Introduction

QR decomposition also known as QR factorization or QU factorization is used to solve linear least squares problem and the basics of eigen value algorithm. An square matrix can be decomposed as

$$A=QR$$

Where A :- Input matrix

Q :- Orthogonal matrix

R :- Upper triangular matrix

If A is a complex square matrix then there Q is an unitary matrix($Q^T * Q = Q * Q^T = I$)

Where I is identity matrix.

Q is responsible for the triangular form of R.

Method:

In project I have used Householder reflection algorithm . It takes a vector and reflects it on to the hyperplane.

Int this algorithm the Q Is calculated as $Q=I - 2VV^T$

Where V is calculated as $V=U/||U||$

$||U||$ is norm of U matrix

U is calculated as $U=X[0] - \alpha e1$

Where e is the vector like this(1 0 0 0 ….)^T

X[0] is the zeroth column in the matrix

First we multiply with the householder matrix with Q1 then we obtain when we choose the first column matrix for X . The result in a matrix Q1A with zeros in the left column.

$$Q_1 A = \begin{bmatrix} \alpha_1 & \star & \dots & \star \\ 0 & & & \\ \vdots & & A' & \\ 0 & & & \end{bmatrix}$$

This can be repeated for $A^|$ (Q1A gives the output by deleting the first column),it will give Q2…etc this will continue until the minor matrix will goes to 1*1.

The R matrix can be calculated as

$$R = Q_t \cdots Q_2 Q_1 A$$

This method gives numerical stability than other decomposition algorithms.

R -matrix parallelization:

Implementation :

In R-Matrix parallelization at first Q matrix is calculated whenever Q matrix calculation is completed the q-flag in the code will get updated and the other cores will start doing R matrix parallelized way.

In householder algorithm the R matrix depends on Q matrix so at first we need to calculate Q matrix to find R matrix. So I first calculated Q matrix and then I am storing the Q matrix values in the shared memory and I have used delay in the code because the shared memory is taking some time to copy the values from one core to other core .

Input matrix I took

```
double input[][16] = {
        {5.124575, 6.716550, 4.749593, 0.745557, 0.835284, 1.061532, 2.406306, 3.394185, 7.122869, 4.096635, 7.461806, 5.104174, 0.995246, 3.524249, 1.343407, 7.898948},
        {4.709401, 5.185006, 2.168096, 1.438314, 3.459950, 4.195184, 6.530458, 4.538858, 1.076255, 2.883427, 6.272343, 5.829943, 2.964446, 0.192133, 7.946213, 1.210326},
        {8.183881, 6.291060, 5.034810, 2.737167, 9.811339, 2.984305, 0.413352, 2.290696, 3.698015, 6.639598, 5.670019, 4.373680, 6.214067, 3.592056, 1.280568, 0.145697},
        {6.742811, 6.390177, 2.078934, 8.330168, 6.574254, 0.563682, 4.178736, 4.793037, 9.529616, 8.307987, 2.311288, 6.081988, 3.150382, 9.572844, 0.373962, 3.856355},
        {3.678255, 8.243491, 5.616988, 8.213211, 1.015898, 1.159368, 1.437121, 6.370590, 2.500651, 7.166724, 9.152067, 4.968377, 4.722142, 2.282590, 6.428624, 5.997973},
        {9.094666, 3.641620, 5.956050, 1.629081, 6.459073, 9.468715, 4.752189, 7.788865, 2.220713, 2.484793, 6.529717, 2.083286, 0.599322, 8.827311, 5.588702, 3.405092},
        {5.301290, 2.267589, 0.140771, 1.161778, 0.592439, 9.798719, 4.314055, 3.972274, 9.824009, 9.260374, 6.768367, 5.079985, 5.533086, 1.848284, 3.462898, 0.671444},
        {9.660500, 7.457226, 2.946092, 4.258254, 2.801475, 0.722540, 7.356618, 6.927009, 8.416201, 6.432596, 7.406141, 4.093572, 9.802859, 9.046292, 8.103308, 0.099300},
        {0.001511, 1.054377, 0.355802, 4.236591, 6.461459, 4.018142, 2.400816, 3.651846, 4.161861, 8.746218, 9.976361, 4.284649, 7.141063, 7.410219, 6.350062, 7.652938},
        {4.751442, 8.985795, 9.131959, 0.385422, 6.837505, 9.114349, 0.238815, 8.198015, 8.454410, 2.955473, 7.821795, 0.187684, 6.068312, 3.552302, 2.452192, 3.747190},
        {7.072122, 1.099953, 0.179495, 1.086232, 9.648564, 4.431138, 2.433835, 4.077998, 4.805562, 1.784756, 2.986027, 2.828004, 8.104572, 1.811495, 0.483971, 3.724873},
        {4.902166, 7.299483, 6.822000, 8.115096, 3.138024, 8.902470, 5.816616, 1.066501, 0.368879, 1.762015, 8.535061, 6.834885, 1.670396, 9.097155, 7.376048, 4.097865},
        {0.370360, 3.752868, 6.622229, 5.945147, 8.541925, 8.859654, 3.349494, 2.638748, 1.283802, 9.154308, 8.949241, 9.176734, 9.049492, 8.284714, 4.103983, 9.309067},
        {8.626863, 3.556053, 0.016668, 6.758221, 5.183024, 8.069129, 0.308171, 5.894849, 4.018287, 6.069993, 7.510052, 0.099244, 8.835959, 4.522346, 4.287552, 8.246637},
        {2.015325, 9.879946, 7.386629, 2.522771, 1.881512, 0.924535, 3.546607, 8.241838, 1.206717, 9.042177, 5.350417, 4.953636, 5.543465, 4.732425, 4.703988, 3.690862},
        {0.150104, 1.012173, 6.372757, 4.470197, 8.672633, 9.264992, 5.326238, 1.308067, 6.909482, 6.954018, 5.243287, 0.047543, 5.374593, 4.245235, 9.933439, 8.686286},
};
```

