

1 Overture SYNTAX AND SEMANTICS

$v \in \mathbb{F}_p$, $w \in \text{String}$, $\iota \in \text{Clients} \subset \mathbb{N}$

$\varepsilon ::= r[w] \mid s[w] \mid m[w] \mid p[w] \mid v \mid \varepsilon - \varepsilon \mid \varepsilon + \varepsilon \mid \varepsilon * \varepsilon$ *expressions*

$x ::= r[w]@_\iota \mid s[w]@_\iota \mid m[w]@_\iota \mid p[w] \mid \text{out}_\iota$ *variables*

$\pi ::= m[w]@_\iota := \varepsilon@_\iota \mid p[w] := e@_\iota \mid \text{out}_\iota := \varepsilon@_\iota \mid \pi; \pi$ *protocols*

$$\begin{aligned} \llbracket \sigma, v \rrbracket_\iota &= v \\ \llbracket \sigma, \varepsilon_1 + \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota + \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 - \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota - \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 * \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota * \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, r[w] \rrbracket_\iota &= \sigma(r[w]@_\iota) \\ \llbracket \sigma, s[w] \rrbracket_\iota &= \sigma(s[w]@_\iota) \\ \llbracket \sigma, m[w] \rrbracket_\iota &= \sigma(m[w]@_\iota) \\ \llbracket \sigma, p[w] \rrbracket_\iota &= \sigma(p[w]) \end{aligned}$$

$$\frac{(\sigma, x := \varepsilon@_\iota) \Rightarrow \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} \quad \frac{(\sigma_1, \pi_1) \Rightarrow \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow \sigma_3}{(\sigma_1, \pi_1; \pi_2) \Rightarrow \sigma_3}}{(\sigma_1, \pi_1; \pi_2) \Rightarrow \sigma_3}$$

2 Overture ADVERSARIAL SEMANTICS

$$(\sigma, x := \varepsilon@_\iota) \Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} \quad \iota \in H$$

$$(\sigma, x := \varepsilon@_\iota) \Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \text{rewrite}_{\mathcal{A}}(\sigma_C, \varepsilon) \rrbracket_\iota\} \quad \iota \in C$$

$$(\sigma, \text{assert}(\varepsilon_1 = \varepsilon_2)@_\iota) \Rightarrow_{\mathcal{A}} \sigma \quad \text{if } \llbracket \sigma, \varepsilon_1 \rrbracket_\iota = \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \text{ or } \iota \in C$$

$$(\sigma, \text{assert}(\phi(\varepsilon))@_\iota) \Rightarrow_{\mathcal{A}} \perp \quad \text{if } \neg\phi(\sigma, \llbracket \sigma, \varepsilon \rrbracket_\iota)$$

$$\frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow_{\mathcal{A}} \sigma_3}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \sigma_3} \quad \frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \perp}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \perp}$$

$$\frac{(\sigma_1, \pi_1) \Rightarrow_{\mathcal{A}} \sigma_2 \quad (\sigma_2, \pi_2) \Rightarrow_{\mathcal{A}} \perp}{(\sigma_1, \pi_1; \pi_2) \Rightarrow_{\mathcal{A}} \perp}$$

3 Overture CONSTRAINT TYPING

$$\phi ::= x \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi$$

$$E ::= \phi \equiv \phi \mid E \wedge E$$

We write $E_1 \models E_2$ iff every model of E_1 is a model of E_2 . Note that this relation is reflexive and transitive.

$$\lfloor x \rfloor = x \quad \lfloor \varepsilon_1 + \varepsilon_2 @_\iota \rfloor = \lfloor \varepsilon_2 @_\iota \rfloor + \lfloor \varepsilon_1 @_\iota \rfloor \quad \lfloor \varepsilon_1 - \varepsilon_2 @_\iota \rfloor = \lfloor \varepsilon_2 @_\iota \rfloor - \lfloor \varepsilon_1 @_\iota \rfloor$$

$$\lfloor \varepsilon_1 * \varepsilon_2 @_\iota \rfloor = \lfloor \varepsilon_2 @_\iota \rfloor * \lfloor \varepsilon_1 @_\iota \rfloor$$

$$\lfloor \text{OT}(\varepsilon_1 @_{\iota_1}, \varepsilon_2, \varepsilon_3) @_{\iota_2} \rfloor = (\lfloor \varepsilon_1 @_{\iota_1} \rfloor \wedge \lfloor \varepsilon_3 @_{\iota_2} \rfloor) \vee (\neg \lfloor \varepsilon_1 @_{\iota_1} \rfloor \wedge \lfloor \varepsilon_2 @_{\iota_2} \rfloor)$$

$$\lfloor x := \varepsilon @_\iota \rfloor = x \equiv \lfloor \varepsilon @_\iota \rfloor$$

$$\lfloor \pi_1; \pi_2 \rfloor = \lfloor \pi_1 \rfloor \wedge \lfloor \pi_2 \rfloor$$

