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Conduction Heat Transfer Solutions

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PREFACE

This text is a collection of solutions to a variety of heat conduction problems found in numerous publications, such as textbooks, handbooks, journals, reports, etc. Its purpose is to assemble these solutions into one source that can facilitate the search for a particular problem solution. Generally, it is intended to be a handbook on the subject of heat conduction.

Engineers, scientists, technologists, and designers of all disciplines should find this material useful, particularly those who design thermal systems or estimate temperatures and heat transfer rates in structures. More than 500 problem solutions and relevant data are tabulated for easy retrieval. Having this kind of material available can save time and effort in reaching design decisions.

There are twelve sections of solutions which correspond with the class of problems found in each. Geometry, state, boundary conditions, and other categories are used to classify the problems. A case number is assigned to each problem for cross-referencing, and also for future reference. Each problem is concisely described by geometry and condition statements, and many times a descriptive sketch is also included. At least one source reference is given so that the user can review the methods used to derive the solutions. Problem solutions are given in the form of equations, graphs, and tables of data, all of which are also identified by problem case numbers and source references.

The introduction presents a synopsis on the theory, differential equations, and boundary conditions for conduction heat transfer. Some discussion is given on the use and interpretation of solutions. Also, some example problem solutions are included. This material may give the user a review, or even some insight, on the phenomenology of heat conduction and its applicability to specific problems.

Supplementary data such as mathematical functions, convection correlations, and thermal properties are included for aiding the user in computing numerical values from the solutions. Property data were taken from some of the latest publications relating to the particular properties listed. Only the international system of units (SI) is used.

Consistency in nomenclature and terminology is used throughout, making this text more readable than a collection of different references. Also, dimensionless parameters are frequently used to generalize the applicability of the solutions and to permit easier evaluation of the effects of problem conditions.

Even though some of the equational solutions are lengthy and include several different mathematical functions, this should not pose a formidable task for most users. Modern computers can make complicated calculations easy to perform. Even many electronic calculators can be used to compute complex functions. If, however, these tools are not available, one can resort to hand computing methods. The table of mathematical functions and constants would be useful in that case.

Heat conduction has been studied extensively, and the number of published solutions is large. In fact, there are many solutions that are not included in this text. For example, some solutions are found by a specific computational process that cannot be described briefly. Moreover, new solutions are constantly appearing in technical journals and reports. Nevertheless, this collection contains most of the published solutions.

The differential equations and boundary-condition equations for heat flow are identical in form to those for other phenomena such as electrical fields, fluid flow, and mass diffusion. This similarity gives additional utility to the heat conduction solutions. The user needs only identify equivalence of conditions and terms when selecting a proper solution. This practice is prescribed in many texts on applied mathematics, electrical theory, heat transfer, and mass transfer.

A search for particular solutions has frequently been a tedious and difficult task. Too often, countless hours have been spent in searching for a problem solution. Locating and obtaining a proper reference can require considerable effort. Also, it is frequently necessary to study a theoretical development in order to find the applicable solution. In so doing, there are sometimes misinterpretations which lead to erroneous results. This text should help alleviate some of these problems.

Science gives us information for reaching new frontiers in technology. It is, thus, appropriate to give something back. I hope this text is at least a small contribution.

James H. VanSant

NOMENCLATURE

A = Area, m^2

b = Time constant, s^{-1}

c = Specific heat, $J/kg \cdot {}^oC$

C = Circumference, m

d, D = Diameter, depth, m

h = Heat transfer coefficient, $W/m^2 \cdot {}^oC$

k = Thermal conductivity, $W/m \cdot {}^oC$

m = $\sqrt{hC/kA}$, m^{-1}

ℓ, L = Length, m

q = Heat flux rate, W/m^2

q_x, q_y, q_z = Heat flux in x, y, z directions, W/m^2

q''' = Volumetric heating rate, W/m^3

Q = Heat transfer rate, W

r, R = Radius, m

t, T = Temperature, ${}^oC$, K

V = Velocity, m/s

w = Width, m

x, y, z, = Cartesian coordinates, m

α = Thermal diffusivity, $k/\rho c$, m^2/s

β = Temperature coefficient, ${}^oC^{-1}$

γ = Heat of evaporation, J/kg

γ = Latent heat of fusion, J/kg

Δ = Difference

ε = Emissivity for thermal radiation

η = Fin effectiveness, $\dfrac{\text{(actual heat transferred)}}{\text{(heat transferred without fins)}}$

ϕ = Fin effectiveness, $\dfrac{\text{(actual heat transferred}}{\text{(heat transferred from infinite conductivity fins)}}$

ρ = Density, kg/m^3

σ = $2\sqrt{\alpha\tau}$, m

σ = Stefan-Boltzmann constant, $W/m^{-2} \cdot K^{-4}$

τ = Time, s

$\mathcal{F}$ = Radiation configuration--emissivity factor

DIMENSIONLESS GROUPS

Bi_ℓ, Bi = Biot modulus = $h\ell/k$

Bf_ℓ, Bf = Bi Fo = $h\tau/\rho c\ell$

Fo_ℓ, Fo = Fourier modulus = $\alpha\tau/\ell^2$

Fo^*_ℓ, Fo^* = Modified Fourier modulus = $1/(2\sqrt{Fo})$

Gr_ℓ, Gr = Grashof number = $g\beta\Delta t\ell^3/\nu^2$

Ki_ℓ, Ki = Kirpichev number = $q\ell/k\Delta t$

Nu_d, Nu = Nusselt number = hd/k

Pd_ℓ, Pd = Predvoditelev modulus = $b\ell^2/\alpha$

Po_ℓ, Po = Pomerantsev modulus = $q'''\ell^2/k\Delta t$

Pr = Prandtl number = ν/α

R = Radius ratio = r/r_o

Re_d, Re = Reynolds number = vd/ν

X = Length ratio = x/ℓ

Y = Width ratio = y/w

Z = Length ratio = z/ℓ

MATHEMATICAL FUNCTIONS

exp = Exponential function

Ei = Exponential integral

erf = Error function

erfc = Complementary error function

i^nerfc = Complementary error function integral

I_n = Modified Bessel function of the first kind

J_n = Bessel function of the first kind

K_n = Modified Bessel function of the second kind

ℓn = Natural log

Y_n = Bessel function of the second kind

P_n = Legendre polynomial of the first kind

Γ = Gamma function

INTRODUCTION

1. HEAT CONDUCTION

Energy in the form of heat has been used by man ever since he began walking on this earth. Moreover, the transfer of heat is essential to our very existence. Not only do our own physiological functions require some form of heat transfer, but so do most life-sustaining processes of nature and many man-controlled activities. The importance of the thermal sciences in the total sphere of science can, thus, hardly be disputed.

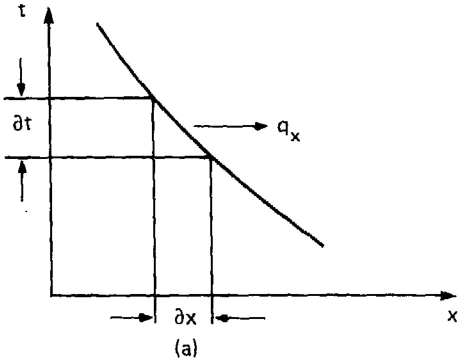
Conduction is one of the three principal heat transfer modes, the others being convection and radiation. It is customarily distinguished as being an energy diffusion process in materials which do not contain molecular convection. Kinetic energy is exchanged between molecules resulting in a net transfer between regions of different energy levels, these energy levels are commonly called temperature. Particularly, heat conduction in metals is mainly attributed to the motion of free electrons and in solid electrical insulators to the longitudinal oscillations of atoms. In fluids, the elastic impact of molecules is considered as the heat conduction process.

The process of heat transfer in materials has been studied for many centuries. Even early Greek philosophers, such as Lucretius (c. 98-55 B.C.), meditated on the subject and recorded their conclusions. Much later, the famous mathematical physicist, Joseph B. J. Fourier (1768-1830), developed a mathematical expression that became the basis of practically all heat conduction solutions. He postulated that a local heat flux rate in a material is proportional to the local temperature gradient in the direction of heat flow:

$$q_x \propto \frac{\partial t}{\partial x} , \tag{1}$$

where q_x is the heat flow in the x-direction per unit area as illustrated in Fig. 1a. Material properties are accounted for by including a proportionality constant:

$$q_x = -k \frac{\partial t}{\partial x} , \tag{2}$$



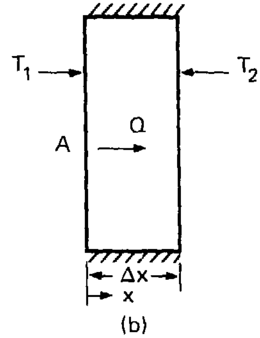


FIG. 1. Illustration of heat flow and temperature gradient.

where the constant k is called thermal conductivity. (The minus sign must be included to satisfy the second law of thermodynamics.) This equation is called Fourier's law for heterogeneous isotropic continua.

A simpler form of Fourier's law is for homogeneous isotropic continua. For example, consider a plate of this type material having isothermal surfaces and insulated edges as shown in Fig. 1. The plate has a width Δx, surface area A, and thermal conductivity k. The heat flow in the plate is expressed as

$$Q_x = -kA \frac{(t_2 - t_1)}{\Delta x} \tag{3}$$

This expression becomes Eq. (2) when Δx diminishes to zero.

$$\frac{Q_x}{A} = q_x = -k \lim_{\Delta x \to 0} \frac{\Delta t}{\Delta x} = -k \frac{\partial t}{\partial x} \quad . \tag{4}$$

The heat flux q is presumed to have both magnitude and direction. Thus, it can be given as a vector, $\bar{q}$ which is normal to an isothermal surface. For example, in Cartesian coordinates

$$\bar{q} = q_x\bar{i} + q_y\bar{j} + q_z\bar{k} \quad . \tag{5}$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$ are unit vectors in the x-, y-, and z-directions, respectively. Since Eq. (4) defines $q_x = -k\partial t/\partial x$, and similarly $q_y = -k\partial t/\partial y$, $q_z = -k\partial t/\partial z$, we can state

$$\bar{q} = -k\ (\bar{i}\partial t/\partial x + \bar{j}\partial t/\partial y + \bar{k}\partial t/\partial z) \tag{6}$$

$$\bar{q} = -k\nabla t \quad . \tag{7}$$

In anisotropic continua the direction of the heat flux vector is not necessarily normal to an isothermal surface. Example materials are crystals, laminates, and oriented fiber composites. In such materials we may assume each component of the heat flux vector to be linearly dependent on all components of the temperature gradient at a point. The vector form of Fourier's law for heterogeneous anisotropic continua becomes

$$\bar{q} = -K \cdot \nabla t \quad , \tag{8}$$

where K is the conductivity tensor; the components of this tensor are called the conductivity coefficients.

In Cartesian form, Eq. (8) is

$$\begin{aligned} q_x &= -\left(k_{11}\frac{\partial t}{\partial x} + k_{12}\frac{\partial t}{\partial y} + k_{13}\frac{\partial t}{\partial z}\right) \\ q_y &= -\left(k_{21}\frac{\partial t}{\partial x} + k_{22}\frac{\partial t}{\partial y} + k_{23}\frac{\partial t}{\partial z}\right) \\ q_z &= -\left(k_{31}\frac{\partial t}{\partial x} + k_{32}\frac{\partial t}{\partial y} + k_{33}\frac{\partial t}{\partial z}\right) \quad . \end{aligned} \tag{9}$$

To compute heat flow by Fourier's law, a thermal conductivity value is needed. It can be estimated from theoretical predictions for some ideal materials, but mostly, it is determined by measurement and Fourier's law, Eq. (2). As illustrated in Fig. 2, thermal conductivity can have a large range, which depends on materials and temperature. For example, copper at 20 K has a thermal conductivity of approximately 1000 W/m•K and diatomaceous earth at 200 K has a conductivity of 0.05 W/m•K. Consequently, heat flow in materials can have a very large range, depending on a combined effect of temperature gradient and material property.

Thermal properties of some selected materials are given in Section 18.

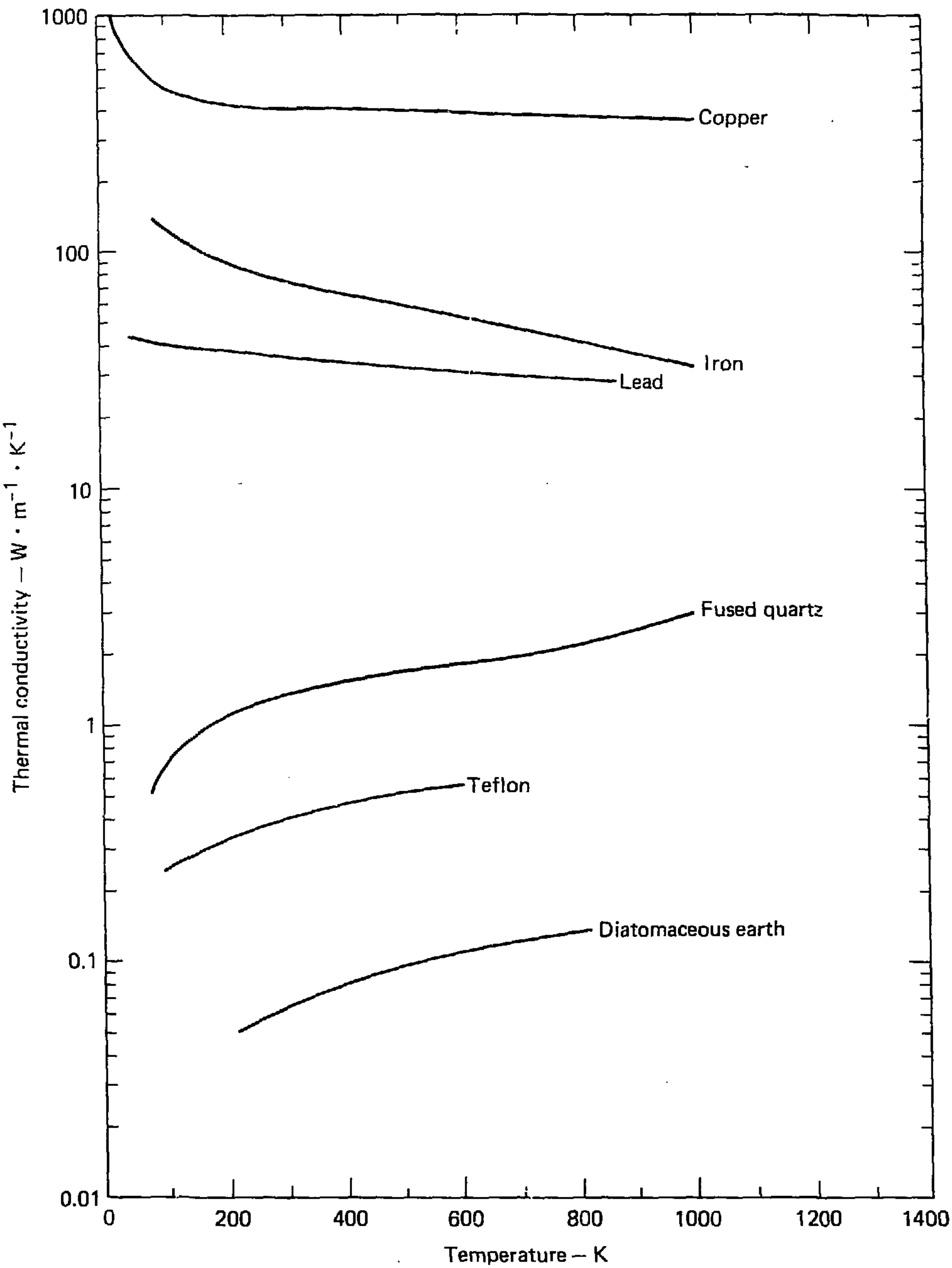


FIG. 2. Thermal conductivity of some selected solids.

2. DIFFERENTIAL EQUATIONS OF HEAT CONDUCTION

Solutions to heat conduction problems are usually found by some mathematical technique which begins with a differential equation of the temperature field. The appropriate equation should include all energy sources and sinks pertinent to a particular problem. Also, the equation should be expressed in terms of a convenient coordinate system such as rectangular, cylindrical, or spherical. Then analytical or differencing methods can be used to solve for temperature or heat flow.

A common method for deriving the generalized differential equation for heat conduction is to apply the first law of thermodynamics (conservation of energy) to a volume element in a selected coordinate system. By accounting for all the thermal energy transferred through the element faces, the change of internal energy and thermal sources or sinks in the element, and by letting the element dimensions approach zero, the differential equation can be derived. This procedure is typified by the following heat energy accounting of the rectangular solid element shown in Fig. 3. The net heat flow though its six faces is

$$Q_{net} = Q_x - Q_{x+\Delta x} + Q_y - Q_{y+\Delta y} + Q_z - Q_{z+\Delta z} \quad , \tag{10}$$

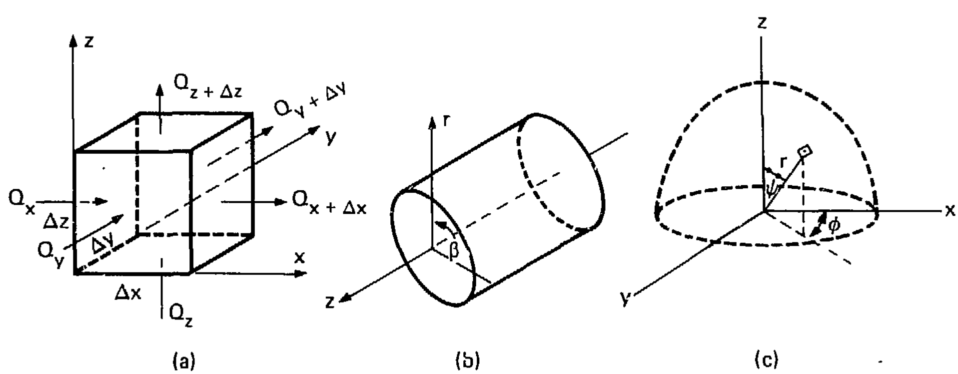


FIG. 3. Coordinate systems for heat conduction equations: (a) rectangular, (b) cylindrical, (c) spherical.

where typically

$$Q_x = -\Delta y \Delta z \left(k_x \frac{\partial t}{\partial x}\right)_x ,$$

$$Q_y = -\Delta x \Delta z \left(k_y \frac{\partial t}{\partial y}\right)_y ,$$

$$Q_z = -\Delta x \Delta y \left(k_z \frac{\partial t}{\partial z}\right)_z ,$$

and k_x, k_y, and k_z are directional conductivities.

An increase in internal energy of the element is represented by

$$\Delta I = \Delta x \Delta y \Delta z \rho c \frac{\partial t}{\partial \tau} , \tag{11}$$

where t is the mean temperature of the element, ρ is the material density, and c is its specific heat.

Internal energy sources can be expressed as

$$Q''' = q''' \Delta x \Delta y \Delta z , \tag{12}$$

where q''' is the unit volume source rate. Examples of internal heating in materials are joule, nuclear, or radiation heating. Summing these energies in accordance with the energy conservation law yields

$$\Delta I = Q_{net} + Q''', \quad \rho c \frac{\partial t}{\partial \tau} = \frac{q_x - q_{x+\Delta x}}{\partial x} + \frac{q_y - q_{q+\Delta y}}{\partial y} + \frac{q_z - q_{z+\Delta z}}{\partial z} + q''' . \tag{13}$$

For the limits Δx, Δy, $\Delta z \to 0$, we obtain

$$\rho c \frac{\partial t}{\partial \tau} = q''' - \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} , \tag{14}$$

where

$$q_x = -k_x \frac{\partial t}{\partial x} ,$$

$$q_y = -k_y \frac{\partial t}{\partial y} ,$$

$$q_z = -k_z \frac{\partial t}{\partial z} .$$

Using Eq. (14) as a general differential equation, we can derive the following specific equations.

2.1 Rectangular Coordinate System

For isotropic heterogeneous media

$$\rho c \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(k \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial t}{\partial z}\right) + q''' \quad . \tag{15}$$

For isotropic homogeneous media this becomes

$$\frac{\partial t}{\partial \tau} = \frac{k}{\rho c}\left[\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q'''}{\rho c} = \alpha \nabla^2 t + \frac{q'''}{\rho c} \quad . \tag{16}$$

When $q''' = 0$, Eq. (16) becomes Fourier's equation.

In steady-state conditions, $\partial t/\partial \tau = 0$ and Eq. (16) becomes the Poisson equation.

When $q''' = \partial t/\partial \tau = 0$, Eq. (16) reduces to the Laplace equation.

Nonisotropic materials, such as laminates, can have directionally sensitive properties. For such materials the conduction differential equation in two dimensions is expressed in the following form:

$$\rho c \frac{\partial t}{\partial \tau} = \left(k_\xi \cos^2 \beta + k_\eta \sin^2 \beta\right) \frac{\partial^2 t}{\partial x^2} + \left(k_\xi \sin^2 \beta + k_\eta \cos^2 \beta\right) \frac{\partial^2 t}{\partial y^2}$$

$$+ (k_\xi - k_\eta)(\sin^2 \beta) \frac{\partial^2 t}{\partial x \partial y} + q''' , \tag{17}$$

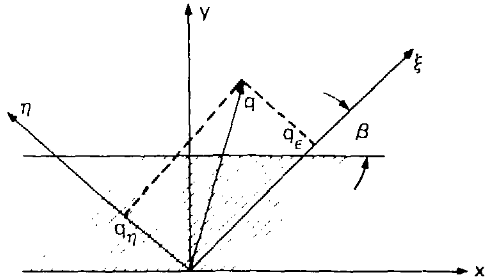


FIG. 4. Coordinate system for a nonisotropic medium.

where k_ξ and k_η are directional thermal conductivities, and β is the angle of laminations as indicated in Fig. 4. When the geometrical axes of the nonisotropic material are oriented with the principal axes of the thermal conductivities, then Eq. (17) simplifies to the form of Eq. (14)

$$\rho c \frac{\partial t}{\partial \tau} = k_x \frac{\partial^2 t}{\partial x^2} + k_y \frac{\partial^2 t}{\partial y^2} + q''' \quad . \tag{18}$$

2.2 Cylindrical Coordinate System

Rectangular coordinates can be transformed into cylindrical coordinates by the relations $x = r \cos \theta$, $y = r \sin \theta$, and $z = z$. The partial differential equations (15) and (16) transformed to cylindrical coordinates are thus

$$\rho c \frac{\partial t}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial t}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(k \frac{\partial t}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial t}{\partial z} \right) + q''' \tag{19}$$

$$\frac{\partial t}{\partial \tau} = \alpha \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{1}{r^2} \frac{\partial^2 t}{\partial \theta^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{q'''}{\rho c} \quad . \tag{20}$$

For nonisotropic materials with the conductivity and geometry axes aligned as in Eq. (18) the differential equation is

$$\rho c \frac{\partial t}{\partial \tau} = \frac{k_r}{r} \frac{\partial}{\partial r} \left(r \frac{\partial t}{\partial r} \right) + \frac{k_\theta}{r^2} \frac{\partial^2 t}{\partial \theta^2} + k_z \frac{\partial^2 t}{\partial z^2} + q''' \quad . \tag{21}$$

2.3 Spherical Coordinate System

A transformation from rectangular to spherical coordinates can be accomplished by substituting the relations $x = r \sin\psi \cos\phi$, $y = r \sin\psi \sin\phi$ and $z = \cos\psi$ into Eqs. (15) and (16), which yield the partial differential equations for isotropic heterogeneous and homogeneous materials, respectively.

$$\rho c \frac{\partial t}{\partial \tau} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial t}{\partial r}\right) + \frac{1}{r^2 \sin^2\psi}\frac{\partial}{\partial \theta}\left(k \frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2 \sin\psi}\frac{\partial}{\partial \psi}\left(k \sin\psi \frac{\partial t}{\partial \psi}\right) + q''' \tag{22}$$

$$\frac{\partial t}{\partial \tau} = \alpha\left(\frac{\partial^2 t}{\partial r^2} + \frac{2}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2 \sin^2\psi}\frac{\partial^2 t}{\partial \theta^2} + \frac{1}{r^2 \tan\psi}\frac{\partial t}{\partial \psi}\right) + \frac{q'''}{\rho c} \quad . \tag{23}$$

The differential equation for nonisotropic materials with aligned conductivity and geometric axes is

$$\rho c \frac{\partial t}{\partial \tau} = \frac{k_r}{r}\frac{\partial^2 t}{\partial r^2} + \frac{k_\phi}{r^2 \sin^2\psi}\frac{\partial^2 t}{\partial \phi^2} + \frac{k_\psi}{r^2 \sin\psi}\frac{\partial}{\partial \psi} \sin\psi \frac{\partial t}{\partial \psi} + q''' \quad . \tag{24}$$

3. SPECIAL DIFFERENTIAL EQUATIONS

Some defining equations can have implied assumptions and boundary conditions. They are usually employed in special cases to simplify the method of solution. However, Fourier's law is the basis for deriving these special equations.

3.1 Combined Conduction-Convection

Thin materials having relatively high thermal conductivity have very small lateral temperature gradients. If the surfaces are convectively heated or cooled, the convection condition becomes part of a heat accounting on a differential element. Referring to Fig. 3, let Δz be the thickness b of a thin solid. On the surface z and $z + \Delta z$, the solid has a convection boundary described by $q = h\,(t - t_f)$, where t_f is the convection fluid temperature.

Applying the same principles used to develop the general equations for rectangular coordinate systems should result in

$$\rho c \frac{\partial t}{\partial \tau} + 2 \frac{h}{b} (t - t_f) = q''' + \frac{\partial}{\partial x}\left(k \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right). \tag{25}$$

If the geometry is a thin rod of circumference C, the appropriate equation is

$$\rho c \frac{\partial t}{\partial \tau} + \frac{hC}{A} (t - t_f) = q''' + \frac{\partial}{\partial z}\left(k \frac{\partial t}{\partial z}\right). \tag{26}$$

3.2 Moving heat sources

The general heat conduction Eq. (15) can also be used for moving heat sources, but a simpler quasi-steady-state equation can be derived by coordinate transformation. If the coordinates are relative to the traveling source, the temperature distributions appear to be stationary. For example, if a point source of strength Q is moving at a velocity U parallel to the x-axis, the transformation would be $x = x' + U\tau$, where x' is the x-direction distance from the source. By substitution in Eq. (16) we can obtain

$$\frac{\partial}{\partial x'}\left(k \frac{\partial t}{\partial x'}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial t}{\partial z}\right) + U\rho c \frac{\partial t}{\partial x'} + q''' = 0 \quad . \tag{27}$$

The applicable equation for a moving source in a thin rod that is convectively cooled is

$$\frac{\partial}{\partial x'}\left(k \frac{\partial t}{\partial x'}\right) + U\rho c \frac{\partial t}{\partial x'} + q''' = \frac{hC}{A} (t - t_f) \quad . \tag{28}$$

The moving source strength is accounted for in the boundary conditions for a particular problem solution.

4. BOUNDARY CONDITIONS

Solutions to heat conduction problems require statements of conditions. For general solutions there must be given at least a definition of the solution region, such as infinite, semi-infinite, quarter-infinite, finite, etc. Additionally, limits can be specified for any of these regions.

If specific solutions are needed, then complete conditions must be defined. They could include, for example, initial, internal, and surface conditions. Other conditions might include property definitions. Whether deriving a solution or searching for existing solutions, one must decide which conditions are applicable to the problem and how they can be suitably expressed.

4.1 Initial Condition

Unsteady-state problems must have an initial condition defined. Mostly, this implies a temperature distribution at $\tau = 0$ but, also, internal or surface conditions could have initial values. A problem solution for $\tau > 0$ depends, of course, on whatever is specified at $\tau = 0$.

4.2 Surface Conditions

The most commonly employed surface conditions in heat conduction problems are prescribed surface convection, temperature, heat flux, or radiation. It is even acceptable to prescribe two of these for the same surface, such as combined radiation and convection. Other surface conditions could include phase change, ablation, chemical reactions, or mass transfer from a porous solid.

4.2.1 Convection boundary

Conduction and convection heat transfer rates on a surface are equated to satisfy continuity of heat flux according to Newton's law:

$$-k \frac{\partial t\,(x_b, \tau)}{\partial x} = h[t(x_b, \tau) - t_f] \quad , \tag{29}$$

where t_f is the temperature of the convection fluid, and x_b is the boundary location. The convection coefficient h must be determined from suitable sources that give predicted values satisfying the conditions of the fluid. (Some correlations of convection coefficients are given in Section 16.) The method for defining h can vary depending on the type of convection or the methods prescribed by those researchers who have supplied values for the coefficient. However, h is usually defined as

TABLE 1. Sample convection coefficient values.

Fluid	Condition	h, $W/m^2 \cdot K$
Air	Free convection on vertical plates	10
Air	Forced convection on plates	100
Air	Forced flow in tubes	200
Steam	Forced flow in tubes	300
Oil	Forced flow in tubes	500
Water	Forced flow in tubes	2,000
Water	Nucleate boiling	5,000
Liquid helium	Nucleate boiling	8,000
Steam	Film condensation	10,000
Liquid metal	Forced flow in tubes	20,000
Steam	Dropwise condensation	50,000
Water	Forced convection boiling	100,000

$$h = \frac{q}{t(x_b, \tau) - t_f} , \tag{30}$$

where t_f can be given as

$$t_f = \beta\left[t\ (x_b, \tau) - t_\infty\right] + t_\infty , \tag{31}$$

and where $\beta \leq 1$, and t_∞ is the temperature outside the thermal boundary layer of the fluid. In this respect, one must take care to use the proper fluid temperature and convection coefficient.

Some typical order-of-magnitude values for the convection coefficient h are given in Table 1.

4.2.2 Surface temperature

Of all boundary conditions, this is probably the simplest in a mathematical sense. It can be variable or constant with respect to position and time. In the real sense, it is very difficult to achieve a prescribed surface temperature, but it can be closely approached by imposing a relatively high convection rate.

4.2.3 Heat flux

Fourier's law defines the flux on a boundary by

$$q = -k \frac{\partial t(x_b,\tau)}{\partial x} \quad . \tag{32}$$

An adiabatic surface can be defined by either setting q in Eq. (32) or h in Eq. (29) to zero. Inversely, if the solid's temperature distribution has been solved, then Eq. (32) can be used to determine the surface heat flux. Understandably, heat flux can be, in particular cases, time and position dependent.

4.2.4 Thermal radiation

Heat transfer from an opaque surface by radiation can be expressed as

$$\sigma\mathscr{F}\left[T^4(x_b,\ \tau) - T_s^4\right] = -k \frac{\partial t(x_b,\ \tau)}{\partial x} \quad , \tag{33}$$

where σ is the Stefan-Boltzmann radiation constant, $\mathscr{F}$ is the combined configuration-emissivity factor for muiltiple-surface radiation exchange, and T_s is the sink or source temperature for radiation. Because Eq. (33) is a nonlinear expression, it is frequently difficult to find exact solutions to problems with this condition.

A common method for dealing with radiation problems is to treat the radiation boundary as a convection boundary. According to Eq. (29) we can write

$$-k \frac{\partial t(x_b,\ \tau)}{\partial x} = h_r\left[t(x_b,\ \tau) - t_s\right] \quad , \tag{34}$$

where

$$h_r = \sigma\mathscr{F}\left[T(x_b,\ \tau) + \tau\right]\left[T^2(x_b,\ \tau) + T_s^2\right] \quad .$$

Using this method means that $T(x_b,\ \tau)$ must first be estimated in order to compute a value of h_r. After a value for $T(x_b,\ \tau)$ has been computed from the problem solution, then the estimated value for h_r can be improved. This, of course, becomes an iterative process.

4.3 Interface Conditions

4.3.1 Contact

Two contacting solids, either similar of dissimilar, will almost always have some interface thermal resistance to heat flow between them. The magnitude of this resistance can depend greatly on the condition of the two contacting surfaces. Properties that can effect the surface condition include cleanliness, roughness, waviness, yield strength, contact pressure, and the thermal conductivities of the solids and interstitual fluid. Since there are so many influences on the contact thermal resistance, it is difficult to theoretically predict its value. Consequently, experimental results are frequently used. Some representative values of the inverse thermal contact resistance, commonly referred to as the thermal contact coefficient, are given in Section 17.

The generally accepted definition of the contact coefficient is

$$h_c = \frac{q}{\Delta t_i} \quad , \tag{35}$$

where q is the steady heat flux corresponding to a fictitious interface temperature drop of Δt_i defined by extrapolating the virtual linear temperature gradient in each solid to the contact centerline. This temperature drop, which is illustrated in Fig. 5, would diminish to zero if the interfaces were in perfect contact.

4.3.2 Phase change

Other interface conditions include those caused by endothermic reactions such as melting, solidification, sublimation, vaporization, and chemical dissociation.

A statement of an interface reaction condition defines the difference of heat flux across the interface. If the interface which separates two phases or a material is located at $x = x_i$, the heat balance for a phase change reaction is given in the form

$$k_2 \frac{\partial t_2(x_i)}{\partial x} - k_1 \frac{\partial t_1(x_i)}{\partial x} = \gamma\rho_2 \frac{dx_i}{d\tau} \quad , \tag{36}$$

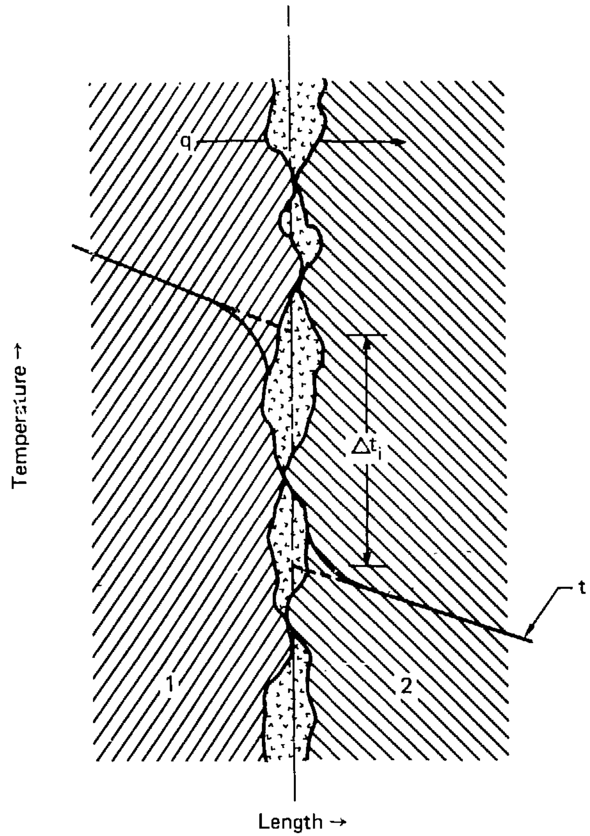


FIG. 5. Illustration of interface contact between solids.

where γ is the latent heat or chemical heat capacity, and subscripts 1 and 2 refer to the two phases.

5.0 Solutions

5.1 Extending solutions

A solution can be retrieved after identifying a problem by boundary conditions, geometry, and other pertinent data. Usually, a temperature solution is given, but heat flow can be derived from the temperature distribution by using Fourier's law, i.e. Eq. (2). If cumulative heat flow is required, a time and surface integration of local heat flux is necessary.

$$Q = \int_s \int_0^\tau \frac{\partial t(s, \tau)}{\partial n} \, d\tau ds \quad , \tag{37}$$

where n is the direction normal to the surface s.

Steady-state solutions can be considered as the infinite-time condition for unsteady-state solutions. That is, problems which have a time-asymptotic solution exhibit steady-state solutions for $\tau \to \infty$. Thus, steady-state solutions can be derived from transient solutions.

A steady surface temperature condition can be implied from a convection boundary condition. For $h \to \infty$, the surface temperature approaches the fluid temperature. Therefore, a solution which includes a convection boundary can be transformed into a constant temperature boundary solution by solving for the implied limiting case.

5.2 Dimensionless parameters

Grouping particular variables yields dimensionless numbers that can be useful. Symbolically, they can shorten an equational expression. But, they can also give insight to the behavior of heat transfer in a particular problem.

One very useful parameter is the Biot number, $Bi = h\ell/k$, which results from convection boundary conditions. This parameter is proportional to the ratio of the conduction resistance to the convection resistance. Thus, we could say that for

> $Bi > 1$, conduction is highest resistance to heat transfer,
>
> $Bi < 1$, convection is highest resistance to heat transfer,
>
> $Bi << 1$, the solid behaves like $k \simeq \infty$.

Another dimensionless parameter is the Fourier number, $Fo = \alpha\tau/\ell^2$, which is found in transient solutions. This number is a dimensionless time value, but it is also considered an indicator of the degree of thermal penetration into a solid. Since $\alpha\tau/\ell^2 = (k\tau\Delta t/\ell)/(\rho c\ell\Delta t)$, it is proportional to the ratio of conduction heat transferred to thermal capacity. Thus, an increasing Fo value implies approaching thermal equilibrium.

The product of Bi and Fo numbers yields the parameter $Bf = h\tau/\rho c\ell$ which occurs in transient problems having a convection boundary. This is also a dimensionless time parameter, but it is based on convection heat transfer instead of conduction as in the Fourier number.

Solutions to problems having an internal heat source q''' usually have a dimensionless heating parameter called the Pomerantsev modulus $Po = q'''\ell^2/k\Delta t$. This number is a ratio of internal heating to heat conduction rates. Large values of Po imply large temperature differences will occur in the solid.

The parameter $Fo^* = \ell/2\sqrt{\alpha\tau}$ is a form of the reciprocal of the Fourier number and occurs in many solutions for transient temperatures in semi-infinite solids.

When time dependent boundary conditions have a time constant, the solution will frequently include a dimensionless group called the Predvoditelev modulus, $Pd = b\ell^2/d$, where b is the inverse time constant. Small values of Pd imply a slow changing condition. It signifies the ratio of the change rate of the boundary condition to the change rate of the solid temperature.

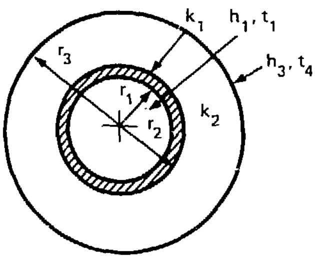
5.3 Example Problems

5.3.1 Steady heat-transfer in a pipe wall

Hot water flows at 0.5 m/s in a 2.5 cm i.d., 2.66 cm o.d. smooth copper pipe. The pipe is horizontal in still air and covered with a 1-cm layer of polystyrene foam insulation. For a 65°C water temperature and 20°C air temperature, estimate the heat loss rate per unit length. The solution given in case 2.1.2 is

$$q = \frac{2\pi\ (t_1 - t_2)}{\frac{1}{k_1}\ \ell n \frac{r_2}{r_1} + \frac{1}{k_2}\ \ell n \frac{r_3}{r_2} + \frac{1}{r_1 h_1} + \frac{1}{r_3 h_3}} .$$

From the problem description



$t_1 = 65^oC$

$t_4 = 20^oC$

$r_1 = 1.25$ cm

$r_2 = 1.33$ cm

$r_3 = 2.33$ cm

$k_1 = 400$ W/m$\cdot$$^oC$ (from Table 18.1)

$k_2 = 0.038$ W/m$\cdot$$^oC$ (from Table 18.2)

$hd/k = 0.0155\ Pr^{0.5}Re^{0.9}$ (from Sect. 16.1)

$h_1 = (k_{water}/2r_1)(0.0155\ Pr^{0.5}Re^{0.9})$

$k_{water} = 0.659$ W/m$\cdot$$^oC$ (at 65^oC)

$Pr = 2.73$

$Re = 2\rho vr1/\mu$

$\rho = 980\ kg/m^3$

$v = 0.5$ m/s

$\mu = 4.3 \times 10^{-4}$ kg/m$\cdot$s

$$Re = \frac{2(980)(0.5)(0.0125)}{4.3 \times 10^{-4}} = 28\ 488$$

$h_1 = (0.659/0.025)(0.0155)(2.73)^{0.5}(28\ 488)^{0.9} = 6895\ W/(m^2\text{-}^oC)$

$h_3 = (k_{air}/2r_3)\ C\ (Gr_dPr)^m$ (from Sect. 16.8)

$k_{air} = 0.025$ W/m$\cdot$$^oC$

$Pr = 0.71$

$Gr = g\beta(t_4 - t_3)(2r_3)^3/\nu^2$

$g = 9.8$ m/s

$\beta = 1/T_4 = 1/293\ K^{-1}$

$\nu = 16.55 \times 10^{-6}\ m^2/s$

$$Gr_dPr = \frac{(9.8)(0.0466)^3(0.71)}{(293)(16.55 \times 10^{-6})^2} = 8774, \quad (t_4 - t_3 = 1^oC)$$

$C = 1.14$, $m = 1/7$ (from Table 16.3)

$h_3 = (0.025/0.0466)(1.14)(8774)^{1/7} = 2.24\ W/m^2\cdot{}^oC$

$$q = \frac{2\pi(65-20)}{\frac{1}{400}\ \ln\left(\frac{1.33}{1.25}\right) + \frac{1}{0.038}\ \ln\left(\frac{2.33}{1.33}\right) + \frac{1}{(0.0125)(6895)} + \frac{1}{(0.233)(2.24)}}$$

$$= \frac{2\pi(45)}{1.55 \times 10^{-4} + 14.76 + 0.012 + 19.16} = 8.33\ W/m$$

$$t_3 - t_4 = \frac{q}{2\pi r_3 h_3} = \frac{8.33}{2\pi(0.0233)(2.24)} = 25.4^{\circ}C \quad .$$

Using this new estimate of $(t_4 - t_3)$, we can recalculate h_3.

$$h_3 = 2.24\ (25.4)^{1/7} = 3.56\ W/m^2{\cdot}{}^{\circ}C,$$

$$q = \underline{10.54\ W/m}.$$

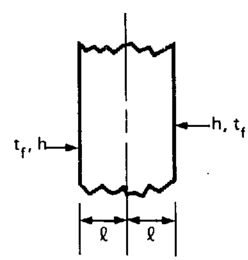
Additional iterations on h_3 would little improve this result.

Note that the copper tube and water film have a small effect on the results because they present little resistance to heat transfer by comparison to the insulation and air film.

5.3.2 Transient heat conduction in a slab

A billet of 304 stainless steel measuring 2 × 2 × 0.1 m thick and having a uniform temperature of $30^{\circ}C$ is heated by sudden immersion into a $450^{\circ}C$ molten salt bath. The mean convection coefficient is 350 $W/m^2{\cdot}{}^{\circ}C$. Determine the time required for the center temperature of the billet to reach $400^{\circ}C$. The solution is found in Section 8.1 (case 8.1.7 and Fig. 8.4a):

$$\frac{t - t_f}{t_i - t_f} = \frac{400 - 450}{30 - 450} = 0.119 \quad .$$



From Table 18.1

$$k = 21\ W/m{\cdot}{}^{\circ}C,$$

$$\alpha = 7 \times 10^{-6}\ m^2/s,$$

$$\frac{k}{h\ell} = \frac{21}{(350)(0.05)} = 1.2 \ .$$

From Fig. 8.4a

$$\alpha\tau/\ell^2 = 3.4,$$

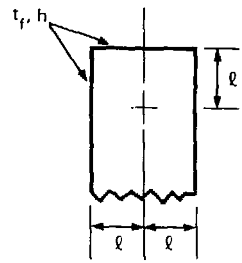
$$\tau = \frac{3.4\ell^2}{\alpha} = \frac{(3.4)(0.05)^2}{7 \times 10^{-6}} = \underline{1214\ s} \ .$$

5.3.3 Transient heat conduction in a semi-infinite plate

For the conditions given in 5.3.2, find the billet center temperature at 0.05 m from the edges and sides of the billet.

The solution is found in case 7.1.21 and Fig. 9.4a for a semi-infinite plate and expressed as T = (P)(Fo)•S(X). Values of P(Fo) are given in Fig. 8.4 and values of S(X) are given in Fig. 7.2.

$$Bi\sqrt{Fo} = \frac{h\sqrt{\alpha\tau}}{k} = \frac{350\sqrt{(7 \times 10^{-6})\ (1214)}}{21} = 1.54$$



$$Fo_x^* = \frac{\ell}{2\sqrt{\alpha\tau}} = \frac{0.05}{2\sqrt{(7 \times 10^{-6})\ (1214)}} = 0.27$$

If $(T = (t - t_f)/(t_i - t_f)$,

$S(X) = 1 - 0.45 = 0.55$ (from Fig. 7.2).

The P(Fo) value is given in 5.3.2. Thus, the solution is

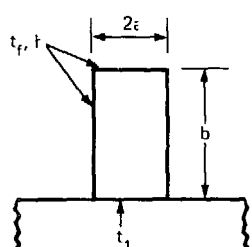
$$\frac{t - t_f}{t_i - t_f} = P(Fo)\ S(X) = (0.119)(0.55) = 0.066$$

$$t = (0.066)(30 - 450) + 450 = \underline{423^\circ C}$$

5.3.4 Extended surface steady-state heat transfer

A 160 °C uniform-temperature copper plate has a long rectangular rib brazed to it. All surfaces are convectively cooled by 30°C air having a convection coefficient of 53 W/(m²•°C). The rib is yellow brass extending 4 cm from the flat surface and 2 cm wide. Estimate the additional heat loss from the flat surface caused by the rib.

The solution for temperature distribution in the rib is given in Case 1.1.17 for $f(x) = t_i$, $w = b$ and $\ell = a$.



$$\frac{t - t_f}{t_1 - t_f} = 2 \sum_{n=1}^{\infty} \frac{Bi \cos(\lambda_n X)\left\{\lambda_n \cosh\left[\lambda_n(B - Y)\right] + Bi \sinh\left[\lambda_n(B - Y)\right]\right\}}{\cos(\lambda_n)(Bi^2 + \lambda_n^2 + Bi)\left[\lambda_n \cosh(\lambda_n B) + Bi \sinh(\lambda_n B)\right]}$$

$\lambda_n \tan(\lambda_n) = Bi$ (characteristic equation)

$B = b/a = 4/1 = 4$

$X = x/a$, $Y = y/a$

$Bi = \frac{ha}{k} = \frac{(53)(0.01)}{130} = 0.004$ (k value from Table 18.1)

From Table 14.1

$\lambda_1 = 0.0632$, $\lambda_2 = 3.1429$, $\lambda_3 = 6.2838$, $\lambda_4 = 9.4252$, $\lambda_5 = 12.5667$.

Heat loss form the convectively cooled surfaces is determined by Eq. 32 applied to the boundary $y = 0$.

$$q = 2k \int_o^a \frac{\partial t(x,o)}{\partial y} dx$$

$$= 4k(t_i - t_f) \sum_{n=1}^{\infty} \frac{Bi \tan(\lambda_n)[\lambda_n \sinh(\lambda_n B) + Bi \cosh(\lambda_n B)]}{(Bi^2 + \lambda_n^2 + Bi)[\lambda_n \cosh(\lambda_n B) + Bi \sinh(\lambda_n B)]}$$

$$= 666 \text{ W/m}$$

Heat loss without the rib attached would be

$q = hA(t_1 - t_f) = 53(0.02)(160 - 30) = 138$ W/m .

The additional heat loss is thus

$\Delta q = 666 - 138 = \underline{528 \text{ W/m}}$.

5.3.5 Rectangular fin heat transfer

Use the straight rectangular fin solution to estimate heat loss from the rib described in 5.3.4.

The solution is found in Case 5.1.4 and Fig. 5.2.

$$m = \sqrt{\frac{h}{ka}} = \sqrt{\frac{53}{(130)(0.01)}} = 6.385 \text{ m}^{-1}$$

$\ell_c = 0.04 + 0.01 = 0.05$ m

$m\ell_c = 0.3193$

5.3.6 Semi-infinite plate heat transfer

Find an equation for the heat transfer rate through the edge of the semi-infinite plate described in case 1.1.5 with $f(x) = t_2$.

Using the given temperature solution and Eqs. (2) and (37) we can find the heat transfer in the following manner:

$$q_Y = -k \frac{\partial t}{\partial y} = -\frac{k}{\ell} (t_2 - t_1) \frac{\partial T}{\partial Y} \text{ , where } T = \frac{t - t_1}{t_2 - t}$$

$$\frac{\partial T}{\partial Y}\bigg|_{Y = 0} = -2 \sum_{n=1}^{\infty} \sin (n\pi X)\left[1 - \cos (n\pi)\right]$$

$$Q_Y\big|_{Y = 0} = \ell\int_0^1 q_Y\big|_{Y = 0} \, dX = 2k(t_2 - t_1)\int_0^1 \sum_{n=1}^{\infty} \sin (n\pi X)\left[1 - \cos (n\pi)\right]dX$$

$$= 2k(t_2 - t_1) \sum_{n=1}^{\infty} \frac{[1 - \cos(n\pi)]^2}{n\pi} = \frac{8}{\pi} k \, (t_2 - t_1) \sum_{n=1}^{\infty} \frac{1}{n} \text{ , } n = 1, 3, 5, \ldots$$

1. Plane Surface — Steady State

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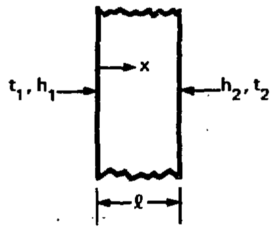
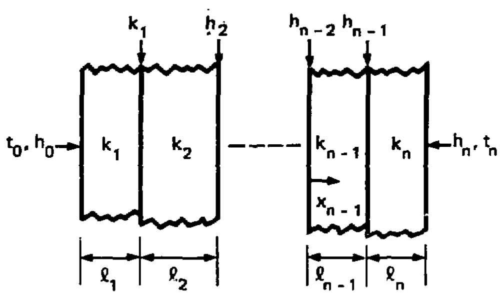
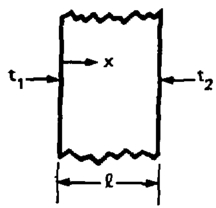
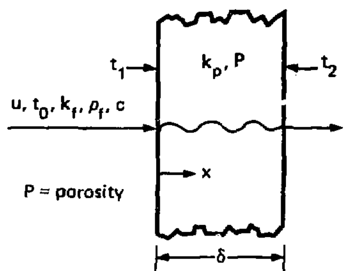
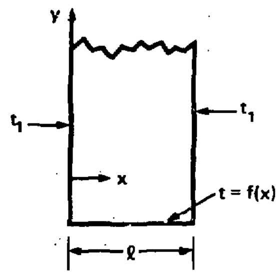
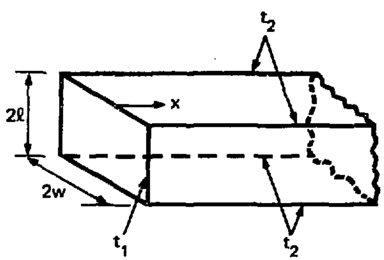
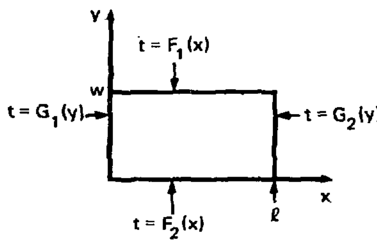
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

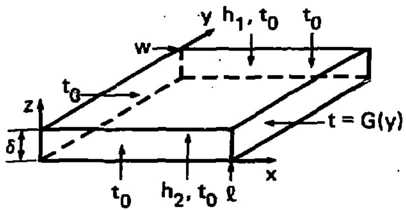
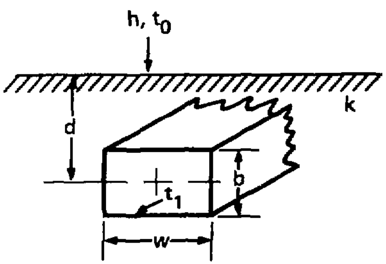
Case No.	References	Description	Solution
1.1.1	19, p. 3-103	Convectively heated and cooled plate. 	$q = \dfrac{h_1(t_1 - t_2)}{Bi_1 + 1 + (h_1/h_2)}$ $\dfrac{t - t_1}{t_2 - t_1} = \dfrac{Bi_1 + 1}{Bi_1 + 1 + (h_1/h_2)}$
1.1.2	19, p. 3-103	The composite plate. 	$q = \dfrac{(t_0 - t_n)}{\sum\limits_{i-1}^{n} (W_i/k_i + 1/h_i) + 1/h_0}$ Temp in the jth layer: $\dfrac{t_j - t_0}{t_n - t_0} = \dfrac{\sum\limits_{i=1}^{j-1} (\ell_i/k_i + 1/h_i) + (x_j/k_j) + (1/h_0)}{\sum\limits_{i=1}^{n} (\ell_i/k_i + 1/h_i) + (1/h_0)}, \ j > 1$

Case No.	References	Description	Solution
1.1.3	1, p. 138	Plate with temperature dependent conductivity. $k = k_1 + \beta(t - t_1)$. 	$q = k_m \dfrac{t_1 - t_2}{\ell}$ $t - t_1 = \dfrac{1}{\beta} \sqrt{k_1^2 + 2\beta k_m (t_2 - t_1)\dfrac{x}{\ell}} - 1$ $k_m = (k_1 + k_2)/2$
1.1.4	2, p. 221	Porous plate with internal fluid flow. $t = t_1$, $x = 0$. $t = t_2$, $x = \delta$. P = Porosity. 	$\dfrac{t - t_0}{t_2 - t_0} = \exp\left[- \xi_p \delta (1 - x/\delta)\right]$, $0 \leq x \leq \delta$ $\dfrac{t - t_0}{t_2 - t_1} = \dfrac{\exp(\xi_f x)}{\exp\left[\xi_p \delta - 1\right]}$, $-\infty \leq x \leq 0$ Mean temp: $\dfrac{t_m - t_0}{t_2 - t_0} = \dfrac{1}{\xi_p \delta}\left[1 - \exp(-\xi_p \delta)\right]$, $0 \leq X \leq \delta$ $\xi_p = \dfrac{\rho_f uc}{k_p(1 - P)}$, $\xi_f = \dfrac{\rho_f uc}{k_f(1 - P)}$ (See Fig. 1.1)

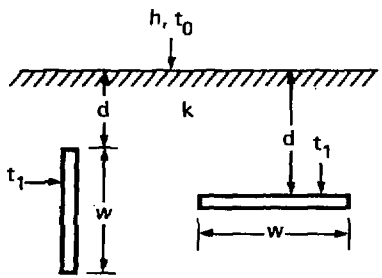
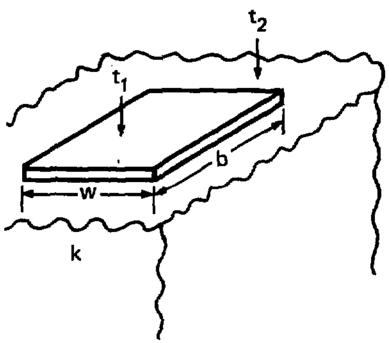
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.5	2, p. 122 9, p. 164	Semi-infinite plate. $t = t_1,\ x = 0,\ \ell,\ y \geq 0.$ $t = f(x),\ 0 > x > \ell,\ y = 0.$  	$t - t_1 = 2 \sum_{n=1}^{\infty} \exp(-n\pi Y) \sin(n\pi X) \times \int_0^1 \left[f(X) - t_i\right] \sin(n\pi X)\,dX$ For $f(x) = t_2$: $\frac{t - t_1}{t_2 - t_1} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp(-n\pi Y) \sin(n\pi X)\left[1 - \cos(n\pi)\right]$
1.1.6	3, p. 250	Rectangular semi-infinite rod. $t = t_1,\ x = 0.$ $t = t_2$ on other surfaces.  	$\frac{t - t_2}{t_1 - t_2} = 4 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^{m+n} \exp\left[-(\lambda_n x/w)^2 + (\lambda_m x/\ell)^2\right]^{\frac{1}{2}}}{\lambda_n \lambda_m} \times \cos(\lambda_n) \cos(\lambda_m)$ $\lambda_n = (2n + 1)\frac{\pi}{2},\ \lambda_m = (2m + 1)\frac{\pi}{2}$

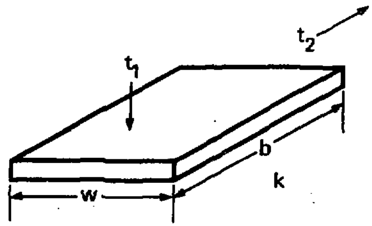
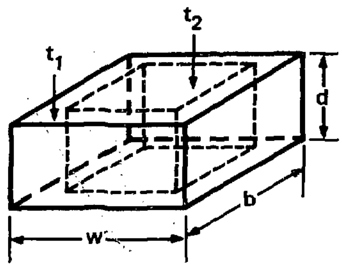
Case No.	References	Description	Solution
1.1.7	2, p. 130	Rectangular infinite rod. $t = F_1(x),\ 0 < x < \ell,\ y = 0.$ $t = F_2(x),\ 0 < x < \ell,\ y = w.$ $t = G_1(x),\ x = 0,\ 0 < y < w.$ $t = G_2(x),\ x = \ell,\ 0 < y < w.$ 	$t = t_I + t_{II} + t_{III} + t_{IV}$ $t_I = 2 \sum_{n=1}^{\infty} \frac{\sinh (n\pi Y/L)}{\sinh (n\pi/L)} \sin (n\pi XL) \int_0^1 F_1(X) \sin (n\pi X) dX$ $t_{II} = 2 \sum_{n=1}^{\infty} \frac{\sinh \left[\frac{n\pi}{L}(1 - Y)\right]}{\sinh (n\pi/L)} \sin (n\pi XL) \int_0^1 F_2(X) \sin (n\pi X) dX$ $t_{III} = 2 \sum_{n=1}^{\infty} \frac{\sinh (n\pi Y/L)}{\sinh (n\pi/L)} \sin (n\pi Y) \int_0^1 G_1(Y) \sin (n\pi Y) dY$ $t_{IV} = 2 \sum_{n=1}^{\infty} \frac{\sinh [n\pi L(1 - X)]}{\sinh (n\pi L)} \sin (n\pi Y) \int_0^1 G_2(Y) \sin (n\pi Y) dY$ $L = \ell/w$ $\frac{t - t_1}{t_2 - t_1} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cosh \left[\frac{n\pi}{L}\left(\frac{1}{2} - Y\right)\right]}{n\cosh\left(\frac{\pi}{2L}\right)} \sin (n\pi X)$, $F_1(x) = F_2(x) = t_2,\ G_2(y) = G_1(y) = t$

Case No.	References	Description	Solution
1.1.8	2, p. 147	Thin rectangular plate. $t = t_0,\ x = 0,\ 0 < y < x.$ $t = t_0,\ 0 < x < \ell,\ y = 0.$ $t = t_0,\ 0 < x < \ell,\ y = w.$ $t = G(y),\ x = \ell,\ 0 < y < w.$ h_1, t_0 at $z = \delta$. h_2, t_0 at $z = 0$. 	$t - t_0 = 2 \sum_{n=1}^{\infty} \frac{\sinh\left[(Bi + n^2\pi^2L^2)^{\frac{1}{2}}X\right]}{\sinh\left[(Bi + n^2\pi^2L^2)^{\frac{1}{2}}\right]} \sin(n\pi Y)$ $\times \int_0^1 \left[G(Y) - t_0\right] \sin(n\pi Y)\, dY$ $Bi = (h_1 + h_2)\ell/k\delta,\ L = \ell/w$
1.1.9	4, p. 41	Infinite rectangular rod in a semi-infinite solid. 	$q = \frac{t_1 - t_0}{R}$ $R \approx \frac{1}{k\left(5.7 + \frac{w}{2b}\right)} - \ln\left[\frac{3.5(d + k/h)}{w^{0.25}\ b^{0.75}}\right]$

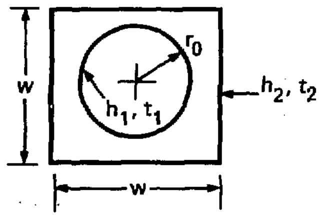
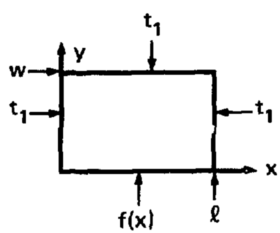
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

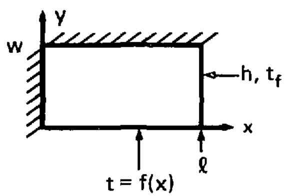
Case No.	References	Description	Solution
1.1.10	4, p. 41	Infinitely long thin plate in a semi-infinite solid. 	Vertical plate: $Q \approx \dfrac{k(t_1 - t_0)}{0.42\left(\dfrac{d}{w} + \dfrac{1}{Bi_w}\right)^{0.24}}, \quad 0.5 < \dfrac{d}{w} < 12$ Horizontal plate: $Q \approx \dfrac{k(t_1 - t_0)}{0.34\left(\dfrac{d}{w} + \dfrac{1}{Bi_w}\right)^{0.3}}, \quad 0.5 < \dfrac{d}{w} < 12$
1.1.11	4, p. 43	Thin rectangular plate on the surface of a semi-infinite solid. 	$Q = \dfrac{kw\pi(t_1 - t_2)}{\ln\left(\dfrac{4w}{b}\right)}$

Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.12	4, p. 44	Thin rectangular plate in an infinite solid. 	$Q = \dfrac{2\pi wk(t_1 - t_2)}{\ell n\left(\dfrac{4w}{b}\right)}$
1.1.13	5, p. 54	Rectangular parallelpiped with wall thickness of δ. 	$Q_t = \left[\dfrac{2k}{\delta}(dw + db + wb) + 2.16(d + w + b) + 1.2\delta\right](t_2 - t_1)$ = total heat flow through six walls

Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.14	4, p. 37	Infinite hollow square rod. 	$q = \dfrac{2\pi k(t_2 - t_1)}{\dfrac{k}{h_1 r_0} + \ell n \dfrac{1.08w}{2r_0} + \dfrac{\pi k}{2h_2 w}}$
1.1.15	9, p. 166	Rectangular infinite rod. $t = f(x),\ 0 < x < a,\ y = 0.$ $t = t_1,\ 0 < x < a,\ y = b.$ $t = t_1,\ x = 0,\ 0 < y < b.$ $t = t_1,\ x = a,\ 0 < y < b.$ 	$t - t = \sum_{n=1}^{\infty} A_n \sin(n\pi X) \sinh\left[(1 - Y)\left(\frac{n\pi}{L}\right)\right] \operatorname{cosech}\left(\frac{n\pi}{L}\right)$ $A_n = 2\int_0^1 \left[f(X) - t_2\right] \sin(n\pi X)\,dX,\ L = \ell/w$ For: $f(X) = t_2$: $\frac{t - t_1}{t_2 - t_1} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi X) \sinh\left[(1 - Y)\left(\frac{n\pi}{L}\right)\right] \operatorname{cosech}\left(\frac{n\pi}{L}\right),$ $n = 1, 3, 5, 7 \ldots\ldots$

Case No.	References	Description	Solution
1.1.16	9, p. 167	Rectangular infinite rod. $t = f(x),\ 0 < x < \ell,\ y = 0.$ $q_y = 0,\ 0 < x < \ell,\ y = w.$ $q_x = 0,\ x = 0,\ 0 < y < w.$ 	$t - t_f = 2\sum_{n=1}^{\infty} \frac{\left(Bi^2+\lambda_n^2\right)\cos(\lambda_n X)\cosh\left[(1-Y)W\lambda_n\right]}{\left[\left(Bi^2+\lambda_n^2\right)+Bi\right]\cosh(\lambda_n W)} \times \int_0^1 \left[f(X)-t_f\right]\cos(\lambda_n X)\,dX$ $\lambda_n \tan(\lambda_n) = Bi\ ,\ Bi = h\ell/k\ ,\ W = w/\ell$ For: $f(x) = t_1$: $\frac{t - t_f}{t_1 - t_f} = 2\,Bi \sum_{n=1}^{\infty} \frac{\cos(\lambda_n X)\cosh\left[(1-Y)W\lambda_n\right]}{\left[\left(Bi^2+\beta_n^2\right)+Bi\right]\cos(\lambda_n)\cosh(\lambda_n W)}$

Case No.	References	Description	Solution
1.1.17	9, p. 168	Case 1.1.16 with $q_y = 0$, $y = w$, $0 < x < \ell$ is replaced by convection boundary h, t_f.	

$$t - t_f = 2 \sum_{n=1}^{\infty} \frac{\left(Bi^2 + \lambda_n^2\right) \cos(\lambda_n X) \left\{\lambda_n \cosh\left[\lambda_n (W - Y)\right] + Bi \sinh\left[\lambda_n (W - Y)\right]\right\}}{\left[\left(Bi^2 + \lambda_n^2\right) + Bi\right] \left\{\lambda_n \cosh(\lambda_n W) + Bi \sinh(\lambda_n W)\right\}}$$

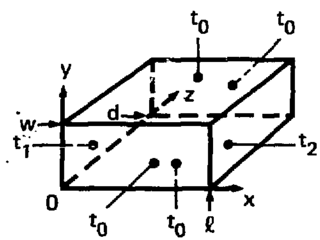
$$\times \int_0^1 \left[f(X) - t_f\right] \cos(\lambda_n X)\, dX$$

$$\lambda_n \tan(\lambda_n) = Bi \;, \quad Bi = h\ell/k \;, \quad W = w/\ell$$

For: $f(x) = t_1$:

$$\frac{t - t_f}{t_1 - t_f} = 2 \sum_{n=1}^{\infty} \frac{Bi \cos(\lambda_n X) \left\{\lambda_n \cosh\left[\lambda_n (W - Y)\right] + Bi \sinh\left[\lambda_n (W - Y)\right]\right\}}{\cos(\lambda_n) \left[\left(Bi^2 + \lambda_n^2\right) + Bi\right] \left[\lambda_n \cosh(\lambda_n W) + Bi \sinh(\lambda_n W)\right]}$$

Case No.	References	Description	Solution
1.1.18	9, p. 168	Case 1.1.16 with $t = t_f$, $y = w$, $0 < x < \ell$.	$t - t_f = 2 \sum_{n=1}^{\infty} \frac{\left(Bi^2 + \lambda_n^2\right) \cos (\lambda_n X) \sinh \left[\lambda_n (W - Y)\right]}{\left[\left(Bi^2 + \lambda_n^2\right) + Bi\right] \sinh (\lambda_n W)} \times \int_0^1 \left[f(X) - t_f\right] \cos (\lambda_n X) dX$ $\lambda_n \tan (\lambda_n) = Bi$, $Bi = h\ell/k$, $W = w/\ell$ For: $f(x) = t_1$: $\frac{t - t_f}{t_1 - t_f} = 2Bi \sum_{n=1}^{\infty} \frac{\cos (\lambda_n X) \sinh \left[\lambda_n (W - Y)\right]}{\left[\left(Bi^2 + \lambda_n^2\right) + Bi\right] \cos (\lambda_n) \sinh (\lambda_n W)}$
1.1.19	9, p. 169	Case 1.1.16 with $t = t_1$, $y = 0$, $0 < x < \ell$. $t = t_2$, $y = w$, $0 < x < \ell$.	$\frac{t - t_f}{t_1 - t_f} = 2Bi \sum_{n=1}^{\infty} \frac{\cos (\lambda_n X) \left\{\sinh \left[\lambda_n (W - Y)\right] - \sinh (\lambda_n Y)(t_2 - t_f)/(t_1 - t_f)\right\}}{\left[\left(Bi^2 + \lambda_n^2\right) + Bi\right] \cos (\lambda_n) \sinh (\lambda_n W)}$ $\lambda_n \tan (\lambda_n) = Bi$, $W = w/\ell$

Case No.	References	Description	Solution
1.1.20	9, p. 178	Rectangular parallelpiped. $t = t_1$, $x = 0$, $0 < y < w$, $0 < z < d$. $t = t_2$, $x = \ell$, $0 < y < w$, $0 < z < d$. Remaining surfaces at t_0.  	$$\frac{t - t_0}{t_1 - t_0} = \frac{16}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{[\sinh\,(L - LX) + T\,\sinh\,(LX)]\,\sin\,(n\pi Y)\,\sin\,(m\pi Z)}{(nm)\,\sinh\,(L)}$$ $n = 1, 3, 5, \ldots\ldots$, $m = 1, 3, 5, \ldots\ldots$ $L^2 = (n\pi\ell/w)^2 + (m\pi\ell/d)^2$, $Z = z/d$ $T = (t_2 - t_0)/(t_1 - t_0)$

Case No.	References	Description	Solution
1.1.21	9, p. 179	Rectangular parallelpiped. $t = t_1$, $x = 0$, $-w < y < +w$, $-d < z < +d$. $t = t_2$, $x = \ell$, $-w < y < +w$, $-d < z < +d$. Remaining surfaces convection boundary with h, t_f.	

$$\frac{t - t_f}{t_1 - t_f} = 4\,Bi^2\,D \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[\sinh(L - X) + T \sinh(LX)] \cos(\lambda_n Y) \cos(\beta_m Z)}{\cos(\lambda_n) \cos(\beta_m) \left[\lambda_n^2 + Bi^2 + Bi\right]\left(\beta_m^2 + Bi^2 D^2 + Bi\ D\right) \sinh(L)}$$

$$\lambda_n \tan(\lambda_n) = Bi, \quad \beta_m \tan(\beta_m) = Bi\ D, \quad Bi = hw/k$$

$$L^2 = \lambda_n^2 \ell^2/w^2 + \beta_m \ell^2/d^2, \quad D = d/w, \quad X = x/\ell, \quad Y = y/w$$

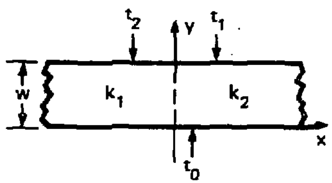
$$Z = z/d\ , \quad T = (t_2 - t_f)/(t_1 - t_f)$$

Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.22	9, p. 180	Case 1.1.21 except $t = t_1$, $x = 0$, $-w < y < +w$, $-c < z < +c$. Remaining faces are correction boundaries with h, t_f.	$$\frac{t - t_f}{t_1 - t_f} = 4\,Bi^2\,D \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{[A\,Bi \sinh(L - LX) + L \cosh(L - LX)] \cos(\lambda_n Y) \cos(\beta_m Z)}{[A\,Bi \sinh(L) + L \cosh(L)]\,NM \cos(\lambda_n) \cos(\beta_m)}$$ $A = \ell/w$, $N = \lambda_n^2 + Bi^2 + Bi$, $M = \beta_m^2 + Bi^2\,C^2 + BiC$. λ_n, β_m, L, D, X, Y, Z, and Bi are defined in case 1.1.21.
1.1.23	2, p. 175	Infinite plate with cylindrical heat source.	Case 2.2.10

Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

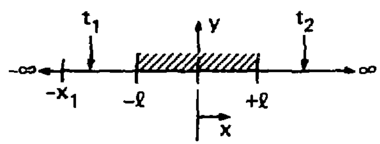
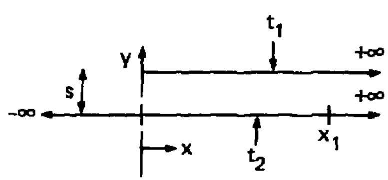
Case No.	References	Description	Solution
1.1.24	9, p. 428	Infinite strip with stepped temp boundary. $t = t_0$, $y = 0$, $-\infty < x < +\infty$. $t = t_1$, $y = w$, $x > 0$. $t = t_2$, $y = w$, $x < 0$. $k = k_1$, $0 < y < w$, $x < 0$. $k = k_2$, $0 < y < w$, $x > 0$.	



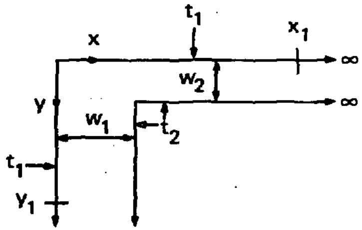
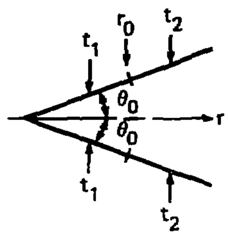
$$\frac{t - t_0}{t_1 - t_0} = Y - \frac{2}{\pi}\frac{\left[(t_2/t_1) - 1\right]k_2}{(k_1 + k_2)}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(n\pi Y)\exp(-n\pi X) \quad , X > 0$$

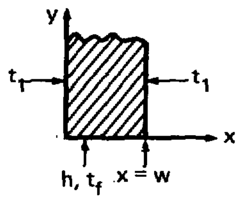
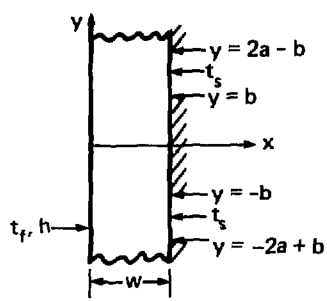
$$\frac{t - t_0}{t_2 - t_1} = Y + \frac{2}{\pi}\frac{\left[1 - (t_1/t_2)\right]k_1}{(k_1 + k_2)}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(n\pi Y)\exp(n\pi X) \quad , X < 0$$

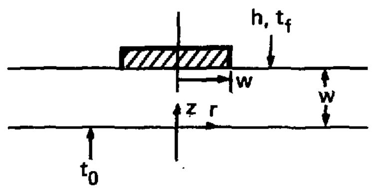
$$X = x/w \ , \ Y = y/w$$

Case No.	References	Description	Solution
1.1.25	9, p. 452	Heated planes on a semi-infinite medium. $t = t_1$, $x \leq -\ell$, $y = 0$. $t = t_2$, $x \geq +\ell$, $y = 0$. $q_y = 0$, $-\ell < x < +\ell$, $y = 0$. 	$Q = \frac{k}{\pi} \cosh^{-1}\left(\frac{x_1}{\ell}\right)(t_1 - t_2)$, $-x_1 < x < -\ell$
1.1.26	9, p. 453	Heated parallel planes in an infinite medium. $t = t_1$, $x \geq 0$, $y = s$. $t = t_2$, $-\infty < x + \infty$. 	Heat flow from bottom side of semi-infinite plane: $Q = k\left[(x_1/s) + (1/\pi)\right](t_1 - t_2)$, $0 < x < x_1$. Heat flow from top side of semi-infinite plane: $Q = \frac{k}{\pi} \ln\left[(\pi x_1/s) + 1\right](t_1 - t_2)$, $0 < x < x_1$.

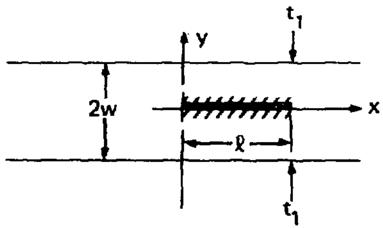
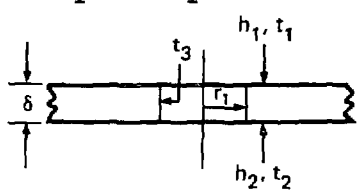
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.27	9, p. 454	Infinite right-angle corner. $t = t_1$, $x > 0$, $y = 0$. $t = t_1$, $x = 0$, $y > 0$. $t = t_2$, $x > w_1$, $y = w_2$. $t = t_2$, $x = w_1$, $y > w_2$. 	$Q = k\left[\frac{x_1 - w_1}{w_2} + \frac{y_1 - w_2}{w_1} + \frac{2w_1}{\pi w_2}\tan^{-1}\left(\frac{w_2}{w_1}\right) + \frac{2}{\pi}\ln\left(\frac{w_1^2 + w_2^2}{4w_1w_2}\right) + \frac{2}{\pi}\frac{w_2}{w_1}\tan^{-1}\left(\frac{w_1}{w_2}\right)\right](t_1 - t_2)$, $0 < x < x_1$, $0 < y < y_1$ For $w_1 = w_2 = w$, $x_1 = y_1 = x$: $Q = k\{2[(x/w) - 1] + 0.559\}(t_1 - t_2)$
1.1.28	9, p. 462	A wedge with stepped surface temp. $t = t_1$, $0 < r < r_0$, $\theta = \pm\theta_0$. $t = t_2$, $r < r_0$, $\theta = \pm\theta_0$ 	$\frac{t - t_1}{t_2 - t_1} = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\sin \lambda\left[\ln(r_0/r)\right] \cosh(\lambda\theta)d\lambda}{\lambda \cosh(\lambda\theta_0)}$

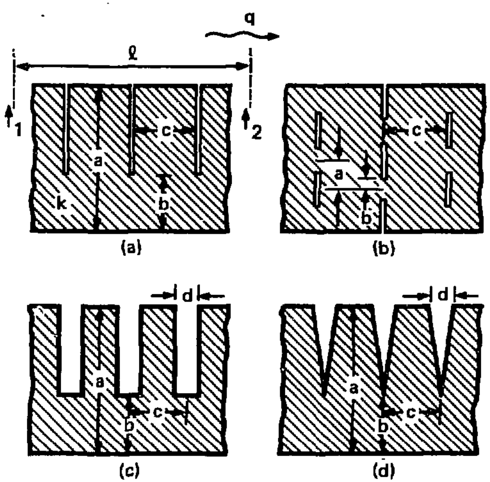
Case No.	References	Description	Solution
1.1.29	15	Semi-infinite strip with convection boundary. $t = t_1$, $x = 0$, w , $y = 0$. Convection boundary at $y = 0$. 	$$\frac{t - t_1}{t_f - t_1} = 4 \sum_{n=1}^{\infty} \frac{\exp(-n\pi Y)\ \sin\ (n\pi X)}{n\pi + (n^2\pi^2/Bi)},\ n = 1, 3, 5, \ldots .$$ Heat transfer into strip at $y = 0$: $$\frac{Q}{k(t_1 - t_f)} = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{1}{n + (n^2\pi/Bi)},\ n = 1, 3, 5, \ldots .$$ See Tables 1.2a and 1.2b.
1.1.30	88 27	Periodic strip heated plate $q_x(w,y) = 0$, on 2b wide strips. $t(w,y) = t_s$, on $(2a - b)$ wide strips spaced 2a on centers. 	See Fig. 1.6a and b for values of maximum differences on $y = 0$ surface, i.e. $[t(0,a) - t(0,0)] = \Delta t_m$, and heat transfer.

Case No.	References	Description	Solution
1.1.31	28	Case 1.1.30 except Q = surface heat flux on $(2a - b)$ wide strips spaced $2a$ on centers.	$\frac{[t(x, y) - t_f]k}{Qa} = \frac{B}{Bi}(1 + BiX) + \frac{2}{\pi^2} \times \sum_{n=1}^{\infty} \frac{\sin(n\pi B)\cos(n\pi Y)\left[\cosh(n\pi X) + \frac{Bi}{n\pi}\sinh(n\pi X)\right]}{n^2\left[\sinh(n\pi W) + \frac{Bi}{n\pi}\cosh(n\pi W)\right]}$ $B = b/a$, $Bi = ha/k$, $W = w/a$, $X = x/a$, $Y = y/a$. See Fig. 1.7 for values of $T_m = t(0 , a) - t(0 , 0)$.
1.1.32	29	Spot insulated infinite plate with constant temp on one face and convection boundary with an insulating spot on the other. 	See Fig. 1.9 for values of $\theta(r/w, z/w)$ = $[t(r , z) - t_f]/[t(\infty , w) - t_f]$ at $r = 0$, $z = w$.

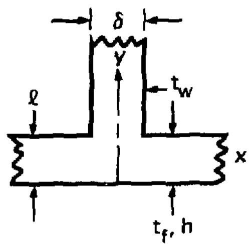
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.33	32	Infinite plate containing an insulating strip. $t = t_1$, $-\infty < x < \infty$, $y = w$. $t = t_0$, $-\infty < x < \infty$, $y = -w$. $q_y = 0$, $0 < x < \ell$, $y = 0$. 	$\frac{t(x\ ,\ y) - t_0}{t_1 - t_0} = \frac{1}{2} + \frac{1}{2\pi} \cos^{-1} \left\{ \frac{\sqrt{2}\ F}{\left[F^2 + G^2 + 1 + \sqrt{(F^2 + G^2 + 1)^2 - 4F^2}\right]^{\frac{1}{2}}} \right\}$ $F = \frac{2\{1 + \cosh [\pi(2X - L)]\ \cos [2\pi Y)]\}}{1 + \cosh (\pi L)}$ $G = \frac{2 \sinh [\pi(2X - L)] \sin (2\pi Y)}{1 + \cosh (\pi L)}$ For $L \to \infty$: $F = 2 \cos (2\pi Y) \exp(-2\pi X) - 1$, $G = 2 \sin (2\pi Y) \exp(-2\pi X) - 1$, $X = x/w$, $Y = y/w$, $L = \ell/w$
1.1.34	19, p. 3-110	Infinite thin plate with heated circular hole. $t = t_1$, $r = r_1$. 	$\frac{t - t_\infty}{t_3 - t_\infty} = \frac{K_0(Br/\delta)}{K_0(Br_1/\delta)}$, $r > r$, $B = \sqrt{Bi_1 + Bi_2}$, $t_\infty = (t_1 + Ht_2)/(1 + H)$, $H = Bi_1/Bi_2$

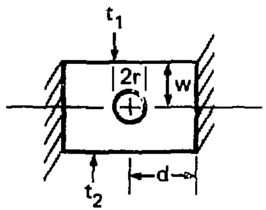
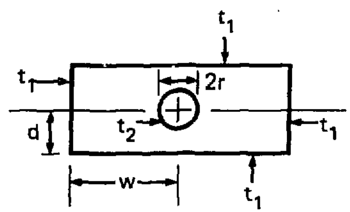
Case No.	References	Description	Solution
1.1.35	19, p. 3-110	Case 1.1.34 with t_3 replaced by a heat source of strength q.	$\frac{k\delta(t - t_\infty)}{q} = \frac{K_0(Br/\delta)}{2\pi(Br_1/\delta)\ K_1(Br_1/\delta)}$, $r > r_1$ B and t_∞ defined in case 1.1.34 .
1.1.36	19, p. 3-111	Case 1.1.34 with $h_2 = 0$.	$\frac{t - t_1}{t_3 - t_1} = \frac{I_0\ (Br/\delta)}{I_0(Br_1/\delta)}$, $r > r_1$ B given in case 1.1.34 .
1.1.37	60 19, p. 3-123	Infinite plate with wall cuts as shown. Heat flow normal to cuts.	See Table 1.3 for conductance data K/K_{uncut}. $K_{uncut} = ka/\ell$ $Q = K\ t_1 - t_2$



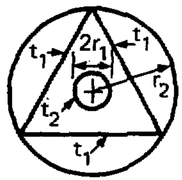
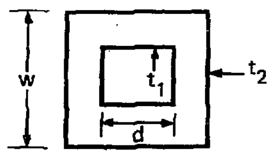
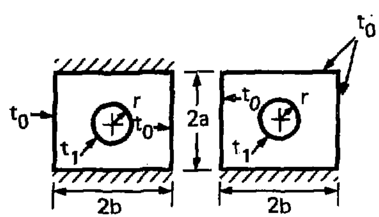
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.37.1	84	Infinite medium with single and multiple insulating cuts.	See Ref. 84 for temperature and heat flow solutions.
1.1.38	61 19, p. 3-124	Case 1.1.37, cut (c), with only one cut.	Conductance: $K = \dfrac{k}{[(\ell - d)/a] + (d/b) + (4/\pi)\ \ell n \{\sec (\pi/2)[1 - (b/a)]\}}$ $Q = K\ t_1 - t_2$
1.1.39	62 19, p. 3-126	Infinite rib on an infinite plate with convection. $t = t_w$, $x = \pm\delta/2$, $y > \ell$. $t = t_w$, $x > \lvert\delta/2\rvert$, $y = \ell$. Convection boundary at $y = 0$.  	See Fig. 1.10 for temp at $x = 0$, $y \geq 0$.

Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
1.1.40	79	Finite plate with centered hole. 	$q = \dfrac{k(t_1 - t_2)\left[\dfrac{\pi d}{2w} + \ln\left(\dfrac{w}{\pi r}\right)\right]}{2\pi}$
1.1.41	79	Tube centered in a finite plate. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ln\left(\dfrac{4d}{\pi r}\right) - C}, \quad r < \dfrac{d}{10}$

w/d	C
1.00	0.1658
1.25	0.0793
1.50	0.0356
2.00	0.0075
2.50	0.0016
3.00	0.0003
4.00	1.4×10^{-5}
∞	0

Case No.	References	Description	Solution
1.1.42	79	Tube in a multi-sided infinite solid. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ell n\left(\dfrac{r_2}{r_1}\right) - C(n)}$ n = No. sides $r_1 < r_2/10$ for $n = 3$
1.1.43	79	Infinite square pipe. 	$q = \dfrac{2\pi k(t_1 - t_2)}{0.93\ \ell n\ (w/d) - 0.0502}$, $w/d > 1.4$ $q = \dfrac{2\pi k(t_1 - t_2)}{0.785\ \ell n\ (w/d)}$, $w/d < 1.4$
1.1.44	87	Partially adiabatic rectangular rod with an isothermal hole. 	$q = Sk(t_1 - t_0)$ See Fig. 1.11 for values of S.

