2021_02_24_EE538_Lecture8_W2021

February 27, 2021

1 EE 538: Analog Integrated Circuit Design

1.1 Winter 2021

1.2 Instructor: Jason Silver

1.3 Python packages/modules

[1]:

```
import matplotlib as mpl
from matplotlib import pyplot as plt
import numpy as np
from scipy import signal
#%matplotlib notebook

mpl.rcParams['font.size'] = 12
mpl.rcParams['legend.fontsize'] = 'large'

def plot_xy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def plot_xy2(x1, y1, x1label, y1label, x2, y2, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].plot(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])
```

```python
def plot_x2y(x, y1, y2, xlabel, ylabel, y1label, y2label):

    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y1, 'b')
    ax.plot(x, y2, 'r')
    ax.legend( [y1label,y2label] ,loc='upper center', ncol=5, fancybox=True,
           shadow=True, bbox_to_anchor=(0.5,1.1))
    ax.grid()
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_xy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].plot(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].plot(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_xlogy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.semilogy(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def plot_logxy2(x1, y1, x2, y2, x1label, y1label, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].semilogx(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])
```

```
def nmos_iv_sweep(V_gs, V_ds, W, L, lmda):
    u_n = 350                   # electron mobility (device parameter)
    e_ox = 3.9*8.854e-12/100; # relative permittivity
    t_ox = 9e-9*100;          # oxide thickness
    C_ox = e_ox/t_ox          # oxide capacitance
    V_thn = 0.7                # threshold voltage (device parameter)
    V_ov = V_gs - V_thn
    Ldn = 0.08e-6
    Leff = L - 2*Ldn

    I_d = []

    for i in range(len(V_ds)):
        I_d.append(np.piecewise(V_ds[i], [V_ds[i] < V_ov, V_ds[i] >= V_ov],
                        [u_n*C_ox*(W/Leff)*(V_gs - V_thn - V_ds[i]/
 ↪2)*V_ds[i]*(1+lmda*V_ds[i]) ,
                         0.5*u_n*C_ox*(W/Leff)*(V_gs -␣
 ↪V_thn)**2*(1+lmda*V_ds[i])]))

    return np.array(I_d)

def pmos_iv_sweep(V_sg, V_sd, W, L, lmda):
    u_p = 100                   # electron mobility (device parameter)
    e_ox = 3.9*8.854e-12/100; # relative permittivity
    t_ox = 9e-9*100;          # oxide thickness
    C_ox = e_ox/t_ox          # oxide capacitance
    V_thp = -0.8                # threshold voltage (device parameter)
    V_ov = V_sg - np.abs(V_thp)
    Ldp = 0.09e-6
    Leff = L - 2*Ldp

    I_d = []

    for i in range(len(V_sd)):
        I_d.append(np.piecewise(V_sd[i], [V_sd[i] < V_ov, V_sd[i] >= V_ov],
                        [u_p*C_ox*(W/Leff)*(V_sg - np.abs(V_thp) - V_sd[i]/
 ↪2)*V_sd[i]*(1+lmda*V_sd[i]) ,
                         0.5*u_p*C_ox*(W/Leff)*(V_sg - np.
 ↪abs(V_thp))**2*(1+lmda*V_sd[i])]))

    return np.array(I_d)

def nmos_iv_sat(V_gs, V_ds, W, L, lmda):
    u_n = 350                   # electron mobility (device parameter)
    e_ox = 3.9*8.854e-12/100; # relative permittivity
    t_ox = 9e-9*100;          # oxide thickness
```

```python
    C_ox = e_ox/t_ox          # oxide capacitance
    V_thn = 0.7                # threshold voltage (device parameter)
    V_ov = V_gs - V_thn
    Ldn = 0.08e-6
    Leff = L - 2*Ldn

    I_d = 0.5*u_n*C_ox*(W/Leff)*(V_gs - V_thn)**2*(1+lmda*V_ds)

    return I_d

def nmos_diff_pair(V_id, I_ss, R_D, W, L, V_dd):
    u_n = 350                 # electron mobility (device parameter)
    e_ox = 3.9*8.854e-12/100; # relative permittivity
    t_ox = 9e-9*100;          # oxide thickness
    C_ox = e_ox/t_ox          # oxide capacitance
    V_thn = 0.7                # threshold voltage (device parameter)
    Ldn = 0.08e-6
    Leff = L - 2*Ldn

    I_dp = I_ss/2 + 0.25*u_n*C_ox*(W/L)*V_id*np.sqrt(4*I_ss/(u_n*C_ox*(W/L)) -␣
 ↪V_id**2)
    I_dm = I_ss/2 - 0.25*u_n*C_ox*(W/L)*V_id*np.sqrt(4*I_ss/(u_n*C_ox*(W/L)) -␣
 ↪V_id**2)

    return I_dp, I_dm
```

2 Lecture 8 - Stability and Frequency Compensation

2.1 Announcements

- Design Project Phase 1 posted, due Sunday March 7
  - PDF submission on Canvas
- Design Project Phase 2 will be posted soon

2.2 Week 8

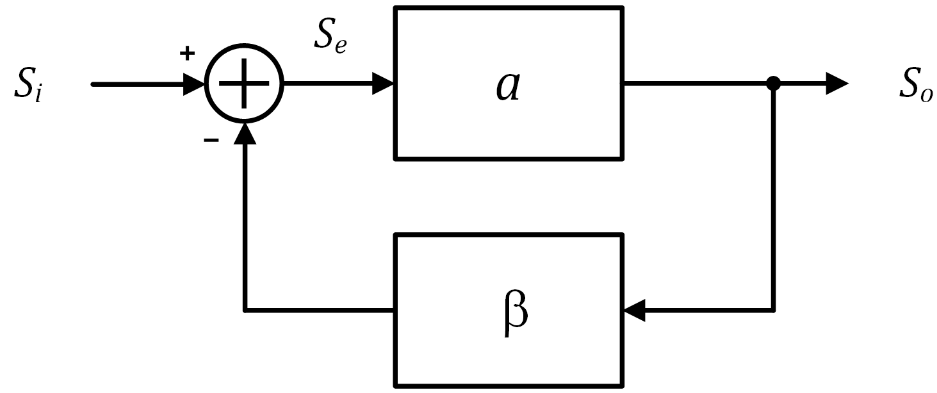
- Chapter 6 of Razavi (Frequency Response)
- Chapter 10 of Razavi (Stability and Frequency Compensation)

2.3 Overview

- Last time…
  - CMOS amplifier design
  - Subthreshold MOS operation
  - g_m/I_D design methodology
- Today…
  - Feedback

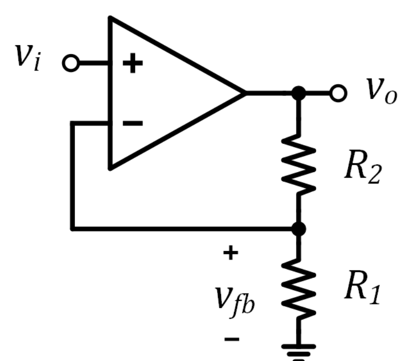
- Stability definition and criteria (Bode/root locus)
- Damping ratio and phase margin
- Mirror poles
- Frequency response of two-stage amplifiers

2.4 Negative feedback



- Negative feedback loop processes the error $S_i(s) - \beta \cdot S_o(s)$
- If the magnitude of a is large, the error is minimized, i.e. $S_i(s) - \beta \cdot S_o(s) \to 0$
- In this sense, negative feedback "desensitizes" the transfer function to the open-loop gain a