The motivating idea is that we can interpret any protocol π as a set of equality constraints $\lfloor \pi \rfloor$ and use an SMT solver to verify properties relevant to correctness, confidentiality, and integrity. Further, we can leverage entailment relation is critical for efficiency– we can use annotations to obtain a weakened precondition for relevant properties. That is, given π , program annotations or other cues can be used to find a minimal E with $\lfloor \pi \rfloor \models E$ for verifying correctness and security.

3.0.1 Example: Correctness of 3-Party Addition.

```

m[s1]@2 := (s[1] - r[local] - r[x])@1
m[s1]@3 := r[x]@1
m[s2]@1 := (s[2] - r[local] - r[x])@2
m[s2]@3 := r[x]@2
m[s3]@1 := (s[3] - r[local] - r[x])@3
m[s3]@2 := r[x]@3
p[1] := (r[local] + m[s2] + m[s3])@1
p[2] := (m[s1] + r[local] + m[s3])@2
p[3] := (m[s1] + m[s2] + r[local])@3
out@1 := (p[1] + p[2] + p[3])@1
out@2 := (p[1] + p[2] + p[3])@2
out@3 := (p[1] + p[2] + p[3])@3

```

Letting π be this protocol, we can verify correctness as:

$$\lfloor \pi \rfloor \models \text{out}@3 \equiv s[1]@1 + s[2]@2 + s[3]@3$$

3.1 Confidentiality Types

$$\begin{array}{c}
\text{DEFTY} \\
\frac{}{\emptyset, E \vdash \phi : \text{vars}(\phi)}
\end{array}
\quad
\frac{\text{ENCODE} \quad E \models \phi \equiv \phi' \oplus r[w]@_i \quad \oplus \in \{+, -\} \quad R, E \vdash \phi' : T}{R; \{r[w]@_i\}, E \vdash \phi : \{c(r[w]@_i, T)\}}$$

$$\begin{array}{c}
\text{SEND} \\
\frac{R, E \vdash \lfloor \varepsilon @_i \rfloor : T}{R, E \vdash x := \varepsilon @_i : (x : T)}
\end{array}
\quad
\frac{\text{SEQ} \quad \frac{R_1, E \vdash \pi_1 : \Gamma_1 \quad R_2, E \vdash \pi_2 : \Gamma_2}{R_1; R_2, E \vdash \pi_1; \pi_2 : \Gamma_1; \Gamma_2}}{}$$

Definition 3.1. $R, E \vdash \pi : \Gamma$ is *valid* iff it is derivable and $\lfloor \pi \rfloor \models E$.

$$\begin{array}{c}
\frac{i \in C}{\Gamma, C \vdash \Gamma(m[w]@_i)} \quad \frac{\Gamma, C \vdash T_1 \cup T_2}{\Gamma, C \vdash T_1} \quad \frac{\Gamma, C \vdash \{m[w]@_i\}}{\Gamma, C \vdash \Gamma(m[w]@_i)} \\
\frac{\Gamma, C \vdash \{r[w]@_i\} \quad \Gamma, C \vdash \{c(r[w]@_i, T)\}}{\Gamma, C \vdash T}
\end{array}$$

THEOREM 3.2. *If $R, E \vdash \pi : \Gamma$ is valid and for all H, C it is not the case that $\Gamma, C \vdash \{s[w]@_i\}$ for $i \in H$, then π satisfies gradual release.*

3.1.1 Examples.

```

m[s1]@2 := (s[1] - r[local] - r[x])@1
m[s1]@3 := r[x]@1

// m[s1]@2 : { c(r[x]@1, { c(r[local]@1, {s[1]@1} ) ) }
// m[s1]@3 : { r[x]@1 }

```

```

50 m[x]@1 := s2(s[x],-r[x],r[x])@2
51
52 // m[x]@1 == s[x]@2 + -r[x]@2
53 // m[x]@1 : { c(r[x]@2, { s[x]@2 }) }
54
55 m[y]@1 := OT(s[y]@1,-r[y],r[y])@2
56
57 // m[y]@1 == s[y]@1 + -r[y]@2
58 // m[y]@1 : { c(r[y]@2, { s[y]@1 }) }
59

