1 Overture SYNTAX AND SEMANTICS

$$v \in \mathbb{F}_p$$
, $w \in \text{String}$, $\iota \in \text{Clients} \subset \mathbb{N}$

$$\varepsilon ::= r[w] | s[w] | m[w] | p[w] | expressions$$
$$v | \varepsilon - \varepsilon | \varepsilon + \varepsilon | \varepsilon * \varepsilon$$

$$x ::= r[w]@i | s[w]@i | m[w]@i | p[w] | out@i$$
 variables

$$\pi ::= m[w]@\iota := \varepsilon @\iota \mid p[w] := e@\iota \mid out@\iota := \varepsilon @\iota \mid \pi; \pi \quad protocols$$

$$\begin{split} & \llbracket \sigma, v \rrbracket_{\iota} &= v \\ & \llbracket \sigma, \varepsilon_{1} + \varepsilon_{2} \rrbracket_{\iota} &= \llbracket \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} + \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \rrbracket \\ & \llbracket \sigma, \varepsilon_{1} - \varepsilon_{2} \rrbracket_{\iota} &= \llbracket \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} - \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \rrbracket \\ & \llbracket \sigma, \varepsilon_{1} * \varepsilon_{2} \rrbracket_{\iota} &= \llbracket \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} * \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \rrbracket \\ & \llbracket \sigma, r[w] \rrbracket_{\iota} &= \sigma(r[w]@\iota) \\ & \llbracket \sigma, s[w] \rrbracket_{\iota} &= \sigma(s[w]@\iota) \\ & \llbracket \sigma, m[w] \rrbracket_{\iota} &= \sigma(m[w]@\iota) \\ & \llbracket \sigma, p[w] \rrbracket_{\iota} &= \sigma(p[w]) \end{split}$$

$$(\sigma, x := \varepsilon \mathfrak{Q}_{l}) \Rightarrow \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_{l}\} \qquad \frac{(\sigma_{1}, \pi_{1}) \Rightarrow \sigma_{2} \qquad (\sigma_{2}, \pi_{2}) \Rightarrow \sigma_{3}}{(\sigma_{1}, \pi_{1}; \pi_{2}) \Rightarrow \sigma_{3}}$$

1.1 Overture Adversarial Semantics

$$\pi ::= \cdots \mid \operatorname{assert}(\varepsilon = \varepsilon)$$

$$(\sigma, x := \varepsilon @ \iota) \implies_{\mathcal{A}} \sigma \{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_{\iota} \} \qquad \iota \in H$$

$$(\sigma, x := \varepsilon @ \iota) \implies_{\mathcal{A}} \sigma \{x \mapsto \llbracket \operatorname{rewrite}_{\mathcal{A}}(\sigma_{C}, \varepsilon) \rrbracket_{\iota} \} \qquad \iota \in C$$

$$(\sigma, \operatorname{assert}(\varepsilon_{1} = \varepsilon_{2}) @ \iota) \implies_{\mathcal{A}} \sigma \qquad \text{if } \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} = \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \text{ or } \iota \in C$$

$$(\sigma, \operatorname{assert}(\phi(\varepsilon)) @ \iota) \implies_{\mathcal{A}} \bot \qquad \text{if } \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} = \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \text{ or } \iota \in C$$

$$(\sigma_{1}, \operatorname{assert}(\phi(\varepsilon)) @ \iota) \implies_{\mathcal{A}} \bot \qquad \text{if } \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} = \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \text{ or } \iota \in C$$

$$(\sigma_{1}, \operatorname{assert}(\phi(\varepsilon)) @ \iota) \implies_{\mathcal{A}} \bot \qquad \text{if } \llbracket \sigma, \varepsilon_{1} \rrbracket_{\iota} = \llbracket \sigma, \varepsilon_{2} \rrbracket_{\iota} \text{ or } \iota \in C$$