2.5 Non-inverting amplifier



$$a = A_0 \tag{1}$$

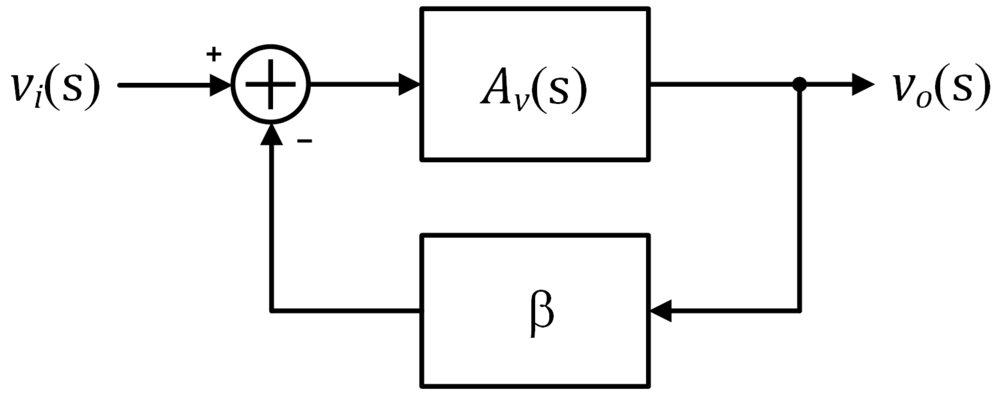
$$\beta = \frac{R_1}{R_1 + R_2} \tag{2}$$

$$\frac{v_o}{v_i} = \frac{A_0}{1 + \beta A_0} \approx \frac{1}{\beta} \tag{3}$$

- A fraction of the output voltage (set by β) is fed back to the inverting input and the error voltage is processed by the amplifier

- Open-loop gain specification is determined by precision requirements (application-dependent)
- Exact value of DC gain A_0 is unimportant as long as it's "large enough"

2.6 Gain-bandwidth product



$$G(s) = \frac{v_o(s)}{v_i(s)} = \frac{A_v(s)}{1 + \beta A_v(s)} \tag{4}$$

$$A_v(s) = \frac{A_0}{1 + s/\omega_0} \tag{5}$$

$$GBW = A_0 \omega_0 \tag{6}$$

- Assuming dominant-pole behavior, we can readily assess the effect of negative feedback on frequency response
- A_0 is the *DC* gain of the *open-loop* amplifier, and ω_0 is the dominant pole frequency (i.e. $3dB$ bandwidth)

- To determine the frequency response of the closed-loop system, we substitute the frequency-dependent expression for $A_v(s)$ into the closed-loop gain expression

$$G(s) = \frac{A_v(s)}{1 + \beta A_v(s)} = \frac{A_0}{1 + s/\omega_0 + \beta A_0} = \frac{\frac{A_0}{1+\beta A_0}}{1 + \frac{s}{\omega_0(1+\beta A_0)}} \tag{7}$$

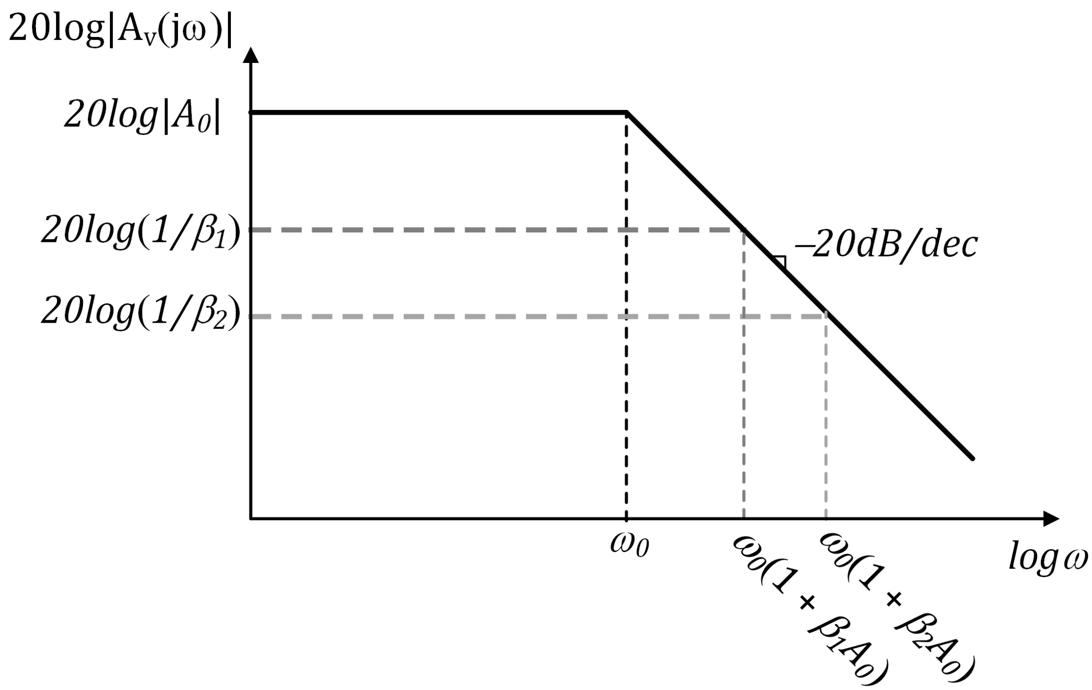
- Solving for the closed-loop pole frequency gives

$$\omega_0^{'} = \omega_0 \cdot (1 + \beta A_0) \tag{8}$$

[2]:
```
def plot_CL_freq(A_dB, f_t, betas, w):
    A_0 = 10**(A_dB/20)
    f_3dB = f_t/A_0
    w_0 = f_3dB*2*np.pi
    A_s = np.array([])
```

```
    fig, axs = plt.subplots(2, figsize=(10.0, 8.0))
    for b in betas:
        Av_cl = signal.TransferFunction([A_0], [1/w_0, 1 + b*A_0])
        w, mag, phase = Av_cl.bode(w=w)       # rad/s, dB, degrees
        f = w/2/np.pi

        # Plot the frequency response for multiple values of beta
        fig.suptitle('Opamp Closed-Loop Frequency Response')
        axs[0].semilogx(f, mag)
        axs[0].grid()
        axs[0].set_ylabel('Magnitude [dB]')
        axs[1].semilogx(f,phase)
        axs[1].grid()
        axs[1].set_ylabel('Phase [deg]')
        axs[1].set_xlabel('Frequency [Hz]')
        fig.align_ylabels(axs[:])

def plot_CL_step(A_dB, f_t, betas, w):
    A_0 = 10**(A_dB/20)
    f_3dB = f_t/A_0
    w_0 = f_3dB*2*np.pi
    A_s = np.array([])

    fig, axs = plt.subplots(2, figsize=(10.0, 8.0))
    for b in betas:
        Av_cl = signal.TransferFunction([A_0], [1/w_0, 1 + b*A_0])
        tin = np.linspace(0,20e-6,100)
        u_step = np.concatenate( (0, np.ones(99)), axis=None)
        tout,vout = signal.step(Av_cl, X0=None, T=tin)

        # Plot the step response for multiple values of beta
        fig.suptitle('Opamp Closed-Loop Step Response')
        axs[0].plot(1e6*tout, b*vout)
        axs[0].grid()
        axs[0].set_ylabel(r'$\beta V_o$ [V]')
        axs[1].plot(1e6*tin,u_step)
        axs[1].grid()
        axs[1].set_ylabel('Input Voltage [V]')
        axs[1].set_xlabel('Time [$\mu $s]')
        fig.align_ylabels(axs[:])
```