```

3.2 Compositional Type Verification in *Prelude*

(*Need to fix the following to allow reduction of x . – Chris*)

$$\begin{array}{c}
\text{MMSG} \\
\frac{e_1 \Rightarrow \varepsilon \quad e_2 \Rightarrow \iota \quad R_1, E \Vdash [\varepsilon @ \iota] : (R_2, T)}{R_1, E \vdash x := e_1 @ e_2 : (x : T, R_1; R_2, E \wedge x \equiv [\varepsilon @ \iota])} \\
\\
\text{ENCODE} \\
\frac{e_1 \Rightarrow \varepsilon \quad e_2 \Rightarrow \iota \quad e_3 \Rightarrow \phi \quad E \models [\varepsilon @ \iota] \equiv \phi \quad R_1, E \Vdash \phi : (R_2, T)}{R_1, E \vdash x := e_1 @ e_2 \text{ as } e_3 : (x : T, R_1; R_2, E \wedge x \equiv \phi)} \\
\\
\text{APP} \\
\frac{\text{sig}(f) = \{E_1\} x_1, \dots, x_n \{\Gamma, R, E_2\} \quad e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad \rho = [v_1/x_1] \cdots [v_n/x_n] \quad E \models \rho(E_1)}{R_1, E \vdash f(e_1, \dots, e_n) : (\rho(\Gamma), R_1; \rho(R), E \wedge \rho(E_2))} \\
\\
\text{SEQ} \\
\frac{R_1, E_1 \vdash \pi_1 : (\Gamma_2, R_2, E_2) \quad R_2, E_2 \vdash \pi_2 : (\Gamma_3, R_3, E_3)}{R_1, E_1 \vdash \pi_1; \pi_2 : (\Gamma_2; \Gamma_3, R_3, E_3)} \\
\\
\text{SIG} \\
\frac{\rho = [v_1/x_1] \cdots [v_n/x_n] \quad C(f) = x_1, \dots, x_n, \mathbf{c} \quad \emptyset, \rho(E_1) \vdash \rho(\mathbf{c}) : (\rho(\Gamma), \rho(R), E) \quad E \models \rho(E_2)}{f : \{E_1\} x_1, \dots, x_n \{\Gamma, R, E_2\}}
\end{array}$$

Definition 3.3. sig is verified iff $f : \text{sig}(f)$ is valid for all $f \in \text{dom}(\text{sig})$.

The following theorem holds for protocols with default preprocessing.

THEOREM 3.4. If sig is verified and $\emptyset, \emptyset \vdash e : (\Gamma, R, E)$ then $e \Rightarrow \pi$ and $R, E \vdash \pi : \Gamma$ is valid.

3.2.1 Examples.

```

90 andtableygc(g,x,y)
91 {
92   let table = (~r[g],~r[g],~r[g],r[g])
93   in permute4(r[x],r[y],table)
94 }
95
96 m[x]@1 := s2(s[x],r[x],~r[x])@2 as s[x]@2 xor r[x]@2
97
98

```

```

99 // m[x]@1 : { c(r[x]@2, { s[x]@2 }) }
100
101 m[y]@1 := OT(s[y]@1,r[y],~r[y])@2 as s[y]@1 xor r[y]@2;
102
103 // m[y]@1 : { c(r[y]@2, { s[y]@1 }) }
104
105 m[ag]@1 := OT4(m[x]@1, m[y]@1, andtable(ag,r[x],r[y]))@2
106   as ~((r[x]@2 = m[x]@1) and (r[y]@2 = m[y]@1)) xor r[ag]@2
107
108 // m[ag]@1 : { c(r[ag]@2, {r[x]@2, r[y]@2, m[x]@1, m[y]@1}) }
109
110 p[o] := OT2(m[ag]@1, perm2(r[ag],(false,true)))@2
111
112 // p[o] : { c(r[ag]@2, {r[x]@2, r[y]@2, m[x]@1, m[y]@1}), r[ag]@2 }
113
114 out@1 := p[o]@1
115
116 // out@1 == s[x] and s[y]
117
118   encodegmw(in, i1, i2) {
119     m[in]@i2 := (s[in] xor r[in])@i1;
120     m[in]@i1 := r[in]@i1
121   }
122
123   andtablegmw(x, y, z) {
124     let r11 = r[z] xor (m[x] xor true) and (m[y] xor true) in
125     let r10 = r[z] xor (m[x] xor true) and (m[y] xor false) in
126     let r01 = r[z] xor (m[x] xor false) and (m[y] xor true) in
127     let r00 = r[z] xor (m[x] xor false) and (m[y] xor false) in
128     { row1 = r11; row2 = r10; row3 = r01; row4 = r00 }
129   }
130
131   andgmw(z, x, y) {
132     let table = andtablegmw(x,y,z) in
133     m[z]@2 := OT4(m[x],m[y],table,2,1)
134     as ~((m[x]@1 xor m[x]@2) and (m[y]@1 xor m[y]@2)) xor r[z]@1;
135     m[z]@1 := r[z]@1
136   }
137
138 // and gate correctness postcondition
139 { } andgmw { m[z]@1 xor m[z]@2 == (m[x]@1 xor m[x]@2) and (m[y]@1 xor m[y]@2) }
140
141 // and gate type
142 andgmw :
143   Pi z,x,y .
144   { }
145   { { r[z]@1 },
146     (m[z]@1 : { r[z]@1 }; m[z]@2 : {c(r[z]@1, { m[x]@1, m[x]@2, m[y]@1, m[y]@2 })}) },
147

