

$v \in \mathbb{F}_p, w \in \text{String}, \iota \in \text{Clients} \subset \mathbb{N}$

$\varepsilon ::= r[w] \mid s[w] \mid m[w] \mid p[w] \mid$ *expressions*
 $v \mid \varepsilon - \varepsilon \mid \varepsilon + \varepsilon \mid \varepsilon * \varepsilon$

$x ::= r[w]@_\iota \mid s[w]@_\iota \mid m[w]@_\iota \mid p[w] \mid \text{out}@_\iota$ *variables*

$\pi ::= m[w]@_\iota := \varepsilon@_\iota \mid p[w] := e@_\iota \mid \text{out}@_\iota := \varepsilon@_\iota \mid \pi; \pi$ *protocols*

$$\begin{aligned} \llbracket \sigma, v \rrbracket_\iota &= v \\ \llbracket \sigma, \varepsilon_1 + \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota + \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 - \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota - \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, \varepsilon_1 * \varepsilon_2 \rrbracket_\iota &= \llbracket \llbracket \sigma, \varepsilon_1 \rrbracket_\iota * \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \\ \llbracket \sigma, r[w] \rrbracket_\iota &= \sigma(r[w]@_\iota) \\ \llbracket \sigma, s[w] \rrbracket_\iota &= \sigma(s[w]@_\iota) \\ \llbracket \sigma, m[w] \rrbracket_\iota &= \sigma(m[w]@_\iota) \\ \llbracket \sigma, p[w] \rrbracket_\iota &= \sigma(p[w]) \end{aligned}$$

$$(\sigma, x := \varepsilon@_\iota) \Rightarrow \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} \quad \frac{(\sigma_1, \varepsilon_1) \Rightarrow \sigma_2 \quad (\sigma_2, \varepsilon_2) \Rightarrow \sigma_3}{(\sigma_1, \varepsilon_1; \varepsilon_2) \Rightarrow \sigma_3}$$

$$\begin{aligned} (\sigma, x := \varepsilon@_\iota) &\Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} & \iota \in H \\ (\sigma, x := \varepsilon@_\iota) &\Rightarrow_{\mathcal{A}} \sigma\{x \mapsto \llbracket \text{rewrite}_{\mathcal{A}}(\sigma_C, \varepsilon) \rrbracket_\iota\} & \iota \in C \end{aligned}$$

$$\begin{aligned} (\sigma, \text{assert}(\varepsilon_1 = \varepsilon_2)@_\iota) &\Rightarrow_{\mathcal{A}} \sigma & \text{if } \llbracket \sigma, \varepsilon_1 \rrbracket_\iota = \llbracket \sigma, \varepsilon_2 \rrbracket_\iota \text{ or } \iota \in C \\ (\sigma, \text{assert}(\phi(\varepsilon))@_\iota) &\Rightarrow_{\mathcal{A}} \perp & \text{if } \neg\phi(\sigma, \llbracket \sigma, \varepsilon \rrbracket_\iota) \end{aligned}$$

$$(\sigma, x := \varepsilon@_\iota) \Rightarrow \sigma\{x \mapsto \llbracket \sigma, \varepsilon \rrbracket_\iota\} \quad \frac{(\sigma_1, \varepsilon_1) \Rightarrow \perp}{(\sigma_1, \varepsilon_1; \varepsilon_2) \Rightarrow \perp}$$

$\ell \in \text{Field}, y \in \text{EVar}, f \in \text{FName}$

$e ::= v \mid r[e] \mid s[e] \mid m[e] \mid p[e] \mid e \text{ binop } e \mid \text{let } y = e \text{ in } e \mid$
 $f(e, \dots, e) \mid \{\ell = e; \dots; \ell = e\} \mid e.\ell$
 $c ::= m[e]@e := e@e \mid p[e] := e@e \mid \text{out}@e := e@e \mid \text{assert}(e = e)@e \mid$
 $f(e, \dots, e) \mid c; c \mid \text{pre}(E) \mid \text{post}(E)$