2.7 Gain-bandwidth (Bode)



$$A_v(s) = \frac{A_0}{1 + s/\omega_0} \tag{9}$$

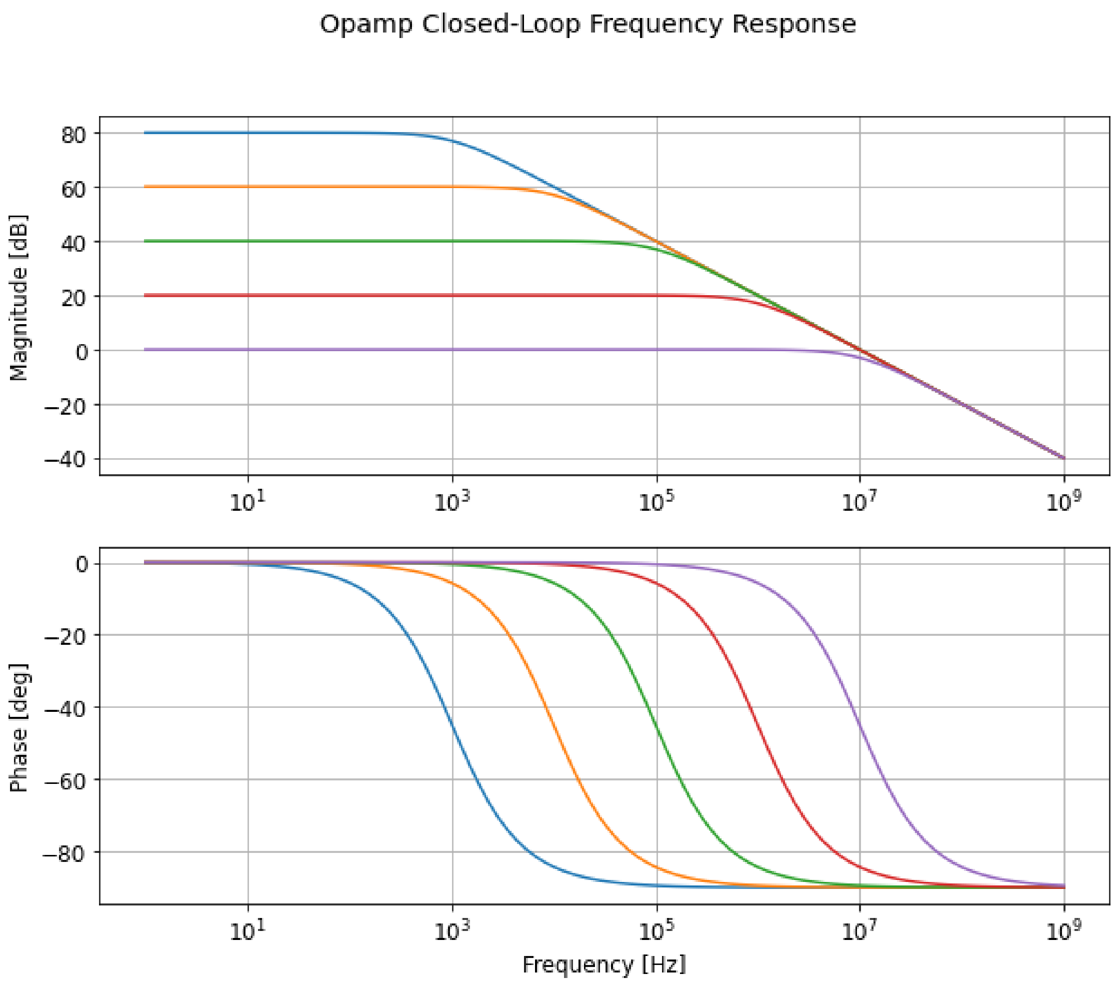
$$G(s) = \frac{\frac{A_0}{1+\beta A_0}}{1 + \frac{s}{\omega_0(1+\beta A_0)}} \tag{10}$$

$$\omega_0^{'} = \omega_0 \cdot (1 + \beta A_0) \tag{11}$$

- For every $20dB$ reduction in closed-loop gain, the $3dB$ frequency increases by 1 decade
- This results from a constant gain-bandwidth product, which is an intrinsic property of the open-loop amplifier
- Note that this assumes that the impedances in the feedback network are purely real (i.e. resistors only)

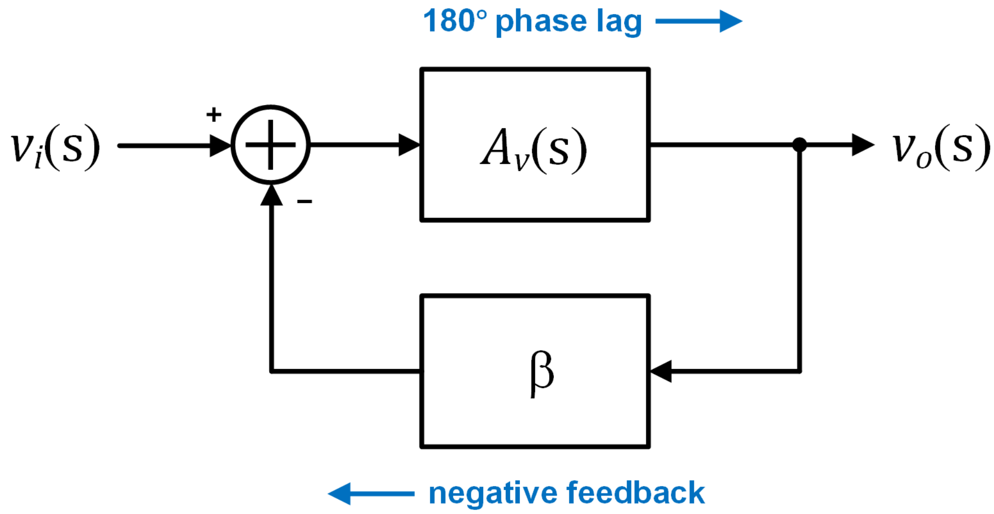
- Let's take a look at the closed-loop frequency response as a function of the feedback factor β

[3]:
```
betas = np.logspace(-4, 0, num=5)
w = 2*np.pi*np.logspace(0,9,num=100)
plot_CL_freq(120, 10e6, betas, w)
```



Opamp Closed-Loop Frequency Response
Magnitude [dB]
80
60
40
20
0
−20
−40
10^1
10^3
10^5
10^7
10^9
Phase [deg]
0
−20
−40
−60
−80
Frequency [Hz]

2.8 Stability: Barkhausen criteria



- In a negative feedback loop, if the loop gain at a given frequency ω_1 is -1, the circuit may oscillate
- This corresponds to a loop gain magnitude $|\beta A_v(j\omega_1)| = 1$ and phase $\angle A_v(j\omega_1) = -180°$

