots, y_n, \mathbf{c} \quad (\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}[\lfloor v_1/y_1 \rfloor \cdots \lfloor v_n/y_n \rfloor]) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2)}{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, f(e_1, \dots, e_n)) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2)}
\end{array}$$

5 EXAMPLES

```

148 encodegmw(in, i1, i2) {
149   m[in]@i2 := (s[in] xor r[in])@i2;
150   m[in]@i1 := r[in]@i2
151 }
152
153 andtablegmw(b1, b2, r) {
154   let r11 = r xor (b1 xor true) and (b2 xor true) in
155   let r10 = r xor (b1 xor true) and (b2 xor false) in
156   let r01 = r xor (b1 xor false) and (b2 xor true) in
157   let r00 = r xor (b1 xor false) and (b2 xor false) in
158   { row1 = r11; row2 = r10; row3 = r01; row4 = r00 }
159 }
160
161 andgmw(z, x, y) {
162   pre();
163   let r = r[z] in
164   let table = andtablegmw(m[x], m[y], r) in
165   m[z]@2 := OT4(m[x], m[y], table, 2, 1);
166   m[z]@1 := r@1;
167   post(m[z]@1 xor m[z]@2 == (m[x]@1 xor m[x]@2) and (m[y]@1 xor m[y]@2))
168 }

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197     }
198
199     xorgmw(z, x, y) {
200         m[z]@1 := (m[x] xor m[y])@1; m[z]@2 := (m[x] xor m[y])@2;
201     }
202
203     decodegmw(z) {
204         p["1"] := m[z]@1; p["2"] := m[z]@2;
205         out@1 := (p["1"] xor p["2"])@1;
206         out@2 := (p["1"] xor p["2"])@2
207     }
208
209     encodegmw("x",2,1);
210     encodegmw("y",2,1);
211     encodegmw("z",1,2);
212     andgmw("g1", "x", "z");
213     xorgmw("g2", "g1", "y");
214     decodegmw("g2")
215     pre();
216     post(out@1 == (s["x"]@1 and s["z"]@2) xor s["y"]@1)
217
218
219     secopen(w1,w2,w3,i1,i2) {
220         pre(m[w1++"m"]@i2 == m[w1++"k"]@i1 + (m["delta"]@i1 * m[w1++"s"]@i2 /\
221             m[w1++"m"]@i2 == m[w1++"k"]@i1 + (m["delta"]@i1 * m[w1++"s"]@i2));
222         let locsum = macsum(macshare(w1), macshare(w2)) in
223         m[w3++"s"]@i1 := (locsum.share)@i2;
224         m[w3++"m"]@i1 := (locsum.mac)@i2;
225         auth(m[w3++"s"],m[w3++"m"],mack(w1) + mack(w2),i1);
226         m[w3]@i1 := (m[w3++"s"] + (locsum.share))@i1
227     }
228
229
230     _open(x,i1,i2){
231         m[x++"exts"]@i1 := m[x++"s"]@i2;
232         m[x++"extm"]@i1 := m[x++"m"]@i2;
233         assert(m[x++"extm"] == m[x++"k"] + (m["delta"] * m[x++"exts"]));
234         m[x]@i1 := (m[x++"exts"] + m[x++"s"]@i2
235     }`
236
237     _sum(z, x, y,i1,i2) {
238         pre(m[x++"m"]@i2 == m[x++"k"]@i1 + (m["delta"]@i1 * m[x++"s"]@i2 /\
239             m[y++"m"]@i2 == m[y++"k"]@i1 + (m["delta"]@i1 * m[y++"s"]@i2));
240         m[z++"s"]@i2 := (m[x++"s"] + m[y++"s"]@i2;
241         m[z++"m"]@i2 := (m[x++"m"] + m[y++"m"]@i2;
242         m[z++"k"]@i1 := (m[x++"k"] + m[y++"k"]@i1;
243         post(m[z++"m"]@i2 == m[z++"k"]@i1 + (m["delta"]@i1 * m[z++"s"]@i2)
244     }
245

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246
247 sum(z,x,y) { _sum(z,x,y,1,2);_sum(z,x,y,2,1) }
248
249 open(x) { _open(x,1,2); _open(x,2,1) }
250
251
252 sum("a","x","d");
253 open("d");
254 sum("b","y","e");
255 open("e");
256 let xys =
257     macsum(macctimes(macshare("b"), m["d"]),
258           macsum(macctimes(macshare("a"), m["e"]),
259                 macshare("c")))
260 let xyk = mack("b") * m["d"] + mack("a") * m["e"] + mack("c")
261
262 secopen("a","x","d",1,2);
263 secopen("a","x","d",2,1);
264 secopen("b","y","e",1,2);
265 secopen("b","y","e",2,1);
266 let xys =
267     macsum(macctimes(macshare("b"), m["d"]),
268           macsum(macctimes(macshare("a"), m["e"]),
269                 macshare("c")))
270 in
271 let xyk = mack("b") * m["d"] + mack("d") * m["d"] + mack("c")
272 in
273 secreveal(xys,xyk,"1",1,2);
274 secreveal(maccsum(xys,m["d"] * m["e"]),
275           xyk - m["d"] * m["e"],
276           "2",2,1);
277 out@1 := (p[1] + p[2])@1;
278 out@2 := (p[1] + p[2])@2;
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