$$(\sigma_{1}, \pi_{1}) \implies_{\mathcal{A}} \sigma_{2} \qquad (\sigma_{2}, \pi_{2}) \implies_{\mathcal{A}} \bot \qquad (\sigma_{1}, \pi_{1}; \pi_{2}) \implies_{\mathcal{A}} \bot$$

$$(\sigma_{1}, \pi_{1}; \pi_{2}) \implies_{\mathcal{A}} \bot$$

$$(\sigma_{1}, \pi_{1}; \pi_{2}) \implies_{\mathcal{A}} \bot$$

2 Overture CONSTRAINT TYPING

2.1 Constraint Satisfiability Modulo Finite Fields

$$\begin{array}{lll} \phi & ::= & x \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi \\ E & ::= & \phi \equiv \phi \mid E \wedge E \end{array}$$

We write $E_1 \models E_2$ iff every model of E_1 is a model of E_2 . Note that this relation is reflexive and transitive.

$$\lfloor \mathsf{OT}(\varepsilon_1 @ \iota_1, \varepsilon_2, \varepsilon_3) @ \iota_2 \rfloor = (\lfloor \varepsilon_1 @ \iota_1 \rfloor \land \lfloor \varepsilon_3 @ \iota_2 \rfloor) \lor (\neg \lfloor \varepsilon_1 @ \iota_1 \rfloor \land \lfloor \varepsilon_2 @ \iota_2 \rfloor)$$

$$|x := \varepsilon \Theta_{\ell}| = x \equiv |\varepsilon \Theta_{\ell}|$$
 | assert $(\varepsilon_1 = \varepsilon_2)_{\ell}| = |\varepsilon_1 \Theta_{\ell}| \equiv |\varepsilon_2 \Theta_{\ell}|$ | $|\pi_1; \pi_2| = |\pi_1| \wedge |\pi_2|$

The motivating idea is that we can interpret any protocol π as a set of equality constraints $\lfloor \pi \rfloor$ and use an SMT solver to verify properties relevant to correctness, confidentiality, and integrity. Further, we can leverage entailment relation is critical for efficiency—we can use annotations to obtain a weakened precondition for relevant properties. That is, given π , program annotations or other cues can be used to find a minimal E with $\lfloor \pi \rfloor \models E$ for verifying correctness and security.

2.1.1 Example: Correctness of 3-Party Addition.

$$\begin{array}{llll} \text{m}[s1]@2 & := & (s[1] - r[local] - r[x])@1 \\ \text{m}[s1]@3 & := & r[x]@1 \\ \text{m}[s2]@1 & := & (s[2] - r[local] - r[x])@2 \\ \text{m}[s2]@3 & := & r[x]@2 \\ \text{m}[s3]@1 & := & (s[3] - r[local] - r[x])@3 \\ \text{m}[s3]@2 & := & r[x]@3 \\ \text{p}[1] & := & (r[local] + m[s2] + m[s3])@1 \\ \text{p}[2] & := & (m[s1] + r[local] + m[s3])@2 \\ \text{p}[3] & := & (m[s1] + m[s2] + r[local])@3 \\ \text{out}@1 & := & (p[1] + p[2] + p[3])@1 \\ \text{out}@2 & := & (p[1] + p[2] + p[3])@2 \\ \text{out}@3 & := & (p[1] + p[2] + p[3])@3 \\ \end{array}$$

Letting π be this protocol, we can verify correctness as:

$$|\pi| \models \text{out@3} \equiv s[1]@1 + s[2]@2 + s[3]@3$$

2.2 Confidentiality Types

$$\begin{array}{cccc} t & ::= & x \mid c(x,T) \\ T & \in & 2^t \\ \Gamma & ::= & \varnothing \mid \Gamma; x:T \end{array}$$

Definition 2.1. R_1 ; $R_2 = R_1 \cup R_2$ iff $R_1 \cap R_2 = \emptyset$.