Case 1.1.42:

n	C(n)
3	0.5696
4	0.2708
5	0.1606
6	0.1067
7	0.0761
8	0.0570
9	0.0442
10	0.0354
∞	0

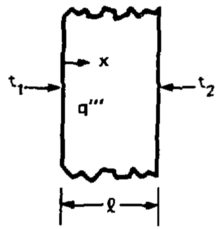
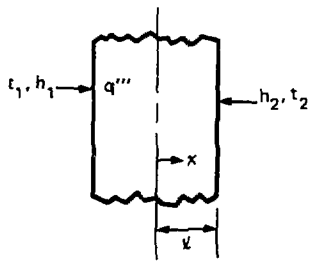
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution

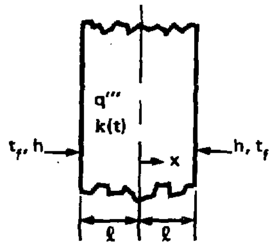
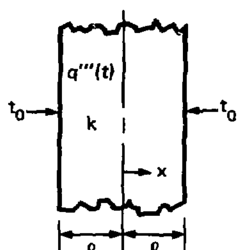
Section 1.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution

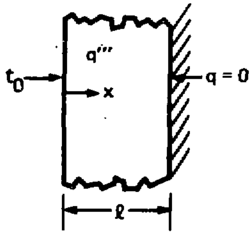
Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution
1.2.1	1, p. 169	Infinite plate. $t = t_1$, $x = 0$. $t = t_2$, $x = \ell$. 	$\frac{t - t_1}{t_2 - t_1} = X + Po\,X(1 - X)/2$
1.2.2	4, p. 50	Infinite plate with convection boundaries. 	$\frac{t - t_2}{t_1 - t_2} = \frac{1 - Po\,(1/Bi_2 + 1)}{1 + Bi_2 + H} + \frac{Po}{Bi_2} + \frac{Po}{2}(1 - X^2) + \frac{Bi_1\left[1 + Po(1/Bi_2 + 1/2)\right](1 - X)}{1 + Bi_1 + H}$ $H = h_2/h_1$

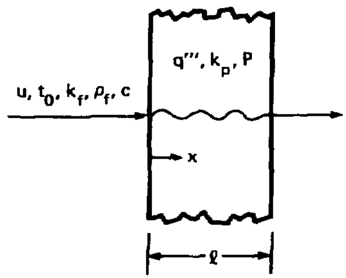
Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution
1.2.3	3, p. 130	Infinite plate with temperature dependent conductivity. $k = k_f + \beta(t - t_f)$. 	$\frac{(t - t_f)k_f}{q'''L^2} = \frac{2}{B}\left[-1 + \sqrt{1 + 2B(1 - X^2)}\right]$ $B = \beta q'''L^2/2k_\infty^2$ See Fig. 1.2.
1.2.4	2, p. 215	Infinite plate with temperature dependent internal heating. $t = t_0$, $x = \pm\ell$, $q''' = q_0''' + \beta(t - t_0)$. 	$\frac{(t - t_0)k}{q_0'''\ell^2} = \frac{1}{Po_\beta}\left[\frac{\cos(\sqrt{Po_\beta}\,X)}{\cos(\sqrt{Po_\beta})} - 1\right]$ $Po_\beta = \beta\ell^2/k$ Mean temp: $\frac{(t_m - t_0)k}{q_0'''\ell^2} = \frac{1}{Po_\beta^{3/2}}\tan(\sqrt{Po_\beta})$

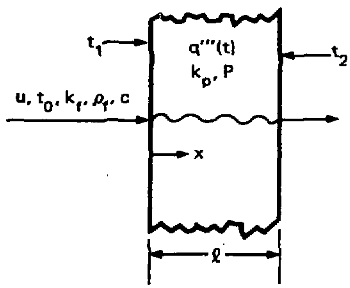
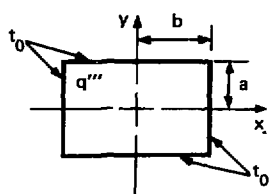
Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution
1.2.5	2, p. 217	Infinite plate with radiation heating. $t = t_0$, $x = 0$, $q''' = \dot{q}\gamma e^{-\gamma x}$. $\dot{q}$ = radiation energy flux. γ = mean radiation absorption coeff. 	$\dfrac{(t - t_0)k\gamma}{\dot{q}} = 1 - \gamma x e^{-\gamma \ell} - e^{-\gamma x}$

Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

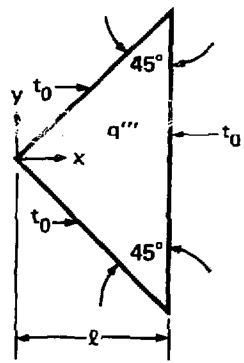
Case No.	References	Description	Solution
1.2.6	2, p. 223	Porous plate with internal fluid flow. P = Porosity.  	$\frac{t - t_0}{\zeta} = \frac{1}{\xi_p}\left[1 - e^{-\xi_p(1-X)}\right] + X \; , \; 0 \leq X \leq 1$ (See Fig 1.3) $\frac{t - t_0}{\zeta} = \frac{1}{\xi_p}\left(1 - e^{\xi_p}\right)e^{\xi_f X} \; , \; -\infty < X < 0$ Mean temp: $\frac{t_m - t_0}{\zeta} = \frac{1}{\xi_p^2}\left(\xi_p + e^{-\xi_p}\right) + \frac{1}{2}$ $\xi_p = \frac{\rho_f u c \ell}{k_p(1 - P)} \; , \; \xi_f = \frac{\rho_f u c \ell}{k_f(1 - P)} \; , \; \zeta = \frac{q''' \ell (1 - P)}{\rho_f u c}$

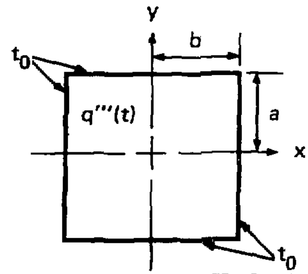
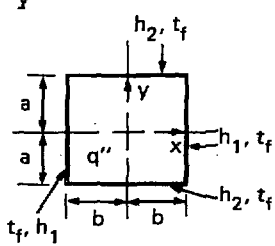
Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

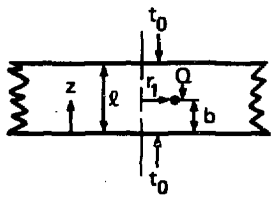
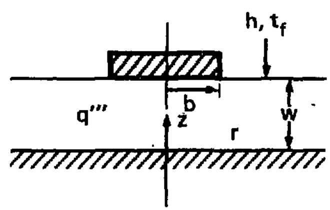
Case No.	References	Description	Solution
1.2.7	2, p. 227	Porous plate with internal fluid flow and temperature dependent internal heating. $t = t_2$, $x = \ell$, $q''' = q_0'''(1 + \beta t)$, P = Porosity.  	$\frac{\beta t + 1}{\beta t_2 + 1} = \frac{e^{D_1 X}}{E}\left(\frac{\beta t_1 + 1}{\beta t_2 + 1} e^{D_2} - 1\right) - \frac{e^{D_2 X}}{E}\left(\frac{\beta t_1 + 1}{\beta t_2 + 1} e^{D_1} - 1\right)$ $E = e^{D_2} - e^{D_1}$ $D_1 = \xi_p/2 + \left[(\xi_p/2)^2 - Po_\beta\right]^{1/2}$ $D_2 = \xi_p/2 - \left[(\xi_p/2)^2 - Po_\beta\right]^{1/2}$ $\beta t_1 = \frac{(D_1 - D_2)(\beta t_2 + 1) - E\beta t_0 \xi_p - E^*}{E^* - E\xi_p}$ $E^* = D_1 e^{D_2} - D_2 e^{D_1}$, $\xi_p = \frac{\rho_f u c \ell}{k_p(1 - P)}$, $Po = q_0'''\beta \ell^2/k_p$
1.2.8	3, p. 220	Rectangular rod. $t = t_0$, $x = \pm b$, $-a < y < +a$. $t = t_0$, $-b < x < +b$, $y = \pm a$.  	$\frac{(t - t_0)k}{q'''a^2} = \frac{1}{2}(1 - Y^2) - 2 \times \sum_{n=0}^{\infty} \frac{(-1)^n \cosh\left[(2n + 1)\frac{\pi}{2}X\right] \cos\left[(2n + 1)\frac{\pi}{2}Y\right]}{\left[(2n + 1)\frac{\pi}{2}\right]^3 \cosh\left[(2n + 1)\frac{\pi}{2}B\right]}$ See Fig. 1.4, Table 1.1. $Y = y/a$, $X = x/a$, $B = b/a$

Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution
1.2.9	3, p. 469	Infinite triangular bar. $t = t_0$, $x = \pm y$. $t = t_0$, $x = \ell$.	$\dfrac{(t - t_0)k}{q'''\ell^2} = \dfrac{5}{16}\,(X^2 - Y^2)\,\ell n\left(\dfrac{1}{X}\right)$ $Y = y/\ell$



Case No.	References	Description	Solution
1.2.10	2, p. 197	Infinite rectangular rod with temperature dependent internal heating. $q''' = q_0'''(1 + \beta t)$. $t = t_0,\ x = \pm b,\ -a < y < +a$. $t = t_0,\ -b < x < +b,\ y = \pm a$.  	$\frac{(t - t_0)}{1/\beta + t_0} = 4Po_\beta \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^{n+m} \cos(\lambda_n X) \cos(\lambda_m Y)}{\lambda_n \lambda_m \left[\lambda_n^2 A + \left(\lambda_m^2/A\right) - Po_\beta\right]}$ $\lambda_n = 2\pi(2n - 1)$, $\lambda_m = 2\pi(2m - 1)$ $X = x/b$, $Y = y/a$, $Po_\beta = q_0'''ab\beta/k$, $A = a/b$ See Fig 1.5 .
1.2.11	9, p. 171	Infinite rectangular rod with convection boundary. k_x = conductivity in x-dir. k_y = conductivity in y-dir.  	$\frac{(t - t_f)k_x}{q'''a^2} = \frac{1}{Bi_1} + \frac{1}{2}(1 - X) - 4 \sum_{n=1}^{\infty}$ $\times \frac{\sin(\lambda_n) \cos(\lambda_n X) \cosh(\lambda_n AY)}{\lambda_n^2 \left[2\lambda_n + \sin(2\lambda_n)\right] \left[\left(\frac{\lambda_n}{ABBi_1}\right) \sinh(A\lambda_n) + \cosh(A\lambda_n)\right]}$ $\lambda_n \tan(\lambda_n) = Bi_1$, $Bi_1 = h_1 a/k_x$, $Bi_2 = h_2 a/k_x$ $A = \frac{a}{b}\sqrt{k_x/k_y}$, $X = x/b$, $Y = y/a$, $B = b/a$

Case No.	References	Description	Solution
1.2.12	9, p. 423	Infinite plate with point source. $t = t_0$, $z = 0$, ℓ. Source of strength Q is located at r_1 , θ_1 , b. 	$\frac{(t - t_0)k\ell}{Q} = \frac{1}{\pi}\sum_{n=1}^{\infty} \sin(n\pi Z)\sin(n\pi B)K_0(n\pi R)$ $R = \frac{1}{\ell^2}\left[r^2 + r_1^2 - 2rr_1\cos(\theta - \theta_1)\right]$ $B = b/\ell$, $Z = z/\ell$
1.2.13	29	Spot insulated infinite plate. Insulated on one side and an insulating spot with convection boundary on the other. 	Solutions of $\left[t(r , z) - t_f\right]/\left[t(\infty , w) - t_f\right] = \theta(r/w , z/w)$ at $r = 0$, $z = 0$, w are given in Fig. 1.8.

Section 1.2. Solids Bounded by Plane Surfaces—With Internal Heating.

Case No.	References	Description	Solution

Section 1.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution

2. Cylindrical Surface — Steady State

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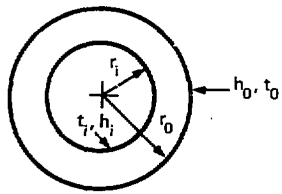
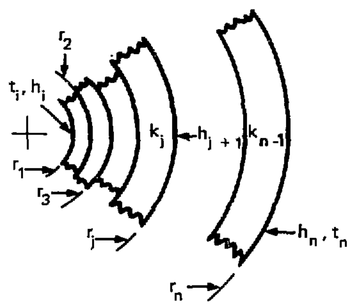
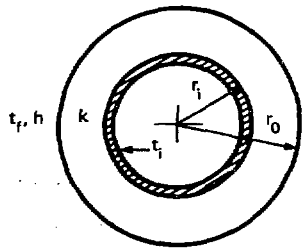
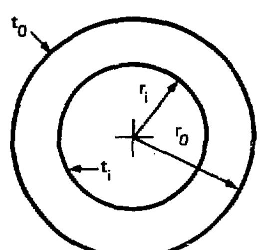
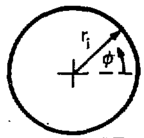
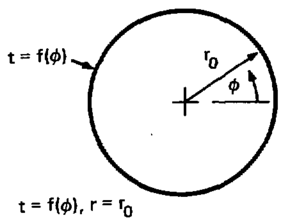
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.1	4, p. 37	Infinite hollow cylinder.  	$q = \dfrac{2\pi k(t_0 - t_i)}{\ell n\ (r_0/r_i) + (1/Bi_i) + (1/Bi_0)}$ $\dfrac{t - t_0}{t_i - t_0} = \dfrac{\ell n\ (r/r_0)}{\ell n\ (r_i/r_0) + (1/Bi_0 + (1/Bo_0)}$ $Bi_i = h_i r_i/k\ ,\ Bi_0 = h_0 r_0/k$
2.1.2	19, p.3-107	The composite cylinder.  	$q = \dfrac{2\pi(t_n - t_1)}{\sum\limits_{i=1}^{n-1} \dfrac{1}{k_i}\ \ell n \left(\dfrac{r_{i+1}}{r_i}\right) + \sum\limits_{i=1}^{n} \dfrac{1}{r_i h_i}}$ Temp in the jth layer: $\dfrac{t_j - t_1}{t_n - t_1} =$ $\dfrac{\sum\limits_{i=1}^{j-1} \left[\dfrac{1}{k_i}\ \ell n \left(\dfrac{r_{i+1}}{r_i}\right) + \dfrac{1}{r_i h_i}\right] + \dfrac{1}{k_j}\ \ell n \left(\dfrac{r}{r_j}\right) + \dfrac{1}{r_j h_j}}{\sum\limits_{i=1}^{n-1} \dfrac{1}{k_i}\ \ell n \left(\dfrac{r_{i+1}}{r_i}\right) + \sum\limits_{i=1}^{n} \dfrac{1}{r_i h_i}},\ j > 1$

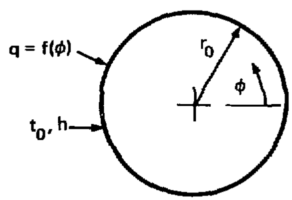
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating

Case No.	References	Description	Solution
2.1.3	3, p. 121	Insulated tubes. 	$q = \dfrac{2\pi k(t_i - t_f)}{\ell n\left(\dfrac{r_0}{r_i}\right) + \dfrac{1}{Bi_i}}$ See Fig. 2.1 Max heat loss occurs when $r_0 = k/h$.
2.1.4	1, p. 138	Infinite cylinder with temperature dependent thermal conductivity. $k = k_0 + \beta(t - t_0)$. $k = k_0$ at r_0. $k = k_i$ at r_i. 	$q = \dfrac{2\pi k_m(t_i - t_0)}{\ell n\ (r_0/r_i)}$ $\dfrac{(t - t_0)\beta}{k_0} = \sqrt{1 + \dfrac{2\beta k_m}{k_0^2} \dfrac{\ell n(r_0/r)}{\ell n(r_0/r_i)} (t_i - t_0) - 1}$ $k_m = (k_0 + k_i)/2$

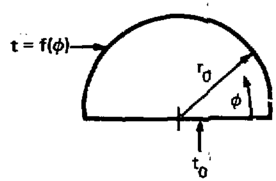
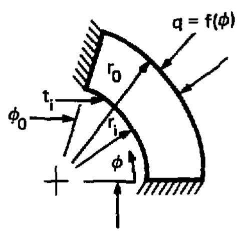
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.5	2, p. 148	Cylindrical surface in an infinite medium. $t = f(\phi)$, $r = r_1$. $t = t_\infty$, $r \to \infty$. 	$t - t_\infty = \sum_{n=1}^{\infty} \left(\frac{r_1}{r}\right)^n \left[A_n \cos (n\phi) + B_n \sin (n\phi)\right], r > r$ $A_n = \frac{1}{\pi} \int_0^{2\pi} \left[f(\phi) - t_\infty\right] \cos (n\phi) d\phi$ $B_n = \frac{1}{\pi} \int_0^{2\pi} \left[f(\phi) - t_\infty\right] \sin (n\phi) d\phi$
2.1.6	3, p. 226	Infinite cylinder with specified surface temperature, $t - f(\phi)$, $r = r_0$. 	$t = a_0 + \sum_{n=1}^{\infty} R^n \left[a_n \cos (n\phi) + b_n \sin (n\phi)\right]$ $a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\phi) d\phi$ $a_n = \frac{1}{\pi} \int_0^{2\pi} f(\phi) \cos (n\phi) d\phi$ $b_n = \frac{1}{\pi} \int_0^{2\pi} f(\phi) \sin (n\phi) d\phi$ For $f(\phi) = \begin{cases} t_0 , & 0 < \phi < \pi \\ 0 , & \pi < \phi < 2\pi \end{cases}$: $\frac{t}{t_0} = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (R)^n \sin (n\phi)$

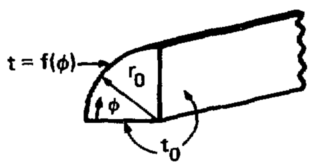
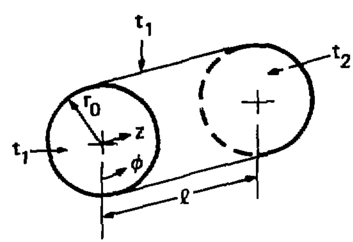
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.7	3, p. 228	Infinite cylinder with specified surface heat flux. $q = f(\phi)$, $r = r_0$.  	$t - t_0 = a_0 + \sum_{n=1}^{\infty} R^n \left[a_n \cos (n\phi) + b_n \sin (n\phi) \right]$ $a_0 = \frac{1}{2\pi h} \int_0^{2\pi} f(\phi) d\phi$, $a_n = \frac{1}{\pi h(1 + n/Bi)} \int_0^{2\pi} f(\phi) \cos (n\phi) d\phi$ $b_n = \frac{1}{\pi h(1 + n/Bi)} \int_0^{2\pi} f(\phi) \sin (n\phi) d\phi$ For $q = \begin{cases} q_0 \sin (\phi) , & 0 < \phi < \pi \\ 0 , & \pi < \phi < 2\pi \end{cases}$: $\frac{(t - t_0)h}{q_0} = \frac{1}{\pi} + \frac{R \sin (\phi)}{2[1 + (1/Bi)]} - \frac{2}{\pi} \sum_{n=1}^{\infty} (R)^{2n} \frac{\cos (2n\pi)}{(4n^2 - 1)\left[1 + (2n/Bi)\right]}$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

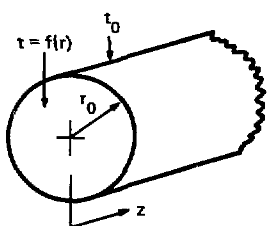
Case No.	References	Description	Solution
2.1.8	2, p. 133	Infinite half-cylinder with specified surface temperature. $t = t_0$, $\phi = 0$ and π. $t = f(\phi)$, $r - r_0$.  	$t - t_0 = \frac{2}{\pi}\sum_{n=1}^{\infty} R^n \sin(n\phi)\int_0^{\pi}\left[f(\phi) - t_0\right]\sin(n\phi)d\phi$ For $f(\phi) = t_1$: $\frac{t - t_0}{t_1 - t_0} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}R^n \sin(n\phi)$ For $f(\phi) = t_1, \sin(\phi)$: $\frac{t - t_0}{t_1 - t_0} = R\sin(\phi) - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}R^n \sin(n\phi)$
2.1.9	3, p. 230	Cylindrical shell section with specified surface heat flux and temperature. $t = t_i$, $r = r_i$. $q = f(\phi)$, $r = r_0$.  	$t - t_i = a_0 \ln(R) + \sum_{n=1}^{\infty} a_n\lambda_n\left[R_0^{\lambda_n} - R_0^{-\lambda_n}\right]\cos(\lambda_n\phi)$ $a_0 = \frac{r_0}{k\phi_0}\int_0^{\phi_0}\left[f(\phi) - t_i\right]d\phi$ $a_n\lambda_n\left(R_0^{\lambda_n} + R_0^{-\lambda_n}\right) = \frac{2r_0}{k\phi_0}\int_0^{\phi_0}\left[f(\phi) - t_i\right]\cos(\lambda_n\phi)d\phi$ $\lambda_n = n\pi/\phi_0$, $R = r/r_i$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

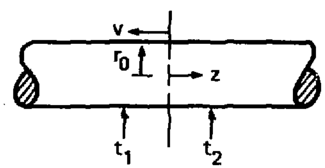
<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr><td>2.1.10</td><td>2, p. 148</td><td>Infinite quarter cylinder.
$t = t_0$, $\phi = 0$ and $\pi/2$.
$t = f(\phi)$, $r = r_0$.

</td><td>$$t - t_0 = \frac{4}{\pi} \sum_{n=1}^{\infty} R^{2n} \sin (2n)\phi \int_0^{\pi/2} \left[f(\phi) - t_0\right] \sin (2n\phi) d\phi$$
For $f(\phi) = t_1$:
$$\frac{t - t_0}{t_1 - t_0} = \frac{2}{\pi} \tan^{-1} \left[\frac{2R^2 \sin (2\phi)}{1 - R^4}\right]$$</td></tr>
<tr><td>2.1.11</td><td>2, p. 133</td><td>Finite cylinder with two surface temperatures.
$t = t_1$, $z = 0$.
$t = t_1$, $r = r_0$.
$t = t_2$, $z = \ell$.

</td><td>$$\frac{t - t_1}{t_2 - t_1} = 2 \sum_{n=1}^{\infty} \frac{\sinh (\lambda_n Z) J_0(\lambda_n R)}{\lambda_n \sinh (\lambda_n L) J_1(\lambda_n)}$$
$J_0(\lambda_n) = 0$, $Z = z/r_0$, $R = r/r_0$, $L = \ell/r_0$
For $t_2 = f(r)$:
$$t - t_1 = \frac{2}{r_0^2} \sum_{n=1}^{\infty} \frac{\sinh (\lambda_n Z) J_0(\lambda_n R)}{\sinh (\lambda_n L) J_1^2(\lambda_n)} \int_0^1 R\left[f(R) - t_1\right] J_0(\lambda_n R) dR$$</td></tr>
</table>

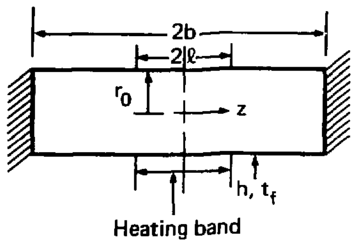
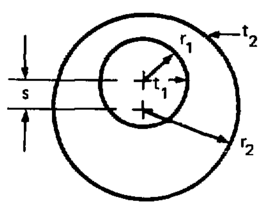
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.12	3, p. 253	Case 2.1.11 with $t = t_1$, $r = 0$, $r = \ell$. $t = t_1$, $r = r_0$, $0 < \phi < \pi$. $t = t_0$, $r = r_0$, $\pi < \phi < 2\pi$.	$\frac{t - t_0}{t_1 - t_0} = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{I_0(\lambda_n R)\sin(\lambda_n Z)}{nI_0(\lambda_n R_0)} + \frac{8}{\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{I_m(\lambda_n R)\sin(\lambda_n Z)\sin(m\phi)}{nmI_n(\lambda_n R_0)}$ $\lambda_n = n\pi$, $R = r/\ell$, $Z = z/\ell$
2.1.13	3, p. 234	Semi-infinite rod with variable end temperature. $t = f(r)$, $z = 0$. $t = t_0$, $r = r_0$.	$t - t_0 = \sum_{n=1}^{\infty} A_n J_0(\lambda_n R)\exp(-\lambda_n Z)$ $A_n = \frac{2\int_0^1 R\left[f(R) - t_0\right]J_0(\lambda_n R)\,dR}{J_1^2(\lambda_n)}$ $Z = z/r_0$, $J_0(\lambda_n) = 0$, $\lambda_n > 0$ If $f(r) = t_1$: $\frac{t - t_0}{t_1 - t_0} = 2\sum_{n=1}^{\infty}\frac{\exp(-\lambda_n Z)J_0(\lambda_n R)}{\lambda_n J_1(\lambda_n)}$

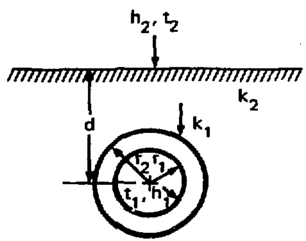
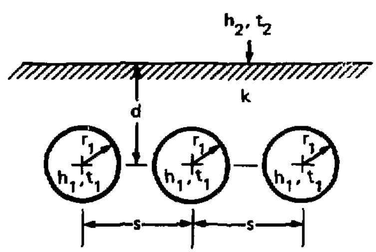


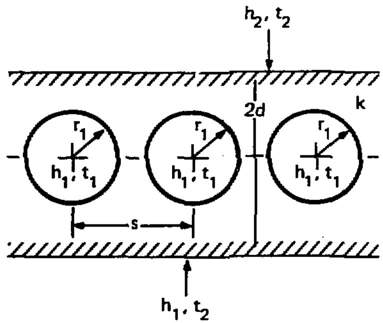
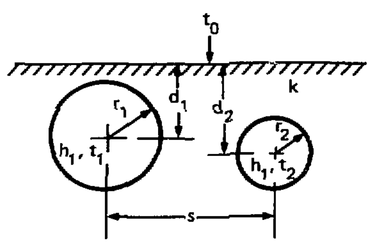
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.14	3, p. 234	Case 2.1.13 with $t = t_1$, $z = 0$, and convection boundary h , t_f at $r = r_0$.	$\frac{t - t_f}{t_1 - t_f} = 2 \sum_{n=1}^{\infty} \frac{Bi\ J_0(\lambda_n R)\ \exp\ (-\lambda_n Z)}{(\lambda_n^2 + Bi^2)\ J_0(\lambda_n)}$ $\lambda_n J_1(\lambda_n) = Bi\ J_0(\lambda_n)$
2.1.15	3, p. 238	Infinite rod with a traveling boundary between two temperature zones. $t = t_1$, $r = r_0$, $z < 0$. $t = t_2$, $r = r_0$, $z > 0$. Velocity of boundary $(z = 0) = v$. 	$\frac{t - t_2}{t_1 - t_2} = 1 - \sum_{n=1}^{\infty} \left[1 - \frac{1}{\left[1 + (\lambda_n/P)^2\right]^{1/2}}\right] \frac{J_0(\mu_n R)}{\lambda_n J_1(\lambda_n)}$ $\times \exp\left[\left\{1 + \left[1 + (\lambda_n/P)^2\right]^{1/2}\right\} P\ Z\right],\ Z < 0.$ $\frac{t - t_2}{t_1 - t_2} = \sum_{n=1}^{\infty} \left[1 + \frac{1}{\left[1 + (\lambda_n/P)^2\right]^{1/2}}\right] \frac{J_0(\lambda_n R)}{\lambda_n J_1(\lambda_n)}$ $\times \exp\left[\left\{1 - \left[1 + (\lambda_n/P)^2\right]^{1/2}\right\} P\ Z\right],\ Z > 0.$ $Z = z/r_0$, $P = vr_0/\alpha$, $J_0(\lambda_n) = 0$

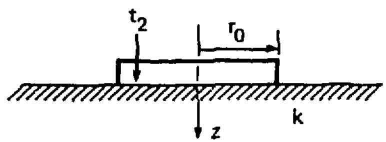
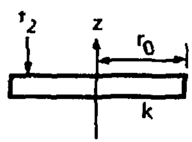
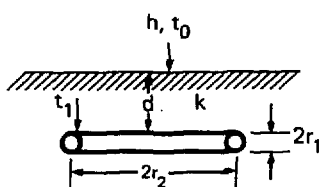
Case No.	References	Description	Solution
2.1.16	3, p. 238	Finite rod with band heating. $q_z = 0$, $z = \pm b$. Convection boundary at $r = r_0$, $-b < z < +b$. Surface heating (q_s) at $r = r_0$, $-\ell < z < +\ell$.  	$\dfrac{t - t_f}{q_s r_0/k} = \dfrac{L}{Bi} + 2\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin(\lambda_n L) I_0(\lambda_n \rho R) \cos(\lambda_n Z)}{\lambda_n[Bi\ I_0(\lambda_n \rho) + (\lambda_n \rho) I_1(\lambda_n \rho)]}$ $\lambda_n = n\pi$, $Bi = hr_0/k$, $L = \ell/b$, $Z = z/b$ $R = r/r_0$, $\rho = r_0/b$
2.1.17	4, p. 37	Eccentric hollow cylinder. $t = t_2$, $r = r_2$. $t = t_1$, $r = r_1$.  	$q = \dfrac{2\pi k(t_2 - t_1)}{\ell n\left[\dfrac{\sqrt{(R+1)^2 - S^2} + \sqrt{(R-1)^2 - S^2}}{\sqrt{(R+1)^2 - S^2} - \sqrt{(R-1)^2 - S^2}}\right]}$ $R = r_2/r_1$, $S = s/r_1$ See Ref. 80 for other solutions to eccentric cylinder.

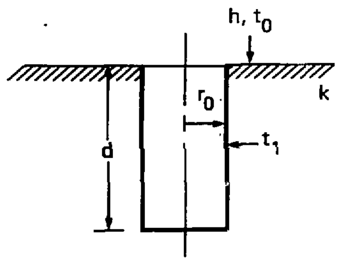
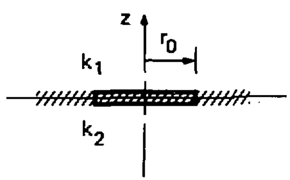
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.18	4, p. 39	Pipe in semi-infinite solid. $d > 4r_1$.  	$q = \dfrac{2\pi k_1(t_2 - t_1)}{\dfrac{1}{Bi_1} + \ell n\left(\dfrac{r_2}{r_1}\right) + \dfrac{1}{K}\ \ell n\left[2D + \dfrac{2KD}{Bi_2}\right]}$ $Bi_1 = \dfrac{h_1 r_1}{k_1},\ Bi_2 = \dfrac{h_2 d}{k_2},\ K = \dfrac{k_2}{k_1},\ D = \dfrac{d}{r_2}$
2.1.19	4, p. 40	Row of pipes in semi-infinite solid.  	$q = \dfrac{2\pi k(t_2 - t_1)}{\dfrac{1}{Bi_1} + \ell n\left[\pi \dfrac{d}{Dr_1}\ \sinh\left(\pi 2\left[D + \dfrac{D}{Bi_2}\right]\right)\right]}$, for one pipe. $Bi_1 = \dfrac{h_1 r_1}{k},\ Bi_2 = \dfrac{h_2 d}{k},\ D = \dfrac{d}{s}$

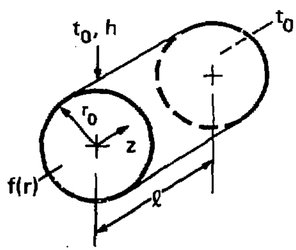
Case No.	References	Description	Solution
2.1.20	4, p. 40	Row of pipes in a wall. 	$q = \dfrac{4\pi k(t_2 - t_1)}{\dfrac{1}{Bi_1} + \ell n\left[\dfrac{d}{\pi r_1 D}\sinh\left(2\pi\left[D + \dfrac{D}{Bi_2}\right]\right)\right]}$, for each pipe. $Bi_1 = \dfrac{h_1 r_1}{k}$, $Bi_2 = \dfrac{h_2 d}{k}$, $D = \dfrac{d}{s}$
2.1.21	4, p. 38	Two pipes in a semi-infinite solid. 	$q_1 = \dfrac{2\pi k(t_1 - t_0)\, Z_1}{Z_2}$ $Z_1 = \ell n\left(\dfrac{2d_2}{r_2}\right) - \dfrac{t_2 - t_0}{t_1 - t_0}\,\ell n\sqrt{\dfrac{s^2 + (d_1 + d_2)^2}{s^2 + (d_1 - d_2)^2}}$ $Z_2 = \ell n\left(\dfrac{2d_1}{r_1}\right)\ell n\,\dfrac{2d_2}{r_2} + \left(\ell n\sqrt{\dfrac{s^2 + (d_1 + d_2)}{s^2 + (d_1 - d_2)^2}}\right)^2$ For q_2, interchange indices 1 and 2.

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.22	4, p. 43 9, p. 215	Circular disk on the surface of a semi-infinite solid. $t = t_1$, $z \to \infty$.  	$q = 4r_0k(t_2 - t_1)$ $\frac{t - t_1}{t_2 - t_1} = \frac{2}{\pi}\sin^{-1}\left[\frac{2}{\left[(R-1)^2 + Z^2\right]^{1/2} + \left[(R+1)^2 + Z^2\right]^{1/2}}\right]$ $Z = z/r_0$
2.1.23	4, p. 44	Circular disk in an infinite solid. $t = t_1$, $z \to \pm\infty$.  	$q = 8r_0k(t_2 - t_1)$
2.1.24	4, p. 42	Circular ring in a semi-infinite solid.  	$q = \frac{4\pi^2 r_2(t_1 - t_0)}{\ln\left(\frac{8r_2}{r_1}\right) + \ln\left[\frac{4r_2}{d\left(1 + \frac{1}{Bi_d}\right)}\right]}$, $r_1 \ll d \ll r_2$

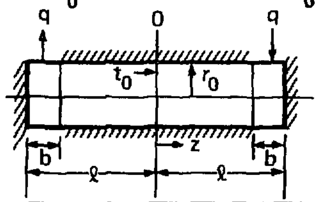
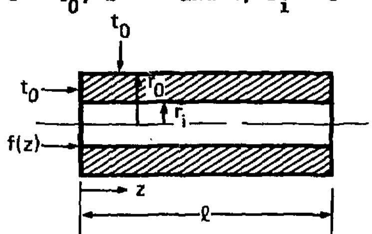
Case No.	References	Description	Solution
2.1.25	4, p. 42	Vertical cylinder in a semi-infinite solid.  	$q = \left\{\dfrac{2D}{\ell n\left[2D\left(1 + \dfrac{1}{Bi_d}\right)\right]} + \dfrac{Bi_d}{D}\right\}\pi r_0 k(t_1 - t_0)$ $D = d/r_0$
2.1.26	9, p. 216	Two semi-infinite regions of different conductivities connected by a circular disk. $t = t_0,\ z \to +\infty.$ $t = t_1,\ z \to -\infty.$ $q_z = 0,\ r > r_0,\ z = 0.$  	$\dfrac{t - t_0}{t_1 - t_0} = \dfrac{2k_2}{\pi(k_1 + k_2)}$ $\times \sin^{-1}\left\{\dfrac{2}{\left[(R - 1)^2 + Z^2\right]^{1/2} + \left[(R + 1)^2 + Z^2\right]^{1/2}}\right\},\ Z < 0 .$ $\dfrac{t - t_0}{t_1 - t_0} = 1 - \dfrac{2k_1}{\pi(k_1 + k_2)}$ $\times \sin^{-1}\left\{\dfrac{2}{\left[(R - 1)^2 + Z^2\right]^{1/2} + \left[(R + 1)^2 + Z^2\right]^{1/2}}\right\},\ Z < 0 .$ $q = \left[4r_0k_1k_2/(k_1 + k_2)\right](t_0 - t_1)\ ,\ Z = z/r_0$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.27	9, p. 218, 69	Finite cylinder with: $t = f(r)$, $z = 0$, $0 < r < r_0$. $t = t_0$, $z = \ell$, $0 < r < r_0$. Convection boundary at $r = r_0$, $0 < z < \ell$, with h, t_0. 	$t - t_0 = \sum_{n=1}^{\infty} C_n \frac{J_0(R\lambda_n) \sinh([1 - Z]L\lambda_n)}{\sinh(L\lambda_n)}$ $C_n = \frac{2\lambda_n^2}{(Bi^2 + \lambda_n^2) J_0^{2\prime}(\lambda_n)} \int_0^1 R\left[f(R) - t_0\right] J_0(R\lambda_n)\,dR$ $L = \ell/r_0$, $Bi = hr_0/k$, $\lambda_n J_0'(\lambda_n) + Bi\, J_0(\lambda_n) = 0$, $\lambda_n > 0$ For $f(r) = t_1$: $\frac{t - t_0}{t_1 - t_0} = 2Bi \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n) \sinh([1 - Z]L\lambda_n)}{\left(Bi^2 + \lambda_n^2\right) J_0(\lambda_n) \sinh(L\lambda_n)}$
2.1.28	9, p. 219	Case 2.1.27 with $t = f(r)$, $z = 0$, $0 < r < r_0$. Remaining surfaces convection boundaries h, t_0.	$t - t_0 = \sum_{n=1}^{\infty} C_n J_0(R\lambda_n)$ $\times \frac{\lambda_n \cosh([1 - Z]L\lambda_n) + Bi \sinh([1 - Z]L\lambda_n)}{\lambda_n \cosh(L\lambda_n) + Bi \sinh(L\lambda_n)}$ C_n, λ_n, L, and Bi are defined in case 2.1.27.

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.29	9, p. 220	Case 2.1.27 with $t = f(z)$. $r = r_0$, $0 < z < \ell$. $t = t_0$, $z = 0, \ell$, $0 < r < r_0$.	$t - t_0 = 2 \sum_{n=1}^{\infty} \frac{I_0(n\pi R)}{I_0(n\pi R_0)} \sin(n\pi Z) \int_0^1 \left[f(Z) - t_0\right] \sin(n\pi Z)\,dz$ $R = r/\ell$
2.1.30	9, p. 221	Case 2.1.27 with $t = f(z)$, $r = r_0$, $0 < z < \ell$. Remaining surfaces convection boundaries h, t_0.	$t - t_0 = 2 \sum_{n=1}^{\infty} \frac{\left[\lambda_n \cos(\lambda_n Z) + Bi_\ell \sin(\lambda_n Z)\ I_0(\lambda_n R)\right]}{\left(Bi_\ell^2 + \lambda_n^2 + 2Bi_\ell\right) I_0(\lambda_n R_0)}$ $\times \int_0^1 f(Z)\left[\lambda_n \cos(\lambda_n Z) + Bi_\ell \sin(\lambda_n Z)\right] dZ$ $\tan(\lambda_n) = (2\lambda_n Bi_\ell)/(\lambda_n^2 + Bi_\ell^2)$, $\lambda_n > 0$, $R = r/\ell$

Case No.	References	Description	Solution
2.1.31	9, p. 220	Finite cylinder with strip heating and cooling. $q_r = -q_1,\ r = r_0,\ \ell > z > \ell - b.$ $q_r = 0,\ r = r_0,\ \ell - b > z > -\ell + b.$ $q_r = +q_1,\ r = r_0,\ -\ell + b > z > -\ell.$ $q_z = 0,\ z = \pm\ell,\ r = r_0.$ $t = t_0,\ z = 0,\ 0 < r < r_0.$ 	$\frac{(t - t_0)k}{q_1\ell} = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n I_0(m\pi R)}{m^2 I_1(m\pi R_0)}\sin(m\pi b/\ell)\sin(m\pi Z)$ $m = 2n + 1,\ R = r/2\ell,\ Z = z/2\ell,\ B = b/2\ell$
2.1.32	9, p. 220	Finite hollow cylinder with: $t = f(z),\ r = r_i,\ 0 < z < \ell.$ $t = t_0,\ r = r_0,\ 0 < z < \ell.$ $t = t_0,\ z = 0$ and $\ell,\ r_i < r < r_0.$ 	$t - t_0 = 2\sum_{n=1}^{\infty}\frac{F_0}{F_1}\sin(n\pi Z)\int_0^1\left[f(Z) - t_0\right]\sin(n\pi Z)dZ$ $F_0 = I_0(n\pi R)\ K_0(n\pi R_0) - K_0(n\pi R)\ I_0(n\pi R_0)$ $F_1 = I_1(n\pi R_i)\ K_1(n\pi R_0) - K_1(n\pi R_i)\ I_1(n\pi R_1)$ $R = r/\ell$

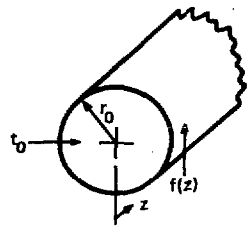
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.33	9, p. 221	Case 2.1.32 with $q_r = f(z)$, $r = r_i$, $0 < z < \ell$. Remaining surfaces at t_0.	$t - t_0 = -\frac{2\ell}{k\pi} \sum_{n=1}^{\infty} \frac{F_0}{nF_1} \sin (n\pi Z) \int_0^1 f(Z) \sin (n\pi Z) dZ$
			F_0 defined in case 2.1.32
			$F_1 = I_1(n\pi R_i)\ K_0(n\pi R_0) + K_1(n\pi R_i)\ I_0(n\pi R_0)$
			For $f(z) = q_1$, $w < z < (\ell - w)$ $= 0$, $0 < z < w$ and $(\ell - w) < z < \ell$:
			$\frac{(t - t_0)k}{q_1\ell} = \sum_{n=1}^{\infty} \frac{F_0}{n^2F_1} \cos (n\pi W) \sin (n\pi Z)$, $n = 1,3,5, \ldots$
			$W = w/\ell$

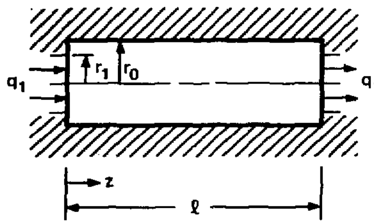
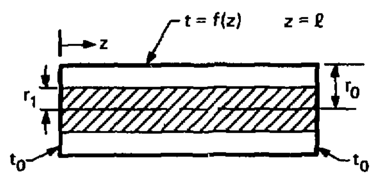
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.34	9, p. 222	Case 2.1.32 with $t = f(r)$, $z = 0$, $r_i < r < r_0$. Remaining surfaces at t_0.	$t - t_0 = \frac{\pi^2}{2} \sum_{n=1}^{\infty} \frac{\lambda_n^2 J_0^2(\lambda_n) U_0(R\lambda_n) \sinh\left[(1 - Z)L\lambda_n\right]}{\left[J_0^2(\lambda_n) - J_0^2(\lambda_n R_0)\right] \sinh(L\lambda_n)}$ $\times \int_0^1 \left[Rf(R) - t_0\right] U_0(R\lambda_n)\,dR$ $U_0(\lambda_n) = 0,\ \lambda_n > 0,\ R = r/r_i,\ Z = z/\ell,\ L = \ell/r_i$ $U_0(R\lambda_n) = J_0(R\lambda_n)\, Y_0(\lambda_n R_0) - J_0(\lambda_n R_0)\, Y_0(R\lambda_n)$
2.1.35	9, p. 222	Case 2.1.32 with $t = f(z)$, $r = r_i$, $0 < z < \ell$. Remaining surfaces convection boundary with h, t_f.	$t - t_f = 2 \sum_{n=1}^{\infty} \frac{\left[\lambda_n \cos(\lambda_n Z) + Bi \sin(\lambda_n Z)\right] G(R,n)}{\left[\lambda_n^2 + Bi_\ell^2 + 2Bi_\ell\right] G(r_0,n)}$ $\times \int_0^1 \left[f(Z) - t_f\right]\left[\lambda_n \cos(\lambda_n Z) + Bi_\ell \sin(\lambda_n Z)\right] dZ$ $\tan(\lambda_n) = 2\lambda_n Bi_\ell / \left(\lambda_n^2 - Bi_\ell^2\right),\ \lambda_n > 0$ $G(R,n) = I_0(R\lambda_n)\left[\lambda_n K_1(\lambda_n R_0) - Bi_\ell\, K_0(\lambda_n R_0)\right]$ $+ K_0(R\lambda_n)\left[\lambda_n I_1(\lambda_n R_0) + Bi\, I_0(\lambda_n R_0)\right]$

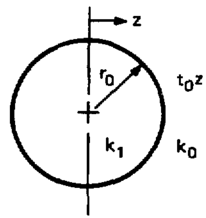
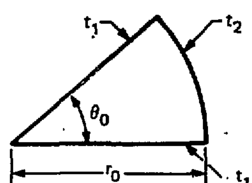
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.36	9, p. 223	Semi-infinite cylinder with: $t = f(z),\ r = r_0,\ z > 0.$ $t = t_0,\ z = 0,\ 0 < r < r_0.$  	$$t - t_0 = \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n)}{J_1(\lambda_n)} \int_0^1 \left[f(\bar{Z}) - t_0\right] \exp(-\lambda_n \bar{Z} + \lambda_n Z) - \exp(-\lambda_n \bar{Z} + \lambda_n Z)\, d\bar{Z}$$ $J_0(\lambda_n) = 0,\ \lambda_n > 0,\ Z = z/r_0$
2.1.37	9, p. 223	Case 2.1.36 with $t = f(z),\ r = r_0,\ z > 0.$ Convection cooling at $z = 0$ with $h,\ t_f$.	$$t - t_f = \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n)}{J_1(\lambda_n)} \int_0^1 \left[f(\bar{Z}) - t_f\right] \left\{\exp(-\lambda_n Z + \lambda_n \bar{Z}) + \left(\frac{\lambda_n - Bi}{\lambda_n + Bi}\right) \exp(-\lambda_n \bar{Z} - \lambda_n Z)\right\} d\bar{Z}$$ $J_0(\lambda_n) = 0,\ \lambda_n > 0$

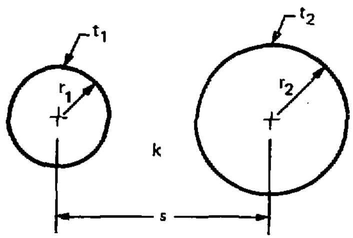
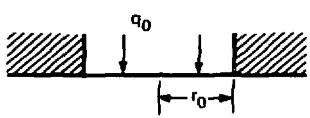
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

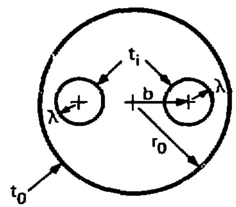
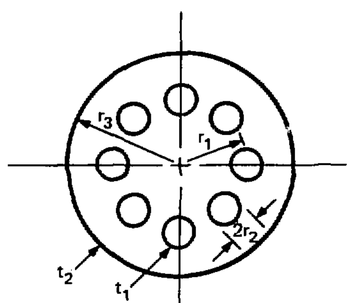
Case No.	References	Description	Solution
2.1.38	9, p. 223	Finite cylinder with heated disks on ends. $q_z = +q_1$, $z = 0, \ell$, $0 < r < r_1$. Remaining surfaces insulated.  	$\frac{(t_{m,0} - t_{m,\ell})k}{q_1 \ell} = 1 - \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{I_1(n\pi R_1)}{n^2 I_1(n\pi R_0)} \left[I_1(n\pi R_0)\ K_1(n\pi R_1) - K_1(n\pi R_0)\ I_1(n\pi R_1)\right]$, $n = 1, 3, 5, \ldots\ldots$ $R = r/\ell$ $t_{m,0}$ = mean temp. over region $0 < r < r_1$, $z = 0$ $t_{m,\ell}$ = mean temp. over region $0 < r < r_1$, $z = \ell$
2.1.39	9, p. 224	Case 2.1.32 with $t = t_1$, $z = 0$ and ℓ, $r_i < r < r_0$. $t = f(z)$, $r = r_0$, $0 < z < \ell$. $q_r = 0$, $r = r_i$, $0 < z < \ell$.  	$t - t_0 = 2 \sum_{n=1}^{\infty} \frac{F_1}{F_2} \sin(n\pi Z) \int_0^1 \left[f(Z) - t_0\right] \sin(n\pi Z)\,dZ$ $F_1 = I_1(n\pi R_i)\ K_0(n\pi R) + K_1(n\pi R_i)\ I_0(n\pi R)$ $F_2 = I_1(n\pi R_i)\ K_0(n\pi R_0) + K_1(n\pi R_i)\ I_0(n\pi R_0)$ $R = r/\ell$, $Z = z/\ell$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

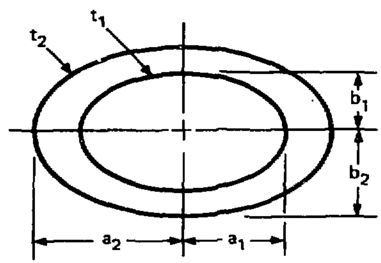
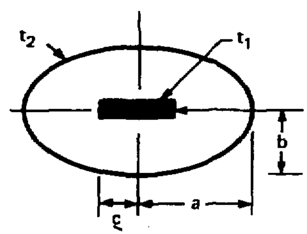
Case No.	References	Description	Solution
2.1.40	9, p. 426	Infinite cylinder in an infinite medium having a linear temp gradient. At large distances from cylinder, $t = t_0 z$. Cylinder, conductivity = k_1. Medium conductivity = k_0. 	$\frac{t}{t_0} = \left(\frac{2k_0}{k_1 + k_0}\right) Z,\ 0 < r < r_0$ $\frac{t}{t_0} = \left[1 - \frac{(k_1 - k_0)}{(k_1 + k_0)}\left(\frac{r_0}{r}\right)^2\right] Z,\ r > r_0$ $Z = z/r_0$
2.1.41	9, p. 434	Sector of a cylinder. $t = t_1,\ 0 < r < r_0,$ $\theta = 0,\ \theta_0$. $t = t_2,\ r = r_0,$ $0 < \theta < \theta_0$. 	$\frac{t - t_1}{t_2 - t_1} = \frac{2}{\pi} \tan^{-1} \left\{\frac{\sin\ (\pi\theta/\theta_0)}{\sinh\left[\frac{\pi}{\theta_0}\ \ln\left(\frac{1}{R}\right)\right]}\right\}$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

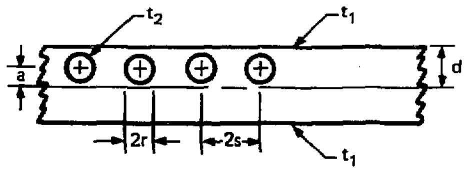
<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr><td>2.1.42</td><td>9, p. 451</td><td>Two infinite cylinders in an infinite medium.

</td><td>$$q = \frac{2\pi k(t_1 - t_2)}{\cosh^{-1}\left(\frac{S^2 - R^2 - 1}{2R}\right)}$$
$R = r_1/r_2$, $S = s/r_2$</td></tr>
<tr><td>2.1.43</td><td>9, p. 462</td><td>Spot heated semi-infinite solid.
$q_z = q_0$, $0 < r < r_0$, $z = 0$.
$q_z = 0$, $r > r_0$, $z = 0$.
$t = t_\infty$, $r > 0$, $z \to \infty$.

</td><td>$$\frac{(t - t_\infty)k}{r_0 q_0} = \int_0^\infty \exp\ (-\lambda Z)\, J_0(\lambda R)\, J_1(\lambda) \frac{d\lambda}{\lambda}$$</td></tr>
</table>

Case No.	References	Description	Solution
2.1.44	73	Infinite cylinder with two internal holes. $t = t_0,\ r = r_0$. $t = t_i$, hole surface . hole radius = λ . 	$q = Sk(t_0 - t_i)$ S = *shape factor* See Fig. 2.6
2.1.45	79	Infinite cylinder with multiple internal holes. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ell n\left(\dfrac{r_3}{r_1}\right) - \dfrac{1}{n}\,\ell n\left(\dfrac{nr_2}{r_1}\right)}\ ,\ r_2 < r_1,\ n > 1$ n = No. holes

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.46	79	Ellipsoidal pipe. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ell n \left(\dfrac{a_2 + b_2}{a_1 + b_1}\right)}$
2.1.47	79	Ellipsoid with a constant temperature slot. 	$Q = \dfrac{2\pi k(t_1 - t_2)}{\ell n \left(\dfrac{a + b}{c}\right)}$

Case No.	References	Description	Solution
2.1.48	79	Off-center tube row in an infinite plate.	$q = \dfrac{2\pi k(t_1 - t_2)}{\ln\left(\dfrac{\theta_2(a/2d)}{\theta_1'(0)}\right) + \ln\left(\dfrac{4d}{\theta}\right)}$



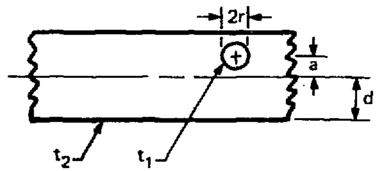
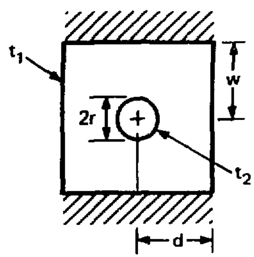
$$\theta_2(a/d) = \sum_{n=1}^{\infty} \exp\left[i\pi s\left(n + \frac{1}{2}\right)^2/2d\right] \cos\,[(2n + 1)\pi a/2d]$$

$$\theta_1'(0) = \pi\theta_2\theta_3\theta_0$$

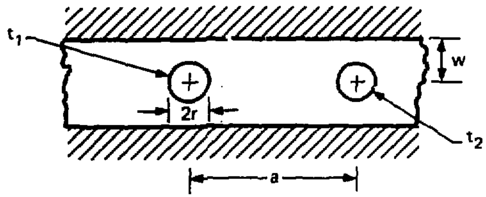
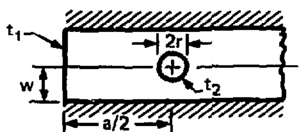
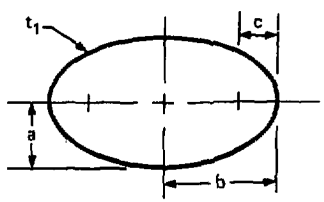
$$\theta_2(0) = \sum_{n=1}^{\infty} \exp\left[i\pi s\left(n + \frac{1}{2}\right)^2/2d\right]$$

$$\theta_3 = 1 + \sum_{n=1}^{\infty} \exp\,(i\pi s n^2/2d)$$

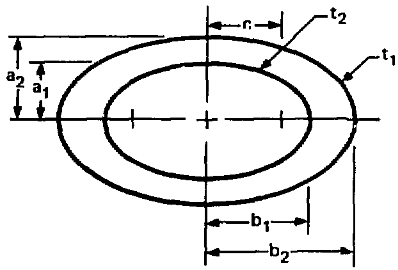
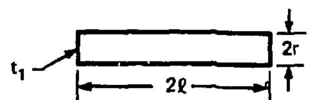
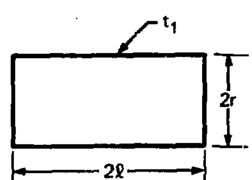
$$\theta_0 = 1 + 2\sum_{n=1}^{\infty} (-1)^n \exp\,(i\pi s n^2/2d)$$

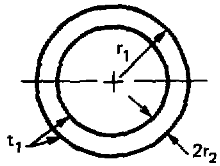
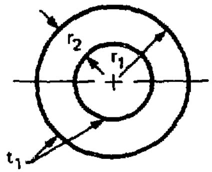
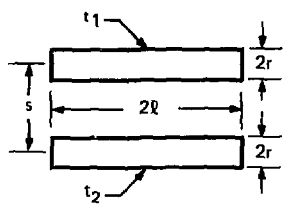
Case No.	References	Description	Solution
2.1.49	79	Single off-center tube in an infinite plate. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ln\left\{\cot\left[\dfrac{\pi(d + a)}{4d}\right]\right\} + \ln\left(\dfrac{4d}{\pi r}\right)}$
2.1.50	73	Tube centered in a finite plate. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\ln\left(\dfrac{w}{\pi r}\right) + \dfrac{\pi d}{2w}}$

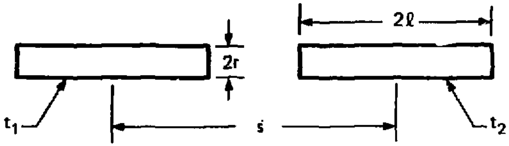
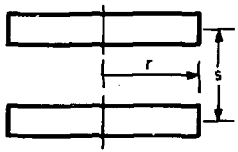
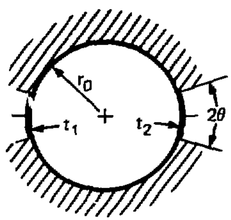
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.51	79	Heat flow between two pipes in an insulated infinite plate. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\dfrac{\pi a}{w} + \ln\left(\dfrac{w}{\pi r}\right)}$
2.1.52	79	Tube in a semi-infinite plate. 	$q = \dfrac{2\pi k(t_1 - t_2)}{\dfrac{\pi a}{2w} + \ln\left(\dfrac{w}{\pi r}\right)}$
2.1.53	79	Ellipsoid in an infinite medium. $t = t_2$; $x, y, z \to \infty$. 	$Q = \dfrac{4\pi\sqrt{1 - a^2/b^2}\; k(t_1 - t_2)}{\operatorname{arctanh}\left(\sqrt{1 - a^2/b^2}\right)}$

Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
2.1.54	79	Ellipsoidal shell.  	$Q = \dfrac{4\pi k(t_1 - t_2)}{\operatorname{arctanh}(c/b_1) - \operatorname{arctanh}(c/b_2)}$ $c = \sqrt{b_1^2 - a_1^2} = \sqrt{b_2^2 - a_2^2}$
2.1.55	79	Rod in an infinite medium. $t = t_2;\ x, y, z \to \infty$.  	$Q = \dfrac{4\pi k(t_1 - t_2)}{\ell n\,(2\ell/r)}\ ,\ \dfrac{r}{\ell} < 0.1$
2.1.56	79	Short cylinder in an infinite medium. $t = t_2;\ x, y, z \to \infty$.  	$Q = ck(t_1 - t_2)$ ℓ/r — c 0 — 8 0.25 — 10.42 0.5 — 12.11 1.0 — 14.97 2.0 — 19.87 4.0 — 27.84

Case No.	References	Description	Solution
2.1.57	79	Toroidal ring in an infinite medium. $t = t_2;\ x,y,z \to \infty$. 	$Q = \dfrac{4\pi^2 k(t_1 - t_2)}{\ell n\ (8r_1/r_2)}\ ,\ r_1/r_2 > 20$
2.1.58	79	Thin flat ring in an infinite medium. $t = t_2;\ x,y,z \to \infty$. 	$Q = \dfrac{4\pi^2 k(t_1 - t_2)}{\ell n\ (16r_1/r_2)}\ ,\ r_1/r_2 > 10$
2.1.59	79	Two parallel rods in an infinite medium. 	$Q = \dfrac{4\pi k(t_1 - t_2)}{2\left[\ell n\left(\dfrac{2\ell}{r}\right)\left(\dfrac{\sqrt{s^2 + \ell^2 - 1}}{s}\right)\right]}\ ,\quad \begin{array}{l} s > 5r \\ \ell \gg r \end{array}$ For parallel strips of width 2w, use $r = w/2$.

Case No.	References	Description	Solution
2.1.60	79	Two aligned rods in an infinite medium. 	$$Q = \frac{4\pi k(t_1 - t_2)}{2\left[\ell n \left(\frac{2\ell}{r}\right)\sqrt{\frac{s-1}{s+1}}\right]} , \quad s - 2\ell > 5r$$ For aligned strips of width 2w, use $r = w/2$.
2.1.61	79	Parallel disks in an infinite medium. 	$$Q = \frac{4\pi k(t_1 - t_2)}{2\left[\frac{\pi}{2} - \arctan (r/s)\right]} , \quad s > 5r$$
2.1.62	86	Infinite cylinder with symmetric isothermal caps. 	$$q \approx \frac{k\pi(t_1 - t_2)}{2\ \ell n\left[2(1 + \cos\theta)/\sin\theta\right]}$$

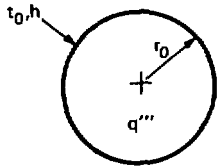
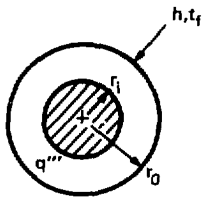
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

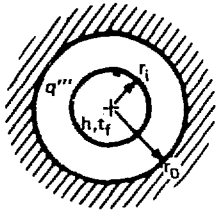
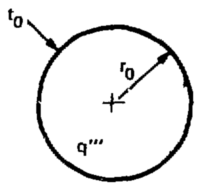
Case No.	References	Description	Solution

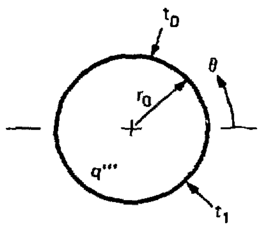
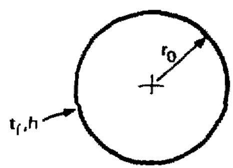
Section 2.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

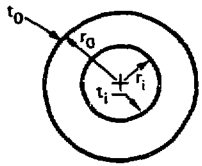
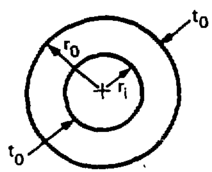
Case No.	References	Description	Solution

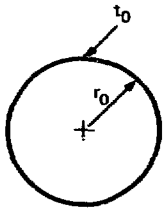
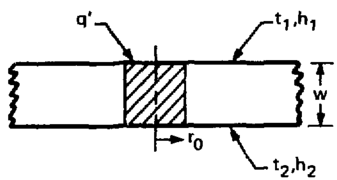
Section 2.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

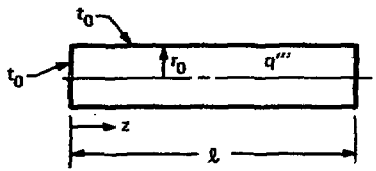
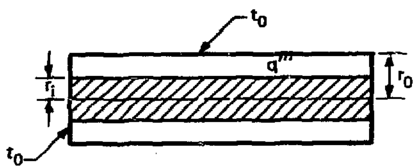
Case No.	References	Description	Solution
2.2.1	4, 51	Infinite cylinder with convection boundary. 	$$\frac{(t - t_0)k}{q'''r_0^2} = \frac{1}{4}\left[\frac{2}{Bi} + 1 - R^2\right]$$
2.2.2	4, 51	Hollow infinite cylinder with with convection boundary on outside surface. $q_r = 0,\ r = r_i$. 	$$\frac{(t - t_f)k}{q'''r_0^2} = \frac{1}{4}\left\{\frac{2}{Bi}\left[1 - R_1^2\right] + 1 - R^2 + 2R_i^2 \ln (R)\right\}$$ $R = r/r_i$, $Bi = hr_0/k$

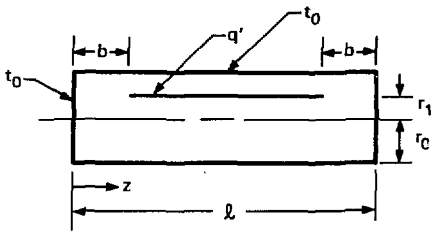
Case No.	References	Description	Solution
2.2.3	4, 51	Hollow infinite cylinder with convection cooled inside surface. $q_r = 0,\ r = r_0$.  	$\frac{(t - t_f)k}{q'''r_i^2} = \frac{1}{4}\left\{\frac{2}{Bi}\left[R_0^2 - 1\right] + 1 - R^2 + 2R_0^2 \ln (R)\right\}$ $R = r/r_i,\ Bi = hr_i/k$
2.2.4	2, p. 189	Infinite cylinder with temperature dependent heat source. $t = t_0,\ r = r_0$. $q''' = q_0'''(1 + \beta t)$.  	$\frac{\beta t + 1}{\beta t_0 + 1} = \frac{J_0(\sqrt{Po_\beta}\ R)}{J_0(\sqrt{Po_\beta})}$, $Po_\beta = \beta q_0''' r_0^2/k$ Mean temp: $\frac{\beta t_m + 1}{\beta t_0 + 1} = \frac{2}{\sqrt{Po_\beta}} \frac{J_1(\sqrt{Po_\beta})}{J_0(\sqrt{Po_\beta})}$ Max temp: $\frac{\beta t_{max} + 1}{\beta t_0 + 1} = \frac{1}{J_0(\sqrt{Po_\beta})}$ See Fig. 2.2 .

Case No.	References	Description	Solution
2.2.5	2, p. 189	Case 2.2.4 with: $t = t_0,\ r = r_0,\ 0 \le \phi \le \pi.$ $t = t_1,\ r = r_0,\ \pi \le \phi \le 2\pi.$ $q''' = q_0'''(1 + \beta t)$. 	$\frac{t}{t_0 + t_1} = \frac{1}{2}\frac{J_o(\sqrt{Po_\beta}\ R)}{J_o(\sqrt{Po_\beta})} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin n\phi}{n}\frac{J_n(\sqrt{Po_\beta}\ R)}{J_n(\sqrt{Po_\beta})}$, $n = 1, 3, 5, \ldots$ $Po_\beta = \beta q_0''' r_0^2/k$
2.2.6	2, pg 191	Case 2.2.4 with convection boundary. 	$\frac{\beta t + 1}{\beta t_f + 1} = \frac{J_0(\sqrt{Po_\beta}\ R)}{J_0(\sqrt{Po_\beta}) - \frac{\sqrt{Po_\beta}}{Bi}J_1(\sqrt{Po_\beta})}$, $Po_\beta = \beta q_0''' r_0^2/k$ Mean temp: $\frac{\beta t_m + 1}{\beta t_0 + 1} = \frac{2J_1(\sqrt{Po_\beta})}{\sqrt{Po_\beta}\left[J_0(\sqrt{Po_\beta}) - \frac{\sqrt{Po_\beta}}{Bi}J_1(\sqrt{Po_\beta})\right]}$ See Ref. 85 for nonuniform convection boundary solution.

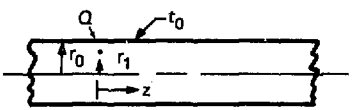
Case No.	References	Description	Solution
2.2.7	2, p. 193	Infinite hollow cylinder with temperature dependent heat source. $t = t_0,\ r = r_0$. $t = t_i,\ r = r_i$. $q''' = q_0'''(1 + \beta t)$.  	$\frac{\beta t + 1}{\beta t_i + 1} = B\left\{\left[Y_0(\sqrt{Po_\beta}) - \frac{(\beta t_0 + 1)}{(\beta t_i + 1)} Y_0(\sqrt{Po_\beta}R_i)\right] J_0(\sqrt{Po_\beta}R) + \left[\frac{(\beta t_0 + 1)}{(\beta t_i + 1)} J_0(\sqrt{Po_\beta}R_i) - J_0(\sqrt{Po_\beta})\right] Y_0(\sqrt{Po_\beta}R)\right\}$ $B = \frac{1}{J_0(R_i\sqrt{Po_\beta})\ Y_0(\sqrt{Po_\beta}) - J_0(\sqrt{Po_\beta})\ Y_0(R_i\sqrt{Po_\beta})}$ $Po_\beta = \beta q_0''' r_0^2/k,\ R = r/r_0$
2.2.8	2, p. 194	Infinite hollow cylinder with temperature dependent thermal conductivity. $t = t_0,\ r = r_i$. $t = t_0,\ r = r_0$. $k = k_0\left[1 + \beta(t = t_0)\right]$.  	$\frac{(t - t_0)k}{q''' r_0^2} = \left\{\left[\frac{1}{Po_\beta^2} + \frac{1}{2Po_\beta}\left[(1 - R^2) - \frac{\left(1 - R_i^2\right) \ln\ (1/R)}{\ln\left(\frac{1}{R_i}\right)}\right]\right]^{1/2} - \frac{1}{Po_\beta}\right\}$ $Po_\beta = q''' r_0^2 \beta/k_0,\ R = r/r_0$

<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr><td>2.2.9</td><td>6, p. 272</td><td>Electrically heated wire with temperature dependent thermal and electrical conductivities.
$t = t_0,\ r = r_0$.
$k_t / k_{t0} = 1 + \beta_t (t - t_0)$.
$k_e / k_{e0} = 1 + \beta_e (t - t_0)$.
</td><td>$$\frac{t - t_0}{B} = R\left\{1 + B\left[\frac{R}{8} - \frac{\beta_e}{\beta_t}\,\frac{(2 + R)}{16}\right]\right\}$$
$$B = \frac{k_{e0} r_0^2 E^2 \beta_t}{k_{t0} L^2}\ ,\quad R = 1 - \left(r^2/r_0^2\right)$$
k_t = thermal conductivity
k_e = electrical conductivity
E = voltage drop over length L</td></tr>
<tr><td>2.2.10</td><td>2, p. 175</td><td>Cylindrical heat source in an infinite plate.
$t = t_\infty,\ r \rightarrow \infty$.
</td><td>$$\frac{t - t_\infty}{q'/kW} = \frac{K_0[RB]}{2\pi\ Bk_1[B]}$$
See Fig. 2.3
$$B = r_0\sqrt{(h_1 + h_2)/kw}$$
$$t_\infty = \frac{h_1 t_1 + h_2 t_2}{h_1 + h_2}$$
q' = heating rate in cylindrical source</td></tr>
</table>

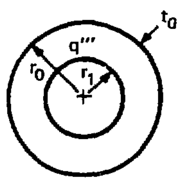
Case No.	References	Description	Solution
2.2.11	9, p. 224	Finite cylinder with steady surface temperature. $t = t_0,\ z = 0$ and $\ell,\ 0 < r < r_0$. $t = t_0,\ r = r_0,\ 0 < z < \ell$. 	$\dfrac{(t - t_0)k}{q'''\ell^2} = \dfrac{Z - Z^2}{2} - \dfrac{4}{\pi^3}\sum_{n=1}^{\infty}\dfrac{I_0(n\pi R)}{n^3 I_0(n\pi R_0)}\sin(n\pi Z)$, $n = 1, 3, 5, \ldots$ $Z = z/\ell,\ R = r/\ell$
2.2.12	9, p. 224	Finite hollow cylinder. $t = t_0,\ r = r_0,\ 0 < z < \ell$. $t = t_0,\ z = 0$ and $\ell,\ r_i < r < r_0$. $q_r = 0,\ r = r_i,\ 0 < z < \ell$. 	$\dfrac{(t - t_0)k}{q'''\ell^2} = \dfrac{Z - Z^2}{2} - \dfrac{4}{\pi^3}\sum_{n=1}^{\infty}\dfrac{\sin(n\pi Z)}{n^3}\cdot\dfrac{F_1}{F_2}$, $n = 1, 3, 5, \ldots$ $F_1 = I_1(n\pi R_i)K_0(n\pi R) + I_0(n\pi R)K_1(n\pi R_i)$ $F_2 = I_1(n\pi R_i)K_0(n\pi R_0) + I_0(n\pi R_0)K_1(n\pi R_i)$ $Z = z/\ell,\ R = r/\ell$

Case No.	References	Description	Solution
2.2.13	9, p. 224	Case 2.2.11 with $q''' = q_0'''[1 + \beta(t - t_0)]$.	$\frac{(t - t_0)k}{q_0'''\ell^2} = \frac{\cos[(\sqrt{Po}/2 - \sqrt{Po}Z)]}{Po \cos(\sqrt{Po}/2)} - \frac{1}{Po}$ $- \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{I_0(\lambda_n R)\sin(n\pi Z)}{I_0(\lambda_n R_0)n\lambda_n^2}$, $n = 1, 3, 5, \ldots.$ $\lambda_n^2 = (2n + 1)^2\pi^2 - Po$, $Po = q_0'''\beta\ell^2/k$, $Z = z/\ell$, $R = r/\ell$
2.2.14	9, p. 423	Finite cylinder with line heat source. $t = t_0$, $r = r_0$, $0 < z < \ell$. $t = t_0$, $0 < r < r_0$, $z = 0, \ell$. Line source of strength q' per unit length is located at r_1, θ_1, $b < z < \ell - b$. 	$\frac{(t - t_0)k}{q'} = \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{\overline{m}}\cos(\overline{m}\pi B)\sin(\overline{m}\pi Z)$ $\times \sum_{n=0}^{\infty}\frac{I_n(\overline{m}\pi R)}{I_0(\overline{m}\pi R_0)}\left[I_n(\overline{m}\pi R_0)K_n(\overline{m}\pi R_1) - K_n(\overline{m}\pi R_0)I_n(\overline{m}\pi R_1)\right]$ $\times E_n \cos[n(\theta - \theta_1)]$, $0 < R < R_1$ $E_n = 1$ if $n = 0$, $E_n = 2$ if $n > 0$, $\overline{m} = 2m - 1$ $B = b/\ell$, $R = r/\ell$, $Z = z/\ell$ For $R_1 < R < R_0$ interchange R for R_1.

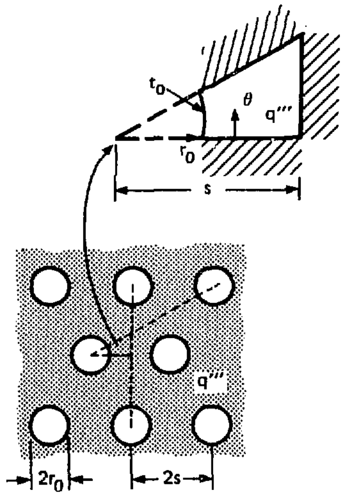
Section 2.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
2.2.15	9, p. 423	Case 2.2.14 with a point source of strength Q located at r_1, θ_1, z_1 .	$\frac{(t - t_0)k\ell}{Q} = \frac{1}{\pi}\sum_{m=1}^{\infty} \sin(m\pi Z)\sin(m\pi Z_1)$ $\times \sum_{n=0}^{\infty} E_n \frac{I_n(m\pi R)}{I_n(m\pi R_0)}\left[I_n(m\pi R)K_n(m\pi R_1) - K_n(m\pi R)I_n(m\pi R_1)\right]$ $\times \cos\left[n(\theta - \theta_1)\right], \; 0 < R < R_1$ For $R_1 < R < R_0$, interchange R for R_1. $R = r/\ell$, $Z = z/\ell$, $E_n = 1$ if $n = 0$, $E_n = 2$ if $n > 0$
2.2.16	9, p. 423	Infinite cylinder with point source of strength Q. $t = t_0$, $r = r_0$. Source located at r_1, θ_1, $z = 0$. 	$\frac{(t - t_0)kr_0}{Q} = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos\left[n(\theta - \theta_1)\right]$ $\times \sum_{\lambda>0}^{\infty} \frac{\exp(-\lambda\lvert Z\rvert)J_n(\lambda_n R)J_n(\lambda_n R_1)}{\lambda\left[\frac{n}{\lambda}J_n(\lambda) - J_{n+1}(\lambda)\right]^2}$ $J_n(\lambda_n) = 0$, $Z = z/r_0$, $R = r/r_0$

Case No.	References	Description	Solution
2.2.17	16	Hollow cylinder with temperature dependent heating and thermal conductivity. $t = t_0,\ r = r_0.$ $q_r = 0,\ r = r_0.$ $k = k_0,\ r = r_0.$ $q''' = q_0''',\ r = 0.$ $k = k_0 + a(t_0 - t).$ $q''' = q_0''' + b(t_0 - t).$	Solution by numerical method . See Fig. 2.4 for values of t at $r = r_i$. $\rho_i = 1 - (r_i/r_0),\ K = k_0/at_0,\ R = at_0/br_0^2,\ G = q_0'''/bt_0$ For a = b = 0: $\frac{(t_0 - t_i)k_0}{q_0'''r_0^2} = \frac{1}{4}(1 - \rho_i)^2 + \frac{1}{2}\,\ell n\left(\frac{1}{1 - \rho_i}\right) - \frac{1}{4}$ (Eq. (5) in Fig. 2.4)

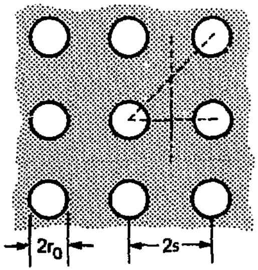


Case No.	References	Description	Solution
2.2.18	64	Triangular arrayed cylindrical cooling surfaces in an infinite solid. $t = t_0,\ r = r_0$.	$\dfrac{(t - t_0)k}{q'''s^2} = \dfrac{\sqrt{3}}{\pi}\,\ell n\,(R) - \dfrac{1}{4}\left[\left(\dfrac{R}{S}\right)^2 - \left(\dfrac{1}{S}\right)^2\right] + \sum_{j=1}^{\infty} \dfrac{\Delta_j}{6j}\left(\dfrac{R}{S}\right)^{6j}\left[1 - \left(\dfrac{1}{R}\right)^{12j}\right]\cos\,(6j\theta)$ See Table 2.1 for values of Δ_j . See Fig. 2.5 for t_{max} values . $S = s/r_0,\ R = r/r_0$



Equilateral triangular array

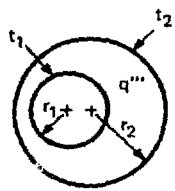
Case No.	References	Description	Solution
2.2.19	64	Case 2.2.18 with a square array of holes.	$\frac{(t - t_0)k}{q'''s^2} = \frac{2}{\pi} \ln (R) - \frac{1}{4}\left[\left(\frac{R}{S}\right)^2 - \left(\frac{1}{S}\right)^2\right] + \sum_{j=1}^{\infty} \frac{\delta_j}{4j} \left(\frac{R}{S}\right)^{4j}\left[1 - \left(\frac{1}{R}\right)^{8j}\right] \cos (4j\theta)$ See Table 2.2 for values of δ_j. See Fig. 2.5 for t_{max} values.



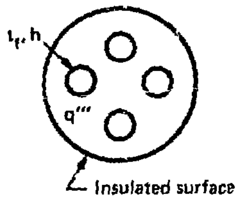
Square array

Section 2.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
2.2.20	67	Eccentric, hollow, infinite cylinder.	See Ref. 67 for equational solution.



2.2.21	73	Cylinder cooled by ring of internal holes.	See Ref. 73 for max temperatures.



Case No.	References	Description	Solution

Section 2.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

3. Spherical

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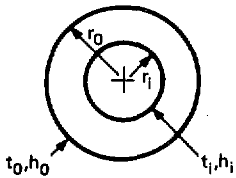
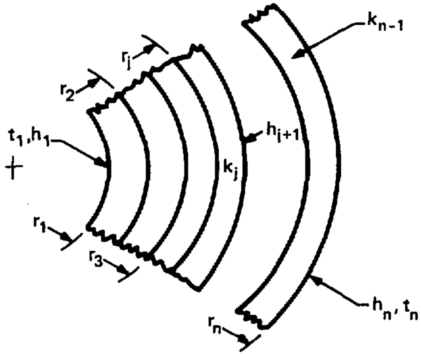
Case No.	References	Description	Solution
3.1.1.	4, p. 37	Spherical shell.	



$$Q = \frac{4\pi r_0 k(t_i - t_0)}{(r_0/r_i) - 1 + (r_0/r_i)(k/h_i r_i) + k/h_0 r_0}$$

$$\frac{t - t_0}{t_i - t_0} = \frac{(r_0/r) - 1}{(r_0/r_i) - 1 + (r_0/r_i)(k/r_i h_i) + k/r_0 h_0}$$

Case No.	References	Description	Solution
3.1.2	92	Composite sphere.	

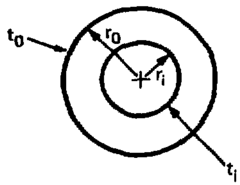
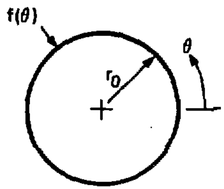


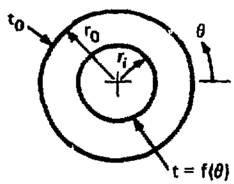
$$Q = \frac{4\pi(t_1 - t_n)}{\sum_{i=1}^{n-1} \frac{1}{k_i}\left(\frac{1}{r_i} - \frac{1}{r_{i+1}}\right) + \sum_{i=1}^{n} \frac{1}{r_i^2 h_i}}$$

$$\frac{t_j - t_1}{t_n - t_1} = \frac{\sum_{i=1}^{j-1}\left[\frac{1}{k_i}\left(\frac{1}{r_i} - \frac{1}{r_{i+1}}\right) + \frac{1}{r_i^2 h_i}\right] + \frac{1}{k_j}\left(\frac{1}{r_j} - \frac{1}{r}\right)}{\sum_{i=1}^{n-1} \frac{1}{k_i}\left(\frac{1}{r_i} - \frac{1}{r_{i+1}}\right) + \sum_{i=1}^{n} \frac{1}{h_i r_i^2}}, \quad j > 1$$

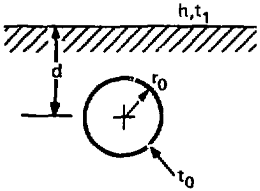
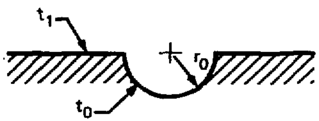
t_j = local temperature in j^{th} layer

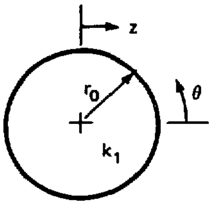
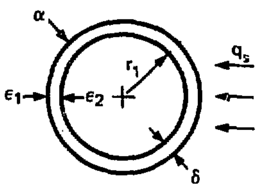
Section 3.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

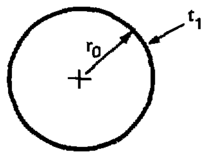
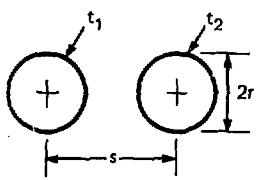
Case No.	References	Description	Solution
3.1.3	1, p. 139	Sphere with temperature-dependent thermal conductivity. $t = t_i,\ r = r_i$. $t = t_0,\ r = r_0$. $k = k_0 + \beta(t - t_0)$.  	$Q = \dfrac{4\pi r_0 k_m (t_i - t_0)}{(r_o/r_i) - 1}$ $k_m = (k_0 + k_i)/2$ $k = k_0,\ t = t_0$ $k = k_i,\ t = t_i$ $\dfrac{t - t_0}{k_0/\beta} = \sqrt{1 + 2\beta(t_i - t_0)\dfrac{k_m}{k_0^2}\,\dfrac{(r_0/r - 1)}{(r_0/r_i - 1)}} - 1$
3.1.4	2, p. 137	Sphere with variable surface temperature. $t = f(\theta),\ r = r_0$.  	$t = \sum_{n=0}^{\infty} a_n r^n P_n(\cos\theta)$ $a_n = \dfrac{2n + 1}{2r_0} \displaystyle\int_0^{\pi} f(\theta)\ P_n(\cos\theta)\ \sin\theta\ d\theta$

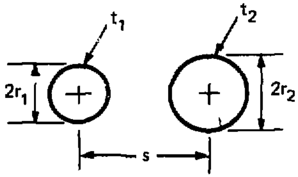
Case No.	References	Description	Solution
3.1.5	2, p. 137	Case 3.1.4 with: $t = t_0,\ 0 < \theta < \pi/2$. $t = 0,\ \pi/2 < \theta < \pi$.	$\frac{t}{t_0} = \frac{1}{2} + \frac{3}{4} R\, P_1(\cos\theta) - \frac{7}{16} R^3\, P_3(\cos\theta)$ $+ \frac{11}{32} R^5\, P_5(\cos\theta)\ .\ .\ .$
3.1.6	2, p. 137	Spherical shell with specified surface temperatures. $t = t_0,\ r = r_0$. $t = f(\theta),\ r = r_i$. 	$t - t_0 = \sum_{n=0}^{\infty} \left(n + \frac{1}{2}\right) \frac{R_0^{2n+1} - R^{2n+1}}{\left(R_0^{2n+1} - 1\right)} \left(\frac{1}{R}\right)^{n+1} P_n(\cos\theta)$ $\times \int_0^{\pi} P_n(\cos\theta)[f(\cos\theta) - 1] \sin\theta\, d\theta$ $R = r/r_i$

Section 3.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
3.1.7	4, p. 41	Sphere in a semi-infinite solid. 	$Q = \dfrac{4\pi r_0 k(t_1 - t_0)}{1 + \dfrac{1}{2\left[(d/r_0) + (1/Bi)\right]}}$ $Bi = hr_0/k$
3.1.8	4, p. 43	Hemisphere in the surface of a semi-infinite solid. 	$Q = 2\pi r_0 k(t_0 - t_1)$

Case No.	References	Description	Solution
3.1.9	9, p. 423	Sphere in an infinite medium having linear temp gradient. For large distances from sphere $t = t_0 z$. Sphere conductivity is k_1 and medium conductivity is k_0. 	$\frac{t}{t_0} = \frac{3k_0}{2k_0 + k_1} Z,\ 0 < R < 1,\ Z = z/r_0$ $\frac{t}{t_0} = \left[R + \left(\frac{1}{R}\right)^2 \frac{(k_0 - k_1)}{(2k_0 + k_1)}\right] \cos\theta,\ R > 1$
3.1.10	59 19, p. 3-111	Irradiated spherical thin shell. q_s = source heat flux . T_s = sink temp . α = absorptivity . 	See Fig. 3.1 for T/T_∞. $T_\infty = \left[\frac{q_s \alpha}{\sigma \epsilon_1}\left(\frac{\epsilon_1/\epsilon_2 + 1/4}{\epsilon_1/\epsilon_2 + 1}\right)\right]^{1/4}$

Case No.	References	Description	Solution
3.1.11	79	Sphere in an infinite medium. $t = t_2$, $r \to \infty$. 	$Q = 4\pi\, r_0 k(t_1 - t_2)$
3.1.12	79	Two spheres separated a large distance in an infinite medium. 	$Q = \dfrac{4\pi rk(t_1 - t_2)}{2(1 - r/s)}$, $s > 5r$ for $s > 2r$, error $\simeq$ 1%

Case No.	References	Description	Solution
3.1.13	79	Two spheres separated a small distance (see case 3.1.12) in an infinite medium.	$Q = 2\pi rk(t_1 - t_2)\left[1 + r/s + (r/s)^2 + (r/s)^3 + 2(r/s)^4 + 3(r/s)^5 + - - -\right], \quad 2r < s < 5s$
3.1.14	79	Two spheres of different radii in an infinite medium. 	$Q = \dfrac{4\pi r_2 k(t_1 - t_2)}{\dfrac{r_2}{r_1} + \left[1 - \dfrac{(r_2/s)^4}{1 - (r_2/s)^2}\right]} - 2r_2/s \quad , \quad s > 5r_2$

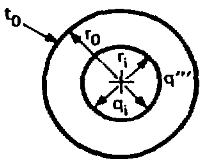
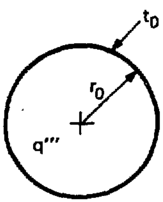
Section 3.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

Case No.	References	Description	Solution

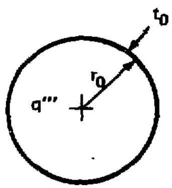
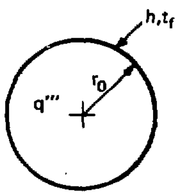
Section 3.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

Case No.	References	Description	Solution

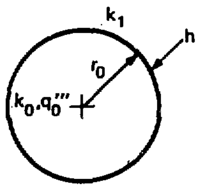
Section 3.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
3.2.1		Spherical shell with specified inside surface heat flux. $t = t_0,\ r = r_0$. $q_r = q_i,\ r = r_i$.  	$$\frac{(t - t_0)k}{q_i r_i} = \frac{q''' r_i}{6 q_i}\left[\frac{2(R - R_0)}{RR_0} + R_0^2 - R^2\right] + \frac{R_0 - R}{RR_0}$$ $R = r/r_i$
3.2.2	1, p. 190	Sphere with temperature dependent heating. $t = t_0,\ r = r_0$. $q''' = q_0''' + \beta(t - t_0)$.  	$$\frac{(t - t_0)\beta}{q_0'''} = \frac{1}{R}\,\frac{\sin(R\sqrt{Po})}{\sin(\sqrt{Po})} - 1\ ,\quad Po = \beta r_0^2/k$$ $$\frac{t - t_0}{t_{max} - t_0} = 1 - R^2,\ Po = 0$$ $$\frac{t - t_0}{t_{max} - t_0} = \frac{\sin(\pi R)}{\pi R}\ ,\quad \sqrt{Po} = \pi$$ $$\frac{(t_{max} - t_0)\beta}{q_0'''} = \frac{\sqrt{Po}}{\sin(\sqrt{Po})} - 1$$

Section 3.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
3.2.3	6, p. 274	Sphere with nuclear type heating. $t = t_0$, $r = r_0$. $q''' = q_0'''\left[1 - \beta(r/r_0)^2\right]$. 	$\dfrac{(t - t_0)k}{q_0'''r_0^2} = \dfrac{1}{6}\left[(1 - R^2) - \dfrac{3\beta}{10}(1 - R^4)\right]$
3.2.4	9, p. 232	Solid sphere with convection boundary. 	$\dfrac{(t - t_f)k}{q'''r_0^2} = \dfrac{1}{6}\left[1 - R^2 + (2/Bi)\right]$ See Ref. 85 for nonuniform convection boundary solution.