2.9 Stability: Root locus

- Open-loop transfer function

$$A_v(s) = \frac{A_0}{1 + \frac{s}{\omega_0}} \tag{12}$$

- Closed-loop transfer function

$$G(s) = \frac{A_0}{1 + s/\omega_0 + \beta A_0} \tag{13}$$

- Open-loop pole

$$s_0 = -\omega_0 \tag{14}$$

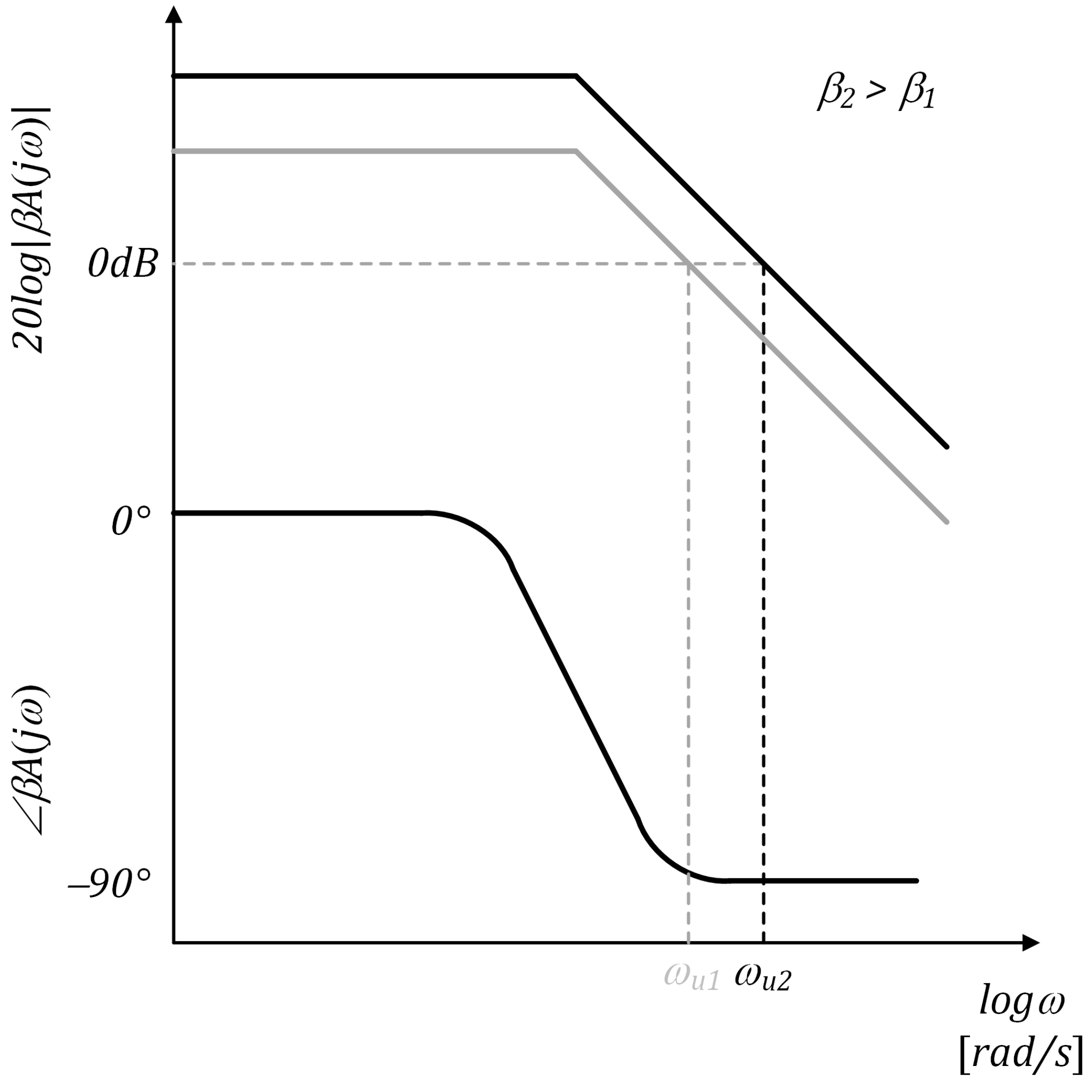
- Closed-loop pole

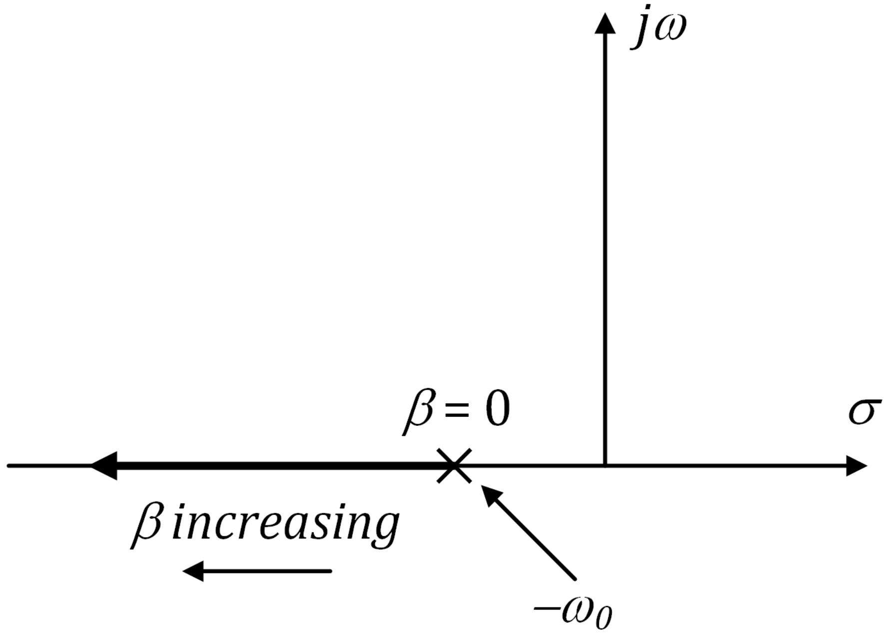
$$s_0^{'} = -\omega_0(1 + \beta A_0) \tag{15}$$

- Root locus plots involve plotting the closed-loop poles in the complex plane to evaluate stability
- If a given pole $s = j\omega + \sigma$ falls in the right half plane (RHP), the system is unstable (phase lag $> 180°$)

- Here, for a single pole system, we have a single, real, LHP pole, so the system is unconditionally stable

2.10 Bode plot vs root locus





- For a single-pole system both methods indicate unconditional stability:
  - Bode plot: Maximum phase lag of 90°
  - Root locus: Purely real, LHP pole
- For higher-order systems, the worst-case scenario arises when $\beta = 1$ (highest possible loop gain)

2.11 Root locus of a second-order system

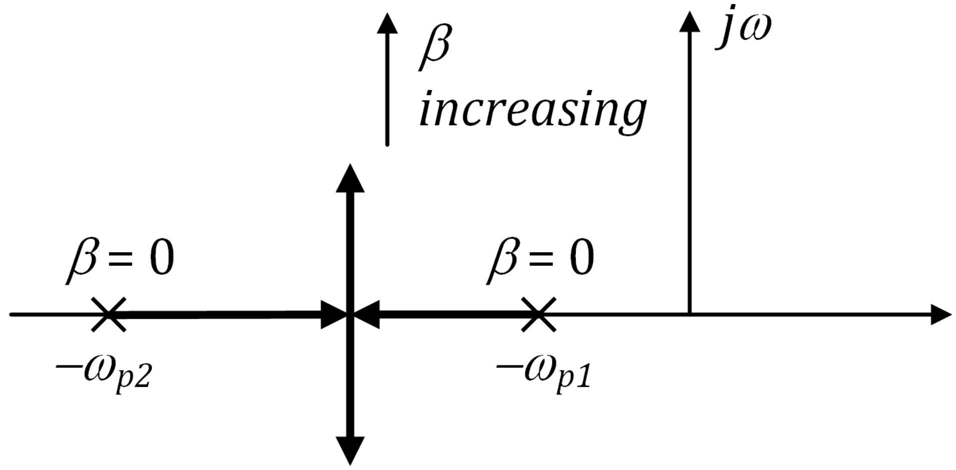
- Open-loop transfer function

$$A_v(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)} \tag{16}$$

- Closed-loop transfer function

$$A_v(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) + \beta A_0} \tag{17}$$

- For a second-order system, the situation is more "complex"
- We can solve for the closed-loop pole locations using the quadratic formula and plot them in the complex plane to evaluate stability

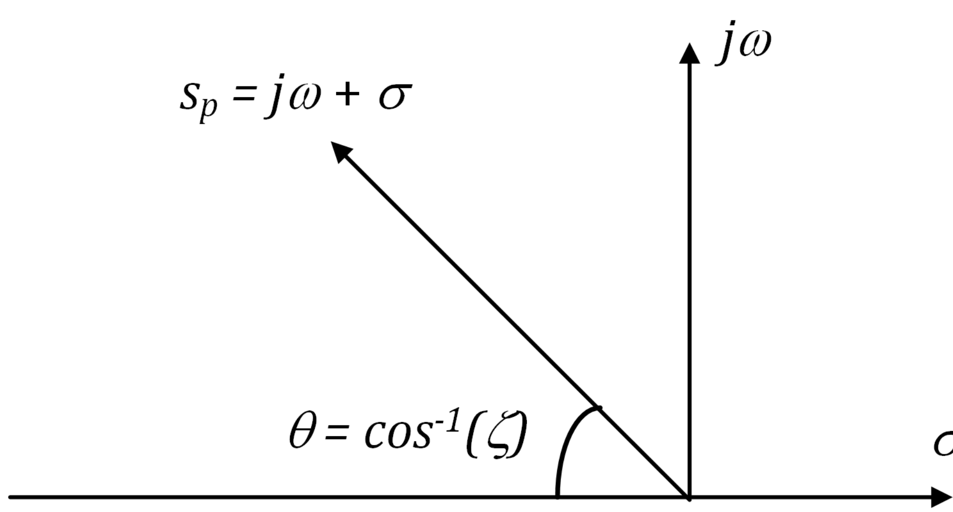


- The closed-loop poles are given by

$$s_{p1,2} = \frac{-(\omega_{p1} + \omega_{p2}) \pm \sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + \beta A_0)\omega_{p1}\omega_{p2}}}{2} \tag{18}$$

- When $\beta = 0$ (no feedback), the closed-loop poles are equal to the open-loop poles
- As β increases, the imaginary components of s_{p1} and s_{p2} increase