DEPTY
$$\emptyset, E \vdash \phi : vars(\phi)$$

$$E \vdash \phi \equiv \phi' \oplus r[w]@\iota \quad \oplus \in \{+, -\} \quad R, E \vdash \phi' : T$$

$$R; \{r[w]@\iota\}, E \vdash \phi : \{c(r[w]@\iota, T)\}$$

$$\frac{SEQ}{R, E \vdash \phi : T} \\ R, E \vdash x \equiv \phi : (x : T)$$

$$\frac{SEQ}{R_1, E \vdash \phi_1 : \Gamma_1 \qquad R_2, E \vdash \phi_2 : \Gamma_2}{R_1; R_2, E \vdash \phi_1 \land \phi_2 : \Gamma_1; \Gamma_2}$$

Definition 2.2. Given preprocessing predicate E_{pre} and protocol π we say $R, E \vdash E_{pre} \land \lfloor \pi \rfloor : \Gamma$ is valid iff it is derivable and $E_{pre} \land \lfloor \pi \rfloor \models E$.

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97 98 VALUE

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\frac{\Gamma, C \vdash_{leak} T_1 \cup T_2}{\Gamma, C \vdash_{leak} T_1} \qquad \frac{\Gamma, C \vdash_{leak} \{\mathfrak{m}[w]@l\}}{\Gamma, C \vdash_{leak} \Gamma(\mathfrak{m}[w]@l)}
                                              \frac{\Gamma, C \vdash_{leak} \{r[w]@l\} \qquad \Gamma, C \vdash_{leak} \{c(r[w]@l, T)\}}{\Gamma, C \vdash_{leak} T}
     THEOREM 2.3. If R, E \vdash E_{pre} \land \lfloor \pi \rfloor : \Gamma is valid and there exists no H, C and s[w]@i for i \in H with
\Gamma, C \vdash_{leak} \{s[w]@\iota\}, then \pi satisfies gradual release.
2.2.1 Examples.
m[s1]@2 := (s[1] - r[local] - r[x])@1
m[s1]@3 := r[x]@1
// m[s1]@2 : { c(r[x]@1, { c(r[local]@1, {s[1]@1} ) }
// m[s1]@3 : { r[x]@1 }
m[x]@1 := s2(s[x], -r[x], r[x])@2
// m[x]@1 == s[x]@2 + -r[x]@2
// m[x]@1 : { c(r[x]@2, { s[x]@2 }) }
m[y]@1 := OT(s[y]@1,-r[y],r[y])@2
// m[y]@1 == s[y]@1 + -r[y]@2
// m[y]@1 : { c(r[y]@2, { s[y]@1 }) }
2.3 Integrity Types
                                                                          \varsigma ::= High | Low \Gamma ::= \varnothing \mid \Gamma; x : \varsigma
                                          SECRET
                                                                                                                                         Mesg
  \Gamma, E \vdash_{\iota} v : \mathrm{High} \qquad \Gamma, E \vdash_{\iota} \mathtt{s[w]} : \mathcal{L}(\iota) \qquad \Gamma, E \vdash_{\iota} \mathtt{r[w]} : \mathcal{L}(\iota) \qquad \Gamma, E \vdash_{\iota} \mathtt{m[w]} : \Gamma(\mathtt{m[w]} @ \iota)
                                                                                 BINOP
                 РивМ
                                                                                 \frac{\Gamma, E \vdash_{\iota} \varepsilon_{1} : \varsigma \qquad \Gamma, E \vdash_{\iota} \varepsilon_{2} : \varsigma \qquad \oplus \in \{+, -, *\}}{\Gamma, E \vdash_{\iota} \varepsilon_{1} \oplus \varepsilon_{2} : \varsigma}
                 \Gamma, E \vdash_{\iota} \mathsf{p[w]} : \Gamma(\mathsf{p[w]})
                                                                       INTEGRITYWEAKEN
                                                                       \frac{\Gamma, E \vdash_{\iota} \varepsilon : \varsigma_{1} \qquad \varsigma_{1} \leq \varsigma_{2}}{\Gamma, E \vdash_{\iota} \varepsilon : \varsigma_{2}}