Case No.	References	Description	Solution
3.2.5	9, p. 232	Solid sphere in an infinite medium. $k = k_0$, $0 < r < r_0$. $k = k_1$, $r > r_0$. $q''' = q_0'''$, $0 < r < r_0$. $q''' = 0$, $r > r_0$. h = contact coefficient at $r = r_0$. $t = t_\infty$, $r > \infty$.	$\dfrac{(t - t_\infty)k_0}{q_0'''r_0^2} = \dfrac{1}{6}(1 - R^2 + 2/Bi + 2k_0/k_1)$, $0 < R < 1$ $\dfrac{(t - t_\infty)k}{q_0'''r_0^2} = 1/3R$, $r > r_0$ $Bi = hr_0/k_0$



Section 3.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

Section 3.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

Traveling Heat Sources

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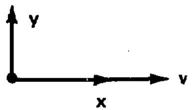
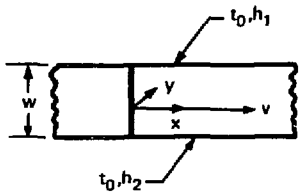
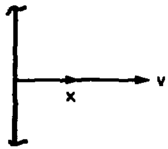
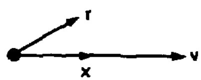
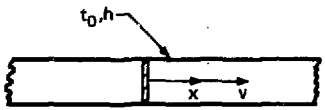
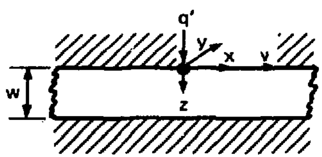
Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution
4.1.1	2, p. 189 9, p. 267	Traveling line heat source in an infinite solid. $t = t_0,\ x \rightarrow \infty,\ y \rightarrow \pm\infty$. 	$\frac{(t - t_0)k}{q'} = \frac{1}{2\pi} \exp\left(-\frac{vx}{2\alpha}\right) K_0\left(\frac{v}{2\alpha}\sqrt{x^2 + y^2}\right)$ v = velocity of line source q' = heating rate in line per unit length
4.1.2	2, p. 291	Traveling line heat source in an infinite plate. $t = t_0,\ x \rightarrow \infty,\ y \rightarrow \pm\infty$. 	$\frac{(t - t_0)w}{q'/k} = \frac{1}{2\pi} \exp\left(-\frac{vx}{2\alpha}\right) K_0\left[\sqrt{\frac{h_1 + h_2}{kw} + \left(\frac{v}{2\alpha}\right)^2}\sqrt{x^2 + y^2}\right]$ v = velocity of line source q' = heating rate of line

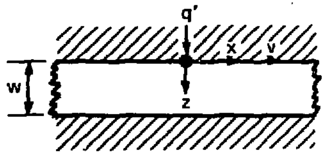
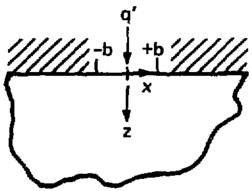
Section 4.1. Traveling Heat Sources.

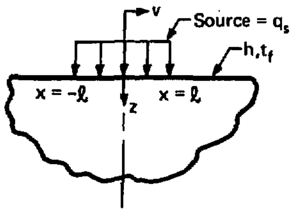
Case No.	References	Description	Solution
4.1.3	2, p. 286	Traveling plane heat source in an infinite medium. $t = t_0,\ x \to \infty.$  	$\dfrac{(t - t_0)kv}{q''\alpha} = \exp\left(-\dfrac{vx}{\alpha}\right),\ x > 0$ $\dfrac{(t - t_0)kv}{q''\alpha} = 1,\ x \ll 0$ q'' = heating rate of the plane
4.1.4	1, p. 352 9, p. 266	Traveling point heat source in an infinite solid. $t = t_0,\ r \to \infty.$  	$\dfrac{t - t_0}{q'/kr} = \dfrac{1}{4\pi} \exp\left[-\dfrac{u(x + y)}{2\alpha}\right]$ q' = heating rate of the point

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution
4.1.5	9, p. 268	Traveling plane source in an infinite rod. $t = t_0,\ x \to \pm\infty$.  	$$\frac{(t - t_0)kC}{q'} = \frac{\alpha C}{AU} \exp\left(\frac{vx - \lvert x \rvert U}{2\alpha}\right)$$ $U = (v^2 + 4\alpha^2\ hC/kA)^{1/2}$ v = velocity of source q' = heating rate of source C = circumference of rod A = cross-sectional area
4.1.6	9, p. 268	Traveling point source on the surface of an infinite plate and no surface loss. $t = t_0,\ r \to \infty$.  	$$\frac{(t - t_0)kw}{q'} = \frac{1}{2\pi} K_0\left(\frac{vr}{2\alpha}\right) + \frac{1}{\pi} \sum_{n=1}^{\infty} K_0\left\{\frac{vr}{2\alpha}\left[1 + \left(\frac{2\alpha n\pi}{vw}\right)^2\right]^{1/2}\right\}$$ $$\times \cos\left(\frac{n\pi Z}{w}\right) \exp(vx/2\alpha) \quad .$$ $r^2 = x^2 + y^2$ v = source velocity in x-direction q' = heating rate of point source

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution
4.1.7	9, p. 268	Traveling line source on the surface of an infinite plate with no surface losses. $t = t_0,\ x \to \pm\infty.$  	$$\frac{(t - t_0)vwk}{q'\alpha} = 1 + \sum_{n=1}^{\infty} \frac{2}{N} \cos(n\pi z/w)(1 - N) \exp(vx/2\alpha)$$ $N = \left[1 + (2\alpha n\pi/vw)^2\right]^{1/2}$ v = source velocity in the x-direction q' = heating rate of source
4.1.8	9, p. 269	Traveling infinite strip source on the surface of a semi-infinite solid with no surface losses. $t = t_0,\ x \to \pm\infty.$  	$$\frac{(t - t_0)kv}{q'\alpha} = \frac{2}{\pi} \int_{X-B}^{X+B} \exp(\lambda) K_0\left[(Z^2 + \lambda^2)^{1/2}\right] d\lambda$$ $X = vx/2\alpha,\ Z = vz/2\alpha,\ B = bv/2\alpha$ See Fig. 4.1 v = strip velocity in the x-direction q' = heating rate of strip

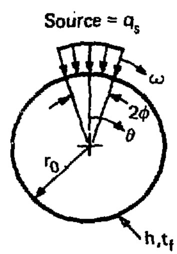
Case No.	References	Description	Solution
4.1.9	30	Traveling infinite strip source on a semi-infinite solid with convection boundary. $t = t_f$, $x \to \pm\infty$, and z.  	$\frac{(t(x,z) - t_f)k\pi v}{2\alpha q_s} = \int_{X-L}^{X+L} K_0 e^{-m}\sqrt{Z^2 + m^2}\,dm$ $- \pi H \exp(HZ) \int_0^\infty \tau \exp(H^2\tau^2)\,\mathrm{erfc}(Z/2\tau + H\tau)$ $\times \{\mathrm{erf}[(X + L)/2\tau + \tau] - \mathrm{erf}[(X - L)/2\tau + \tau]\}d\tau$ v = velocity of strip $H = 2\alpha h/kv$, $L = v\ell/2\alpha$, $X = vx/2\alpha$, $Z = vz/2\alpha$ See Fig. 4.2.

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution
4.1.10	30	Traveling band source on an infinite cylinder with convection cooling.	See below

$$\frac{(t - t_f)\pi k}{r_0 q_s} \simeq \frac{\phi}{Bi} + \frac{2}{\sqrt{R}} \sum_{n=1}^{\infty} \sin (n\phi) \exp [-W(1 - R)/\sqrt{2}]$$

$$\times \left(\frac{W \cos \{W^2 Fo - n\phi - [W(1 - R)/\sqrt{2}] - \pi/4\} + Bi \cos [W^2 Fo - n\phi - W(1 - R)/\sqrt{2}]}{n(W^2 + Bi^2 + \sqrt{2}\, W\, Bi)}\right)$$

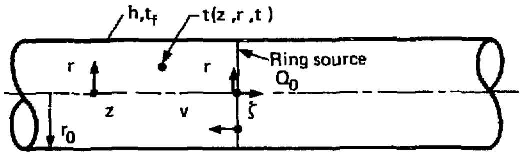


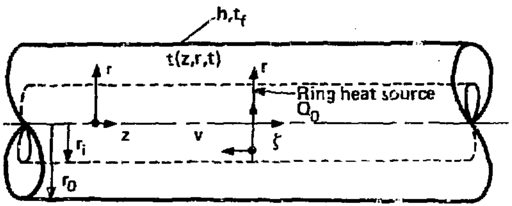
$W = r_0\sqrt{n\omega/\alpha}$

ω = rotation speed of source

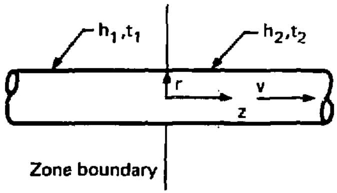
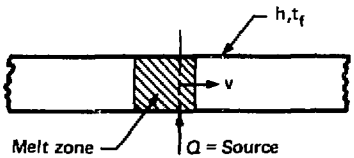
2ϕ = angular width of source

See Fig. 4.3 a and b.

Case No.	References	Description	Solution
4.1.11	34	Infinite cylinder with moving ring heat source on its surface. 	$$\frac{(t - t_f)2\pi r_0 k}{Q_0} = \sum_{n=1}^{\infty} \frac{\lambda_n^2}{(Nu^2/4) + \lambda_n^2} \frac{J_0(\lambda_n R)}{J_0(\lambda_n)}$$ $$\times \frac{\exp\left(\frac{Pe}{4} \pm \sqrt{(Pe^2/16) + \lambda_n^2}\right)\zeta}{\sqrt{(Pe^2/16) + \lambda_n^2}}, \quad \begin{array}{l} + \text{ for } \zeta < 0 \\ - \text{ for } \zeta > 0 \end{array}$$ $Nu = 2hr_0/k$, $Pe = 2vr_0/\alpha$, $\zeta = \frac{Z + v\tau}{r_0}$, $R = r/r_0$ $Nu\, J_0(\lambda_n) = 2\lambda_n J_1(\lambda_n)$ See Figs. 4.4 a and b.

Case No.	References	Description	Solution
4.1.12	34	Infinite cylinder with a moving ring heat source on its inner surface.  	$\frac{(t - t_f)2\pi r_i k}{Q_0} = \sum_{n=1}^{\infty} A_n\left[J_0(\lambda_n R) - \frac{J_1(\lambda_n)}{Y_1(\lambda_n)} Y_0(\lambda_n R)\right].$ $\lambda_n\left[J_1(\lambda_n R_0)Y_1(\lambda_n) - J_1(\lambda_n)Y_1(\lambda_n R_0)\right]$ $= \frac{1}{2} Nu\left[J_0(\lambda_n R_0)Y_1(\lambda_n) - J_1(\lambda_n)Y_0(\lambda_n R_0)\right].$ $A_n = \frac{C_n/2}{\sqrt{(Pe^2/16) + \lambda_n^2}} \exp\left[(Pe/4) \pm \sqrt{(Pe^2/16) + \lambda_n^2}\right],$ + for $\zeta < 0$ − for $\zeta > 0$ $C_n = \frac{\left[J_0(\lambda_n)Y_1(\lambda_n) - J_1(\lambda_n)Y_0(\lambda_n)\right] Y_1(\lambda_n)}{B_n}$ $B_n = \frac{R_0^2}{2}\left\{\left[J_0(\lambda_n R_0)Y_1(\lambda_n) - J_1(\lambda_n)Y_0(\lambda_n R_0)\right]^2\right.$ $\left. + \left[J_1(\lambda_n)Y_1(\lambda_n R_0) - Y_1(\lambda_n)J_1(\lambda_n R_0)\right]^2 \lambda_n^2\right\}$ $- \frac{1}{2}\left[J_0(\lambda_n)Y_1(\lambda_n) - J_1(\lambda_n)Y_0(\lambda_n)\right]^2$ $R = r/r_i$, $Nu = 2hr_i/k$, $Pe = 2vr_i/\alpha$, $\zeta = \frac{z + v\tau}{r_i}$ See Figs. 4.5 a and b.

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution
4.1.13	70	Infinite cylinder traveling through temperature zones. 	Temperature solution given in source Ref. 70.
4.1.14	71	Traveling plane source in a thin rod with change of phase. 	Temperature solution given in source Ref. 71.
4.1.15	71	Traveling plane source in an infinite medium with change of phase.	Temperature solution given in source Ref. 71.

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution

Section 4.1. Traveling Heat Sources.

Case No.	References	Description	Solution

5. Extended Surface — Steady State

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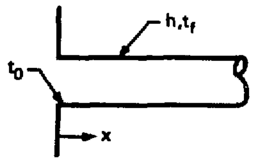
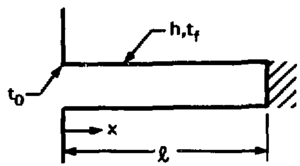
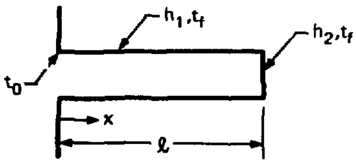
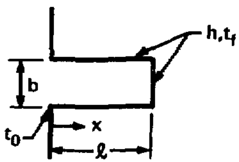
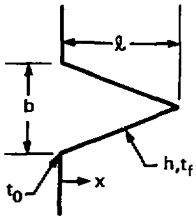
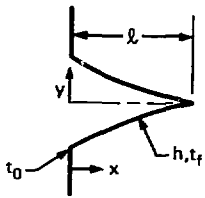
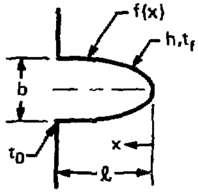
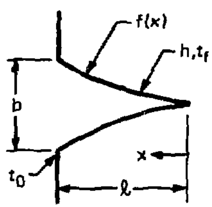
Case No.	References	Description	Solution
5.1.1	1, p. 209	Infinite rod. $t = t_0,\ x = 0.$ $t = t_f,\ x \to \infty.$ 	$\dfrac{t - t_f}{t_0 - t_f} = \exp\ (-mx)$ $m = \sqrt{\dfrac{hC}{kA}}$
5.1.2	7, p. 43	Finite rod, insulated end. $t = t_0,\ x = 0.$ $q_x = 0,\ x = \ell.$ 	Total heat loss: $Q = \sqrt{hCkA}\ \tan\ (m\ell)\,(t_0 - t_f)$ $\dfrac{t - t_f}{t_0 - t_f} = \dfrac{\cosh\ [m(\ell - x)]}{\cosh\ (m\ell)}$ See Fig. 5.1.

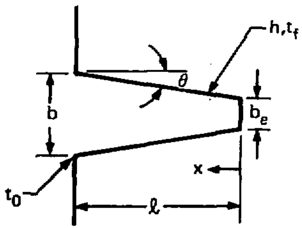
Section 5.1. Extended Surfaces--No Internal Heating.

Case No.	References	Description	Solution
5.1.3	7, p. 43	Finite rod with end loss. $t = t_0,\ x = 0.$ 	$Q = \dfrac{mkA\left[(h_2/mk) + \tan\ (m\ell)\right](t_0 - t_f)}{1 + (h_2/mk)\ \tan\ (m\ell)}$ $\dfrac{t - t_f}{t_0 - t_f} = \dfrac{\cosh\ [m(\ell - x)] + (h_2/mk)\ \sinh\ [m(\ell - x)]}{\cosh\ (m\ell) + (h_2/mk)\ \sinh\ (m\ell)}$ $m = \sqrt{\dfrac{h_1 C}{kA}}$
5.1.4	7, p. 44	Straight rectangular fin. 	Use $m = \sqrt{\dfrac{2h}{kb}}$ in case 5.1.3. $\phi = \dfrac{\tanh\ (m\ell_c)}{m\ell_c}$, $\ell_c = \ell + \dfrac{b}{2}$, See Fig. 5.2. Recommended design criteria: $2k/hb > 5$. Minimum weight criteria: $2\ell/b = 1.419\sqrt{2k/hb}$.

Case No.	References	Description	Solution
5.1.5	7, p. 52	Straight triangular fin. 	$q = \sqrt{2hkb}\,\dfrac{I_1(2\sqrt{\beta\ell})}{I_0(2\sqrt{\beta\ell})}$, $\beta = \dfrac{2h\ell}{kb}$ $\dfrac{t - t_f}{t_0 - t_f} = \dfrac{I_0(2\sqrt{\beta x})}{I_0(2\sqrt{\beta\ell})}$, $\phi = \dfrac{1}{\sqrt{\beta\ell}}\,\dfrac{I_1(2\sqrt{\beta\ell})}{I_0(2\sqrt{\beta\ell})}$, See Fig. 5.2. Optimum ℓ/b ratio: $\ell/b = 0.655\sqrt{\dfrac{2k}{hb}}$.
5.1.6	1, p. 228	Straight fin of minimum mass. 	$\dfrac{t - t_f}{t_0 - t_f} = 1 - \dfrac{h(t_0 - t_f)x}{q'}$, q' = total heat loss. Profile: $y = \dfrac{h}{2k}\left(x - \dfrac{q'}{h(t_0 - t_f)}\right)^2$ $\ell = \dfrac{q'}{h(t_0 - t_f)}$

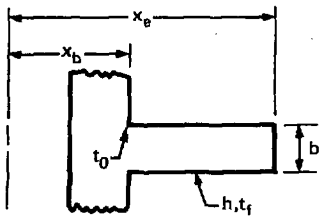
Section 5.1. Extended Surfaces--No Internal Heating.

Case No.	References	Description	Solution
5.1.7	10, p. 95	Straight fin of convex parabolic profile. $f(x) = \frac{b}{2}\sqrt{\frac{x}{\ell}}$ 	$\frac{t - t_f}{t_0 - t_f}\ f = \left(\frac{x}{\rho}\right)^{1/4} \frac{I_{-1/3}\left(\frac{4}{3}\ m\ell^{1/4} x^{3/4}\right)}{I_{-1/3}\left(\frac{4}{3}\ m\ell\right)}$ $m = \sqrt{2h/kb}$ $q = kbm(t_0 - t_f)\ \frac{I_{2/3}\left(\frac{4}{3}\ m\ell\right)}{I_{-1/3}\left(\frac{4}{3}\ m\ell\right)}$ $\phi = \frac{1}{m\ell}\ \frac{I_{2/3}\left(\frac{4}{3}\ m\ell\right)}{I_{-1/3}\left(\frac{4}{3}\ m\ell\right)}$, See Fig. 5.2
5.1.8	10, p. 175	Straight fin of trapezoidal parabolic profile. $f(x) = \frac{b}{2}\left(\frac{x}{\ell}\right)^2$ 	$\frac{t - t_f}{t_0 - t_f} = \left(\frac{x}{\ell}\right)^{\beta}$ $\beta = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\ m^2\ell^2}$ $q = \frac{kb\beta}{\ell}\ (t_0 - t_f)$ $\phi = \frac{1}{\beta + 1}$, See Fig. 5.2

Case No.	References	Description	Solution
5.1.9	10, p. 175	Straight fin of trapezoidal profile.  	$\frac{t - t_f}{t_0 - t_f} = \frac{K_1(\beta_e)I_0(\beta) + I_1(\beta_e)K_0(\beta)}{I_0(\beta_c)K_1(\beta_e) + I_1(\beta_e)K_0(\beta_c)}$ $q = \frac{h\beta_c(t_0 - t_f)}{H^2\cos\theta}\left[\frac{K_1(\beta_e)I_1(\beta_c) - I_1(\beta_e)K_1(\beta_c)}{K_1(\beta_e)I_0(\beta_c) + I_1(\beta_e)K_0(\beta_c)}\right]$ $\phi = \frac{\beta_c}{2H^2\ell_c}\ \frac{K_1(\beta_e)I_1(\beta_c) - I_1(\beta_e)K_1(\beta_c)}{K_1(\beta_e)I_0(\beta_c) + I_1(\beta_e)K_0(\beta_c)}$ $\beta = 2H\sqrt{x + \frac{b_e(1 - \tan\theta)}{2\tan\theta}}\ ,\ \beta_e = 2H\sqrt{\frac{b_e(1 - \tan\theta)}{2\tan\theta}}$ $\beta_c = 2H\sqrt{\ell_c + \frac{b_e(1 - \tan\theta)}{2\tan\theta}}\ ,\ H = \sqrt{h/k\ \sin\theta},$ $\ell_c = \ell + \frac{b_e}{2}$

Case No.	References	Description	Solution
5.1.10	7, p. 54	Cylindrical fin--rectangular profile.	See below
5.1.11	7, p. 58 91	Cylindrical fin--triangular profile.	See below

5.1.10 7, p. 54 Cylindrical fin--rectangular profile.



$$Q = 2\pi x_b bkm(t_0 - t_f)\,\frac{I_1(x_e m)K_1(x_b m) - K_1(x_e m)\,I_1(x_b m)}{I_1(x_e m)K_0(x_b m) + K_1(x_e m)I_0(x_b m)}$$

$$m = \sqrt{2h/kb}$$

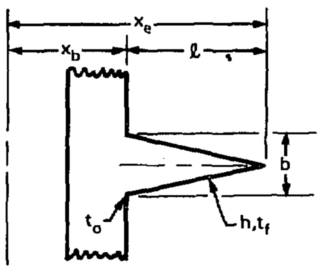
$$\frac{t - t_f}{t_0 - t_f} = \frac{K_1(x_e m)I_0(rm) + I_1(x_e m)K_0(rm)}{K_1(x_e m)I_0(x_b m) + I_1(x_e m)K_0(x_b m)}$$

$$\phi = \frac{2}{x_b m\left[1 - (x_e/x_b)^2\right]}\left[\frac{I_1(x_b m) - \beta K_1(x_b m)}{I_0(x_b m) + \beta K_0(x_b m)}\right], \quad \text{See Fig. 5.3}$$

$$\beta = I_1(x_e/x_b)/K_1(x_e/x_b)$$

Minimum material: use Fig. 5.4.

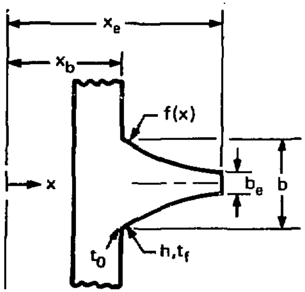
5.1.11 7, p. 58; 91 Cylindrical fin--triangular profile.



$$\phi = \frac{2}{x_b m\left[1 - (x_e/x_b)^2\right]}\left[\frac{I_{-2/3}(\delta) + \beta I_{2/3}(\delta)}{I_{1/3}(\delta) + \beta I_{-1/3}(\delta)}\right]$$

$$m = \sqrt{2h/kb}, \quad \delta = \frac{2}{3}\,x_b m, \qquad \beta = \frac{-I_{-2/3}\left[\left(\frac{x_e}{x_b}\right)^{3/2}\right]}{I_{2/3}\left[\left(\frac{x_e}{x_b}\right)^{3/2}\right]}$$

See Fig. 5.5.

Case No.	References	Description	Solution
5.1.12	10, p. 106	Cylindrical fin--hyperbolic profile. $f(x) = \dfrac{bx_b}{2x}$ 	see below

$$Q = 2\pi kMbx_b^{3/2}(t_0 - t_f)\frac{\left[I_{2/3}(B_e)I_{-2/3}(B_b) - I_{-2/3}(B_e)I_{2/3}(B_b)\right]}{\left[I_{-2/3}(B_e)I_{-1/3}(B_b) - I_{2/3}(B_e)I_{1/3}(B_b)\right]}$$

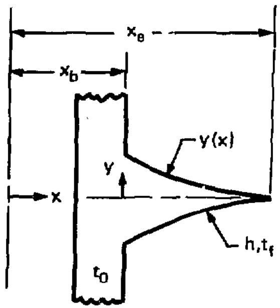
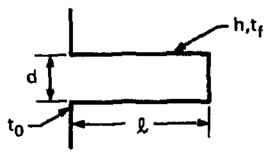
$$\frac{(t - t_f)}{(t_0 - t_f)} = \left(\frac{x}{x_b}\right)^{1/2} \frac{\left[I_{2/3}(B_e)I_{1/3}(B) - I_{-2/3}(B_e)I_{-1/3}(B)\right]}{\left[I_{-2/3}(B_e)I_{1/3}(B_b) - I_{-2/3}(B_e)I_{-1/3}(B_b)\right]}$$

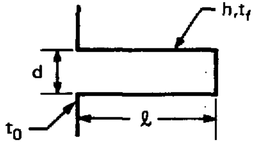
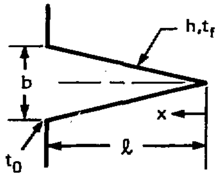
$$\phi = \frac{1}{\eta}\left(\frac{4\rho(1 - \rho)}{(1 + \rho)^2 \ln (1/\rho)}\right)^{1/2} \frac{\left[I_{2/3}(B_e)I_{-2/3}(B_b) - I_{-2/3}(B_e)I_{2/3}(B_b)\right]}{\left[I_{-2/3}(B_e)I_{-1/3}(B_b) - I_{2/3}(B_e)I_{1/3}(B_b)\right]}$$

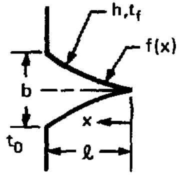
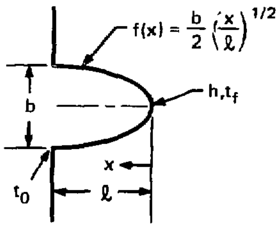
$$M = \sqrt{2h/kbx_b},\quad B_e = \frac{2}{3}Mx_e^{3/2},\quad B_b = \frac{2}{3}Mx_b^{3/2},\quad B = \frac{2}{3}Mx^{3/2}$$

$$\eta = (x_e - x_b)^{3/2} M/\sqrt{\ln 1/\rho},\quad \rho = x_b/x_e$$

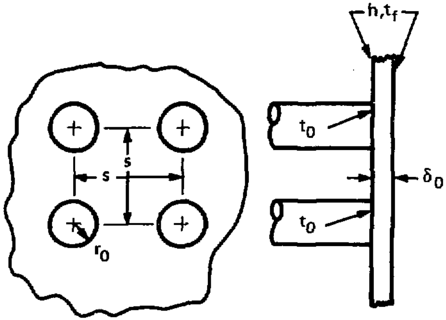
Section 5.1. Extended Surfaces--No Internal Heating.

Case No.	References	Description	Solution
5.1.13	1, p. 234	Cylindrical fin of minimum material. 	$Q = \frac{h\pi x_b^2}{3}(x_e/x_b + 2)(x_e/x_b - 1)(t_0 - t_f)$ Profile: $y(x) = \frac{hr_e^2}{k}\left[\frac{1}{3}\left(\frac{x}{x_e}\right)^2 - \frac{1}{2}\frac{x}{x_e} + \frac{1}{6}\frac{x_e}{x}\right]$
5.1.14	10, p. 114	Pin fins--cylindrical type. 	$Q = \frac{\pi}{4} k d^2 m \tanh(m\ell)(t_0 - t_f)$ $\phi = \frac{\tanh(m\ell)}{m\ell}$, $m = \sqrt{4h/kd}$, See Fig. 5.1

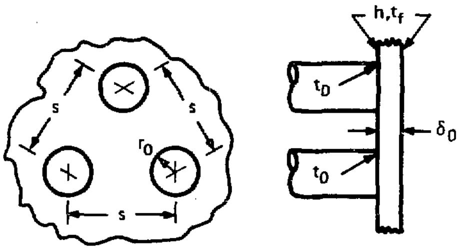
Case No.	References	Description	Solution
5.1.15	10, p. 115	Pin fins--rectangular type. Depth of fin = b.  	Same as for case 5.1.14 except $\left(\frac{\pi}{4} d^2\right)$ is replaced by (bd).
5.1.16	10, p. 117	Pin fins--conical type.  	$Q = \frac{\pi k b^2 M}{8\ell} \cdot \frac{I_2(M)}{I_1(M)} \cdot (t_0 - t_f)$ $\frac{t - t_f}{t_0 - t_f} = \sqrt{\frac{\ell}{x}} \; \frac{I_1(M\sqrt{x/\ell})}{I_1(M)}$, See Fig. 5.1 $\phi = \frac{4I_2(M)}{MI_1(M)}$, $M = 2\sqrt{2}\, m\ell$, $m = \sqrt{2h/kb}$

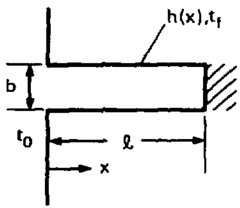
Case No.	References	Description	Solution
5.1.17	10, p. 119	Pin fins--concave parabolic type.  	$Q = \frac{\pi k b^2}{8\ell}\left[\sqrt{9 + 4M^2} - 3\right](t_0 - t_f)$ $\frac{t - t_f}{t_0 - t_f} = \left(\frac{x}{\ell}\right)^{-\frac{3}{2}+\frac{1}{2}\sqrt{9+4M^2}}$ $\phi = \frac{2}{1 + \left(1 + \frac{4}{9}M^2\right)^{1/2}}$, $M = \sqrt{4h/kb}\,\ell$, See Fig. 5.1
5.1.18	10, p. 121	Pin fins--convex parabolic type.  	$Q = \frac{3k\pi b^2 M}{16\ell}\,\frac{I_1(M)}{I_0(M)}\,(t_0 - t_f)$ $\frac{t - t_f}{t_0 - t_f} = \frac{I_0[M(x/\ell)]^{3/4}}{I_0(M)}$, $M = \frac{8\ell}{3}\sqrt{\frac{k}{kb}}$ $\phi = \frac{2}{M}\,\frac{I_1(M)}{I_0(M)}$, See Fig. 5.1

Case No.	References	Description	Solution
5.1.19	10, p. 134	Infinite fin heated by square arrayed round rods.	See Fig. 5.6

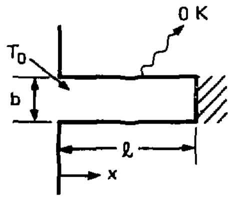


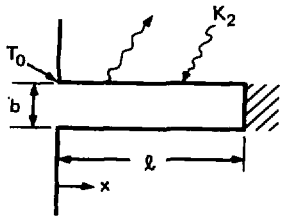
5.1.20	10, p. 135	Infinite fin heated by equilateral-triangular arrayed round rods.	See Fig. 5.7

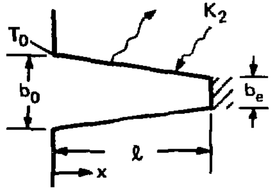
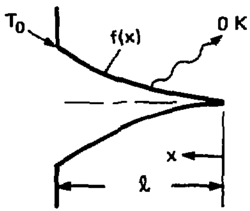


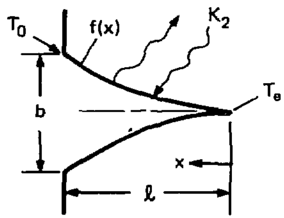
Case No.	References	Description	Solution
5.1.21	10, p. 185	Straight rectangular fin with nonuniform cooling. $h(x) = (\gamma + 1)h_a\left(\frac{x}{\ell}\right)^\gamma$. h_a = av coeff.  	$\frac{t - t_f}{t_0 - t_f} = \frac{\Gamma\left(\frac{\gamma + 1}{\gamma + 2}\right)}{2^{1/(\gamma+2)}} U^{1/(\gamma+2)}\left[I_{-\frac{1}{\gamma+2}}(U) + \frac{I_{-\frac{\gamma+1}{\gamma+2}}(U_\ell)}{I_{-\frac{\gamma+1}{\gamma+2}}(U_\ell)} I_{\frac{1}{\gamma+2}}(U)\right]$ $\phi = \left[\frac{(\gamma + 2)^\gamma(\gamma + 1)}{(m\ell)^{2(\gamma+1)}}\right]^{1/(\gamma+2)}\left[\frac{I_{(\gamma+1)/(\gamma+2)}(U_\ell)}{I_{-(\gamma+1)/(\gamma+2)}(U_\ell)}\right]\left[\frac{\Gamma\,\frac{\gamma+1}{\gamma+2}}{\Gamma\,\frac{1}{\gamma+2}}\right]$ $U = \frac{2\sqrt{\gamma + 1}}{\gamma + 2}\, m\ell^{-\gamma/2}\, x^{(\gamma+2)/2}$ $U_\ell = \frac{2\sqrt{\gamma + 1}}{\gamma + 2}\, m\ell,\ m = \sqrt{2h_a/kb}$
5.1.22	10, p. 191	Same conditions as case 5.1.21 with: $h(x) = h_a\left[\frac{(1 + a)(x/\ell + c)^a}{(1 + c)^{a+1} - c^{a+1}}\right]$.	$\frac{t - t_f}{t_0 - t_f} = \left(\frac{u}{u_0}\right)^n\left[\frac{I_n(U)I_{1-n}(U_\ell) - I_{-n}(U)I_{n-1}(U_\ell)}{I_n(U_0)I_{1-n}(U_\ell) - I_{-n}(U_0)I_{n-1}(U_\ell)}\right]$ $\phi = \frac{2(1 - n)}{u_0\left[1 - (u_\ell/u_0)^{2(1-n)}\right]}\left[\frac{I_{n-1}(u_0)I_{1-n}(u_\ell) - I_{1-n}(u_0)I_{n-1}(u_\ell)}{I_n(u_0)I_{1-n}(u_\ell) - I_{-n}(u_0)I_{n-1}(u_\ell)}\right]$ $u = 2n\left(\frac{1 - n}{n}\right)^{1/2}\left[\frac{(x/\ell + c)^{1/n}}{(1 + c)^{(1-n)/n} - c^{(1-n)/n}}\right]m\ell$ $u_0 = u,\ x = 0;\ u_\ell = u,\ x = \ell,\ n = \frac{1}{2a},\ m = \sqrt{2h_a/kb}$

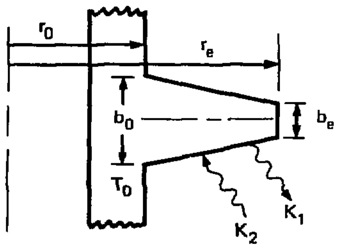
Case No.	References	Description	Solution
5.1.23	10, p. 187	Same conditions as case 5.1.22 with: $h(x) = h_a\left[\dfrac{1 - a\exp(-cx/\ell)}{1 - (a/c)\left[1 - \exp(-c)\right]}\right].$	$n = \dfrac{2m\ell}{c},\ u = n\sqrt{a}\exp\left(-\dfrac{xc}{2\ell}\right)$ $\psi = n\sqrt{a}\exp(c),\ \beta = n\sqrt{a}$ If n is an integer: $Q = \dfrac{\sqrt{a}}{2m\ell\left\{1 - (a/c)\left[1 - \exp(-c)\right]\right\}} \cdot \dfrac{A_1 - A_2}{A_3 - A_4}$ $A_1 = \left[Y_{n-1}(\psi) - Y_{n+1}(\psi)\right]\left[J_{n-1}(\beta) - J_{n+1}(\beta)\right]$ $A_2 = \left[J_{n-1}(\psi) - J_{n+1}(\psi)\right]\left[Y_{n-1}(\beta) - Y_{n+1}(\beta)\right]$ $A_3 = J_n(\beta)\left[Y_{n-1}(\psi) - Y_{n+1}(\psi)\right]$ $A_4 = Y_m(\beta)\left[J_{n-1}(\psi) - J_{n+1}(\psi)\right]$ If n is not an integer: $\phi = \dfrac{c}{2(m\ell)^2\left\{1 - (a/c)\left[1 - \exp(-c)\right]\right\}} \cdot \dfrac{A_1 - A_2}{A_3 - A_4}$ $A_1 = \left[-nJ_{-n}(\psi) - \psi J_{1-n}(\psi)\right]\left[-nJ_n(\beta) + \beta J_{n-1}(\beta)\right]$ $A_2 = \left[-J_n(\psi) + \psi J_{n-1}(\psi)\right]\left[-nJ_{-n}(\beta) - \beta J_{1-n}(\beta)\right]$ $A_3 = J_n(\beta)\left[-nJ_{-n}(\psi) - \psi J_{1-n}(\psi)\right]$ $A_4 = J_{-n}(\beta)\left[-nJ_n(\psi) + \psi J_{n-1}(\psi)\right]$

Case No.	References	Description	Solution
5.1.24	10, p. 205	Straight fin of rectangular profile radiating to free space. $t = t_0$, $x = 0$. $T = T_e$, $x = \ell$. Space temp = 0 K.  	$\ell\sqrt{\frac{20\sigma\epsilon T_e^3}{kb}} = B(0.3, 0.5) - B_u(0.3, 0.5)$ B = complete beta function B_u = incomplete beta function $Q = 2kb\sqrt{\frac{\sigma\epsilon}{5kb}}\left(T_0^5 - T_e^5\right)$, see Figs. 5.8 and 5.9 $\phi = \frac{2}{\ell}\sqrt{\frac{1/Z^3 - 1/Z^8}{20\sigma\epsilon T_e^3/kb}}$, $Z = T_0/T_e$ Optimum length: $b/\ell^2 = 2.486\sigma\epsilon T_0^3/k$

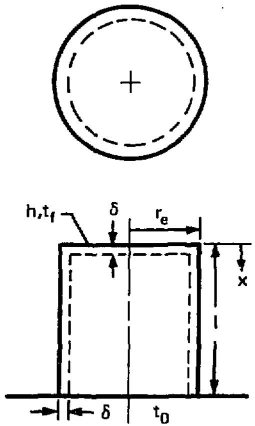
Case No.	References	Description	Solution
5.1.25	10, p. 211	Straight fin of rectangular profile radiating to nonfree space. $T = T_0$, $x = 0$. $T = T_e$, $x = \ell$. K_2 = incident radiation absorbed from surroundings. 	$\dfrac{Q\ell}{kbT_0} = \ell\sqrt{\dfrac{2K_1T_0^3}{5kbZ^5}}\sqrt{Z-1}\left[Z^4 + Z^3 + Z^2 + Z + 1 - \dfrac{5Z^4K_2}{K_1T_0^4}\right]^{1/2}$ $\phi = \dfrac{Q\ell/kbT_0}{2\sigma\varepsilon\ell^2T_0^3/kb}$, $K_1 = 2\sigma\varepsilon$, $Z = T_0/T_e$ See Figs. 5.10 and 5.11 Optimum length: $b/\ell^2 = 2.365\sigma\varepsilon T_0^3/k$

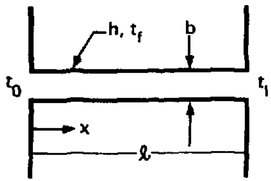
Case No.	References	Description	Solution
5.1.26	10, p. 218	Straight trapezoidal fin radiating to nonfree space. $T = T$, $x = 0$. K_2 = incident radiation absorbed from surroundings. 	See Figs. 5.12a, 5.12b, 5.12c, and 5.12d. $\lambda = b_e/b_0 = 0.75, 0.50, 0.25$, and 0 (triangular fin). $K_1 = 2\ \sigma\epsilon$.
5.1.27	10, p. 228	Straight fin of least material radiating to free space. $T = 0$, $x = 0$. 	Profile: $f(x) = \dfrac{3Q^2}{2k\sigma\epsilon T_0^5}\left(\dfrac{x}{\ell}\right)^{7/2}, \ \ell = \dfrac{3Q}{2\sigma\epsilon T_0^4}$ $\dfrac{T}{T_0} = \left(\dfrac{x}{\ell}\right)^{1/2}$

Case No.	References	Description	Solution
5.1.28	10, p. 230	Straight fin with constant temperature gradient radiating to nonfree space. $\frac{dT}{dx} = \frac{T_0 - T_e}{\ell}$.  	Profile: $f(x) = \frac{K_1 \ell^2 T_0^3}{10kZ^3(Z - 1)^2} \left\{\left[1 + \frac{x}{\ell}(Z - 1)\right]^5 - 1 - \frac{5K_2 Z^4}{K_1 T_0^4}\left[\frac{x}{\ell}(Z - 1)\right]\right\}$ $Q = \frac{K_1 \ell T_e^4 (Z^5 - 1)}{5(Z - 1)} - K_2 \ell$ $Z = T_0/T_e$ $K_1 = 2\,\sigma\varepsilon$ $\phi = \frac{Z^5 - 1}{5Z^5(Z - 1)} - \frac{K_2}{K_1 T_0^4}$ K_2 = incident radiation absorbed $T_{e,min} = (K_2/K_1)^{1/4}$, See Figs. 5.13 and 5.14

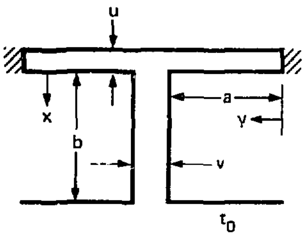
Case No.	References	Description	Solution
5.1.29	10, p. 247	Radial fin of trapezoidal profile radiating to nonfree space. $T = T_0$, $r = r_0$. 	Fin efficiency: (See Figs. 5.15a-5.15ℓ) $\lambda = b_e/b_0 = 0, 0.5, 0.75, 1.0$ $K_2/K_1T_0^4 = 0, 0.2, 0.4$ Profile No. $= K_1T_0^3r_0^2/kb_0$ $K_1 = \sigma\varepsilon$, K_2 = incident radiation absorbed

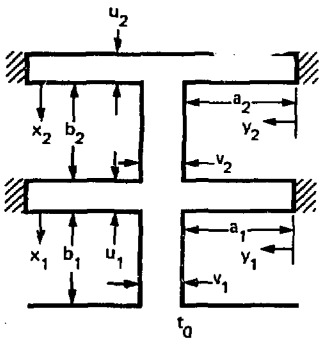
Case No.	References	Description	Solution
5.1.30	10, p. 275	The capped cylinder, convection boundary. $t = t_0,\ x = \ell$. Insulated inside surfaces.	Side: $\dfrac{t - t_f}{t_0 - t_f} = \dfrac{\cosh(mx) + \sinh(mx)\left[I_1(mr_e)/I_0(mr_e)\right]}{\cosh(m\ell) + \sinh(m\ell)\left[I_1(mr_e)/I_0(mr_e)\right]}$ $\phi = \dfrac{\sinh(m\ell) + \left[\cosh(m\ell) - 1\right]\left[I_1(mr_e)/I_0(mr_e)\right]}{m\ell\left\{\cosh(m\ell) + \sinh(m\ell)\left[I_1(mr_e)/I_0(mr_e)\right]\right\}}$ Top: $\dfrac{t - t_f}{t_0 - t_f} = \dfrac{I_0(mr)/I_0(mr_e)}{\cosh(m\ell) + \sinh(m\ell)\left[I_1(mr_e)/I_0(mr_e)\right]}$ $\phi = \dfrac{2I_1(mr_e)/I_0(mr_e)}{mr_e\left\{\cosh(m\ell) + \sinh(m\ell)\left[I_1(mr_e)/I_0(mr_e)\right]\right\}}$ $m = \sqrt{h/k\delta}$, See Figs. 5.16a, and 5.16b



Case No.	References	Description	Solution
5.1.31	10, p. 407	Doubly heated straight, rectangular fin. $t = t_0$, $x = 0$. $t = t_\ell$, $x = \ell$.  	$\dfrac{t - t_f}{t_0 - t_f} = \exp(mx) - 2B \sinh(mx)$ $q_0 = kbm(2B - 1)(t_0 - t_f)$ $q_\ell = kbm\left[\exp(m\ell) - 2B \cosh(m\ell)\ (t_0 - t_f)\right]$ $B = \dfrac{\exp(m\ell) - (t_\ell - t_f)/(t_0 - t_f)}{2 \sinh(m\ell)}$, $m = \sqrt{2h/kb}$ See Fig. 5.17

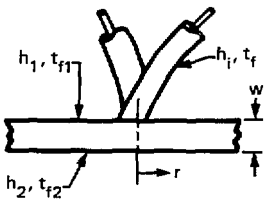
Case No.	References	Description	Solution
5.1.32	10, p. 394	Single T, straight, rectangular fin. $t = t_0$, $x = b$. All surfaces convectively cooled by h, t_f.	Vertical section: $\frac{t - t_f}{t_0 - t_f} = F\left[\cosh(m_x x) + \sqrt{2u/v}\tanh(m_y a)\sinh(m_x x)\right]$ $\phi_x = \frac{F}{m_x b}\left\{\sinh(m_x b) + \sqrt{2u/v}\tanh(m_y a)\left[\cosh(m_x b) - 1\right]\right\}$ Horizontal section: $\frac{t - t_f}{t_0 - t_f} = F\frac{\cosh(m_y y)}{\cosh(m_y a)}$ $\phi_y = \frac{F\tanh(m_y a)}{m_y a}$, See Figs. 5.18a, 5.18b $F = \frac{1}{\cosh(m_x b) + \sqrt{2u/v}\tanh(m_y a)\sinh(m_x b)}$ $m_x = \sqrt{h/kv}$, $m_y = \sqrt{h/ku}$



Case No.	References	Description	Solution
5.1.33	10, p. 413	Double T, straight, rectangular fin. $t = t_0,\ x_1 = b_1$. All surfaces convectively cooled by $h,\ t_f$. 	Vertical sections: $\frac{t - t_f}{t_0 - t_f} = F_1\left[\cosh (m_{x1}x_1) + K_1 \sinh (m_{x1}x_1)\right],\ 0 < x_1 < b_1$. $\frac{t - t_f}{t_0 - t_f} = F_1F_2\left[\cosh (m_{x2}x_2) + \sqrt{2u_2/v_2}\tanh (m_{y2}a_2) \times \sinh (m_{x2}x_2)\right],\ 0 < x_2 < b_2$. $\phi_{x1} = \frac{F_1}{m_{x1}b_1}\left[\sinh (m_{x1}b_1) + K_1(\cosh (m_{x1}b_1) - 1)\right],\ 0 < x_1 < b_1$. $\phi_{x2} = \frac{F_1F_2}{m_{x2}b_2}\left[\sinh (m_{x2}b_2) + \sqrt{2u_2/v_2}\tanh (m_{y2}a_2) \times (\cosh (m_{x2}b_2) - 1)\right],\ 0 < x_2 < b_2$. Horizontal section: $\frac{t - t_f}{t_0 - t_f} = F_1 \frac{\cosh (m_{y1}y_1)}{\cosh (m_{y1}a_1)},\ 0 < y_1 < a_1$ $\frac{t - t_f}{t_0 - t_f} = F_1F_2 \frac{\cosh (m_{y2}y_2)}{\cosh (m_{y2}a_2)},\ 0 < y_2 < a_2$ $\phi_{y1} = \frac{F_1}{m_{y1}a_1}\tanh (m_{y1}a_1),\ 0 < y_1 < a_1$ $\phi_{y2} = \frac{F_1F_2}{m_{y2}a_2}\tanh (m_{y2}a_2),\ 0 < y_2 < a_2$

Case No. References	Description	Solution
5.1.33 continued		$F_1 = \dfrac{1}{\cosh(m_{x1}b_1) + B\sinh(m_{x1}b_1)}$
		$F_2 = \dfrac{1}{\cosh(m_{x2}b_2) + \sqrt{2u_2/v_2}\tanh(m_{y2}a_2)\sinh(m_{x2}b_2)}$
		$B = F_2\sqrt{v_2/v_1}\left[\sinh(m_{x2}b_2) + \sqrt{2u_2/v_2}\tanh(m_{y2}a_2) \times \cosh(m_{x2}b_2)\right] + 2\sqrt{u_1/v_1}\tanh(m_{y1}b_1)$
		$m_{x1} = \sqrt{2h/kv_1},\ m_{x2} = \sqrt{2h/kv_2}$
		$m_{y1} = \sqrt{2h/ku_1},\ m_{y2} = \sqrt{2h/ku_2}$

Case No.	References	Description	Solution
5.1.34	2, p. 179	Insulated wires adjoined on a convectively cooled plate (e.g., thermocouple wires). $t = t_\infty$, $r \to \infty$. Wire radius = r_w. Wire insulation thickness = δ. Wire therm. cond. = k_w. Insulation therm. cond. = k_i. Plate therm. cond. = k_p.	$\dfrac{t_0 - t_\infty}{t_{f1} - t_\infty} = \dfrac{1}{1 + \left[B\ K_1(r_w H)/K_0(r_w H)\right]}$ $B = \dfrac{2}{\sqrt{k_{w1}} + \sqrt{k_{w2}}} \left[\dfrac{k_p w(h_1 + h_2)(1/h_i + \delta/k_i)}{r_w}\right]^{1/2}$ $H = \sqrt{2(h_1 + h_2)/kw}$



Case No. | References | Description | Solution

5.1.35 9, p. 157

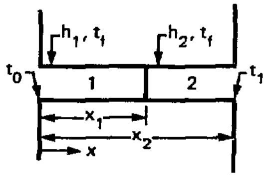
Composite finite rod.

$t = t_0$, $x = 0$.

$t = t_2$, $x = x_2$.

A_1, A_2 = areas.

C_1, C_2 = circumferences.



$$\frac{T}{T_0} = \frac{\cosh (A) \sinh (B) + H \sinh (A) \cosh (B) + (T_2/T_0) \sinh (Dx/x_1)}{\sinh (D) \cosh (A) + H \cosh (D) \sinh (A)}$$

$0 < x < x_1$

$A = m_2(x_2 - x_1)$, $B = m_1(x_1 - x)$, $D = m_1 x_1$, $T = t - t_f$

$H = \sqrt{h_1C_1k_1A_1/h_2C_2k_2A_2}$, $m_1 = \sqrt{h_1C_1/k_1A_1}$, $m_2 = \sqrt{h_2C_2/k_2A_2}$

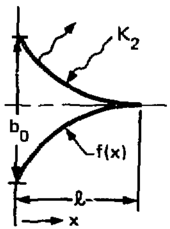
$$\frac{T}{T_2} = \frac{\cosh (A) \sinh (B) + H \sinh (A) \cosh (B) + (T_0/T_2) \sinh (m_2 b - m_2 x)}{\sinh (D) \cosh (A) + H \cosh (D) \sinh (A)}$$

$x_1 < x < x_2$

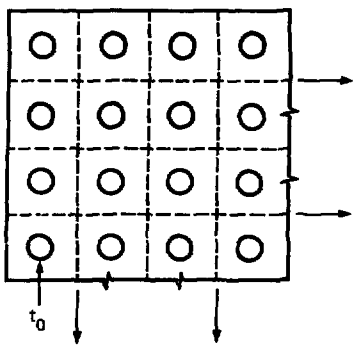
$A = m_1 x_1$, $B = m_2(x_2 - x)$, $D = m_2(x_2 - x_1)$, $T = t - t_f$

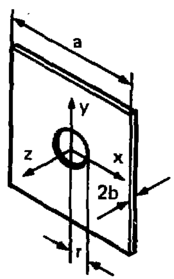
$H = \sqrt{h_2C_2k_2A_2/h_1C_1k_1A_1}$, m_1 and m_2 as above.

Case No.	References	Description	Solution
5.1.36	9, p. 157	Case 5.1.35 with: $q = 0$, $x = x_2$.	$\dfrac{t - t_f}{t_2 - t_f} = \dfrac{\cosh (m_1x_1 - m_1x)}{\cosh (m_1x_1) \cosh (m_2x_2 - m_2x_1) + H \sinh (m_1x_1) \sinh (m_2x_2 - m_2x_1)}$, $0 < x < x_1$. $\dfrac{(t - t_f)}{t_0 - t_f} = \dfrac{\cosh (m_2x_2 - m_2x)}{H \cosh (m_1x_1) \cosh (m_2x_2 - m_2x_1) + \sinh (m_1x_1) \sinh (m_2x_2 - m_1x_1)}$, $x_1 < x < x_2$. $H = \sqrt{h_2C_2k_2A_2/h_1C_1k_1A_1}$, $m_1 = \sqrt{h_1C_1/k_1A_1}$, $m_2 = \sqrt{h_2C_2/k_2A_2}$

Case No.	References	Description	Solution
5.1.37	31	Concave parabolic fin radiating to non-free space. $T = T_b$, $x = 0$. K_2 = incident absorbed radiation. Profile: $f(x) = \frac{b_0}{2}(1 - x/\ell)^2$. 	See Fig. 5.19 for values at $\phi(N_c, N_s)$. $N_c = (2\varepsilon\sigma F\ell^2 T_b^3)/kb_0$, $N_s = K_2/2\varepsilon\sigma F T_b^4$. F = view factor.
5.1.38	31	Convex parabolic fin radiating to non-free space. Conditions same as for case 5.1.37 except profile is: $f(x) = \frac{b_0}{2}\sqrt{1 - x/\ell}$.	See Fig. 5.20 for values of $\phi(N_c, N_s)$. N_c and N_s given in case 5.1.37.
5.1.39	81	Straight rectangular fin of variable conductivity. $k = k_0[1 + \beta(t - t_0)]$.	See Ref. 81 for approximate temperature and efficiency solutions.

Case No.	References	Description	Solution
5.1.40	83	Sheet fin for square array tubes with convection boundary, $h_f t_f$.	





$$\frac{t - t_f}{t_0 - t_f} = \frac{\sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} K_0\left\{\sqrt{Bi}\left[(X - mA)^2 + (Y - nA)^2\right]^{1/2}\right\}}{K_0(R\sqrt{Bi}) + I_0(R\sqrt{Bi}) \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} K_0\left[A\sqrt{Bi}(m^2 + n^2)\right]^{1/2}} ,$$

$n, m \neq 0$.

$$\phi = \frac{2\pi}{R\sqrt{Bi}\left[(A/R)^2 - \pi\right]}$$

$$\times \frac{K_1(R\sqrt{Bi}) - I_1(R\sqrt{Bi}) \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} K_0\left[A\sqrt{Bi}(m^2 + n^2)\right]^{1/2}}{K_0(R\sqrt{Bi}) + I_0(R\sqrt{Bi}) \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} K_0\left[A\sqrt{Bi}(m^2 + n^2)\right]^{1/2}} ,$$

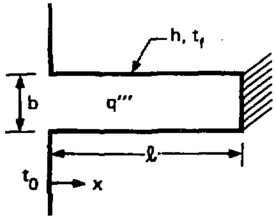
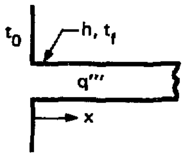
$n, m \neq 0$.

$X = x/b$, $Y = y/b$, $R = r/b$, $Bi = hb/k$, $A = a/b$.

Case No.	References	Description	Solution

Section 5.1. Extended Surfaces--No Internal Heating.

Case No.	References	Description	Solution

<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr><td>5.2.1</td><td>10,
p. 194</td><td>Straight, rectangular fin.
$q = q_0$, $x = 0$.
$q_x = 0$, $x = b$.

</td><td>$$\frac{(t - t_f)}{q_0} mkb = \frac{\cosh (mx)}{\tanh (m\ell)} - \sinh (mx) + \frac{q'''b}{q_0 m}$$
Optimum profile:
$$\frac{\sinh (2m\ell)}{2m\ell} = \frac{1 - q_0/hb(t_0 - t_f)}{1/3 - q_0/hb(t_0 - t_f)}, \quad 0 \le q_0/hb(t_0 - t_f) < 1/3$$
$$m = \sqrt{2h/kb}, \quad t = t_0, \quad x = 0$$</td></tr>
<tr><td>5.2.2</td><td>1, p. 246</td><td>Straight infinite rod.
$t = t_0$, $x = 0$.

</td><td>$$\frac{t - t_f}{t_0 - t_f} = 1 + 2\left[1 - \frac{q'''}{km^2(t_0 - t_f)}\right]\left[\sinh\left(\frac{mx}{2}\right)\right]^2$$</td></tr>
</table>

Section 5.2. Extended Surfaces--With Internal Heating.

Case No.	References	Description	Solution

Case No.	References	Description	Solution

6. Infinite Solids — Transient

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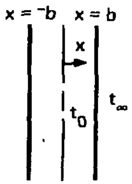
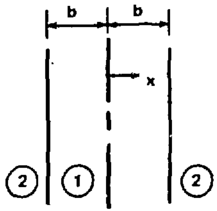
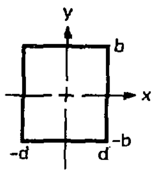
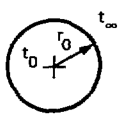
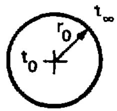
Section 6.1. Infinite Solids--No Inte al Heating.

Case No.	References	Description	Solution
6.1.1	9, p. 54	Infinite plate source. $t = t_0,\ -b < x < b,\ \tau = 0.$ $t = t_\infty,\ b < x < -b,\ \tau = 0.$ 	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{2}\left[\operatorname{erf}\left(Fo_b^* - Fo_x^*\right) + \operatorname{erf}\left(Fo_b^* + Fo_x^*\right)\right],\ -\infty < x < \infty$ See Fig. 6.1

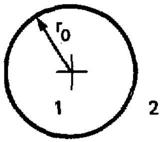
Case No.	References	Description	Solution
6.1.1.1	74, p. 424	Case 6.1.1 with a plate of different properties. 	$\dfrac{t - t_\infty}{t_0 - t_\infty} = 1 - \dfrac{1}{1 + K} \sum_{n=1}^{\infty} (-H)^{n-1} \left\{ \text{erfc}\left[\dfrac{(2n - 1) - X}{2\sqrt{Fo_{b1}}}\right] + \text{erfc}\left[\dfrac{(2n - 1) + X}{2\sqrt{Fo_{b1}}}\right]\right\}, \quad -1 < X < +1$ $\dfrac{t - t_\infty}{t_0 - t_\infty} = \dfrac{K}{K + 1} \text{erfc}\left(\dfrac{X - 1}{2\sqrt{Fo_{b2}}}\right) - \dfrac{K(1 + H)}{1 + K} \sum_{n=1}^{\infty} (1H)^{n-1} \times \text{erfc}\left(\dfrac{X - 1 + 2n\sqrt{\alpha_2/\alpha_1}}{2\sqrt{Fo_{b2}}}\right), \quad 1 < X < -1$ $K = \dfrac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2}, \quad H = \dfrac{1 - K}{1 + K}$ **Interface temp:** $\dfrac{t - t_\infty}{t_0 - t_\infty} = \dfrac{K}{1 + K} - \dfrac{2K}{(1 + K)^2} \sum_{n=1}^{\infty} (-H)^{n-1} \text{erfc}\left(Fo_{b1}^{*}\right), \quad X = 1$ For Fo_{b2} small: $\dfrac{t_2 - t_\infty}{t_0 - t_\infty} \simeq \dfrac{K}{1 + K} \text{erfc}\left(\dfrac{X - 1}{2\sqrt{Fo_{b2}}}\right) - \dfrac{2K}{(1 + K)^2} \text{erfc}\left(\dfrac{X - 1 + 2\sqrt{\alpha_2/\alpha_1}}{2\sqrt{Fo_{b2}}}\right)$

Case No.	References	Description	Solution
6.1.1.2	74, p. 427	Case 6.1.1 with a pulse source of heat of strength Q J/m^2 at $x = 0$, $\tau = 0$. $t = t_0$, $-\infty < x < \infty$, $\tau = 0$.	$\frac{(t - t_i)\rho_1 c_1 b}{Q} = \frac{1}{\pi\sqrt{Fo_1}}\left(\exp\left(-\frac{X^2}{Fo_1}\right) - H\sum_{n=1}^{\infty}(-H)^{n-1} \times \left\{\exp\left[\frac{(2n - X)^2}{4\ Fo_1}\right] + \exp\left[-\frac{(2n + X)^2}{4\ Fo_1}\right]\right\}\right), \quad -1 < X < 1$ $\frac{(t - t_i)\rho_1 c_1 b}{Q} = \frac{K}{\sqrt{\alpha_1/\alpha_2}(1 + K)\sqrt{\pi\ Fo_2}}\sum_{n=1}^{\infty}(-H)^{n-1} \times \exp\left\{-\frac{\left[X_1 - 1 + (2n - 1)\sqrt{\alpha_2/\alpha_1}\right]^2}{4\ Fo_2}\right\}, \quad 1 < X < -1$ $K = \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2}, \quad H = \frac{1 - K}{1 + K}$
6.1.2	9, p. 54	Case 6.1.1 with: $t = t_0(b - x)/b$, $-b < x < b$, $\tau = 0$. $t = t_\infty$, $b < x < -b$, $\tau = 0$.	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1 - X}{2}\,\mathrm{erf}\left(Fo_b^* - Fo_x^*\right) + \frac{1 + X}{2}\,\mathrm{erf}\left(Fo_b^* + Fo_x^*\right) - X\,\mathrm{erf}\left(Fo_x^*\right) + \sqrt{\frac{\alpha\tau}{b^2\pi}}\left\{\exp\left[\frac{-(x + b)^2}{\sigma^2}\right] + \exp\left[\frac{-(x - b)^2}{\sigma^2}\right] - 2\exp\left(\frac{-x^2}{\sigma^2}\right)\right\}$

Case No.	References	Description	Solution
6.1.3	9, p. 56	Infinite rod source. $t = t_0$, $\lvert x\rvert < d$, $\lvert y\rvert < b$, $\tau = 0$. $t = t_\infty$, $\lvert x\rvert > d$, $\lvert y\rvert > b$, $\tau = 0$. 	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{4}\left[\mathrm{erf}\left(Fo_d^* - Fo_x^*\right) + \mathrm{erf}\left(Fo_d^* + Fo_x^*\right)\right] \times \left[\mathrm{erf}\left(Fo_b^* - Fo_y^*\right) + \mathrm{erf}\left(Fo_b^* + Fo_y^*\right)\right]$
6.1.4	74, p. 432	Infinite cylinder source. $t = t_0$, $r < r_0$, $\tau = 0$. $t = t_\infty$, $r > r_0$, $\tau = 0$. 	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{2}{\pi}\int_0^\infty \exp(-\lambda^2 Fo)\left[\frac{J_0(\lambda R)J_1(\lambda)}{J_1(\lambda)Y_0(\lambda) - J_0(\lambda)Y_1(\lambda)}\right]\frac{d\lambda}{\lambda}$, $R > 1$ $\frac{t - t_\infty}{t_0 - t_\infty} = \frac{4}{\pi^2}\int_0^\infty \exp(-\lambda^2 Fo)\left[\frac{J_0(\lambda R)J_1(\lambda)}{\left[J_1(\lambda)Y_0(\lambda)-J_0(\lambda)Y_1(\lambda)\right]^2}\right]\frac{d\lambda}{\lambda^2}$, $R < 1$ See Fig. 6.2

Case No.	References	Description	Solution
6.1.5	9, p. 55	Spherical source. $t = t_0,\ r < b,\ \tau = 0.$ $t = t_\infty,\ r > b,\ \tau = 0.$ 	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{2}\,\mathrm{erf}\left(Fo_r^* - Fo_{r_0}^*\right) - \frac{1}{2}\,\mathrm{erf}\left(Fo_r^* + Fo_{r_0}^*\right)$ $- \frac{1}{2\,Fo_r^*\sqrt{\pi}}\left\{\exp\left[\left(Fo_r^* - Fo_{r_0}^*\right)^2\right]\exp\left[\left(Fo_r^* + Fo_{r_0}^*\right)^2\right]\right\}$ See Fig. 6.3

Case No.	References	Description	Solution
6.1.5.1	74, p. 429	Case 6.1.5 with a sphere of different thermal properties.	$\frac{t - t_0}{t_\infty - t_0} = \frac{1}{R}\left(\frac{1}{K_2 - 1}\right)\left\{\frac{K_2 + K_1}{1 + K_1}\exp\left[P_1^2 Fo_1 - P_1(1 \mp R)\right] \times \operatorname{erfc}\left(\frac{1 \mp R}{2\sqrt{Fo_1}} - P_1\sqrt{Fo_1}\right) - \operatorname{erf}\left(\frac{1 \mp R}{2\sqrt{Fo_1}}\right)\right\}, \quad R \le 1 .$



$$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{R}\left\{\operatorname{erfc}\left(\frac{R - 1}{2\sqrt{Fo_2}}\right) + \frac{1}{K_2 - 1}\operatorname{erfc}\left(\frac{R - 1}{2\sqrt{Fo_2}}\right)\right.$$

$$- \frac{K_2 + K_1}{(K_2 - 1)(K_1 + 1)}\exp\left(P_2^2 Fo_2 - P_2 R - 1\right)\operatorname{erfc}\left(\frac{R - 1}{2\sqrt{Fo_2}} - P_2\sqrt{Fo_2}\right)$$

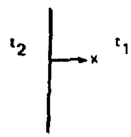
$$- \frac{1}{K_2 - 1}\operatorname{erfc}\left(\frac{R - 1 + 2\sqrt{\alpha_2/\alpha_1}}{2\sqrt{Fo_2}}\right) + \frac{K_2 + K_1}{(K_2 - 1)(K_1 + 1)}$$

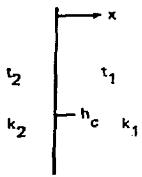
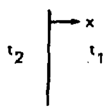
$$\times \exp\left(P_2^2 Fo_2 - P_2 R - 1 + 2\sqrt{\alpha_2/\alpha_1}\right)\operatorname{erfc}\left(\frac{R - 1 + 2\sqrt{\alpha_2/\alpha_1}}{2\sqrt{Fo_2}}\right.$$

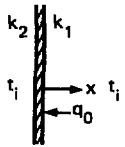
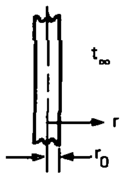
$$\left.\left. - P_2\sqrt{Fo_2}\right)\right\}, \quad R > 1$$

$$K_1 = \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2}, \quad K_2 = \frac{k_1}{k_2}, \quad P_1 = \frac{K_2 - 1}{K_2 + K_2/K_1}, \quad P_2 = \frac{K_2 - 1}{1 + K_1}$$

$$f(\mp R) = f(-R) - f(+R)$$

Case No.	References	Description	Solution
6.1.6	9, p. 61	Adjoined semi-infinite regions. $t = t_1,\ x > 0,\ \tau = 0.$ $t = t_2,\ x < 0,\ \tau = 0.$  	$\dfrac{t - t_2}{t_1 - t_2} = \dfrac{1}{2}\left[\operatorname{erf}\left(Fo_x^*\right) + 1\right],\ -\infty < x < \infty$
6.1.7	7, p. 89	Specified temperature distribution. $t = f(x),\ -\infty < x < \infty,\ \tau = 0.$	$t = \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\infty}^{\infty} f(x + \lambda\sigma)\exp(-\lambda^2)\,d\lambda$

Case No.	References	Description	Solution
6.1.8	9, p. 88	Adjoined semi-infinite regions with contact resistance. $t = t_1,\ x > 0,\ \tau = 0.$ $t = t_2,\ x < 0,\ \tau = 0.$ 	$\frac{t - t_2}{t_1 - t_2} = \frac{1}{1 + A}\left\{1 + A\left[\mathrm{erf}\left(Fo_x^*\right) + \exp\left(H_1x + H_1^2\sigma_1^2/4\right) \times \mathrm{erfc}\left(Fo_x^* + H_1\sigma_1/2\right)\right]\right\}, \quad x > 0$ $\frac{t - t_2}{t_1 - t_2} = \frac{1}{1 + A}\left[\mathrm{erf}\ \lvert Fo_x^*\rvert - \exp\left(H_2x + H_2^2\sigma_2^2/4\right) \times \mathrm{erfc}\left(\lvert Fo_x^*\rvert + H_2\sigma_2/2\right)\right], \quad x < 0$ $A = \frac{k_2}{k_1}\sqrt{\frac{\alpha_1}{\alpha_2}},\ H_1 = \frac{h(1 + A)}{k_1A},\ H_2 = \frac{h}{k_2}(1 + A)$
6.1.9	9, p. 88	Adjoined semi-infinite regions--no contact resistance. $t = t_1,\ x > 0,\ \tau = 0.$ $t = t_2,\ x < 0,\ \tau = 0.$ 	$\frac{t - t_2}{t_1 - t_2} = \frac{1}{1 + A}\left[1 + A\ \mathrm{erf}\left(Fo_x^*\right)\right], \quad x > 0$ $\frac{t - t_2}{t_1 - t_2} = \frac{1}{1 + A}\ \mathrm{erf}\left(\lvert Fo_x^*\rvert\right), \quad x < 0$ $A = \frac{k_2}{k_1}\sqrt{\frac{\alpha_1}{\alpha_2}}$

Case No.	References	Description	Solution
6.1.10	9, p. 89	Plane source of heating between adjoined semi-infinite regions. $t = t_i$, $-\infty < x < \infty$, $\tau = 0$. $q_x = q_0$, $x = 0$, $\tau > 0$ = heat source strength.  	$\frac{(t - t_i)k_1}{q_0\sigma_1} = \frac{1}{1 + A}\,\text{ierfc}\left(Fo_x^*\right)$, $x > 0$ $\frac{(t - t_i)k_2}{q_0\sigma_2} = \frac{1}{1 + A}\,\text{ierfc}\left(\lvert Fo_x^*\rvert\right)$, $x < 0$ $A = \frac{k_2}{k_1}\sqrt{\frac{\sigma_1}{\sigma_2}}$
6.1.11	1, p. 341 9, p. 261	Line source of heat. $t = t_0$, $0 < r < r_0$, $\tau = 0$. $t = t_\infty$, $r > r_0$, $\tau = 0$.  	$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{\pi\, Fo}\exp\left(-Fo_r^{*2}\right)$, $r > r_0$, $r_0 \to 0$ If line is a steady source of strength q_0: $\frac{(t - t_\infty)k}{q_0} = \frac{1}{2\pi}\int_{1/\sqrt{\tau}}^{\infty}\frac{1}{\lambda}\exp\left(-\lambda^2 Fo_r^{*2}\right)d\lambda = -\frac{1}{4\pi}\,\text{Ei}\left(-Fo_r^{*2}\right)$

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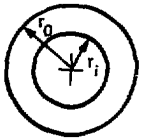
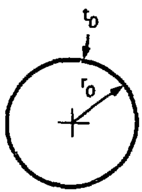
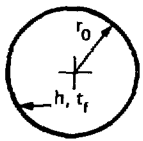
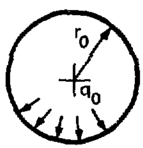
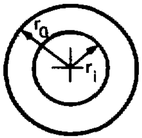
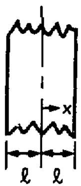
Case No.	References	Description	Solution
6.1.12	1, p. 342	Cylindrical shell source. $t = t_0,\ r_i < r < r_0,\ \tau = 0.$ $t = t_\infty,\ r > r_0,\ \tau = 0.$ 	$\dfrac{t - t_\infty}{t_0 - t_\infty} = \left(Fo^{*2}_{r_0} - Fo^{*2}_{r_i}\right) \exp\left[-\left(Fo^{*2}_{r} + Fo^{*2}_{r_i}\right)\right] J_0\left(2i\ Fo^{*}_{r}\ Fo_{r_i}\right)$ $i = \sqrt{-1}$
6.1.13	9, p. 247	Spherical surface at steady temperature. $t = t_i,\ r > r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ 	$\dfrac{t - t_i}{t_0 - t_i} = \dfrac{1}{R}\ \mathrm{erfc}\left(Fo^{*}_{r} - Fo^{*}_{r_0}\right)$

Case No.	References	Description	Solution
6.1.14	9, p. 247	Spherical surface with convection boundary. $t = t_i,\ r > r_0,\ \tau = 0.$  	$$\frac{t - t_i}{t_f - t_i} = \frac{Bi}{(Bi + 1)R} \left\{ erfc\left(Fo_r^* - Fo_{r_0}^*\right) - \exp[(Bi + 1)(R - 1) + (Bi + 1)Fo] erfc\left[Fo_r^* - Fo_{r_0}^* + (Bi + 1)\sqrt{Fo}\right] \right\}.$$
6.1.15	9, p. 248	Spherical surface with steady heat flux. $t = t_i,\ r = r_0,\ \tau = 0.$ $q_r = q_0,\ r = r_0.$  	$$\frac{(t - t_i)k}{q_0 r_0} = \frac{1}{R}\left[erfc\left(Fo_r^* - Fo_{r_0}^*\right) - \exp(R - 1 + Fo) erfc\left(Fo_r^* - Fo_{r_0}^* + \sqrt{Fo}\right)\right]$$

Case No.	References	Description	Solution
6.1.16	1, p. 336	Point source. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_\infty,\ r > r_0,\ \tau = 0.$ 	$\frac{t - t_\infty}{t_i - t_\infty} = \frac{r_0^3}{6\sqrt{\pi\alpha^3\tau^3}} \exp\left(- Fo_r^{*2}\right),\ r > r_0,\ r_0 \to 0$ If point is a steady source of strength Q_0: $\frac{(t - t_\infty)k}{Q_0/r_0} = \frac{r_0}{4\pi^{3/2}\sqrt{\alpha}} \int_{1/\sqrt{\tau}}^{\infty} \exp\left(- \lambda^2 Fo_r^{*2}\right) d\lambda$ For $\tau \to \infty$: $\frac{(t - t_\infty)kr}{Q_0} = \frac{1}{4\pi}$

Case No.	References	Description	Solution
6.1.17.1	1, p. 340	Spherical shell source. $t = t_i$, $r_i < r < r_0$, $\tau = 0$. $t = t_\infty$, $r > r_0$, $\tau = 0$.  	$\frac{t - t_\infty}{t_i - t_\infty} = \frac{R_0 - 1}{6\sqrt{\pi}R\sqrt{Fo}} \exp\left[-\frac{(R-1)^2}{4Fo}\right] - \exp\left[-\frac{(R+1)^2}{4Fo}\right]$, $R > R_0$, $R_0 \to 0$ $R = r/r_i$ If the shell is a steady source of strength Q_0: $\frac{(t - t_\infty)k}{Q_0/r_i} = \frac{\sqrt{\alpha}}{4\pi^{3/2}r} \int_{1/\sqrt{\tau}}^{\infty} \frac{1}{\lambda^2}\left\{\exp\left[-\frac{(r - r_i)^2\lambda^2}{4\alpha}\right] - \exp\left[-\frac{(r + r_i)^2\lambda^2}{4\alpha}\right]\right\} d\lambda$ For $\tau \to \infty$: $\frac{(t - t_\infty)kr}{Q_0} = \frac{1}{4\pi}$
6.1.17.2	1, p. 342 9, p. 263	Plane source. $t = t_i$, $-\ell < x < +\ell$, $\tau = 0$. $t = t_\infty$, $\lvert x \rvert > \ell$, $\tau = 0$.  	$\frac{t - t_\infty}{t_i - t_\infty} = \frac{1}{\pi Fo} \exp\left(-Fo_x^{*2}\right)$, $\lvert x \rvert > \ell$, $\ell \to 0$ If the plane is a steady source of strength q_0: $\frac{(t - t_\infty)k}{q_0\ell} = \frac{\sqrt{\alpha}}{\ell\sqrt{\pi}} \int_{1/\sqrt{\tau}}^{\infty} \frac{1}{\lambda^2} \exp(-x^2\lambda^2/4\alpha)\, d\lambda$ $= \sqrt{Fo} \exp\left(-Fo_x^{*2}\right) - \frac{\lvert x \rvert}{2} \operatorname{erfc}\left(Fo^*_{\lvert x \rvert}\right)$

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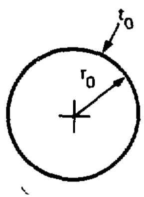
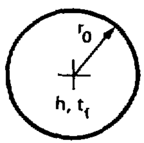
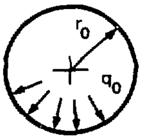
Case No.	References	Description	Solution
6.1.18	9, p. 335	Cylinder with steady surface temp. $t = t_i,\ r > r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$  	$\frac{t - t_i}{t_0 - t_i} = 1 + \frac{2}{\pi}\int_0^\infty \exp(-\lambda^2\ Fo)\frac{J_0(\lambda R)Y_0(\lambda) - Y_0(\lambda R)J_0(\lambda)}{J_0^2(\lambda) + Y_0^2(\lambda)}\frac{d\lambda}{\lambda}$ See Fig. 6.4 $\frac{qr_0}{k(t_0 - t_i)} = (\pi\ Fo)^{-1/2} + \frac{1}{2} - \frac{1}{4}\left(\frac{Fo}{\pi}\right)^{1/2} + \frac{1}{8}\ Fo\ .\ .\ .\ , \qquad R = 1$ For $Fo < 0.02$: $\frac{(t - t_i)}{(t_0 - t_i)} = \frac{1}{\sqrt{R}}\ \mathrm{erfc}\left(\frac{R - 1}{2\sqrt{Fo}}\right) + \frac{(R - 1)\sqrt{Fo}}{4R^{3/2}}\ \mathrm{ierfc}\left(\frac{R - 1}{2\sqrt{Fo}}\right) + \frac{(9 - 2R - 7R^2)}{32R^{5/2}}\ i^2\mathrm{erfc}\left(\frac{R - 1}{2\sqrt{Fo}}\right) + \ .\ .\ .$
6.1.19	9, p. 337	Cylindrical surface with convection boundary. $t = t_i,\ r > r_0,\ \tau = 0.$  	$\frac{t - t_f}{t_i - t_f} = -2\ \frac{Bi}{\pi}\int_0^\infty \exp(-\lambda^2 Fo)$ $\times\ \frac{J_0(\lambda R)\left[\lambda Y_1(\lambda) + Bi\ Y_0(\lambda)\right] - Y_0(\lambda R)\left[\lambda J_1(\lambda) + Bi\ J_0(\lambda)\right]}{\left[\lambda J_1(\lambda) + Bi\ J_0(\lambda)\right]^2 + \left[\lambda Y_1(\lambda) + Bi\ Y_0(\lambda)\right]^2}\ \frac{d\lambda}{\lambda}$

Section 6.1. Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
6.1.20	9, p. 338	Cylindrical surface with steady heat flux. $t = t_i,\ r > r_0,\ \tau = 0.$ $q_r = q_0,\ r = r_0,\ \tau > 0.$ 	$$\frac{(t - t_i)k}{q_0 r_0} = -\frac{2}{\pi}\int_0^\infty \left[1 - \exp(\lambda^2 Fo)\right] \frac{J_0(\lambda R)Y_1(\lambda) - Y_0(\lambda R)J_1(\lambda)\,d\lambda}{\left[J_1^2(\lambda) + Y_1^2(\lambda)\right]\lambda^2}$$ For Fo < 0.02: $$\frac{(t - t_i)k}{q_0 r_0} = 2\sqrt{\frac{Fo}{R}}\ \mathrm{ierfc}\left(\frac{R - 1}{2\sqrt{Fo}}\right) - \frac{3R + 1}{4R}\sqrt{Fo}\ i^2\mathrm{erfc}\left(\frac{R - 1}{2\sqrt{Fo}}\right) + \ldots$$ For Fo >> 1: $$\frac{(t - t_i)k}{q_0 r_0} = \ln\left(\frac{4Fo}{R^2}\right) - 0.57722$$

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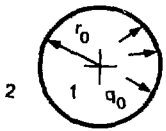
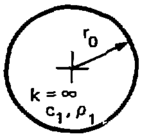
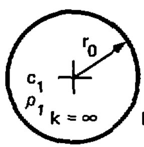
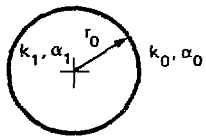
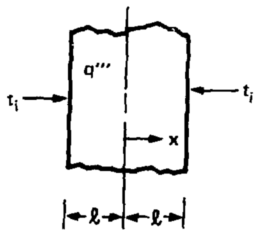
Case No.	References	Description	Solution
6.1.20.1	74, p. 434	Case 6.1.20 with two different thermal materials. $t = t_i$, $r > 0$, $\tau = 0$. $q_r = q_0$, $r = r_0$, $\tau > 0$.	$\frac{(t - t_i)k_1}{q_0 r_0} = \frac{4}{\pi^2} \int_0^\infty \left[1 - \exp\left(-\lambda^2 Fo_1\right)\right] \frac{J_0(\lambda^2 R) J_1(\lambda)\, d\lambda}{\lambda^4(\phi^2 + \psi^2)}$, $R < 1$
			$\frac{(t - t_i)k_2}{q_0 r_0} = \frac{2}{\pi^2} \int_0^\infty \left[1 - \exp\left(-\lambda^2 Fo_1\right)\right] \frac{J_1(\lambda)}{\lambda^3(\phi^2 + \psi^2)}$
			$\times \left[J_0\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda R\right)\phi - Y_0\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda R\right)\psi\right] d\lambda$, $R > 1$
			$\phi = K J_1(\lambda) Y_0\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda\right) - \sqrt{\frac{\alpha_1}{\alpha_2}}\, J_0(\lambda) Y_1\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda\right)$
			$\psi = K J_1(\lambda) J_0\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda\right) - \sqrt{\frac{\alpha_1}{\alpha_2}}\, J_0(\lambda) J_1\left(\sqrt{\frac{\alpha_1}{\alpha_2}}\, \lambda\right)$
			$K = \frac{k_1}{k_2}$

Section 6.1. Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
6.1.21	9, p. 342	Infinite conductivity cylinder in an infinite medium. $t = t_0,\ r > r_0,\ \tau = 0.$ $t = t_1,\ r < r_0,\ \tau = 0.$ 	$\frac{t - t_0}{t_1 - t_0} = \frac{4B}{\pi^2}\int_0^\infty \exp(-\lambda^2\ \mathrm{Fo})\frac{d\lambda}{\lambda\Delta}$, $R < 1$ $B = 2\rho_0 c_0/\rho_1 c_1$ $\Delta = \left[\lambda J_0(\lambda) - BJ_1(\lambda)\right]^2 + \left[\lambda Y_0(\lambda) - BY_1(\lambda)\right]^2$ See Fig. 6.5
6.1.22	9, p. 342	Infinite conductivity cylinder with steady heat source in an infinite medium. $t = t_0,\ r > 0,\ \tau = 0.$ Heat rate = Q per unit length. 	$\frac{(t - t_0)k}{Q} = \frac{B^2}{\pi^3}\int_0^\infty \left[1 - \exp(-\lambda^2\ \mathrm{Fo})\right]\frac{d\lambda}{\lambda^3\Delta}$, $R < 1$ B and Δ defined in case 6.1.21. See Fig. 6.6.