```

```

148     m[z]@1 xor m[z]@2 == (m[x]@1 xor m[x]@2) and (m[y]@1 xor m[y]@2)}
149
150 xorgmw(z, x, y) {
151     m[z]@1 := (m[x] xor m[y])@1; m[z]@2 := (m[x] xor m[y])@2;
152 }
153
154 decodegmw(z) {
155     p["1"] := m[z]@1; p["2"] := m[z]@2;
156     out@1 := (p["1"] xor p["2"])@1;
157     out@2 := (p["1"] xor p["2"])@2
158 }
159
160 prot() {
161     encodegmw("x", 2, 1);
162     encodegmw("y", 2, 1);
163     encodegmw("z", 1, 2);
164     andgmw("g1", "x", "z");
165     xorgmw("g2", "g1", "y");
166     decodegmw("g2")
167 }
168
169 {} prot { out@1 == (s["x"]@1 and s["z"]@2) xor s["y"]@1 }

```

3.3 Integrity Types

181	VALUE	SECRET	RANDO
182	$\Gamma, \emptyset, E \vdash v : \emptyset \cdot \text{High}$	$\Gamma, \emptyset, E \vdash s[w] : \{s[w]@_i\} \cdot \mathcal{L}(i)$	$\Gamma, \emptyset, E \vdash r[w] : \{r[w]@_i\} \cdot \mathcal{L}(i)$
184	MESG	PUBM	INTEGRITYWEAKEN
185	$\Gamma, \emptyset, E \vdash m[w] : \Gamma(m[w]@_i)$	$\Gamma, \emptyset, E \vdash p[w] : \Gamma(p[w])$	$\frac{\Gamma, R, E \vdash_i \varepsilon : T \cdot \zeta_1 \quad \zeta_1 \leq \zeta_2}{\Gamma, R, E \vdash_i \varepsilon : T \cdot \zeta_2}$
188	$\frac{\text{ENCODE} \quad \Gamma, \emptyset, E \vdash_i \varepsilon : T \cdot \zeta \quad E \models [\varepsilon@_i] = \phi \oplus r[w]@_{i'} \quad \oplus \in \{+, -\}}{\Gamma, r[w]@_i, E \vdash_i \varepsilon : \{c(r[w]@_{i'}, \Gamma(\phi))\} \cdot \zeta}$		
192	$\frac{\text{BINOP} \quad \Gamma, R_1, E \vdash_i \varepsilon_1 : T_1 \cdot \zeta \quad \Gamma, R_2, E \vdash_i \varepsilon_2 : T_2 \cdot \zeta \quad \oplus \in \{+, -, *\}}{\Gamma, R_1; R_2, E \vdash_i \varepsilon_1 \oplus \varepsilon_2 : T_1 \cup T_2 \cdot \zeta}$		