                           \frac{\Gamma, E \vdash_{\iota} \varepsilon : \mathcal{L}(\iota)}{\Gamma, E \vdash_{\iota} \varepsilon : \varepsilon e_{\iota} : \Gamma; x : \mathcal{L}(\iota)} \qquad \frac{\Gamma_{1}, E \vdash_{\iota} \pi_{1} : \Gamma_{2}}{\Gamma_{1}, E \vdash_{\iota} \pi_{1} : \pi_{2} : \Gamma_{3}}
                                               MAC
                                                                  E \models [\mathsf{assert}(\psi_{BDOZ}(w))@\iota]
                                                \Gamma, E \vdash \mathsf{assert}(\psi_{BDOZ}(w))@\iota : \Gamma; \mathsf{m}[w\mathsf{s}]@\iota : \mathsf{High}
```

 $\psi_{BDOZ}(w) \triangleq m[wm] = m[wk] + (m[delta] * m[ws])$

Definition 2.4. Given H, C and pre-processing predicate E_{pre} defining M, define $init(E_{pre}) \triangleq \Gamma$ where for all $m[w]@\iota \in M$ we have $\Gamma(m[w]@\iota) = \mathcal{L}(\iota)$.

Definition 2.5. Given pre-processing predicate E_{pre} and protocol π , for all H, C the judgement $init(E_{pre}), E \vdash \pi : \Gamma$ is valid iff it is derivable and $E_{pre} \land \lfloor \pi \rfloor \models E$.

Theorem 2.6. Given pre-processing predicate E_{pre} and protocol π , if for all H, C the judgement $init(E_{pre}), E \vdash \pi : \Gamma$ is valid and for all $x \in V_{H \triangleright C}$ we have $\Gamma(x) = \text{High}$, then cheating is detectable in π .

COMPOSITIONAL TYPE VERIFICATION IN Prelude

Note the redefinition of x impacts the definition of T, Γ , ϕ , and E.

eternation of x impacts the defination of Y, Y,
$$\phi$$
, and E.

 $x ::= r[e]@e \mid s[e]@e \mid m[e]@e \mid p[e] \mid out@e$
 $\ell \in \text{Field}, y \in \text{EVar}, f \in \text{FName}$
 $e ::= v \mid r[e] \mid s[e] \mid m[e] \mid p[e] \mid e \text{ binop } e \mid \text{let } y = e \text{ in } e \mid$
 $f(e, ..., e) \mid \{\ell = e; ...; \ell = e\} \mid e.\ell$
 $c ::= m[e]@e := e@e \mid p[e] := e@e \mid \text{out@e} := e@e \mid \text{assert}(e = e)@e \mid$
 $f(e, ..., e) \mid c; c \mid m[e]@e \text{ as } \phi$
 $\text{binop} ::= + \mid -\mid *\mid ++$
 $v ::= w \mid \iota \mid \varepsilon \mid \{\ell = v; ...; \ell = v\}$
 $fn ::= f(y, ..., y)\{e\} \mid f(y, ..., y)\{c\}$
 $R \Vdash \phi : (R_1, T) \qquad r[w]@\iota \notin R \qquad \oplus \in \{+, -\}$

$$R \Vdash x : (\varnothing, \{x\}) \qquad \qquad \frac{R \Vdash \phi : (R_1, T) \qquad \texttt{r[w]@} \iota \notin R \qquad \oplus \in \{+, -\}}{R_1 \Vdash \phi \oplus \texttt{r[w]@} \iota : (R_1 \cup \{\texttt{r[w]@} \iota\}, \{c(\texttt{r[w]@} \iota, T)\})}$$

$$\frac{R \Vdash \phi_1 : (R_1, T_1) \qquad R \Vdash \phi_2 : (R_2, T_2) \qquad \oplus \in \{+, -, *\}}{R_1 \Vdash \phi_1 \oplus \phi_2 : (R_1; R_2, T_1 \cup T_2)}$$

$$\frac{e[v/y] \Rightarrow v'}{\text{let } y = v \text{ in } e \Rightarrow v'}$$

$$\frac{C(f) = y_1, \dots, y_n, e \qquad e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \qquad e[v_1/y_1] \cdots [v_n/y_n] \Rightarrow v}{f(e_1, \dots, e_n) \Rightarrow v}$$