Case No.	References	Description	Solution
6.1.23	9, p. 346	Cylinder with properties different from surrounding medium. $t = t_1,\ r < r_0,\ \tau = 0.$ $t = t_0,\ r > r_0,\ \tau = 0.$ 	$\frac{t - t_0}{t_1 - t_0} = \frac{4\gamma}{\pi^2}\int_0^\infty \exp\left(-\lambda^2 Fo_1\right)\frac{J_0(\lambda R)J_1(\lambda)\,d\lambda}{\lambda^2\left[\phi^2 + \psi^2\right]},\ R < 1.$ $\frac{t - t_0}{t_1 - t_0} = \frac{2\gamma}{\pi}\int_0^\infty \exp\left(-\lambda^2 Fo_1\right)\frac{J_1(\lambda)\left[J_0(\beta\lambda R)\phi - Y_0(\beta\lambda R)\psi\right]d\lambda}{\lambda(\phi^2 + \psi^2)},$ $R > 1.$ $\psi = \gamma J_1(\lambda)J_0(\beta\lambda) - \beta J_0(\lambda)J_1(\beta\lambda)$ $\phi = \gamma J_1(\lambda)Y_0(\beta\lambda) - \beta J_0(\lambda)Y_1(\beta\lambda)$ $\beta = \sqrt{\alpha_1/\alpha_0},\ \gamma = k_1/k_0$ See case 6.1.4 for equal material properties.
6.1.24	9, p. 402	Point source of rectangular periodic wave heating. $Q(\tau) = 0,\ \tau < 0.$ $Q(\tau) = Q_0,\ n\tau_0 < \tau < n\tau_0 + \tau_1,\ n = 0, 1, \ldots$ $Q(\tau) = 0,\ n\tau_0 + \tau < \tau < (n + 1)\tau_0.$ n = Number of cycles. $t = t_\infty,\ r \to \infty.$ 	$\frac{(t - t_\infty)4\pi rk}{Q_0} = \operatorname{erfc}\left(\frac{Fo^*}{2\sqrt{T}}\right) - T_1 + 1$ $+ \frac{2}{\pi}\int_0^\infty \frac{\exp(-T\lambda^2)\left[\exp\left(-\lambda^2 + T_1\lambda^2\right) - \exp(-\lambda^2)\right]\sin(Fo^*\lambda)\,d\lambda}{\lambda\left[1 - \exp(-\lambda^2)\right]},$ $0 < T < T_1.$ $T_1 = \tau_1/\tau_0,\ T = \tau/\tau_0,\ Fo^* = r/\sqrt{\sigma\tau_0}$

Section 8.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution
8.2.3	9, p. 131	Infinite plate with time dependent internal heating. $t = t_i$, $-\ell < x < \pm\ell$, $\tau = 0$. $t = t_i$, $x = \pm\ell$, $\tau > 0$. $q''' = f(\tau)$. 	$\left(t - t_i\right) = \frac{4\alpha}{\pi k} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \cos\left[\frac{(2n+1)}{2} \pi X\right]$ $\times \int_0^{\tau} f(\tau) \exp\left[-\pi^2 (2n+1)^2 \alpha(\tau - \tau')/\ell^2\right] d\tau'$
8.2.4	9, p. 132	Case 8.2.3 with $f(\tau) = q_0''' e^{-b\tau}$.	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{1}{Pd}\left[\frac{\cos(X\sqrt{Pd})}{\cos(\sqrt{Pd})} - 1\right] \exp(-b\tau)$ $+ \frac{2}{\pi Pd} \sum_{n=0}^{\infty} \frac{(-1)^n \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n\left(1 - \lambda_n^2/Pd\right)}$ $\lambda_n = (2n + 1)\pi/2$

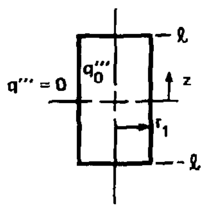
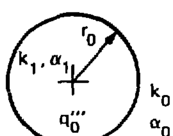
Section 6.1. Infinite Solids--No internal heating.

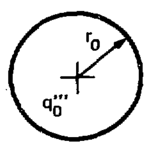
Case No.	References	Description	Solution

Section 6.1. Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution

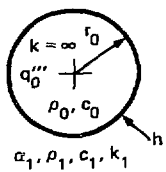
Section 6.2. Infinite Solids--With Internal Heating.

Case No.	References	Description	Solution
6.2.1	9, p. 265	Cylindrical source of heating. $q''' = q_0'''$, $0 < r < r_1$, $-\ell < z < +\ell$, $\tau > 0$. $q''' = 0$, $r > r_i$, $\|z\| > \ell$, $\tau > 0$. $t = t_i$, $r > 0$, $\|z\| > 0$, $\tau = 0$. 	$\frac{(t - t_i)k}{q_0''' r_1^2} = \frac{\alpha}{r_i^2} \int_0^{\tau} \left[1 - \exp\left(-r_1^2/4\alpha u\right)\right] \mathrm{erf}\left(\frac{\ell}{2\sqrt{\alpha u}}\right) du$, $r = 0$, $Z = 0$ For $\ell \to \infty$: $\frac{(t - t_i)k}{q_0''' r_1^2} = \mathrm{Fo}\left[1 - \exp\left(-\frac{1}{4\,\mathrm{Fo}}\right)\right] - \frac{1}{4}\,\mathrm{Ei}\left(-\frac{1}{4\,\mathrm{Fo}}\right)$, $r = 0$, $Z = 0$ $\mathrm{Fo} = \alpha\tau/r_1^2$
6.2.2	9, p. 347	Case 6.1.23 with a cylinder and internal heating. $t = t_0$, $r > 0$, $\tau = 0$. $q''' = q_0'''$, $0 < r < r_0$, $\tau > 0$. 	$\frac{(t - t_0)k_0}{q_0''' r_0^2} = \frac{4}{\pi^2} \int_0^{\infty} \frac{\left[1 - \exp(-\lambda^2 \mathrm{Fo}_1)\right] J_0(\lambda R) J_1(\lambda)\, d\lambda}{\lambda^4\left[\phi^2 + \psi^2\right]}$, $R < 1$. $\frac{(t - t_0)k_0}{q''' r_0^2} = \frac{2}{\pi} \int_0^{\infty} \frac{\left[1 - \exp(-\lambda\ \mathrm{Fo}_1)\right]\left[J_0(\beta\lambda R)\phi - Y_0(\beta\lambda R)\psi\right] d\lambda}{\lambda^3\left[\phi^2 + \psi^2\right]}$, $R > 1$. β, ϕ, and ψ defined in case 6.1.23.

Case No.	References	Description	Solution
6.2.3	9, p. 348	Sphere with internal heating in an infinite medium. $t = t_0,\ r > 0,\ \tau = 0.$ $q''' = q_0''',\ 0 < r < r_0,$ $\tau > 0.$ $q''' = 0,\ r > r_0,$ $\tau > 0.$ 	$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{Fo^{*2}}{4}\left[1 - 2R\ i^2\mathrm{erfc}\left(\frac{1 - R}{Fo^*}\right) - 2R\ i^2\mathrm{erfc}\left(\frac{1 + R}{Fo^*}\right) - \frac{2\ Fo^*}{R}\ i^3\mathrm{erfc}\left(\frac{1 - R}{Fo^*}\right) + \frac{2\ Fo^*}{R}\ i^3\mathrm{erfc}\left(\frac{1 + R}{Fo^*}\right)\right],$ $0 < R < 1.$ $\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{Fo^{*2}}{2R}\left[i^2\mathrm{erfc}\left(\frac{R - 1}{Fo^*}\right) + i^2\mathrm{erfc}\left(\frac{R + 1}{Fo^*}\right) - Fo^*\ i^3\mathrm{erfc}\left(\frac{R - 1}{Fo^*}\right) + Fo^*\ i^3\mathrm{erfc}\left(\frac{R + 1}{X}\right)\right],\ R > 1.$ $Fo^* = 2\sqrt{\alpha\tau}/r_0$

Section 6.2. Infinite Solids--With Internal Heating.

Case No.	References	Description	Solution
6.2.4	9, p. 349	Infinite conductivity sphere in an infinite medium with contact resistance. $t = t_0,\ r > 0,\ \tau = 0.$ $q''' = q_0''',\ 0 < r < r_0,\ \tau > 0.$ $q''' = 0,\ r > 0,\ \tau > 0.$	$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{1}{3}\left[\frac{1 + Bi}{Bi} - \frac{6}{\pi} K^2\ Bi^2 \times \int_0^\infty \frac{\exp(Fo\ u^2)du}{\left[u^2(1 + Bi) - K\ Bi\right]^2 + \left[u^3 - K\ Bi\ u\right]^2}\right],$ $0 < R < 1.$



For small values of τ:

$$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{K\ Fo}{3}[1 - (K\ Bi\ Fo)/2 + \ldots]$$

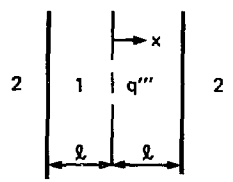
For large values of τ:

$$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{1}{3}\left[\frac{1 + Bi}{Bi} - \frac{1}{\pi Fo} - \frac{2 + Bi(2 - K)}{2\ Bi\ K\ \pi\ \sqrt{Fo^3}} + \ldots\right]$$

$$K = \frac{3\rho_1 c_1}{\rho_0 c_0}$$

Case No.	References	Description	Solution
6.2.5	74, p. 426	Infinite plate with internal heating in an *infinite* medium. $t = t_0,\ -\ell < x < \ell,\ \tau = 0.$ $t = t_\infty,\ \ell < x < -\ell,\ \tau = 0.$	

$$\frac{t - t_\infty}{t_0 - t_\infty} = 1 + \text{Po Fo}_1 - \frac{1}{1 + K} \sum_{n=1}^{\infty} (-H)^{n-1} \left\{ \text{erfc} \left[\frac{(2n - 1) \mp X}{2\sqrt{\text{Fo}_1}} \right] + 4 \text{ Po Fo}_1 \ i^2\text{erfc} \left[\frac{(2n - 1) \mp X}{2\sqrt{\text{Fo}_1}} \right] \right\},\ -1 < X < 1.$$



$$\frac{t - t_\infty}{t_0 - t_\infty} = \frac{1}{1 + K} \left[\text{erfc} \left(\frac{X + 1}{2\sqrt{\text{Fo}_2}} \right) + 4 \text{ Po Fo}_2 \ i^2\text{erfc} \left(\frac{X - 1}{2\sqrt{\text{Fo}_2}} \right) \right]$$

$$- \frac{K(1 + H)}{1 + K} \sum_{n=1}^{\infty} (-H)^{n-1} \left[\text{erfc} \left(\frac{2n\sqrt{\alpha_2/\alpha_1} + X - 1}{2\sqrt{\text{Fo}_2}} \right) \right.$$

$$\left. + 4 \text{ Po Fo}_2 \ i^2\text{erfc} \left(\frac{2n\sqrt{\alpha_2/\alpha_1} + X - 1}{2\sqrt{\text{Fo}_2}} \right) \right],\ 1 < X < -1$$

$$K = \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2},\quad H = \frac{1 - K}{1 + K},\quad f(\mp X) = f(-X) - f(+X)$$

Section 6.2. Infinite Solids--With internal heating.

Case No.	References	Description	Solution

Section 6.2. Infinite Solids--With Internal Heating.

Case No.	References	Description	Solution

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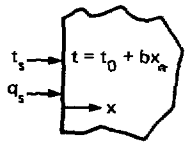
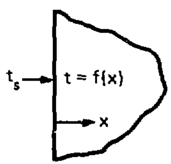
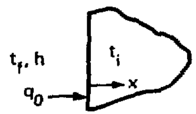
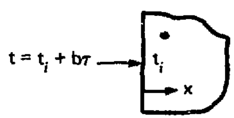
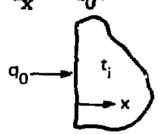
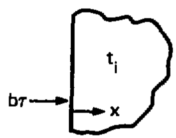
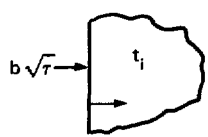
Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.1	9, p. 61	Constant surface temperature. $t = t_0 + bx,\ x > 0,\ \tau = 0.$ $t = t_s,\ x = 0,\ \tau > 0.$  	$\frac{t - t_s}{t_0 - t_s} = \operatorname{erf}\left(Fo_x^*\right) + \frac{bx}{t_0 - t_s}$ *For b = 0:* $\frac{q_s \sigma}{k(t_s - t_0)} = \frac{2}{\sqrt{\pi}}$ See Fig. 7.1
7.1.2	7, p. 89	Variable initial temperature. $t = f(x),\ x > 0,\ \tau = 0.$ $t = t_s,\ x = 0,\ \tau > 0.$  	$t - t_s = \frac{1}{\sigma\sqrt{\pi}} \int_0^\infty f(\beta)\left\{\exp\left[-\left(\frac{x-\beta}{\sigma}\right)^2\right] - \exp\left[-\left(\frac{x+\beta}{\sigma}\right)^2\right]\right\} d\beta$ $f(\beta) = f(x)$

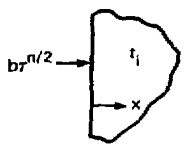
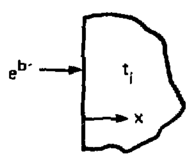
Section 7.1. Semi-Infinite Solids--No Internal Heating.

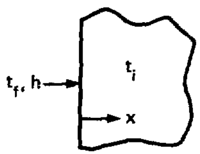
Case No.	References	Description	Solution
7.1.3	5, p. 80 74, *p. 206*	Convection boundary. $t = t_i,\ x > 0,\ \tau = 0.$  	$\frac{t - t_i}{t_f - t_i} = 1 - \mathrm{erf}\left(Fo_x^*\right) - \exp\left(Bi_x + Bi_x^2\ Fo_x\right)\left[1 - \mathrm{erf}\left(Fo_x^* + Bi_x\sqrt{Fo_x}\right)\right]$ $\frac{q_0}{h(t_f - t_i)} = \exp(\alpha\tau h^2/k^2)\,\mathrm{erfc}(\sqrt{\alpha\tau}\ h/k)$ See Fig. 7.2
7.1.4	2, p. 275	Ramp surface temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t = t_i + b\tau,\ x = 0,\ \tau \geq 0.$  	$\frac{t - t_i}{b\tau} = \left(1 + 2\ Fo_x^{*2}\right)\ \mathrm{erfc}\left(Fo_x^*\right) - \frac{2}{\sqrt{\pi}}\ Fo_x^*\ \exp\left(-Fo_x^{*2}\right)$
7.1.5	1, p. 258	Steady surface heat flux. $t = t_i,\ x \geq 0,\ \tau = 0.$ $q_x = q_0,\ x = 0,\ \tau > 0.$  	$\frac{(t - t_i)k}{q_0\sqrt{\alpha\tau}} = 2\sqrt{\pi}\ Fo_x^{*2}\ \mathrm{erfc}\left(Fo_x^*\right) + \exp\left(Fo_x^*\right)$ $\frac{(t - t_i)k}{q_0\sqrt{\alpha\tau}} = \frac{2}{\sqrt{\pi}}\ ,\ x = 0$ See Fig. 7.7

Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.6	9, p. 62	Time dependent surface temperature. $t = t_0,\ x = 0,\ 0 < \tau < \tau_0.$ $t = t_1,\ x = 0,\ \tau > \tau_0.$ $t = t_1,\ x > 0,\ \tau = 0.$	$\frac{t - t_i}{t_0 - t_i} = \operatorname{erfc}\left(Fo_x^*\right),\ 0 < \tau < \tau_0$ $\frac{t - t_i}{t_0 - t_i} = \operatorname{erfc}\left(Fo_x^*\right) + \frac{t_1 - t_0}{t_0 - t_i} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha(\tau - \tau_0)}}\right),\ \tau > \tau_0$
7.1.7	9, p. 63	Time dependent surface temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t = b\tau,\ x = 0,\ \tau > 0.$ 	$\frac{t - t_i}{b\tau} = \left(1 + 2\ Fo_x^{*2}\right)\operatorname{erfc}\left(Fo_x^*\right) - \frac{2}{\sqrt{\pi}}\ Fo_x^*\ \exp\left(-Fo_x^{*2}\right)$
7.1.8	9, p. 63	Time dependent surface temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t = b\sqrt{\tau},\ x = 0,\ \tau > 0.$ 	$\frac{t - t_i}{b\tau} = \exp\left(-2\ Fo_x^{*2}\right) - \sqrt{\pi}\ Fo_x^*\ \operatorname{erfc}\left(Fo_x^*\right)$

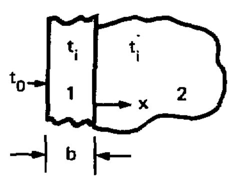
Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.9	9, p. 63	Time dependent surface temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t = t_i + b\tau^{n/2},\ x = 0,\ \tau > 0.$ 	$\frac{t - t_i}{b\tau^{n/2}} = 2^n\Gamma\left(\frac{n}{2} + 1\right) i^n\mathrm{erfc}\left(Fo_x^*\right)$ See Fig. 7.6
7.1.10	9, p. 64	Time dependent surface temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t = e^{b\tau},\ x = 0,\ \tau > 0.$ 	$\frac{t - t_i}{e^{b\tau}} = \frac{1}{2}\exp(-\ x\sqrt{b/\alpha})\,\mathrm{erfc}\left(Fo_x^* - \sqrt{b\tau}\right)$ $+ \exp(x\sqrt{b/\alpha})\,\mathrm{erfc}\left(Fo_x^* + \sqrt{b\tau}\right)$

Case No.	References	Description	Solution
7.1.11	9, p. 74	Convection boundary with step fluid temperature. $t = t_i,\ x > 0,\ \tau = 0.$ $t_f = t_1,\ 0 \le \tau \le \tau_1.$ $t_f = t_2,\ \tau > \tau_1.$  	$\frac{t - t_i}{t_1 - t_i} = \operatorname{erfc}\left(Fo_x^*\right) - \exp\left[Bi_x + (H^2/4)\right]\operatorname{erfc}\left[Fo_x^* + (H/2)\right] \ ,\ 0 < \tau < \tau_1 .$ $\frac{t - t_i}{t_1 - t_i} = \operatorname{erfc}\left(Fo_x^*\right) - \exp\left[Bi_x + (H^2/4)\right]\operatorname{erfc}\left[Fo_x^* + (H/2)\right]$ $+ \frac{t_2 - t_1}{t_1 - t_i}\left\{\operatorname{erfc}\left[\frac{x}{2\sqrt{\alpha(\tau - \tau_0)}}\right] - \exp\left[Bi_x + \frac{h\alpha(\tau - \tau_0)}{k}\right]\right.$ $\left. \times \operatorname{erfc}\left[\frac{x}{2\sqrt{\alpha(\tau - \tau_0)}} + \frac{h\sqrt{\alpha(\tau - \tau_0)}}{k}\right]\right\} \ ,\tau > \tau_0 .$ $H = \frac{2h\sqrt{\alpha\tau}}{k}$

Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.12	9, p. 76	Step surface heat flux. $t = t_i,\ x > 0,\ \tau = 0.$ $q_x = q_0,\ x = 0,\ 0 < \tau < \tau_0.$ $q_x = 0,\ x = 0,\ \tau > \tau_0.$	$\dfrac{(t - t_i)k}{q_0\sqrt{\alpha\tau}} = 2\ \mathrm{ierfc}\left(Fo_x^*\right) - 2\sqrt{\dfrac{(\tau - \tau_0)}{\tau}}\ \mathrm{ierfc}\left(\dfrac{x}{2\sqrt{\alpha(\tau - \tau_0)}}\right),$ $\tau > \tau_0.$ See case 7.1.5 for $0 < \tau < \tau_0$.
7.1.13	9, p. 76	Time dependent surface heat flux. $t = t_i,\ x > 0,\ \tau = 0.$ $q_x = k/\sqrt{\alpha\tau\pi},\ x = 0,$ $0 < \tau < \tau_0.$ $q_x = 0,\ x = 0,\ \tau > \tau_0.$	$t - t_i = 1,\ x = 0,\ 0 < \tau < \tau_0$ $t - t_i = \frac{2}{\pi}\sin^{-1}(\sqrt{\tau_0/\tau})\ ,\ x = 0,\ \tau > \tau_0$ $t - t_i = \dfrac{2}{\pi}\displaystyle\int_0^{\sqrt{\tau_0/(\tau-\tau_0)}} \dfrac{\exp\left[-Fo_x^{*2}(1 + u^2)\right]}{1 + u^2}\,du,\ \tau > \tau_0$

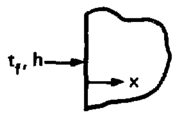
Case No.	References	Description	Solution
7.1.14	9, p. 77	Time dependent surface heat flux. $t = t_i,\ x > 0,\ \tau = 0.$ $q_x = q_0\tau^{n/2},\ x = 0,\ \tau > 0,$ $n = -1, 0, 1, 2 \ldots$	$\frac{(t - t_i)k}{q_0\sqrt{\alpha\tau}\,\tau^{n/2}} = \frac{\Gamma(1 + n/2)}{\Gamma\left(\frac{3}{2} + \frac{n}{2}\right)},\ x = 0,\ \tau > 0$ $\frac{(t - t_i)k}{q_0\sqrt{\alpha\tau}\,\tau^{n/2}} = 2(4)^{n/2}\Gamma(1 + n/2)\,i^{n+1}\mathrm{erfc}\left(Fo_x^*\right)$ See Fig. 7.8 for $n = 2$.
7.1.15	3, p. 402	Plate and semi-infinite solid composite with constant surface temperature. $t = t_i,\ -b < x < \infty,\ \tau = 0.$ $t = t_0,\ x = -b,\ \tau > 0.$ 	$\frac{t_1 - t_i}{t_0 - t_i} = \sum_{n=0}^{\infty} \beta^n \left\{\mathrm{erfc}\left[\frac{(2n + 1)b + x}{\sigma_1}\right] - \beta\ \mathrm{erfc}\left[\frac{(2n + 1)b - x}{\sigma_1}\right]\right\},\ -b < x < 0.$ $\frac{t_2 - t_i}{t_0 - t_i} = \frac{2}{1 + \lambda} \sum_{n=0}^{\infty} \beta^n\ \mathrm{erfc}\left[\frac{(2n + 1)b + \delta x}{\sigma_1}\right],\ x > 0.$ $\delta = \sqrt{\alpha_1/\alpha_2},\ \lambda = k_2\delta/k_1,\ \beta = \frac{\lambda - 1}{\lambda + 1}$

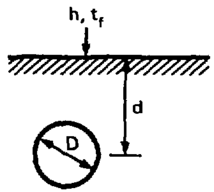
Section 7.1. Semi-Infinite Solids--No Internal Heating.

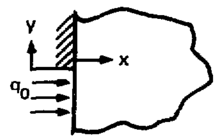
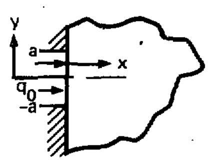
Case No.	References	Description	Solution
7.1.15.1	74, p. 409	Case 7.1.15 with: $t = t_1, 0 < x < b$, $\tau = 0$. $t = t_2$, $x > b$, $\tau = 0$. $t = t_o$, $x = 0$, $\tau > 0$.	$\frac{t - t_1}{t_o - t_1} = \text{erfc}\left(Fo_x^*\right) \mp H \sum_{n=1}^{\infty} H^{n-1}\,\text{erfc}\left(2n\,Fo_b^* \mp Fo_x^*\right)$ $\mp \frac{t_1 - t_2}{(t_o - t_1)(1 + K)} \sum_{n=1}^{\infty} H^{n-1}\,\text{erfc}\left[(2n - 1)\,Fo_b^* \mp Fo_x^*\right],$ $o < X < 1.$ $\frac{t - t_2}{t_o - t_1} = \frac{2K}{1 + K} \sum_{n=1}^{\infty} H^{n-1}\,\text{erfc}\left[Fo_x^* - Fo_x^* + Fo_b^* + (2n - 1)B\,Fo_b^*\right]$ $+ \frac{(t_1 - t_2)K}{(t_0 - t_1)(1 + K)}\,\text{erfc}\left[Fo_x^* - Fo_b^* + \frac{2K}{(1 + K)^2 (t_0 - t_1)}\right]$ $\times \sum_{n=1}^{\infty} H^{n-1}\,\text{erfc}\left[Fo_x^* - Fo_b^* + 2nB\,Fo_b^*\right],\ X > 1.$ $K = \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2},\ H = \frac{1 - K}{1 + K},\ B = \sqrt{\frac{\alpha_2}{\alpha_1}}$

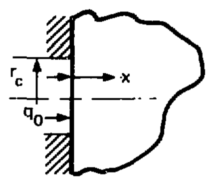
Case No.	References	Description	Solution
7.1.16	7, p. 99	Periodic surface temperature. $t = (t_M - t_0) \cos\left(\frac{2\pi n\tau}{\tau_o}\right)$, $x = 0,\ \tau > 0.$ $t = t_0,\ x > 0,\ t = 0.$  	$\frac{t - t_0}{t_M - t_0} = \exp\left(-\sqrt{\frac{\omega}{2\alpha}}\,x\right) \cos\left(\omega\tau - \sqrt{\frac{\omega}{2\alpha}}\,x\right)$, See Fig 7.3 T_m = maximum surface temperature τ_0 = period of periodic temperature $\omega = 2\pi n/\tau_0$ Maximum temp: $\frac{t_{max} - t_0}{t_M - t_0} = \exp\left(-\sqrt{\frac{\omega}{2\alpha}}\,x\right)$ Time for t_{max} to reach x: $\frac{t_M}{t_0} = \frac{m}{n} + \frac{x}{2\sqrt{\alpha n\pi\tau_0}}$, m = 0, 1, 2, ... Heat transferred into solid, $\tau_1 < \tau < \tau_2$: $\frac{q\sqrt{\alpha/\tau_0}}{k(t_M - t_0)} = \frac{1}{\sqrt{2\pi}}\left[\cos\left(\omega\tau_1 - \frac{\pi}{4}\right) - \cos\left(\omega\tau_2 - \frac{\pi}{4}\right)\right]$

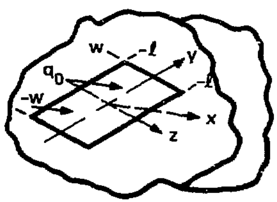
Section 7.1. Semi-In⌞inite Solids--No Internal Heating.

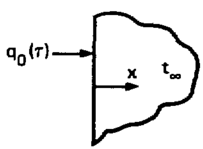
Case No.	References	Description	Solution
7.1.17	7, p. 106	Periodic fluid temperature. $t_f = (t_{fM} - t_0)\cos\left(\frac{2\pi n\tau}{\tau_0}\right)$, $\tau > 0$. $t = t_0$, $x > 0$, $\tau = 0$.  	$\frac{t - t_0}{t_{fM} - t_0} = \frac{\exp\left(-\sqrt{\frac{\omega}{2\alpha}}\,x\right)\cos\left[\omega\tau - \sqrt{\frac{\omega}{2\alpha}}\,x - \tan^{-1}\left(\frac{1}{1 + H}\right)\right]}{\left[1 + (2/H) + (2/H^2)\right]^{1/2}}$ $H = \frac{\alpha\tau_o h^2}{n\pi k^2}$, $\omega = 2n\pi/\tau_0$ t_{fM} = max fluid temperature τ_0 = period of periodic fluid temperature Maximum temp: $\frac{t_M - t_o}{(t_{fM} - t_o)} = \frac{\exp\left(-\sqrt{\frac{\omega}{2\alpha}}\,x\right)}{\left[1 + (2/H) + (2/H^2)\right]^{1/2}}$
7.1.18	9, p. 68	Square wave surface temperature. $t = t_m + t_D$, $x = 0$, $2m\tau_0 < \tau < (2m + 1)\tau_0$, $m = 0, 1, 2, \ldots$. $t = t_m - t_D$, $x = 0$, $(2m + 1)\tau_0 < \tau <$ $(2m + 2)\tau_0$.	$\frac{t - t_D}{t_m - t_D} = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n + 1}\exp\left(-x\sqrt{\frac{(2n + 1)\pi}{2\alpha\tau_o}}\right)\sin\left[\frac{(2n + 1)\pi\tau}{t_o} - x\sqrt{\frac{(2n + 1)}{2\alpha\tau_0}}\right]$ τ_0 = period of periodic temp t_m = mean surface temp t_D = deviation of surface temp from t_m
7.1.19	2, p. 248	Quarter-infinite solid.	Dimensionless temperatures equal product of solution for semi-infinite solid (case 7.1.1 or 7.1.3) and solution for infinite plate (case 8.1.6 or 8.1.7). See Figs. 9.4a and 9.4b.
7.1.20	2, p. 248	Eighth-infinite solid.	
7.1.21	2, p. 248	Semi-infinite plate.	

Case No.	References	Description	Solution
7.1.22	4, p. 84	Infinite cylinder in a semi-infinite solid. Cylinder assumed massless.  	Time for soil to reach steady state: $\frac{\alpha\tau}{D^2} \simeq C(d/D + 1/Bi_D)^{1.44}$. C = 4.6 for const. cylinder temperature. C = 6.0 for const. heat flux from cylinder. The heat transferred during the time given above: $\frac{Q\alpha}{KD^2(t_c - t_f)} = 12(d/D + 1/Bi_D)^{1.25}$. Temperature of cylinder = t_c. Initial temperature of semi-infinite solid = t_f. $t_c > t_f$. The time for the cylinder to return to temp t after steady state is achieved and heating is stopped: $\frac{\alpha\tau}{D^2} = (d/D + 1/Bi_D)^{2.3}\, f\left(\frac{t - t_f}{t_c - t_f}\right)$. See Fig. 7.4 for values of f.

Case No.	References	Description	Solution
7.1.23	9, p. 264	Semi-infinite surface heating. $t = t_i$, $0 < x < \infty$, $-\infty < y < +\infty$, $\tau = 0$. $q_x = q_0$, $x = 0$, $-\infty < y < 0$. $q_x = 0$, $x = 0$, $0 < y < +\infty$. 	$\frac{(t - t_i)k}{q_o\sqrt{\alpha\tau}} = \frac{1}{\sqrt{\pi}}\left[\mathrm{erfc}\left(Fo_y^*\right) + \frac{Fo_y^*}{\sqrt{\pi}}\,\mathrm{Ei}\left(-Fo_y^{*2}\right)\right]$, $x = 0$.
7.1.24	9, p. 264	Infinite strip heated surface. $t = t_i$, $0 < x < \infty$, $-\infty < y < +\infty$, $\tau = 0$. $q_x = q_o$, $x = 0$, $-a < y < +a$. $q_x = 0$, $x = 0$, $y > a$, $y < -a$. 	$\frac{(t - t_i)k}{q_o a} = \frac{\sqrt{Fo_a}}{\sqrt{\pi}}\left\{\mathrm{erf}\left(\frac{1 + Y}{2\sqrt{Fo_a}}\right) + \mathrm{erf}\left(\frac{1 - Y}{2\sqrt{Fo_a}}\right) - \left(\frac{1 + Y}{2\sqrt{Fo_\alpha}\pi}\right)\mathrm{Ei}\left[-\left(\frac{1 + Y}{2\sqrt{Fo_\alpha}}\right)^2\right] - \left(\frac{1 - Y}{2\sqrt{Fo_\alpha}}\right)^2 \mathrm{Ei}\left[-\left(\frac{1 - Y}{2\sqrt{Fo_\alpha}}\right)^2\right]\right\}$, $x = 0$. See Fig. 7.5

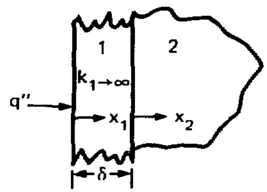
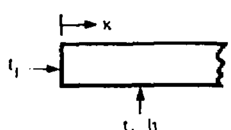
Case No.	References	Description	Solution
7.1.25	9, p. 264	Heating through a circular surface. $q_x = q_0$, $0 < r < r_c$, $x = 0$, $\tau > 0$. $q_x = 0$, $r > r_c$, $x = 0$, $\tau > 0$. $t = t_i$, $0 < r < \infty$, $0 < x < \infty$, $\tau = 0$. 	$\frac{(t - t_i)k}{q_0 r_c} = 2\sqrt{Fo_{r_c}}\left\{\mathrm{ierfc}\left(Fo_x^*\right) - \mathrm{ierfc}\left[Fo_x^*\sqrt{1 + (r_c/x)^2}\right]\right\}$, $r = 0$

Case No.	References	Description	Solution
7.1.26	9, p. 265	Heating through a rectangular surface. $t = t_i$, $x \geq 0$, $y \geq 0$, $z \geq 0$, $\tau = 0$. $q_x = q_0$, $-w < y < +w$, $-\ell < z < +\ell$, $x = 0$, $\tau > 0$. $q_x = 0$, $x = 0$ $\lvert y\rvert > w$, $\lvert z\rvert > \ell$, $\tau > 0$. 	Maximum temp: $\frac{(t_{max} - t_i)k}{q_0 w} = \frac{2}{\pi}\left[\sinh^{-1}(L) + L\sinh^{-1}\left(\frac{1}{L}\right)\right]$, $-w < y < +w$, $-\ell < z < +\ell$, $x = 0$. Average temp: $\frac{(t_m - t_i)k}{q_0 w} = \frac{2}{\pi}\left\{\sinh^{-1}(L) + L\sinh^{-1}\left(\frac{1}{L}\right) + \frac{1}{3}\left[\frac{1}{L} + L^2 - \frac{(L^2+1)^{3/2}}{L}\right]\right\}$, $-w < y < +w$, $-\ell < z + \ell$, $x = 0$. $L = \ell/w$

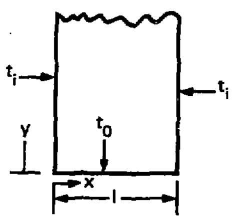
Case No.	References	Description	Solution
7.1.27	9, p. 401	Rectangular wave surface heat flux. $t = t_\infty,\ x \to \infty,\ \tau > 0.$ $q = q_0(\tau),\ x = 0,\ \tau > 0.$ $q_0(\tau) = 0,\ \tau > 0.$ $q_0(\tau) = q_0,$ $n\tau_0 < \tau < n\tau_0 + \tau_1,$ $n = 0, 1, \ldots$ $q_0(\tau) = 0,\ n\tau_0 +$ $\tau_1 < \tau < (n + 1)\tau_0.$ n = No. of cycles.  	At $x = 0$: $\dfrac{(t - t_\infty)k}{q_0\sqrt{\alpha\tau_0}} = \dfrac{2}{\sqrt{\pi}} T_1 \sqrt{T} + \dfrac{2}{\pi}\dfrac{T}{T_1}\left[(1 - T_1)\sqrt{T} - I(T_1, T)/\sqrt{\pi}\right],$ $0 < T < T_1.$ $\dfrac{(t - t_\infty)k}{q_0\sqrt{\alpha\tau}} = \dfrac{2}{\pi} T_1 \sqrt{T} + \dfrac{2}{\sqrt{\pi}}\dfrac{T}{T_1}\left[(1 - T_1)\sqrt{T} - (T - T_1)^{1/2} - I(T_1, T)/\sqrt{\pi}\right],$ $T_1 < T < 1.$ $I(T_1, T) = \displaystyle\int_0^\infty \frac{\exp(-T\lambda^2)\left[(1 - T_1)\exp(-\lambda^2) - \exp(-\lambda^2 + \lambda^2 T_1) + T_1\right]}{\lambda^2\left[1 - \exp(-\lambda^2)\right]}\, d\lambda$ $T = \tau/\tau_0\ ,\ T_1 = \tau_1/\tau_0$

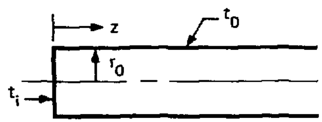
Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.28	9, p. 419	Semi-infinite cylinder with convection boundary. $t = t_0,\ 0 < r_0 < r,\ z > 0,\ \tau = 0.$ $t = t_1,\ 0 < r < r_0,\ z = 0,\ \tau > 0.$ Convection boundary of h, t_0 at $r = r_0,\ z > 0,\ \tau > 0.$	See case 9.1.20
7.1.29	9, p. 419	Case 7.1.28 with: $t = t_1,\ r = r_0,\ z > 0,$ $\tau > 0.$ $t = t_0,\ 0 < r < r_0,\ z = 0,\ \tau > 0.$	See case 9.1.21

Case No.	References	Description	Solution
7.1.30	19, p. 3-83	Plate and semi-infinite solid composite with constant surface heat flux. $t = t_0$, $0 < x_1 < \delta$, $0 < x_2 < \infty$, $\tau = 0$. $k_1 \to \infty$. $q = q''$, $x = 0$, $\tau > 0$. 	$\frac{k_2(t_1 - t_0)}{2\, q'' \sqrt{\alpha_2 \tau}} = \frac{1}{2\, m \sqrt{Fo_2}} \left[\exp\left(m^2\, Fo_2\right) \mathrm{erfc}\,(m \sqrt{Fo_2}) - 1 \right] + \frac{1}{\sqrt{\pi}}, \quad 0 < x_1 < \delta.$ Heat flux at interface: $q / q'' = 1 - \exp\left(m^2\, Fo_2\right)\, \mathrm{erfc}\,(m \sqrt{Fo_2}), \quad x_1 = \delta.$ $m = (\rho_2\, c_2 / \rho_1\, c_1)$ See Fig. 7.9
7.1.31	74, p. 211	Semi-infinite rod with convection boundary. $t = t_i$, $x > 0$, $\tau = 0$. $t = t_f$, $x = 0$, $\tau > 0$. 	$\frac{t - t_i}{t_f - t_i} = \frac{1}{2} \exp\,(\sqrt{Bi_d}\, X)\ \mathrm{erfc}\left(\frac{X}{2\sqrt{Fo_d}} - Bi_d\, Fo_d\right) + \exp\,(Bi_d\, X)\ \mathrm{erfc}\left(\frac{X}{2\sqrt{Fo_d}} + \sqrt{Bi_d\, Fo_d}\right)$ $d = \frac{\text{cross section area}}{\text{perimeter}}, \quad X = x/d$ Heat flux at $x = 0$: $\frac{q_0 d}{k(t_f - t_i)} = \frac{1}{\pi \sqrt{Fo_d}} \exp\,(-Bi_d\, Fo_d) + \sqrt{Bi_d}\ \mathrm{erfc}\,(\sqrt{Bi_d\, Fo_d})$

Section 7.1. Semi-Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
7.1.32	74, p. 465	Semi-infinite plate with fixed surface temperatures. $t = t_i,\ 0 < x < \ell,$ $0 < y < \infty,\ \tau = 0.$ $t = t_i,\ x = 0,\ \ell,$ $0 < y < \infty,\ \tau > 0.$ $t = t_0,\ 0 < x < \ell,$ $y = 0,\ \tau > 0.$ 	$\frac{t - t_i}{t_o - t_i} = X + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(\lambda_n X)\exp\left(-\lambda_n^2\,Fo\right)$ $+ \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin(\lambda_n X)\Big[\exp(\lambda_n Y)\,\mathrm{erfc}$ $\times\left(Fo_y^* + \lambda_n\sqrt{Fo}\right) + \exp(-\lambda_n Y)\,\mathrm{erfc}\left(Fo_y^* - \lambda_n\sqrt{Fo}\right)$ $- 2\exp\left(-\lambda_n^2\,Fo\right)\mathrm{erfc}\left(Fo_y^*\right)\Big].$ $\lambda_n = n\pi.$

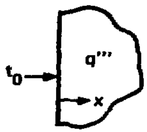
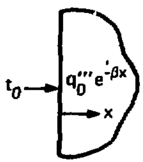
Case No.	References	Description	Solution
7.1.33	74, p. 466	Semi-infinite cylinder with fixed surface temperatures. $t = t_i$, $0 < r < r_0$, $0 < y < \infty$, $\tau = 0$. $t = t_i$, $0 < r < r_0$, $y = 0$, $\tau > 0$. $t = t_0$, $r = r_0$, $0 < y < \infty$, $\tau > 0$. 	$\frac{t - t_i}{t_0 - t_i} = 1 - \sum_{n=1}^{\infty} \frac{J_0(\lambda_n R)}{\lambda_n J_1(\lambda_n)} \left[2 \exp\left(-\lambda_n^2 Fo\right) \mathrm{erf}\left(Fo_z^*\right) + \exp(\lambda_n Z)\,\mathrm{erfc}\left(Fo_z^* + \lambda_n \sqrt{Fo}\right) + \exp(-\lambda_n Z)\,\mathrm{erfc}\left(Fo_z^* - \lambda_n \sqrt{Fo}\right)\right].$ $J_0(\lambda_n) = 0$
7.1.34	74, p. 466	Case 7.1.33 with: convection boundary h, t_i at $r = r_0$, and $t = t_0$, $0 < r < r_0$, $z = 0$, $\tau > 0$.	$\frac{t - t_i}{t_0 - t_i} = \sum_{n=1}^{\infty} \frac{2\,Bi\,J_0(\lambda_n R) \exp(-\lambda_n Z)}{J_0(\lambda_n)\left(Bi^2 + \lambda_n^2\right)} + \sum_{n=1}^{\infty} \frac{Bi\,J_0(\lambda_n R)}{J_0(\lambda_n)\left(Bi^2 + \lambda_n^2\right)} \left[\exp(\lambda_n Z)\,\mathrm{erfc}\left(\lambda_n \sqrt{Fo} + Fo_z^*\right) - \exp(-\lambda_n Z)\,\mathrm{erfc}\left(\lambda_n \sqrt{Fo} - Fo_z^*\right)\right].$ $\frac{J_0(\lambda_n)}{J_1(\lambda_n)} = \frac{\lambda_n}{Bi}$

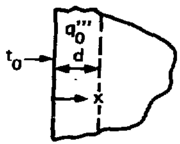
Section 7.1. Semi-infinite Solids--No Internal Heating.

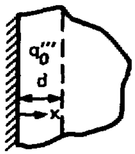
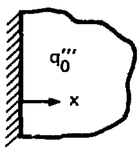
Case No.	References	Description	Solution

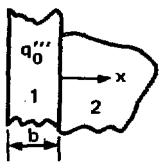
Section 7.1. Semi-infinite Solids--No Internal Heating.

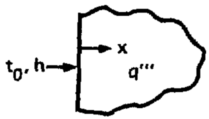
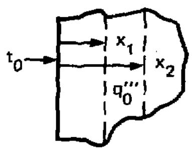
Case No.	References	Description	Solution

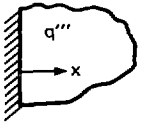
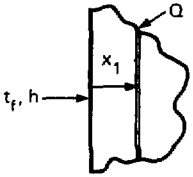
Case No.	References	Description	Solution
7.2.1	9, p. 79	Steady surface temperature and initial temperature distribution. $t = T_1 + bx,\ x \geq 0,$ $\tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$ 	$\frac{t - t_0}{T_1} = (1 + Po^* + Po^* Fo^{*2})\ \mathrm{erf}\left(Fo_x^*\right) + \frac{2}{\sqrt{\pi}} Po^* Fo_x^* \exp\left(- Fo_x^{*2}\right) + \frac{bx}{T_1} - Po^* Fo_x^{*2}.$ $Po^* = \frac{q''' \alpha\tau}{kT_1}$
7.2.2	9, p. 79	Case 7.2.1--variable heating. $t = T_1 + bx,\ x \geq 0,$ $\tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$ $q''' = q_0''' e^{-\beta x}.$ 	$\frac{(t - t_o) k\beta^2}{q_0'''} = \frac{bx}{T_1\ Po^*} + \left(\frac{1}{Po^*} - 1\right) \mathrm{erf}\ (Fo^*) + 1 - e^{-\beta x} + \frac{1}{2} e^{\lambda-\beta x}\ \mathrm{erfc}\ (\sqrt{\lambda} - Fo^*) - \frac{1}{2} e^{\lambda+\beta x}\ \mathrm{erfc}\ (\sqrt{\lambda} + Fo^*).$ $Po^* = \frac{q_0'''}{kT_1\beta^2},\quad \lambda = \alpha\beta^2\tau$

Case No.	References	Description	Solution
7.2.3	9, p. 79	Case 7.2.1 with band heating. $t = T_1 + bx,\ x \geq 0,$ $\tau = 0.$ $t = t_o,\ x = 0,\ \tau > 0.$ $q''' = q_0''',$ $0 < x < d.$ $q''' = 0,\ x > d.$ 	$\frac{t - t_0}{T_1} = Po^* \Big\{1 - 4\ i^2\text{erfc}\left(Fo_x^*\right) + 2\ i^2\text{erfc}\left[Fo_d^*\ (1 + X)\right] - 2\ i^2\text{erfc}\left[Fo_d^*(1 - X)\right]\Big\} + \frac{bx}{T_1} + \text{erf}\left(Fo_x^*\right),\ 0 < X < 1 .$ $\frac{t - t_o}{T_1} = 2\ Po^* \Big\{i^2\text{erfc}\left[Fo_d^*(X - 1)\right] + i^2\text{erfc}\left[Fo_d^*(X + 1)\right] - 2\ i^2\text{erfc}\left(Fo_x^*\right)\Big\} + \frac{bx}{T_1} + \text{erf}\left(Fo_x^*\right),\ X > 1 .$ $Po^* = \frac{q_0'''\ \alpha\tau}{kT_1}$

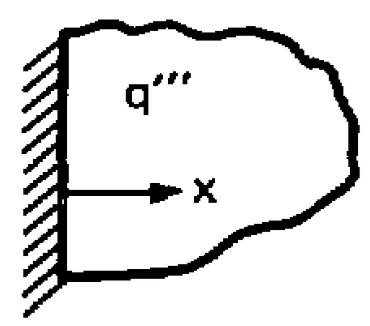
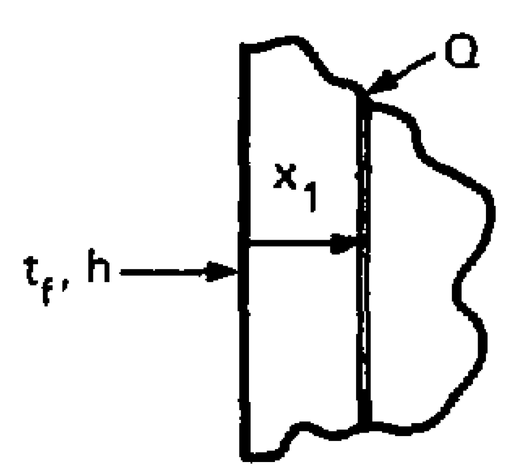
Case No.	References	Description	Solution
7.2.4	9, p. 80	Insulated boundary and band heating. $t = t_i,\ x \geq 0,\ \tau = 0.$ $q_x = 0,\ x = 0,\ \tau > 0.$ $q''' = q_0''',\ 0 < x < d,$ $\tau > 0.$ $q''' = 0,\ x > d.$ 	$\frac{(t - t_i)k}{q_0'''\alpha\tau} = 1 - 2\ i^2\mathrm{erfc}\left[Fo_d^*(1 - X)\right] - 2\ i^2\mathrm{erfc}\left[Fo_d^*(1 + X)\right],\quad 0 < X < 1.$ $\frac{(t - t_i)k}{q_0'''\alpha\tau} = 2\left\{i^2\mathrm{erfc}\left[Fo_d^*(X - 1)\right] - i^2\mathrm{erfc}\left[Fo_d^*(X + 1)\right]\right\},\quad X > 1.$
7.2.5	9, p. 80	Insulated boundary and variable heating. $t = t_0 + bx,\ x > 0,$ $\tau = 0.$ $q_x = 0,\ x = 0,\ \tau > 0.$ $q''' = q_0'''e^{-\beta x},$ $x > 0,\ \tau > 0.$ 	$\frac{(t - t_0)k\beta^2}{q_0'''} = \frac{\lambda^2}{Po_x^*} - \exp(-\beta x) + \left(\frac{\lambda^2}{Fo_x^*\ Po_x^*} + 2\lambda\right)\mathrm{erfc}\left(Fo_x^*\right)$ $+ \frac{1}{2}\exp(\lambda^2 - \beta x)\ \mathrm{erfc}\left(\lambda - Fo_x^*\right) + \frac{1}{2}\exp(\lambda^2 + \beta x)\ \mathrm{erfc}\left(\lambda + Fo_x^*\right)$ $Po_x^* = \frac{q_0'''\alpha\tau}{kbx},\ \lambda = \beta\sqrt{\alpha\tau}$

Case No.	References	Description	Solution
7.2.6	9, p. 323	Plate and semi-infinite solid composite. $t = t_0$, $-b < x < \infty$, $\tau = 0$. $t = t_0$, $x = -b$, $\tau > 0$. $q''' = q_0'''$, $-b < x < 0$, $\tau > 0$. $q''' = 0$, $x < 0$, $\tau > 0$. 	$\frac{(t_1 - t_0)k_1}{q_0'''\alpha\tau} = 1 - 4 \sum_{n=0}^{\infty} \beta^n \left[i^2\mathrm{erfc}\left(\frac{(2n+1) + X}{2\sqrt{Fo}}\right) - \beta\, i^2\mathrm{erfc}\left[\frac{(2n+1) - X}{2\sqrt{Fo}}\right] + \frac{\lambda}{1+\lambda}\, i^2\mathrm{erfc}\left(\frac{2n + X}{2\sqrt{Fo}}\right) - \frac{\lambda}{1+\lambda}\, i^2\mathrm{erfc}\left(\frac{(2n+2) + X}{2\sqrt{Fo}}\right)\right], \quad -1 < X < 0.$ $\frac{(t_2 - t_0)k_1}{q_0'''\alpha\tau} = \frac{4}{(1+\lambda)} \sum_{n=0}^{\infty} \beta^n \left\{ i^2\mathrm{erfc}\left(\frac{2n + \delta X}{2\sqrt{Fo}}\right) + i^2\mathrm{erfc}\left[\frac{(2n+2) + \delta X}{2\sqrt{Fo}}\right] - 2\, i^2\mathrm{erfc}\left[\frac{(2n+1) + \delta X}{2\sqrt{Fo}}\right]\right\}, \quad X > 0.$ $\delta = \sqrt{\alpha_1/\alpha_2}$, $\lambda = k_2\delta/k_1$, $\beta = \frac{\lambda - 1}{\lambda + 1}$, $Fo = \frac{\alpha_1\tau}{b^2}$, $X = \frac{x}{b}$

Case No.	References	Description	Solution
7.2.7	9, p. 307	Time dependent heating and convection boundary. $t = t_0,\ x > 0,\ \tau = 0.$ $q''' = q_0'''\tau^{s/2},$ $s = -1, 0, 1, \ldots$ 	$$\frac{(t - t_0)k}{q_0'''\alpha\tau^{1+(s/2)}} = \frac{1}{[1 + (s/2)]} + \frac{\Gamma[1 + (s/2)]}{Fo_x^{(s/2)+1}\,(-Bi_x)^{s+2}} \times \left[\exp\left(Bi_x + Bi_x^2\,Fo_x\right)\mathrm{erfc}\left(Fo_x^* + Bi_x\sqrt{Fo_x}\right) - \sum_{n=0}^{s+2}\left(-\frac{Bi_x}{Fo_x^*}\right)^n i^n\mathrm{erfc}\left(Fo_x^*\right)\right]$$
7.2.8	9, p. 308	Constant temperature surface and band heating. $t = t_0,\ x > 0,\ \tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$ $q''' = 0,\ 0 < x < x_1,$ $x > x_2.$ $q''' = q_0'''$ $x_1 < x < x_2.$ 	Surface heat flux: $$\frac{q_s}{q_0'''\,\alpha\tau} = 2\left[\mathrm{ierfc}\left(Fo_{x_1}^*\right) - \mathrm{ierfc}\left(Fo_{x_2}^*\right)\right],\ x = 0$$

Case No.	References	Description	Solution
7.2.9	54 19, p. 384	Exponential heating. $t = t_0,\ x > 0,\ \tau = 0.$ $q''' = q_0''' \exp(-\mu x).$  	$\frac{k(t - t_0)}{2\,q_0'''\sqrt{\alpha\tau}} = \operatorname{ierfc}\left(Fo_x^*\right) - \frac{\exp\left(-2\,M\,Fo_x^*\right)}{M}\left[1 - \exp\,(M)^2\right]$ $- \frac{\exp\,(M)^2}{4M}\left[\exp\left(-2\,M\,Fo_x^*\right)\operatorname{erfc}\left(Fo_x^* - M\right) - \exp\left(2\,M\,Fo_x^*\right)\operatorname{erfc}\left(Fo_x^* + M\right)\right].$ Surface temp: $\frac{k(t_w - t_0)}{2\,q_0'''\sqrt{\alpha\tau}} = 0.5642 - \frac{1 - \exp\,(M^2)\ \operatorname{erfc}\,(M)}{2M},\quad x = 0.$ $M = \mu\sqrt{\alpha\tau}$ See Fig. 7.10
7.2.10	74, p. 383	Planar heat pulse and convection boundary. Instantaneous heat pulse occurs at $\tau = 0$, $x = x_1$ with strength $Q\ \frac{J}{m^2}$. $t = t_f,\ x > 0,\ \tau = 0.$  	$\frac{(t - t_f)kx_1}{Q\alpha} = \frac{1}{2\sqrt{\pi Fo}}\left\{\exp\left[-\frac{(X-1)^2}{4\,Fo}\right] + \exp\left[-\frac{(X-1)^2}{4\,Fo}\right]\right.$ $\left. - Bi\exp\left[Bi\,(X - 1) + Bi^2\,Fo\right]\operatorname{erfc}\left[\frac{X+1}{2\,Fo} + Bi\sqrt{Fo}\right]\right\}$

Section 7.2. Semi-infinite Solids--With Internal Heating.

Case No.	References	Description	Solution
7.2.9	54 19, p. 384	Exponential heating. $t = t_0$, $x > 0$, $\tau = 0$. $q''' = q_0''' \exp(-\mu x)$. 	$\frac{k(t - t_0)}{2\, q_0'''\sqrt{\alpha\tau}} = \mathrm{ierfc}\left(Fo_x^*\right) - \frac{\exp\left(-2\, M\, Fo_x^*\right)}{M}\left[1 - \exp\,(M)^2\right]$ $- \frac{\exp\,(M)^2}{4M}\left[\exp\left(-2\, M\, Fo_x^*\right)\mathrm{erfc}\left(Fo_x^* - M\right)\right.$ $\left. - \exp\left(2\, M\, Fo_x^*\right)\mathrm{erfc}\left(Fo_x^* + M\right)\right]$. Surface temp: $\frac{k(t_w - t_0)}{2\, q_0'''\sqrt{\alpha\tau}} = 0.5642 - \frac{1 - \exp\,(M^2)\,\mathrm{erfc}\,(M)}{2M}, \quad x = 0$. $M = \mu\sqrt{\alpha\tau}$ See Fig. 7.10
7.2.10	74, p. 383	Planar heat pulse and convection boundary. Instantaneous heat pulse occurs at $\tau = 0$, $x = x$, with strength $Q\,\frac{J}{m^2}$. $t = t_f$, $x > 0$, $\tau = 0$. 	$\frac{(t - t_f)kx_1}{Q\alpha} = \frac{1}{2\sqrt{\pi Fo}}\left\{\exp\left[-\frac{(X-1)^2}{4\,Fo}\right] + \exp\left[-\frac{(X-1)^2}{4\,Fo}\right]\right.$ $\left. - Bi\,\exp\left[Bi\,(X-1) + Bi^2\,Fo\right]\mathrm{erfc}\left[\frac{X+1}{2\,Fo} + Bi\sqrt{Fo}\right]\right\}$

Section 7.2. Semi-infinite Solids--With Internal Heating.

Case No.	References	Description	Solution

Section 7.2. Semi-infinite Solids--With Internal Heating.

Case No.	References	Description	Solution

8. Plane Surface — Transient

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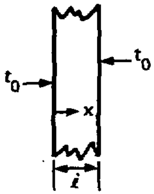
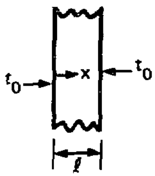
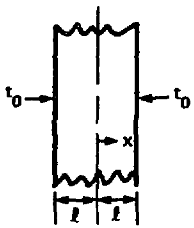
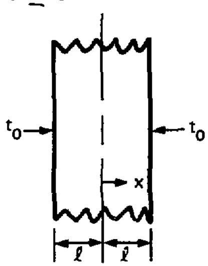
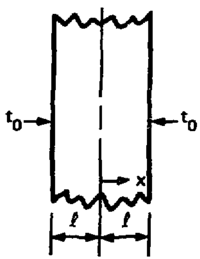
Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

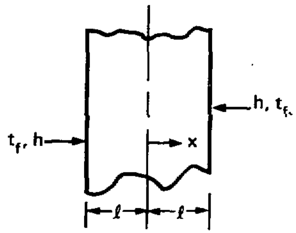
Case No.	References	Description	Solution
8.1.1	9, p. 94	Infinite plate with steady surface temperature. $t = t(x),\ 0 < x < \ell,$ $\tau = 0.$ $t = t_0,\ x = 0,\ \ell,\ \tau \geq 0.$ 	$t - t_0 = 2 \sum_{n=1}^{\infty} A_n \sin (n\pi X) \exp \left(-n^2\pi^2 \text{ Fo}\right)$ $A_n = \int_0^1 f(X) \sin (n\pi X)\, dX$ See case 8.1.25 for convection cooling at $x = \ell$.
8.1.2	9, p. 96	Infinite plate with steady surface temperature and linear initial temperature. $t = t_0 + bx,\ 0 < x < \ell,$ $\tau = 0.$ $t = t_0,\ x = 0,\ \ell,\ \tau \geq 0.$ 	$\frac{t - t_0}{b\ell - t_0} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin (n\pi X) \exp (-n^2\pi^2 \text{ Fo})$

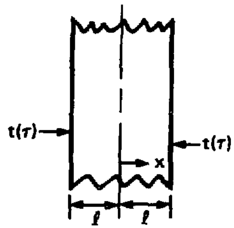
Case No.	References	Description	Solution
8.1.3	9, p. 97	Infinite plate with steady surface temperature and bilinear initial temperature. $t = (t_c - t_0)(\ell - \lvert x\rvert)/(\ell + t_0)$, $-\ell < x < +\ell$, $\tau = 0$. $t = t_0$, $x = \pm\ell$, $\tau > 0$. 	$\frac{t - t_0}{t_c - t_0} = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\left[\frac{(2n+1)}{2}\pi X\right]\exp\left[-\frac{(2n+1)^2}{4}\pi^2 Fo\right]$ $= 1 - \lvert X\rvert - 2\sqrt{Fo}\sum_{n=0}^{\infty}(-1)^n\left\{\mathrm{ierfc}\left(\frac{2n + \lvert X\rvert}{2\sqrt{Fo}}\right) - \mathrm{ierfc}\left[\frac{(2n+2) - \lvert X\rvert}{2\sqrt{Fo}}\right]\right\}$ See Fig. 8.1

Case No.	References	Description	Solution
8.1.4	9, p. 98	Infinite plate with steady surface temperature and bi-parabolic initial temperature. $t = (t_c - t_0)(\ell^2 - x^2)/\ell^2 + t_c$, $-\ell < x < +\ell$, $\tau = 0$. $t = t_0$, $x = \pm\ell$, $\tau \geq 0$ (see case 8.1.3).	$\dfrac{t - t_0}{t_c - t_0} = \dfrac{32}{\pi^2} \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n+1)^3} \cos\left[\dfrac{(2n+1)}{2} \pi X\right] \exp\left[- \dfrac{(2n+1)^2}{4} \pi^2 \, Fo\right]$ See Fig. 8.2
8.1.5	9, p. 100	Infinite plate with steady surface temperature and cosine initial temperature. $t - t_0 = (t_c - t_0) \cos\left(\dfrac{\pi x}{2\ell}\right) + t_c$, $-\ell < x < +\ell$ $\tau = 0$. $t = t_0$, $x = \pm\ell$, $\tau \geq 0$. 	$\dfrac{t - t_0}{t_c - t_0} = \cos\left(\dfrac{\pi}{2} X\right) \exp\left(- \dfrac{\pi^2}{4} Fo\right)$

Case No.	References	Description	Solution
8.1.6	9, p. 97 74, p. 113	Infinite plate with steady temperature and constant initial temperature. $t = t_i,\ -\ell < x < \ell,$ $\tau = 0.$ $t = t_0,\ x = \pm\ell,$ $\tau \geq 0.$ 	$\frac{t - t_0}{t_i - t_0} = 2 \sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_n} \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo\right)$ Mean temp: $\frac{t_m - t_0}{t_i - t_0} = 2 \sum_{n=0}^{\infty} \frac{1}{\lambda_n^2} \exp\left(-\lambda_n^2 Fo\right)$ $\frac{t_m - t_0}{t_i - t_0} = 1 - 2\sqrt{\frac{Fo}{\pi}},\ Fo < 0.1$ Heat loss: $\frac{Q}{2\ell\rho c\,(t_i - t_0)} = 2 \sum_{n=1}^{\infty} \frac{\sin^2\left\{\lambda_n\left[1 - \exp\left(-\lambda_n^2 Fo\right)\right]\right\}}{\lambda_n\left[\lambda_n + \sin(\lambda_n)\cos(\lambda_n)\right]}$ $\lambda = (2n + 1)\pi/2,$ See Fig. 8.3

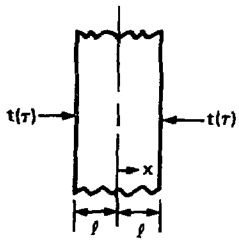
Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
8.1.7.1	74, p. 220	Infinite plate with convection boundary (general). $t = f(x)$, $-\ell < x < +\ell$, $\tau = 0$.  	$t - t_f = 2 \sum_{n=1}^{\infty} \frac{\lambda_n \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo_\ell\right)}{\lambda_n + \sin(\lambda_n) \cos(\lambda_n)} \int_0^1 \left[f(X) - t_f\right] \cos(\lambda_n X)\, dX$ $\cot(\lambda_n) = \lambda_n/Bi_\ell$ λ_n given in Table 14.1
8.1.7.2	3, p. 294 74, p. 223	Infinite plate with convection boundary--constant initial temp. $t = t_i$, $-\ell < x < +\ell$, $\tau = 0$. (see case 8.1.7.1)	$\frac{t - t_f}{t_i - t_f} = 2 \sum_{n=1}^{\infty} \frac{\sin(\lambda_n) \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n + \sin(\lambda_n) \cos(\lambda_n)}$ $\cot(\lambda_n) = \lambda_n/Bi$ $\frac{t - t_f}{t_i - t_f} = 1 - \cos(\sqrt{Bi}\, X) \exp(-Bi\, Fo)$, $Bi < 0.1$ $\frac{t - t_f}{t_i - t_f} = \text{erf}\left(\frac{1 - X}{2\sqrt{Fo}}\right) - \exp\left[Bi(1 - X) + Bi^2 Fo\right] \text{erfc}\left(\frac{1 - X}{2\sqrt{Fo}} + Bi\sqrt{Fo}\right)$ $+ \text{erfc}\left(\frac{1 + X}{2\sqrt{Fo}}\right) - \exp\left[Bi(1 + X) + Bi^2 Fo\right] \text{erfc}\left(\frac{1 + X}{2\sqrt{Fo}} + Bi\sqrt{Fo}\right)$, $Fo < 0.1$. See Figs. 8.4a, b, c, d and e and 9.1d.

Case No.	References	Description	Solution
8.1.8	1, p. 268	Infinite plate with time varying surface temperature. $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i + b\tau$, $x = \pm\,\ell$, $\tau > 0$. 	$\frac{(t - t_i)\,\alpha}{b\ell^2} = Fo + \frac{X^2 - 1}{2} + \frac{16}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\exp\left[-\left(\frac{2n+1}{2\pi}\right)^2 Fo\right] \times \cos\left[(2n+1)\frac{\pi}{2}X\right]$ See Table 8.1, Fig. 8.16 and Fig. 8.25.
8.1.9	9, p. 104	Infinite plate with time varying surface temperature (general). $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i + f(\tau)$, $x = \pm\,\ell$, $\tau > 0$.	$t - t_i = \pi\sum_{n=0}^{\infty}(2n+1)(-1)^n\,Fo\,\exp\left[-\frac{(2n+1)^2\pi^2}{4}Fo\right] \times \cos\left[\frac{(2n+1)\pi}{2}X\right]\int_0^{\tau}\exp\left[\frac{(2n+1)^2\pi^2\alpha\tau'}{4\ell^2}\right]\left[t_i + f(\tau')\right]d\tau'$

Case No.	References	Description	Solution
8.1.10	9, p. 104	Infinite plate with exponentially varying surface temperature. $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i + T\,(1 - e^{-b\tau})$, $x = \pm\ell$, $\tau > 0$.	$\frac{t - t_i}{T - t_i} = 1 - e^{-b\tau}\frac{\cos\,(X\sqrt{Pd})}{\cos\,(\sqrt{Pd})} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\left[1 - (2n+1)^2\,\pi^2/4Pd^2\right]}$ $\times \exp\left[-\frac{(2n+1)^2\pi^2}{4}\,Fo\right]\cos\left[\frac{(2n+1)\pi}{2}\,X\right]$, $b \neq (2n+1)^2\,\pi^2\alpha/4\ell^2$
8.1.11	9, p. 105	Infinite plate with exponentially varying surface temperature. $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i + Te^{b\tau}$, $x = \pm\ell$, $\tau > 0$.	$\frac{t - t_i}{T - t_i} = e^{b\tau}\frac{\cosh\,(X\sqrt{Pd})}{\cosh\,(\sqrt{Pd})} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\left[1 + 4Pd/\pi^2(2n+1)^2\right]}$ $\times \exp\left[-\frac{(2n+1)^2\pi^2}{4}\,Fo\right]\cos\left[\frac{(2n+1)\pi}{2}\,X\right]$

Case No.	References	Description	Solution
8.1.12	1, p. 299	Infinite plate with periodic surface temperature. $t = (t_{mx} - t_{mn}) \cos\left(\frac{2\pi\tau}{\tau_0}\right)$, $x = \pm\ell$.	See below



$$\frac{t - t_{mn}}{t_{mx} - t_{mn}} = \zeta \cos\left[\left(2\pi\tau/\tau_0\right) + \phi\right]$$

t_{mx} = max surface temp

t_{mn} = mean surface temp

t_0 = cycle time

$$\zeta = \left[\frac{f_1^2(X\sigma) + f_2^2(X\sigma)}{f_1^2(\sigma) + f_2^2(\sigma)}\right]^{1/2}$$, See Table 8.2

$$\phi = \tan^{-1}\left[\frac{f_1(\sigma)\ f_2(X\sigma) - f_2(\sigma)\ f_1(X\sigma)}{f_1(\sigma)\ f_1(X\sigma) + f_2(\sigma)\ f_2(X\sigma)}\right]$$

$$f_1(\sigma) = \cos(\sigma)\cosh(\sigma),\quad f_2(\sigma) = \sin(\sigma)\sinh(\sigma)$$

$$\sigma = \sqrt{\pi Pd} = \ell\sqrt{\pi/\alpha\tau_0}$$

Heat stored during half cycle:

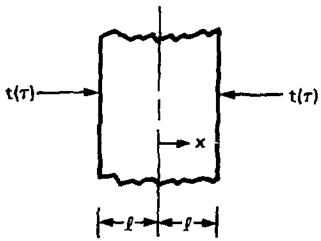
$$Q = 2\ell\rho c\left(t_{mx} - t_{mn}\right) F(\sigma)/\sigma$$

$$F(\sigma) = \left[\frac{df_1^2(\sigma) + df_2^2(\sigma)}{f_1^2(\sigma) + f_2^2(\sigma)}\right]^{1/2}$$ See Fig. 8.5

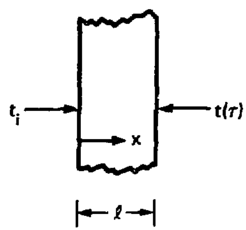
$$df_1(\sigma) = \cos(\sigma)\sinh(\sigma) - \sin(\sigma)\cosh(\sigma)$$

$$df_2(\sigma) = \sin(\sigma)\cosh(\sigma) + \cos(\sigma)\sinh(\sigma)$$

Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

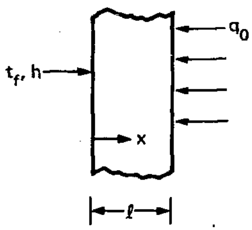
Case No.	References	Description	Solution
8.1.13	9, p. 105	Infinite plate with periodic surface temperature. $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i + (t_m - t_i)$ $\sin(\omega\tau + \theta)$, $x = \pm\ell$, $\tau > 0$. t_m = *maximum surface* temperature. 	$\frac{t - t_i}{t_m - t_i} = A \sin(\omega\tau + \theta + \phi)$ $+ 4\pi \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)\left[4\,Pd\cos(\theta) - (2n+1)^2\pi^2\sin(\theta)\right]}{16\,Pd^2 + (2n+1)^4\pi^4}$ $\times \exp\left[-(2n+1)^2\pi^2 Fo/4\right] \cos\left[\frac{(2n+1)\pi}{2} X\right]$. $A = \left[\frac{\cosh(X\sqrt{2\,Pd}) + \cos(X\sqrt{2\,Pd})}{\cosh(\sqrt{2\,Pd}) + \cos(\sqrt{2\,Pd})}\right]^{1/2}$, See Fig. 8.6a. $\phi = \arg\left[\frac{\cosh\left(X\sqrt{\frac{Pd}{2}} + X\frac{Pd}{2}i\right)}{\cosh\left(\sqrt{\frac{Pd}{2}} + \sqrt{\frac{Pd}{2}}i\right)}\right]$, See Fig. 8.6b. $Pd = \ell^2\omega/\alpha$.

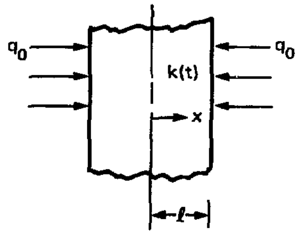
Case No.	References	Description	Solution
8.1.14	9, p. 105	Infinite plate with periodic surface temperature on one side, constant temperature on the other. $t = t_i,\ 0 < x < \ell,$ $\tau = 0.$ $t = t_i + (t_m - t_i)$ $\sin(\omega\tau + \theta),\ x = \ell,$ $\tau > 0.$ $t = t_i,\ x = 0,\ \tau > 0.$ t_m = maximum surface temperature.	$\frac{t - t_i}{t_m - t_i} = A \sin(\omega\tau + \theta + \phi)$ $+ 2\pi \sum_{n=1}^{\infty} \frac{n(-1)^n \left[n^2\pi^2 \sin(\theta) - \cos(\theta)\,Pd\right]}{n^4\pi^4 + \omega^2\ell^4/\alpha^2}$ $\times \exp\left(-n^2\pi^2 Fo\right) \sin(n\pi X)$ $A = \left[\frac{\cosh(X\sqrt{2\,Pd}) - \cos(X\sqrt{2\,Pd})}{\cosh(\sqrt{2\,Pd}) - \cos(\sqrt{2\,Pd})}\right]^{1/2}$ $\phi = \arg\left[\frac{\sinh\left(X\sqrt{\frac{Pd}{2}} + X\sqrt{\frac{Pd}{2}}\,i\right)}{\sinh\left(\sqrt{\frac{Pd}{2}} + \sqrt{\frac{Pd}{2}}\,i\right)}\right],\ Pd = \ell^2\omega/\alpha$

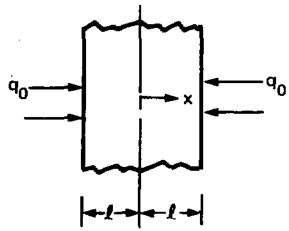


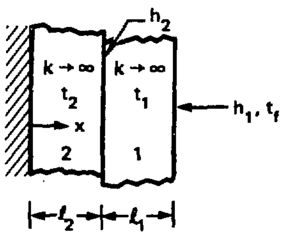
Case No.	References	Description	Solution
8.1.15	9, p. 108	Infinite plate with steady periodic surface temperature on one side, constant temperature on the other. $t = t_0,\ x = 0,\ \tau > 0.$ $t = t_1,\ x = \ell,$ $mT < \tau < mT + T_1,$ $m = 0, 1, 2, \ldots .$ $t = t_0,\ x = \ell,\ mT + T < \tau < (m + 1)\ T.$ T = period of cycle.	$\frac{t - t_0}{t_1 - t_0} = X + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n\pi X) \frac{e^{\lambda_n (T_1 - \tau')} - e^{\lambda_n (T - \tau')}}{1 - e^{\lambda_n T}},$ $\tau' = \tau - mT,\ m >> 1 .$ $\frac{t - t_0}{t_1 - t_0} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(n\pi X) \frac{e^{\lambda_n (T - T_1 - \tau'')} - e^{\lambda_n (T - \tau'')}}{1 - e^{\lambda_n T}},$ $\tau'' = \tau - (mT + T_1),\ m >> 1 .$ $\lambda_n = \alpha n^2 \pi^2 / \ell^2$

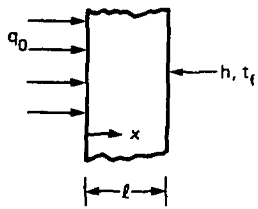
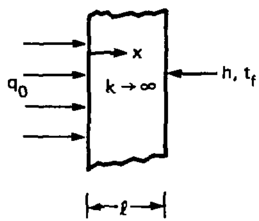
Case No.	References	Description	Solution
8.1.16	9, p. 109	Infinite plate with steady surface temperature on one side, insulated on the other. $q_x = 0$, $x = 0$, $\tau > 0$. $t = t_1$, $x = \ell$, $mT < \tau < mT + T_1$, $m = 0, 1, 2, \ldots$. $t = t_0$, $x = \ell$, $mT + T_1 < \tau < (m + 1)T$. T = *period of cycle*.	$\frac{t - t_0}{t_1 - t_0} \approx \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n + 1} \cos \left[\frac{(2n + 1)\pi}{2} x\right] \times \left[1 - \frac{e^{\lambda_n(T_1 - \tau')} - e^{\lambda_n(T - \tau')}}{1 - e^{\lambda_n \tau}}\right]$, $\tau' = \tau - mT$, $m >> 1$. $\frac{t - t_0}{t_1 - t_0} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n + 1)} \cos \left[\frac{(2n + 1)\pi}{2} x\right] \times \left[\frac{e^{\lambda_n(T - T_1 - \tau'')} - e^{\lambda_n(T - \tau'')}}{1 - e^{\lambda_n T}}\right]$, $\tau'' = \tau - (mT + T_1)$, $m >> 1$. $\lambda_n = \alpha\, (2n + 1)^2\, \pi^2/4\ell^2$

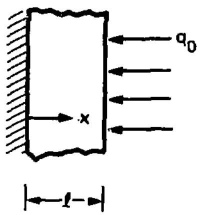
Case No.	References	Description	Solution
8.1.17	4, p. 82 48	Infinite plate with steady surface heat flux and convection boundaries. $t = t_0,\ 0 < x < \ell,\ \tau = 0.$ $q_x = q_0,\ x = \ell,\ \tau > 0.$ $t_f = t_0,\ \tau > 0.$  	$\frac{(t - t_0)h}{q_0} = \frac{Bi}{2}\sqrt{\frac{Fo}{\pi}}\left\{\exp\left[-\frac{(X-1)^2}{4\ Fo}\right] + \exp\left[-\frac{(X+1)^2}{4\ Fo}\right]\right\}$ $- Bi^2\ Fo\left[\exp\left(Bi_x + Bi + 4\ Bi^2\ Fo\right)\ \mathrm{erfc}\left(\frac{X+1}{4\sqrt{Fo}} + Bi\sqrt{Fo}\right)\right].$ See Fig. 8.18. For $Bi \to \infty$: $\frac{(t - t_0)k}{q_0\ell} = \frac{\sqrt{Fo}}{2\sqrt{\pi}}\left\{\exp\left[-\frac{(X-1)^2}{4\ Fo}\right] + \exp\left[-\frac{(X+1)^2}{4\ Fo}\right]\right\}.$ See case 8.1.20 for special solutions.

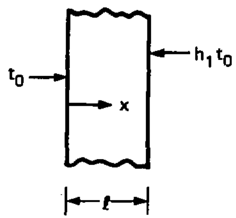
Case No.	References	Description	Solution
8.1.18	14 47 74, p. 174	Infinite plate with steady surface heat flux and variable thermal conductivity. $t = t_0$, $-\ell < x < +\ell$, $\tau = 0$. $k = k(t)$. α is constant. $q_x = q_0$, $x = \ell, -\ell$. 	$\frac{q_0 \ell}{k_0 t_0}\left[\frac{1}{2}(X-1)^2 + (X-1) + \frac{1}{3} + Fo - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left[n\pi(X-1)\right] \times \exp(-n^2\pi^2 Fo)\right] = \frac{1}{k_0}\int_0^T k(T)\,dT$ $T = (t - t_0)/t_0$, $k_0 = k(t_0)$ For $k(t) = k_0\left[1 + \beta(t - t_0)\right]$: $\frac{1}{k_0}\int_0^T k(T)\,dT = \frac{t - t_0}{t_0} + \frac{\beta t_0}{2}\left(\frac{t - t_0}{t_0}\right)^2$ For $\beta = 0$, see Fig. 8.7. For $\beta = 0$ and q_0 terminated at $\tau = D$, see Fig. 8.17.

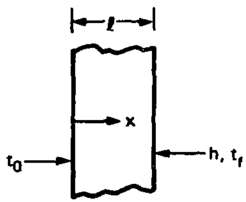
Case No.	References	Description	Solution
8.1.18.1	3, p. 279	Infinite plate with steady surface heat flux. $t = t_i$, $-\ell < x < +\ell$, $\tau = 0.$ $q_x = q_0$, $x = \pm\ell$, $\tau > 0.$  	$\dfrac{(t - t_i)k}{q_0\ell} = Fo + \dfrac{X^2}{2} - \dfrac{1}{6} - 2 \cdot \sum_{n=1}^{\infty} \dfrac{(-1)^n}{\lambda_n^2} \exp\left(-\lambda_n^2\, Fo\right) \cos\,(\lambda_n X)$ $\lambda_n = n\pi$

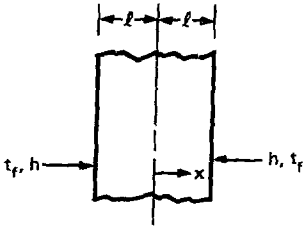
Case No.	References	Description	Solution
8.1.19	3, p. 350	Infinite composite plates with convection boundary and infinite thermal conductivity. $t = t_2, 0 < x < \ell_2, \tau > 0.$ $t = t_1, \ell_2 < x < \ell_2 + \ell_1, \tau > 0.$ $t = t_0, 0 < x < \ell_1 + \ell_2, \tau = 0.$ $q_x = 0, x = 0,$ h_2 is contact coefficient between 1 and 2.  	$\frac{t_1 - t_f}{t_0 - t_f} = \frac{\lambda_1 e^{-\lambda_2 \tau} - \lambda_2 e^{-\lambda_1 \tau}}{\lambda_1 - \lambda_2}$ $\frac{t_2 - t_f}{t_0 - t_f} = \frac{t_1 - t_f}{t_0 - t_f} - b_3 \left(\frac{e^{-\lambda_2 \tau} - e^{-\lambda_1 \tau}}{\lambda_1 - \lambda_2} \right)$ $\lambda_1 + \lambda_2 = b_1 + b_2 + b_3$ $\lambda_1 \lambda_2 = b_1 b_3$ $b_1 = h_2/\rho_2 c_2 \ell_2,\ b_2 = h_2/\rho_1 c_1 \ell_1,\ b_3 = h_1/\rho_1 c_1 \ell_1$

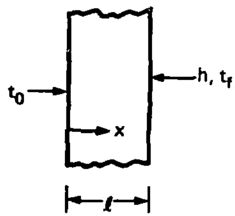
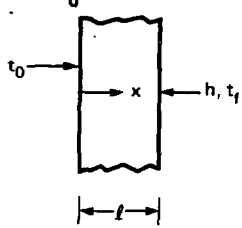
Case No.	References	Description	Solution
8.1.20	3, p. 476	Infinite plate with steady surface heat flux and convection boundary. $t = t_f,\ 0 < x < \ell,\ \tau = 0.$ $q_x = q_0\ ,\ x = 0,\ \tau > 0.$ 	$\dfrac{(t - t_f)k}{q_0\ell} = \dfrac{\sqrt{5\ Fo}}{2}\left(1 - \dfrac{X}{\sqrt{5\ Fo}}\right)^2\ ,\ 0 \leq \tau \leq \tau_0 .$ $\dfrac{(t - t_f)k}{q_0\ell} = \dfrac{1}{2}\left(1 - X\right)^2 + \left[\dfrac{1}{Bi} + \dfrac{1}{2}\ (1 - X^2)\right]$ $\times \left\{1 - \exp\left[- \dfrac{\left[1 + (Bi/3)\right] Bi\ Fo\ \left[1 - \left(\tau_0/\tau\right)\right]}{1 + 2(Bi/3) + 2(Bi^2/15)}\right]\right\}\ ,\ \tau > \tau_0 .$ τ_0 = penetration time to X = 1 $= \ell^2/5\alpha$ See case 8.1.17 for general solution.
8.1.21	3, p. 349	Infinite plate with unsteady surface heat flux; convection boundary and infinite thermal conductivity. $t = t_f,\ 0 < x < \ell,\ \tau = 0.$ $q_x = q_0 \cos\ (\omega\tau),\ x = 0.$ 	$\dfrac{(t - t_f)h}{q_0} = \dfrac{m}{\sqrt{m^2 + \omega^2}} \cos\ (\omega\tau - b) - \dfrac{m^2}{m^2 + \omega^2} \exp\ (-m\tau)$ $m = h/\rho c\ell,\ b = \tan^{-1}\ (\omega/m)$

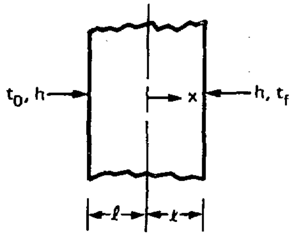
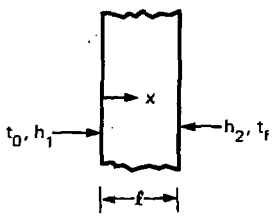
Case No.	References	Description	Solution
8.1.22	2, p. 248	Semi-infinite plate, quarter-infinite plate, infinite rectangular bar, semi-infinite rectangular bar, rectangular parallelepiped.	Dimensionless temperatures equal the product of solution for semi-infinite solid (case 7.1.1 or 7.1.3) and solution for infinite plate (case 8.1.6 or 8.1.7). See Figs. 9.4a and 9.4b.
8.1.23	9, p. 113	Infinite plate with unsteady surface heat flux. $t = t_i$, $0 < x < \ell$, $\tau = 0$. $q_x = 0$, $x = 0$, $\tau > 0$. $q_x = q_0\tau^{m/2}$, $x = \ell$. $\tau > 0$, $m = -1, 0, 1$. 	$\frac{(t - t_i)k}{q_0\ell\tau^{m/2}} = 2^{m+1}\,\Gamma\,(m/2 + 1)\sqrt{Fo}\sum_{n=0}^{\infty} i^{m+1}\mathrm{erfc}\left[\frac{(2n + 1) - X}{2\sqrt{Fo}}\right] + i^{m+1}\mathrm{erfc}\left[\frac{(2n + 1) + X}{2\sqrt{Fo}}\right]$

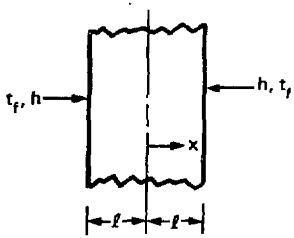
Case No.	References	Description	Solution
8.1.24	9, p. 114	Case 8.1.23 with $t = t_i$, $x = 0$, $\tau > 0$.	$\frac{(t - t_i)k}{q_0 \ell \tau^{m/2}} = 2^{m+1}\, \Gamma\, (m/2 + 1)\, \sqrt{Fo} \sum_{n=0}^{\infty} i^{m+1} \times \mathrm{erfc}\left[\frac{(2n + 1) - X}{2\ Fo}\right] - i^{m+1}\, \mathrm{erfc}\left[\frac{(2n + 1) + X}{2\sqrt{Fo}}\right]$
8.1.25.1	9, p. 120	Infinite plate with steady surface temperature, convection boundary and variable initial temperature. $t = f(x)$, $0 < x < \ell$, $\tau = 0$. $t = t_0$, $x = 0$, $\tau < 0$. 	$(t - t_0) = 2 \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right) \frac{\left(Bi^2 + \lambda_n^2\right) \sin\, (\lambda_n X)}{Bi^2 + \lambda_n^2 + Bi} \times \int_0^1 \left[t_0 - f\ (X)\right] \sin\, (\lambda_n X)\ dX$ $Bi = h\ell/k$, $\lambda_n \cot\, (\lambda_n) + Bi = 0$ See case 8.1.1 for $t = t_0$, $x = \ell$, $\tau > 0$

Case No.	References	Description	Solution
8.1.25.2	74, p. 239	Infinite plate with steady surface temp and convection boundaries. $t = t_0,\ 0 < x < \ell,$ $\tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$ 	$\frac{t - t_0}{t_f - t_0} = 2 \sum_{n=1}^{\infty} \frac{\left(Bi^2 + \lambda_n^2\right)\left[1 - \cos(\lambda_n)\right]}{\left(Bi^2 + Bi + \lambda_n^2\right)} \sin(\lambda_n X) \exp\left(-\lambda_n^2 Fo\right)$ $\lambda_n \cot\left(\lambda_n\right) + Bi = 0$ See Fig. 8.24.