$\text{binop} ::= + \mid - \mid * \mid ++$

$v ::= w \mid \iota \mid \varepsilon \mid \{\ell = v; \dots; \ell = v\}$

$fn ::= f(y, \dots, y)\{e\} \mid f(y, \dots, y)\{c\}$

$\phi ::= r[e]@e \mid s[e]@e \mid m[e]@e \mid p[e] \mid \text{out}@e \mid \phi + \phi \mid \phi - \phi \mid \phi * \phi$

$E ::= \phi = \phi \mid E \wedge E$

$$\begin{array}{c}
\frac{e[v/y] \Rightarrow v'}{\text{let } y = v \text{ in } e \Rightarrow v'} \\
\\
\frac{C(f) = y_1, \dots, y_n, e \quad e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad e[v_1/y_1] \cdots [v_n/y_n] \Rightarrow v}{f(e_1, \dots, e_n) \Rightarrow v} \\
\\
\frac{e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n}{\{\ell_1 = e_1; \dots; \ell_n = e_n\} \Rightarrow \{\ell_1 = v_1; \dots; \ell_n = v_n\}} \quad \frac{e \Rightarrow \{\dots; \ell = v; \dots\}}{e.\ell \Rightarrow v} \quad \frac{e_1 \Rightarrow w_1 \quad e_2 \Rightarrow w_2}{e_1 ++ e_2 \Rightarrow w_1 w_2} \\
\\
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{on}, \text{assert}(e_1 = e_2) @ e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2) @ \iota, (E_1, E_2 \wedge \lfloor \varepsilon_1 @ \iota \rfloor = \lfloor \varepsilon_2 @ \iota \rfloor), \text{on})} \\
\\
\frac{e_1 \Rightarrow \varepsilon_1 \quad e_2 \Rightarrow \varepsilon_2 \quad e \Rightarrow \iota}{(\pi, (E_1, E_2), \text{off}, \text{assert}(e_1 = e_2) @ e) \Rightarrow (\pi; \text{assert}(\varepsilon_1 = \varepsilon_2) @ \iota, (E_1, E_2, \text{off}))} \\
\\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{on}, m[e_1] @ e_2 := e_3 @ e_4) \Rightarrow (\pi; m[w] @ \iota_1 := \varepsilon @ \iota_2, (E_1 \wedge m[w] @ \iota_1 = \lfloor \varepsilon @ \iota_2 \rfloor, E_2), \text{on})} \\
\\
\frac{e_1 \Rightarrow w \quad e_2 \Rightarrow \iota_1 \quad e_3 \Rightarrow \varepsilon \quad e_4 \Rightarrow \iota_2}{(\pi, (E_1, E_2), \text{off}, m[e_1] @ e_2 := e_3 @ e_4) \Rightarrow (\pi; m[w] @ \iota_1 := \varepsilon @ \iota_2, (E_1, E_1), \text{off})} \\
\\
(\pi, (E_1, E_2), \text{on}, \text{pre}(E)) \Rightarrow (\pi, E_1, E_2 \wedge E, \text{off}) \\
\\
(\pi, (E_1, E_2), \text{off}, \text{post}(E)) \Rightarrow (\pi, (E_1 \wedge E, E_2), \text{on}) \\
\\
\frac{(\pi_1, (E_{11}, E_{12}), sw_1, \mathbf{c}_1) \Rightarrow (\pi_2, (E_{21}, E_{22}), sw_2) \quad (\pi_2, (E_{21}, E_{22}), sw_2, \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), sw_3)}{(\pi_1, (E_{11}, E_{12}), sw_1, \mathbf{c}_1; \mathbf{c}_2) \Rightarrow (\pi_3, (E_{31}, E_{32}), sw_3)} \\
\\
\frac{C(f) = y_1, \dots, y_n, \mathbf{c} \quad e_1 \Rightarrow v_1 \cdots e_n \Rightarrow v_n \quad (\pi_1, (E_{11}, E_{12}), sw_1, \mathbf{c}[v_1/y_1, \dots, v_n/y_n]) \Rightarrow (\pi_2, (E_{21}, E_{22}), sw_2)}{(\pi_1, (E_{11}, E_{12}), sw_1, f(e_1, \dots, e_n)) \Rightarrow (\pi_2, (E_{21}, E_{22}), sw_2)}
\end{array}$$