2.12 Damping ratio and phase margin

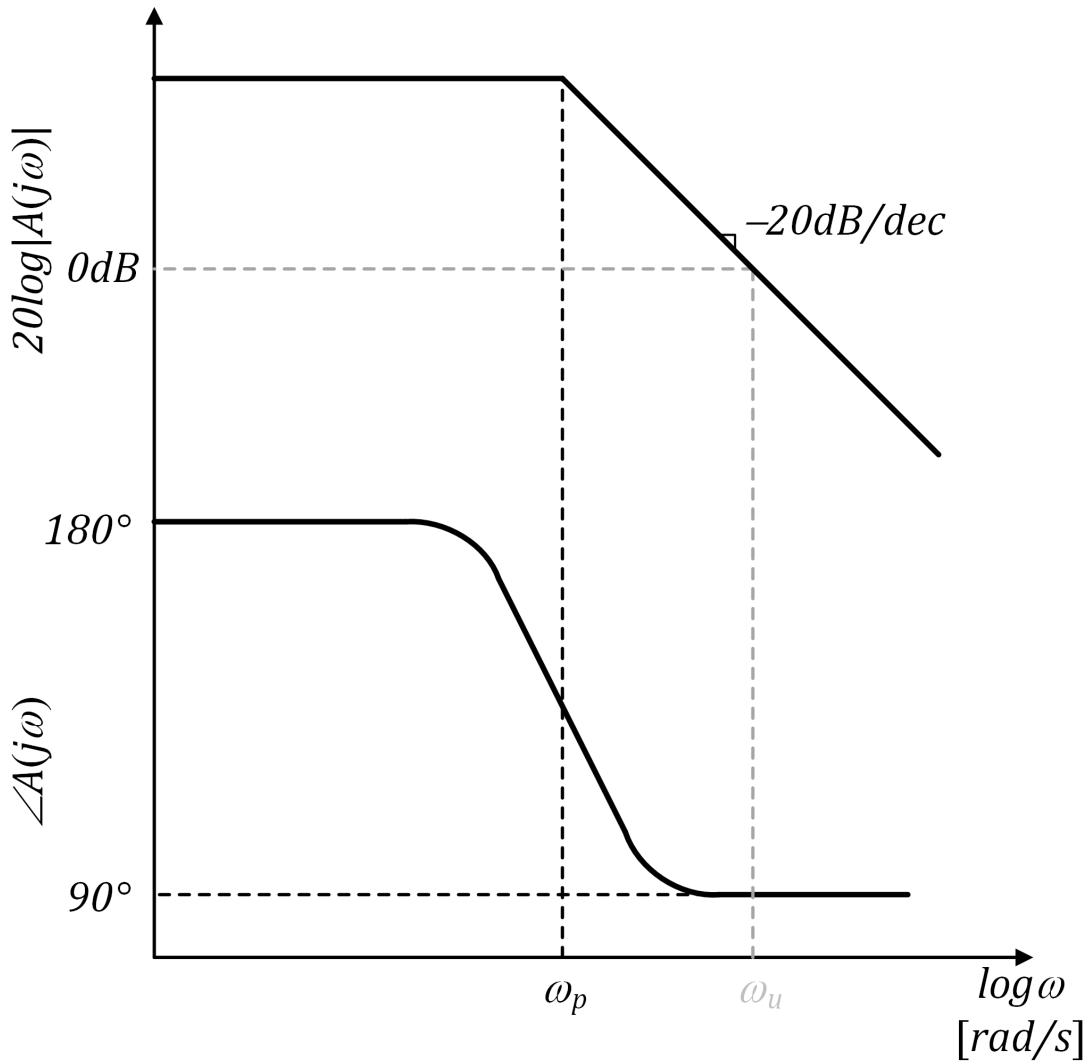


$$PM = \tan^{-1} \frac{2\zeta}{-2\zeta^2 + \sqrt{1 + 4\zeta^2}} \tag{19}$$

- As β increases, the angle $\cos^{-1}(\zeta)$ increases, the magnitude of the imaginary component increases relative to that of the real component
- ζ is referred to as the "damping factor," and can be used to evaluate the qualitative behavior of the impulse response

- Another means of evaluating this behavior is by looking at the phase margin, which enables use of the Bode plot

2.13 Phase margin of a single-pole system



- Here, phase margin is defined as

$$PM = \angle A(j\omega_u) - 0^\circ \tag{20}$$

- For a single-pole system, the phase margin is

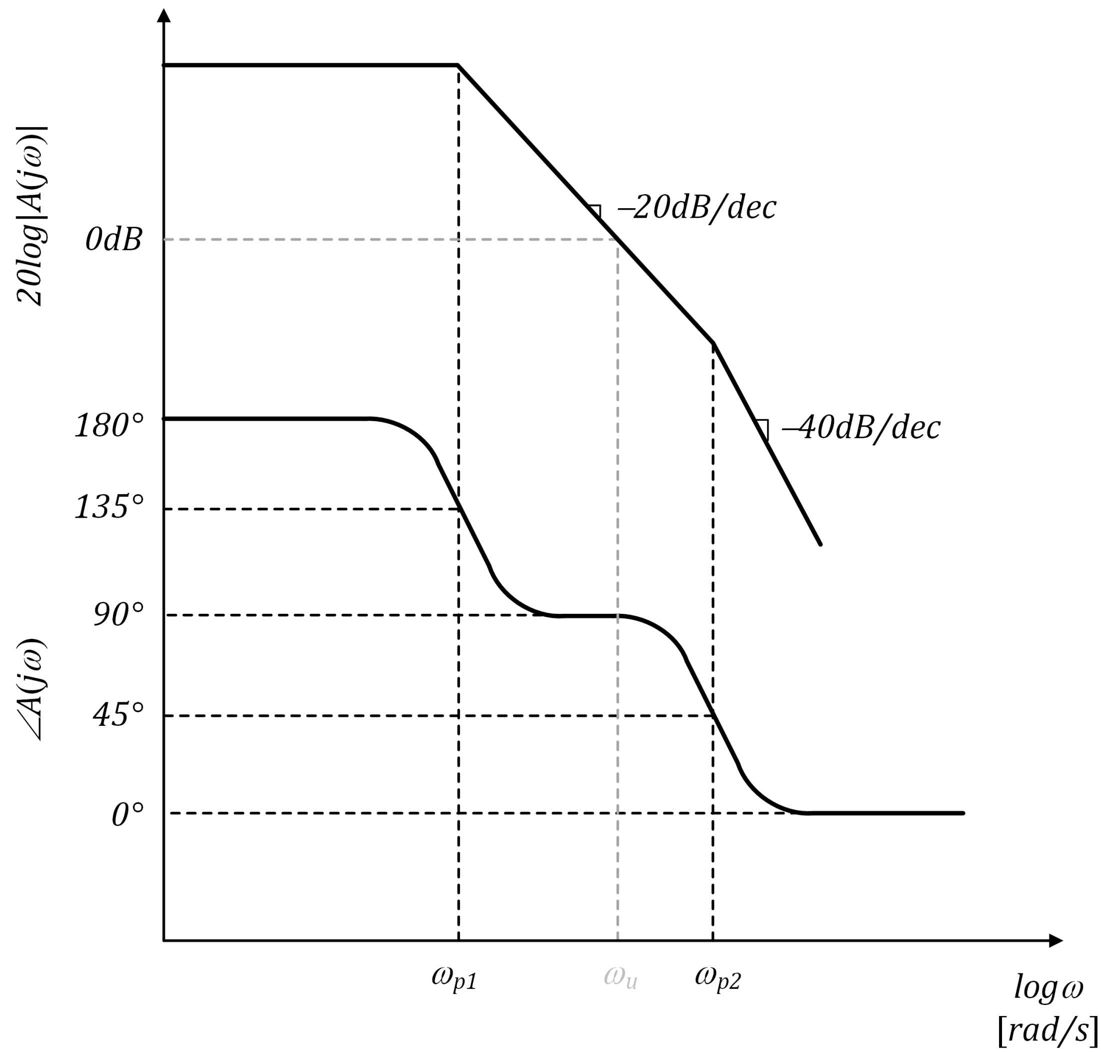
$$PM = \angle A(j\omega_u) = 180^\circ - \tan^{-1}\frac{\omega_u}{\omega_{p1}} \tag{21}$$