Case No.	References	Description	Solution
8.1.26	74, p. 236	Infinite plate with parabolic initial temperature and convection boundary. $t = t_c - (t_c - t_s)(x/\ell)^2$, $0 < x < \ell$, $\tau = 0$. $t = t_c$, $x = 0$, $\tau = 0$. $t = t_s$, $x = \pm\ell$, $\tau = 0$. 	$\frac{t - t_f}{t_s - t_f} = 1 + 4\,\frac{t_s - t_c}{t_s - t_f}\sum_{n=1}^{\infty}\left(\frac{1}{Bi_\ell} - \frac{1}{\lambda_n^2}\right)\left[\frac{\sin(\lambda_n)\cos(\lambda_n X)}{\lambda_n + \sin(\lambda_n)\cos(\lambda_n)}\right]\exp\left(-\lambda_n^2\,Fo_\ell\right)$ $\lambda_n \tan(\lambda_n) = Bi_\ell$

Case No.	References	Description	Solution
8.1.27	9, p. 125	Infinite plate with steady surface temp and convection boundary. $t = t_0,\ 0 < x < \ell,\ \tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$  	$\frac{t - t_0}{t_f - t_0} = \frac{Bi\ X}{1 + Bi} - 2\ Bi \sum_{n=1}^{\infty} \frac{\sin(\lambda_n X)\ \exp\left(-\lambda_n^2\ Fo\right)}{\left(Bi + Bi^2 + \lambda_n^2\right)\sin(\lambda_n)}$ $\lambda_n \cot\left(\lambda_n\right) + Bi = 0$
8.1.28	9, p. 126	Infinite plate with a steady surface temp. Convection boundary and linear initial temperature. $t = (t_1 - t_0)x/\ell + t_0,$ $0 < x < \ell,\ \tau = 0.$ $t = t_0,\ x = 0,\ \tau > 0.$  	$\frac{t - t_0}{t_1 - t_0} = X + \frac{X\left[Bi\ (t_f - t_0)/(t_1 - t_0) - Bi - 1\right]}{Bi + 1}$ $- 2\left\{Bi\ \left[(t_f - t_0)/(t_1 - t_0)\right] - Bi - 1\right\} \sum_{n=1}^{\infty} \frac{\sin(\lambda_n X)\ \exp\left(-\lambda_n^2\ Fo\right)}{\left(Bi^2 + Bi + \lambda_n^2\right)\sin(\lambda_n)}\ .$ $\lambda_n \cot(\lambda_n) + Bi = 0$

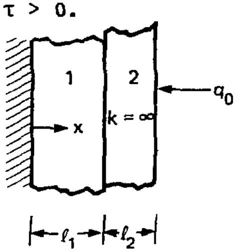
Case No.	References	Description	Solution
8.1.29.1	9, p. 126	Infinite plate with two convection temperatures. $t = t_0,\ -\ell < x < +\ell,\ \tau = 0.$ 	$\dfrac{t - t_0}{t_f - t_0} = \dfrac{Bi\ X}{2\ (Bi + 1)} - Bi \displaystyle\sum_{n=1}^{\infty} \dfrac{\sin\ (\beta_n X)\ \exp\left(-\beta_n^2\ Fo\right)}{\left(Bi^2 + Bi + \beta_n^2\right) \sin\ (\beta_n)}$ $+ \dfrac{1}{2} - Bi \displaystyle\sum_{n=1}^{\infty} \dfrac{\cos\ \left(\lambda_n X\right)\ \exp\ \left(-\lambda_n^2\ Fo\right)}{\left(Bi^2 + Bi + \lambda_n^2\right) \cos\ (\lambda_n)}$ $\lambda_n \tan\ (\lambda_n) = Bi,\ \beta_n\ \cot\ (\beta_n) + Bi = 0$
8.1.29.2	74, p. 239	Infinite plate with two convection coefficients. $t = t_0,\ 0 < x < +\ell,$ $\tau = 0.$ 	$\dfrac{t - t_0}{t_f - t_0} = \dfrac{1 + Bi_{\ell 1} X}{1 + Bi_{\ell 1} + (Bi_{\ell 1} / Bi_{\ell 2})}$ $- \displaystyle\sum_{n=1}^{\infty} A_n \left[\cos\ (\lambda_n X) + \dfrac{Bi_{\ell 1}}{\lambda_n} \sin\ (\lambda_n X)\right] \exp\ \left(-\lambda_n^2\ Fo_\ell\right).$ $A_n = \left\{\left[1 + \left(Bi_{\ell 1} / Bi_{\ell 2}\right)\right] \dfrac{\sin\ (\lambda_n)\ \cos\ (\lambda_n) + \lambda_n}{2\ \sin\ (\lambda_n)} + \dfrac{Bi_{\ell 1}}{\lambda_n} \sin\ (\lambda_n)\right\}^{-1}.$ $Bi_{\ell 1} = h_1 \ell / k,\ Bi_{\ell 2} = h_2 \ell / k$ $\cot\ (\lambda_n) = \left(\dfrac{\lambda_n}{Bi_{\ell 2}} - \dfrac{Bi_{\ell 1}}{\lambda_n}\right) \left(1 + \dfrac{Bi_{\ell 1}}{Bi_{\ell 2}}\right)^{-1}$

Case No.	References	Description	Solution
8.1.30	9, p. 127	Infinite plate with convection boundaries and time dependent fluid temperature. $t = t_i,\ -\ell < x < +\ell,\ \tau = 0.$ $t_f = \phi(\tau).$  	$t - t_i = 2\ \mathrm{Bi}\ \frac{\alpha}{\ell^2} \sum_{n=1}^{\infty} \frac{\lambda_n^2 \cos(\lambda_n X) \exp\left(-\lambda_n^2\ \mathrm{Fo}\right)}{\left(\mathrm{Bi} + \lambda_n^2 + \mathrm{Bi}^2\right) \cos(\lambda_n)}$ $\times \int_0^{\tau} \exp\left(\alpha \lambda_n^2 \tau'/\ell^2\right) \phi(\tau')\ d\tau'$ $\lambda_n \tan(\lambda_n) = \mathrm{Bi}.$

Case No.	References	Description	Solution
8.1.30.1	74, p. 316	Case 8.1.30 with $t_f = t_{fm} - (t_{fm} - t_i)\ e^{-b\tau}$.	$\frac{t - t_i}{t_{fm} - t_i} = 1 - \frac{\cos(\sqrt{Pd}\ X)\ \exp(-Pd\ Fo)}{\cos(\sqrt{Pd}) - \frac{1}{Bi}\sqrt{Pd}\ \sin(\sqrt{Pd})}$ $- \sum_{n=1}^{\infty} \frac{A_n \cos(\lambda_n X)\ \exp\left(-\lambda_n^2\ Fo\right)}{1 - \lambda_n^2/Pd}$ $A_n = \frac{(-1)^{n+1}\ 2\ Bi\left(Bi^2 + \lambda_n^2\right)^{1/2}}{\lambda_n\left(Bi^2 + Bi + \lambda_n^2\right)}$ Mean Temp: $\frac{t_m - t_i}{t_{fm} - t_i} = 1 - \frac{\exp(-Pd\ Fo)}{\sqrt{Pd}\left[\cot(\sqrt{Pd}) - \frac{1}{Bi}\sqrt{Pd}\right]}$ $- 2\ Bi^2 \sum_{n=1}^{\infty} \frac{\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^2\left(Bi^2 + Bi + \lambda_n^2\right)\left[1 - \left(\lambda_n^2/Pd\right)\right]}$

Case No.	References	Description	Solution
8.1.31	9, p. 127	Case 8.1.30 with $t_f = \begin{cases} t_0, & 0 < \tau < \tau_0 \\ t_1, & \tau > \tau_0. \end{cases}$	$\frac{t - t_i}{t_0 - t_i} = 1 - 2\,\mathrm{Bi} \sum_{n=1}^{\infty} \frac{\cos(\lambda_n X)\exp\left(-\lambda_n^2\,\mathrm{Fo}\right)}{\left(\mathrm{Bi}^2 + \lambda_n^2 + \mathrm{Bi}\right)\cos(\lambda n)}, \quad 0 < \tau < \tau_0$ $\frac{t - t_i}{t_1 - t_i} = 1 - 2\,\mathrm{Bi} \sum_{n=1}^{\infty} \frac{\left[T + (1 - T)\exp\left(\lambda_n^2\,\mathrm{Fo}_0\right)\right]}{\left(\mathrm{Bi}^2 + \lambda_n^2 + \mathrm{Bi}\right)\cos(\lambda_n)} \times \cos(\lambda_n X)\exp\left(-\lambda_n^2\,\mathrm{Fo}\right)$ $T = (t_0 - t_i)/(t_1 - t_i),\ \mathrm{Fo}_0 = \alpha\tau_0/\ell^2,\ \lambda_n \tan(\lambda_n) = \mathrm{Bi}$

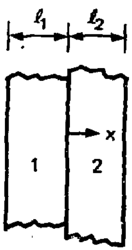
Case No.	References	Description	Solution
8.1.32	9, p. 127 74, p. 325	Case 8.1.30 with $t_f = (t_0 - t_i) \sin (\omega\tau + \varepsilon) + t_i$, $\tau > 0$.	$\dfrac{t - t_i}{t_0 - t_i} = \dfrac{Bi\ M_o}{M_1} \sin (\omega\tau + \varepsilon + \beta_0 - \beta_1)$ $+ 2\ Bi \sum_{n=1}^{\infty} \dfrac{\left[\dfrac{2\ Fo^{*2}}{\lambda_n^2} \cos (\varepsilon) - \sin (\varepsilon)\right] \cos (\lambda_n X) \exp \left(-\lambda_n^2\ Fo\right)}{\left(1 + 4\ Fo^{*4}/\lambda_n^4\right)\left(\lambda_n^2 + Bi^2 + Bi\right) \cos (\lambda_n)}$ $M_0 e^{i\beta_0} = \cosh (X\ Fo^*) \cos (X\ Fo^*) + i \sinh (X\ Fo^*)\ (X\ Fo^*) \sin (X\ Fo^*)$ $M_1 e^{i\beta_1} = (Fo^*) \sinh (Fo^*) \cos (Fo^*) - Fo^* \cosh (Fo^*) \sin (Fo^*) + Bi \cosh (Fo^*)$ $\times \cos (Fo^*) + i \left[Fo^* \sinh (Fo^*) \cos (Fo^*) + Fo^* \cosh (Fo^*) \sin (Fo^*)\right.$ $\left. + Bi \sinh (L) \sin (L)\right]$ $Fo^* = \ell \sqrt{\omega/2\alpha}$, $\lambda_n \tan (\lambda_n) = Bi$

Case No.	References	Description	Solution
8.1.33	9, p. 127 89	Case 8.1.30 with $t_f = t_i + b\tau,\ \tau > 0.$	$\frac{(t - t_i)\alpha}{b\ell^2} = Fo + \frac{Bi\ X^2 - Bi - 2}{2\ Bi}$ $+ 2\ Bi \sum_{n=1}^{\infty} \frac{\cos(\lambda_n X) \exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^2 \left(Bi^2 + \lambda_n^2 + Bi\right) \cos(\lambda_n)}$ $\lambda_n \tan(\lambda_n) = Bi$, See Fig. 8.26. (See Fig. 8.1.25 for $Bi = \infty$.)
8.1.34	9, p. 128	Infinite plate in perfect contact with an infinite plate of infinite conductivity. $t = t_i,\ 0 < x < \ell_1 + \ell_2,$ $\tau = 0.$ $q_x = 0,\ x = 0,\ \tau > 0.$ $-q_x = q_0,\ x = \ell_1 + \ell_2,$ $\tau > 0.$ 	$\frac{(t - t_i)k}{q_0 \ell_1} = \frac{M}{1 + M}\left[Fo_1 + \frac{X^2}{2} - \frac{3 + M}{6(1 + M)}\right]$ $- 2M^2 \sum_{n=1}^{\infty} \frac{\cos\left(\lambda_n X\right) \exp\left(-\lambda_n^2\ Fo_1\right)}{\lambda_n^2 \left(\lambda_n^2 + M^2 + M\right) \cos(\lambda_n)}$ $\lambda_n \cot(\lambda_n) = -M$ $M = \frac{\rho_1 c_1 \ell_1}{\rho_2 c_2 \ell_2},\ Fo_1 = \alpha_1 \tau / \ell_1^2,\ X = x/\ell_1$

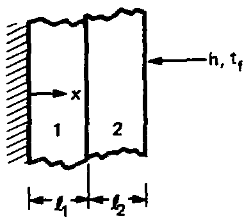
Case No.	References	Description	Solution
8.1.35	9, p. 129	Case 8.1.34 with $q_x = 0,\ x = \ell_1 + \ell_2,\ \tau > 0.$ $t = t_2,\ \ell_1 < x < \ell_2,\ \tau = 0.$ $t = t_i,\ 0 < x < \ell_1,\ \tau = 0.$	$\frac{(t - t_i)}{(t_2 - t_i)} = \frac{1}{1 + M} + 2M \sum_{n=1}^{\infty} \frac{\cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo_1\right)}{\left(\lambda_n^2 + M^2 + M\right) \cos(\lambda_n)}$ $\lambda_n \cot(\lambda_n) = -M$
8.1.36	9, p. 128	Case 8.1.34 with $t = t_i,\ x = 0,\ \tau > 0.$	$\frac{(t - t_i)k}{q_0 \ell_1} = X - 2M \sum_{n=1}^{\infty} \frac{\sin(\lambda_n X) \exp\left(-\lambda_n^2 Fo_1\right)}{\lambda_n \left(\lambda_n^2 + M^2 + M\right) \cos(\lambda_n)}$ $\lambda_n \tan(\lambda_n) = M$
8.1.37	9, p. 128	Case 8.1.34 with $t = t_i,\ x = 0,\ \tau > 0.$ $t = t_2,\ \ell_1 < x < \ell_2,\ \tau = 0.$ $t = t_i,\ 0 < x < \ell_1,\ \tau = 0.$ $q_x = 0,\ x = \ell_1 + \ell_2,\ \tau > 0.$	$\frac{(t - t_i)}{(t_2 - t_i)} = 2M \sum_{n=1}^{\infty} \frac{\sin(\lambda_n X) \exp\left(-\lambda_n^2 Fo_1\right)}{\left(\lambda_n^2 + M^2 + M\right) \sin(\lambda_n)}$ $\lambda_n \tan(\lambda_n) = M$
8.1.38	9, p. 129	Case 8.1.34 with $t = t_0,\ x = 0,\ \tau > 0.$ $q_x = 0,\ x = \ell_1 + \ell_2,\ \tau > 0.$	$\frac{(t - t_i)}{(t_0 - t_i)} = 1 - 2 \sum_{n=1}^{\infty} \frac{\left(\lambda_n^2 + M^2\right) \sin\left(\lambda_n X\right) \exp\left(-\lambda_n^2 Fo_1\right)}{\lambda_n \left(\lambda_n^2 + M^2 + M\right)}$ $\lambda_n \tan(\lambda_n) = M$

Case No.	References	Description	Solution
8.1.39	9, p. 129	Case 8.1.34 with $q_x = 0$, $x = \ell_1 + \ell_2$, $\tau > 0$. $q_x = q_0$, $x = 0$, $\tau > 0$.	$\frac{(t - t_i)k(1 + M)}{q_0 \ell_1} = M\,Fo + 1 - X + \frac{M}{2}(1 - X)^2 - \frac{M(3 + M)}{6(1 + M)} - 2(1 + M)\sum_{n=1}^{\infty} \frac{\left(M^2 + \lambda_n^2\right)\cos(\lambda_n X)\exp\left(-\lambda_n^2\,Fo_1\right)}{\lambda_n^2\left(\lambda_n^2 + M^2 + M\right)}$. $\lambda_n \cot(\lambda_n) + M = 0$
8.1.40	9, p. 129	Case 8.1.34 with convection boundary h, t_f at $x = \ell_1 + \ell_2$. $t = t_2$, $\ell_1 < x < \ell_2$, $\tau = 0$.	$\frac{(t - t_2)}{(t_i - t_2)} = 2\sum_{n=1}^{\infty} \frac{\left(Bi_1 - \lambda_n^2/M\right)\cos(\lambda_n X)\exp\left(-\lambda_n^2\,Fo_1\right)}{\left[\left(Bi_1 - \lambda_n^2/M\right)^2 + \lambda_n^2 + \lambda_n^2/M + Bi_1\right]\cos(\lambda_n)}$ $\lambda_n \tan(\lambda_n) = Bi_1 - \lambda_n^2/M$, $Bi = h\ell_1/k_1$ (See Ref. 82 for solution of material 1 of low conductivity and material 2 of high conductivity.)

Case No.	References	Description	Solution
8.1.41	9, p. 129	Case 8.1.40 with $-q_x = f(\tau)$, $x = 0$, $\tau > 0$.	$$(t - t_i) = \frac{2\alpha}{k\ell^3} \sum_{n=1}^{\infty} \frac{\lambda_n^2 \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo_1\right)}{\left[\left(Bi_1 - \lambda_n^2/M\right)^2 + (1 + 1/M) \lambda_n^2 + Bi\right] \cos(\lambda_n)}$$ $$\times \int_0^{\tau} f(\tau') \exp\left(\lambda_n^2 \alpha\tau'/\ell^2\right) d\tau' .$$ $$\lambda_n \tan(\lambda_n) = Bi_1 - \lambda_n^2/M .$$
8.1.42	9, p. 129	Case 8.1.34 with a contact resistance h_c between the two plates.	$$\frac{(t - t_i)(M + 1)k}{q_0 \ell_1 M} = Fo_1 + \frac{1}{2}(1 - X)^2 - \frac{3\,Bi_c + 6 + M\,Bi_c}{6\,Bi_c (1 + M)}$$ $$+ 2\,Bi_c (1 + M) \sum_{n=1}^{\infty} \frac{\lambda_n^2 - M\,Bi_c}{P_n \cos(\lambda_n)} \cos(\lambda_n - \lambda_n X) \exp\left(-\lambda_n^2 Fo\right) .$$ $$\left(\lambda_n^2 - M\,Bi_c\right) \tan(\lambda_n) = Bi_c\,\lambda_n .$$ $$P_n = \lambda_n^6 + \lambda_n^4 \left(Bi_c^2 + Bi_c - 2\,M\,Bi_c\right) + M\,Bi_c^2 (1 + M) \lambda_n^2 .$$ $$Bi_c = h_c \ell_1 / k_1 .$$

Case No.	References	Description	Solution
8.1.43.1	9, p. 324	Composite plates of different materials. $t = t_0$, $-\ell_1 < x < \ell_2$, $\tau = 0$. $t = t_1$, $x = -\ell_1$, $\tau < 0$. $t = t_0$, $x = \ell_2$, $\tau > 0$.  	Region 1, $-\ell_1 < x < 0$: $\frac{(t - t_0)}{(t_1 - t_0)} = \frac{(L - KX)}{(L + K)} - 2 \sum_{n=1}^{\infty} \exp -\lambda_n^2 Fo_1 \times \frac{\cos(\lambda_n X) \sin(\beta L\lambda_n) - \sigma \sin(\lambda_n X) \cos(\beta L\lambda_n)}{\lambda_n \left[(1 + K\beta^2 L) \sin(\lambda_n) \sin(\beta L\lambda_n) - \beta (K + L) \cos(\lambda_n) \cos(\beta L\lambda_n)\right]}$. Region 2, $0 < x < \ell_2$: $\frac{(t - t_0)}{(t_1 - t_0)} = \frac{L - X}{L + K} - 2 \sum_{n=1}^{\infty} \exp -\lambda_n^2 Fo_1 \times \frac{\sin(\beta L\lambda_n - \beta\lambda_n X)}{\lambda_n \left[(1 + K\beta^2 L) \sin(\lambda_n) \sin(\beta L\lambda_n) - \beta (K + L) \cos(\lambda_n) \cos(\beta L\lambda_n)\right]}$. $\cot(\lambda_n) + \sigma \cot(\beta L\lambda_n) = 0$ $\beta = \sqrt{\alpha_1/\alpha_2}$, $K = k_2/k_1$, $Fo_1 = \alpha_1\tau/\ell_1^2$, $L = \ell_2/\ell_1$, $X = x/\ell_1$

Case No.	References	Description	Solution
8.1.43.2	74, p. 441	Composite plates of different materials with insulated surface and convection boundary. $t = t_i$, $0 < x < \ell_1 + \ell_2$, $\tau = 0$. $q_x = 0$, $x = 0$, $\tau > 0$.	see below



$$\frac{t - t_i}{t_f - t_i} = 1 - 2\sum_{n=1}^{\infty} \frac{1}{\lambda_n\psi} \cos\left(\lambda_n X/\sqrt{A}\right) \exp\left(-\lambda_n^2 L^2 Fo_1/A\right), \quad 0 < X < L .$$

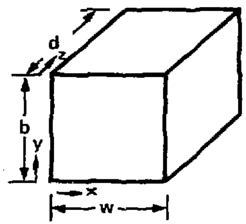
$$\frac{t - t_i}{t_f - t_i} = 1 - 2\sum_{n=1}^{\infty} \frac{1}{\lambda_n\psi} \left\{\cos\left[\lambda_n \left(X - L\right)\right] \cos\left(\lambda_n L/\sqrt{A}\right) - K \sin\left[\lambda_n (X - L)\right] \sin\left(\lambda_n L/\sqrt{A}\right)\right\} \exp\left(-\lambda_n^2 L^2 Fo_1/A\right), \quad L < X < 1 .$$

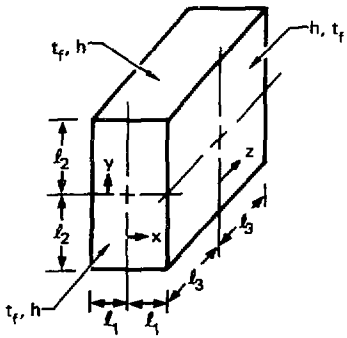
$$\psi = \left[\left(1 + KL/\sqrt{A} + \frac{1 + L}{Bi}\right) \sin(\lambda_n) + \lambda_n \left(\frac{1 + L}{Bi}\right) \times (1 + KL/\sqrt{A}) \cos(\lambda_m)\right] \cos\left(\lambda_n L/\sqrt{A}\right) + \left[\left(1 + KL/\sqrt{A} + \frac{1 + L}{Bi}\right) \cos(\lambda_n) - \lambda_n \left(\frac{1 + L}{Bi}\right)\left(1 + \frac{L}{K\sqrt{A}}\right) \sin(\lambda_n)\right] K \sin\left(\lambda_n L/\sqrt{A}\right) .$$

$$\frac{L\lambda_n}{Bi} (1 + L) \tan\left(\lambda_n L/\sqrt{A}\right) = 1 - \frac{\lambda_n}{Bi} (1 + L) \tan(\lambda_n) - K \tan(\lambda_n) \tan\left(\lambda_n L/\sqrt{A}\right) .$$

$$Bi = h\ell_1/K_2, \quad L = \ell_1/\ell_2, \quad Fo_1 = \alpha_1\tau/\ell_1^2, \quad A = \alpha_1/\alpha_2, \quad K = \frac{\rho_1 c_1 k_1}{\rho_2 c_2 k_2}$$

Case No.	References	Description	Solution
8.1.44.1	9, p. 416	Rectangular parallelpiped. $t = t_i$, $0 < x < w$, $0 < y < b$, $0 < z < d$, $\tau = 0$. $t = t_0$, $x = 0$, $0 < y < b$, $0 < z < d$, $\tau > 0$. $t = t_i$, all other faces, $\tau > 0$.	$\frac{t - t_i}{t_0 - t_i} = \frac{16}{\pi^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\sin(\bar{m}\pi Y)\sin(\bar{n}\pi Z)\sinh\left[(1 - X)\sqrt{\lambda_{m,n,o}}\right]}{\bar{m}\,\bar{n}\sinh(\lambda_{m,n,o})}$ $- \frac{32}{\pi} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{\ell \sin(\bar{m}\pi Y)\sin(\bar{n}\pi Z)\sin(\ell\pi X)\exp(-\lambda_{m,n,\ell} Fo_w)}{\bar{m}\,\bar{n}\,\lambda_{m,n,\ell}}$ $\lambda_{m,n,\ell} = \ell^2\pi^2 + (\bar{m}\pi w/b^2) + (\bar{n}\pi w/d)^2$ $\bar{m} = 2m + 1$, $\bar{n} = 2n + 1$, $X = x/w$, $Y = y/b$, $Z = z/d$



Case No.	References	Description	Solution
8.1.44.2	74, p. 287	Rectangular parallelepiped with convection boundary. $t = t_i$, $-\ell_1 < x < +\ell_1$, $-\ell_2 < y < +\ell_2$, $-\ell_3 < z < +\ell_3$, $\tau = 0$. 	$\frac{t - t_0}{t_f - t_0} = 1 - \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{s=1}^{\infty} A_{n1} A_{m2} A_{s3} \cos(\lambda_{n1} X) \cos(\lambda_{m2} Y)$ $\times \cos\left(\lambda_{s3} Z\right) \exp\left(\lambda_{n1}^2 Fo_1 + \lambda_{m2}^2 Fo_2 + \lambda_{s3}^2 Fo_3\right)$ $A_{n,m,s,i} = \frac{(-1)^{(n,m,s)+1}\, 2\, Bi_i \left(Bi_i^2 + \lambda_{n,m,s,i}^2\right)^{1/2}}{\lambda_{n,m,s,i} \left(Bi_i^2 + Bi_i + \lambda_{n,m,s,i}^2\right)}$ $\cot(\lambda_{n,m,s,i}) = \left(\frac{1}{Bi_i}\right) \lambda_{n,m,s,i}$, $Fo_i = \alpha\tau/\ell_i^2$ $Bi_i = h\ell_i/k$, $i = 1, 2, 3$, $X = x/\ell_1$, $Y = y/\ell_2$, $Z = z/\ell_3$

Case No.	References	Description	Solution
8.1.45	9, p. 417	Case 8.1.44.1 with $t = t_0 \sin(\omega\tau + \varepsilon)$, $x = 0$, $0 < y < b$, $0 < z < d$, $\tau > 0$.	

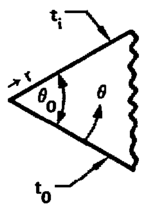
$$\frac{t - t_i}{t_0 - t_i} = \frac{16}{\pi^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\sin(\bar{m}\pi Y)\,\sin(\bar{n}\pi Z)\,M_{m,n}\sin(\omega\tau + \varepsilon + \phi_{m,n})}{\bar{m}\,\bar{n}}$$

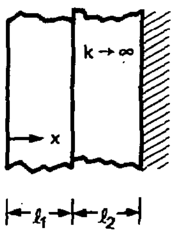
$$- \frac{32}{\pi} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{\ell \sin(\bar{m}\pi Y)\,\sin(\bar{n}\pi Z)\,\sin(\ell\pi X)\,(W\cos\varepsilon - \lambda_{m,n,\ell}\sin\varepsilon)\,\exp(-\lambda\alpha\tau/w^2)}{\left(W^2 + \lambda_{m,n,\ell}^2\right)\bar{m}\,\bar{n}}$$

$$M_{m,n}\exp(i\phi_{m,n}) = \frac{\sinh\left\{(1-X)\left[(\bar{m}\pi w/b)^2 + (\bar{n}\pi w/d)^2 + iW\right]^{1/2}\right\}}{\sinh\left\{\left[(\bar{m}\pi w/b)^2 + (\bar{n}\pi w/d)^2 + iW\right]^{1/2}\right\}}$$

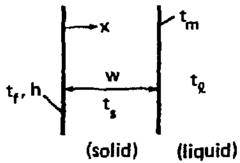
$W = w^2\omega/\alpha$, See case 8.1.44.1 for $\lambda_{m,n,\ell}$, X, Y, Z, $\bar{m}$, $\bar{n}$

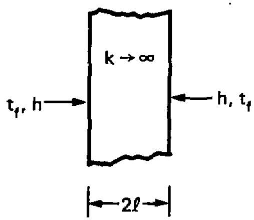
Case No.	References	Description	Solution
8.1.46	9, p. 419	Wedge with steady surface temperature. $t = t_i$, $0 < \theta < \theta_0$, $\tau = 0$. $t = t_0$, $\theta = 0$, $\tau > 0$. $t = t_i$, $\theta = \theta_0$, $\tau > 0$.	$\frac{t - t_i}{t_0 - t_i} = \frac{\theta}{\theta_0} + \frac{2}{\theta_0} \sum_{n=1}^{\infty} (-1)^n \left[\sin \ (2n + 1)\pi\theta/\theta_0\right]$ $\times \int_0^{\infty} \frac{\exp\ (-\alpha\tau u^2/r^2)}{u} J_s(u)\ du$ $s = (2n + 1)\ \pi/\theta_0$ For $t = t_0$, $\theta = 0$, θ_0, $\tau > 0$: $\frac{t - t_i}{t_i - t_i} = 1 - \frac{4}{\theta_0} \sum_{n=0}^{\infty} \sin\ (s\theta) \int_0^{\infty} \exp\ (-\alpha\tau u^2/r^2)\ \frac{J_s(u)du}{u}$

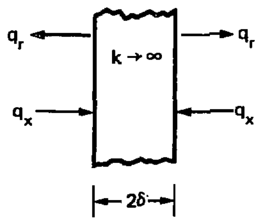


Case No.	References	Description	Solution
8.1.47	33	Composite slabs with ramp surface temperature. $t = t_i,\ 0\ x < \ell_1 + \ell_2,$ $\tau = 0.$ $t = t_i + (t_0 - t_i)\, b\tau,$ $x = 0,\ 0 < \tau < 1/b.$ $t = t_o,\ x = 0,\ \tau \geq 1/b.$ $q_x = 0,\ x = \ell_1 + \ell_2 .$ 	$$\frac{t - t_0}{t_i - t_0} = 2 \sum_{n=1}^{\infty} \frac{Pd\left(\lambda_n^2 + M^2\right) \exp\left(-\lambda_n^2\, Fo_1\right)\left[1 - \exp\left(\lambda_n^2/Pd\right)\right] \sin\left(\lambda_n X\right)}{\lambda_n^3\left(\lambda_n^2 + M^2 + M\right)}$$ $\lambda_n \tan(\lambda_n) = M,\ M = \ell_1\rho_1 c_1/\ell_2\rho_2 c_2,\ X = x/\ell_1$ See cases 8.1.34 to 8.1.42 for other conditions.

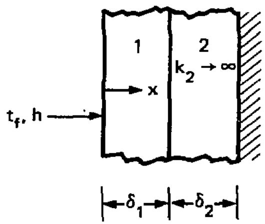
Section 11.1. *Change of Phase--Plane Interface.*

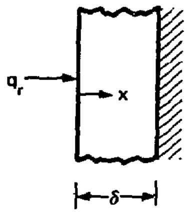
Case No.	References	Description	Solution
11.1.9	9, p. 292	Convection boundary at original solid-liquid boundary. $t = t_1,\ x > 0,\ \tau = 0,\ t_1 > t_m$. t_m = melting temp. 	$\dfrac{t_s - t_m}{t_m - t_f} = Bi_x - \dfrac{F\,Bi_x^2}{2}(1 + 2!\,Fo_x) - \dfrac{F\,Bi_x^3}{3!}(1 + 3!\,Fo_x) + \ldots,\ 0 < x < w.$ $w = F\,Bi_x\,Fo_x - \dfrac{F^2\,Bi_x^3}{2}(1 + F)\,Fo_x^2 + \ldots$ $F = k_s(t_m - t_f)/\alpha_s\rho\gamma\ ,\ Fo_x = \alpha_s\tau/x^2\ ,\ Bi_x = hx/k_s$

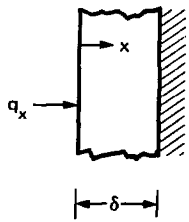
Case No.	References	Description	Solution
8.1.49	19, p. 3-30	Infinite plate of infinite conductivity with convection boundary and harmonic fluid temperature. $t_f = t_{mn} + (t_{mx} - t_{mn}) \cos(\omega\tau)$. t_{mn} = mean temp. t_{mx} = max temp. 	$$\frac{t - t_{mn}}{t_{mx} - t_{mn}} = \frac{1}{\sqrt{\theta^2 + 1}} \cos(\tau\omega - \psi) - \frac{1}{\theta^2 + 1} \exp\left(- \frac{h\tau}{\rho c \ell}\right)$$ $$\theta = \omega\rho c\ell/h, \; \psi = \tan^{-1}(\theta)$$ Instantaneous surface heat flux: $$\frac{q}{h(t_{mx} - t_{mn})} = \pm \frac{\theta}{\sqrt{1 + \theta^2}} \sin(\tau\omega - \psi)$$ See Fig. 8.9

Case No.	References	Description	Solution
8.1.50	40	Infinite plate of infinite conductivity with surface reradiation and circular pulse heating. $t = t_0,\ \tau = 0,$ $q_r = \sigma \mathscr{F} T^4.$ $q_x = q_{max} \sin^2 \pi\tau / D.$ 	See Fig. 8.12 for maximum temperature values: $\frac{t_{max}}{t_0} = f\left(\frac{q_{max} D}{\rho c \ell t_0}\right)$ D = heating duration

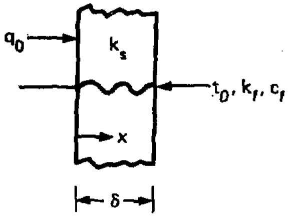
Case No.	References	Description	Solution
8.1.51	19, p. 3-44 42 43 66	Infinite composite plates in perfect contact, convection boundary on one side, insulated on other side. $t = t_0$, $0 < x < \delta_1 + \delta_2$, $\tau = 0$. $q_x = 0$, $x = \delta_1 + \delta_2$, $\tau > 0$.	See Fig. 8.14 $\dfrac{\rho_1\delta_1}{\sqrt{\rho_1 k_1 D/c_1}} = f\left(\dfrac{t_{2max} - t_0}{t_f - t_0}\right)$ D = heating duration

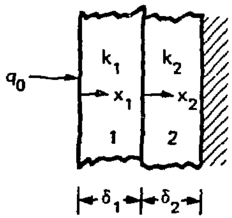


Case No.	References	Description	Solution
8.1.52	19, p. 3-49 44 68	Infinite plate with radiation heating or cooling. $t = t_0,\ 0 < x < \delta,$ $\tau = 0.$ $q_r = \sigma \mathscr{F}\left(T_s^4 - T_{x=0}^4\right).$ T = source temp.  	See Figs. 8.15a-e for heating. See Figs. 8.15f-1 for cooling. $\frac{t - t_0}{t_s - t_0} = f(Fo)$

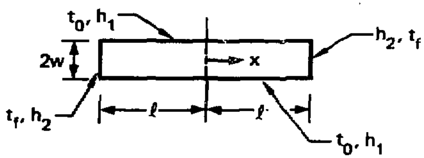
Case No.	References	Description	Solution
8.1.53	19, p. 3-68 49	Infinite plate with time-linear surface heat flux. $t = t_0$, $0 < x < \delta$, $\tau = 0$. $q_x = q_{max}\tau/D$, $\tau > 0$.  	$\frac{k\,(t - t_0)}{q_0\delta} = \frac{Fo}{Fo_D}\left(\frac{Fo}{2} - \frac{X^3}{12} + \frac{5X^2}{8} - X + \frac{5}{16}\right)$, $\frac{1}{2} < Fo < Fo_D$, $Fo_D \geq 1$. $Fo_D = \alpha D/\delta^2$, $X - x/\delta$ For $q_x = q_{max}(1 - \tau/D)$: $\frac{k\,(t - t_0)}{q_0\delta} = Fo + \frac{X^2}{2} - X + \frac{1}{3} + \left(\text{solution for } q_x = q_{max}\tau/D\right)$, $\frac{1}{2} < Fo < Fo_D$, $Fo_D \geq 1$. D = duration of heating.
8.1.54	19, p. 3-68 150	Case 8.1.53 with $q_x = q_{max}\sin^2\pi\tau/D$.	See Figs. 8.19a & b $\frac{\pi\rho c\delta}{Q_0}(t - t_0) = f(2\pi\tau/D)$, $Q_0 = q_x D$

Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution
8.1.55	19, p. 3-73 51	Porous infinite plate with steady surface heat flux and fluid flow. $t = t_0$, $0 < x < \delta$, $\tau = 0$. t_0 = fluid source temp. 	See Figs. 8.20a & b $\frac{k_e F(t - t_0)}{q_0 \delta} = f\left(\alpha_e \tau/\delta^2\right)$ $F = G_f c_f \delta/k_e$ $k_e = Pk_f + (1 - P)k_s$ G_f = fluid mass flux P = plate porosity Steady state solution: $\frac{k_e (t - t_0)}{q''\delta} = \frac{1}{F} \exp(-Fx/\delta)$

Case No.	References	Description	Solution
8.1.56	19, p. 3-75 52	Two infinite plates in perfect contact with a steady surface heat flux. $t = t_0,\ 0 < x_1 < \delta_1,$ $0 < x_2 < \delta_2,\ \tau = 0.$ $q_x = q_0,\ x_1 = 0,\ \tau > 0.$ 	See Figs. 8.21 a-d $\dfrac{(t - t_0)k_1}{q_0\delta_1} = f(Fo_1)$
8.1.57	19, p. 3-75 48	Case 8.1.56 with $k_2 \to \infty$.	See Fig. 8.22 $\dfrac{k_1(t - t_0)}{q_0\delta_1} = f(Fo_1)$
8.1.58	19, p. 3-75 53	Case 8.1.56 with $k_2 \to \infty$ and $q_x = q_{max} \sin^2 \pi\tau/D$.	See Figs. 8.23 a, b, and c $\dfrac{t - t_0}{t_{eqb} - t_0} = f(2\pi\tau/D)$ D = duration of pulse

Case No.	References	Description	Solution
8.1.59	74, p. 245	Thin plate with two convection conditions. $t = t_0$, $-\ell < x < +\ell$, $\tau = 0$.	



$$\frac{t - t_0}{t_f - t_0} = \frac{\cosh\left(Bi_{1w}^{1/2}\ X\right)}{\cosh\left(Bi_{1w}^{1/2}\ L\right) + \dfrac{L\sqrt{Bi_{1w}}}{Bi_{2w}} \sinh\left(Bi_{1w}^{1/2}\right)L}$$

$$- 2 \sum_{n=1}^{\infty} \frac{\lambda_n^2 \sin \lambda_n \cos(\lambda_n X) \exp\left[-\left(\lambda_n^2 + Bi_{1w} L^2\right) Fo_w\right]}{\left(\lambda_n^2 + Bi_{1w} L^2\right)\left[\lambda_n + \sin(\lambda_n)\cos(\lambda_n)\right]}$$

$$\lambda_n \tan(\lambda_n) = Bi_{2w},\ Bi_{1w} = h_1 w/k,\ Bi_{2w} = h_2 w/k$$

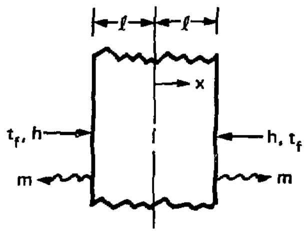
$$X = x/w,\ L = \ell/w$$

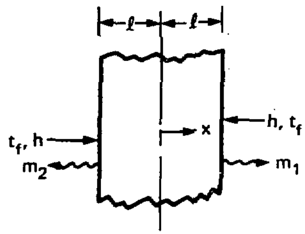
Mean temp:

$$\frac{t_m - t_0}{t_f - t_0} = \frac{\tanh\left(Bi_{1w}^{1/2}\ L\right)}{Bi_{1w}^{1/2}\ L + \dfrac{Bi_{1w}}{Bi_{2w}} L^2 \tanh\left(Bi_{1w}^{1/2}\ L\right)}$$

$$- 2 \sum_{n=1}^{\infty} \frac{\lambda_n^2 B_n \exp\left[-\left(\lambda_n^2 + Bi_{1w} L^2\right) Fo_w\right]}{\lambda_n^2 + Bi_{1w} L^2}$$

$$B_n = \frac{Bi_{1w}}{\lambda_n^2 \left(Bi_{1w}^2 + Bi_{1w} + \lambda_n^2\right)}$$

Case No.	References	Description	Solution
8.1.60.1	74, p. 320	Infinite plate with evaporation and convection boundary. $t = t_i, -\ell < x < +\ell,$ $\tau = 0.$ $m = m_0 e^{-b\tau}, \ x = \pm\ell.$ m = evaporation rate. 	$\frac{t - t_i}{t_f - t_i} = 1 - \frac{M \cos(\sqrt{Pd}X) \exp(-Pd\ Fo)}{\cos(\sqrt{Pd}) - \frac{1}{Bi}\sqrt{Pd} \sin(\sqrt{Pd})}$ $- \sum_{n=1}^{\infty}\left[1 - \frac{M}{1 - Pd/\lambda_n^2}\right] A_n \cos(\lambda_n X) \exp -\lambda_n^2\ Fo$ $A_n = \frac{2\sin(\lambda_n)}{\lambda_n + \sin(\lambda_n)\cos(\lambda_n)}, \ M = \frac{\gamma m_0}{h(t_f - t_i)} = \frac{t_f - t_{wb}}{t_f - t_i}$ $\cot(\lambda_n) = \lambda_n/Bi, \ t_{wb}$ = wet bulb temp.

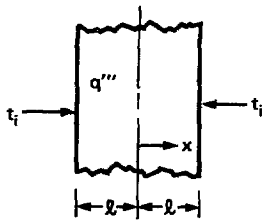
Case No.	References	Description	Solution
8.1.60.2	74, p. 321	Case 8.1.60.1 with asymmetrical boundary conditions. 	$\frac{t - t_i}{t_f - t_i} = 1 - \frac{M_1 + M_2}{2} - \sum_{n=1}^{\infty} \left(1 - \frac{M_1 + M_2}{2}\right) A_n \cos(\lambda_n X) \exp\left(-\lambda_n Fo\right)$ $- \frac{(M_1 - M_2) Bi\ X}{2(1 + Bi)} - \sum_{m=1}^{\infty} \frac{(M_1 - M_2)}{2} A_m \sin(\lambda_m X) \exp\left(-\lambda_m Fo\right)$ $A_n = \frac{2 \sin(\lambda_n)}{\lambda_n + \sin(\lambda_n)\cos(\lambda_n)}, \quad A_m = \frac{2 \cos(\lambda_m)}{\lambda_m - \cos(\lambda_m)\sin(\lambda_m)}$ $\cot(\lambda_n) = \lambda_n / Bi, \quad \tan(\lambda_m) = -\lambda_m / Bi$ Mean temp: $\frac{t_m - t_i}{t_f - t_i} = 1 - \frac{M_1 + M_2}{2} - 2 \sum_{n=1}^{\infty} \left(1 - \frac{M_1 + M_2}{2}\right) \frac{Bi^2 \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n^2 \left(Bi^2 + Bi + \lambda_n^2\right)}$

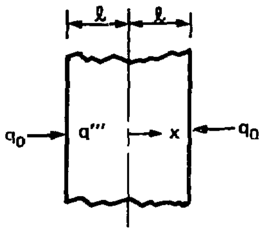
Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

Case No.	References	Description	Solution

Section 8.1. Solids Bounded by Plane Surfaces--No Internal Heating.

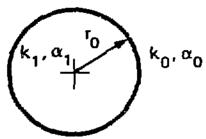
Case No.	References	Description	Solution

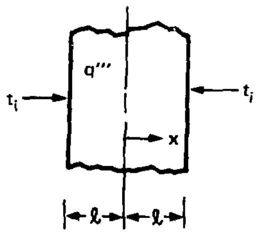
Case No.	References	Description	Solution
8.2.1	3, p. 276	Infinite plate with uniform internal heating. $t = t_i$, $-\ell < x < \ell$, $\tau = 0$. $t = t_i$, $x = \pm\ell$, $\tau > 0$, $q''' = q_0'''$.  	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{1}{2}(1 - X^2) - 2\sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_n^3} \exp\left(-\lambda_n^2 \text{ Fo}\right) \cos(\lambda_n X)$ $\lambda_n = (2n + 1)\pi/2$ See Fig. 8.8

Case No.	References	Description	Solution
8.2.2.1	74, p. 363	Infinite plate with steady surface heat flux and variable internal heating. $t = t_i$, $-\ell < x < +\ell$, $\tau = 0$. $q''' = q_0''' (1 - x/\ell)$. $q_x = q_0$, $x = \pm\ell$, $\tau > 0$. 	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{Fo}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos\left[(2n - 1)\pi X\right] \cos(\lambda_n X)}{(2n - 1)^4} \times \left[1 - \exp\left(-\lambda_n^2 Fo\right)\right] + \Phi$ $\Phi = \frac{q_0}{q_0''' \ell}\left[Fo - \frac{1}{6} + \frac{X^2}{2}\right] - \sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{\lambda_n^2}\right) \cos(\lambda_n X) \times \exp\left(\lambda_n^2 Fo\right)$ $\lambda_n = n\pi$
8.2.2.2	74, p. 363	Case 8.2.2.1 with $q''' = q_0''' (1 - X^2)$.	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{2}{3} Fo - \sum_{n=1}^{\infty} (-1)^n \frac{4}{\lambda_n^2} \cos(\lambda_n X) \left[1 - \exp\left(-\lambda_n^2 Fo\right)\right] + \Phi$ λ_n and Φ given in case 8.2.2.1

Case No.	References	Description	Solution
8.2.2.3	74, p. 364	Case 8.2.2.1 with $q''' = q_0''' \exp(-bX)$.	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{Fo}{b}\left[1 - \exp(-b)\right] + \sum_{n=1}^{\infty}\left[1 - (-1)^n \exp(-b)\right] \frac{2b}{\lambda_n^2\left(b^2 + \lambda_n^2\right)}$ $\times \cos(\lambda_n X)\left[1 - \exp\left(-\lambda_n^2 Fo\right)\right] + \Phi$ λ_n and Φ given in case 8.2.2.1
8.2.2.4	74, p. 364	Case 8.2.2.1 with $q''' = q_0'''(1 + b\tau)$.	$\frac{(t - t_i)k}{q_0''' \ell^2} = Fo\,(1 + Pd\,Fo) + \Phi$ Φ given in case 8.2.2.1

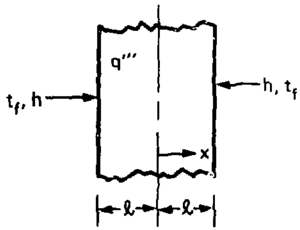
Section 6.1. Infinite Solids--No Internal Heating.

Case No.	References	Description	Solution
6.1.23	9, p. 346	Cylinder with properties different from surrounding medium. $t = t_1,\ r < r_0,\ \tau = 0.$ $t = t_0,\ r > r_0,\ \tau = 0.$  	$\frac{t - t_0}{t_1 - t_0} = \frac{4\gamma}{\pi^2}\int_0^\infty \exp\left(-\lambda^2 Fo_1\right)\frac{J_0(\lambda R)J_1(\lambda)\,d\lambda}{\lambda^2\left[\phi^2 + \psi^2\right]},\ R < 1.$ $\frac{t - t_0}{t_1 - t_0} = \frac{2\gamma}{\pi}\int_0^\infty \exp\left(-\lambda^2 Fo_1\right)\frac{J_1(\lambda)\left[J_0(\beta\lambda R)\phi - Y_0(\beta\lambda R)\psi\right]d\lambda}{\lambda(\phi^2 + \psi^2)},$ $R > 1.$ $\psi = \gamma J_1(\lambda)J_0(\beta\lambda) - \beta J_0(\lambda)J_1(\beta\lambda)$ $\phi = \gamma J_1(\lambda)Y_0(\beta\lambda) - \beta J_0(\lambda)Y_1(\beta\lambda)$ $\beta = \sqrt{\alpha_1/\alpha_0},\ \gamma = k_1/k_0$ See case 6.1.4 for equal material properties.
6.1.24	9, p. 402	Point source of rectangular periodic wave heating. $Q(\tau) = 0,\ \tau < 0.$ $Q(\tau) = Q_0,\ n\tau_0 < \tau < n\tau_0 + \tau_1,\ n = 0, 1, \ldots$ $Q(\tau) = 0,\ n\tau_0 + \tau < \tau < (n + 1)\tau_0.$ n = Number of cycles. $t = t_\infty,\ r \to \infty.$  	$\frac{(t - t_\infty)4\pi rk}{Q_0} = \mathrm{erfc}\left(\frac{Fo^*}{2\sqrt{T}}\right) - T_1 + 1$ $+ \frac{2}{\pi}\int_0^\infty \frac{\exp(-T\lambda^2)\left[\exp\left(-\lambda^2 + T_1\lambda^2\right) - \exp(-\lambda^2)\right]\sin(Fo^*\lambda)\,d\lambda}{\lambda\left[1 - \exp(-\lambda^2)\right]},$ $0 < T < T_1.$ $T_1 = \tau_1/\tau_0,\ T = \tau/\tau_0,\ Fo^* = r/\sqrt{\alpha\tau_0}$

Case No.	References	Description	Solution
8.2.3	9, p. 131	Infinite plate with time dependent internal heating. $t = t_i$, $-\ell < x < \pm\ell$, $\tau = 0$. $t = t_i$, $x = \pm\ell$, $\tau > 0$. $q''' = f(\tau)$. 	$\left(t - t_i\right) = \frac{4\alpha}{\pi k} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \cos\left[\frac{(2n+1)}{2} \pi X\right] \times \int_0^{\tau} f(\tau) \exp\left[-\pi^2 (2n+1)^2 \alpha(\tau - \tau')/\ell^2\right] d\tau'$
8.2.4	9, p. 132	Case 8.2.3 with $f(\tau) = q_0''' e^{-b\tau}$.	$\frac{(t - t_i)k}{q_0''' \ell^2} = \frac{1}{Pd}\left[\frac{\cos(X\sqrt{Pd})}{\cos(\sqrt{Pd})} - 1\right] \exp(-b\tau) + \frac{2}{\pi Pd} \sum_{n=0}^{\infty} \frac{(-1)^n \cos(\lambda_n X) \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n\left(1 - \lambda_n^2/Pd\right)}$ $\lambda_n = (2n + 1)\,\pi/2$

Case No.	References	Description	Solution
8.2.5	9, p. 132	Case 8.2.1 with $q''' = f(x)$.	$(t - t_i) = \frac{4\ell^2}{\pi^2 k} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{n\pi}{2} X\right) \left[1 - \exp\left(- \frac{n^2\pi^2}{4} Fo\right)\right] \times \int_{-1}^{1} f(X) \cos\left(\frac{n\pi}{2} X\right) dX$

<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr>
<td>8.2.6</td>
<td>9, p. 132
41</td>
<td>Infinite plate with uniform internal heating and convection boundary.
$t = t_f$, $-\ell < x < +\ell$, $\tau = 0$.
$q''' = q_0'''$.



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$$\frac{(t - t_f)k\,Bi}{q_0'''\ell^2} = 1 + \frac{Bi}{2} - \frac{Bi\,X^2}{2} - 2\,Bi^2 \sum_{n=1}^{\infty} \frac{\cos(\lambda_n X)}{\lambda_n^2\left(\lambda_n^2 + Bi^2 + Bi\right)\cos(\lambda_n)} \exp\left(-\lambda_n^2\,Fo\right)$$

$\lambda_n \tan(\lambda_n) = Bi$, See Fig. 8.13
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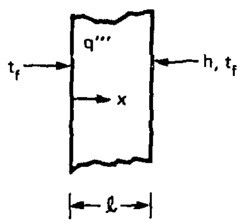
<tr>
<td>8.2.7</td>
<td>9, p. 132</td>
<td>Infinite plate with uniform internal heating and convection boundary on one side.
$t = t_f$, $0 < x < \ell$, $\tau = 0$.
$t = t_f$, $x = 0$, $\tau > 0$.
$q''' = q_0'''$.



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$$\frac{(t - t_f)k}{q'''\ell^2} = \frac{(1 + Bi/2)X}{1 + Bi} - \frac{X^2}{2} + 4\,Bi \sum_{n=1}^{\infty} \frac{\sin(\lambda_n X)\left[1 - \cos(\lambda_n)\right]}{\lambda_n^2\left(\lambda_n^2 + Bi^2 + Bi\right)\sin(2\lambda_n)} \exp\left(-\lambda_n^2\,Fo\right)$$

$\lambda_n \cot(\lambda_n) + Bi = 0$
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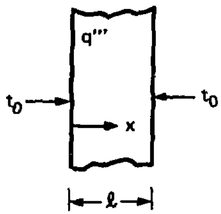
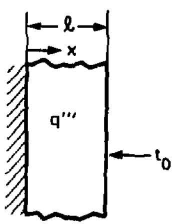
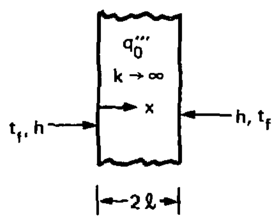
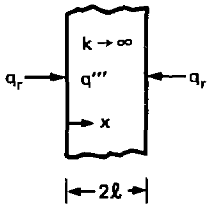
Case No.	References	Description	Solution
8.2.8	9, p. 311	Infinite plate with time-dependent heating. $t = t_0,\ 0 < x < \ell,\ \tau = 0.$ $t = t_0,\ x = 0,\ \ell,\ \tau > 0.$ $q''' = \beta\tau^{s/2},$ $s = -1, 0, 1, 2, \ldots.$ 	$\frac{(t - t_0)k}{\beta\alpha\tau^{1+s/2}} = \frac{1}{(1 + s/2)}\left\{1 - \Gamma(2 + s/2)\,2^{s+2} \times \sum_{n=0}^{\infty} (-1)^n \left[i^{s+2}\operatorname{erfc}\left(\frac{n + X}{2\sqrt{Fo}}\right) + i^{s+2}\operatorname{erfc}\left(\frac{n + 1 - X}{2\sqrt{Fo}}\right)\right]\right\}$
8.2.9	9, p. 404	Infinite plate with variable internal heating. $t = t_0,\ 0 < x < \ell,\ \tau = 0.$ $t = t_0,\ x = \ell,\ \tau > 0.$ $q''' = q_0''' + \beta(t - t_0).$ $q_x = 0,\ x = 0,\ \tau \geq 0.$ 	$\frac{(t - t_0)k}{q_0'''\ell^2} = \frac{1}{B}\left\{\frac{\cos(\sqrt{B}X)}{\cos(\sqrt{B})} - 1\right\} + \frac{16}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n \exp\left\{\left[-(2n + 1)^2\pi^2 + 4B\right] Fo/4\right\} \cos\left[(2n + 1)\pi X/2\right]}{\left[4B - (2n + 1)^2\pi^2\right](2n + 1)}$ $B = \beta\ell^2/k$

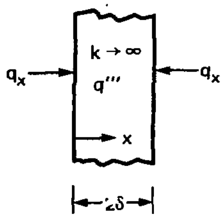
Section 8.2. Solids Bounded by Plane Surfaces--With Internal Heating.

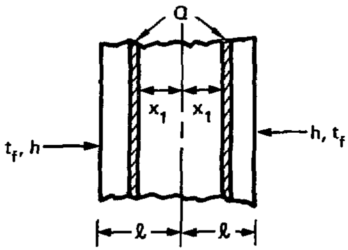
Case No.	References	Description	Solution
8.2.10	9, p. 405	Case 8.2.9 with convection boundary h, t_0 at $x = \ell$.	$\frac{(t - t_0)k}{q_0'''\ell^2} = \frac{Bi \cos(\sqrt{BX})}{B\left[Bi \cos(\sqrt{B}) - \sqrt{B}\sin(\sqrt{B})\right]} - \frac{1}{B}$ $+ 2\,Bi \sum_{n=0}^{\infty} \frac{\cos(\lambda_n X)\exp\left[\left(B - \lambda_n^2\right) Fo\right]}{\left(B - \lambda_n^2\right)\left[\lambda_n^2 + Bi(Bi + 1)\right]\cos(\lambda_n)}$ $\lambda_n \tan(\lambda_n) = Bi$, $B = \beta\ell^2/k$
8.2.11	19, p. 3-29	Infinite plate of infinite conductivity, variable specific heat, convection boundaries and steady heating. $t = t_0$, $0 < x < 2\ell$, $\tau = 0$. $c = c_0 + \beta(t - t_0)$. $q''' = q_0'''$. 	$\bar{G}\,\ell n\left[1 - (\theta/G)\right] + \theta = LH/\bar{C}$ $\bar{G} = (1 + q_0'''\,\ell)/hL(t - t_f)$, $\bar{C} = (1 - c_f)/c_o$, $H = h\tau/\rho c\ell$ H evaluated at t_0, c_f evaluated at t_f $L = \ell \times$ (surface area/volume) $= 1$ $\theta = (t - t_0)/(t_f - t_0)$

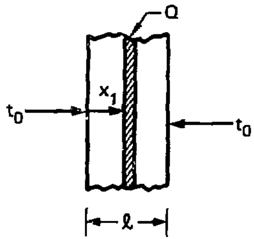
Case No.	References	Description	Solution
8.2.12	19, p. 3-29	Case 8.2.11 for infinite square rod and cube.	See case 8.2.11, set: L = 2 for square rod L = 3 for cube
8.2.13	19, p. 3-32	Infinite plate of infinite conductivity with surface radiation and steady heating. $t = t_0,\ 0 < x < 2\ell,\ \tau = 0.$ $q_r = \sigma\mathcal{F}\left(T_s^4 - T^4\right).$ T_s = source temp. $q_0''' = q_0'''.$ 	$\ell n\left[\frac{(\theta + N)(1 - N)}{(\theta + N)(1 + N)}\right] + 2\tan^{-1}(\theta/N) = 2\tan^{-1}(1/N) + 4MN^3$ $\theta = T/T_0,\ N = \left(q_0'''\ell/\sigma\mathcal{F}T_0^4 + \theta_s^4\right)^{1/4},\ M = \sigma\mathcal{F}T_0^3\tau/\rho c\ell$

Case No.	References	Description	Solution
8.2.14	19, p. 3-33	Case 8.2.13 with simultaneous surface convection and radiation. $q_x = h(t_f - t) + \sigma\mathscr{F}\left(T_s^4 - T^4\right)$, $x = 0, 2\ell, \tau > 0$.	$M = A_1 \ln\left(\frac{\theta - R_1}{\theta + R_1}\right) + A_2 \ln\left(\frac{\theta - R_2}{\theta + R_2}\right) + \frac{A_3}{2} \ln\left[\frac{(\theta - \eta)^2 + \phi^2}{(1 - \eta)^2 + \phi^2}\right]$ $+ \frac{A_4}{\phi}\left[\tan^{-1}\left(\frac{\theta - \eta}{\phi}\right) - \tan^{-1}\left(\frac{1 - \eta}{\phi}\right)\right]$ $A_1 = \left\{8\sqrt{\nu - \eta^2}\left[\eta\sqrt{\nu - \eta^2} + (\nu/2 + \eta^2)\right]\right\}^{-1}$ $A_2 = \left\{8\sqrt{\nu - \eta^2}\left[\eta\sqrt{\nu - \eta^2} - (\nu/2 + \eta^2)\right]\right\}^{-1}$ $A_3 = \eta/(\nu^2 + 8\eta^4)$ $A_4 = (\nu/2 - \nu^2)/(\nu^2 + 8\eta^4)$ $R_1 = -\eta - \sqrt{\nu - \eta^2},\ R_2 = -\eta + \sqrt{\nu - \eta^2}$ $\phi = \sqrt{\eta + \beta/4},\ \eta = \sqrt{\beta/2}$ $\beta = \left(\frac{s^2}{2}\sqrt{\frac{s^4}{4} + \frac{64}{27}N^{12}}\right)^{1/3} + \left(\frac{s^2}{4} - \sqrt{\frac{s^4}{4} + \frac{64}{27}N^{12}}\right)^{1/3}$ $\nu = \sqrt{N_2^4 + \beta^2/4}$ $s = h/\sigma\mathscr{F}T_0^3$ $N = \left[q'''\ell/\sigma\mathscr{F}T_0^4 + \theta_s^4 + s\theta_f\right]^{1/4}$

Case No.	References	Description	Solution
8.2.15	19, p. 3-34	Infinite plate of infinite conductivity with prescribed heating rates. $t = t_0$, $\tau = 0$.	
		a. step input; $\frac{q'''}{q'''_{max}} = 1$, $\frac{Q'''}{q'''_{max}D} = 1$	a. $\frac{\rho C\delta}{Q'''}\,\theta = \bar{\tau}$
		b. linear input; $\frac{q'''}{q'''_{max}} = \bar{\tau}$, $\frac{Q'''}{q'''_{max}D} = 0.5$	b. $\frac{\rho C\delta}{Q'''}\,\theta = \bar{\tau}^2$
		c. linear input; $\frac{q'''}{q'''_{max}} = (1 - \bar{\tau})$, $\frac{Q'''}{q'''_{max}D} = 0.5$	c. $\frac{\rho C\delta}{Q'''}\,\theta = \bar{\tau}(2 - \bar{\tau})$
		d. circular pulse; $\frac{q'''}{q'''_{max}} = \sin^2 \pi\bar{\tau}$, $\frac{Q'''}{q'''_{max}D} = 0.5$	d. $\frac{\rho C\delta}{Q'''}\,\theta = \bar{\tau} - \left(\frac{1}{2\pi}\right)\sin 2\pi\bar{\tau}$
		e. power pulse; $\frac{q'''}{q'''_{max}} = (3\bar{\tau})^{-3}\, e^{-(1/\bar{\tau}-3)}$, $\frac{Q'''}{q'''_{max}D} = 0.5473$	e. $\frac{\rho C\delta}{Q'''}\,\theta = 0.06767\left(\frac{1}{\bar{\tau}} + 1\right) e^{-(1/\bar{\tau}-3)}$
		f. power pulse, $\frac{q'''}{q'''_{max}} = (5\bar{\tau})^{-5}\, e^{-(1/\bar{\tau}-5)}$, $\frac{Q'''}{q'''_{max}D} = 0.2795$	f. $\frac{\rho C\delta}{Q'''}\,\theta = 0.006869\left(\frac{1}{6\bar{\tau}^3} + \frac{1}{2\bar{\tau}^2} + \frac{1}{\bar{\tau}} + 1\right) e^{-(1/\bar{\tau}-5)}$
		g. exponential pulse; $\frac{q'''}{q'''_{max}} = (10\bar{\tau})\, e^{1-10\bar{\tau}}$, $\frac{Q'''}{q'''_{max}D} = 0.2717$	g. $\frac{\rho C\delta}{Q'''}\,\theta = 1.001\left[1 - (10\bar{\tau} + 1)\, e^{-10\bar{\tau}}\right]$
			$\theta = t - t_0$, D = heating duration $\bar{\tau} = \tau/D$, Q''' = total heat input per unit volume during time D See Fig. 8.10

Case No.	References	Description	Solution
8.2.16	19, p. 3-35	Infinite plate of infinite conductivity with surface reradiation, steady surface heating, and steady internal heating. $t = t_0,\ \tau = 0.$ $q_x = q'' + \sigma\mathcal{F}\left(T_s^4 - T^4\right),\ x = 0,\ 2\delta.$ T_s = source temp. 	See Fig. 8.11

Case No.	References	Description	Solution
8.2.17	74, p. 386	Infinite plate with two symmetrical planar heat pulses and convection boundaries. $t = t_f$, $-\ell < x < +\ell$, $\tau = 0$. Instantaneous pulse occurs at $\tau = 0$, $x = \pm x$, with strength $Q(J/m^2)$.  	$\displaystyle \frac{(t - t_f)k\ell}{Q\alpha} = 2 \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n + \sin(\lambda_n)\cos(\lambda_n)} \cos(\lambda_n X_1) \cos(\lambda_n X) \times \exp\left(-\lambda_n^2 \, Fo\right).$ $\lambda_n \tan(\lambda_n) = Bi.$ For $t = f(x)$, $-\ell < x < +\ell$, $\tau = 0$: $\displaystyle \frac{(t - t_f)k\ell}{Q\alpha} = 2 \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n + \sin(\lambda_n)\cos(\lambda_n)} \cos(\lambda_n X) \exp\left(-\lambda_n^2 \, Fo\right) \times \int_0^1 \left[f(X) - t_f\right] \cos(\lambda_n X) dX.$ Mean temp: $\displaystyle \frac{(t_m - t_f)k\ell}{Q\alpha} = 2 \sum_{n=1}^{\infty} \frac{Bi^2 \cos(\lambda_n X_1) \exp\left(-\lambda_n^2 \, Fo\right)}{\sin(\lambda_n)\lambda_n\left(Bi^2 + Bi + \lambda_n^2\right)}.$ For $Bi = \infty$: $\displaystyle \frac{(t - t_f)k\ell}{Q\alpha} = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \cos\left[(2n - 1)\pi X_1\right] \cos\left[(2n - 1)\pi X\right] \times \exp\left[-(2n - 1)^2 \pi^2 \, Fo/4\right].$

Case No.	References	Description	Solution
8.2.18	74, p. 387	Infinite plate with a planar heat pulse. $t = t_0$, $0 < x < \ell$, $\tau = 0$. Instantaneous pulse occurs at $x = x_1$, $\tau = 0$ with strength $Q (J/m^2)$. 	$\frac{(t - t_0)k\ell}{Q\alpha} = 2 \sum_{n=1}^{\infty} \sin(n\pi X_1) \sin(n\pi X) \exp(-n^2\pi^2 \text{ Fo})$

Case No.	References	Description	Solution

Section 8.2. Solids Bounded by Plane Surfaces--With Internal Heating.

Case No.	References	Description	Solution

9. Cylindrical Surface — Transient

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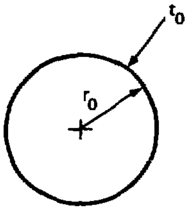
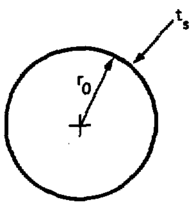
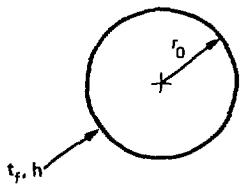
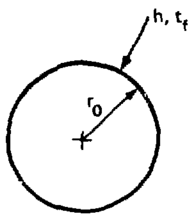
Section 9.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
9.1.1	2, p. 240 74, p. 131	Infinite cylinder with steady surface temp. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ 	$$\frac{t - t_0}{t_i - t_0} = 2 \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \exp\left(-\lambda_n^2\ Fo\right) \frac{J_0(\lambda_n R)}{J_1(\lambda_n)}$$ $J_0(\lambda_n) = 0$, see Fig. 9.2 Cumulative heating: $$\frac{Q}{Q_0} = 4\ R \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \left[1 - \exp\left(-\lambda_n^2\ Fo\right) \frac{J_1(\lambda_n R)}{J_0(\lambda_n)}\right]$$ $Q_0 = \pi r_0^2 \rho c (t_0 - t_i)$
9.1.2	1, p. 269 9, p. 328	Infinite cylinder with time dependent surface temp. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t_s = t_i + c\tau,\ r = r_0,\ \tau > 0.$ 	$$\frac{t - t_i}{c r_0^2/\alpha} = Fo + \frac{1}{4}\ (R^2 - 1) + 2 \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right) \frac{J_0(\lambda_n R)}{\lambda_n^3\ J_1(\lambda_n)}$$ $J_0(\lambda_n) = 0$ See Table 9.1 and Fig. 9.8

<table>
<tr><th>Case No.</th><th>References</th><th>Description</th><th>Solution</th></tr>
<tr><td>9.1.3.1</td><td>74, p. 270</td><td>Infinite cylinder with convection boundary and variable initial temp.
$t = f(r)$, $0 < r < r_0$, $\tau = 0$.

</td><td>

$$t - t_f = 2 \sum_{n=1}^{\infty} \frac{J_0(\lambda_n R) \exp\left(-\lambda_n^2 Fo\right)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} \int_0^1 R\left[f(r) - t_f\right] J_0(\lambda_n R)\, dR$$

$$Bi\, J_0(\lambda_n) = \lambda_n J_1(\lambda_m)$$

</td></tr>
<tr><td>9.1.3.2</td><td>3, p. 298</td><td>Infinite cylinder with convection cooling.
$t = t_i$, $0 < r < r_0$, $\tau = 0$.

</td><td>

$$\frac{t - t_f}{t_i - t_f} = 2 \sum_{n=1}^{\infty} \frac{Bi\, J_0(\lambda_n R) \exp\left(-\lambda_n^2 Fo\right)}{\left(\lambda_n^2 + Bi^2\right) J_0(\lambda_n)}$$

$$\lambda_n J_1(\lambda_n) + Bi\, J_0(\lambda_n) = 0$$

See Figs. 9.1a-c

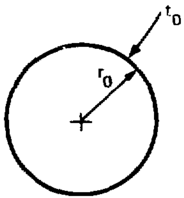
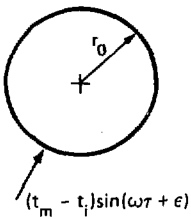
Cumulative heating:

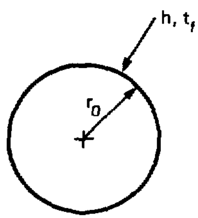
$$\frac{Q}{Q_0} = 4 \sum_{n=1}^{\infty} \frac{Bi^2 \left[1 - \exp\left(-\lambda_n^2 Fo\right)\right]}{\lambda_n^2 \left(\lambda_n^2 + Bi^2\right)}, \quad Q_0 = \pi r_0^2 \rho c (t_i - t_f)$$

For $Fo < 0.02$, $R >> 0$:

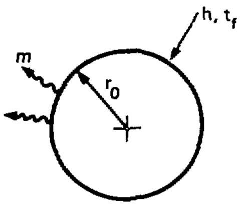
$$\frac{(t - t_i)}{(t_f - t_i)} = 2\, Bi \sqrt{\frac{Fo}{R}}\, \text{ierfc}\left(\frac{1 - R}{2\sqrt{Fo}}\right) + 4\, Bi \frac{Fo}{\sqrt{R}} \times \left(\frac{1}{8R} + \frac{3}{8} - Bi\right) i^2\text{erfc}\left(\frac{1 - R}{2\sqrt{Fo}}\right) + \ldots.$$

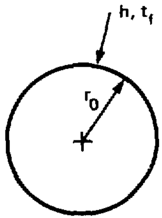
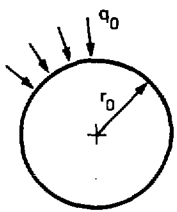
</td></tr>
</table>

Case No.	References	Description	Solution
9.1.4	9, p. 199	Infinite cylinder with initial temp varying with radius. $t = t_i - br^2,$ $0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ 	$\dfrac{t - t_0}{br_0^2} = 2 \sum_{n=1}^{\infty} \dfrac{\exp\left(-\lambda_n^2\ Fo\right) J_0(R\lambda_n)}{\lambda_n^2 J_1^2(\lambda_n)} \times \left[\lambda_n \left(\dfrac{t_i - t_0}{br_0^2} - 1\right) J_1(\lambda_n) + 2\ J_2(\lambda_n)\right]$ $J_0(\lambda_n) = 0$
9.1.5	9, p. 201	Infinite cylinder with periodic surface temperature. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = (t_m - t_i)\sin(w\tau + \varepsilon),$ $r = r_0,\ \tau > 0.$ t_m = max fluid temp. 	$\dfrac{t - t_i}{t_m - t_i} = \text{real}\left\{\dfrac{I_0(R\sqrt{i\ Pd})}{iI_0(\sqrt{i\ Pd})} \exp\left[i(w\tau + \varepsilon)\right]\right\} + 2 \sum_{n=1}^{\infty} \dfrac{\exp\left(-\lambda_n^2\ Fo\right) \lambda_n \left[Pd\cos(\varepsilon) - \lambda_n^2 \sin(\varepsilon)\right] J_0(R\lambda_n)}{\left(\lambda_n^4 + Pd^2\right) J_1(\lambda_n)}$ $J_0(\lambda_n) = 0,\ Pd = r_0^2\omega/\alpha$

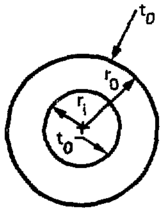
Case No.	References	Description	Solution
9.1.6.1	9, p. 202 89	Infinite cylinder with convection cooling and linear time-dependent fluid temp. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t_f = b\tau + t_i.$  	$\dfrac{(t - t_i)\alpha}{b r_0^2} = Fo - \dfrac{1}{4}\left(1 - R^2 + \dfrac{2}{Bi}\right) + 2\ Bi \displaystyle\sum_{n=1}^{\infty} \dfrac{\exp\left(-\lambda_n^2\ Fo\right)\ J_0(\lambda_n R)}{\lambda_n^2 \left(Bi^2 + \lambda_n^2\right) J_0(\lambda_n)}$ $\lambda_n J_1(\lambda_n) = Bi\ J_0(\lambda_n)$ Mean temp: $\dfrac{t_m - t_i}{b\tau} = Fo - \dfrac{1}{8}\left(1 + \dfrac{4}{Bi}\right) + 4\ Bi^2 \displaystyle\sum_{n=1}^{\infty} \dfrac{\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^4 \left(\lambda_n^2 + Bi^2\right)}$ See Fig. 9.9

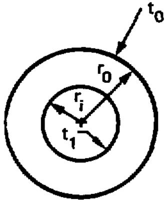
Case No.	References	Description	Solution
9.1.6.2	74, p. 317	Case 9.1.6.1 with $t_f = t_{fm} - (t_{fm} - t_i)e^{-b\tau}$.	$\dfrac{t - t_i}{t_{fm} - t_i} = 1 - \dfrac{J_0(\sqrt{Pd}\,R)\ \exp\,(-Pd\ Fo)}{\sqrt{Pd}\ J_0(\sqrt{Pd}) - \frac{1}{Bi}\ Pd\ J_1(\sqrt{Pd})}$ $- 2\ Bi \displaystyle\sum_{n=1}^{\infty} \dfrac{J_0(\lambda_n R)\ \exp\left(-\lambda_n^2\ Fo\right)}{J_0(\lambda_n)\ \left(\lambda_n^2 + Bi^2\right)\left[1 - \left(\lambda_n^2/Pd\right)\right]}$ Mean temp : $\dfrac{t_m - t_i}{t_{fm} - t_i} = 1 - \dfrac{2J_1(\sqrt{Pd})\ \exp\,(-Pd\ Fo)}{\sqrt{Pd}\ J_0(\sqrt{Pd}) - \frac{1}{Bi}\ Pd\ J_1\ \sqrt{Pd}}$ $- 4\ Bi^2 \displaystyle\sum_{n=1}^{\infty} \dfrac{\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^2\left(\lambda_n^2 + Bi^2\right)\left[1 - \left(\lambda_n^2/Pd\right)\right]}$

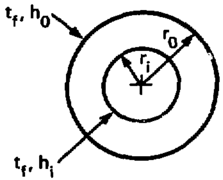
Case No.	References	Description	Solution
9.1.6.3	74, p. 320	Infinite cylinder with convection and evaporation boundary. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $m = m_0 e^{-b\tau}$ = evaporation rate. 	$\frac{t - t_i}{t_f - t_i} = 1 - \frac{M J_0(\sqrt{Pd}\,R)\ \exp(-Pd\ Fo)}{J_0(\sqrt{Pd}) - (\sqrt{Pd}/Bi) J_1(\sqrt{Pd})} - 2 \sum_{n=1}^{\infty} \left\{1 - \frac{M}{\left[1 - \left(Pd/\lambda_n^2\right)\right]}\right\} A_n \cos(\lambda_n R) \exp\left(-\lambda_n^2\ Fo\right)$ $A_n = \frac{Bi}{J_0(\lambda_n)\left(\lambda_n^2 + Bi^2\right)},\quad M = \frac{\gamma(m_0)}{h(t_f - t_i)},$ $J_0(\lambda_n)/J_1(\lambda_n) = \lambda_n/Bi$

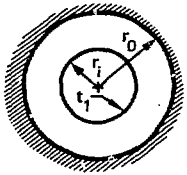
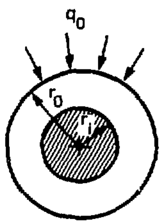
Case No.	References	Description	Solution
9.1.7	9, p. 202	Infinite cylinder with periodic convection boundary. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t_f = (t_m - t_i)\sin(\omega\tau + \varepsilon).$ t_m = max fluid temp. 	$\frac{t - t_i}{t_m - t_i} = \text{real}\left\{\frac{Bi\ I_0(R\sqrt{i\ Pd})\ \exp(i\omega\tau + i\varepsilon)}{i\left[\sqrt{i\ Pd}\ I_1(\sqrt{i\ Pd}) + Bi\ I_0(\sqrt{i\ Pd})\right]}\right\} + 2\ Bi \times \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right) \frac{\lambda_n^2\left[Pd\ \cos(\varepsilon) - \sin(\varepsilon)\right] J_0(\lambda_n R)}{\left(\lambda_n^4 + Pd^2\right)\left(\lambda_n^2 + Bi^2\right) J_0(\lambda_n)}$ $\lambda_n J_1(\lambda_n) = Bi\ J_0(\lambda_m)$, $Pd = r_0^2\omega/\alpha$
9.1.8	9, p. 203	Infinite cylinder with steady surface heat flux. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $q_r = q_0,\ r = r_0,\ \tau > 0.$ 	$\frac{(t - t_i)k}{q_0 r_0} = 2\ Fo + (R^2/2) - (1/4) - 2\sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right)\frac{J_0(\lambda_n R)}{\lambda_n^2 J_0(\lambda_n)}$ $J_1(\lambda_n) = 0$, See Fig. 9.3 For $Fo < 0.02$, $R >> 0$: $\frac{(t - t_i)k}{q_0 r_0} = 2\sqrt{\frac{Fo}{R}}\ \text{ierfc}\left(\frac{1 - R}{2\sqrt{Fo}}\right) + \frac{Fo(1 + 3R)}{2R^{3/2}}\ i^2\text{erfc}\left(\frac{1 - R}{2\sqrt{Fo}}\right) + \ldots$

Section 9.1. *Solids Bounded by Cylindrical Surfaces--No Internal Heating.*

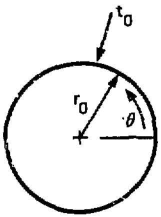
Case No.	References	Description	Solution
9.1.9	9, p. 207	Infinite hollow cylinder with steady surface temp. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_i,\ r_0,\ \tau > 0.$ 	$\frac{t - t_0}{t_i - t_0} = \pi \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right) \times \frac{J_0(R_i\lambda_n)\left[J_0(\lambda_n R)Y_0(\lambda_n) - J_0(\lambda_n)Y_0(\lambda_n R)\right]}{J_0(\lambda_n R_i) + J_0(\lambda_n)}$ $J_0(\lambda_n R_i)Y_0(\lambda_n) - J_0(\lambda_n)Y_0(\lambda_n R_i) = 0$ $R = r/r_0,\ Fo = \alpha\tau/r_0^2$

Case No.	References	Description	Solution
9.1.10.1	9, p. 207	Infinite hollow cylinder with different surface temp. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ $t = t_1,\ r = r_i,\ \tau > 0.$ 	$$\frac{T}{T_i} = \pi \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right)\frac{J_0(a_n)U_0(R\lambda_n)}{J_0(a_n) + J_0(b_n)} - \pi\frac{T_0}{T_i} \times \sum_{n=1}^{\infty}\frac{\left[J_0(a_n) - (T_1/T_0)J_0(b_n)\right]\ J_0(a_n)U_0(R\lambda_n)}{J_0^2(a_n) - J_0^2(b_n)} \exp\left(-\lambda_n^2\ Fo\right) + \frac{T_0\ \left[(T_1/T_0)\ \ell n(1/R) + \ell n(R/R_i)\right]}{T_i\ \ell n(1/R_i)}$$ $a_n = R_i\lambda_n,\ b_n = \lambda_n,\ U_0(R\lambda_n) = J_0(\lambda_n R)Y_0(b_n) - J_0(b_n)Y_0(\lambda_n R)$ $J_0(\lambda_n R_i)Y_0(\lambda_n) - J_0(\lambda_n)Y_0(\lambda_n)Y_0(\lambda_n R_i) = 0$ $R = r/r_0,\ Fo = \alpha\tau/r_0^2$ See Fig. 9.7a for $t_1 = t_i$, and Fig. 9.7b for $t_0 = t_i$.

