Homework # 2

Galactic Astronomy (Astr 511); Winter Quarter 2021

In this homework you will explore some simple potentials and orbits relevant to galactic dynamics. We will also introduce a Python library for galactic dynamics, `galpy`.

1. In Lecture 6 (Slide 12), we've introduced the isochrone potential:

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}} \tag{1}$$

   Show that it behaves as a harmonic oscillator in the core (for $r << b$) and as the potential of a point mass at large distances (for $r >> b$). Expand it to second order in Taylor series around $r = 0$ for the former, and observe the limit $\Phi(r)$ as $r \to \infty$ for the latter.

2. Install the `galpy` orbital dynamics library (`https://galpy.readthedocs.io`). If you're using the Anaconda Python Distribution, it is easily obtainable from `conda-forge` by running `conda install -c defaults -c conda-forge galpy`). Otherwise `pip install galpy` should work as well.

   `galpy` library includes all the potentials we've discussed so far, as well as a number of interesting potentials we haven't had time to look into. Particularly interesting ones are the Myamoto-Nagai potential (a good approximation to the potential of galactic disks), Navarro-Frank-White (NFW) potential (which describes the potential of dark matter halos), and the Hernquist potential (a good description of bulges and spherical galaxies).

   Using `galpy`'s `galpy.potential.plotRotcurve` function, plot the normalized rotation curve given by the sum of these three potentials in the range of $[0, 10]$ effective radii. Plot their individual contributions to circular speed on the same plot. Use the following parameters: $a = 0.5, b = 0.0375, normalize = .6$ for Myamoto-Nagai, $a = 4.5, normalize = .35$ for NFW, and $a = 0.6/8, normalize = 0.05$ for Hernquist.

   Discuss how the combined rotation curve compares to the observed rotation curves of galaxies. Which component makes the dominant contribution to circular velocity at $R = 1$ (inner parts of a galaxy)? What about at $R = 10$?

   Note: You should find the section on 'Rotation Curves' in the `galpy` documentation **very** helpful.

3. Use `galpy`'s orbit plotting routines (see the 'Orbit Integration' section in `galpy` documentation) to plot the R, z plane motion of a particle orbiting in an isochrone potential with $b = 1$ whose initial coordinates are $(R, z) = (1, 0)$, and the radial, tangential, and z velocity $(v_R, v_T, v_z) = (1, 1.1, 0.1)$.

   What are the initial velocities that results in a circular orbit? Demonstrate by plotting it (it's fine to plot it in either in $R - z$ or $x - y$ planes).