$$\frac{e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n}{\{\ell_1 = e_1; \dots; \ell_n = e_n\} \Rightarrow \{\ell_1 = v_1; \dots; \ell_n = v_n\}} \qquad \frac{e \Rightarrow \{\dots; \ell = v; \dots\}}{e.\ell \Rightarrow v}$$

$$\frac{e_1 \Rightarrow x_1 \qquad e_2 \Rightarrow x_2}{e_1 * e_2 \Rightarrow x_1 * x_2} \qquad \frac{e_1 \Rightarrow \phi_1 \qquad e_2 \Rightarrow \phi_2}{e_1 \equiv e_2 \Rightarrow \phi_1 \equiv \phi_2} \qquad \frac{e_1 \Rightarrow E_1 \qquad e_2 \Rightarrow E_2}{e_1 \land e_2 \Rightarrow E_1 \land E_2}$$

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149
                                                                          MESG
                                                                          \frac{e_1 \Rightarrow \varepsilon \qquad e_2 \Rightarrow \iota \qquad R_1 \Vdash \lfloor \varepsilon \Theta \iota \rfloor : (R_2, T)}{R_1 \vdash x := e_1 \Theta e_2 : \{E\} \ x : T, R_1; R_2 \ \{E \land x \equiv \lfloor \varepsilon \Theta \iota \rfloor\}}
150
151
                                                           ENCODE
153
                                                              e_1 \Rightarrow w e_2 \Rightarrow \iota E \models \lfloor \varepsilon @ \iota \rfloor \equiv \phi R_1 \Vdash \phi : (R_2, T)
155
                                                           R_1 \vdash \mathsf{m}[e_1]@e_2 \text{ as } \phi : \{E\} \mathsf{m}[w]@\iota : T, R_1; R_2 \{E \land \mathsf{m}[w]@\iota \equiv \phi\}
                                                       Арр
157
                                                                                          sig(f) = \Pi x_1, \dots, x_n.\{E_1\} \Gamma, R\{E_2\}
                                                       \frac{e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n}{R_1 + f(e_1, \dots, e_n) : \{E\} \ \rho(\Gamma), R_1; \rho(R) \ \{E \land \rho(E_2)\}}
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                                                                    \frac{R_1 \vdash \pi_1 : \{E_1\} \; \Gamma_2, R_2 \; \{E_2\} \qquad R_1 \vdash \pi_2 : \{E_2\} \; \Gamma_3, R_3 \; \{E_3\}}{R_1 \vdash \pi_1; \pi_2 : \{E_1\} \; \Gamma_2; \Gamma_3, R_2; R_3 \; \{E_3\}}
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165
                                       Sig
                                                                                                              C(f) = x_1, \ldots, x_n, \mathbf{c}
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                                        \underline{\rho = [\nu_1/x_1] \cdots [\nu_n/x_n]} \qquad \varnothing \vdash \underline{\rho(\mathbf{c})} : \{\rho(E_1)\} \ \rho(\Gamma), \rho(R) \ \{E\} \qquad E \models \rho(E_2)
                                                                                                  f: \Pi x_1, \dots, x_n, \{E_1\} \Gamma, R \{E_2\}
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Definition 3.1. sig is *verified* iff f : sig(f) is valid for all $f \in dom(sig)$.