$$\begin{array}{c}
\text{SEND} \\
\frac{\Gamma, R, E \vdash_t \varepsilon : T \cdot \mathcal{L}(t) \quad E' \models E \wedge x = \lfloor \varepsilon @ t \rfloor}{\Gamma, R, E \vdash x := \varepsilon @ t : \Gamma; x : T \cdot \mathcal{L}(t), E'} \\
\\
\text{ASSERT} \\
\frac{E \models \lfloor \varepsilon_1 @ t \rfloor = \lfloor \varepsilon_2 @ t \rfloor}{\Gamma, R, E \vdash \text{assert}(\varepsilon_1 = \varepsilon_2) @ t : \Gamma, E} \\
\\
\text{SEQ} \\
\frac{\Gamma_1, R_1, E_1 \vdash \pi_1 : \Gamma_2, E_2 \quad \Gamma_2, R_2, E_2 \vdash \pi_2 : \Gamma_3, E_3}{\Gamma_1, R_1; R_2, E_1 \vdash \pi_1; \pi_2 : \Gamma_3, E_3} \\
\\
\text{CONSTRAINT} \\
\frac{\Gamma_1, R, E_1 \vdash \pi : \Gamma_2, E_2 \quad E'_1 \models E'_1 \quad E_2 \models E'_2}{\Gamma_1, R, E'_1 \vdash \pi : \Gamma_2, E'_2} \\
\\
\text{MAC} \\
\frac{E \models m[\text{wm}] @ t = m[\text{wk}] @ t + (m[\text{delta}] @ t * m[\text{ws}] @ t) \quad \Gamma(m[\text{ws}] @ t) = T \cdot \zeta}{\Gamma, R, E \vdash \text{assert}(m[\text{wm}] = m[\text{wk}] + (m[\text{delta}] * m[\text{ws}])) @ t : \Gamma; m[\text{ws}] @ t : T \cdot \text{High}, E}
\end{array}$$

4 Prelude SYNTAX AND SEMANTICS

$$\begin{array}{l}
\ell \in \text{Field}, y \in \text{EVar}, f \in \text{FName} \\
e ::= v \mid r[e] \mid s[e] \mid m[e] \mid p[e] \mid e \text{ binop } e \mid \text{let } y = e \text{ in } e \mid \\
\quad f(e, \dots, e) \mid \{\ell = e; \dots; \ell = e\} \mid e.\ell \\
c ::= m[e] @ e := e @ e \mid p[e] := e @ e \mid \text{out} @ e := e @ e \mid \text{assert}(e = e) @ e \mid \\
\quad f(e, \dots, e) \mid c; c \mid \text{pre}(E) \mid \text{post}(E) \\
\text{binop} ::= + \mid - \mid * \mid ++ \\
v ::= w \mid t \mid \varepsilon \mid \{\ell = v; \dots; \ell = v\} \\
fn ::= f(y, \dots, y)\{e\} \mid f(y, \dots, y)\{c\} \\
\phi ::= r[e] @ e \mid s[e] @ e \mid m[e] @ e \mid p[e] \mid \text{out} @ e \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi \\
E ::= \phi \equiv \phi \mid E \wedge E
\end{array}$$

$$\frac{e[v/y] \Rightarrow v'}{\text{let } y = v \text{ in } e \Rightarrow v'}$$

$$\frac{C(f) = y_1, \dots, y_n, e \quad e_1 \Rightarrow v_1 \dots e_n \Rightarrow v_n \quad e[v_1/y_1] \dots [v_n/y_n] \Rightarrow v}{f(e_1, \dots, e_n) \Rightarrow v}$$

$$\frac{e_1 \Rightarrow v_1 \dots e_n \Rightarrow v_n}{\{\ell_1 = e_1; \dots; \ell_n = e_n\} \Rightarrow \{\ell_1 = v_1; \dots; \ell_n = v_n\}} \quad \frac{e \Rightarrow \{\dots; \ell = v; \dots\}}{e.\ell \Rightarrow v} \quad \frac{e_1 \Rightarrow w_1 \quad e_2 \Rightarrow w_2}{e_1 ++ e_2 \Rightarrow w_1 w_2}$$