- This has a minimum value of

$$PM \geq 180^\circ - 90^\circ = 90^\circ \tag{22}$$

- This guarantees stability and a "well behaved" step response

2.14 Phase margin of a two-pole system



- For stability (i.e. no oscillation), we need

$$\angle A(j\omega_u) > 0^\circ \tag{23}$$

- For a well-behaved response, we prefer to have

$$\angle A(j\omega_u) \geq 60^\circ \tag{24}$$

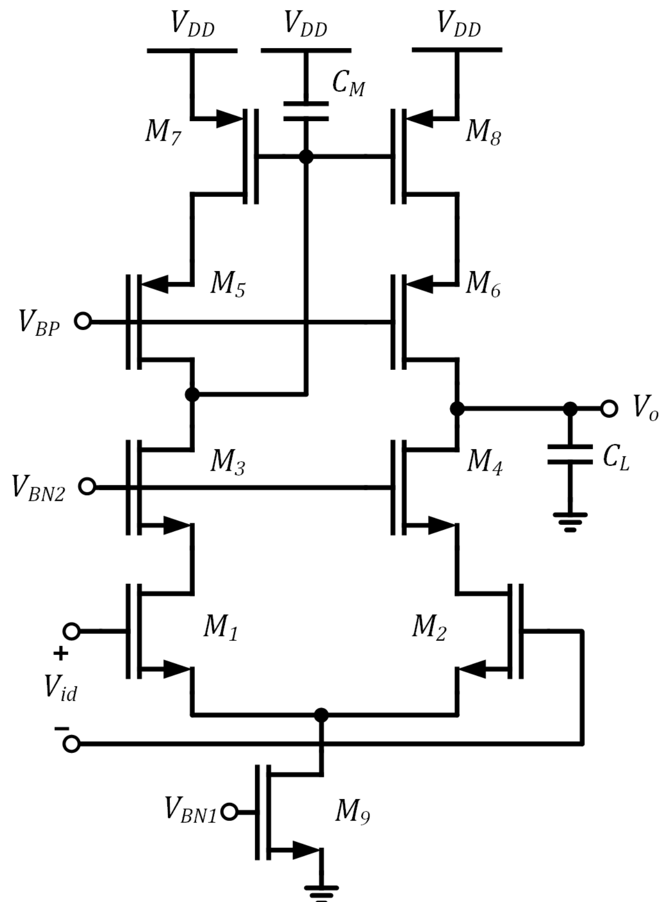
$$PM = \angle A(j\omega_u) = 180^\circ - \tan^{-1}\frac{\omega_u}{\omega_{p1}} - \tan^{-1}\frac{\omega_u}{\omega_{p2}} \tag{25}$$

- This places a requirement on ω_{p2} of

$$\tan^{-1}\frac{\omega_u}{\omega_{p2}} \leq 30^\circ \tag{26}$$

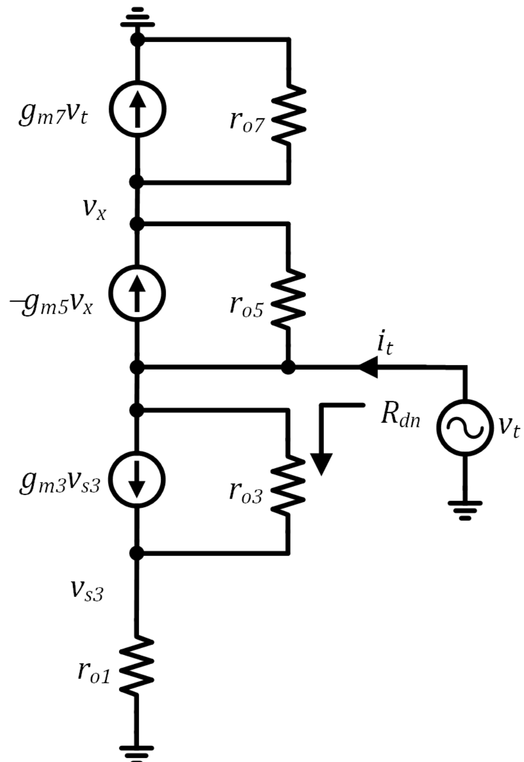
$$\boxed{\omega_{p2} \geq 1.73\omega_u} \tag{27}$$

2.15 Telescopic cascode amplifier



- C_M represents parasitic capacitance at the gate node of M_7, M_8
- C_M may be large if $V_{OV7,8}$ is small
- M_7 gate connection functions as a diode-connected MOS in the small-signal model
- The $3dB$ frequency can be found by ZVTC analysis, but assuming $C_L >> C_M$, it can be approximated as the inverse of product of R_o and C_L

2.16 M7 diode connection (small signal analysis)



$$i_{up} = \frac{v_t - v_x}{r_{o5}} - g_{m5}v_x \qquad v_x = \frac{v_t - i_{up}r_{o5}}{g_{m5}r_{o5} + 1} \tag{28}$$

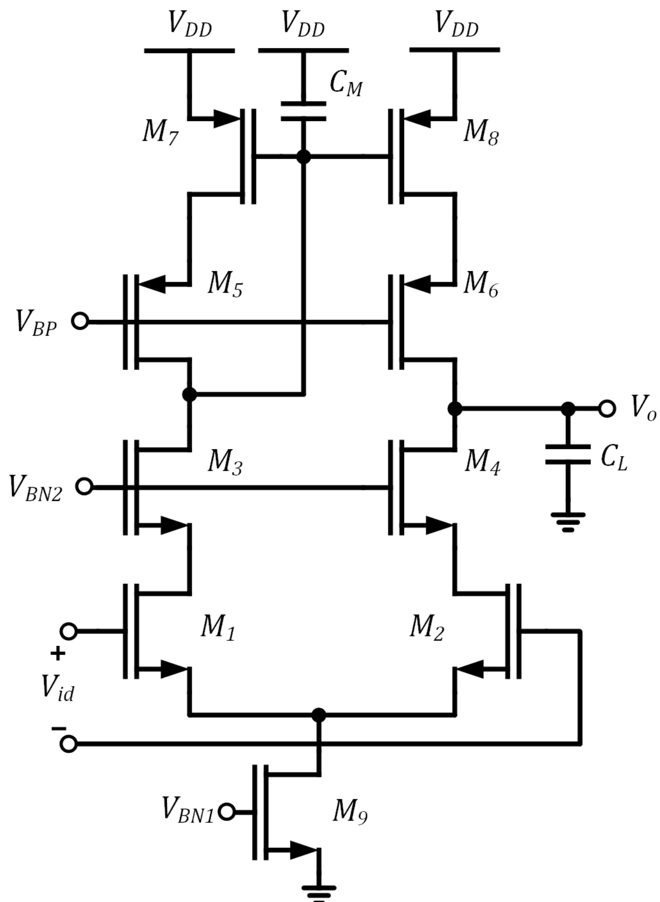
$$i_{up} = g_{m7}v_t + \frac{v_x}{r_{o7}} = g_{m7}v_t + \frac{v_t - i_{up}r_{o5}}{r_{o7}(g_{m5}r_{o5} + 1)} \tag{29}$$

$$i_{up}\left(1 + \frac{r_{o5}}{r_{o7}(g_{m5}r_{o5} + 1)}\right) = v_t\left(g_{m7} + \frac{1}{r_{o7}(g_{m5}r_{o5} + 1)}\right) \tag{30}$$

- The output resistance is thus

$$\boxed{R_{up} = \frac{v_t}{i_{up}} \approx \frac{1}{g_{m7}} \approx \frac{v_t}{i_t}} \tag{31}$$

2.17 Telescopic amplifier frequency response



- The dominant pole frequency is

$$\omega_{p1} \approx \frac{1}{g_{m6}r_{o6}r_{o8}||g_{m4}r_{o4}r_{o2} \cdot C_L} \tag{32}$$