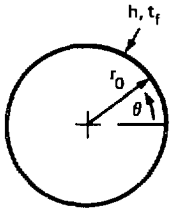
Case No.	References	Description	Solution
9.1.10.2	74, p. 282	Infinite hollow cylinder with two convection boundaries and variable initial temperature-general case. $t = f(R),\ r_i < r < r_0,\ \tau = 0.$  	$t - t_f = \sum_{n=1}^{\infty} E_n \exp\left(-\lambda_n^2\, \mathrm{Fo}\right) W(\lambda_n, R).$ $W(\lambda_n R) = -\left[Bi_i\, Y_0(\lambda_n) + \lambda_n Y_1(\lambda_n)\right] J_0(\lambda_n R) + \left[\lambda_n J_1(\lambda_n) + Bi_i\, J_0(\lambda_n)\right] Y_0(\lambda_n R).$ $E_n = \frac{\pi^2}{2} \lambda_n^2 \left[Bi_0\, J_0(\lambda_n R_0) - \lambda_n J_1(\lambda_n R_0)\right]^2 \times \int_1^{R_0} R\left[f(R) - t_f\right] W(\lambda_n, R)\, dR \left\{\left(\lambda_n^2 + Bi_0^2\right)\left[Bi_i\, J_0(\lambda_n) + \lambda_n J_1(\lambda_n)\right]^2 - \left(\lambda_n^2 + Bi_i^2\right)\left[Bi_i\, J_0(\lambda_n R_0) - \lambda_n J_1(\lambda_n R_0)\right]^2\right\}^{-1}.$ $\left[Bi_i\, J_0(\lambda_n) + \lambda_n J_1(\lambda_n)\right]\left[Bi_0\, Y_0(\lambda_n R_0) - \lambda_n Y_1(\lambda_n R_0)\right] - \left[Bi_0\, J_0(\lambda_n R_0) - \lambda_n J_1(\lambda_n R_0)\right]\left[Bi_i\, Y_0(\lambda_n) - \lambda_n Y_1(\lambda_n)\right] = 0.$ $Bi_i = h_i r_i/k,\ Bi_0 = h_0 r_i/k,\ R = r/r_0$

Case No.	References	Description	Solution
9.1.11	3, p. 409	Infinite hollow cylinder with steady inside surface temperature. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $t = t_1,\ r = r_i,\ \tau > 0.$ $q_r = 0,\ r = r_0,\ \tau > 0.$  	$\frac{t - t_i}{t_1 - t_i} = 1 - 2 \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\ Fo\right) \frac{Y_1(\lambda_n)J_0(\lambda_n R) - J_1(\lambda_n)Y_0(\lambda_n R)}{\lambda_n \Delta_0(R_i) - \lambda_n R_i \Delta_1(R_i)}$ $\Delta_0(R_i) = Y_0(\lambda_n)J_0(\lambda_n R_i) - J_0(\lambda_n)Y_0(\lambda_n R_i)$ $\Delta_1(R_i) = Y_1(\lambda_n)J_1(\lambda_n R_i) - J_1(\lambda_n)Y_1(\lambda_n R_i)$ $Y_1(\lambda_n)J_0(\lambda_n R_i) - J_1(\lambda_n)Y_0(\lambda_n R_i) = 0$ $Fo = \alpha\tau/r_0^2,\ R = r/r_0$
9.1.12	74, p. 200	Infinite hollow cylinder with steady surface heat flux. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $-q_r = q_0,\ r = r_0,\ \tau > 0.$ $q_r = 0,\ r = r_i,\ \tau > 0.$  	$\frac{(t - t_i)k}{q_0 r_0} = \frac{1}{1 - R_i^2} \left\{2\ Fo - \frac{1}{4} + \frac{R^2}{2} - R_i^2\ \ln\left[\left(\frac{R}{R_i} + \frac{1}{1 - R_i^2}\right) \times \ln(R_i) + \frac{3}{4}\right]\right\} + \sum_{n=1}^{\infty} \frac{\pi}{\lambda_n} \frac{J_1(\lambda_n R_i)J_1(\lambda_n)}{J_1^2(\lambda_n R_i) - J_1^2(\lambda_n)}$ $\times \left[J_0(\lambda_n R)Y_1(\lambda_n R_i) - Y_0(\lambda_n R)J_1(\lambda_n R_i)\right] \exp\left(-\lambda_n^2\ Fo\right)$ $Fo = \alpha\tau/r_0^2,\ R = r/r_0$ $J_1(\lambda_n R_i)Y_1(\lambda_n) = Y_1(\lambda_n R_i)J_1(\lambda_n)$

Section 9.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
9.1.13	9, p. 210	Infinite cylinder with variable initial temperature and constant surface temp. $t = f(r,\theta),\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$  	$t - t_0 = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \left[A_{n,m} \cos(n\theta) + B_{n,m} \sin(n\theta)\right]$ $\times J_n(\lambda_m R) \exp\left(-\lambda_m^2 Fo^2\right)$ $A_{o,m} = \frac{1}{\pi \left[J_0'(\lambda_m)\right]^2} \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_0\right] J_0(\lambda_m R)\, dR\, d\theta$ $A_{n,m} = \frac{2}{\pi \left[J_n'(\lambda_m)\right]^2} \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_0\right] \cos(n\theta) J_n(\lambda_m r)\, dR\, d\theta$ $B_{n,m} = \frac{2}{\pi \left[J_n'(\lambda_m)\right]^2} \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_0\right] \sin(n\theta) J_n(\lambda_m R)\, dR\, d\theta$ $J_n(\lambda_m) = 0$

Case No.	References	Description	Solution
9.1.14	9, p. 211	Infinite cylinder with variable initial temp and convection boundary-general case. $t = f(r,\theta)$, $0 < r < r_0$, $\tau = 0$.	$t - t_f = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \left[A_{n,m} \cos (n\theta) + C_{n,m} \sin (n\theta)\right] J_n(\lambda_n R) \exp \left(-\lambda_n^2 \mathrm{Fo}\right)$



$$A_{o,m} = \frac{\lambda_m^2}{\pi\left(\lambda_m^2 + \mathrm{Bi}^2\right) \left[J_0(\lambda_m)\right]^2} \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_f\right] J_0(\lambda_m R)\, dR\, d\theta$$

$$A_{n,m} = \frac{2\lambda_m^2}{\pi\left(\lambda_m^2 + \mathrm{Bi}^2 - n^2\right) \left[J_n(\lambda_m)\right]^2}$$

$$\times \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_f\right] \cos (n\theta) J_n(\lambda_m R)\, dR\, d\theta$$

$$B_{n,m} = \frac{2\lambda_m^2}{\pi\left(\lambda_m^2 + \mathrm{Bi}^2 - n^2\right) \left[J_n(\lambda_m)\right]^2}$$

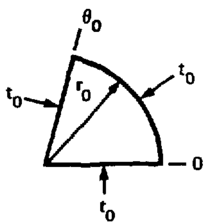
$$\times \int_0^1 \int_{-\pi}^{\pi} R\left[f(R,\theta) - t_f\right] \sin (n\theta) J_n(\lambda_m R)\, dR\, d\theta$$

$$\lambda_m J_n'(\lambda_m) + \mathrm{Bi}\, J_n(\lambda_m) = 0$$

See case 9.1.3.2 for $f(r,\theta) = t_i$

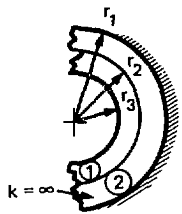
or

case 9.1.3.1 for $f(r,\theta) = f(r)$

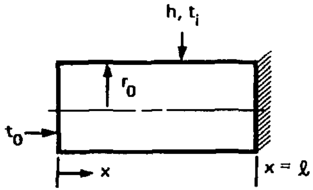
Case No.	References	Description	Solution
9.1.15	9, p. 212	Infinite cylindrical sector with steady surface temperature. $t = t_i$, $0 < r < r_0$, $0 < \theta < \theta_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\theta = 0$, $\tau > 0$. 	$\frac{t - t_0}{t_i - t_0} = \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{\sin(j\theta)}{2n + 1} \sum_{m=1}^{\infty} \exp\left(-\lambda_m^2 \, Fo\right) \frac{J_j(R\lambda_m)}{\left[J_j'(\lambda_m)\right]^2} \int_0^1 RJ_j(R\lambda_m)\,dR$ $j = (2n + 1)\pi/\theta_0$, $J_j(\lambda_m) = 0$
9.1.16	2, p. 248	Semi-infinite cylinder.	Dimensionless temperature equals product of solution for semi-infinite solid (case 7.1.1 or 7.1.3) and solution for infinite cylinder (case 9.1.1 or 9.1.3). See Figs. 9.4a and 9.4b.
9.1.17	2, p. 248	Finite cylinder.	Dimensionless temperature equals product of solution for infinite plate (case 8.1.6 or 8.1.7) and solution for infinite cylinder (case 9.1.1 or 9.1.3). See Figs. 9.4a and 9.4b.
9.1.18		Infinite cylinder in a semi-infinite solid.	See case 7.1.22

Case No.	References	Description	Solution
9.1.19	9, p. 418	Finite cylinder. $t = t_0$, $0 < r < r_0$, $0 < z < \ell$, $\tau = 0$. $t = t_1$, $0 < r < r_0$, $z = 0$, $\tau > 0$. $t = t_0$, $0 < r < r_0$, $z = \ell$, $\tau > 0$. Convection boundary h, t_0 at $r = r_0$, $\tau > 0$.	$\frac{t - t_0}{t_1 - t_0} = \sum_{n=1}^{\infty} \frac{Bi\, J_0(R\lambda_n) \sinh\left[(1 - Z)\lambda_n L\right]}{\left(Bi^2 + \lambda_n^2\right) J_0(\lambda_n) \sinh(\lambda_n L)} - 4\pi\, Bi \times \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{m J_0(R\lambda_n) \sin(m\pi Z) \exp\left\{-\left[\lambda_n^2 + (m\pi/L)^2\right] Fo\right\}}{\left[(\lambda_n L)^2 + (m\pi)^2\right]\left[Bi^2 + \lambda_n^2\right] J_0(\lambda_n)}$ $\lambda_n J_1(\lambda_n) = Bi\, J_0(\lambda_n)$ $Bi = hr_0/k$, $R = r/r_0$, $Z = z/\ell$, $Fo = \alpha\tau/r_0^2$, $L = \ell/r_0$
9.1.20	9, p. 418	Case 9.1.18 with convection boundary h, t_0 at $z = \ell$, $\tau > 0$.	$\frac{t - t_0}{t_1 - t_0} = 2\, Bi \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n)\left\{\lambda_n \cosh\left[\lambda_n L(1 - Z)\right] + Bi \sinh\left[\lambda_n L(1 - Z)\right]\right\}}{\left(Bi^2 + \lambda_n^2\right) J_0(\lambda_n)\left[\lambda_n \cosh(L\lambda_n) + Bi \sinh(L\lambda_m)\right]} - 4\, Bi \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\beta_m \left(Bi^2 L^2 + \beta_m^2\right) J_0(R\lambda_n) \sin(Z\beta_m) \exp\left[-\left(\lambda_n^2 + \beta_m^2/L^2\right) Fo\right]}{\left(Bi^2 + \lambda_n^2\right)\left(Bi^2 L^2 + \beta_m^2 + Bi\, L\right)\left(L^2\lambda_n^2 + \beta_m^2\right) J_0(\lambda_n)}$ $\beta_m \cot\beta_m = -Bi\, L$, $L = \ell/r_0$ See case 9.1.19 for λ_n, Bi, R, Z, and Fo

Case No.	References	Description	Solution
9.1.21	9, p. 419	Semi-infinite cylinder with convection boundary. $t = t_0$, $0 < r < r_0$, $z > 0$, $\tau = 0$. $t = t_1$, $0 < r < r_0$, $z = 0$, $\tau > 0$. Convection boundary h, t_0 at $r = r_0$, $z > 0$, $\tau > 0$.	$\frac{t - t_0}{t_1 - t_0} = \mathrm{Bi} \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n)}{\left(\mathrm{Bi}^2 + \lambda_n^2\right) J_0(\lambda_n)} \left\{ 2 \exp(-\lambda_n Z) + \exp(\lambda_n Z) \times \operatorname{erfc}\left(\frac{\lambda_n}{2\,\mathrm{Fo}^*} + Z\,\mathrm{Fo}^*\right) - \exp(-\lambda_n Z) \operatorname{erfc}\left(\frac{\lambda_n}{2\,\mathrm{Fo}^*} - Z\,\mathrm{Fo}^*\right) \right\}$ $\lambda_n J_1(\lambda_n) = \mathrm{Bi}\, J_0(\lambda_n)$ $\mathrm{Bi} = hr_0/k$, $R = r/r_0$, $Z = z/r_0$, $\mathrm{Fo}^* = r_0/2\sqrt{\alpha\tau}$
9.1.22	9, p. 419	Semi-infinite cylinder with steady surface temperature. $t = t_0$, $0 < r < r_0$, $z > 0$, $\tau = 0$. $t = t_1$, $r = r_0$, $z > 0$, $\tau > 0$. $t = t_0$, $0 < r < r_0$, $z = 0$, $\tau > 0$.	$\frac{t - t_0}{t_1 - t_0} = 1 - \sum_{n=1}^{\infty} \frac{J_0(R\lambda_n)}{\lambda_n J_1(\lambda_n)} \left\{ 2 \exp\left(-\lambda_n^2\,\mathrm{Fo}\right) \operatorname{erf}(Z\,\mathrm{Fo}^*) + \exp(\lambda_n Z) \times \operatorname{erfc}\left(Z\,\mathrm{Fo}^* + \frac{\lambda_n}{2\mathrm{Fo}^*}\right) + \exp(-Z\lambda_n) \operatorname{erfc}\left(Z\,\mathrm{Fo}^* - \frac{\lambda_n}{2\,\mathrm{Fo}^*}\right) \right\}$ See case 9.1.20 for λ_n, Fo^*, R, and Z

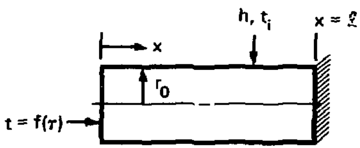
Case No.	References	Description	Solution
9.1.23	9, p. 420	Cone with steady surface temperature. $t = t_i,\ 0 < \theta < \theta_0,\ \tau = 0.$ $t = t_0,\ \theta = \theta_0,\ \tau > 0.$	$\frac{t - t_i}{t_0 - t_i} = 1 + \sqrt{2} \sum_{n=0}^{\infty} \left[\frac{(2n + 1)\Gamma[(n + 1)/2]\, P_n(\lambda)}{n\Gamma(n/2)\,[dP_n(\lambda)/dn]}\right]_{\lambda=\lambda_0}$ $\times \int_0^{\infty} \frac{\exp(-\alpha\tau u/r^2) J_{n+1/2}(u)\, du}{u^{3/2}}$ $P_n(\lambda_0) = 0$ (Legendre polynomial)
9.1.24	33	Infinite composite cylinder. $t = t_i,\ r_3 < r < r_1,\ \tau = 0.$ $t = t_0,\ r = r_3,\ \tau > 0.$ $q_r = 0,\ r = r_1.$  	$\frac{t - t_0}{t_i - t_0} = \pi \sum_{n=1}^{\infty} \exp\left(-\lambda_n^2\, Fo\right) FG\left[\lambda_n^2 J_0(R_2\lambda_n) - N\lambda_n J_1(R_2\lambda_n)\right]$ $F = \frac{J_0(\lambda_n R_3)}{\left[J_0(\lambda_n R_3)\right]^2\left[\lambda_n^2 - N(N - 2/R_2)\lambda_n^2\right] - \left[\lambda_n^2 J_0(R_2\lambda_n) - N\lambda_n J_1(R_2\lambda_n)\right]^2}$ $G = J_0(R\lambda_n)\left[\lambda_n^2 Y_0(R_2\lambda_n) - NY_1(R_2\lambda_n)\right] - Y_0(R\lambda_n)$ $\times\left[\lambda_n^2 J_0(R_2\lambda_n) - N\lambda_n J_1(R_2\lambda_n)\right]$ $R = r/r_1,\ N = \frac{(R_2 - 1)^2\rho_1 c_1}{(R_3 - 1)^3\rho_2 c_2},\ J_0(R_3\lambda_n) = 0$

Case No.	References	Description	Solution
9.1.25	33	Finite cylinder with convection boundary and insulated end. $t = t_i$, $0 < r < r_0$, $0 < x < \ell$, $\tau = 0$. $t = t_0$, $0 < r < r_0$, $x = 0$, $\tau > 0$. $q_x = 0$, $0 < r < r_0$, $x = \ell$, $\tau > 0$. Convection boundary h, h_2, t_0: at $r = r_0$.	$$\frac{t - t_i}{t_0 - t_i} = 2\sum_{n=1}^{\infty} \frac{Bi\, J_0(\lambda_n R)\cosh\left[\lambda_n L(1 - X)\right]}{\left(\lambda_n^2 + Bi^2\right) J_0(\lambda_n)\cosh(\lambda_n L)} - 2\pi \times \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \frac{(2m + 1) Bi J_0(\lambda_n R)\cos\left[\left(\frac{2m - 1}{2}\right)(1 - X)\pi\right]}{L^2\left[\lambda_n^2 + (2m + 1)^2\pi^2/4L^2\right]\left(\lambda_n^2 + Bi^2\right) J_0(\lambda_n)} \times \exp\left\{-Fo\left[\lambda_n^2 + \frac{(2m + 1)^2\pi^2}{4L^2}\right]\right\}.$$ $\lambda_n J_1(\lambda_n) = Bi\, J_0(\lambda_n)$ $Bi = hr_0/k$, $L = \ell/r_0$, $R = r/r_0$, $X = x/\ell$, $Fo = \alpha\tau/r_0^2$



Case No.	References	Description	Solution
9.1.26	33	Case 9.1.25 with convection h_1, t_i at $r = r_0$. Convection boundary h_2, t_0: at $x = 0$.	$\dfrac{t - t_i}{t_0 - t_i} = 2 \sum\limits_{n=1}^{\infty} \dfrac{Bi_1\ Bi_2\ J_0(\lambda_n R)\ \cosh\left[\lambda_n L(1 - X)\right]}{\left(\lambda_n^2 + Bi_1^2\right) J_0(\lambda_n)\ \left[\lambda_n \sinh(\lambda_n L) + Bi_2 \cosh(\lambda_n L)\right]} - 4 \sum\limits_{m=1}^{\infty} \sum\limits_{n=1}^{\infty} \dfrac{\lambda_m\ Bi_1\ Bi_2 \sqrt{\lambda_m^2 + Bi_2^2}\ J_0(\lambda_n R)\ \cos\left[\lambda_m L(1 - X)\right]}{\left(\lambda_n^2 L^2 + \lambda_m^2\right)\left(\lambda_n^2 + Bi_1^2\right) J_0(\lambda_n)\ \left[\left(\lambda_m^2 + Bi_2^2\right) + Bi_2\right]} \times \exp\left[-Fo\left(\lambda_n^2 L^2 + \lambda_m^2\right)\right]$ $\lambda_n J_1(\lambda_n) = Bi_1\ J_0(\lambda_n)$, $\lambda_m \tan(\lambda_m) = Bi_2$ $Bi_1 = h_1 r_0/k$, $Bi_2 = h_2 \ell/k$, $L = \ell/r_0$, $R = r/r_0$, $X = x/\ell$

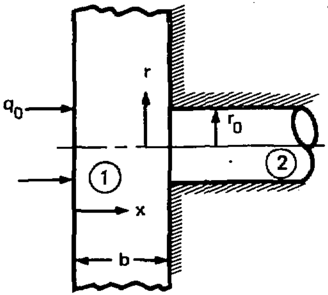
Case No.	References	Description	Solution
9.1.27	33	Finite cylinder with a linear time dependent temperature on one end. $t = t_i$, $0 < r < r_0$, $0 < x < \ell$, $\tau = 0$. $t = t_i + b(t_0 - t_i)\tau$, $x = 0$, $0 < \tau < b$. $t = t_0$, $x = 0$, $\tau > b$. $q_x = 0$, $0 < r < r_0$, $x = \ell$, $\tau > 0$. Convection boundary h, t_i: at $r = r_0$.	$\frac{t - t_i}{t_0 - t_i} = 2 \sum_{n=1}^{\infty} \frac{Bi\, J_0(\beta_n R) \cosh \left[\beta_n L(1 - X)\right]}{\left(\beta_n^2 + Bi^2\right) J_0(\beta_n) \cosh (\beta_n L)} - \frac{4}{Pd}$ $\times \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\beta_m Bi\, J_0'(\beta_n R) \cosh \left[\beta_m L(1 - X)\right]}{\left(\beta_n^2 L^2 + \beta_m\right) \left(\beta_n^2 + Bi^2\right) J_0(\beta_n) \sin (\beta_m)}$ $\times \left\{1 - \exp \left[\frac{1}{Pd} \left(\beta_n^2 + \beta_m^2/L^2\right)\right] \exp \left[-Fo \left(\beta_n^2 + \beta_m^2/L^2\right)\right]\right\}$ $\beta_m = (2m + 1)\pi$, $\beta_n J_1(\beta_n) = Bi\, J_0(\beta_n)$ $Fo = \alpha\tau/r_0^2$, $Bi = hr_0/k$, $L = \ell/a$, $R = r/r_0$, $X = x/\ell$, $Pd = r_0^2 b/\alpha$
9.1.28	33	Case 9.1.27 with convection boundary h_2, t_f at $x = 0$. $t_f = t_i + b(t_0 - t_i)\tau$, $0 < \tau < 1/b$. $t = t_0$, $\tau > 1/b$. Convection boundary h_1, t_i: at $r = r_0$.	Solution identical to that given in case 4.1.25 except multiply the second summation series by $Pd \left\{\exp \left[\frac{1}{Pd} \left(\lambda_n^2 + \lambda_m^2/L^2\right)\right] - 1\right\}$.



Case No.	References	Description	Solution
9.1.29	65	Infinite rod attached to an infinite plate. $t = t_0$, $0 \le x \le \infty$, $0 \le r \le \infty$, $\tau = 0$. $q_x = q_0$, $x = 0$, $\tau > 0$.	see below

$$\frac{(t - t_0)k_1}{q_0 b} = Fo + \frac{X^2}{2} - X + \frac{1}{3} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\exp(-n^2\pi^2\, Fo),$$

$$X = 1,\ R \to \infty .$$



$$\frac{(t - t_0)k_1}{q_0 b} = \left[\frac{(t - t_0)k_1}{q_0 b}\right]_{\substack{X=1\\R\to\infty}} - \left(\frac{k_2\rho_2 c_2}{k_1\rho_1 c_1}\right)^{1/2}$$

$$\times \sum_{n=0}^{\infty} Fo \left\{4\ i^2\mathrm{erfc}\left(\frac{n}{\sqrt{Fo}}\right) + 4\ i^2\mathrm{erfc}\left(\frac{n+1}{\sqrt{Fo}}\right) - 4\ i^2\mathrm{erfc}\left[\frac{\left(4n^2 + R_0^2\right)^{1/2}}{2\sqrt{Fo}}\right]\right.$$

$$\left. - 4\ i^2\mathrm{erfc}\left(\left[\frac{(2n+2)^2 + R_0^2}{4\ Fo}\right]^{1/2}\right)\right\} - \frac{1}{6}\left\{\mathrm{erfc}\left(\frac{n}{\sqrt{Fo}}\right) + \mathrm{erfc}\left(\frac{n+1}{\sqrt{Fo}}\right)\right.$$

$$\left. - \mathrm{erfc}\left[\frac{\left(4n^2 + R_0^2\right)^{1/2}}{2\sqrt{Fo}}\right] - \mathrm{erfc}\left(\left[\frac{(2n+2)^2 + R_0^2}{4\ Fo}\right]^{1/2}\right)\right\},\ X = 1,\ R = 0 .$$

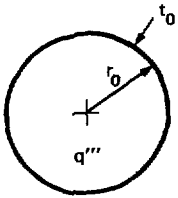
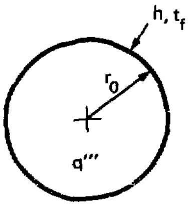
$$Fo = \alpha_1\tau/b^2,\ X = x/b,\ R_0 = r_0/b$$

Section 9.1. Solids Bounded by Cylindrical Surfaces--No Internal Heating.

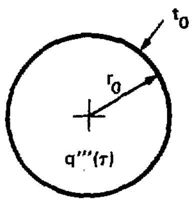
Case No.	References	Description	Solution

Case No.	References	Description	Solution

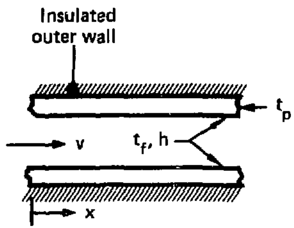
Section 9.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
9.2.1	9, p. 204	Infinite cylinder with steady internal heating and constant surface temperature. $t = t_0,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ $q''' = q_0'''.$ 	$\frac{(t - t_0)k}{r_0^2 q_0'''} = \frac{1}{4}(1 - R^2) - 2\sum_{n=1}^{\infty}\frac{J_0(R\lambda_n)\exp\left(-\lambda_n^2\ Fo\right)}{J_1(\lambda_n)\lambda_n^3}$ $J_0(\lambda_n) = 0$ See Fig. 9.5
9.2.2	9, p. 205 90	Infinite cylinder with steady internal heating and convection boundary. $t = t_f,\ 0 < r < r_0,\ \tau = 0.$ $q''' = q_0'''.$ 	$\frac{(t - t_f)k}{r_0^2 q_0'''} = \frac{1}{4}(1 - R^2 + 2/Bi) - 2\ Bi\sum_{n=1}^{\infty}\frac{\exp\left(-\lambda_n^2\ Fo\right)J_0(R\lambda_n)}{\lambda_n^2\left(Bi^2 + \lambda_n^2\right)J_0(\lambda_n)}$ $Bi\ J_0(\lambda_n) = \lambda_n J_1(\lambda_n),$ See Fig. 9.10.

Case No.	References	Description	Solution
9.2.2.1	74, p. 372	Case 9.2.2 with $q''' = q_0''' e^{-b\tau}$, $t = t_i$, $0 < r < r_0$, $\tau = 0$.	$\dfrac{t - t_i}{t_f - t_i} = 1 - \dfrac{Po}{Pd}\left[1 - \dfrac{J_0(\sqrt{Pd}\,R)}{J_0(\sqrt{Pd}) - \dfrac{\sqrt{Pd}}{Bi} J_1(\sqrt{Pd})}\right] \exp(-Pd\,Fo) - \sum_{n=1}^{\infty}\left(1 - \dfrac{Po}{Pd - \lambda_n^2}\right) A_n J_0(\lambda_n R) \exp\left(-\lambda_n^2\,Fo\right)$ $A_n = \dfrac{2\,Bi}{J_0(\lambda_n)\left(\lambda_n^2 + Bi^2\right)}, \quad \dfrac{J_0(\lambda_n)}{J_1(\lambda_n)} = \dfrac{\lambda_n}{Bi}$
9.2.3	9, p. 204	Infinite cylinder with exponential time-dependent internal heating. $t = t_0$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. $q''' = q_0''' e^{-b\tau}$.	$\dfrac{(t - t_0)k}{r_0^2 q_0'''} = \dfrac{\exp(-b\tau)}{Pd}\left\{\dfrac{J_0(R\sqrt{Pd})}{J_0(\sqrt{Pd})} - 1\right\} - \dfrac{2}{Pd}\sum_{n=1}^{\infty}\dfrac{\exp\left(-Fo\,\lambda_n^2\right) J_0(R\lambda_n)}{\lambda_n\left(\lambda_n^2/Pd - 1\right) J_1(\lambda_n)}$



Case No.	References	Description	Solution
9.2.4	3, p. 363	Infinite pipe with steady fluid flow and sudden heat generation in the wall. Fluid: $t = t_i$, $\tau = 0$. Pipe: $t = t_i$, $\tau = 0$. Fluid velocity = V. Wall heating: $q''' = q_0'''$, $\tau > 0$. No axial conduction.	Pipe temp:



Pipe temp:

$$\frac{(t_p - t_i)h\,P_i}{q_0'''A_p(\lambda - 1)/\lambda^2} = \lambda\tau^+ + \left(\frac{1}{\lambda - 1}\right)(1 - e^{\lambda\tau^+}) - e^{-\xi}\Big[(\lambda\tau^* - 1) \times \psi(\xi,\tau^*) - \lambda\phi(\xi,\tau^*) + \left(\frac{1}{\lambda - 1}\right)e^{-\lambda\tau^*}\Lambda(\xi,\tau^*,\lambda)\Big], \quad \tau^+ \geq (\lambda - 1)\xi.$$

Fluid temp:

$$\frac{(t_f - t_i)h\,P_i}{q_0'''A_p(\lambda - 1)/\lambda^2} = \lambda\tau^+ - (1 - e^{-\lambda\tau^+}) - e^{-\xi} \times \Big[(\lambda\tau^* + \lambda - 1)\psi(\xi,\tau^*) - \lambda\phi(u,\tau^*) - e^{-\lambda\tau^*}\Lambda(\xi,\tau^*,\lambda)\Big], \quad \tau^+ \geq (\lambda - 1)\xi.$$

$$\psi(\xi,\tau^*) = \sum_{n=0}^{\infty} \frac{\xi^n}{(n!)^2}\int_0^{\tau^*} \delta e^{-\delta}\,d\delta$$

$$\phi(\xi,\tau^*) = \sum_{n=0}^{\infty} \frac{\xi^n}{(n!)^2}\int_0^{\tau^*} \delta^{n+1}e^{-\delta}\,d\delta$$

$$\Lambda(\xi,\tau^*,\lambda) = \sum_{n=0}^{\infty} \frac{[\xi/(\lambda - 1)]^n}{(n!)^2}\int_0^{(\lambda-1)\tau^*} \delta^n e^{\delta}\,d\delta$$

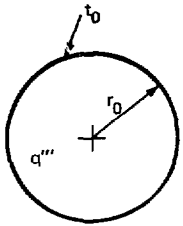
$\xi = b_f x/v$, $\lambda = 1 + b_f/b_p$, $\tau^* = b_p(\tau - x/v)$,

$b_f = h\,P_i/\rho_f c_f A_f$, $b_p = h\,p_i/\rho_p c_p A_p$, $\tau^+ = b_p\tau$

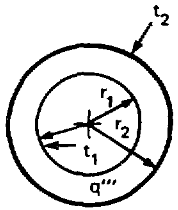
A_f = flow area, A_p = pipe section area,

P_i = pipe inside perimeter.

See Figs. 9.6a, 9.6b, and 9.6c.

Case No.	References	Description	Solution
9.2.5	9, p. 405	Infinite cylinder with linear temperature-dependent internal heating. $t = t_0,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ $q''' = q_0''' + \beta(t - t_0),$ $\tau > 0.$ 	$\frac{(t - t_0)k}{q_0''' r_0^2} = \frac{1}{\beta}\frac{J_0(R\sqrt{Po})}{J_0(\sqrt{Po})} - \frac{1}{Po} + 2\sum_{n=1}^{\infty}\frac{\exp\left[\left(-\lambda_n^2 + Po\right)Fo\right]J_0(R\lambda_n)}{\lambda_n\left(Po - \lambda_n^2\right)J_1(\lambda_n)}$ $J_0(\lambda_n) = 0,\ Po = \beta r_0^2/k,\ R = r/r_0$ If $\beta > 0$, steady state solution exists for $Po < \lambda_1^2$.
9.2.6	19, p. 3-29	Infinite cylinder of infinite conductivity, variable specific heat, convection boundary, and uniform heating.	See case 8.2.11, set L = 2, radius = ℓ.
9.2.7	19, p. 3-29	Case 9.2.6 with a short cylinder of length 2ℓ.	See case 8.2.11, set L = 3, radius = ℓ.

Case No.	References	Description	Solution
9.2.8	63	Hollow infinite cylinder with exponentially space varying heat generation. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $t = t_1(\tau),\ r = r_1,\ \tau > 0.$ $t = t_2(\tau),\ r = r_2,\ \tau > 0.$ $q''' = m\tau \exp\left[-\beta(r - r_1)\right].$	See below



$$\frac{(t - t_i)k\beta^2\alpha}{mr_1^2} = \pi \sum_{n=1}^{\infty} \frac{\lambda_n^2 J_0^2(\lambda_n R_2)}{J_0^2(\lambda_n) - J_0^2(\lambda_n R_2)} B_0(\lambda_n R) \exp\left(-\lambda_n^2 Fo\right)$$

$$\times \left\{ \int_0^{Fo} \left[T_2(\tau) \frac{J_0(\lambda_n)}{J_0(\lambda_n R_2)} - T_1(\tau)\right] \exp\left(\lambda_n^2 Fo\right) d(Fo)\right.$$

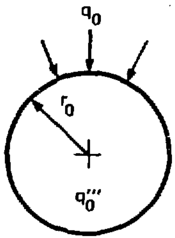
$$\left. + \frac{\pi}{2} \bar{f}(\lambda_n)(\beta r_1)^2 \int_0^{Fo} m\tau \exp\left(\lambda_n^2 Fo\right) d(Fo)\right\}$$

$$R = r/r_1, \quad Fo = \alpha\tau/r_1^2, \quad T(\tau) = \frac{(t - t_i)k\beta^2\alpha}{mr_1^2}$$

$$\bar{f}(\lambda_n) = \int_1^{R_2} \exp\left[-\beta r_1(R - 1)\right] RB_0(\lambda_n R)\, dR$$

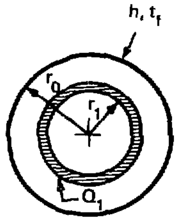
$$B_0(\lambda_n R) = J_0(\lambda_n R)Y_0(\lambda_n) - Y_0(\lambda_n R)J_0(\lambda_n)$$

$$J_0(\lambda_n R_2)Y_0(\lambda_n) = J_0(\lambda_n)Y_0(\lambda_n R_2)$$

Case No.	References	Description	Solution
9.2.9	74, p. 375	Infinite cylinder with a steady surface heat flux and steady internal heating. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $q''' = q_0'''\ 0 < r < r_0,\ \tau > 0.$ $q_r = q_0,\ r = r_0,\ \tau > 0.$ 	$\dfrac{(t - t_i)k}{q_0'''r_0^2} = Fo + \phi$ $\phi = \dfrac{Ki}{Fo}\left[2\ Fo - \dfrac{1}{4}(1 - 2R^2) - 2\sum_{n=1}^{\infty}\dfrac{J_0(\lambda_n R)\ \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n^2 J_0(\lambda_n)}\right]$ $J_0'(\lambda_n) = 0$
9.2.10	74, p. 375	Case 9.2.9 with $q''' = q_0'''(1 - R^2)$.	$\dfrac{(t - t_i)k}{q_0'''r_0^2} = \dfrac{1}{3}\ Fo + 4\sum_{n=1}^{\infty}\dfrac{\left[2J_1'(\lambda_n) - J_0(\lambda_n)\right]}{\lambda_n^3 J_0^2(\lambda_n)}$ $\times\ J_0(\lambda_n R)\ \left[1 - \exp\left(-\lambda_n^2\ Fo\right)\right] + \phi$ ϕ given in case 9.2.9

Section 9.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution
9.2.11	74, p. 375	Case 9.2.9 with $q''' = q_0'''(1 + b\tau)$.	$\dfrac{(t - t_i)k}{q_0'''r_0^2} = \mathrm{Fo}\left(1 + \dfrac{1}{2}\,\mathrm{Pd\ Fo}\right) + \phi$ ϕ given in case 9.2.9
9.2.12	74, p. 376	Case 9.2.9 with $q''' = q_0'''\exp(-b\tau)$.	$\dfrac{(t - t_i)k}{q_0'''r_0^2} = \dfrac{1}{\mathrm{Pd}}\left[1 - \exp(-\mathrm{Pd\ Fo})\right] + \phi$ ϕ given in case 9.2.9
9.2.13	74, p. 376	Case 9.2.9 with $q''' = q_0'''\cos(b\tau)$.	$\dfrac{(t - t_i)k}{q'''r_0^2} = \dfrac{1}{\mathrm{Pd}}\sin(\mathrm{Pd\ Fo}) + \phi$ ϕ given in case 9.2.9
9.2.14	74, p. 376	Case 9.2.9 with $q''' = q_0'''b\tau^n$.	$\dfrac{(t - t_i)k}{q'''r_0^2} = \dfrac{(\mathrm{Pd\ Fo})^n}{n + 1}\mathrm{Fo} + \phi$ ϕ given in case 9.2.9

Case No.	References	Description	Solution
9.2.15	74, p. 393	Infinite cylinder with pulse heating on a cylindrical surface and convection boundary h, t_f, $0 < r < r_0$, $\tau = 0$. Instantaneous pulse heating of strength Q_1 occurs at $r = r_1$, $\tau = 0$. 	$\frac{(t - t_f)\rho c r_0^2}{Q_1} = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\lambda_n^2 J_0(\lambda_n R_1) J_0(\lambda_n R) \exp\left(-\lambda_n^2 \, Fo\right)}{\left(Bi^2 + \lambda_n^2\right) J_0^2(\lambda_n)}$ $J_0(\lambda_n) = 0, \; R = r/r_0$ For $Bi \to \infty$: $\frac{(t - t_f)\rho c r_0^2}{Q_1} = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n R) J_0(\lambda_n R_1) \exp\left(-\lambda_n^2 \, Fo\right)}{J_1^2(\lambda_n)}$ Mean temp: $\frac{(t_m - t_f)\rho c r_0^2}{Q_1} = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\lambda_n J_0(\lambda_n R_1) B_n \exp\left(-\lambda_n^2 \, Fo\right)}{J_1^2(\lambda_n)}$ $B_n = \frac{4\, Bi^2}{\lambda_n^2 \left(\lambda_n^2 + Bi^2\right)}$

Section 9.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

Section 9.2. Solids Bounded by Cylindrical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

10. Spherical Surface — Transient

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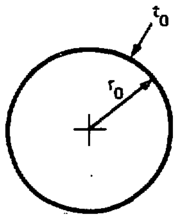
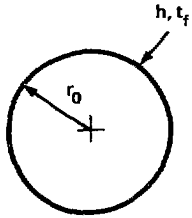
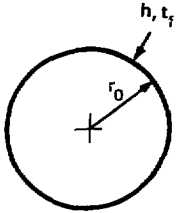
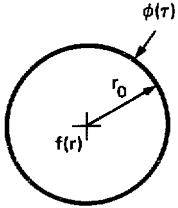
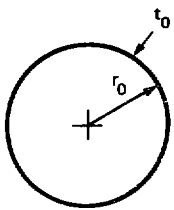
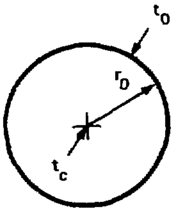
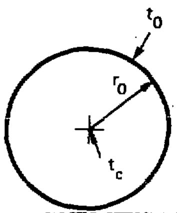
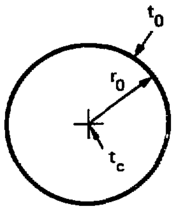
Section 10.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

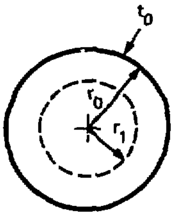
Case No.	References	Description	Solution
10.1.1	2, p. 239 74, p. 126	Sphere with steady surface temperature. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ 	$\dfrac{t - t_0}{t_i - t_0} = \dfrac{2}{\pi} \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n} \exp\,(-n^2\pi^2\ \mathrm{Fo})\ \sin\,(n\pi R)$ $= 1 - \dfrac{1}{R} \sum_{n=1}^{\infty} \left[\mathrm{erfc}\left(\dfrac{2n - 1 - R}{2\sqrt{\mathrm{Fo}}}\right) - \mathrm{erfc}\left(\dfrac{2n - 1 + R}{2\sqrt{\mathrm{Fo}}}\right)\right],\ \mathrm{Fo} < 1.$ Mean temp: $\dfrac{t_m - t_0}{t_i - t_0} = \dfrac{6}{\pi^2} \sum_{n=1}^{\infty} \dfrac{1}{n^2} \exp\,(-n^2\pi^2\ \mathrm{Fo}).$ Cumulative heat rate: $\dfrac{Q}{Q_0} = \dfrac{6}{\pi^3} \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^3} \left[1 - \exp\,(-n^2\pi^2\ \mathrm{Fo})\right],\ Q_0 = \dfrac{4}{3}\pi r_0^3 \rho c (t_0 - t_i).$ See Fig. 10.1

Case No.	References	Description	Solution
10.1.2.1	74, p. 254	Sphere with convection boundary-general case $t = f(r),\ 0 < r < r_0,$ $\tau = 0.$ 	$$t - t_f = 2\sum_{n=1}^{\infty} \frac{\lambda_n \sin(\lambda_n R)\exp\left(-\lambda_n^2\, Fo\right)}{\left[\lambda_n - \sin(\lambda_n)\cos(\lambda_n)\right]R}\int_0^1 R\left[f(r) - t_f\right] \times \sin(\lambda_n R)\, dR.$$ $$\tan(\lambda_n) = \frac{\lambda_n}{(1 - Bi)}$$ See Table 14.2 of roots
10.1.2.2	3, p. 298	Sphere with convection boundary. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ 	$$\frac{t - t_f}{t_i - t_f} = 2\sum_{n=1}^{\infty} \frac{Bi \sin(\lambda_n)\exp\left(-\lambda_n^2\, Fo\right)\sin(\lambda_n R)}{\lambda_n - \sin(\lambda_n)\cos(\lambda_n)\lambda_n R}$$ $$\lambda_n \cos(\lambda_n) = (1 - Bi)\sin(\lambda_n)$$ Cumulative heat rate: $$\frac{Q}{Q_0} = 6\sum_{n=1}^{\infty} \frac{\left[\sin(\lambda_n) - \lambda_n\cos(\lambda_n)\right]\left[1 - \exp\left(-\lambda_n^2\, Fo\right)\right]}{\lambda_n^3\left[\lambda_n - \sin(\lambda_n)\cos(\lambda_n)\right]}$$ $$Q_0 = \frac{4}{3}\pi r_0^3 \rho c (t_i - t_f)$$ See Figs. 10.2a, 10.2b, and 10.2c

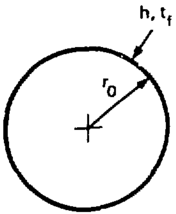
Case No.	References	Description	Solution
10.1.3	9, p. 233	Sphere with initial temp $f(r)$ and surface temp $\phi(\tau)$, general case. 	$T = \frac{2}{R} \sum_{n=1}^{\infty} \exp(-n^2\pi^2 \text{ Fo}) \sin(n\pi R) \times \left\{ \int_0^1 \bar{R} f(\bar{R}) \sin(n\pi\bar{R}) d\bar{R} - n\pi(-1)^n \text{ Fo} \int_0^1 \exp(n^2\pi^2 \text{ Fo } \lambda)\phi(\lambda) d\lambda \right\}$
10.1.4	9, p. 235	Sphere with surface temp proportional to time. $t = t_i$, $0 < r < r_0$, $\tau = 0$. $t_0 = b\tau + t_i$, $r = r_0$, $\tau > 0$. 	$\frac{t - t_i}{t_0 - t_i} = \left(1 + \frac{R^2 - 1}{6 \text{ Fo}}\right) - \frac{2}{\pi^3 \text{ Fo } R} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \exp(-n^2\pi^2 \text{ Fo}) \sin(n\pi R)$ See Table 10.1 and Fig. 10.8

Case No.	References	Description	Solution
10.1.5	9, p. 235	Sphere with variable initial temp. $t = (t_c - t_0)(r_0 - r)/(r_0 + t_0)$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. 	$\frac{t - t_0}{t_c - t_0} = \frac{8}{\pi^3 R} \sum_{n=0}^{\infty} \frac{1}{(2n + 1)^3} \exp\left[-\pi^2 (2n + 1)^2 \, Fo\right] \sin\left[(2n + 1)\, \pi R\right]$
10.1.6	9, p. 236	Sphere with variable initial temp (parabolic). $t = (t_c - t_0)\left(r_0^2 - r^2\right)/\left(r_0^2 + t_0\right)$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. 	$\frac{t - t_0}{t_c - t_0} = \frac{12}{\pi^3 R} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} \sin(n\pi R) \exp(-n^2\pi^2 \, Fo)$

Case No.	References	Description	Solution
10.1.7	9, p. 237	Sphere with variable initial temp (trigonometric). $t = t_0 + (t_c/r)\sin(\pi r/r_0)$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. 	$\dfrac{t - t_0}{t_c - t_0} = \dfrac{1}{R}\sin(\pi R)\exp(-\pi^2\,Fo)$
10.1.8	9, p. 237	Sphere with variable initial temp (exponential). $t = t_0 + (t_c - t_0)\exp\left[b(r - r_0)\right]$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$.	$\dfrac{t - t_0}{t_c - t_0} = \dfrac{2\pi}{R}\sum_{n=1}^{\infty}\dfrac{n}{(n^2\pi^2 + B^2)^2}\exp(-n^2\pi^2\,Fo)$ $\times \sin(n\pi R)\left[(-1)^{n+1}(B^2 - 2B + n^2\pi^2) - 2B\exp(-B)\right]$ $B = br_0$

Case No.	References	Description	Solution
10.1.9	9, p. 236	Sphere with two initial temperatures. $t = t_1,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r_1 < r < r_0,\ \tau = 0.$ $t = t_0\ r = r_0,\ \tau > 0.$  	$\frac{t - t_0}{t_1 - t_0} = \frac{2}{n\pi R} \sum_{n=1}^{\infty} \left[\frac{\sin(n\pi R_1)}{n\pi} - R_1 \sin(n\pi R_1)\right] \times \sin(n\pi R) \exp(-n^2\pi^2\ Fo)$
10.1.10	9, p. 237	Sphere with polynominal initial temperature. $t = t_1 - t_0 + b_1 r + b_2 r^2 + b_3 r^3 + \ldots,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$	$\frac{t - t_0}{t_1 - t_0} = 2 \sum_{n=1}^{\infty} \sin(n\pi R) \left\{\frac{1}{n\pi R}(-1)^{n+1} + \frac{b_1^*}{n^3\pi^3}\left[(n^2\pi^2 - 2) \times (-1)^{n+1} - 2\right] + \frac{b_2^*}{n^4\pi^4}(n^3\pi^3 - 6n\pi)(-1)^{n+1} + \frac{b_3^*}{n^5\pi^5} \times \left[24 - (n^4\pi^4 - 12n^2\pi^2 + 24)(-1)^n + \ldots\right]\right\} \exp(-n^2\pi^2\ Fo)$ $b_1^* = b_1 r_0/R,\ b_2^* = b_2 r_0^2/R,\ b_3^* = b_3 r_0^3/R,\ \ldots$

Case No.	References	Description	Solution
10.1.11	9, p. 238 89	Sphere with convection boundary and changing fluid temperature. $t = t_i$, $0 < r < r_0$, $\tau = 0$. $t_f = b\tau + t_i$, $\tau > 0$.	see below



$$\frac{(t - t_i)\alpha}{br_0^2} = Fo + \frac{R^2\ Bi - 2 + Bi}{6\ Bi} + \frac{2\ Bi}{R}$$

$$\times \sum_{n=1}^{\infty} \frac{\sin(R\lambda_n)\exp\left(-Fo\ \lambda_n^2\right)}{\lambda_n^2\left[\lambda_n^2 + Bi(Bi - 1)\right]\ \sin(\lambda_n)}$$

$$\lambda_n \cot(\lambda_n) + Bi = 1$$

Mean temp:

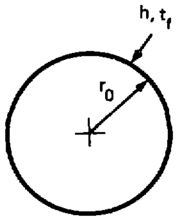
$$\frac{(t_m - t_i)}{b\tau} = Fo - \frac{1}{15}\left(1 - \frac{5}{Bi}\right) + 6\ Bi^2 \sum_{n=1}^{\infty} \frac{\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^4\left(\lambda_n^2 + Bi^2 - Bi\right)}$$

See Fig. 10.9

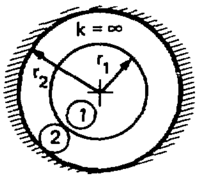
Section 10.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

Case No.	References	Description	Solution
10.1.11.1	74, p. 317	Case 10.1.11 with $t_f = t_{fm} - (t_{fm} - t_i)e^{-b\tau}$.	$\frac{t - t_i}{t_{fm} - t_i} = 1 - \frac{R\ Bi\ \sin(\sqrt{Pd}\ R)\ \exp(-Pd\ Fo)}{(Bi - 1)\sin(\sqrt{Pd}) + \sqrt{Pd}\cos(\sqrt{Pd})}$ $- 2\ Bi \sum_{n=1}^{\infty} \frac{A_n R \sin(\lambda_n R)\exp\left(-\lambda_n^2 Fo\right)}{\left(1 - \lambda_n^2/Pd\right)\lambda_n}$ $A_n = \frac{(-1)^{n+1}\left[\lambda_n^2 + (Bi - 1)^2\right]^{1/2}}{\left(\lambda_n^2 + Bi^2 - Bi\right)}$ Mean temp: $\frac{t_m - t_i}{t_{fm} - t_i} = 1 - \frac{3\ Bi\left[\tan(\sqrt{Pd}) - \sqrt{Pd}\right]\exp(-Pd\ Fo)}{Pd\left[(Bi - 1)\tan(\sqrt{Pd}) + \sqrt{Pd}\right]}$ $- 6\ Bi^2 \sum_{n=1}^{\infty} \frac{\exp\left(-\lambda_n^2 Fo\right)}{\lambda_n^2\left(\lambda_n^2 + Bi^2 - Bi\right)\left(1 - \lambda_n^2/Pd\right)}$

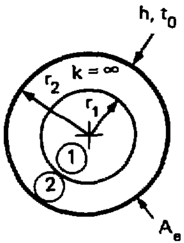
Case No.	References	Description	Solution
10.1.12	9, p. 238	Sphere with convection boundary and changing fluid temp. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t_f = (t_m - t_i)\sin(\omega\tau + \varepsilon),\ \tau > 0.$ t_m = max fluid temp.	$\frac{t - t_i}{t_m - t_i} = \frac{2\,Bi}{Pd\,R} \times \left\{ \sum_{n=1}^{\infty} \frac{\frac{\lambda_n}{Pd}\left[\frac{\lambda_n^2}{Pd}\sin(\varepsilon) - \cos(\varepsilon)\right](Bi - 1)\sin(R\lambda_n)}{\left[\left(\lambda_n^4/Pd\right) + 1\right]\left(\lambda_n^2 + Bi^2 - Bi\right)\cos(\lambda_n)} \times \exp\left(-Fo\,\lambda_n^2\right) + \frac{A_1}{2A_2}\sin(\omega\tau + \varepsilon + \phi_1 + \phi_2)\right\}$ $Pd = \omega r_0^2/\alpha$ $\lambda_n \cot(\lambda_n) + Bi = 1$ $A_1 e^{i\phi_1} = \sinh(\omega_1)\cos(\omega_1) + i\cosh(\omega_1)\sin(\omega_1)$ $A_2 e^{i\phi_2} = \omega_2\cosh(\omega_2) + (Bi - 1)\sinh(\omega_2)$ $\omega_1 = R\sqrt{Pd/2},\ \omega_2 = (1 + i)\sqrt{Pd/2}$

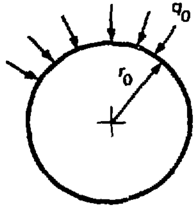
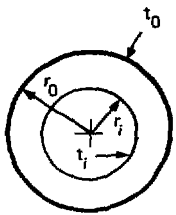


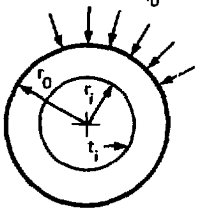
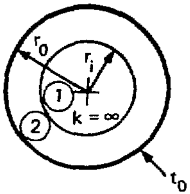
Case No.	References	Description	Solution
10.1.13	9, p. 240	Sphere enclosed in a finite medium of infinite conductivity. $t = t_i,\ 0 < r < r_1,\ \tau = 0.$ $t = t_0,\ r_1 < r < r_2,\ \tau = 0.$ $q_r = 0,\ r = r_2,\ \tau > 0.$	Temp of the sphere: $\dfrac{t - t_0}{t_i - t_0} = \dfrac{1}{M + 1} - \dfrac{2M}{3R} \sum_{n=1}^{\infty} \dfrac{M^2\lambda_n^4 + 3(2M + 3)\lambda_n^2 + 9}{M^2\lambda_n^4 + 9(M + 1)\lambda_n^2} \sin(R\lambda_n) \times \sin(\lambda_n) \exp\left(-\lambda_n^2 Fo_1\right),\ 0 < R < 1.$ Temp of the surrounding medium: $\dfrac{t - t_0}{t_i - t_0} = \dfrac{1}{M + 1} - 6M \sum_{n=1}^{\infty} \dfrac{1}{M^2\lambda_n^2 + 9(M + 1)} \exp\left(-\lambda_n^2 Fo_1\right),\ 1 < R < R_2.$ $M = \dfrac{\rho_2 c_2}{\rho_1 c_1}\left(R_2^3 - 1\right),\ R = r/r_1,\ \tan(\lambda_n) = \dfrac{3\lambda_n}{3 + M\lambda_n^2}$ See Fig. 10.3



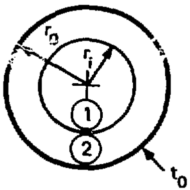
Case No.	References	Description	Solution
10.1.14	9, p. 241	Case 10.1.13, except the surrounding medium transfers heat by convection to a fluid at temp t_0. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r > r_0,\ \tau = 0.$ Exterior surface area = A_e.	Temp of surrounding medium: $\dfrac{t - t_0}{t_i - t_0} = 6 \sum_{n=1}^{\infty} \dfrac{\left(3\ \mathrm{Bi}\ R_2 M \lambda_n^2\right) \exp\left(-\lambda_n^2\ \mathrm{Fo}_1\right)}{M^2\lambda_n^4 + 3\lambda_n^2(3 + 3M - 2M\ \mathrm{Bi}\ R_2) - 9\ \mathrm{Bi}\ R_2(1 - \mathrm{Bi}\ R_2)}$ $\mathrm{Bi} = hr_2/k_i,\ M = \dfrac{\rho_2 c_2}{\rho_1 c_1}\left(R_2^3 - 1\right),\ R = r/r_1$ $\tan(\lambda_n) = \dfrac{3\lambda_n}{3(1 - \mathrm{Bi}\ R_2) + M\lambda_n^2}$



Case No.	References	Description	Solution
10.1.15	9, p. 242	Sphere with steady surface heat flux. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $q_r = q_0,\ r = r_0,\ \tau > 0.$  	$\frac{(t - t_i)k}{q_0 r_0} = 3\ Fo + \frac{5R^2 - 3}{10} - \frac{2}{R} \sum_{n=1}^{\infty} \frac{\sin(R_2\lambda_n)\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^2 \sin(\lambda_n)}$ $\tan(\lambda_n) = \lambda_n$ See Fig. 10.4
10.1.16	9, p. 246	Hollow sphere with two surface temperatures. $t = f(r),\ r_i < r < r_0,\ \tau = 0.$ $t = t_i,\ r = r_i,\ \tau > 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$  	$\frac{T}{T_i} = \frac{R_i}{R} + \frac{(T_0/T_i - R_i)R^*}{R} + \frac{2}{R\pi} \sum_{n=1}^{\infty} \frac{T_0/T_i \cos(n\pi) - R_i}{n}$ $\times \sin(n\pi R^*) \exp(-n^2\pi^2\ Fo) + \frac{2(1 - R_i)}{R} \sum_{n=1}^{\infty} \sin(n\pi R^*)$ $\times \exp(-n^2\pi^2\ Fo) \int_0^1 (R^* + R) f(R^*) \sin(n\pi R^*)\ dR^*$ $R^* = (r - r_i)/(r_0 - r_i),\ R = r/r_0,\ Fo = \alpha\tau/(r_0 - r_i)^2$

Case No.	References	Description	Solution
10.1.17	9, p. 247	Hollow sphere with steady surface flux. $t = t_i,\ r_i < r < r_0,\ \tau = 0.$ $t = t_i,\ r = r_i,\ \tau > 0.$ $-q_r = q_0,\ r = r_0,\ \tau > 0.$  	$\frac{(t - t_i)k}{q_0 r_i} = \frac{(R_0 - R)}{R_0 R} + \frac{2}{R}\sum_{n=1}^{\infty} \frac{\left(1 + \lambda_n^2\right)^{1/2} \sin(\lambda_n R_0 - \lambda_n R)}{\lambda_n\left[R_0 + (R_0 - 1)\lambda_n^2\right]} \times \exp\left(-\lambda_n^2\, Fo\right)$ $\tan(\lambda_n R_0 - \lambda_n) + \lambda_n = 0$ $R = r/r_i,\ Fo = \alpha\tau/r_i^2$
10.1.18	9, p. 350	*Sphere of infinite* conductivity enclosed by shell of *finite* conductivity. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$  	$\frac{t - t_0}{t_i - t_0} = \frac{4K}{R}$ $\times \sum_{n=1}^{\infty} \frac{\sin\left[\lambda_n(1 - R)\right] \exp\left(-\lambda_n^2\, Fo\right)}{2K(1 - R_i)\lambda_n + 4R_i\lambda_n \sin^2\left[\lambda_n(1 - R_i)\right] - K \sin\left[2\lambda_n(1 - R_i)\right]},$ $R_i < R < 1.$ $R_i K \lambda_n \cos\left[\lambda_n(1 - R_i)\right] = \left(\lambda_n^2 R_i^2 - K\right) \sin\left[\lambda_n(1 - R_i)\right].$ $K = 3\rho_2 c_2/\rho_1 c_1,\ R = r/r_0,\ Fo = \alpha\tau/r_0^2$

Case No.	References	Description	Solution
10.1.19	9, p. 351	Composite sphere. $t = t_i$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$.	



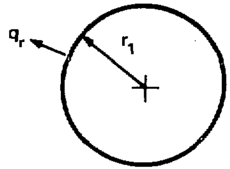
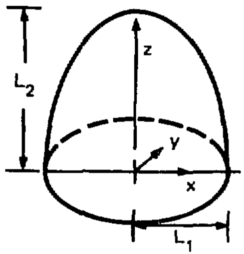
$$\frac{t - t_0}{t_i - t_0} = \frac{2R_0}{R} \sum_{n=1}^{\infty} \frac{1}{\phi(\lambda_n)} \sin(R\lambda_n) \sin(\lambda_n) \sin\left[A(R_0 - 1)\lambda_n\right] \times \exp\left(-\lambda_n^2 Fo_1\right), \quad 0 < R < 1 .$$

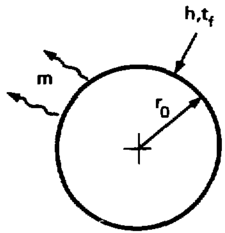
$$\frac{t - t_0}{t_i - t_0} = \frac{2R_0}{R} \sum_{n=1}^{\infty} \frac{1}{\phi(\lambda_n)} \sin^2(\lambda_n) \sin\left[A(R_0 - 1)\lambda_n\right] \times \exp\left(-\lambda_n^2 Fo_2\right), \quad 1 < R < R_0 .$$

$$\phi(\lambda_n) = K\lambda_n \sin^2\left[A\lambda_n(R_0 - 1)\right] + A\lambda_n(R_0 - 1)\sin^2(\lambda_n) + \left[(1 - KA)/A\lambda_n\right]\sin^2(\lambda_n)\sin^2\left[A\lambda_n(R_0 - 1)\right].$$

$$\lambda_n \cot\left[A\lambda_n(R_0 - 1)\right] + 1/A + K\left[\lambda_n \cot(\lambda_n) - 1\right] = 0 .$$

$Fo = \alpha\tau/r_i^2$, $A = \sqrt{\alpha_1/\alpha_2}$, $K = k_1/k_2$, $R = r/r_i$, $A(R_0 - 1)$ irrational

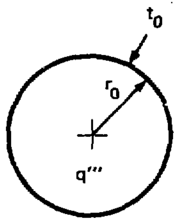
Case No.	References	Description	Solution
10.1.20	45 19, p. 3-57	Sphere with radiation cooling $t = t_0,\ 0 < r < r_1,\ \tau = 0.$ $q_r = \sigma \mathscr{F}\left(T^4_{r=r_1} - T^4_s\right),\ r = r_1.$ T_s = sink temp.  	See Fig. 10.6
10.1.21	72	Prolate spheriod with steady surface temp $t = t_i$, throughout solid, $\tau = 0.$ $t = t_w$, on surface of solid, $\tau > 0.$  	See Figs. 10.7a & b for $\dfrac{t - t_i}{t_w - t_i}$ vs Fo

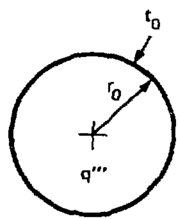
Case No.	References	Description	Solution
10.1.22	74, p. 320	A sphere with convection and evaporation boundary. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $m = m_0 e^{-b\tau}$ = evaporation rate. 	$\frac{t - t_i}{t_f - t_i} = 1 - \frac{R\ M\ Bi\ \sin(\sqrt{Pd}\ R)\ \exp(-Pd\ Fo)}{(Bi - 1)\sin(\sqrt{Pd}) + \sqrt{Pd}\cos(\sqrt{Pd})} - 2\sum_{n=1}^{\infty}\left[1 - \frac{M}{1 - \left(Pd/\lambda_n^2\right)}\right]A_n R \sin(\lambda_n R)\exp\left(-\lambda_n^2\ Fo\right)$ $A_n = \frac{(-1)^{n+1} Bi\left[\lambda_n^2 + (Bi - 1)^2\right]^{1/2}}{\lambda_n^2 + Bi^2 - Bi},\ M = \gamma m_0/h(t_f - t_i)$ $\lambda_n \cot(\lambda_n) = 1 - Bi$

Case No.	References	Description	Solution

Section 10.1. Solids Bounded by Spherical Surfaces--No Internal Heating.

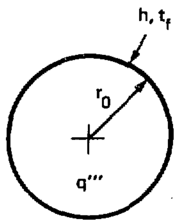
Case No.	References	Description	Solution

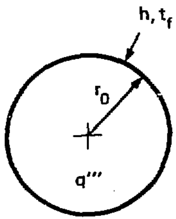
Case No.	References	Description	Solution
10.2.1	1, p. 307	Sphere with steady surface temp and internal heating. $t = t_i$, $0 < r < r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. $q''' = q_0'''$, $0 > r > r_0$, $\tau > 0$. 	$\frac{(t - t_0)k}{q_0''' r_0^2} = \frac{1}{6}(1 - R^2) + \frac{2}{\pi^3 R}\sum_{n=1}^{\infty}\left(\frac{1}{n^3} - \frac{\pi^2}{n\ Po}\right)(-1)^n \times \sin(n\pi R)\exp(-n^2\pi^2 Fo)$. See Fig. 10.5 for condition $t_i = t_0$.

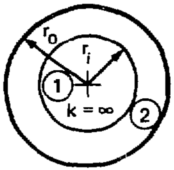
Case No.	References	Description	Solution
10.2.2	9, p. 243	Sphere with linear internal *heating distribution.* $t = t_0,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$ $q''' = q_0'''(r_0 - r)/r_0,$ $0 < r < r_0,\ \tau > 0.$ 	$\frac{(t - t_0)k}{q_0''' r_0^2} = \frac{1}{12}(1 - 2R^2 + R^3) - \frac{8}{\pi^5 R}\sum_{n=0}^{\infty}\frac{1}{(2n + 1)^5} \times \exp\left[-(2n + 1)^2\pi^2\, Fo\right]\sin\left[(2n + 1)\,\pi R\right]$
10.2.2.1	74, p. 366	*Case 10.2.2 with* $q''' = q_0'''\tau^{b/2},$ $b = 1, 0, 1, 2, \ldots$ $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $t = t_0,\ r = r_0,\ \tau > 0.$	$\frac{t - t_i}{t_0 - t_i} = \sum_{n=1}^{\infty}\frac{1}{R}\left[\mathrm{erfc}\left(\frac{2n - 1 - R}{2\sqrt{Fo}}\right) - \mathrm{erfc}\left(\frac{2n - 1 + R}{2\sqrt{Fo}}\right)\right] + \frac{Po\ Fo}{1 + (b/2)}\left\{1 - \Gamma\left(2 + \frac{b}{2}\right)2^{b+2} \times \sum_{n=1}^{\infty}\frac{1}{R}\left[i^{b+2}\mathrm{erfc}\left(\frac{2n - 1 - R}{2\sqrt{Fo}}\right) - i^{b+2}\mathrm{erfc}\left(\frac{2n - 1 + R}{2\sqrt{Fo}}\right)\right]\right\}$ $Po = q_0'''\tau^{b/2}r_0^2/b(t_0 - t_i)$

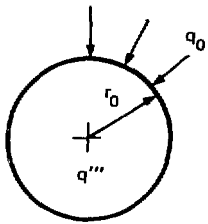
Case No.	References	Description	Solution
10.2.3	9, p. 243	Case 10.2.2, with $q''' = q_0''' \left(r_0^2 - r^2\right)/r_0^2$, $0 < r < r_0$, $\tau > 0$.	$\dfrac{(t - t_0)k}{q_0''' r_0^2} = \dfrac{1}{60}(1 - R^2)(7 - 3R^2) - \dfrac{12}{\pi^5 R} \sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^5} \times \exp(-n^2\pi^2\,\mathrm{Fo}) \sin(n\pi R)$
10.2.4	9, p. 244	Case 10.2.2, with $q''' = \dfrac{q_0''' r_0}{r} \sin(\pi r/r_0)$, $0 < r < r_0$, $\tau > 0$.	$\dfrac{(t - t_0)k}{q_0''' r_0^2} = \dfrac{1}{\pi^2 R}\left[1 - \exp(-\pi^2\,\mathrm{Fo})\right] \sin(\pi R)$
10.2.5	9, p. 244	Case 10.2.2 with $q''' = q_0''' \exp\left[b(r - r_0)\right]$.	$\dfrac{(t - t_0)k}{q''' r_0^2} = \dfrac{1}{B^2}\left\{1 - \dfrac{2}{B} - \left(1 - \dfrac{2}{BR}\right)\exp(BR - B) - \dfrac{2}{B}\left(\dfrac{1}{R} - 1\right) e^{-B}\right\} - \dfrac{2}{\pi R} \sum_{n=1}^{\infty} \dfrac{\sin(n\pi R)}{n(n^2\pi^2 + B^2)} \exp(-n^2\pi^2\,\mathrm{Fo}) \times \left[(-1)^n(2B - n^2\pi^2 - B^2) - 2Be^{-B}\right].$ $B = r_0 b$.

Case No.	References	Description	Solution
10.2.6	9, p. 245	Case 10.2.2 with $q''' = q_0'''e^{-b\tau}$.	$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{1}{Pd} e^{-b\tau} \left[\frac{\sin (R\sqrt{Pd})}{R \sin(\sqrt{Pd})} - 1\right] + \frac{2}{\pi^3 R} \times \sum_{n=1}^{\infty} \frac{(-1)^n \sin (n\pi R)}{n(n^2\pi^2 - Pd)} \exp(-n^2\pi^2 Fo).$ $Pd = r_0^2 b/\alpha$.
10.2.7	9, p. 245	Case 10.2.2 with $q''' = q_0'''e^{-b\tau}$, $r_1 < r < r_0$. $q''' = 0,\ 0 < r < r_1$.	$\frac{(t - t_0)k}{q_0'''r_0^2} = \frac{e^{-b\tau}}{R\ Pd} \left\{\left[\frac{\sin (R\sqrt{Pd})}{\sin (\sqrt{Pd})} - R\right] + \frac{\sin (\sqrt{Pd} - R\sqrt{Pd})}{\sin (\sqrt{Pd})}\right\} \times \left[R_1 \cos (R_1\sqrt{Pd}) - \frac{1}{\sqrt{Pd}} \sin (R_1\sqrt{Pd})\right] + \frac{2}{\pi R} \sum_{n=1}^{\infty} \frac{1}{n(n^2\pi^2 - Pd)} \left[- (-1)^n + \frac{1}{n\pi} \sin (n\pi R_1) - R_1 \cos (n\pi R_1)\right] \sin (n\pi R) \exp(-n^2\pi^2 Fo)$. $Pd = r_0^2 b/\alpha$.

Case No.	References	Description	Solution
10.2.8	9, p. 245	Case 10.2.2 with $q''' = f(r)$, $0 < r < r_0$, $\tau > 0$.	$\frac{(t - t_0)k}{r_0^2} = \frac{2}{\pi^2 R} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-n^2\pi^2 \text{ Fo}) \sin(n\pi R) \int_0^1 R'f(R') \sin(n\pi R')dR' + \frac{1}{R}\int_0^R R'^2 f(R')\, dR' + \int_R^1 R'f(R')\, dR' - \int_0^1 R'^2 f(R')\, dR'$
10.2.9	9, p. 245 90	Sphere with steady internal heating and convection boundary. $t = t_f$, $0 < r < r_0$, $\tau = 0$. $q''' = q_0'''$, $0 < r < r_0$, $\tau > 0$.  	$\frac{(t - t_f)k}{q_0''' r_0^2} = \frac{1}{6}(1 - R^2 + 2/\text{Bi}) - 2\frac{\text{Bi}}{R} \sum_{n=1}^{\infty} \frac{\sin(\lambda_n R) \exp\left(-\lambda_n^2 \text{ Fo}\right)}{\lambda_n^2\left(\lambda_n^2 + \text{Bi}^2 - \text{Bi}\right)\sin(\lambda_n)}$ $\lambda_n \cot(\lambda_n) = 1 - \text{Bi}$ See Fig. 10.10

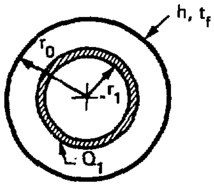
Case No.	References	Description	Solution
10.2.9.1	74, p. 366	Case 8.2.9 with $q''' = q_0'''e^{-b\tau}$.	$\frac{t - t_i}{t_f - t_i} = 1 - \frac{Po}{Pd}\left[1 - \frac{Bi \sin(\sqrt{Pd}\,R)}{R\,[(Bi - 1)\sin(\sqrt{Pd}) + \sqrt{Pd}\cos(\sqrt{Pd})]}\right]$ $\times \exp(-Pd\,Fo) - \sum_{n=1}^{\infty}\left(1 - \frac{Po}{Pd - \lambda_n^2}\right)\frac{A_n \sin(\lambda_n R)\exp\left(-\lambda_n^2 Fo\right)}{R\lambda_n}$ $A_n = \frac{2\left[\sin(\lambda_n) - \lambda_n \cos(\lambda_n)\right]}{\lambda_n - \sin(\lambda_n)\cos(\lambda_n)}$, $\tan(\lambda_n) = \frac{\lambda_n}{1 - Bi}$
10.2.10	9, p. 246	Sphere with parabolic initial temp and convection boundary. $t = q_0'''\left(r_0^2 - r^2\right)/6k + t_1$, $0 < r < r_0$, $\tau = 0$ (temp dist. for steady state with const. surface temp = t_1). $q''' = q_0'''$, $0 < r < r_0$, $\tau > 0$. 	$\frac{(t - t_1)k}{q_0'''r_0^2} = \frac{1}{6}(1 - R^2) + \left[\frac{1}{Po} + \frac{1}{3\,Bi}\right]\left[1 - \frac{2\,Bi}{R}\right.$ $\left.\times \sum_{n=1}^{\infty}\frac{\sin(\lambda_n R)\exp\left(-\lambda_n^2 Fo\right)}{\left(\lambda_n^2 + Bi^2 - Bi\right)\sin(\lambda_n)}\right]$. $\lambda_n \cot(\lambda_n) = 1 - Bi$, $Po = \frac{q_0'''r_0^2}{k(t_f - t_1)}$

Case No.	References	Description	Solution
10.2.11	9, p. 350	Sphere of infinite conductivity enclosed by a shell of finite conductivity. $t = t_0$, $0 < r < r_0$, $\tau = 0$. $q''' = q_0'''$, $0 < r < r_0$, $\tau > 0$. $q''' = 0$, $r_i < r < r_0$, $\tau > 0$.  	$\frac{(t - t_0)k}{q_0''' r_0^2} = \frac{1}{3}\left(\frac{1}{R} - \frac{1}{R_0}\right) - \frac{4K}{R}$ $\times \sum_{n=1}^{\infty} \frac{\sin\left[\lambda_n(R_0 - 1)\right] \sin\left[\lambda_n(R_0 - R)\right] \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n \left\{2K(R_0 - 1)\lambda_n + 4\lambda_n \sin^2\left[\lambda_n(R_0 - 1)\right] - K \sin\left[2\lambda_n(R_0 - 1)\right]\right\}}$. $K\lambda_n \cos\left[\lambda_n(R_0 - 1)\right] = \left(\lambda_n^2 - K\right) \sin\left[\lambda_n(R_0 - 1)\right]$, $1 < R < R_0$. $K = 3\rho_2 c_2/\rho_1 c_1$, $R = r/r_i$, $Fo = \alpha_2 \tau/r_i^2$
10.2.12	19, pp. 3-29	Sphere of infinite conductivity, variable specific heat, convective boundary, and steady heating.	See case 8.2.11, set L = 3, radius = ℓ

Case No.	References	Description	Solution
10.2.13	74, p. 370	Sphere with a steady internal heating and steady surface flux. $t = t_i,\ 0 < r < r_0,\ \tau = 0.$ $-q_r = q_0,\ r = r_0,\ \tau > 0.$ $q''' = q_0''',\ 0 < r < r_0,$ $\tau > 0.$ 	$\dfrac{(t - t_i)k}{q''' r_0^2} = Fo + \Phi$ $\Phi = \dfrac{Ki}{Po}\left[3\ Fo - \dfrac{1}{10}(3 - 5R^2) - 2\sum_{n=1}^{\infty}\dfrac{\sin(\lambda_n R)\exp\left(-\lambda_n^2\ Fo\right)}{\lambda_n^2 R\sin(\lambda_n)}\right]$ $\tan(\lambda_n) = \lambda_n$
10.2.13.1	74, p. 370	Case 10.2.13 with $q''' = q_0'''(1 - r/r_0)$.	$\dfrac{(t - t_i)k}{q_0''' r_0^2} = \dfrac{Fo}{4} + 2\sum_{n=1}^{\infty}\dfrac{2 - \left(2 + \lambda_n^2\right)\cos(\lambda_n)\sin(\lambda_n R)}{\lambda_n^5 R\sin^2(\lambda_n)}$ $\times\left[1 - \exp\left(-\lambda_n^2\ Fo\right)\right] + \Phi$ Φ given in case 10.2.13.

Case No.	References	Description	Solution
10.2.13.2	74, p. 370	Case 10.2.13 with $q''' = q_0'''(1 - R^2)$.	$\frac{(t - t_i)k}{q_0'''r_0^2} = \frac{2\ Fo}{5} - 2 \sum_{n=1}^{\infty} \frac{\sin(\lambda_n R)\left[1 - \exp\left(-\lambda_n^2\ Fo\right)\right]}{\lambda_m^4 R \sin(\lambda_m)} + \Phi$ Φ given in case 10.2.13.
10.2.13.3	74, p. 370	Case 10.2.13 with $q''' = q_0''' \exp(-bR)$.	$\frac{(t - t_i)k}{q_0'''r_0^2} = \frac{3\ Fo\left[2 - \exp(-b)(b^2 + 2b + 2)\right]}{b^3}$ $+ 2b \sum_{n=1}^{\infty} \frac{2\lambda_n - \exp(-b)\left(2 + b^2 + 2b + \lambda_n^2\right) \sin(\lambda_n) \sin(\lambda_n R)}{\lambda_n^2 \sin^2(\lambda_n)\left(\lambda_n^2 + b^2\right)^2 R}$ $\times \left[1 - \exp\left(-\lambda_n^2\ Fo\right)\right] + \Phi$ Φ given in case 10.2.13.
10.2.13.4	74, p. 370	Case 10.2.13 with $q''' = q_0'''(1 + b\tau)$.	$\frac{(t - t_i)k}{q_0'''r_0^2} = Fo\left(1 + \frac{1}{2}\ Pd\ Fo\right) + \Phi$ Φ given in case 10.2.13.
10.2.13.5	74, p. 371	Case 10.2.13 with $q''' = q_0''' \exp(-b\tau)$.	$\frac{(t - t_i)k}{q_0'''r_0^2} = \frac{1}{Pd}\left[1 - \exp(-Pd\ Fo)\right] + \Phi$ Φ given in case 10.2.13.

Case No.	References	Description	Solution
10.2.13.6	74, p. 371	Case 10.2.13 with $q''' = q_0'''\cos(b\tau)$.	$\frac{(t - t_i)k}{q'''r_0^2} = \frac{1}{Pd}\sin(Pd\ Fo) + \Phi$ Φ given in case 10.2.13.
10.2.13.7	74, p. 371	Case 10.2.13 with $q''' = q_0'''b\tau^n$.	$\frac{(t - t_i)k}{q'''r_0^2} = \frac{(Pd\ Fo)^n}{n + 1} Fo + \Phi$ Φ given in case 10.2.13.

Case No.	References	Description	Solution
10.2.14	74, p. 390	Sphere with pulse heating on a spherical surface and a convection boundary. $t = t_f,\ 0 < r < r_0,\ \tau = 0.$ Instantaneous pulse of strength Q_1 occurs at $r = r_1,\ \tau = 0.$  	$\frac{(t - t_f)\rho c r_0^3}{Q_1} = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{\left[(Bi - 1)^2 + \lambda_n^2\right]^{1/2} A_n \sin(\lambda_n R_1) \sin(\lambda_n R)}{Bi\ R_i\ R} \times \exp\left(-\lambda_n^2 Fo\right).$ $A_n = (-1)^{n+1} \frac{2\ Bi \left[\lambda_n^2 + (Bi - 1)^2\right]^{1/2}}{\lambda_n^2 + Bi^2 - Bi}.$ $\tan(\lambda_n) = \frac{\lambda_n}{1 - Bi},\ R = r/r_0$ For $Bi \to \infty$: $\frac{(t - t_f)\rho c r_0^3}{Q_1} = \frac{1}{2\pi} \sum_{n=1}^{\infty} \frac{1}{R_1 R} \sin(n\pi R_1) \sin(n\pi R) \exp(-n^2\pi^2 Fo).$ For $r_1 = 0$: $\frac{(t - t_f)\rho c r_0^3}{Q_1} = \frac{1}{2\pi R} \sum_{n=1}^{\infty} \frac{\lambda_n^2 \sin(\lambda_n R) \exp\left(-\lambda_n^2 Fo\right)}{\lambda_n - \sin(\lambda_n)\cos(\lambda_n)}.$ Mean temp: $\frac{(t_m - t_f)\rho c r_0^3}{Q_1} = \frac{1}{4\pi R_1\ Bi} \sum_{n=1}^{\infty} \left[(Bi - 1)^2 + \lambda_n^2\right]^{1/2} \lambda_n B_n \sin(\lambda_n R_1) \times \exp\left(-\lambda_n^2 Fo\right).$ $B_n = \frac{6\ Bi^2}{\lambda_n^2 \left(\lambda_n^2 + Bi^2 - Bi\right)}.$

Section 10.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

Section 10.2. Solids Bounded by Spherical Surfaces--With Internal Heating.

Case No.	References	Description	Solution

11. Change of Phase

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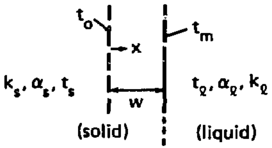
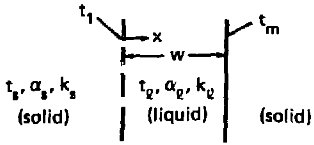
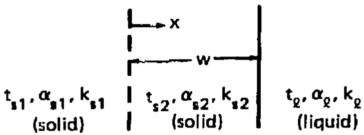
Case No.	References	Description	Solution
11.1.1	9, p. 285	Initial surface maintained at constant temp: freezing liquid. $t = t_0,\ x = 0,\ \tau > 0.$ $t = t_1,\ x > 0,\ \tau = 0.$ t_m = melting temp. 	Solid temp: $\dfrac{t_s - t_0}{t_m - t_0} = \mathrm{erf}\left(Fo^*_{xs}\right)/\mathrm{erf}(\lambda),\ 0 < x < w.$ Liquid temp; $\dfrac{t_\ell - t_1}{t_m - t_1} = \mathrm{erfc}\left(Fo^*_{x\ell}\right)/\mathrm{erfc}(\lambda\sqrt{\alpha_s/\alpha_\ell}),\ x > w.$ $\dfrac{\lambda\gamma\sqrt{\pi}}{c_s(t_m - t_0)} = \dfrac{\exp(-\lambda^2)}{\mathrm{erf}(\lambda)} + \dfrac{(t_m - t_\ell)}{(t_m - t_0)}\dfrac{k_\ell}{k_s}\dfrac{\exp\left(-\lambda^2\alpha_s/\alpha_\ell\right)}{\mathrm{erfc}(\lambda)}$ $w = 2\lambda\sqrt{\alpha_s\tau}$
11.1.2	9, p. 287	Solid region maintained at melting temp: freezing liquid. $t = t_m,\ x < w,\ \tau > 0.$ $t = t_1,\ x > 0,\ \tau = 0.$ $t_1 < t_m,\ t_m$ = melting temp.	Liquid temp: $\dfrac{(t_\ell - t_1)c_\ell}{\gamma} = \lambda\sqrt{\pi}\ \mathrm{erfc}\left(Fo^*_{x\ell}\right)/\exp(-\lambda^2),\ x > w.$ $w = 2\lambda\sqrt{\alpha_\ell\tau}$ $\lambda\ \exp(\lambda^2)\ \mathrm{erfc}(\lambda) = (t_m - t_1)\ c_\ell/\gamma\sqrt{\pi}$

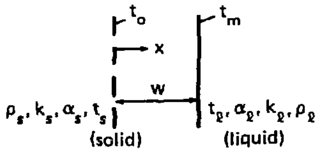
Section 11.1. Change of Phase--Plane Interface.

Case No.	References	Description	Solution
11.1.3	9, p. 287	Melting of a solid in contact with another solid. $t = t_0$, $x > 0$, $\tau = 0$. $t = t_1 > t_m$, $x = 0$, $\tau > 0$. t_m = melting temp. 	Solid temp: $\dfrac{t_s - t_0}{t_m - t_0} = \text{erfc}\ (Fo^*_{xs})/\text{erfc}\ (\lambda\sqrt{\alpha_\ell/\alpha_s})$, $x > w$. Liquid temp: $\dfrac{t_\ell - t_1}{t_m - t_1} = \text{erf}\ (Fo^*_{x\ell})/\text{erf}\ (\lambda)$, $0 < x < w$. $\dfrac{\exp(-\lambda^2)}{\text{erf}\ (\lambda)} + \dfrac{k_s\sqrt{\alpha_\ell}(t_0 - t_m)\ \exp(-\lambda^2\alpha_\ell/\alpha_s)}{k_\ell\sqrt{\alpha_s}(t_1 - t_m)\ \text{erfc}\ (\lambda\sqrt{\alpha_\ell/\alpha_s})} = \dfrac{\lambda\gamma\sqrt{\pi}}{c_\ell(t_1 - t_m)}$ $w = 2\lambda\sqrt{\alpha_\ell\tau}$

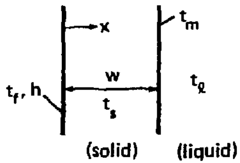
Case No.	References	Description	Solution
11.1.4	9, p. 288	Liquid freezing on solid--properties of solidified liquid different from original solid. $t = t_0$, $x < 0$, $\tau = 0$. $t = t_1$, $x > 0$, $\tau = 0$. t_m = melting temp. 	Original solid: $\frac{t_{s1} - t_0}{t_m - t_0} = \frac{\sqrt{A}}{\sqrt{A} + K\ \mathrm{erf}\ (\lambda)} \left[1 + \mathrm{erf}\ (Fo^*_{xs1})\right]$, $x < 0$. Solidified liquid: $\frac{t_{s2} - t_0}{t_m - t_0} = \frac{\sqrt{A} + K\ \mathrm{erf}\ (Fo^*_{xs2})}{\sqrt{A} + K\ \mathrm{erf}\ (\lambda)}$, $0 < x < w$. Liquid: $\frac{t_\ell - t_1}{t_m - t_1} = \mathrm{erfc}\ (Fo^*_{x\ell})/\mathrm{erfc}\ (\lambda \sqrt{\alpha_{s2}/\alpha_\ell})$, $x > w$. $\frac{\lambda \gamma \sqrt{\pi}}{c_{s2}(t_m - t_0)} = \frac{K\ \exp(-\lambda^2)}{\sqrt{A} + K\ \mathrm{erf}\ (\lambda)} + \frac{k_\ell \sqrt{\alpha_{s2}}(t_m - t_1)\ \exp(-\lambda^2 \alpha_{s2}/\alpha_\ell)}{k_{s2}\sqrt{\alpha_\ell}(t_m - t_0)\ \mathrm{erfc}\ (\lambda\sqrt{\alpha_{s2}/\alpha_\ell})}$ $w = 2\lambda\sqrt{\alpha_{s2}\tau}$, $K = k_{s1}/k_{s2}$, $A = \alpha_{s1}/\alpha_{s2}$

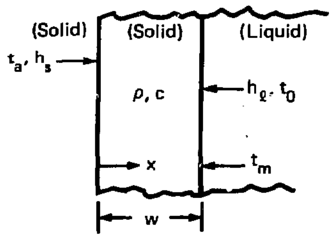
Section 11.1. Change of Phase--Plane Interface.