$$\begin{array}{c}
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{on}, \text{assert}(e_1 = e_2)@e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2)@e, (E_1, E_2 \wedge \lfloor \varepsilon_1 @ \iota \rfloor = \lfloor \varepsilon_2 @ \iota \rfloor), \text{on})} \\
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{off}, \text{assert}(e_1 = e_2)@e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2)@e, (E_1, E_2, \text{off}))} \\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{on}, m[e_1]@e_2 := e_3@e_4) \Rightarrow (\pi; m[w]@_{\iota_1} := \varepsilon@_{\iota_2}, (E_1 \wedge m[w]@_{\iota_1} = \lfloor \varepsilon @_{\iota_2} \rfloor, E_2), \text{on})} \\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{off}, m[e_1]@e_2 := e_3@e_4) \Rightarrow (\pi; m[w]@_{\iota_1} := \varepsilon@_{\iota_2}, (E_1, E_1), \text{off})} \\
(\pi, (E_1, E_2), \text{on}, \text{pre}(E)) \Rightarrow (\pi, E_1, E_2 \wedge E, \text{off}) \\
(\pi, (E_1, E_2), \text{off}, \text{post}(E)) \Rightarrow (\pi, (E_1 \wedge E, E_2), \text{on}) \\
\frac{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}_1) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2) \quad (\pi_2, (E_{21}, E_{22}), \text{sw}_2, \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), \text{sw}_3)}{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}_1; \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), \text{sw}_3)} \\
\frac{e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad C(f) = y_1, \dots, y_n, \mathbf{c} \quad (\pi_1, (E_{11}, E_{12}), \text{sw}_1, \mathbf{c}[\lfloor v_1 / y_1 \rfloor \cdots \lfloor v_n / y_n \rfloor]) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2)}{(\pi_1, (E_{11}, E_{12}), \text{sw}_1, f(e_1, \dots, e_n)) \Rightarrow (\pi_2, (E_{21}, E_{22}), \text{sw}_2)}
\end{array}$$

5 EXAMPLES

```

secopen(w1,w2,w3,i1,i2) {
  pre(m[w1++"m"]@i2 == m[w1++"k"]@i1 + (m["delta"]@i1 * m[w1++"s"]@i2 /\
    m[w1++"m"]@i2 == m[w1++"k"]@i1 + (m["delta"]@i1 * m[w1++"s"]@i2)));
  let locsum = macsum(macshare(w1), macshare(w2)) in
  m[w3++"s"]@i1 := (locsum.share)@i2;
  m[w3++"m"]@i1 := (locsum.mac)@i2;
  auth(m[w3++"s"],m[w3++"m"],mack(w1) + mack(w2),i1);
  m[w3]@i1 := (m[w3++"s"] + (locsum.share))@i1
}

_open(x,i1,i2){
  m[x++"exts"]@i1 := m[x++"s"]@i2;
  m[x++"extm"]@i1 := m[x++"m"]@i2;
  assert(m[x++"extm"] == m[x++"k"] + (m["delta"] * m[x++"exts"]));
  m[x]@i1 := (m[x++"exts"] + m[x++"s"]@i2)@i2
}

_sum(z, x, y,i1,i2) {
  pre(m[x++"m"]@i2 == m[x++"k"]@i1 + (m["delta"]@i1 * m[x++"s"]@i2 /\

```

```

295         m[y++"m"]@i2 == m[y++"k"]@i1 + (m["delta"]@i1 * m[y++"s"]@i2));
296     m[z++"s"]@i2 := (m[x++"s"] + m[y++"s"]@i2);
297     m[z++"m"]@i2 := (m[x++"m"] + m[y++"m"]@i2);
298     m[z++"k"]@i1 := (m[x++"k"] + m[y++"k"]@i1);
299     post(m[z++"m"]@i2 == m[z++"k"]@i1 + (m["delta"]@i1 * m[z++"s"]@i2)
300 }
301
302 sum(z,x,y) { _sum(z,x,y,1,2);_sum(z,x,y,2,1) }
303
304 open(x) { _open(x,1,2); _open(x,2,1) }
305
306
307 sum("a","x","d");
308 open("d");
309 sum("b","y","e");
310 open("e");
311 let xys =
312     macsum(macctimes(macshare("b"), m["d"]),
313         macsum(macctimes(macshare("a"), m["e"]),
314             macshare("c")))
315 let xyk = mack("b") * m["d"] + mack("a") * m["e"] + mack("c")
316
317 secopen("a","x","d",1,2);
318 secopen("a","x","d",2,1);
319 secopen("b","y","e",1,2);
320 secopen("b","y","e",2,1);
321 let xys =
322     macsum(macctimes(macshare("b"), m["d"]),
323         macsum(macctimes(macshare("a"), m["e"]),
324             macshare("c")))
325 in
326 let xyk = mack("b") * m["d"] + mack("d") * m["d"] + mack("c")
327 in
328 secreveal(xys,xyk,"1",1,2);
329 secreveal(maccsum(xys,m["d"] * m["e"]),
330     xyk - m["d"] * m["e"],
331     "2",2,1);
332 out@1 := (p[1] + p[2])@1;
333 out@2 := (p[1] + p[2])@2;
334
335
336
337
338
339
340
341
342
343

```