Results:

Arm processors results

4 X 4 matrix output :

```
Q
    0.578     0.560    -0.301     0.512
    0.094     0.083     0.929     0.349
    0.803    -0.504     0.079    -0.308
    0.112     0.652     0.201    -0.722

R
    8.870     7.665     9.113     5.735
    0.000     4.085    -0.023     3.280
   -0.000    -0.000     1.669     4.924
    0.000    -0.000     0.000    -5.712
```

8 X 8 matrix output :

```
Q
    0.288     0.396     0.034    -0.505    -0.399    -0.543    -0.065     0.213
    0.400    -0.466     0.611     0.026    -0.333    -0.003    -0.075    -0.365
    0.264     0.172    -0.166    -0.231     0.034     0.189     0.781    -0.416
    0.060     0.369     0.558     0.415     0.003     0.091     0.372     0.479
    0.459    -0.228     0.110    -0.320     0.728    -0.032    -0.041     0.299
    0.208     0.633     0.159     0.013     0.202     0.320    -0.474    -0.403
    0.379     0.046    -0.280     0.629     0.153    -0.562     0.010    -0.203
    0.535    -0.075    -0.414     0.144    -0.366     0.491    -0.125     0.354

R
   17.811    16.249    10.817    11.561    11.705    10.287     5.674    10.369
    0.000     5.157     3.460     2.395     3.327     0.480     5.265    -1.601
   -0.000    -0.000     7.776     2.305     0.637    -1.621     3.183     2.201
   -0.000     0.000    -0.000     7.138     1.564    -0.506     3.171     0.785
   -0.000     0.000     0.000     0.000     7.717    -1.975    -0.098    -2.804
    0.000     0.000     0.000     0.000    -0.000     5.666    -1.120    -1.726
   -0.000    -0.000    -0.000    -0.000    -0.000    -0.000     7.171     2.585
    0.000    -0.000    -0.000    -0.000     0.000    -0.000     0.000    -2.457
```