Case No.	References	Description	Solution
11.1.5	9, p. 289	Melting of a solid in contact with another solid--properties of liquid different than original solid. $t = t_1 > t_m,\ x < 0,\ \tau = 0.$ $t = t_0,\ x > 0,\ \tau = 0.$	$w = 2\lambda\sqrt{\alpha_\ell \tau}$ $\dfrac{\lambda\gamma\sqrt{\pi}}{c_\ell(t_1 - t_m)} = \dfrac{k_{s1}\sqrt{\alpha_\ell}\exp(-\lambda^2)}{k_\ell\sqrt{\alpha_{s1}} + k_{s1}\sqrt{\alpha_\ell}\,\mathrm{erf}(\lambda)} + \dfrac{k_{s2}\sqrt{\alpha_\ell}(t_0 - t_m \exp\left(-\lambda^2\alpha_\ell/\alpha_{s2}\right)}{k_\ell\sqrt{\alpha_{s2}}(t_1 - t_m)\,\mathrm{erfc}\,(\lambda\sqrt{\alpha_\ell/\alpha_{s1}})}\ .$ See case 11.1.4 for temperatures t_m = melting temperature ,
11.1.6	9, p. 289	Solid melts over temp range of t_{m1} to t_{m2}. $t = t_{m2},\ x > 0,\ \tau = 0.$ $c_\ell^* = c_\ell + \gamma/(t_{m2} - t_{m1}).$	For case 11.1.1, change equation for λ to: $\dfrac{\exp\left[(\alpha_s - \alpha_\ell)\lambda^2/\alpha_\ell\right]\mathrm{erfc}\,(\lambda\sqrt{\alpha_s/\alpha_\ell})}{\mathrm{erf}\ (\lambda)} = \dfrac{(t_{m2} - t_{m1})k_\ell\sqrt{\alpha_s}}{(t_{m1} - t_0)k_s\sqrt{\alpha_\ell}}\ .$ For case 11.1.4, change equation for λ to: $\dfrac{K\ \exp\left[(\alpha_{s2} - \alpha_\ell)\lambda^2/\alpha_\ell\right]\mathrm{erfc}\,(\lambda\sqrt{\alpha_{s1}/\alpha_\ell})}{\sqrt{A} + K\ \mathrm{erf}\ (\lambda)} = \dfrac{(t_{m2} - t_{m1})k_\ell\sqrt{\alpha_{s2}}}{(t_{m1} - t_0)\sqrt{\alpha_\ell}}\ .$ Use c_ℓ^* for c_ℓ in the above equations and solutions given in cases 11.1.1 and 11.1.4.

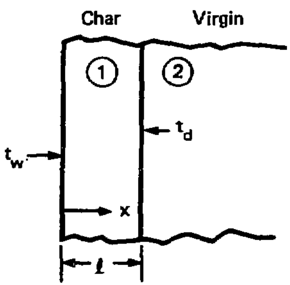
Case No.	References	Description	Solution
11.1.7	9, p. 290	Change of volume during solidification. $t = t_0,\ x = 0,\ \tau > 0.$ $t = t_1,\ x > 0,\ \tau = 0.$ $\rho_s > \rho_\ell.$ t_m = melting temperature.  	$\frac{t_\ell - t_1}{t_m - t_1} = \text{erfc}\left\{Fo^*_{x\ell} + \frac{\lambda(\rho_s - \rho_\ell)}{\rho_\ell}\sqrt{\frac{\alpha_s}{\alpha_\ell}}\right\}/\text{erfc}\left(\lambda\frac{\rho_s}{\rho_\ell}\sqrt{\frac{\alpha_s}{\alpha_\ell}}\right),\ x > w.$ $\frac{\exp(-\lambda^2)}{\text{erf}\ (\lambda)} = \frac{(t_1 - t_m)k_\ell\sqrt{\alpha_s}\ \exp\left(-\lambda^2\rho_s^2\alpha_s/\rho_\ell^2\alpha_1^2\right)}{(t_m - t_0)k_s\sqrt{\alpha_\ell}\ \text{erfc}\ (\lambda\rho_s\sqrt{\alpha_s}/\rho_\ell\sqrt{\alpha_\ell})} = \frac{\lambda\gamma\sqrt{\pi}}{c_s(t_m - t_0)}$ $w = 2\lambda\sqrt{\alpha_s\tau}$
11.1.8	9, p. 291	Constant heat flux at original solid-liquid boundary. $t - t_m,\ x > 0,\ \tau = 0.$ $q_x = q_0,\ x = 0,\ \tau > 0.$ t_s = solid temp, $t_s < t_m$. t_m = melting temp.	$\frac{(t_s - t_m)k_s}{\alpha_s\rho_s\gamma} = Qx - \frac{Q^2}{2x^2}(1 + 2\ Fo_x) + \frac{Q^4}{12x^4}(1 + 12\ Fo_x + 12\ Fo_x^2) - \dots,\ 0 < x < w.$ $w = QT - \frac{1}{2}\,Q^3T^2 + \frac{5}{6}\,Q^5T^3 - \dots$ $Q = q_0/\alpha_s\rho_s\gamma,\ Fo_x = \alpha_s\tau/x^2,\ T = \alpha_s\tau$

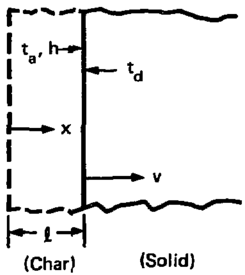
Section 11.1. *Change of Phase--Plane Interface.*

Case No.	References	Description	Solution
11.1.9	9, p. 292	Convection boundary at original solid-liquid boundary. $t = t_1,\ x > 0,\ \tau = 0,\ t_1 > t_m$. t_m = melting temp.  	$\dfrac{t_s - t_m}{t_m - t_f} = Bi_x - \dfrac{F\ Bi_x^2}{2}(1 + 2!\ Fo_x) - \dfrac{F\ Bi_x^3}{3!}(1 + 3!\ Fo_x) + \ldots,\ 0 < x < w.$ $w = F\ Bi_x\ Fo_x - \dfrac{F^2\ Bi_x^3}{2}(1 + F)\ Fo_x^2 + \ldots$ $F = k_s(t_m - t_f)/\alpha_s\rho\gamma\ ,\ Fo_x = \alpha_s\tau/x^2\ ,\ Bi_x = hx/k_s$

Case No.	References	Description	Solution
11.1.10	55 19, p. 3-87	Solidification on plane semi-infinite solid with convection coefficient between solid-solid and liquid-solid. $t = t_0$, $x < 0$, $\tau = 0$. $c = 0$. t_m = melting temperature.  	$\dfrac{h_s^2(t_m - t_a)}{\rho\gamma k} = \dfrac{1}{N^2}\,\ell n\left[\dfrac{N - 1}{N(Bi + 1) - 1}\right] - \dfrac{Bi}{N}$ $N = \dfrac{h_\ell(t_0 - t_m)}{h_s(t_m - t_a)}$, $Bi = \dfrac{h_s w}{k_s}$ See Fig. 11.1
11.1.11	56 19, p. 3-88	Case 11.1.10 with solidified layer of finite heat capacity and heat flow at $x = 0$ given as $q = bt_w$, where t_w is temp at $x = 0$.	See Figs. 11.2a and b

Section 11.1. Change of Phase--Plane Interface.

Case No.	References	Description	Solution
11.1.12	19, 77 p. 3-88	Case 11.1.10 with heat flow at $x = 0$ given as $q = bt_w^4$.	See Figs. 11.3a and b
11.1.13	19, p. 3-88	Decomposition of a semi-infinite solid. $t = t_0,\ x \geq 0,\ \tau = 0.$ $t = t_w,\ x = 0,\ \tau > 0.$ t_d = decomposition temp. $\ell = b\sqrt{\tau}.$ 	Char layer: $\dfrac{t - t_d}{t_w - t_d} = 1 - \dfrac{\mathrm{erf}\left(Fo_{x1}^*\right)}{\mathrm{erf}\ (b/2\sqrt{\alpha_1})}\ ,\ 0 \leq x \leq \ell\ .$ Virgin layer: $\dfrac{t - t_d}{t_w - t_d} = \dfrac{\mathrm{erfc}\left(Fo_{x2}^*\right)}{\mathrm{erfc}\ (b/2\sqrt{\alpha_2})}\ ,\ x \geq \ell\ .$ $\dfrac{\exp\left(-b^2/4\alpha_1\right)}{\mathrm{erf}\ (b/2\sqrt{\alpha_1})} - \sqrt{\dfrac{\rho_2 c_2 k_2}{\rho_1 c_1 k_1}}\left(\dfrac{t_d - t_0}{t_w - t_d}\right)\dfrac{\exp\left(-b^2/4\alpha_2\right)}{\mathrm{erfc}\ (b/2\sqrt{\alpha_2})} = \dfrac{\sqrt{\pi\rho_1/c_1k_1}\ \gamma b}{2(t_w - t_d)}$ See Figs. 11.4a and b

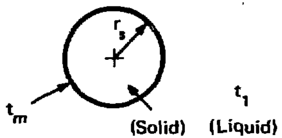
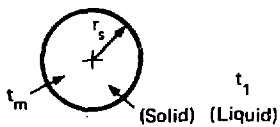
Case No.	References	Description	Solution
11.1.14	57 19, p. 3-93	Ablation of a semi-infinite solid by convection heating. $t = t_0,\ x \geq 0,\ \tau = 0.$ t_d = decomposition temp.  	See Figs. 11.5a, b and c $\frac{t_d - t_0}{t_a - t_0} = 1 - \exp\left[(h/k)^2 \alpha\tau_d\right] \operatorname{erfc}\left[(h/k)\sqrt{\alpha\tau_d}\right]$ τ_d = Time for free surface to reach temp t_d Steady state ablation velocity and temp distribution: $v_{ss} = \frac{h(t_a - t_d)}{\rho\left[c(t_d - t_0) + H\right]}$ $\frac{t - t_0}{t_d - t_0} = \exp\left[-\frac{v_{ss}}{\alpha}(x - \ell)\right]$ H = heat of ablation

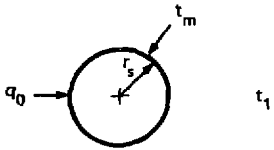
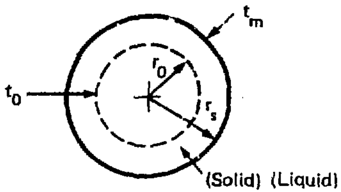
Case No.	References	Description	Solution
11.1.15	58 19, p. 3-95	Case 11.1.15 with heating by a constant surface flux q''.	See Fig. 11.6 Preablation time: $\tau_d = \frac{\pi}{4}\rho c k\left(\frac{t_d - t_0}{q''}\right)^2$ Ablation velocity: $v_{ss} = \frac{q''}{\rho\left[c(t_d - t_0) + H\right]}$

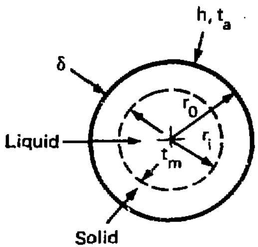
Case No.	References	Description	Solution

Section 11.1. Change of Phase--Plane interface.

Case No.	References	Description	Solution

Case No.	References	Description	Solution
11.2.1	9, p. 29	Liquid freezing on a cylinder. $t = t_m,\ 0 < r < R.$ $t = t_1,\ r \rightarrow \infty,\ t_1 < t_m.$ t_m = melting temp.  	Liquid temp: $\dfrac{t_\ell - t_1}{t_m - t_1} = \mathrm{Ei}\left(-\mathrm{Fo}^{*2}_{r\ell}\right) / \mathrm{Ei}\ (-\lambda^2),\ r > r_s.$ $\lambda^2 \exp(\lambda^2)\ \mathrm{Ei}\ (-\lambda^2) + c_\ell(t_m - t_1)/\gamma = 0.$ $r_s = 2\lambda\sqrt{\alpha_\ell \tau}$
11.2.2	9, p. 295	Liquid freezing on a sphere. $t = t_m,\ 0 < r < R.$ $t = t_1,\ r \rightarrow \infty,\ t_1 < t_m.$  	Liquid temp: $\dfrac{t_\ell - t_1}{t_m - t_1} = \dfrac{2\lambda}{\exp(-\lambda^2) - \lambda\sqrt{\pi}\ \mathrm{erfc}\ (\lambda)} \left\{ \dfrac{1}{2\ \mathrm{Fo}^{*}_{r\ell}} \exp\left(-\mathrm{Fo}^{*2}_{r\ell}\right) - \dfrac{\sqrt{\pi}}{2}\ \mathrm{erfc}\left(\mathrm{Fo}^{*}_{r\ell}\right) \right\},\ r > r_s.$ $\lambda^2 \exp(\lambda^2) \left\{ \exp(-\lambda^2) - \lambda\pi\ \mathrm{erfc}\ (\lambda) \right\} = \dfrac{c_\ell}{2\gamma}\ (t_m - t_1).$ $r_s = 2\lambda\sqrt{\alpha_\ell \tau}$

Case No.	References	Description	Solution
11.2.3	9, p. 296	Steady line heat sink $t = t_1$, $r > 0$, $\tau = 0$, $t_1 > t_m$. sink rate = q_0. t_m = melting temp.  	Solid temp: $\frac{(t_s - t_m)k_s}{q_0'} = 4\pi\ \mathrm{Ei}\left(-Fo_{rs}^{*2}\right) - \mathrm{Ei}(-\lambda^2),\ 0 < r < r_s$. $r_s = 2\lambda\sqrt{\alpha_s\tau}$ Liquid temp: $\frac{(t_\ell - t_1)}{t_m - t_1} = \mathrm{Ei}\left(-Fo_{r\ell}^{*2}\right)/\mathrm{Ei}\left(-\lambda^2\alpha_s/\alpha_\ell\right),\ r > r_s$. $\exp(-\lambda^2)/4\pi + \frac{k_\ell}{q_0}(t_1 - t_m)\ \exp\left(-\lambda^2\alpha_s/\alpha_\ell\right)/\mathrm{Ei}\left(-\lambda^2\alpha_s/\alpha_\ell\right) = \lambda^2\alpha_s\rho\gamma/q_0$.
11.2.4	9, p. 296	Freezing on steady temp cylinder. $t = t_0$, $r = r_0$, $\tau > 0$, $t_0 < t_m$. $t = t_m$, $r > r_0$, $\tau = 0$. t_m = melting temp.  	Solid temp: $\frac{t_s - t_0}{t_m - t_0} = \ell n(r/r_0)/\ell n(r_s/r_0),\ r_0 < r < r_s$ $2r_s^2\ \ell n(r_s/r_0) - r_s^2 + a^2 = 4\ k_s(t_m - t_0)\tau/\gamma\rho$

Case No.	References	Description	Solution
11.2.5	19, p. 3-87	Freezing inside a tube, tube resistance neglected, convection cooling at $r = r_0$. $t = t_m$, $r = r_i$. t_m = melting temp. 	$\frac{h^2(t_m - t_a)}{\rho\gamma k} = \frac{Bi^2}{2}\left[\left(\frac{1}{Bi} + \frac{1}{2}\right)(1 - R^2) - R^2 \ln\left(\frac{1}{R}\right)\right]$, $R \leq 1$. $Bi = h\delta/k$, $R = r_i/r_0$
11.2.6	19, p. 3-87	Case 11.2.5 with freezing on outside of tube and convection cooling on inside.	$\frac{h^2(t_m - t_a)}{\rho\gamma k} = \frac{Bi^2}{2}\left[\left(\frac{1}{Bi} - \frac{1}{2}\right)(R^2 - 1) + R^2 \ln(R)\right]$, $R \geq 1$
11.2.7	19, p. 3-88	Case 11.2.5 with freezing inside a sphere.	$\frac{h^2(t_m - t_a)}{\rho\gamma k} = \frac{Bi^2}{3}\left[\left(\frac{1}{Bi} - 1\right)(1 - R^3) + \frac{3}{2}(1 - R^2)\right]$, $R \leq 1$

Case No.	References	Description	Solution
11.2.8	19, p. 3-88	Case 11.2.6 with freezing on outside of sphere.	$\frac{h^2(t_m - t_a)}{\rho\gamma k} = \frac{Bi^2}{3}\left[\left(\frac{1}{Bi} + 1\right)(R^3 - 1) - \frac{3}{2}(R^2 - 1)\right], \; R \geq 1$

Case No.	References	Description	Solution

Section 11.2. Change of Phase--Nonplanar Interface.

Case No.	References	Description	Solution

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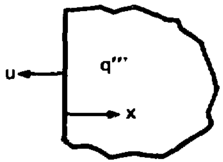
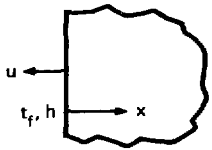
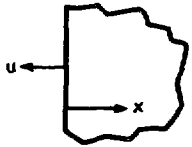
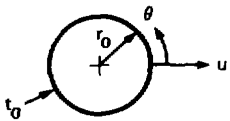
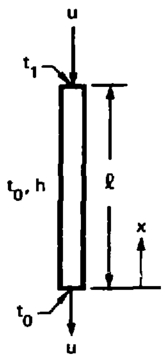
Section 12.1. Traveling Boundaries.

Case No.	References	Description	Solution
12.1.1	9, p. 389	Semi-infinite solid with linear initial temp, time dependent surface temp and internal heating. $t = t_0 + dx,\ x > 0,\ \tau = 0.$ $t = t_1 + b\tau,\ x = 0,\ \tau > 0.$ Solid increasing at velocity u.  	$\frac{t - t_0}{t_1 - t_0} = D(1 - U) + G + \frac{1}{2}\operatorname{erfc}[X(1 - U)] + \frac{1}{2}\exp(4UX^2)\operatorname{erfc}[X(1 + U)] + \frac{1}{2}\left(\frac{B}{U} + D + \frac{G}{U}\right) \times \left\{(1 + U)\exp(4UX^2)\operatorname{erfc}[X(1 + U)] - (1 - U)\operatorname{erfc}[X(1 - U)]\right\}.$ $B = b\tau/(t_1 - t_0),\ D = dx/(t_1 - t_0),\ G = q'''\alpha\tau/k(t_1 - t_0)$ $U = u\tau/x,\ X = x/2\sqrt{\alpha\tau}$
12.1.2	9, p. 389	Semi-infinite solid with convection boundary. $t = t_0,\ x > 0,\ \tau = 0.$ Solid increasing at velocity u.  	$\frac{t - t_0}{t_f - t_0} = \frac{1}{2}\left\{\operatorname{erfc}[X(1 - U)] + \frac{Fo\,Bi}{Fo\,Bi - U}\exp(U/Fo)\operatorname{erfc}[X(1 + U)] + \frac{2\,Fo\,Bi - U}{Fo\,Bi - U}\exp[Bi(1 - U + Fo\,Bi)]\operatorname{erfc}[X(2\,Fo\,Bi + 1 - U)]\right.$. $Bi = hx/k,\ Fo = \alpha\tau/x^2,\ X = x/2\sqrt{\alpha\tau},\ U = u\tau/x$

Case No.	References	Description	Solution
12.1.3	9, p. 389	Semi-infinite solid with steady periodic temp. $t = (t_0 - t_m)\cos(\omega\tau + \theta)$, $x = 0$, $\tau > 0$. Solid increasing at velocity u. 	$\frac{t - t_m}{t_0 - t_m} = \exp\left[ux/2\alpha - x\sqrt{b}\cos(\phi/2)\right]\cos\left[\omega\tau - x\sqrt{b}\sin(\phi/2) + \theta\right]$ $\frac{u^2}{4\alpha^2} + \frac{i\omega}{\alpha} = b\exp(i\phi)$
12.1.4	9, p. 389	Cylindrical boundary traveling in an infinite solid. $t = t_i$, $r \geq r_0$, $\tau = 0$. $t = t_0$, $r = r_0$, $\tau > 0$. Cylinder traveling at velocity u. 	$\frac{t - t_0}{t_0 - t_i} = \sum_{n=0}^{\infty} \frac{\varepsilon_n I_n(U) K_n(UR)\cos(n\theta)}{K_n(U)}$ $+ \frac{2}{\pi}\sum_{n=0}^{\infty} \exp(-U^2\,Fo)\,\varepsilon_n\cos(n\theta)\, I_n(U)$ $\times \int_0^{\infty} \frac{\exp(-Fo\,\lambda^2)\left[J_n(\lambda R)Y_n(\lambda) - Y_n(\lambda R)J_n(\lambda)\right]\lambda d\lambda}{(\lambda^2 + U^2)\left[J_n^2(\lambda) + Y_n^2(\lambda)\right]}$ $\varepsilon_0 = 1$, $\varepsilon_n = 2$ if $n \geq 1$, $Fo = \alpha\tau/r_0^2$, $R = r/r_0$ $U = ur_0/2\alpha$

Case No.	References	Description	Solution
12.1.5	9, p. 391	Thin rod with fixed end temps and convection cooling. $t = t_0,\ 0 < x < \ell,\ \tau = 0.$ $t = t_1,\ x = \ell,\ \tau > 0.$ $t = t_0,\ x = 0,\ \tau > 0.$ The end boundaries traveling at velocity u. p = rod perimeter. A = rod section area.	$\frac{t - t_0}{t_1 - t_0} = \frac{\sinh\left[\sqrt{U^2X^2 + Bi\ X^2}\right]\exp(UX - U)}{\sinh\left[\sqrt{U^2 + Bi}\right]}$ $+ 2\pi \exp(UX - U) \sum_{n=1}^{\infty} \frac{(-1)^n n \sin(n\pi X)}{Bi + U^2 + n^2\pi^2}$ $\times \exp\left[-Fo\ (Bi + U^2 + n^2\pi^2)\right]$ $Bi = hp\ell^2/kA,\ Fo = \alpha\tau/\ell^2,\ U = u\ell/2\alpha,\ X = x/\ell$



Section 12.1. Traveling Boundaries.

Case No.	References	Description	Solution

Case No.	References	Description	Solution

Solution Figs. and Tables

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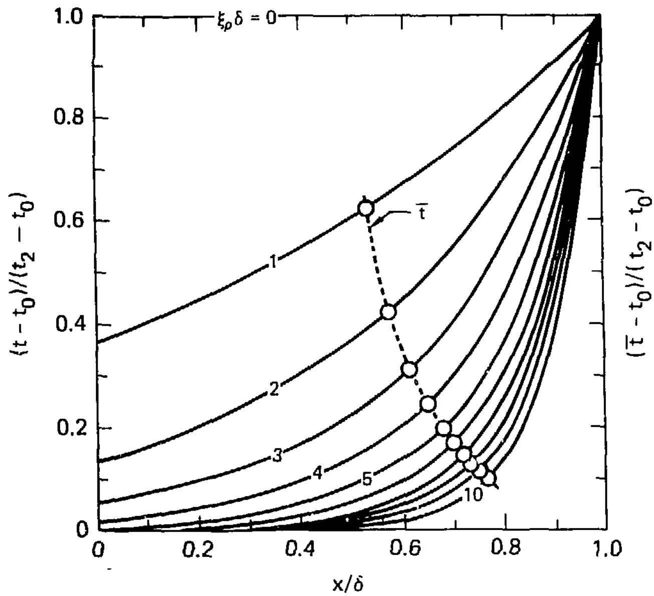


FIG. 1.1. Temperature distribution and mean temperature in a porous plate (case 1.1.4, source: Ref. 2, p. 221, Fig. 9.2).

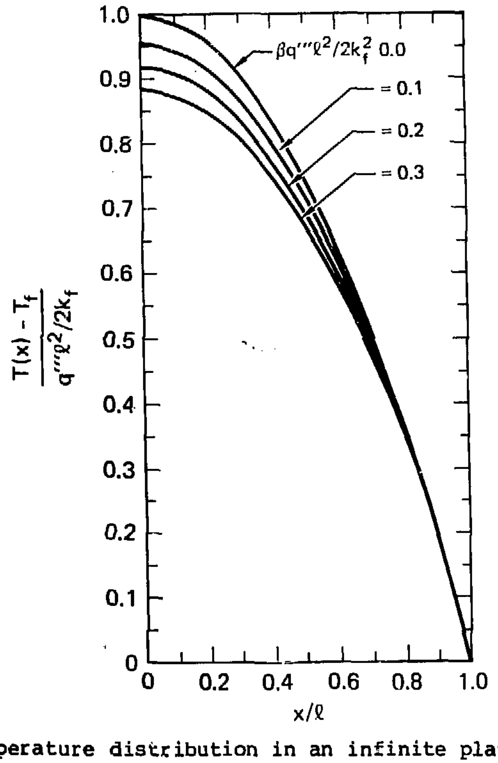


FIG. 1.2. Temperature distribution in an infinite plate with internal heating and temperature dependent conductivity (case 1.2.3, source: Ref. 3, p. 132, Fig. 3.23).

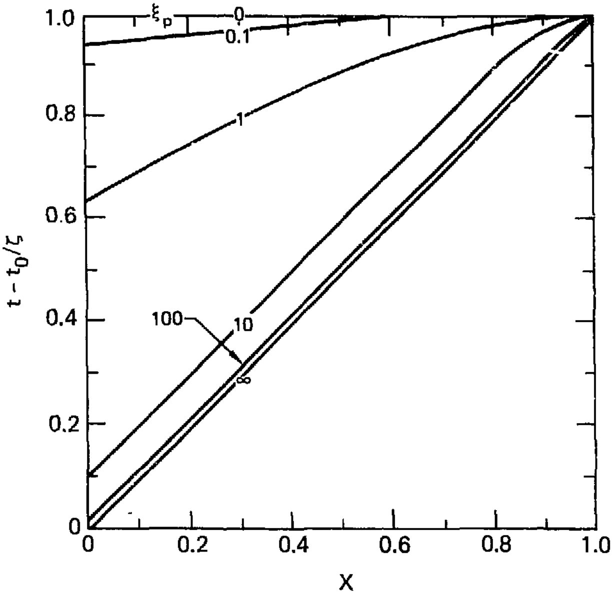


FIG. 1.3. Temperature distribution in a heat generating porous plate (case 1.2.6, source: Ref. 2, p. 223, Fig. 9.3).

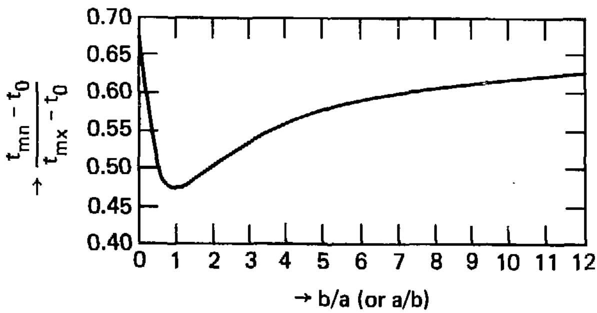


FIG. 1.4. Ratio of mean and maximum temperature excesses in an electrical coil of rectangular cross section (case 1.2.8, source: Ref. 1, p. 180, Fig. 10.5).

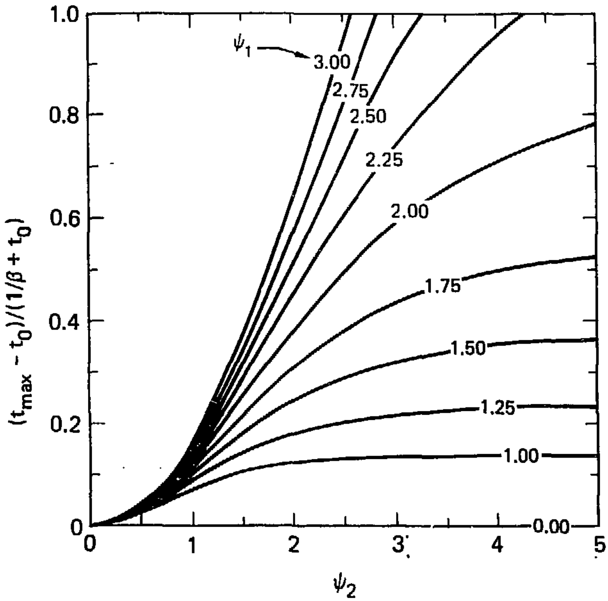


FIG. 1.5. First-term approximation to the maximum temperature in a solid rectangular rod with internal heating, $\psi_1 = \sqrt{q_0'''/\beta k}\ 2b$, $\psi_2 = \sqrt{q_0'''/\beta k}\ 2a$ (case 1.2.10, source: Ref. 2, p. 198, Fig. 8.16).

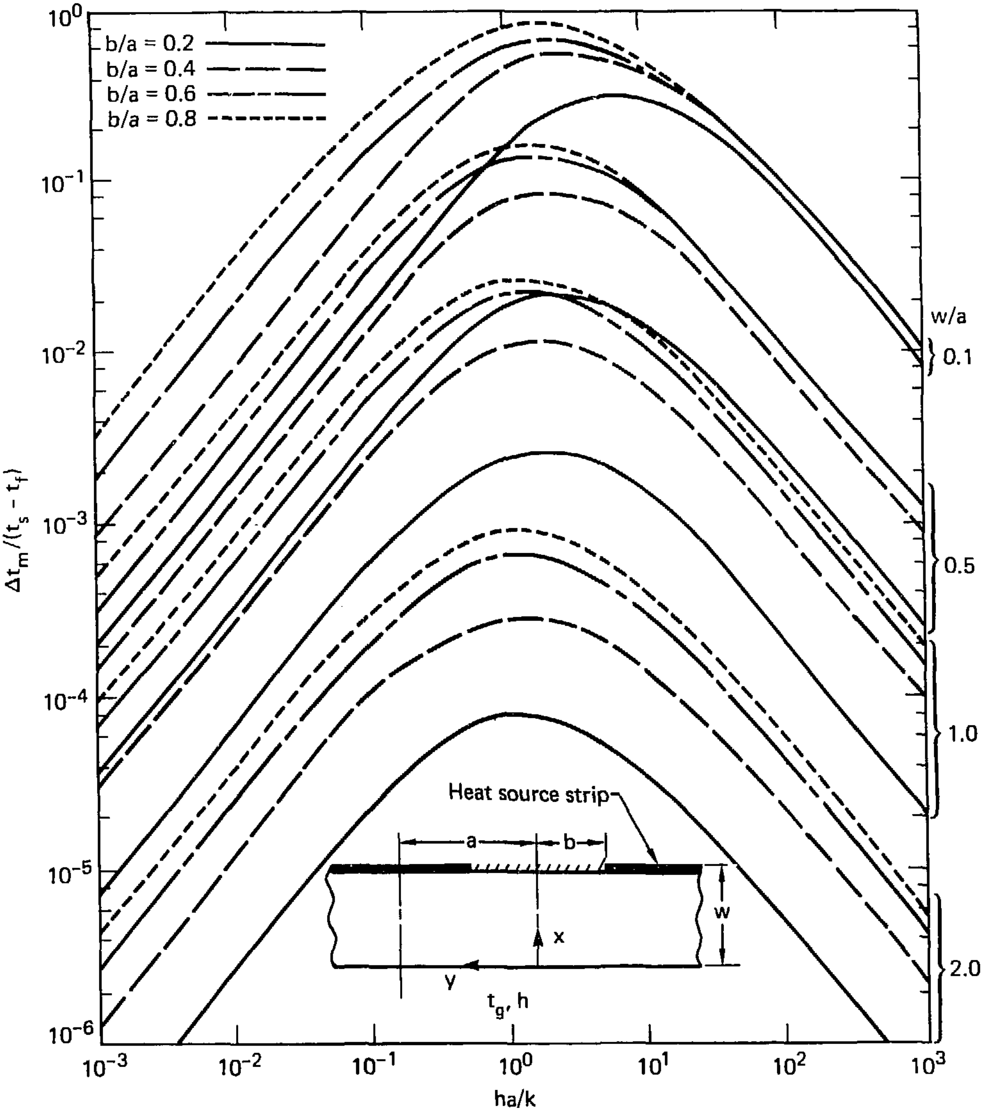


FIG. 1.6a. Maximum temperature variations on the cooled surface of a flat plate having equally spaced adiabatic and constant temperature strips on the opposite surface (case 1.1.30, source: Ref. 27).

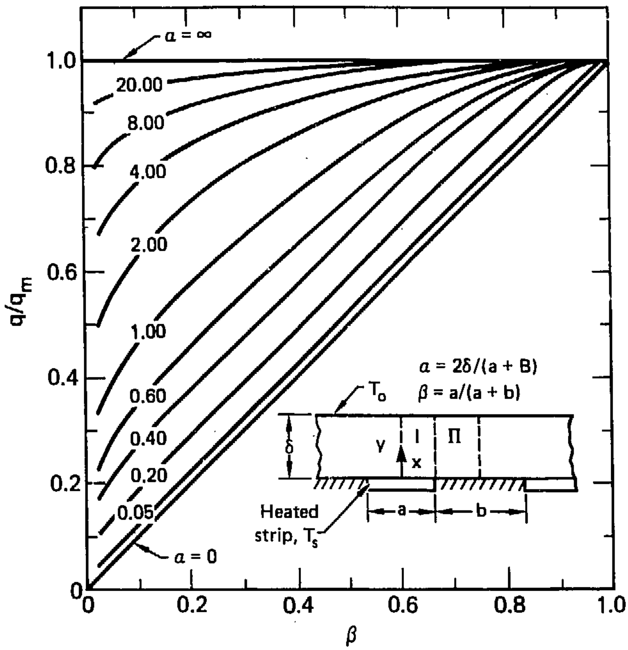


FIG. 1.6b. Heat flux through slabs held at a uniform temperature on one surface and having equally spaced constant temperature strips on the other (case 1.1.30, source: Ref. 88).

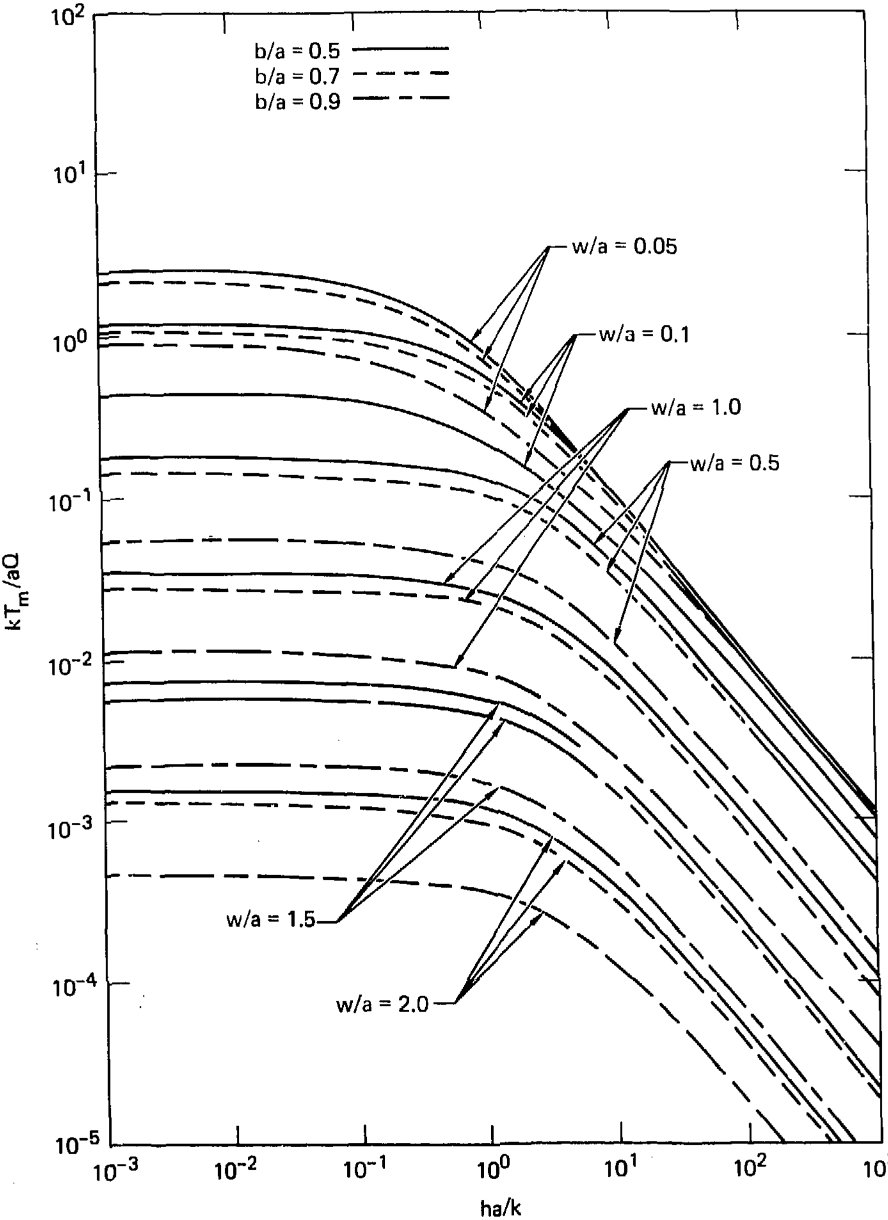


FIG. ?.7. Maximum temperature variation of the cooled surface of a flat plate having alternating adiabatic and constant heat-flux strips on the opposite surface (case 1.1.31, source: Ref. 28).

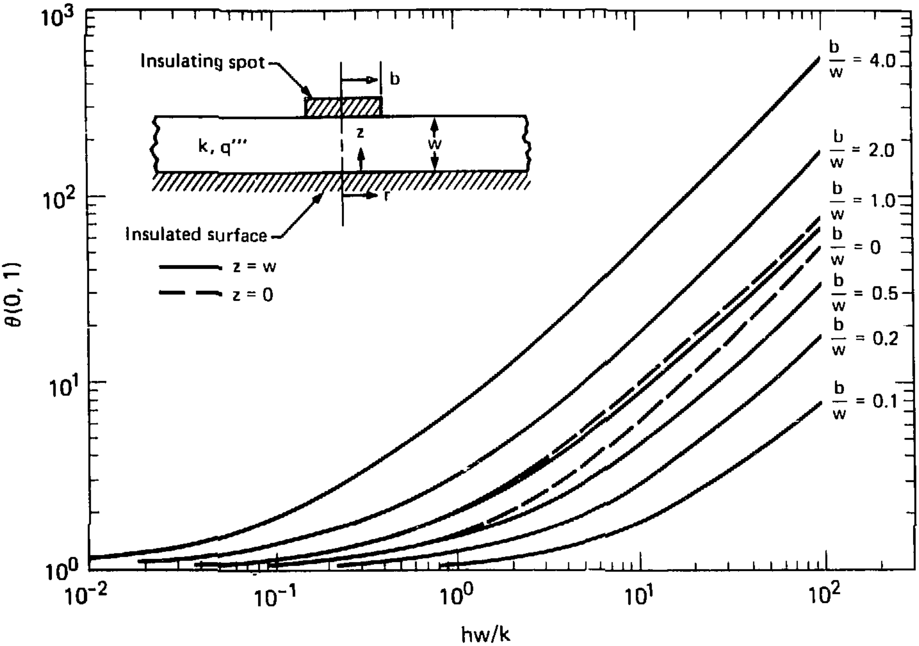


FIG. 1.8. Temperatures at the spot center of a spot-insulated plate having a uniform internal heat source (case 1.2.13, source: Ref. 29).

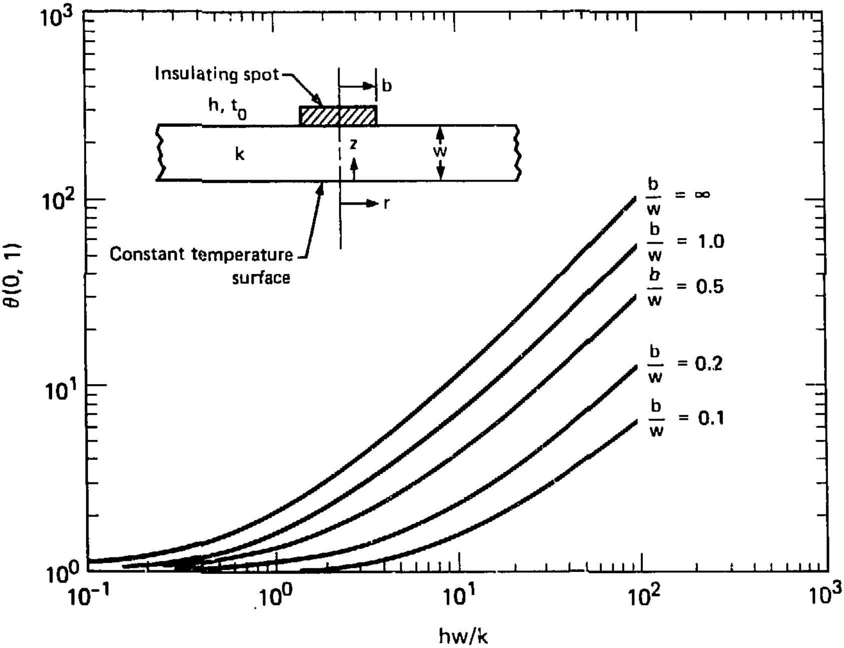


FIG. 1.9. Temperatures on the cooled surface and at the spot center of a spot-insulated plate having a constant temperature heat source on one face (case 1.1.32, source: Ref. 29).

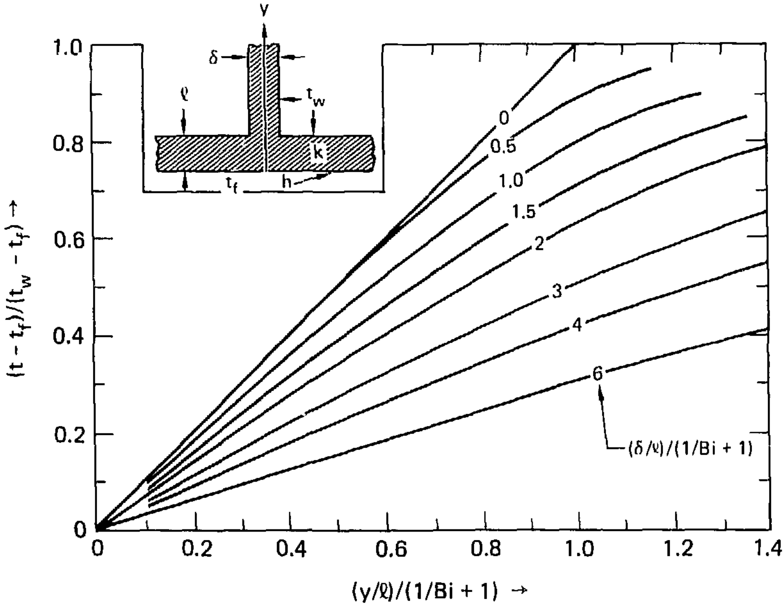


FIG. 1.10. Hotspot temperatures along plate/rib centerline ($Bi = h\ell/k$) (case 1.1.39, source: Ref. 19, p. 3-126, Fig. 77).

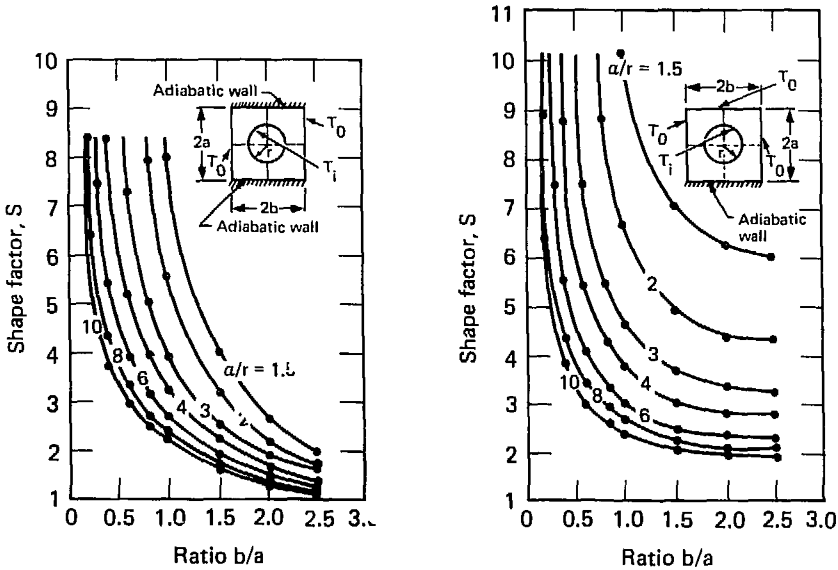


FIG. 1.11. Conductive shape factors for a rectangular section containing a constant temperature tube (case 1.1.44, source: Ref. 87).

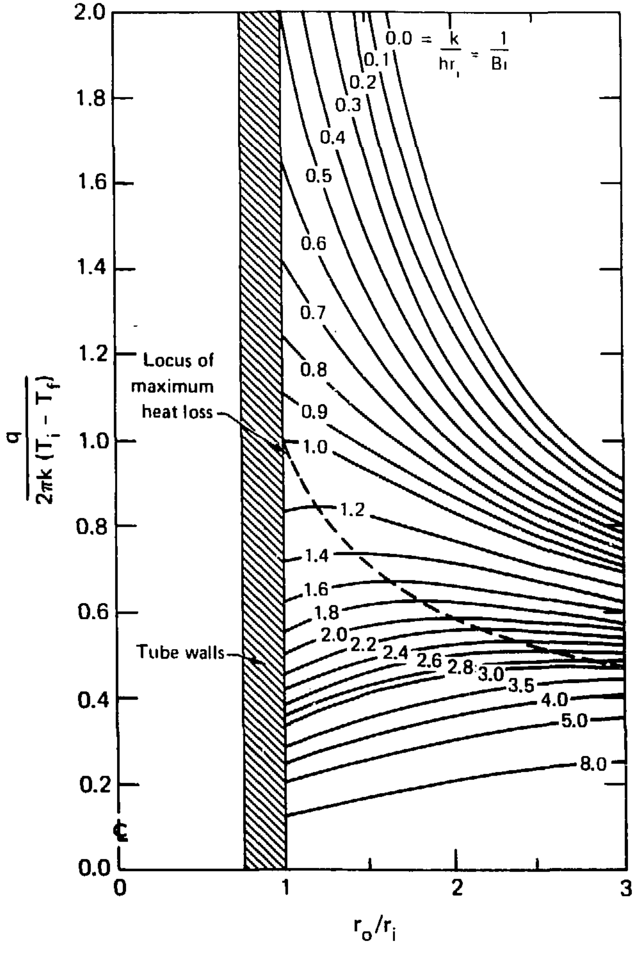


FIG. 2.1. Heat loss from insulated tubes (case 2.1.3, source: Ref. 3, p. 123, Fig. 3.18).

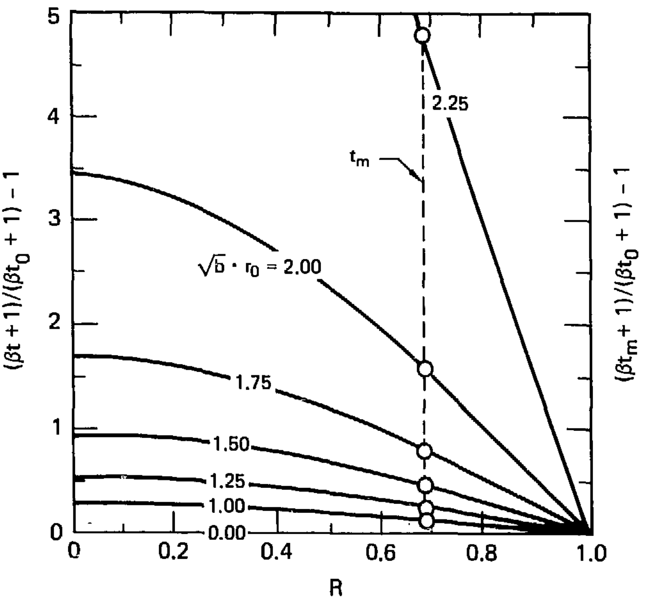


FIG. 2.2. Temperature distribution and mean temperature in a cylinder with temperature dependent heat source (case 2.2.4, source: Ref. 2, p. 189, Fig. 8.11).

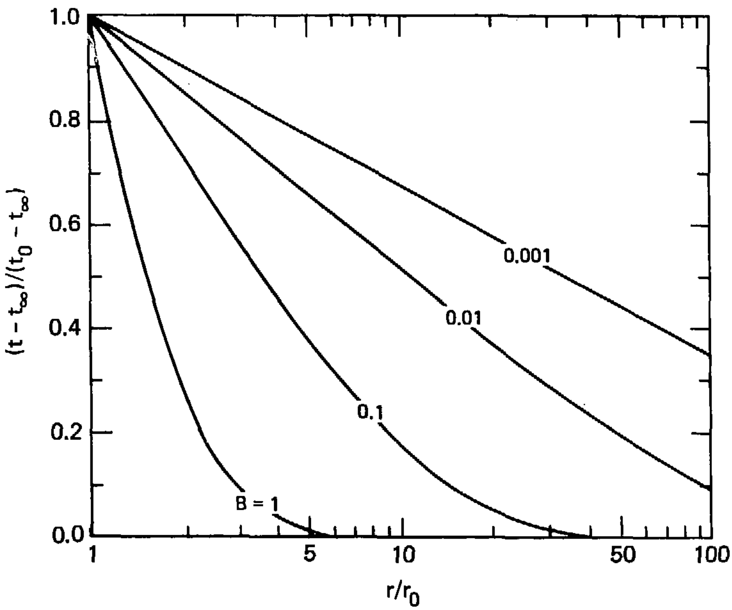


FIG. 2.3. Temperature distribution in an infinite plate with a cylindrical heat source, $B = \sqrt{(h_1 + h_2)/kw}\ r_0$, t_0 = t at r_0, t_∞ = t at r = ∞ (case 2.2.1 source: Ref. 2, p. 175, Fig. 8.2).

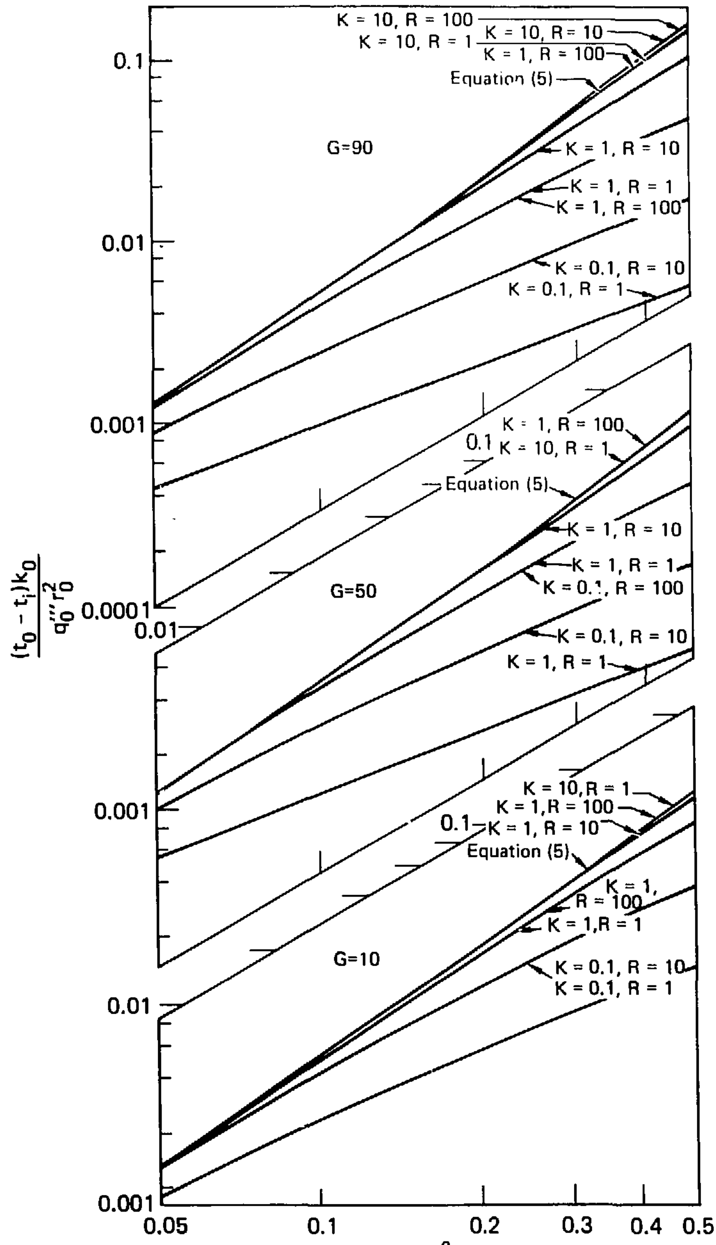


FIG. 2.4. The inside surface temperature of an infinite tube with temperature dependent conductivity and heating (case 2.2.17, source: Ref. 16, Fig. 1). Equation (5) given in case solution.

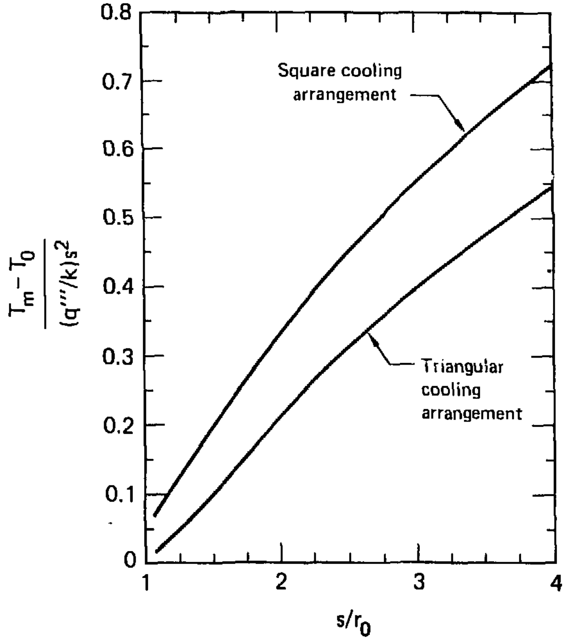


FIG. 2.5. Maximum (hot spot) temperatures in the cross section of a heat-generating solid (cases 2.2.18 and 2.2.19, source: Ref. 64, Fig. 3).

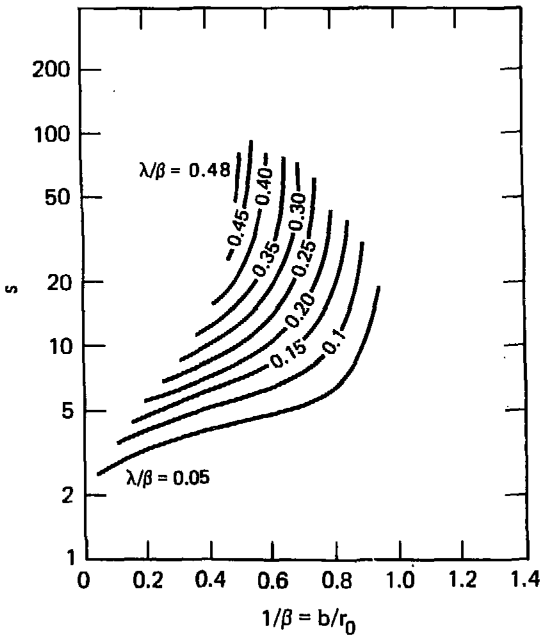


FIG. 2.6. Shape factor for a cylinder with two longitudinal holes (case 2.1.44, source: Ref. 73, Fig. 6).

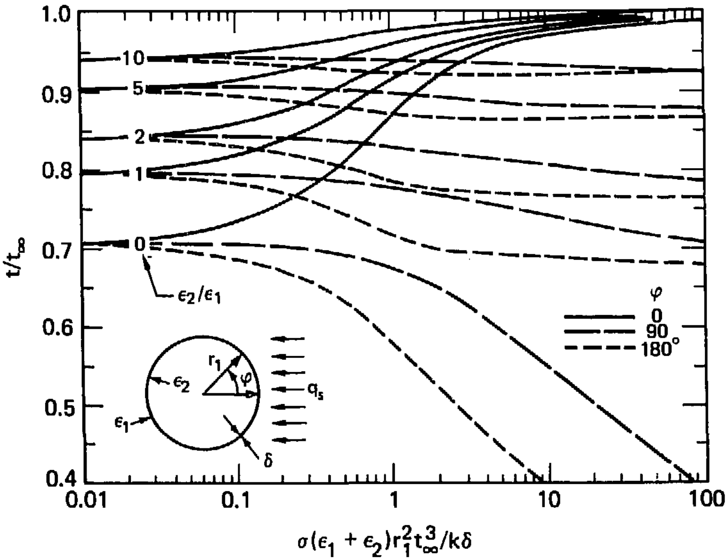


FIG. 3.1. Steady temperature of thin, nonrotating spherical shell in uniform radiation field (case 3.1.10, source: Ref. 19, pp. 3-112, Fig. 68).

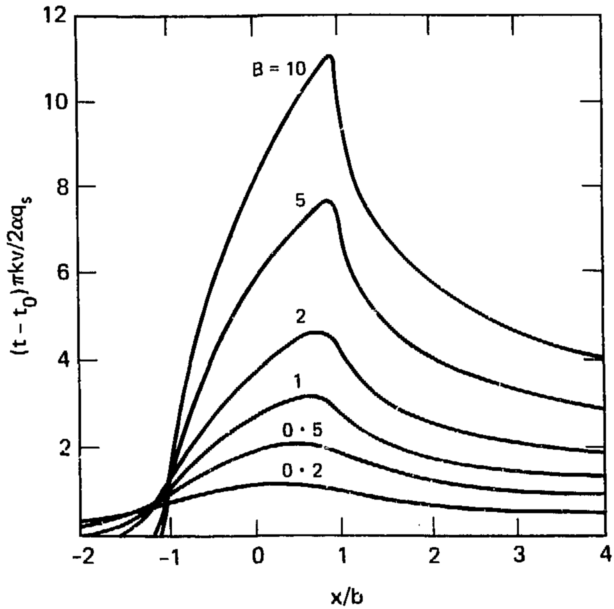


FIG. 4.1. Surface temperature of a semi-infinite solid with heating on the surface over width 2b, which moves at velocity u (case 4.1.8, source: Ref. 9, p. 270, Fig. 34).

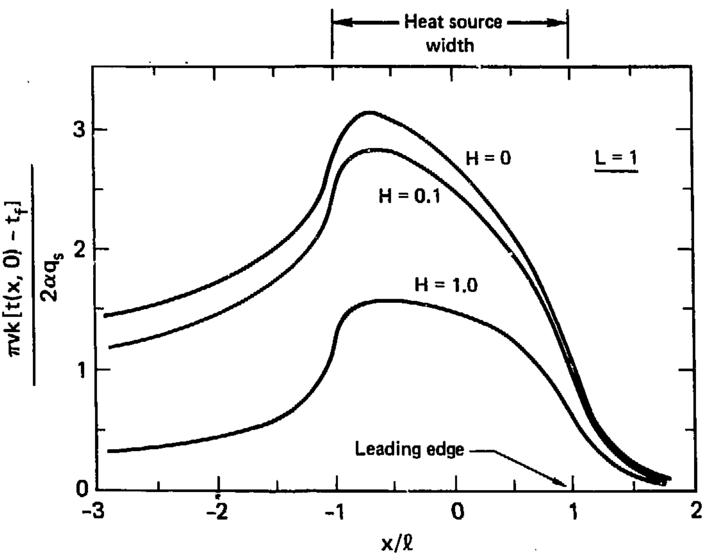


FIG. 4.2. Surface temperature of a convectively cooled semi-infinite solid with a traveling strip heat source (case 4.1.9, source: Ref. 30).

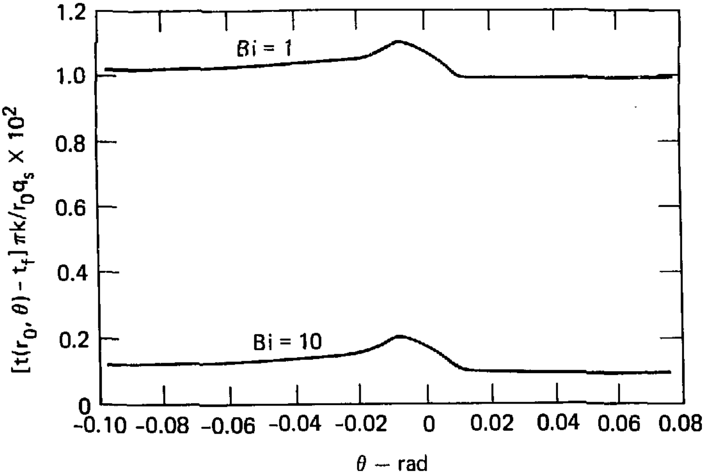


FIG. 4.3a. Surface temperature of an infinite cylinder with a rotating surface band source, $\phi = 0.01$ (case 4.1.10, source: Ref. 30).

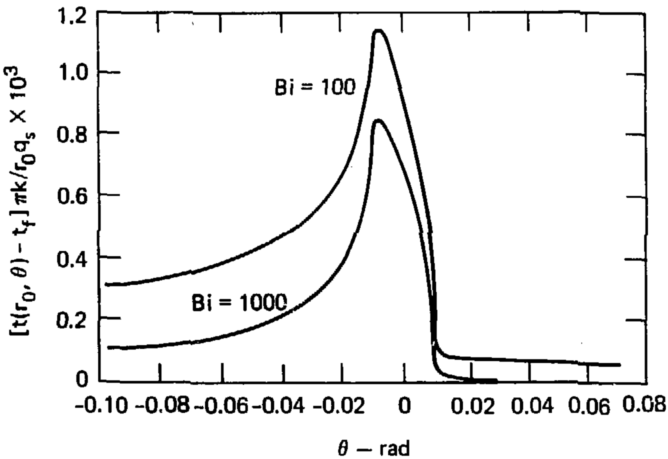


FIG. 4.3b. (Same conditions as for Fig. 4.3a).

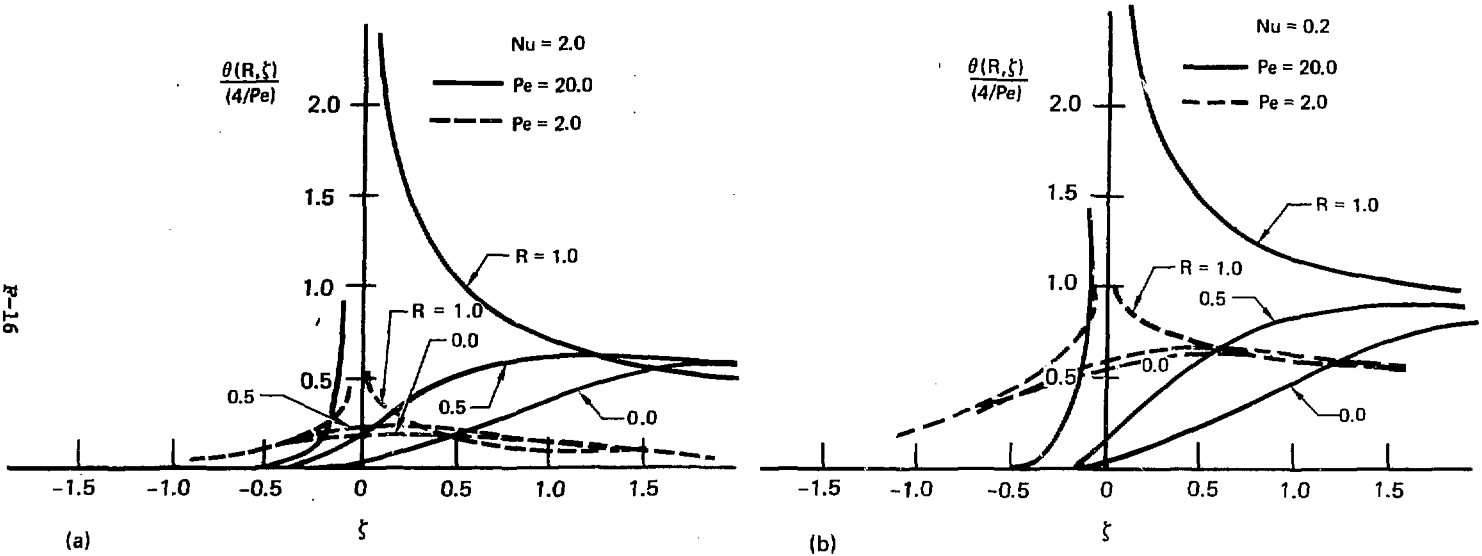


FIG. 4.4. Temperature distribution in a cylinder with a ring heat source (case 4.1.11, source: Ref. 34). $\theta = (t - t_f) 2\pi r_0 k / Q_0$.

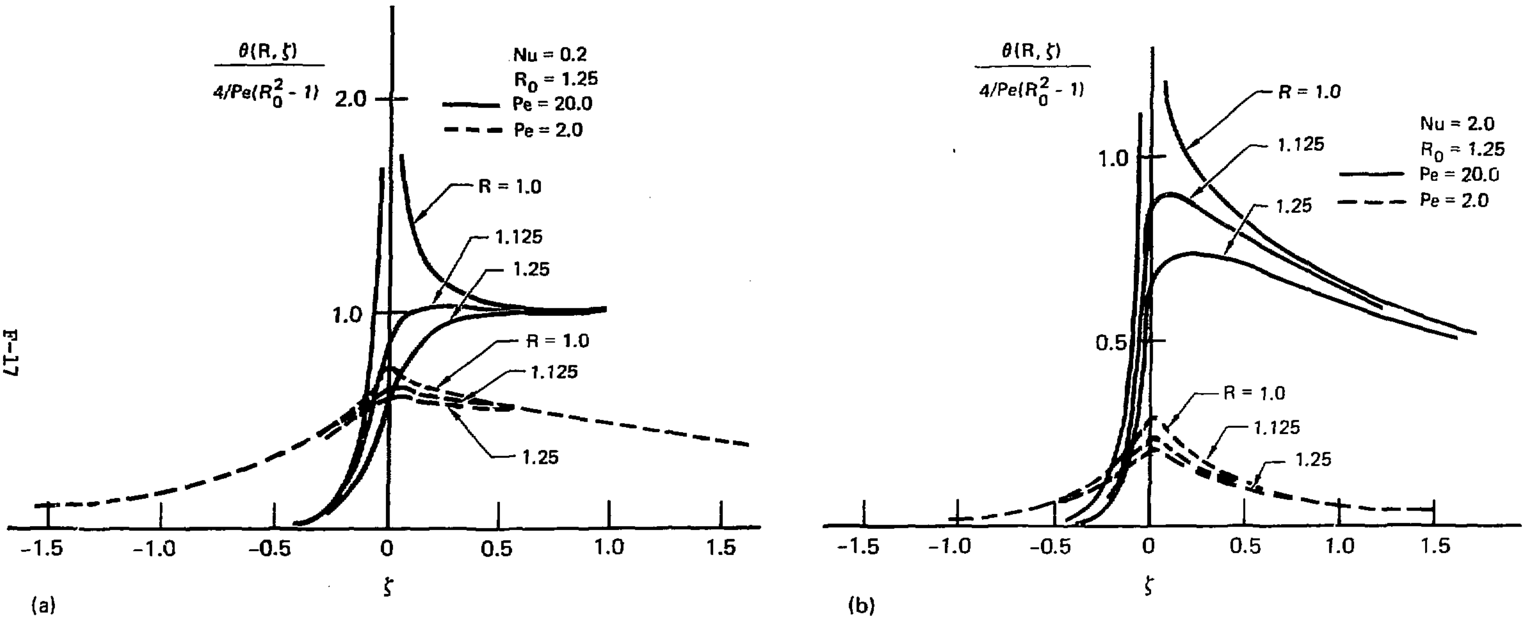


FIG. 4.5. Temperature distributions in a hollow cylinder with an inside ring heat source (case 4.1.12, source: Ref. 34). θ defined in Fig. 4.4.

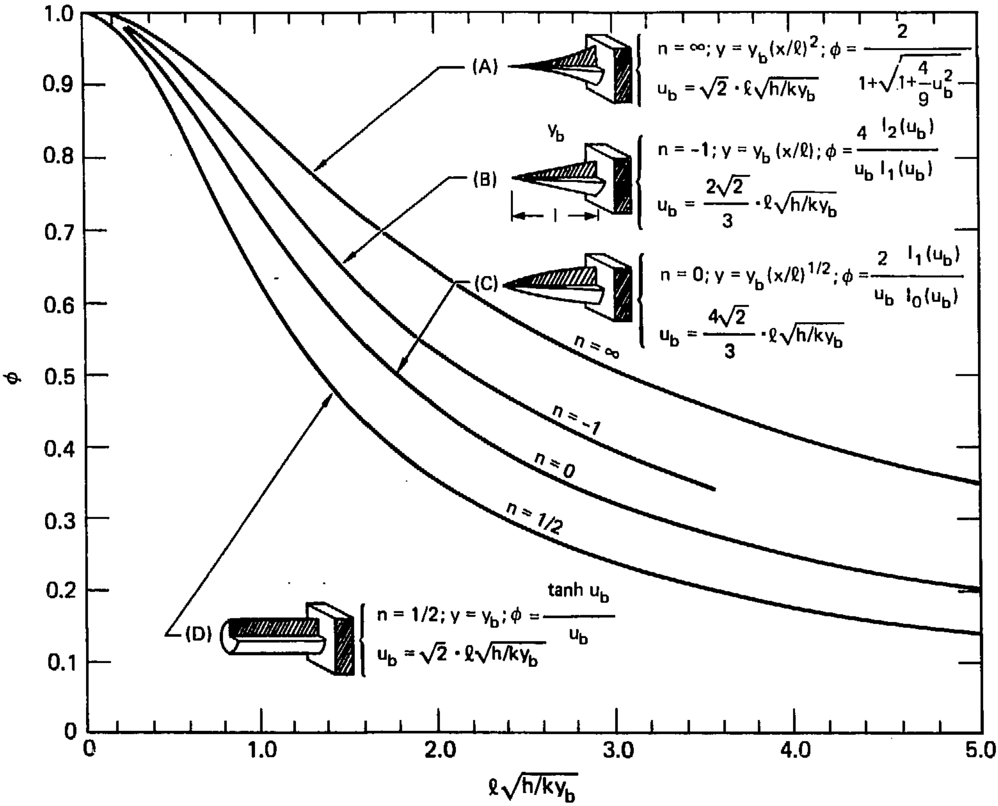


FIG. 5.1. Performance of pin fins (cases 5.1.2, 5.1.14, and 5.1.16 through 5.1.18, source: Ref. 8 and Ref. 7, p. 55, Fig. 3.14.

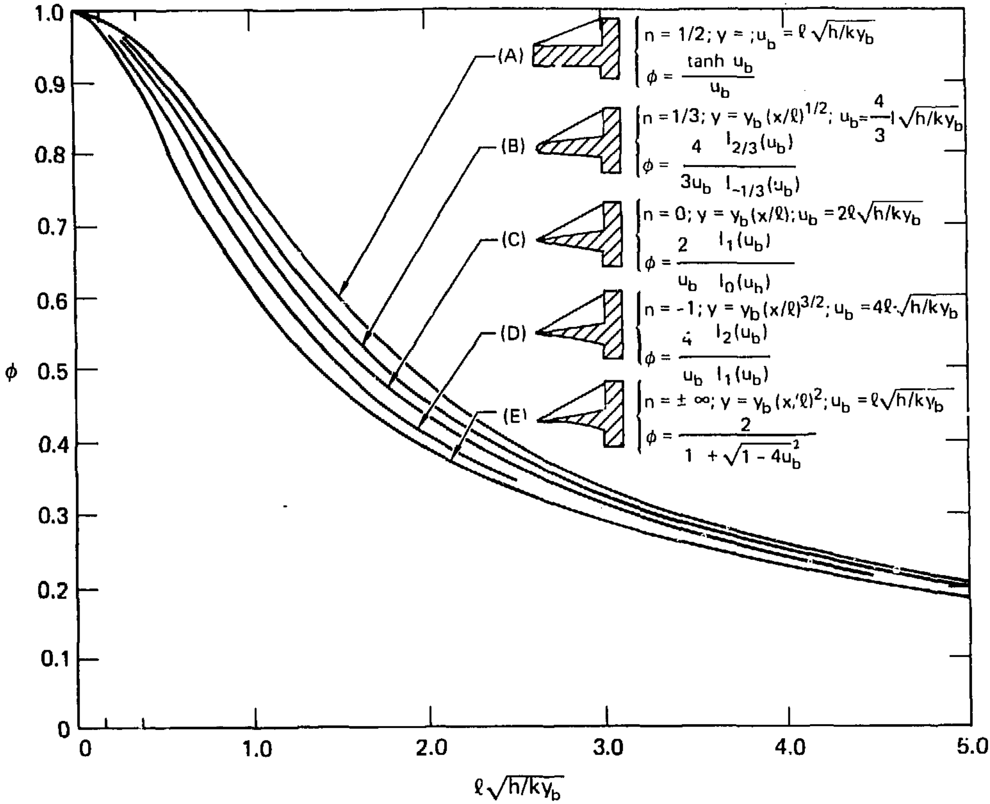


FIG. 5.2. Performance of straight fins (cases 5.1.4, 5.1.5, 5.1.7, 5.1.8, source: Ref. 8 and Ref. 7, p. 56, Fig. 3.15).

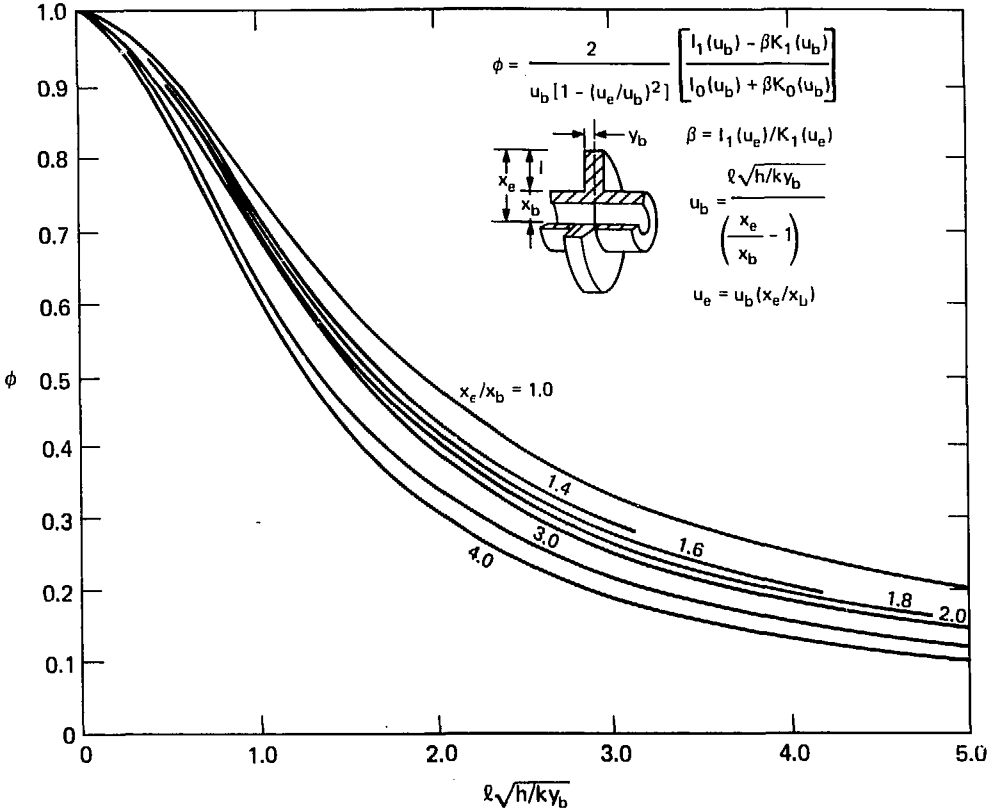


FIG. 5.3. Performance of circumferential fins of rectangular cross section (case 5.1.10, source: Ref. 8 and Ref. 7, p. 57, Fig. 3.16).

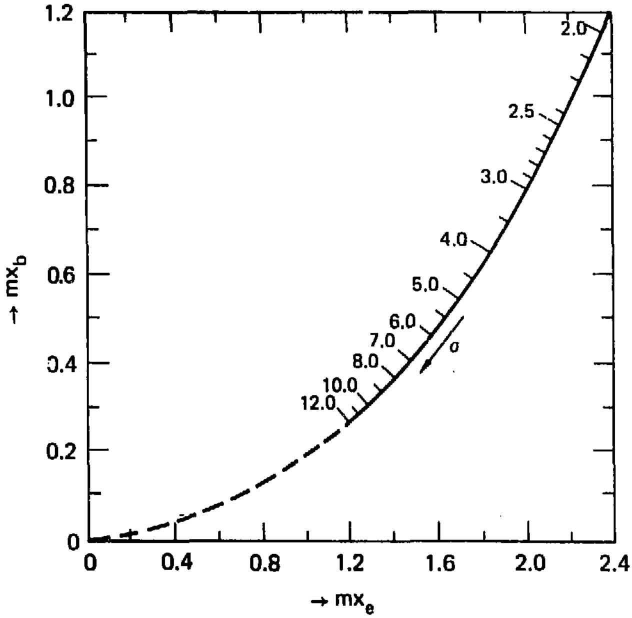


FIG. 5.4. Curve for calculating dimensions of circular fin of rectangular profile requiring least material (case 5.1.10, source: Ref. 1, p. 234, Fig. 11.11). $\sigma = Q/\pi h x_b^2 (t_0 - t_f)$.

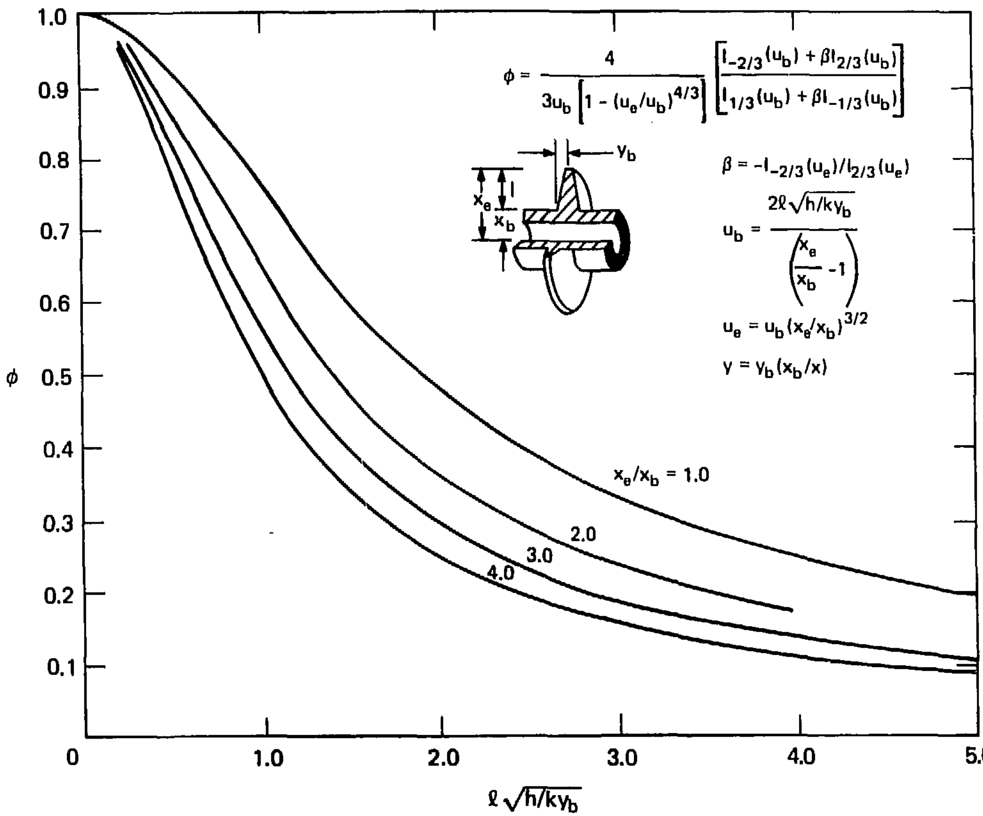


FIG. 5.5. Performance of cylindrical fins of triangular profile (case 5.1.11, source: Ref. 8 and Ref. 7, p. 58, Fig. 3.17).

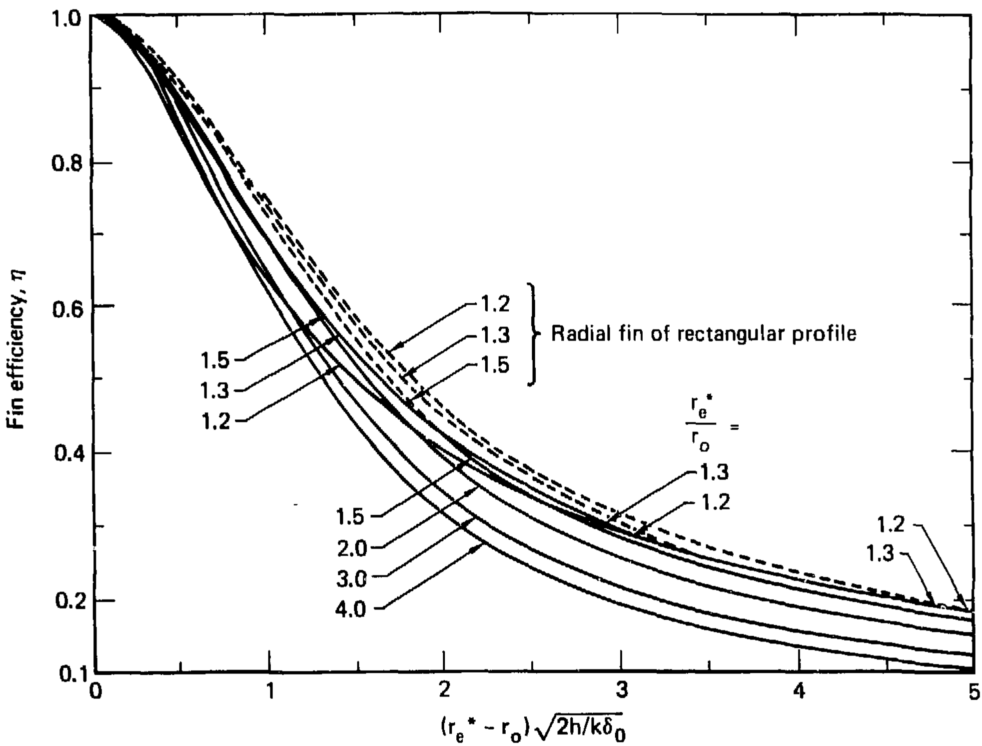


FIG. 5.6. Efficiency of an infinite fin heated by square arrayed round rods (case 5.1.19, source: Ref. 11 and Ref. 10, p. 135, Fig. 2.22). $r_e^* = (2/\sqrt{\pi})s$.

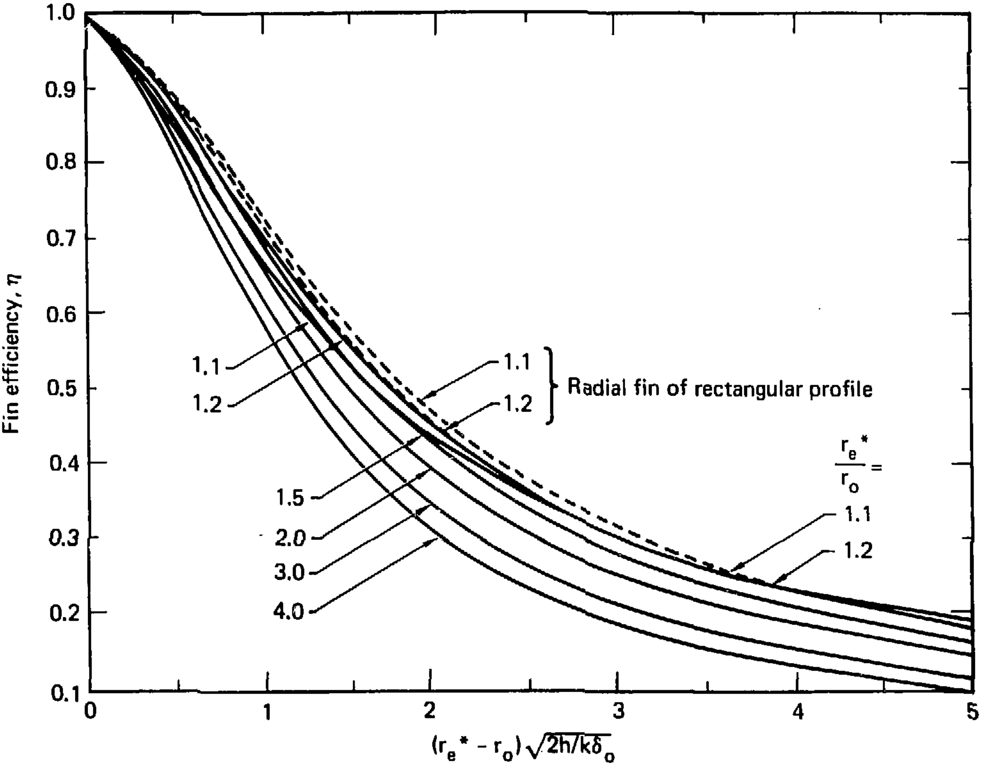


FIG. 5.7. Efficiency of an infinite fin heated by equilateral triangular arrayed round rods (case 5.1.20, source: Ref. 11 and Ref. 10, p. 136, Fig. 2.23). $r_e^* = (2\sqrt{3}/\pi)^{1/2}s$.