The following theorem holds for protocols with default preprocessing.

Theorem 3.2. If sig is verified and $\emptyset \vdash e : \{\emptyset\} \Gamma, R \{E\} \text{ then } e \Rightarrow \pi \text{ and } R, E \vdash \pi : \Gamma \text{ is valid.}$

3.1 Confidentiality Examples

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174175

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```
and table ygc(g, x, y)
177
178
         let table = (r[g], r[g], r[g], r[g])
179
         in permute4(r[x],r[y],table)
180
181
     }
182
     m[x]@1 := s2(s[x],r[x],~r[x])@2;
183
     m[x]@1 as s[x]@2 xor r[x]@2;
184
185
     // m[x]@1 : { c(r[x]@2, { s[x]@2 }) }
186
187
     m[y]@1 := OT(s[y]@1,r[y],~r[y])@2;
188
     m[y]@1 as s[y]@1 xor r[y]@2;
189
190
     // m[y]@1 : { c(r[y]@2, { s[y]@1 }) }
191
192
     m[ag]@1 := OT4(m[x]@1, m[y]@1, andtable(ag,r[x],r[y]))@2;
193
     m[ag]@1 as \sim((r[x]@2 = m[x]@1) and (r[y]@2 = m[y]@1)) xor r[ag]@2;
194
195
```

```
// m[ag]@1 : { c(r[ag]@2, {r[x]@2, r[y]@2, m[x]@1, m[y]@1} }
197
198
      p[o] := OT2(m[ag]@1, perm2(r[ag],(false,true)))@2
199
200
      // p[o] : { c(r[ag]@2, {r[x]@2, r[y]@2, m[x]@1, m[y]@1}), r[ag]@2 }
201
202
      out@1 := p[o]@1
203
204
      // \text{ out@1 == s[x] and s[y]}
205
206
           encodegmw(in, i1, i2) {
207
             m[in]@i2 := (s[in] xor r[in])@i1;
             m[in]@i1 := r[in]@i1
209
           }
210
211
           andtablegmw(x, y, z) \{
212
             let r11 = r[z] \times r(m[x] \times r) and (m[y] \times r) in
213
             let r10 = r[z] xor (m[x] xor true) and (m[y] xor false) in
             let r01 = r[z] xor (m[x] xor false) and (m[y] xor true) in
             let r00 = r[z] \times r(m[x] \times r \text{ false}) and (m[y] \times r \text{ false}) in
             \{ \text{ row1} = \text{r11}; \text{ row2} = \text{r10}; \text{ row3} = \text{r01}; \text{ row4} = \text{r00} \}
           }
           andgmw(z, x, y) \{
             let table = andtablegmw(x,y,z) in
             m[z]@2 := OT4(m[x], m[y], table, 2, 1);
222
             m[z]@2 as \sim((m[x]@1 \text{ xor } m[x]@2)) and (m[y]@1 \text{ xor } m[y]@2)) xor r[z]@1);
             m[z]@1 := r[z]@1
224
           }
225
226
           // and gate correctness postcondition
          \{\}\ andgmw \{\ m[z]@1\ xor\ m[z]@2\ ==\ (m[x]@1\ xor\ m[x]@2)\ and\ (m[y]@1\ xor\ m[y]@2)\ \}
228
           // and gate type
230
           andgmw :
231
            Pi z, x, y.
232
            {}
233
            \{ \{ r[z]@1 \}, \}
234
            (m[z]@1 : { r[z]@1 }; m[z]@2 : {c(r[z]@1, { m[x]@1, m[x]@2, m[y]@1, m[y]@2 })} ),
235
              m[z]@1 \text{ xor } m[z]@2 == (m[x]@1 \text{ xor } m[x]@2) \text{ and } (m[y]@1 \text{ xor } m[y]@2)
236
237
           xorgmw(z, x, y)  {
238
             m[z]@1 := (m[x] xor m[y])@1; m[z]@2 := (m[x] xor m[y])@2;
239
           }
240
241
           decodegmw(z) {
242
             p["1"] := m[z]@1; p["2"] := m[z]@2;
243
             out@1 := (p["1"] xor p["2"])@1;