16 X 16 output :

```
Q
    0.288     0.396     0.034    -0.505    -0.399    -0.543    -0.065     0.213
    0.400    -0.466     0.611     0.026    -0.333    -0.003    -0.075    -0.365
    0.264     0.172    -0.166    -0.231     0.034     0.189     0.781    -0.416
    0.060     0.369     0.558     0.415     0.003     0.091     0.372     0.479
    0.459    -0.228     0.110    -0.320     0.728    -0.032    -0.041     0.299
    0.208     0.633     0.159     0.013     0.202     0.320    -0.474    -0.403
    0.379     0.046    -0.280     0.629     0.153    -0.562     0.010    -0.203
    0.535    -0.075    -0.414     0.144    -0.366     0.491    -0.125     0.354

R
   17.811    16.249    10.817    11.561    11.705    10.287     5.674    10.369
    0.000     5.157     3.460     2.395     3.327     0.480     5.265    -1.601
   -0.000    -0.000     7.776     2.305     0.637    -1.621     3.183     2.201
   -0.000     0.000    -0.000     7.138     1.564    -0.506     3.171     0.785
   -0.000     0.000     0.000     0.000     7.717    -1.975    -0.098    -2.804
    0.000     0.000     0.000     0.000    -0.000     5.666    -1.120    -1.726
   -0.000    -0.000    -0.000    -0.000    -0.000    -0.000     7.171     2.585
    0.000    -0.000    -0.000    -0.000     0.000    -0.000     0.000    -2.457
```

R-Matrix results:

1 core results:

4 X 4 output :

Total cycles=2319349253

```
Q Matrix
0.577714    0.094165    0.802989    0.112198
0.560173    0.083169    -0.503901    0.652206
-0.300873    0.928780    0.079416    0.201336
0.511790    0.348688    -0.308197    -0.722151
R matrix
8.870442    7.665168    9.112971    5.735174
0.000000    4.084952    -0.023112    3.279678
-0.000000    -0.000000    1.668970    4.923829
0.000000    -0.000000    0.000000    -5.712243
completeddddd
```

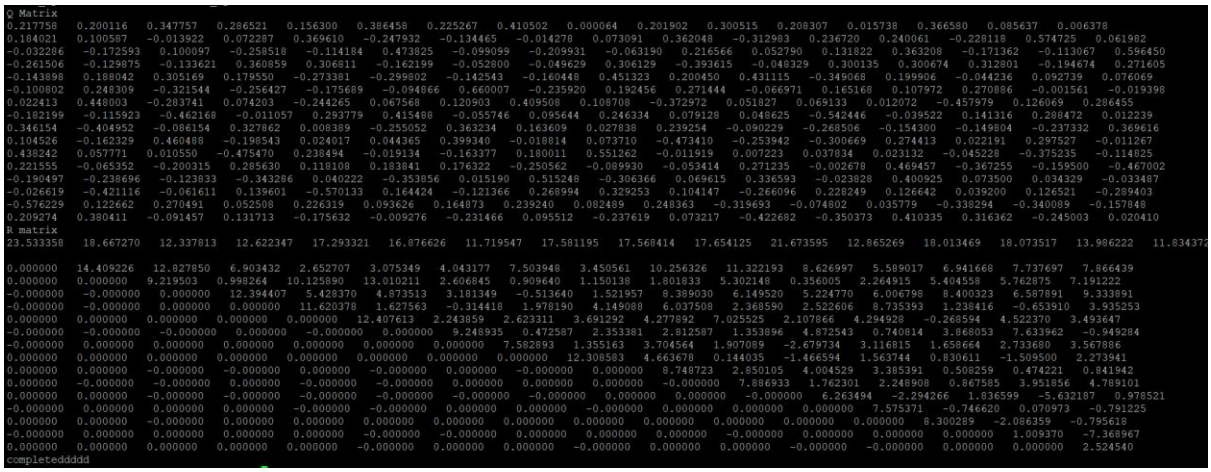
8 X 8 output:

Total cycles = 326468851

```
Q Matrix
0.287714   0.399906   0.264404   0.060425   0.459475   0.207621   0.378568   0.534974
0.395902   -0.465658   0.172353   0.368756   -0.227816   0.633346   0.046337   -0.074587
0.034396   0.610537   -0.165686   0.558488   0.109705   0.158516   -0.279886   -0.413766
-0.505463   0.026423   -0.231055   0.414771   -0.319708   0.012762   0.628660   0.144211
-0.399221   -0.332555   0.033514   0.003392   0.728457   0.201636   0.153425   -0.366127
-0.543035   -0.003416   0.189351   0.091020   -0.031511   0.320081   -0.562180   0.491399
-0.064808   -0.074929   0.780662   0.372474   -0.040913   -0.474006   0.009654   -0.124771
0.212937   -0.365466   -0.416097   0.478948   0.299027   -0.403344   -0.203250   0.353767
R matrix
17.811360   16.248665   10.816890   11.561239   11.704696   10.287237   5.674191   10.368926
0.000000   5.156791   3.460376   2.394935   3.326864   0.480324   5.265464   -1.600962
-0.000000   -0.000000   7.775812   2.305135   0.636525   -1.621480   3.182859   2.200535
-0.000000   0.000000   -0.000000   7.138436   1.564049   -0.506463   3.171285   0.785185
-0.000000   0.000000   0.000000   0.000000   7.716916   -1.974703   -0.098065   -2.804130
0.000000   0.000000   0.000000   0.000000   -0.000000   5.666267   -1.120062   -1.725626
-0.000000   -0.000000   -0.000000   -0.000000   -0.000000   -0.000000   7.171003   2.584629
0.000000   -0.000000   -0.000000   -0.000000   0.000000   -0.000000   0.000000   -2.456694
completeddddd
```

16 X 16 output:

Total cycles = 531967459



2 cores results:

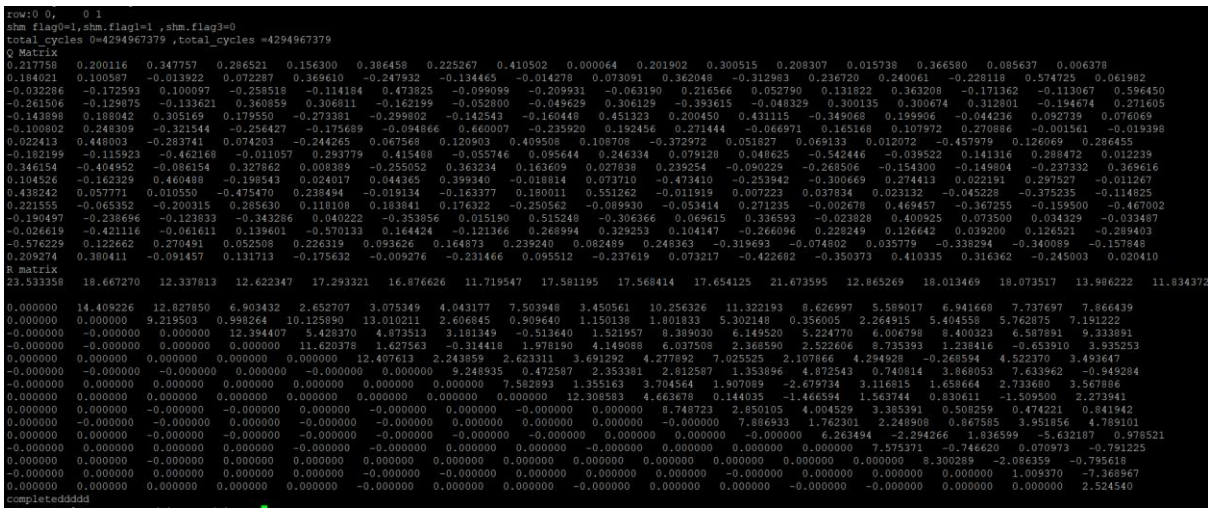
4 X 4 output :

```
row:0 0,    0 1
shm flag0=1,shm.flag1=1 ,shm.flag3=0
total_cycles 0=1299338157 ,total_cycles =1289349253
Q Matrix
0.577714   0.094165   0.802989   0.112198
0.560173   0.083169   -0.503901   0.652206
-0.300873   0.928780   0.079416   0.201336
0.511790   0.348688   -0.308197   -0.722151
R matrix
8.870442   7.665168   9.112971   5.735174
0.000000   4.084952   -0.023112   3.279678
-0.000000   -0.000000   1.668970   4.923829
0.000000   -0.000000   0.000000   -5.712243
completedddddd
```