- The non-dominant pole is given by

$$\omega_{p2} \approx \frac{g_{m7}}{C_M} \tag{33}$$

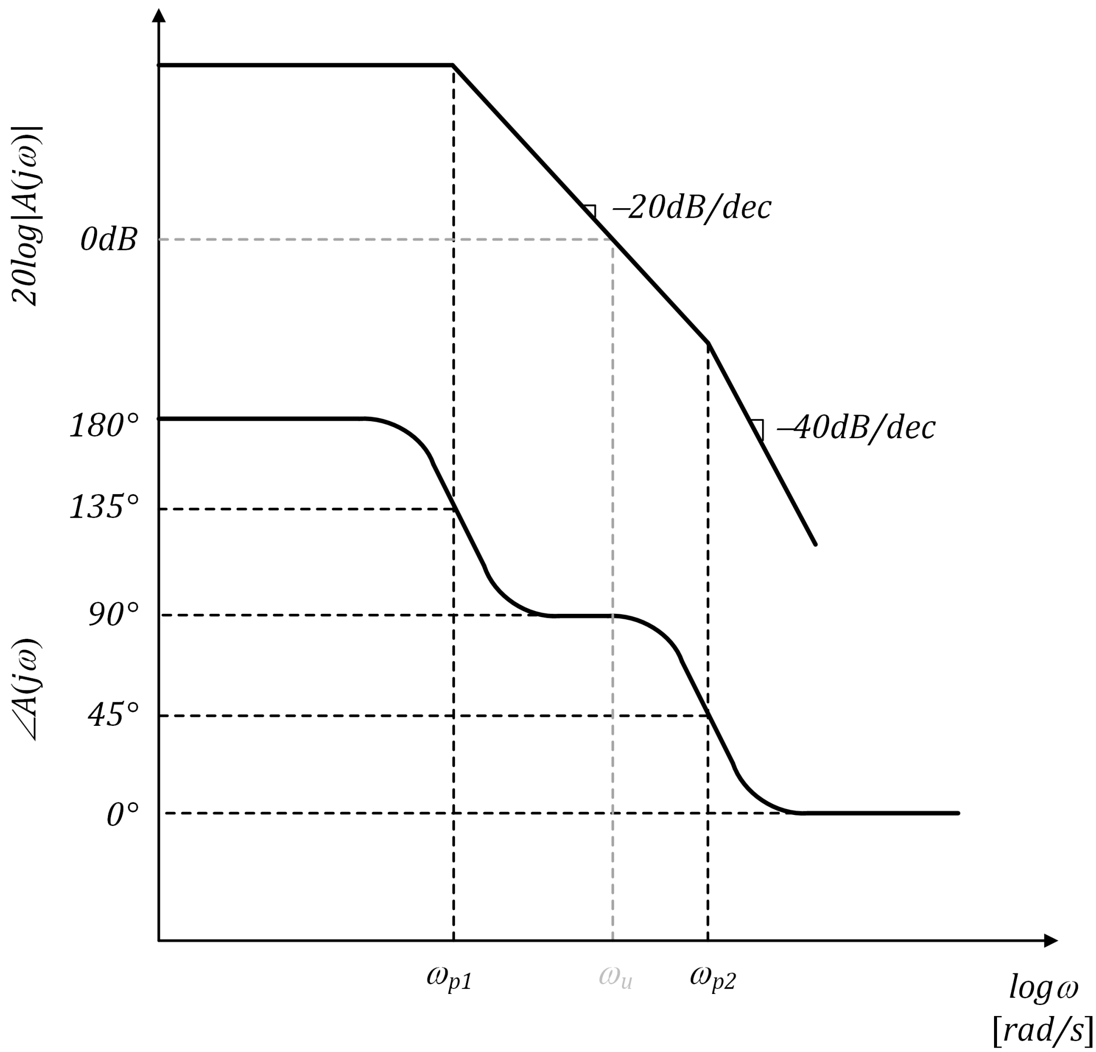
- Assuming $\omega_{p2} >> \omega p1$, the gain-bandwidth is

$$\omega_u \approx \frac{g_{m1,2}}{C_L} \tag{34}$$

- The phase is

$$\angle A(j\omega) = 180^\circ - \tan^{-1}\frac{\omega}{\omega_{p1}} - \tan^{-1}\frac{\omega}{\omega_{p2}} \tag{35}$$

2.18 Telescopic amplifier phase margin



- For a well-behaved response we need

$$A(j\omega_u) \geq 60^\circ \tag{36}$$

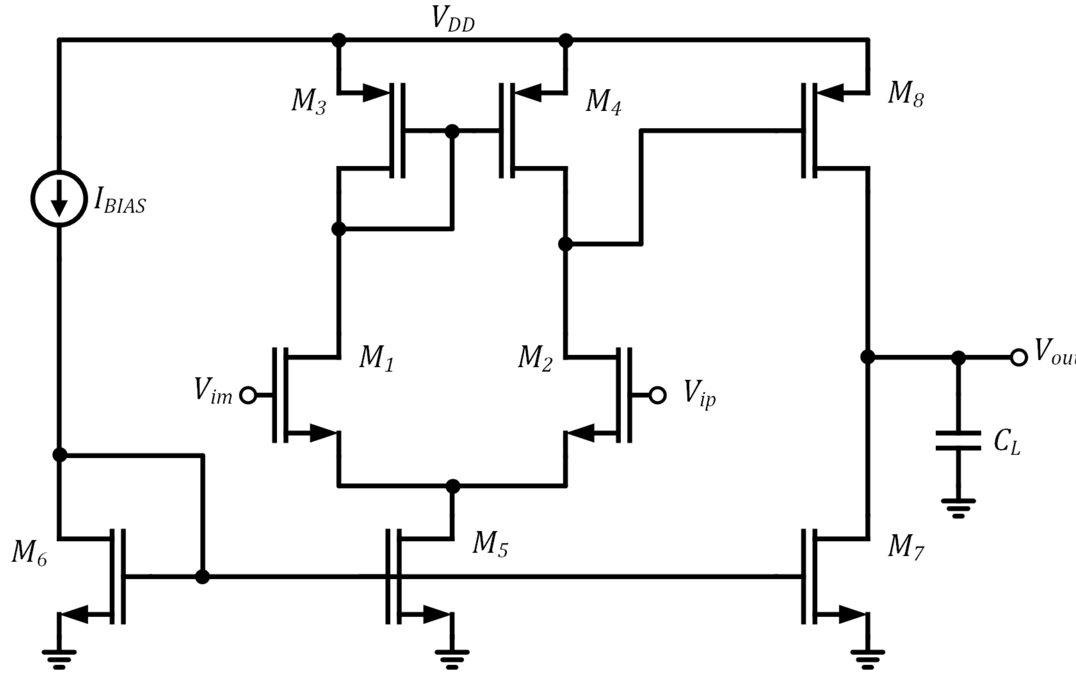
- Assuming a two-pole amplifier, this requires

$$\frac{g_{m7}}{C_M} \geq 1.73\frac{g_{m1,2}}{C_L} \tag{37}$$

- Using the long-channel model, the non-dominant pole can be expressed as

$$\frac{g_{m7}}{C_M} \approx \frac{\mu_p C_{ox}\frac{W}{L}V_{OV7,8}}{2\cdot\frac{2}{3}WLC_{ox}} = \frac{\mu_p V_{OV7,8}}{\frac{4}{3}L^2} \tag{38}$$

2.19 Frequency response of a 2-stage amplifier



- A 2-stage amplifier can be analyzed in the same manner as a common-source amplifier
- Here we primarily focus on the second stage, due to the high output impedance of the first stage ($R_{o1} = r_{o2}||r_{o4}$) and the Miller effect of M_8
- The DC gain of the amplifier is given by the product of the gains of the individual stages:

$$\begin{aligned} A_0 &= G_{m1}R_{o1}G_{m2}R_{o2} && (39) \\ &= g_{m1}r_{o2}||r_{o4} \cdot g_{m8}r_{o8}||r_{o7} && (40) \end{aligned}$$

- Let's take a look at the frequency response...
- Assuming the mirror pole is well above the unity-gain bandwidth of the amplifier, the 2-stage CMOS OTA can be analyzed as a common-source amplifier with $R_{o1} = r_{o2}||r_{o4}$ as the output impedance of the driving stage
- In this case, the transfer function is given as

$$A_v(s) = g_{m1}R_{o1}\frac{(sC_{GD} - g_{m8})R_{o2}}{R_{o1}R_{o2}\xi s^2 + [R_{o1}(1 + g_{m8}R_{o2})C_{GD} + R_{o1}C_{GS} + R_{o2}(C_{GD} + C_L)]s + 1} \quad (41)$$

$$(42)$$

- If we allow $R_{o2} \to \infty$ (this places the dominant pole at the origin), this becomes

$$\lim_{R_{o2}\to\infty} A_v(s) \approx g_{m1}R_{o1}\frac{(sC_{GD}-g_{m8})}{s[R_{o1}(C_{GS}C_{GD}+C_{GS}C_L+C_{GD}C_L)s+g_{m8}R_{o1}C_{GD}+(C_{GD}+C_L)]} \tag{43}$$