244
245
```

```
out@2 := (p["1"] \times p["2"])@2
246
          }
247
248
         prot() {
249
            encodegmw("x",2,1);
250
            encodegmw("y",2,1);
251
            encodegmw("z",1,2);
253
            andgmw("g1", "x", "z");
            xorgmw("g2","g1","y");
            decodegmw("g2")
255
          }
257
          {} prot { out@1 == (s["x"]@1 \text{ and } s["z"]@2) \text{ xor } s["y"]@1 }
259
     3.2 Integrity Examples
260
261
       secopen(w1,w2,w3,i1,i2) {
            pre(m[w1+++w]]@i2 == m[w1+++w]]@i1 + (m[wdelta]]@i1 * m[w1+++w]]@i2 /\
262
263
                m[w1++"m"]@i2 == m[w1++"k"]@i1 + (m["delta"]@i1 * m[w1++"s"]@i2));
            let locsum = macsum(macshare(w1), macshare(w2)) in
            m[w3++"s"]@i1 := (locsum.share)@i2;
            m[w3++"m"]@i1 := (locsum.mac)@i2;
            auth(m[w3++"s"],m[w3++"m"],mack(w1) + mack(w2),i1);
            m[w3]@i1 := (m[w3++"s"] + (locsum.share))@i1
       }
270
271
       _{open(x,i1,i2)}
272
         m[x++"exts"]@i1 := m[x++"s"]@i2;
273
274
         m[x++"extm"]@i1 := m[x++"m"]@i2;
          assert(m[x++"extm"] == m[x++"k"] + (m["delta"] * m[x++"exts"]));
275
         m[x]@i1 := (m[x++"exts"] + m[x++"s"])@i2
276
       }`
277
279
       _{\text{sum}}(z, x, y, i1, i2) \{
            pre(m[x++"m"]@i2 == m[x++"k"]@i1 + (m["delta"]@i1 * m[x++"s"]@i2 /\
280
                m[y++"m"]@i2 == m[y++"k"]@i1 + (m["delta"]@i1 * m[y++"s"]@i2));
281
            m[z++"s"]@i2 := (m[x++"s"] + m[y++"s"])@i2;
282
            m[z++"m"]@i2 := (m[x++"m"] + m[y++"m"])@i2;
283
            m[z++"k"]@i1 := (m[x++"k"] + m[y++"k"])@i1;
284
            post(m[z++"m"]@i2 == m[z++"k"]@i1 + (m["delta"]@i1 * m[z++"s"]@i2)
285
       }
286
287
       sum(z,x,y) \{ sum(z,x,y,1,2); sum(z,x,y,2,1) \}
288
289
       open(x) { _{open}(x,1,2); _{open}(x,2,1) }
290
291
292
       sum("a", "x", "d");
293
294
```

```
open("d");
295
        sum("b", "y", "e");
296
297
        open("e");
        let xys =
298
            macsum(macctimes(macshare("b"), m["d"]),
299
                     macsum(macctimes(macshare("a"), m["e"]),
300
                             macshare("c")))
301
        let xyk = mack("b") * m["d"] + mack("a") * m["e"] + mack("c")
302
303
        secopen("a", "x", "d", 1, 2);
304
          secopen("a", "x", "d", 2, 1);
305
          secopen("b", "y", "e", 1, 2);
306
          secopen("b","y","e",2,1);
307
          let xys =
308
            macsum(macctimes(macshare("b"), m["d"]),
309
                     macsum(macctimes(macshare("a"), m["e"]),
310
                             macshare("c")))
311
312
          in
          let xyk = mack("b") * m["d"] + mack("d") * m["d"] + mack("c")
314
          secreveal(xys,xyk,"1",1,2);
316
          secreveal(maccsum(xys,m["d"] * m["e"]),
                      xyk - m["d"] * m["e"],
317
                      "2",2,1);
318
319
          out@1 := (p[1] + p[2])@1;
320
          out@2 := (p[1] + p[2])@2;
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
```