8 X 8 output :

```
row:0 0,    0 1
shm flag0=1,shm.flag1=1 ,shm.flag3=0
total_cycles 0=2186467949 ,total_cycles =2178711437
Q Matrix
0.287714   0.399906   0.264404   0.060425   0.459475   0.207621   0.378568   0.534974
0.395902   -0.465658   0.172353   0.368756   -0.227816   0.633346   0.046337   -0.074587
0.034396   0.610537   -0.165686   0.558488   0.109705   0.158516   -0.279886   -0.413766
-0.505463   0.026423   -0.231055   0.414771   -0.319708   0.012762   0.628660   0.144211
-0.399221   -0.332555   0.033514   0.003392   0.728457   0.201636   0.153425   -0.366127
-0.543035   -0.003416   0.189351   0.091020   -0.031511   0.320081   -0.562180   0.491399
-0.064808   -0.074929   0.780662   0.372474   -0.040913   -0.474006   0.009654   -0.124771
0.212937   -0.365466   -0.416097   0.478948   0.299027   -0.403344   -0.203250   0.353767
R matrix
17.811360   16.248665   10.816890   11.561239   11.704696   10.287237   5.674191   10.368926
0.000000   5.156791   3.460376   2.394935   3.326864   0.480324   5.265464   -1.600962
-0.000000   -0.000000   7.775812   2.305135   0.636525   -1.621480   3.182859   2.200535
-0.000000   0.000000   -0.000000   7.138436   1.564049   -0.506463   3.171285   0.785185
-0.000000   0.000000   0.000000   0.000000   7.716916   -1.974703   -0.098065   -2.804130
0.000000   0.000000   0.000000   0.000000   -0.000000   5.666267   -1.120062   -1.725626
-0.000000   -0.000000   -0.000000   -0.000000   -0.000000   -0.000000   7.171003   2.584629
0.000000   -0.000000   -0.000000   -0.000000   0.000000   -0.000000   0.000000   -2.456694
completeddddd
```

16 X 16 output :



Q – Matrix parallelization :-

Implementation :

In Q matrix parallelization

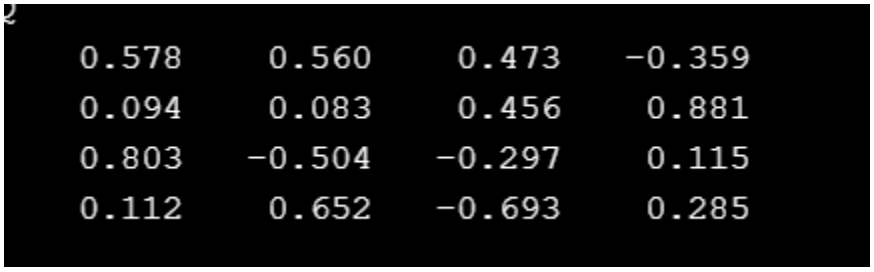
we know that R can be calculated as

$$R = Q_t \cdots Q_2 Q_1 A$$

In householder algorithm we will calculate Q = (Q1*Q2*…..*Qn). We know the commutative formula in matrices (A*B*C=A*(B*C) OR (A*B)*C) like this I taught to calculate Q also Qc=(Q1*Q2*……..*Qa) in one core and Qd=(Qa*Qa+1*….*Qn) in other core and multiplying Qc*Qd finally, but while doing like this my flags are not getting updated properly I don't know the reason why. But when trying to calculate Qc or Qd in single core the values and flags are coming properly. Externally I tried to do multiplication of Qc and Qd I got exact Q matrix what I did in serial execution.

I took 4 X 4 matrix to do this

Qc=Q1*Q2



Qd=Q3*Q4

```
1.000     0.000     0.000     0.000
0.000     1.000     0.000     0.000
0.000     0.000     0.984    -0.177
0.000     0.000     0.177     0.984
```

Q=Qc * Qd

```
0.578     0.560    -0.301     0.512
0.094     0.083     0.929     0.349
0.803    -0.504     0.079    -0.308
0.112     0.652     0.201    -0.722
```

Conclusion:

In householder matrix the R matrix is nothing but multiplication of Q and input matrix(A)

While doing R matrix parallelization the computation time is decreased by 1000000000 nearly ,in R matrix parallelization the Q matrix is calculated in sequential manner after calculation of Q matrix the q_flag will get updated to 1 . whenever the q_flag updates to 1 the final multiplication of Q and A is done parallelly .

While performing Q matrix parallelization the flags are not updated but I when I calculated the cycles of Qc or Qd the cycles decreased by half.

When I tried to increase number of cores from 2 to 4 the flags are not updating at all. I don't know why this is happening like that.

References:

https://en.wikipedia.org/wiki/QR_decomposition#Using_Householder_reflections