(44)

- The assumption that $R_{o2}\to\infty$ is equivalent to the second stage being a "perfect integrator" (i.e. infinite gain).
- We can solve for the non-dominant pole by setting the denominator equal to zero and solving for s. This gives

$$\omega_{p2} \approx \frac{(g_{m8}R_{o1}+1)C_{GD}+C_L}{R_{o1}(C_{GS}C_{GD}+C_{GS}C_L+C_{GD}C_L)} \tag{45}$$

- This can be further approximated by assuming $g_m R_{o1}C_{GD} >> C_L$

$$\omega_{p2} \approx \frac{g_{m8}R_{o1}C_{GD}}{R_{o1}(C_{GD}(C_{GS}+C_L)+C_{GS}C_L)} \tag{46}$$

- We have previously shown the dominant pole of the transfer function to be well-approximated as

$$\omega_{p1} \approx \frac{1}{R_{o1}(1+g_{m8}R_{o2})C_{GD}} \tag{47}$$

2.20 Pole splitting

- With the two poles of the transfer function given by

$$\omega_{p1} \approx \frac{1}{R_{o1}(1+g_{m8}R_{o2})C_{GD}} \qquad \omega_{p2} \approx \frac{g_{m8}C_{GD}}{C_{GD}(C_{GS}+C_L)+C_{GS}C_L} \tag{48}$$

- We can make some qualitative observations about their behavior:
  - As C_{GD} increases, ω_{p1} decreases, lowering bandwidth
  - ω_{p2} simultaneously increases as the C_{GD} term in the denominator becomes dominant
  - ω_{p2} is ultimately limited by $g_{m8}/(C_{GS}+C_L) \approx g_{m8}/C_L$
- This behavior is referred to as "pole splitting," since ω_{p1} and ω_{p2} are moving in opposite directions as C_{GD} is increased

2.21 2-stage amplifier RHP zero

- The full transfer function is once again given by

$$A_v(s) = \frac{(sC_{GD}-g_{m8})R_{o2}}{R_{o1}R_{o2}\xi s^2+[R_{o1}(1+g_{m8}R_{o2})C_{GD}+R_{o1}C_{GS}+R_{o2}(C_{GD}+C_L)]s+1} \tag{49}$$

(50)

- The expression in the numerator, $N(j\omega) = (sC_{GD} - g_{m8})R_{o2}$ results in a zero in the right half of the complex plane

$$\omega_z = \frac{g_{m8}}{C_{GD}} \tag{51}$$

- A zero in the right-half plane increases phase lag as well as gain magnitude, which can be detrimental to stability

$$\angle N(j\omega) = \tan^{-1}\left(-\frac{\omega C_{GD}}{g_{m8}}\right) \tag{52}$$

2.22 Phase margin

- If we assume dominant-pole behavior, we can approximate the unity-gain frequency as

$$\omega_u \approx g_{m1}R_{o1}g_{m8}R_{o2} \cdot \frac{1}{g_{m8}R_{o2}R_{o1}C_{GD}} = \frac{g_{m1}}{C_{GD}} \tag{53}$$

- The phase margin can then be approximated as

$$PM \approx 90^\circ - \tan^{-1}\frac{\omega_u}{\omega_{p2}} - \tan^{-1}\frac{\omega_u}{\omega_z} \tag{54}$$

$$= \boxed{90^\circ - \tan^{-1}\frac{g_{m1}C_L}{g_{m8}C_{GD}} - \tan^{-1}\frac{g_{m1}}{g_{m8}}} \tag{55}$$

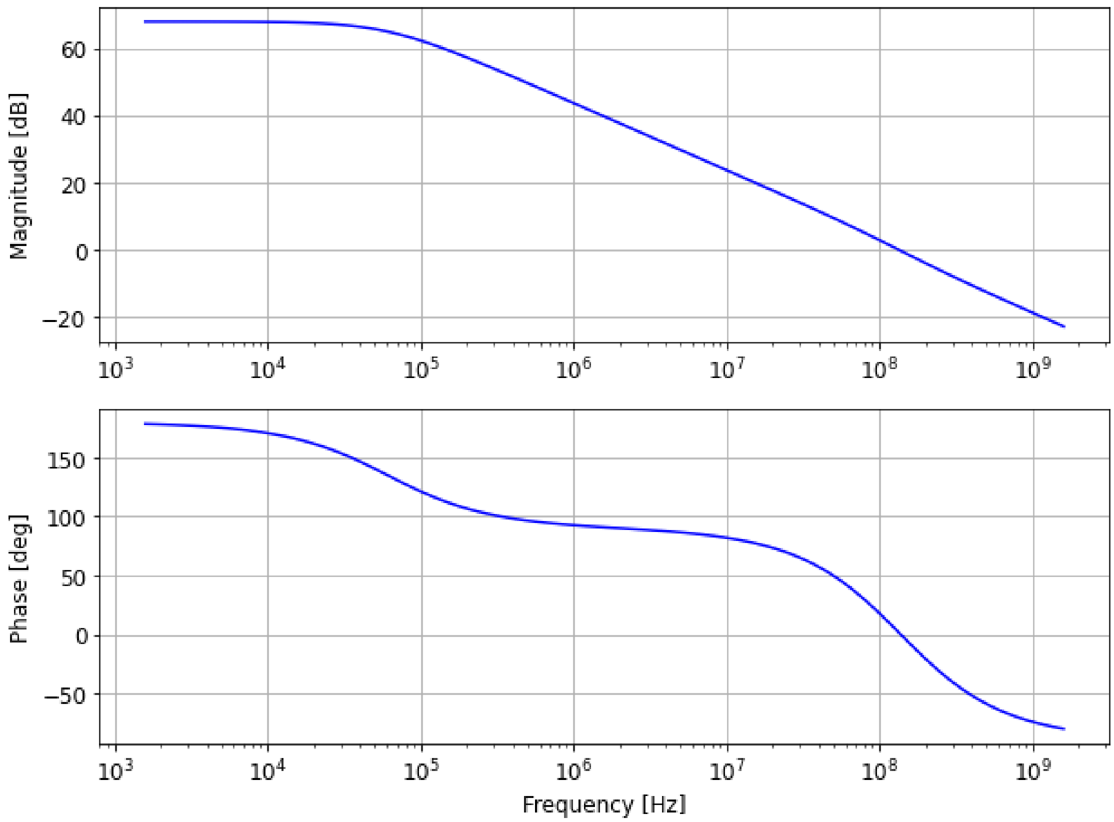
(56)

- Note that if g_{m1} and g_{m8} are comparable, and if $C_L \geq C_{GD}$, the phase margin will be zero, or even negative!

2.23 Frequency response

[5]:
```
gm1 = 1e-3
gm8 = 1e-3
ro = 100e3
R_o1 = ro/2
R_o2 = ro/2
C_GD = 1e-12
C_GS = .2e-12
C_L = 1e-12
zeta = C_GS*C_GD + C_GS*C_L+C_GD*C_L
num = [C_GD*R_o2*gm1*R_o1, -gm8*R_o2*gm1*R_o1]
den = [R_o1*R_o2*zeta, gm1*R_o1*R_o2*C_GD+R_o1*C_GS+R_o2*(C_GD+C_L), 1]
tf_CS = signal.TransferFunction(num,  den)
w, mag, phase = tf_CS.bode()
f = w/2/np.pi
```

```
[6]: plot_logxy2(f, mag, f, phase, 'Frequency [Hz]', 'Magnitude [dB]',
                 'Frequency [Hz]', 'Phase [deg]')
```



2.24 Summary

- The frequency response of closed-loop amplifiers relies on characteristics of the open-loop response
- Stability of negative-feedback systems requires a phase lag of less than 180° at the transit (unity-gain) frequency
- Bode plots and root loci can be used to evaluate stability of closed-loop systems
- To ensure "well-behaved" closed-loop responses, phase margin should be kept above ~60° (overdamped, no overshoot)
- Mirror pole can degrade phase margin in single-ended OTAs
- 2-stage OTAs have multiple poles and a RHP zero
- Next time, we'll look at compensation of 2-stage CMOS OTAs