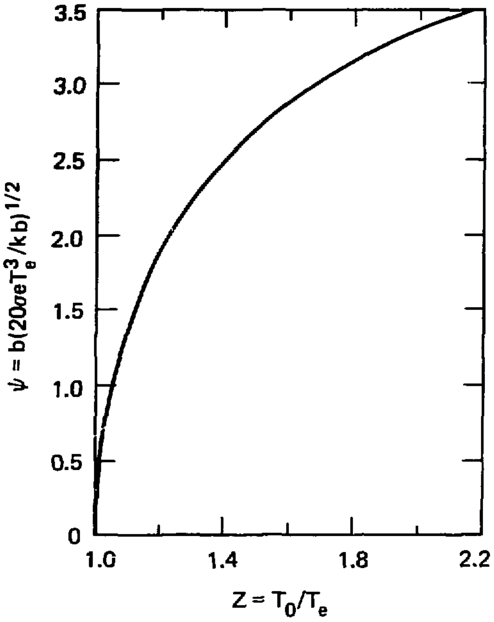


FIG. 5.8. Fin parameter as a function of Z for a straight fin radiating to free space (case 5.1.24, source: Ref. 10, p. 208, Fig. 4.3).

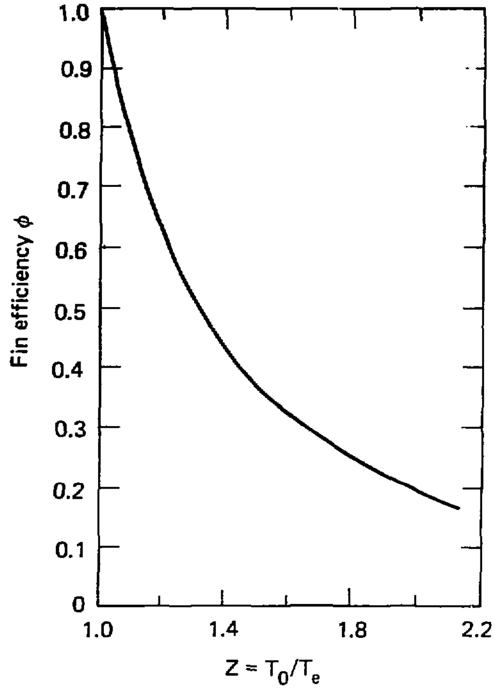


FIG. 5.9. Fin efficiency for a straight fin radiation to free space (case 5.1.24, source: Ref. 10, p. 209, Fig. 4.4).

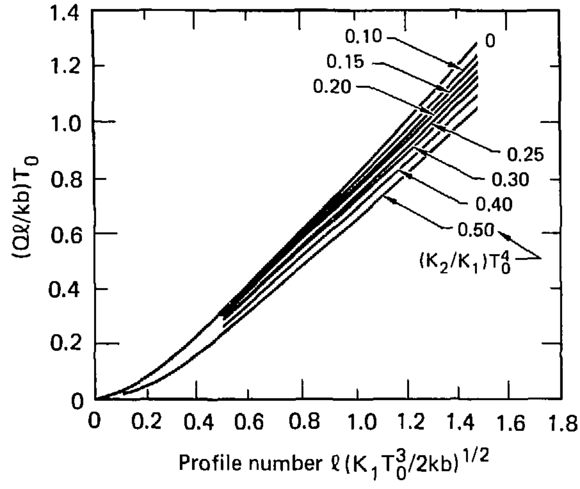


FIG. 5.10. Heat flow relationship for a straight fin of rectangular profile radiating to nonfree space (case 5.1.25, source: Ref. 10, p. 216, Fig. 4.8).

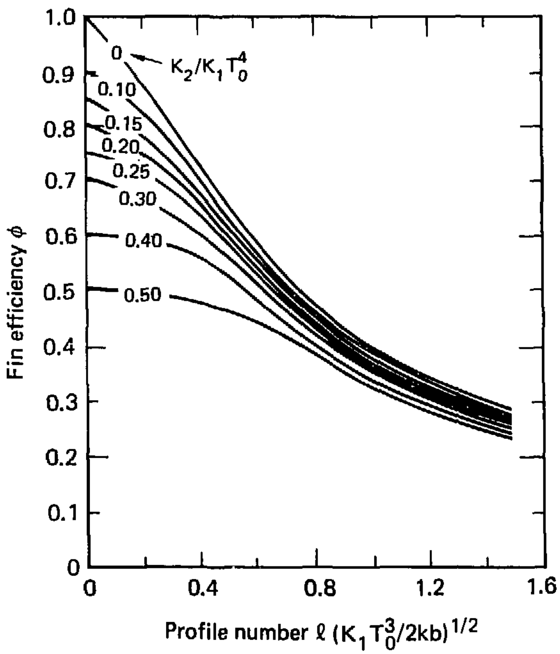


FIG. 5.11. Efficiency of a straight fin of rectangular profile radiating to nonfree space (case 5.1.25, source: Ref. 10, p. 216, Fig. 4.9).

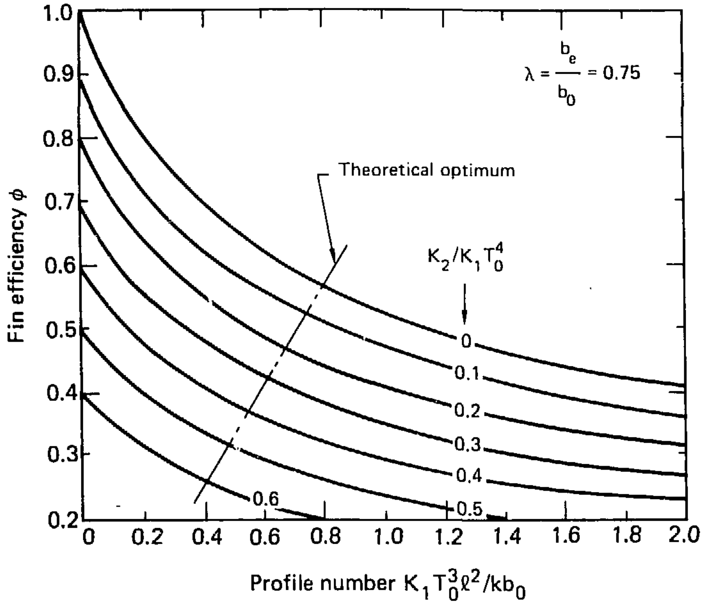


FIG. 5.12a. Fin efficiency for the longitudinal radiating fin of trapezoidal profile with a taper ratio of 0.75 (case 5.1.26, source: Ref. 10, p. 223, Fig. 4.12).

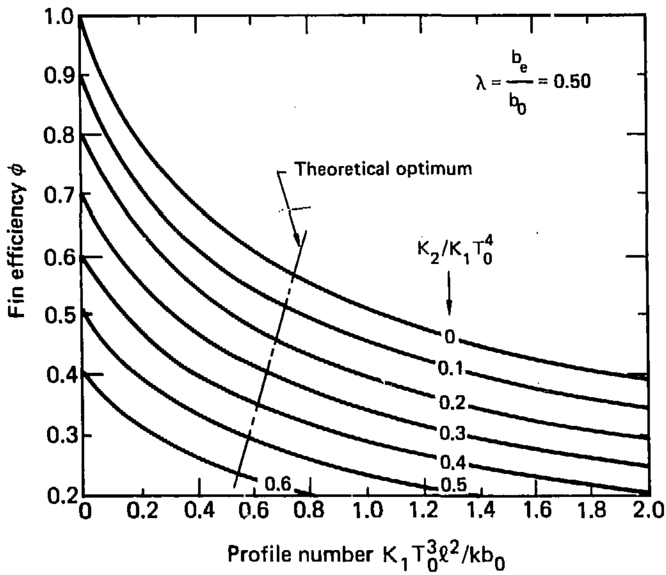


FIG. 5.12b. Fin efficiency for the longitudinal radiating fin of trapezoidal profile with a taper ratio of 0.50 (case 5.1.26, source: Ref. 10, p. 224, Fig. 4.13).

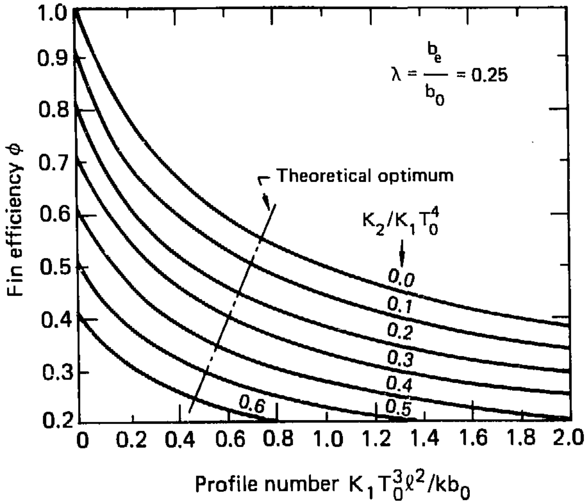


FIG. 5.12c. Fin efficiency for the longitudinal radiating fin of trapezoidal profile with a taper ratio of 0.25 (case 5.1.26, source: Ref. 10, p. 224, Fig. 4.14).

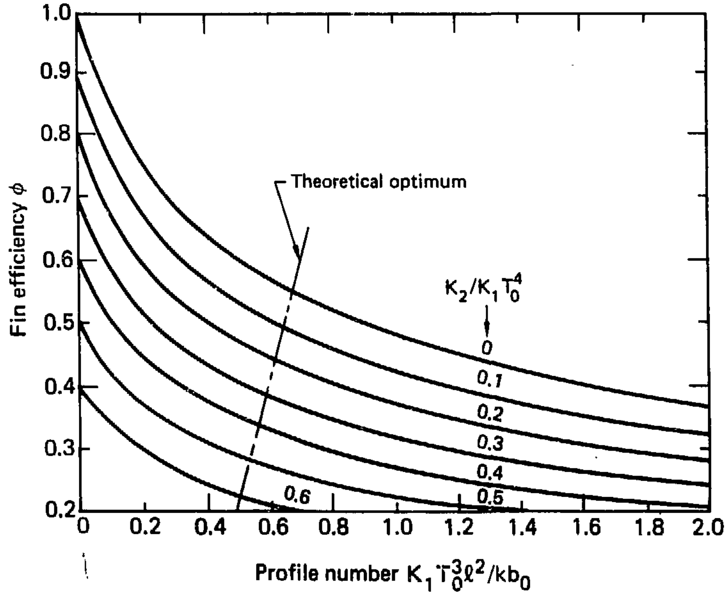


FIG. 5.12d. Fin efficiency for the longitudinal radiating fin of triangular profile (case 5.1.26, source: Ref. 10, p. 225, Fig. 4.15).

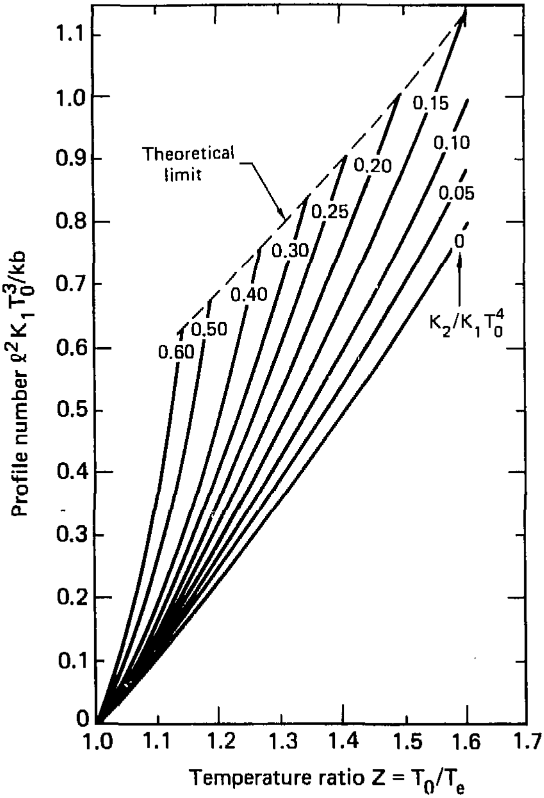


FIG. 5.13. Profile number of constant-temperature-gradient longitudinal radiating fin as a function of base-to-tip temperature ratio (case 5.1.28, source: Ref. 10, p. 233, Fig. 4.18).

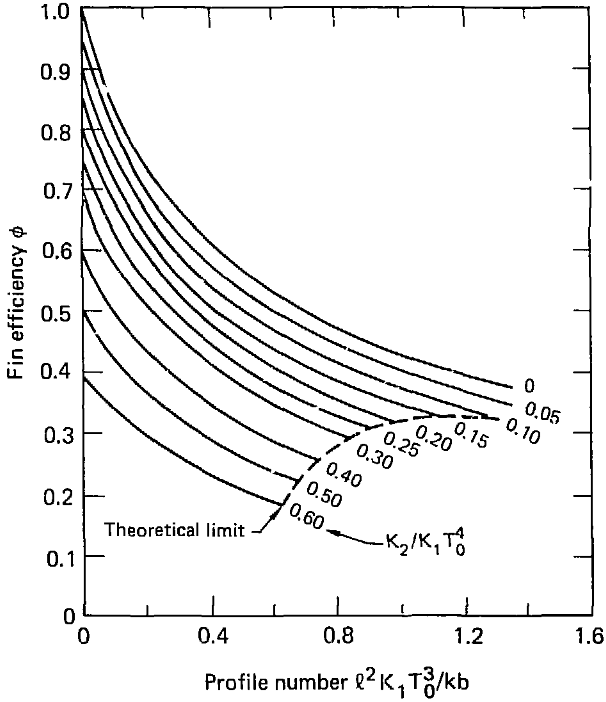


FIG. 5.14. Efficiency of constant-temperature-gradient longitudinal radiating fin as a function of profile number (case 5.1.28, source: Ref. 10, p. 232, Fig. 4.17).

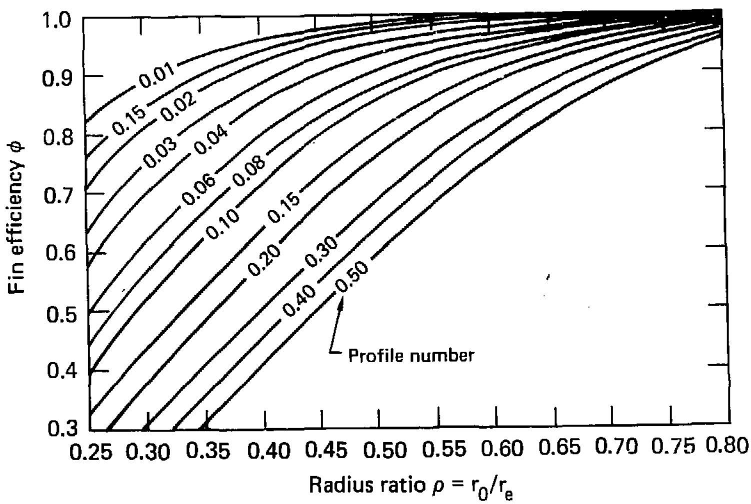


FIG. 5.15a. Radiation fin efficiency of radial fin of rectangular profile. Taper ratio, $\lambda = 1.00$; environmental factor, $K_2/K_1T_0^4 = 0.00$ (case 5.1.29, source: Ref. 10, p. 250, Fig. 4.24).

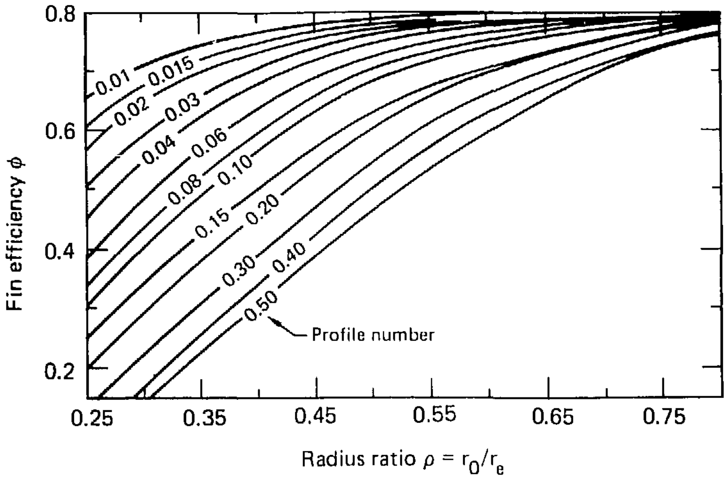


FIG. 5.15b. Radiation fin efficiency of radial fin of rectangular profile. Taper ratio, $\lambda = 1.00$; environmental factor, $K_2/K_1T_0^4 = 0.20$ (case 5.1.29, source: Ref. 10, p. 251, Fig. 4.25).

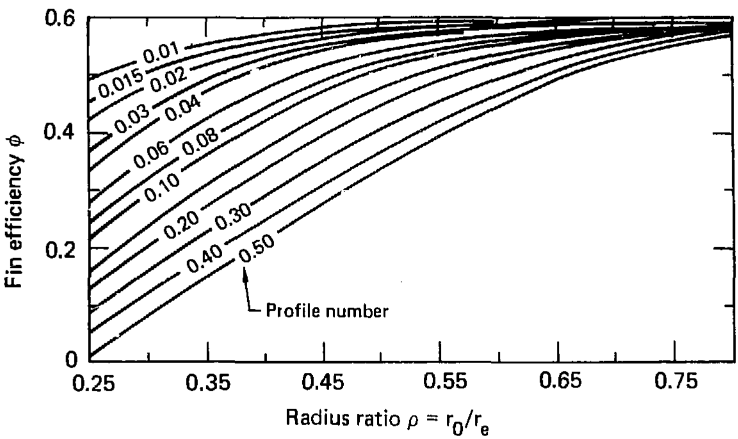


FIG. 5.15c. Radiation fin efficiency of radial fin of rectangular profile. Taper ratio, $\lambda = 1.00$; environmental factor, $K_2/K_1T_0^4 = 0.40$ (case 5.1.29, source: Ref. 10, p. 251, Fig. 4.26).

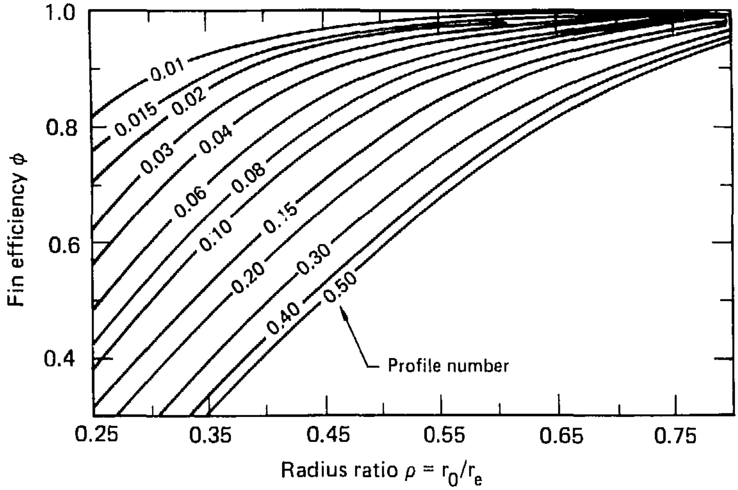


FIG. 5.15d. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.75$; environmental factor, $K_2/K_1T_0^4 = 0.00$ (case 5.1.29, source: Ref. 10, p. 252, Fig. 4.27).

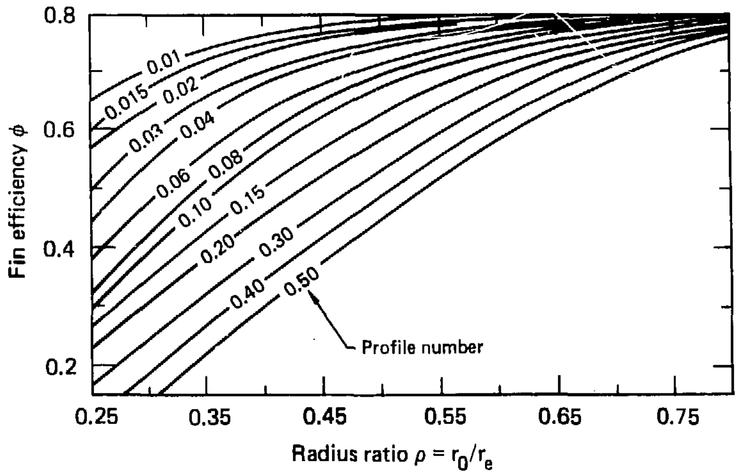


FIG. 5.15e. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.75$; environmental factor, $K_2/K_1T_0^4 = 0.20$ (case 5.1.29, source: Ref. 10, p. 252, Fig. 4.28).

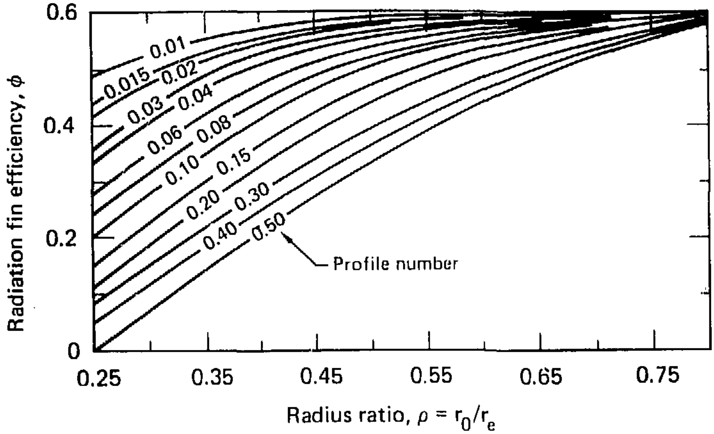


FIG. 5.15f. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.75$; environmental factor, $K_2/K_1T_0^4 = 0.40$ (case 5.1.29, source: Ref. 10, p. 253, Fig. 4.29).

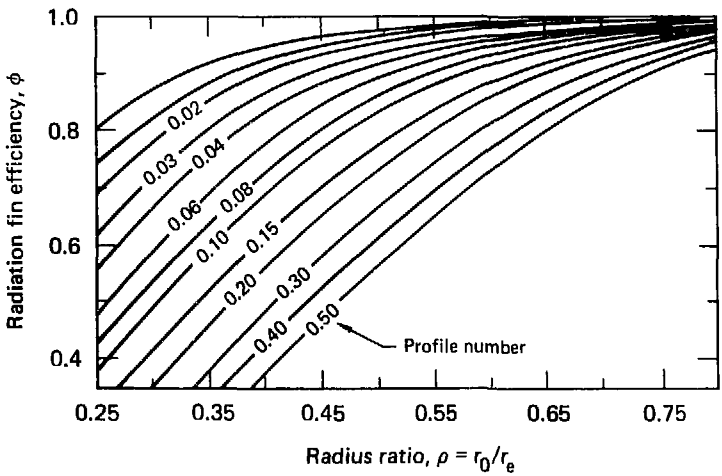


FIG. 5.15g. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.50$; environmental factor, $K_2/K_1T_0^4 = 0.00$ (case 5.1.29, source: Ref. 10, p. 253, Fig. 4.30).

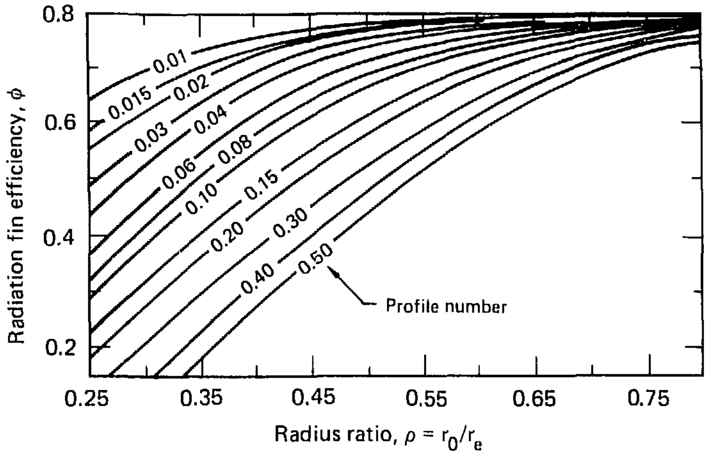


FIG. 5.15h. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.50$; environmental factor, $K_2/K_1T_0^4 = 0.20$ (case 5.1.29, source: Ref. 10, p. 254, Fig. 4.31).

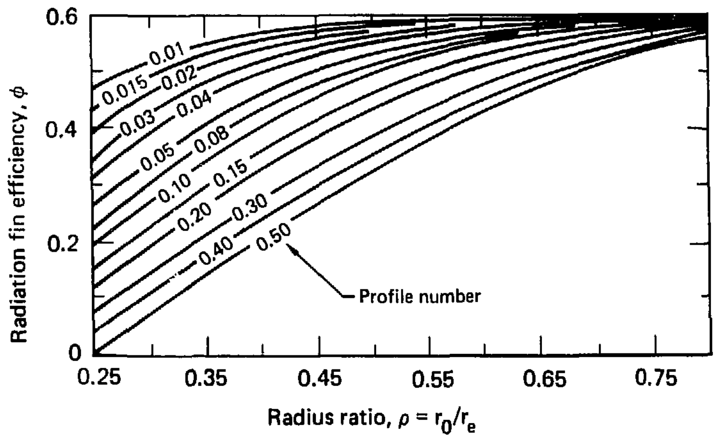


FIG. 5.15i. Radiation fin efficiency of radial fin of trapezoidal profile. Taper ratio, $\lambda = 0.50$; environmental factor, $K_2/K_1T_0^4 = 0.40$ (case 5.1.29, source: Ref. 10, p. 254, Fig. 4.32).

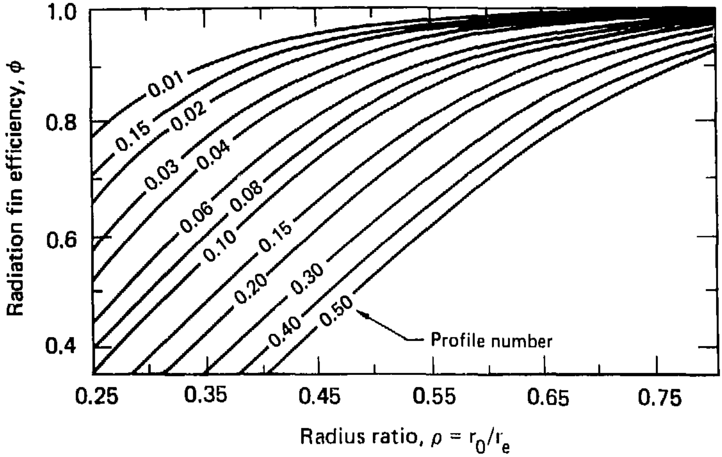


FIG. 5.15j. Radiation fin efficiency of radial fin of triangular profile. Taper ratio, λ = 0.00; environmental factor, $K_2/K_1T_0^4$ = 0.00 (case 5.1.29, source: Ref. 10, p. 255, Fig. 4.33).

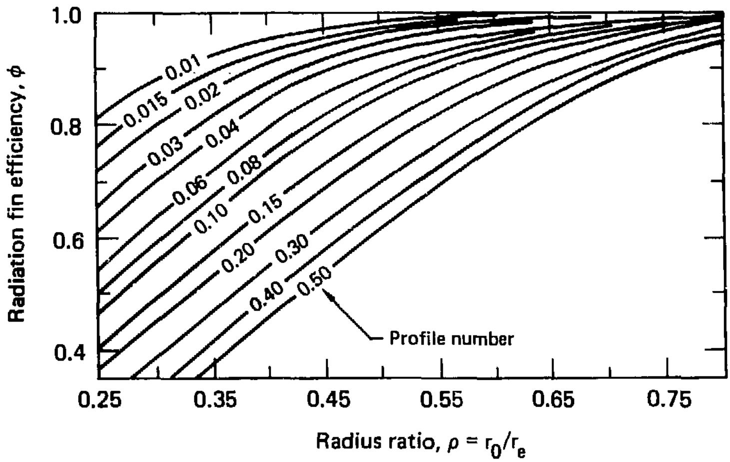


FIG. 5.15k. Radiation fin efficiency of radial fin of triangular profile. Taper ratio, λ = 0.00; environmental factor, $K_2/K_1T_0^4$ = 0.20 (case 5.1.29, source: Ref. 10, p. 255, Fig. 4.34).

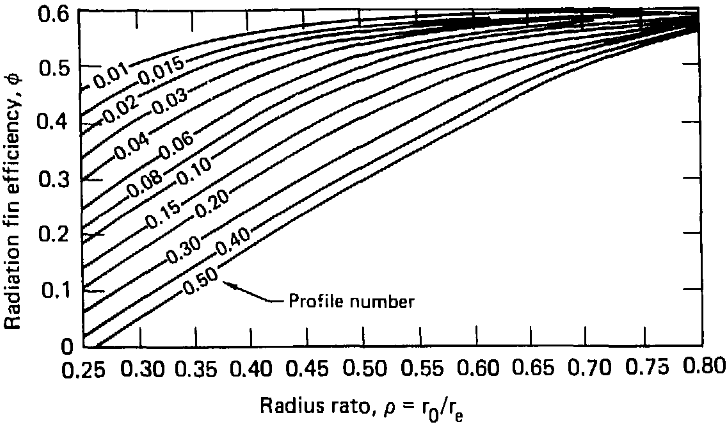


FIG. 5.15ℓ. Radiation fin efficiency of radial fin of triangular profile. Taper ratio, $\lambda = 0.00$; environmental factor, $K_2/K_1T_0^4 = 0.40$ (case 5.1.29, source: Ref. 10, p. 256, Fig. 4.35).

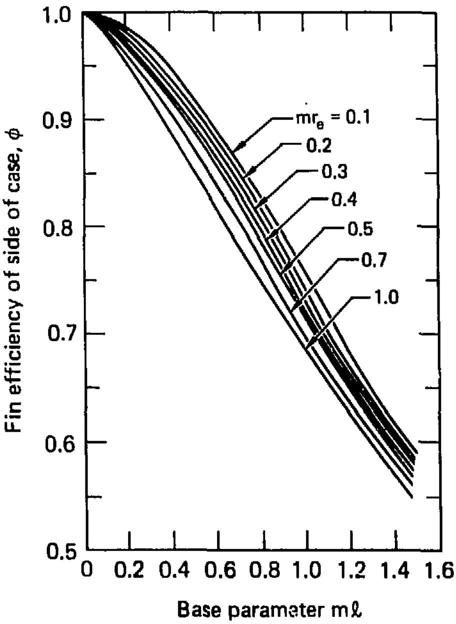


FIG. 5.16a. Efficiency of the side of a capped cylinder fin (case 5.1.30, source: Ref. 10, p. 276, Fig. 5.6).

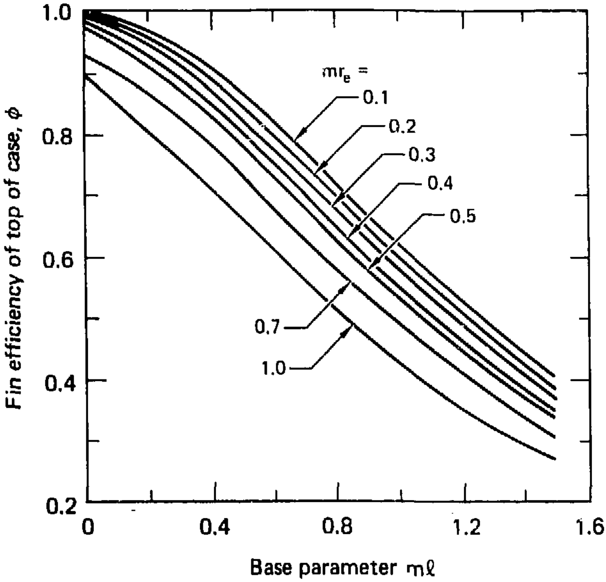


FIG. 5.16b. Efficiency of the top of a capped cylinder fin (case 5.1.30, source: Ref. 10, p. 277, Fig. 5.7).

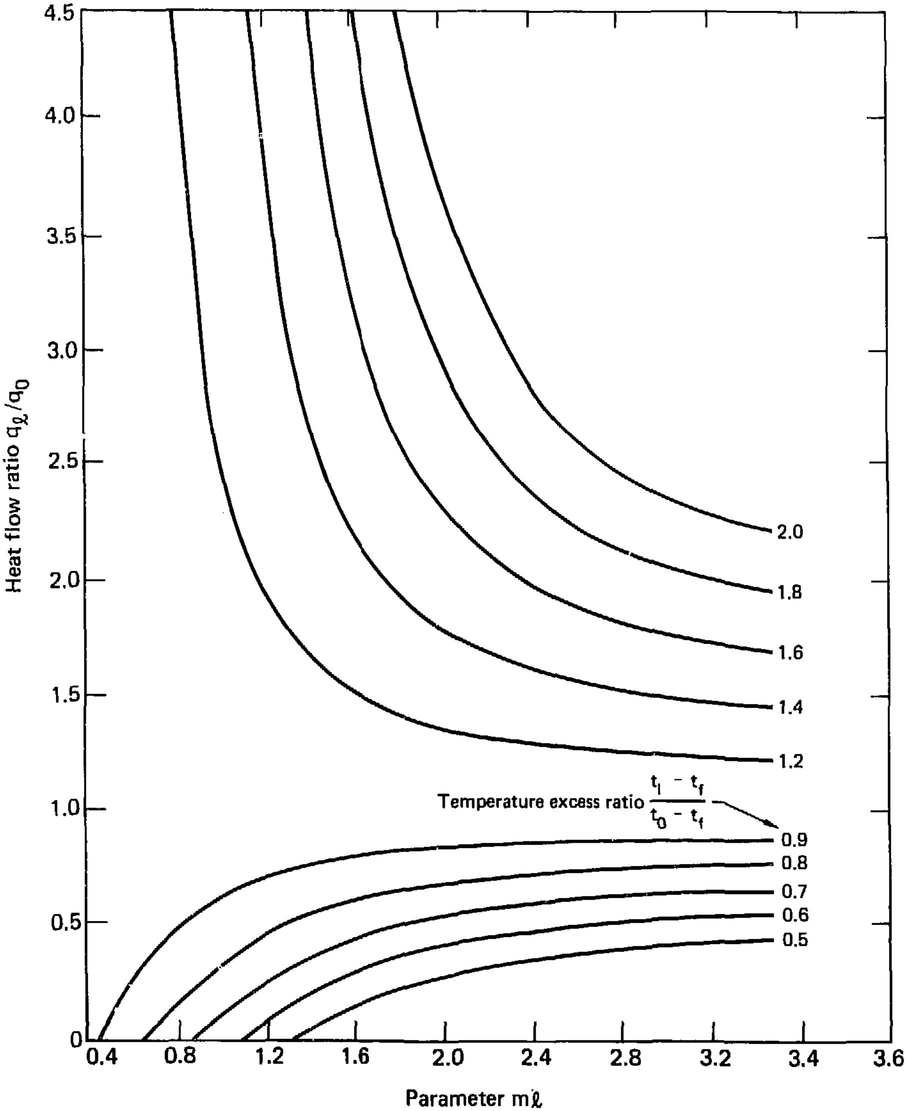


FIG. 5.17. Heat flow ratio q_ℓ/q_0 as a function of mb and temperature excess ratio $(t_1 - t_f)/(t_0 - \tilde{f}_f)$ for the doubly heated rectangular fin (case 5.1.32, source: Ref. 10, p. 410, Fig. 8.10).

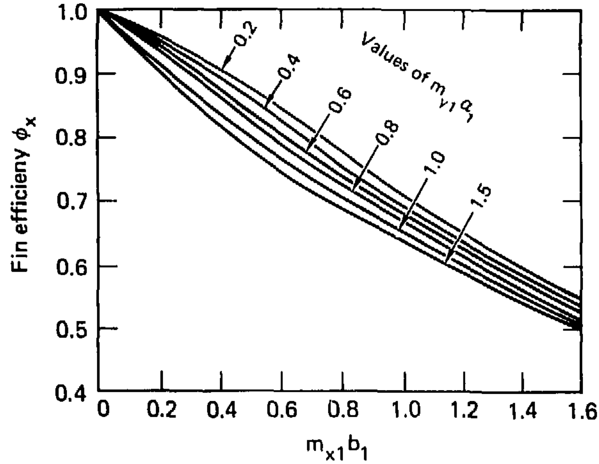


FIG. 5.18a. Efficiency of the vertical section of a straight, single Tee fin for u = v (case 5.1.32, source: Ref. 10, p. 398, Fig. 8.4).

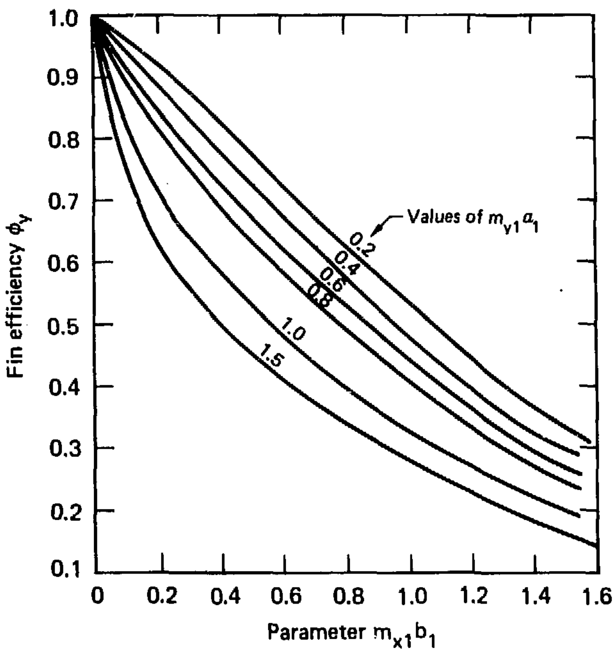


FIG. 5.18b. Efficiency of the horizontal section of a straight, single Tee fin for u = v (case 5.1.32, source: Ref. 10, p. 399, Fig. 8.5).

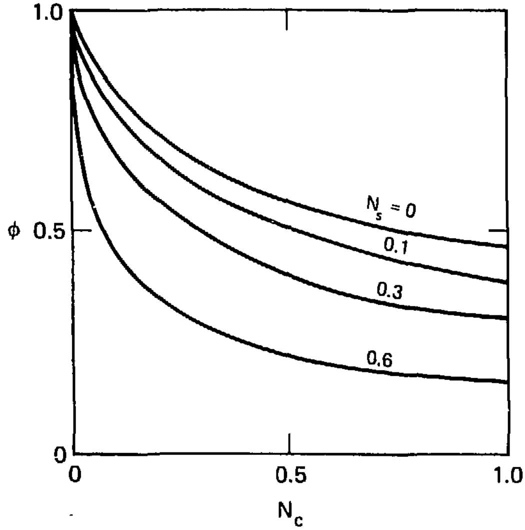


FIG. 5.19. Effectiveness of the concave parabolic fin radiating to non-free space (case 5.1.37, source: Ref. 31).

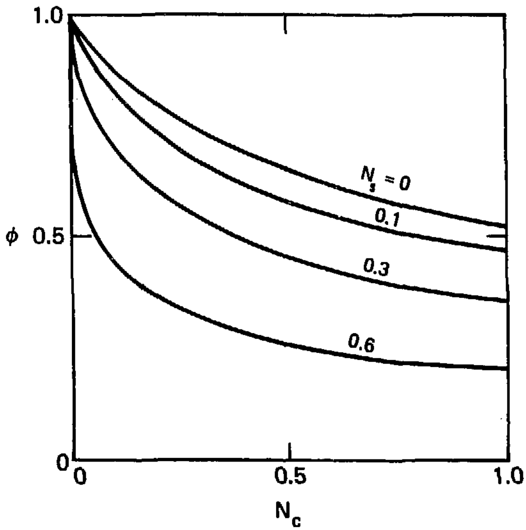


FIG. 5.20. Effectiveness of the convex parabolic fin radiating to non-free space (case 5.1.38, source: Ref. 31).

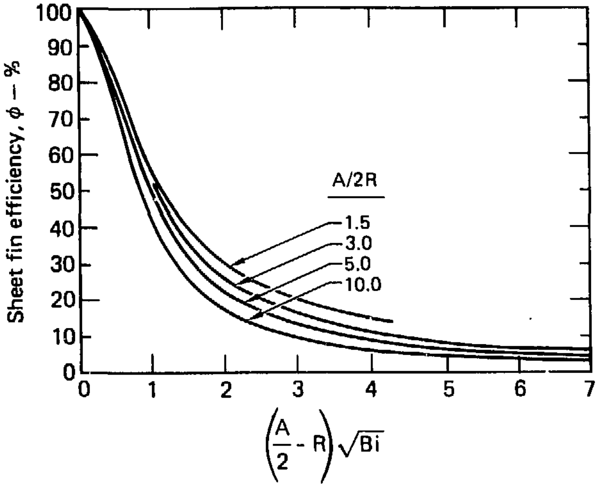


FIG. 5.21. Effectiveness of sheet fin with square array tubes (case 5.1.40, source: Ref. 83, p. 294, Fig. 2).

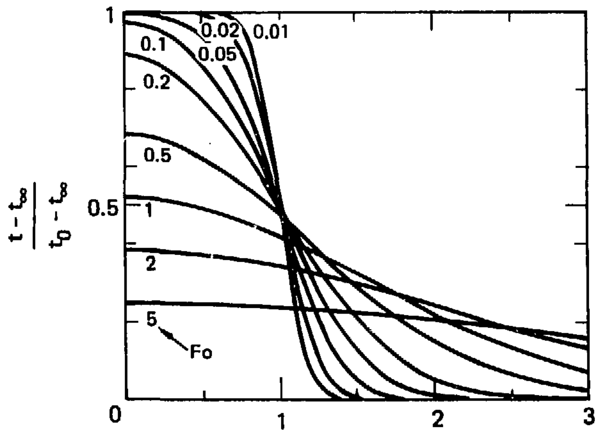


FIG. 6.1. Temperatures in an infinite region of which the region $|x| < b$ is initially at temperature t_0 (case 6.1.1, source: Ref. 9, p. 55, Fig. 4a).

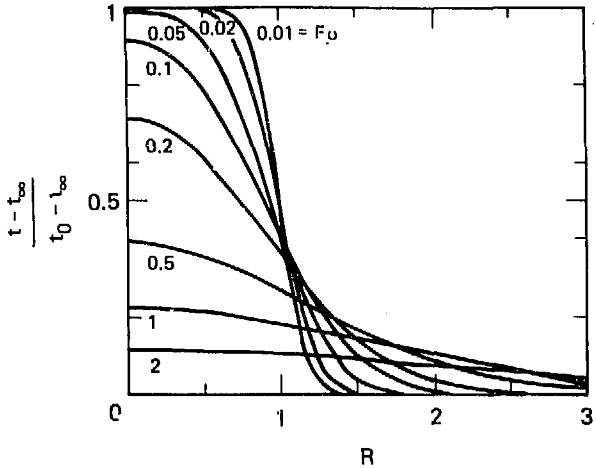


FIG. 6.2. Temperatures in an infinite region of which the region $r < r_0$ is initially at temperature t_0 (case 6.1.4, source: Ref. 9, p. 55, Fig. 4b).

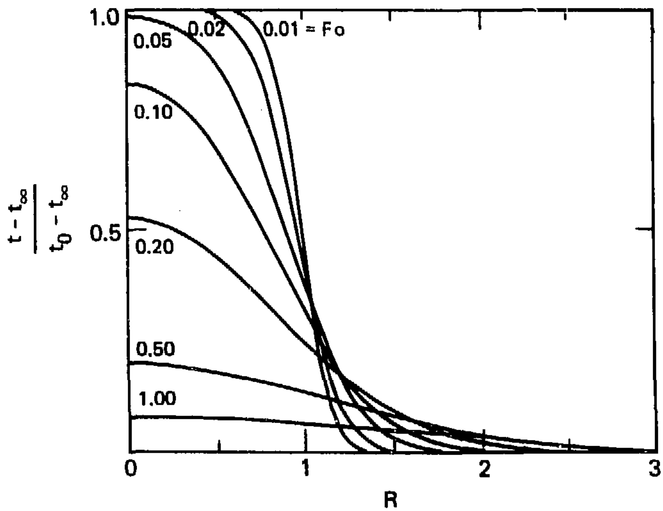


FIG. 6.3. Temperatures in an infinite region of which the region $r < r_0$ is initially at temperature t_0 (case 6.1.5, source: Ref. 9, p. 55, Fig. 4c).

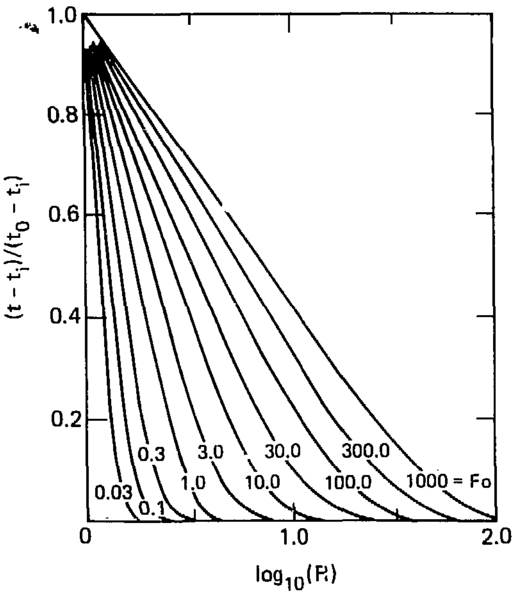


FIG. 6.4. Temperatures in an infinite region with steady temperature t_0 on the surface $r = r_0$ (case 6.1.18, source: Ref. 9, p. 337, Fig. 41).

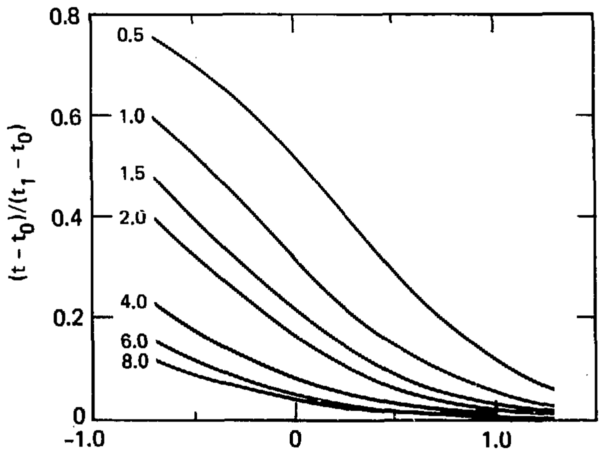


FIG. 6.5. Temperature in a cylinder of infinite conductivity, initially at temperature t_1, in an infinite medium inditially at t_0. Numbers on the curves are values of $2\rho_0 c_0/\rho_1 c_1$ (case 6.1.21, source: Ref. 9, p. 342, Fig. 45).

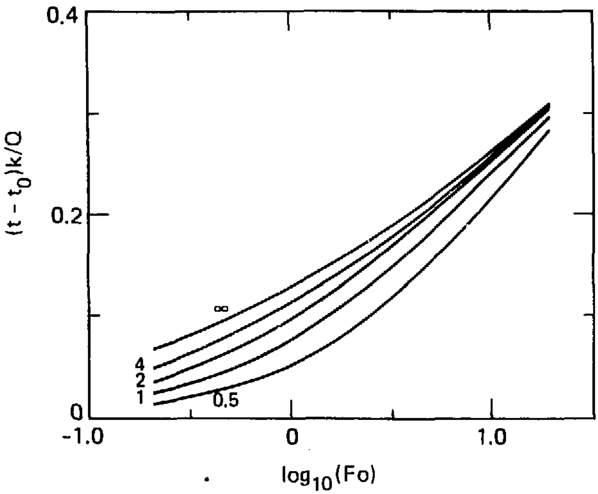


FIG. 6.6. Temperature in a cylinder of infinite conductivity, initially at temperature t_0, in an infinite medium. Numbers on the curves are values of $2\rho_0 c_0/\rho_1 c_1$ (case 6.1.22, source: ref. 9, p. 343, Fig. 46).

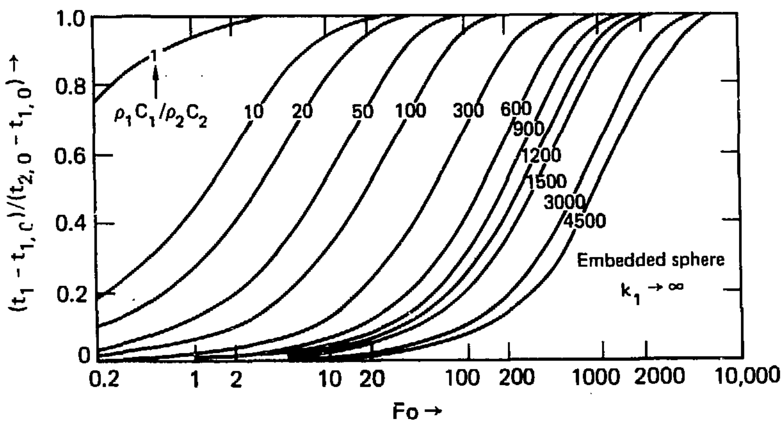


FIG. 6.7. Temperature response of solid sphere $0 \leq r \leq r_1$, with $k_1 \rightarrow \infty$ and initially at $t_{1,0}$, embedded in an infinite solid ($r > r_1$) initially at $t_{2,0}$ (case 6.1.26, source: Ref. 19, pp. 3-64, Fig. 37).

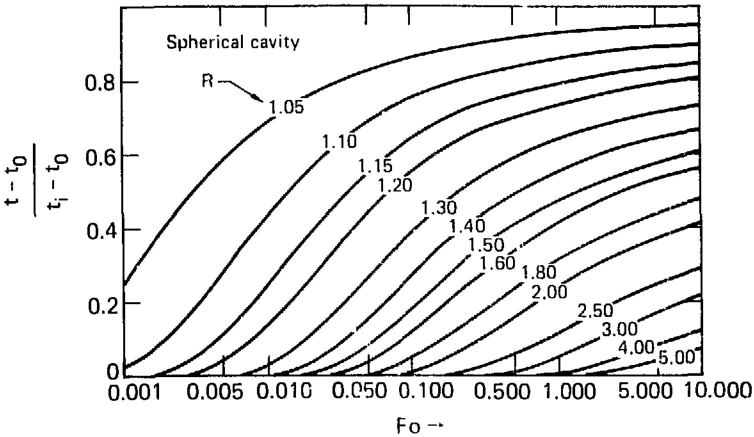


FIG. 6.7. Temperature response of an infinite solid with a constant spherical surface temperature of t_0 (case 6.1.13, source: Ref 74, p. 430, Fig. 10.9).

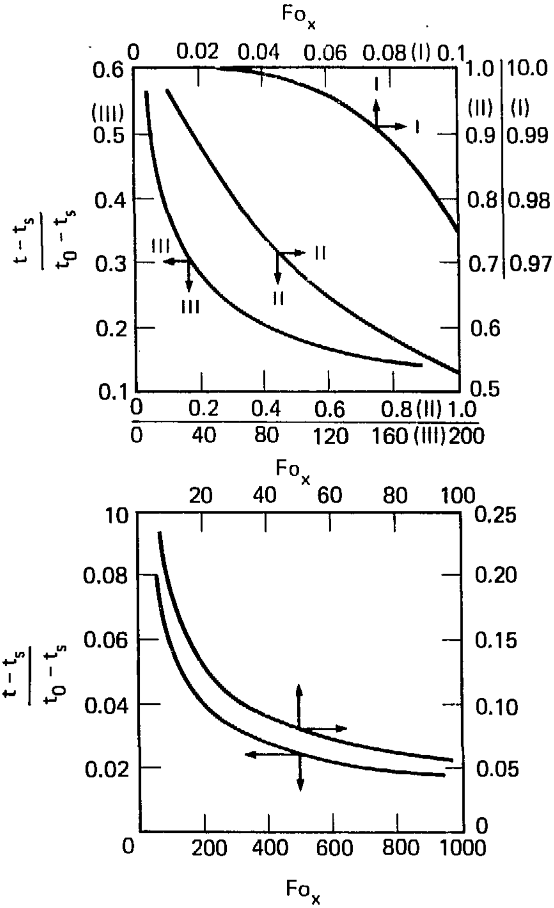


FIG. 7.1. Temperature distribution in the semi-infinite solid having a steady surface temperature (case 7.1.1 b = 0, source: Ref. 74, p. 95, Figs. 4.4 and 4.5).

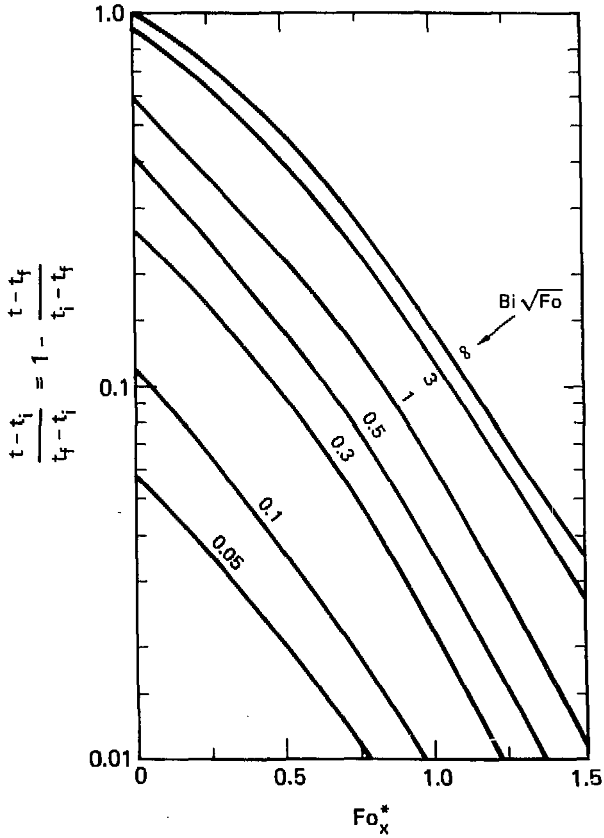


FIG. 7.2a. Temperature distribution in the semi-infinite solid with convection boundary condition (case 7.1.3, source: Ref. 5, p. 82, Fig. 4.6).

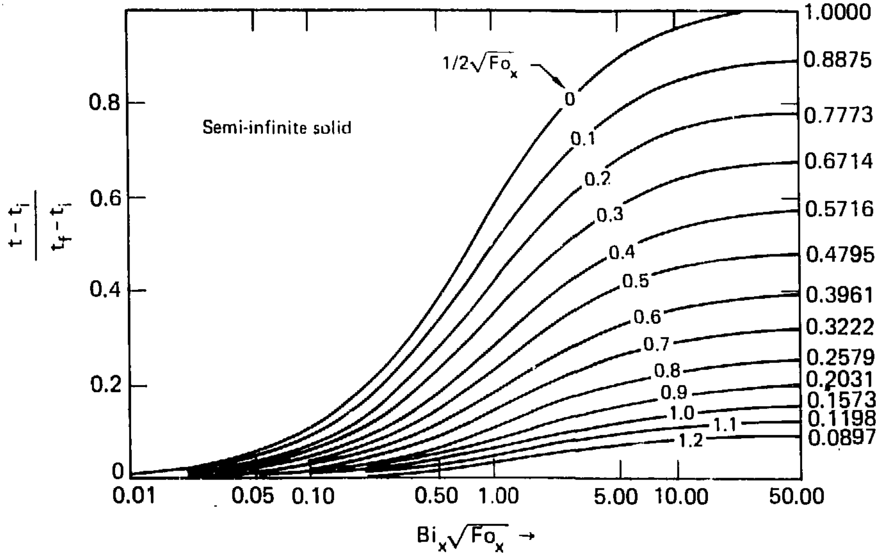


FIG. 7.2b. The dimensionless excess temperature vs the number $Bi_x(Fo_x)^{1/2}$ and various Fourier numbers for semi-infinite solid with a convection boundary (case 7.1.3, source: Ref. 74, p. 207, Fig. 6.2).

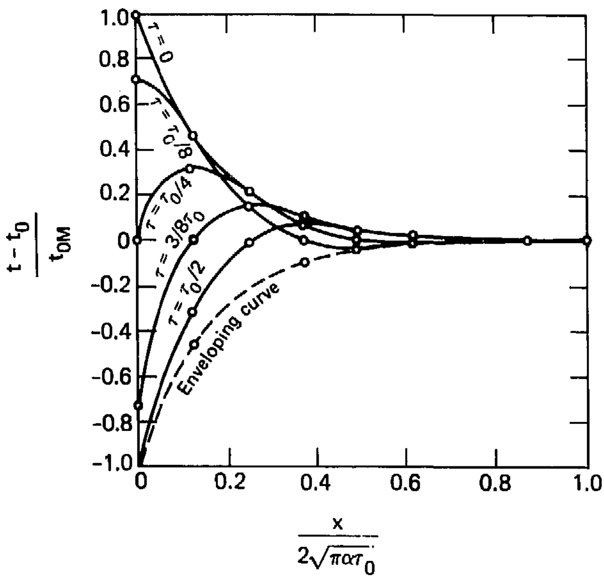


FIG. 7.3. Temperature distribution in a semi-infinite region when the surface temperature is harmonic (case 7.1.16, source: Ref. 7, p. 99, Fig. 4-19).

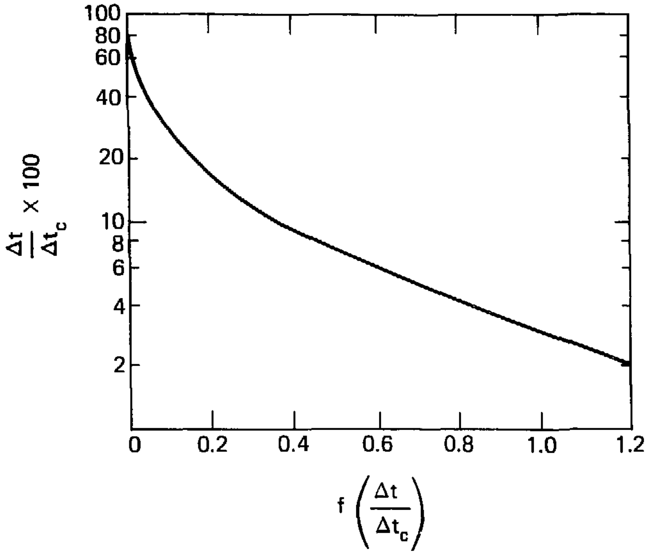


FIG. 7.4. Function $f[(t-t_f)/(t_c-t_f)]$ (case 7.1.22, source: Ref. 4, p.84, Fig. 4.15).

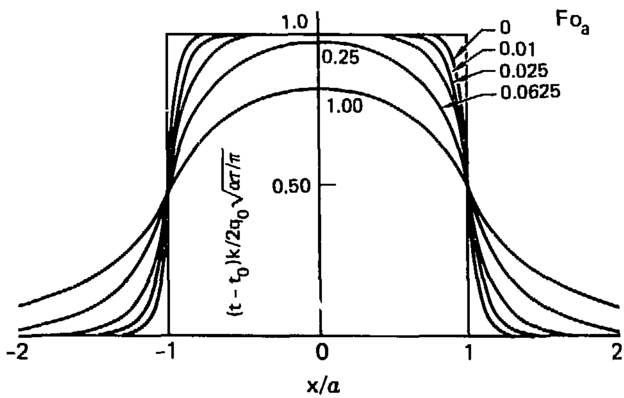


FIG. 7.5. Temperature distribution across a heated strip of width 2a on the surface of a semi-infinite solid (case 7.1.24, source: Ref. 9, p. 265, Fig. 33).

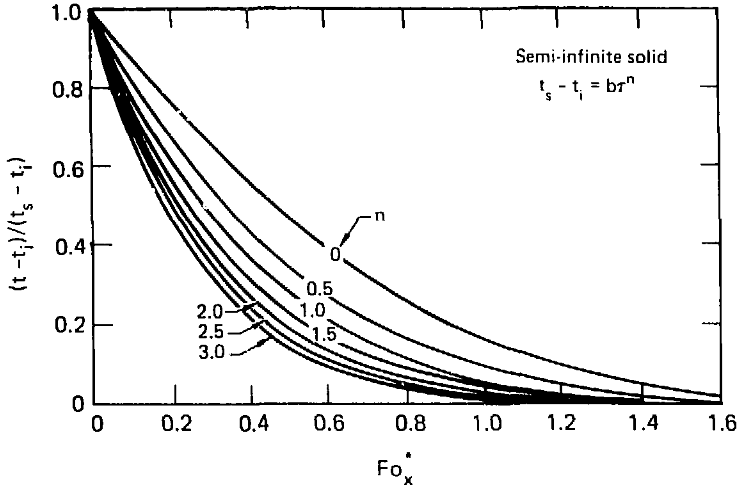


FIG. 7.6. Temperature response of semi-infinite solid ($x \geq 0$) with surface temperature t_s suddenly increasing as power function of time, $t_s = t_i + b\tau^n$, (case 7.1.9, source: Ref. 19, pp. 3-66, Fig. 39).

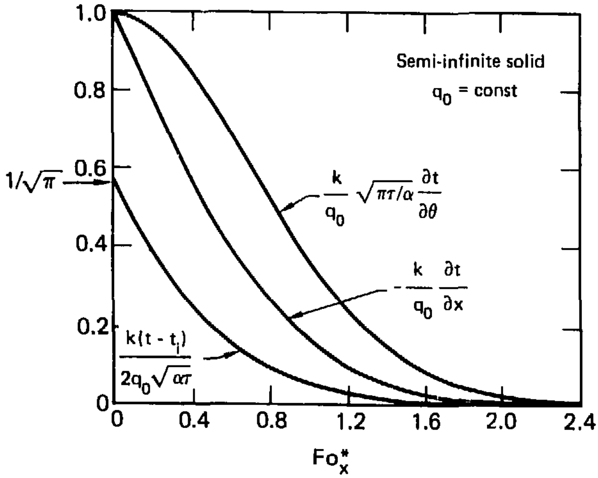


FIG. 7.7. Temperature response, temperature gradient, and heating rate in a semi-infinite solid after exposure to a constant surface heat flux q_0 (case 7.15, source: Ref. 19, pp. 3-81, Fig. 49).

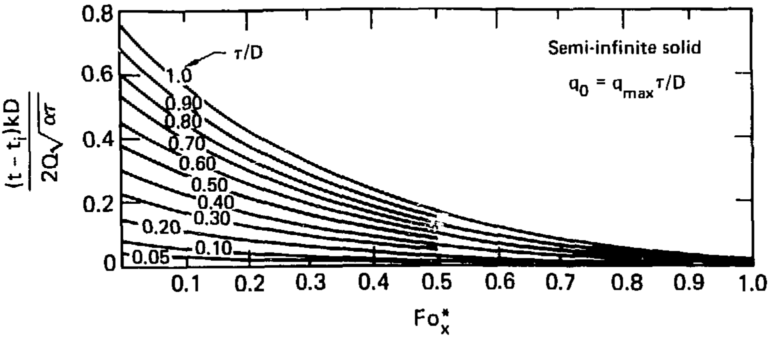


FIG. 7.8. Temperature response of a semi-infinite solid after sudden exposure to a linearly increasing surface heat flux for a duration D, $Q = D\ q_{max}/2$, $q_0 = q_{max}\ \tau/D$ (case 7.1.14, source: Ref. 19, pp. 3-82, Fig. 50).

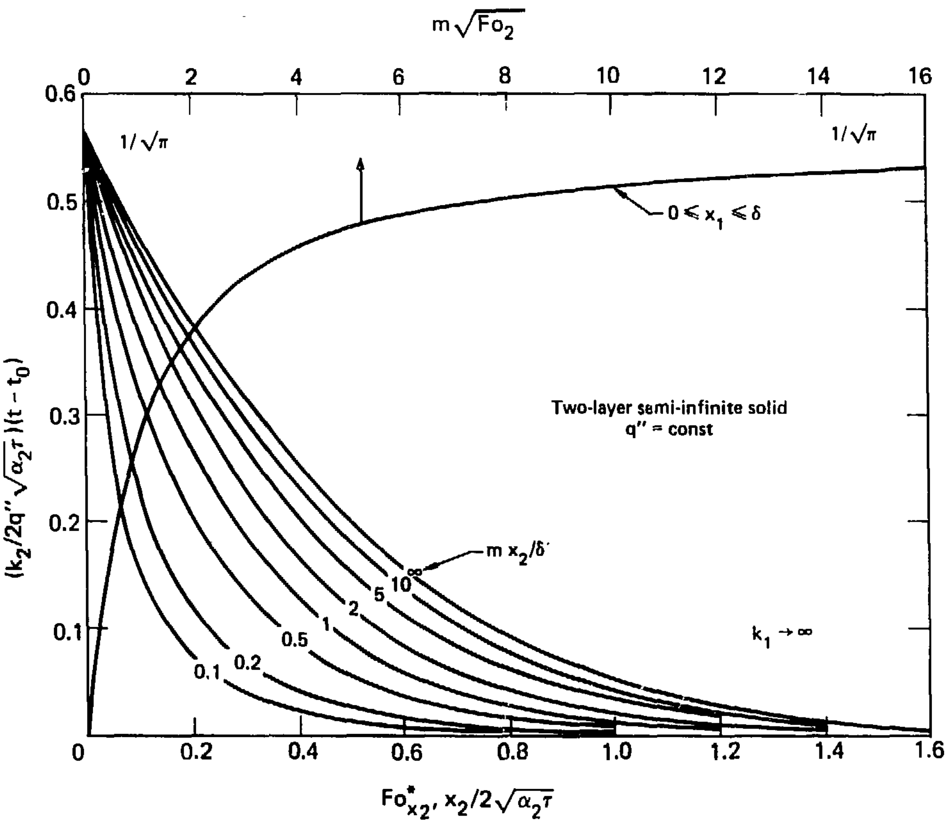


FIG. 7.9. Temperature response of an infinite conductivity plate and semi-infinite solid composite with a constant heat flux at $x_1 = 0$ (case 7.1.30, source: Ref. 19, pp. 3-84, Fig. 51).

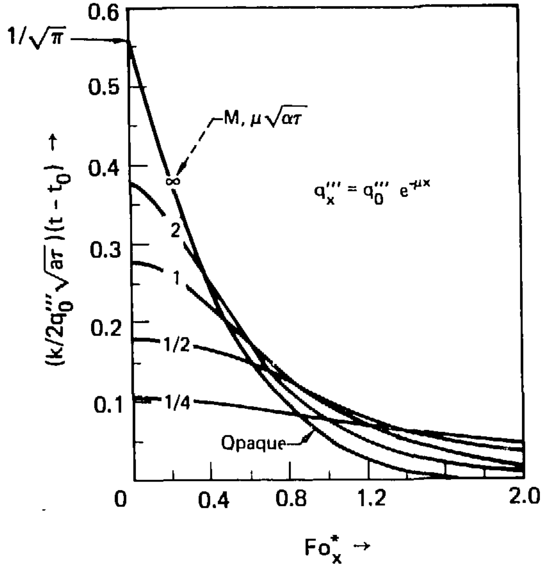


FIG. 7.10. Temperature distributions in a semi-infinite solid with exponential heating (case 7.2.9, source: Ref. 19, pp. 3-85, Fig. 52).

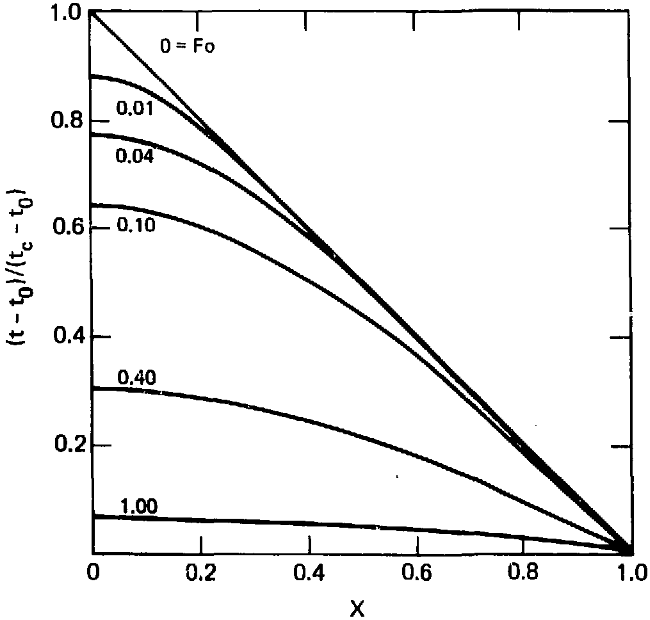


FIG. 8.1. Temperatures in a slab with a linear initial temperature distribution (case 8.1.3, source: Ref. 9, p. 97, Fig. 10).

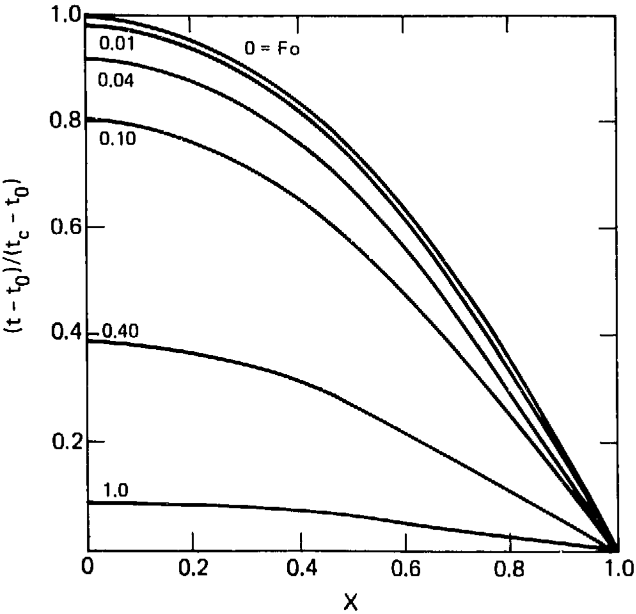


FIG. 8.2. Temperatures in a slab with a parabolic initial temperature distribution (case 8.1.4, source: Ref. 9, p. 98, Fig. 10).

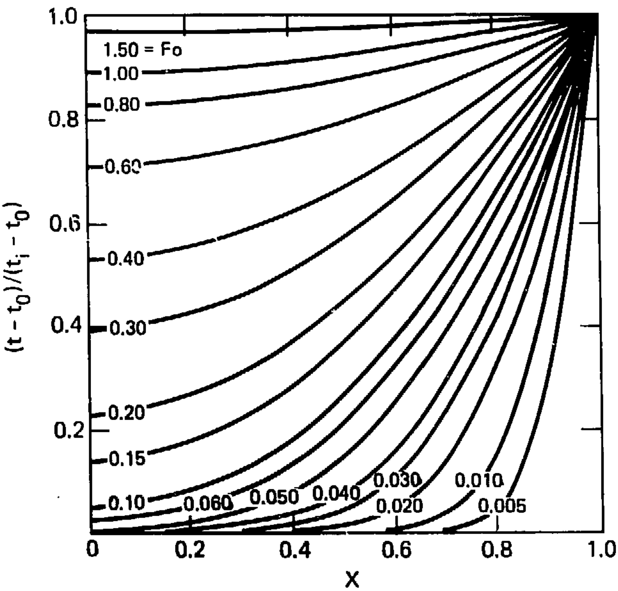


FIG. 8.3. Temperatures in the infinite plate with a constant intial temperature and steady surface temperature (case 8.1.6, source: Ref. 9, p. 101, Fig. 11).

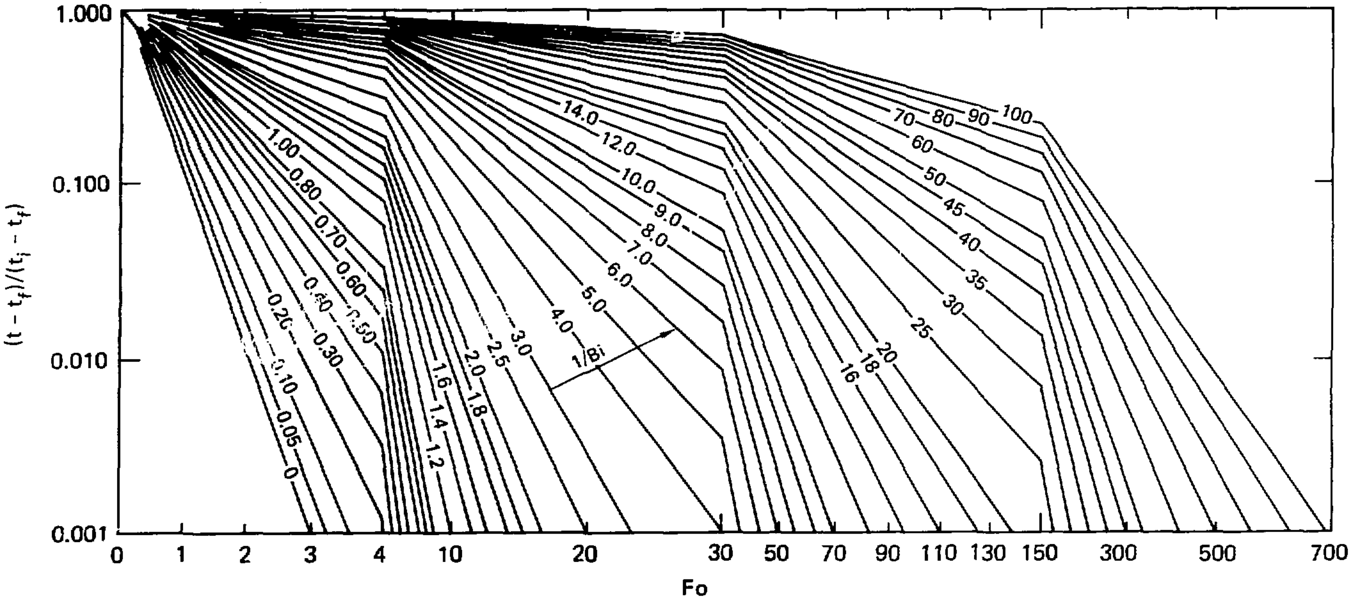


FIG. 8.4a. Midplane temperature of an infinite plate of thickness 2ℓ and convectively cooled. (case 8.1.7, source: Ref. 12, p. 227 and source: Ref. 5, p. 83, Fig. 4-7).

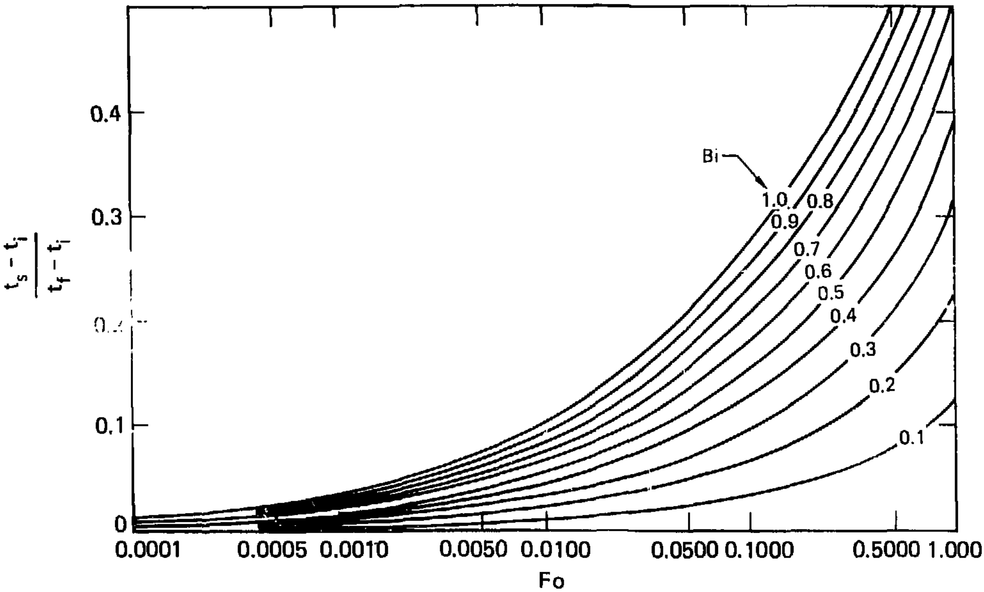


FIG. 8.4b. Surface temperature of an infinite plate of thickness 2ℓ and correction boundary (case 8.1.7.2, source: Ref. 74 p. 226, Fig. 6.6c).

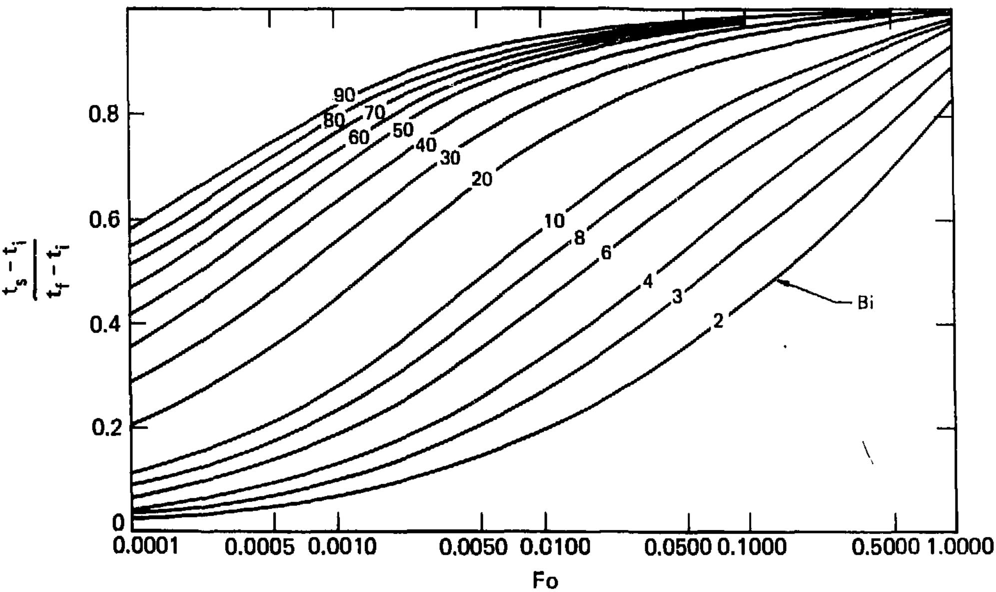


FIG. 8.4c. Surface temperature of an infinite plate of thickness 2ℓ and convection boundary (case 8.1.7.2, source: Ref. 74, p. 226, Fig. 6.6d).

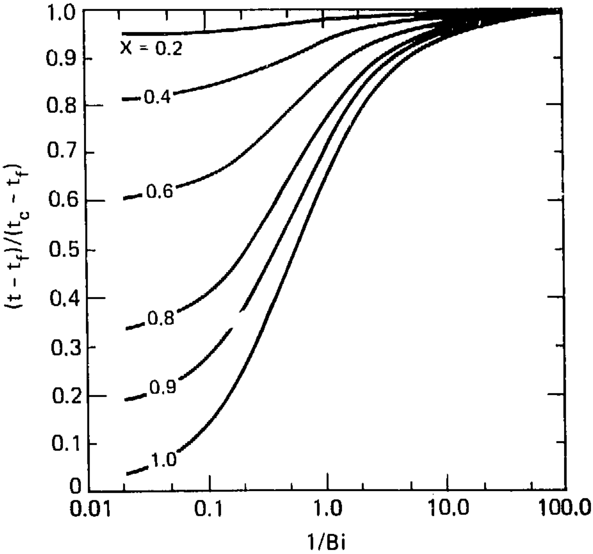


FIG. 8.4d. Temperature as a function of midplane temperature for an infinite plate of thickness 2ℓ and convectively cooled (case 8.1.7, source: Ref. 5, p. 86, Fig. 4-10).

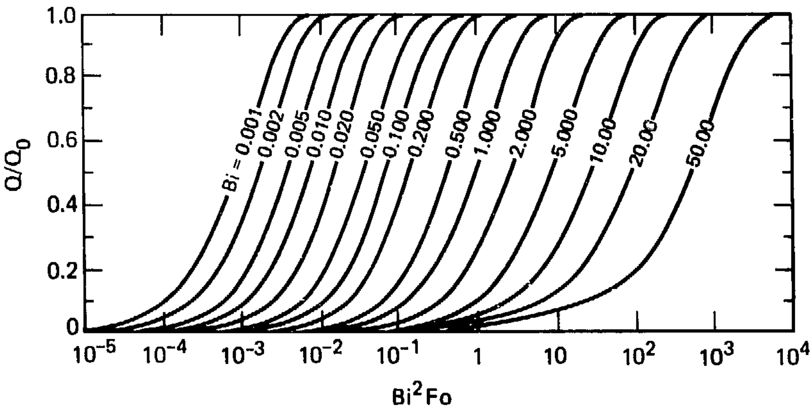


FIG. 8.4e. Relative heat loss from an infinite plate of thickness 2ℓ and convectively cooled (case 8.1.7, source: Ref. 5, p. 90, Fig. 4-14 and Ref. 13).

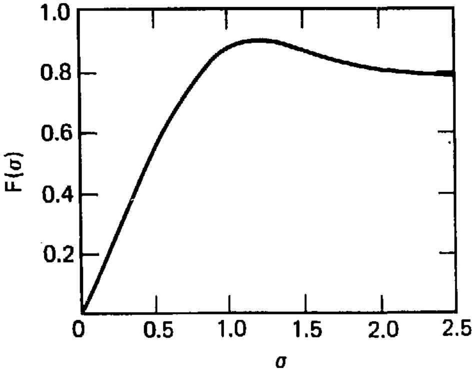


FIG. 8.5. The function F(σ) (case 8.1.12, source: Ref. 1. p. 301, Fig. 14-3).

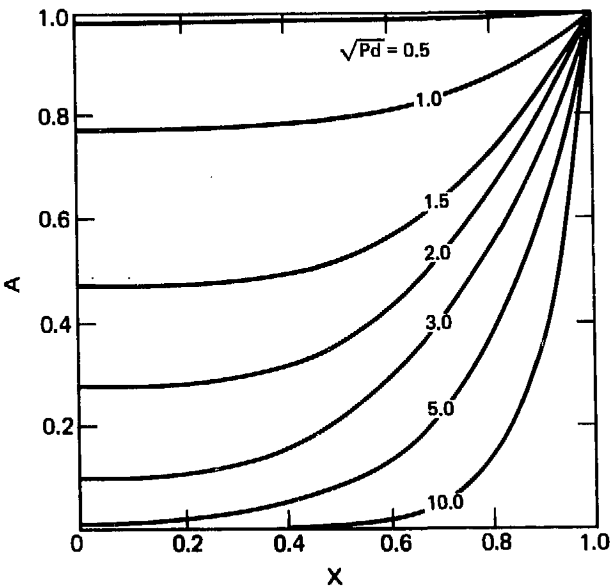


FIG. 8.6a. Variation of amplitude of the steady oscillation of temperature in an infinite plate caused by harmonic surface temperature (case 8.1.13, source: Ref. 9, p. 106, Fig. 13).

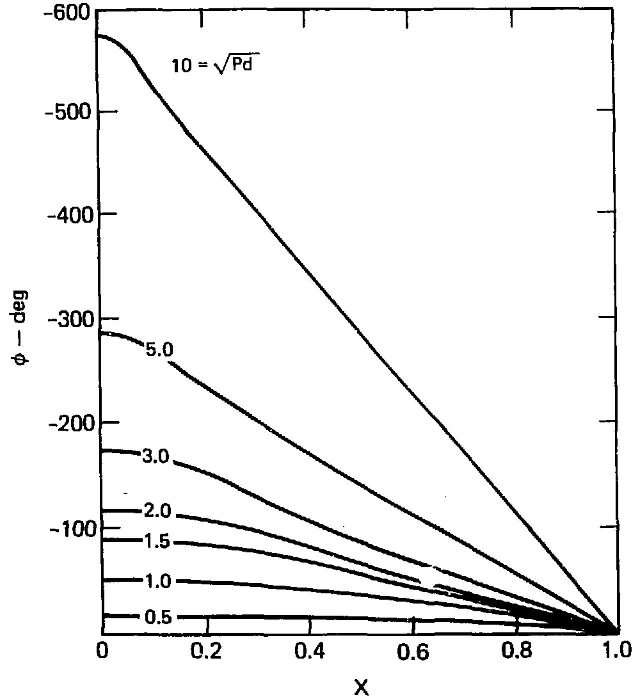


FIG. 8.6b. Variation in phase of the steady oscillation of temperature in an infinite plate caused by harmonic surface temperature (case 8.1.13, source: Ref. 9, p. 107, Fig. 14).

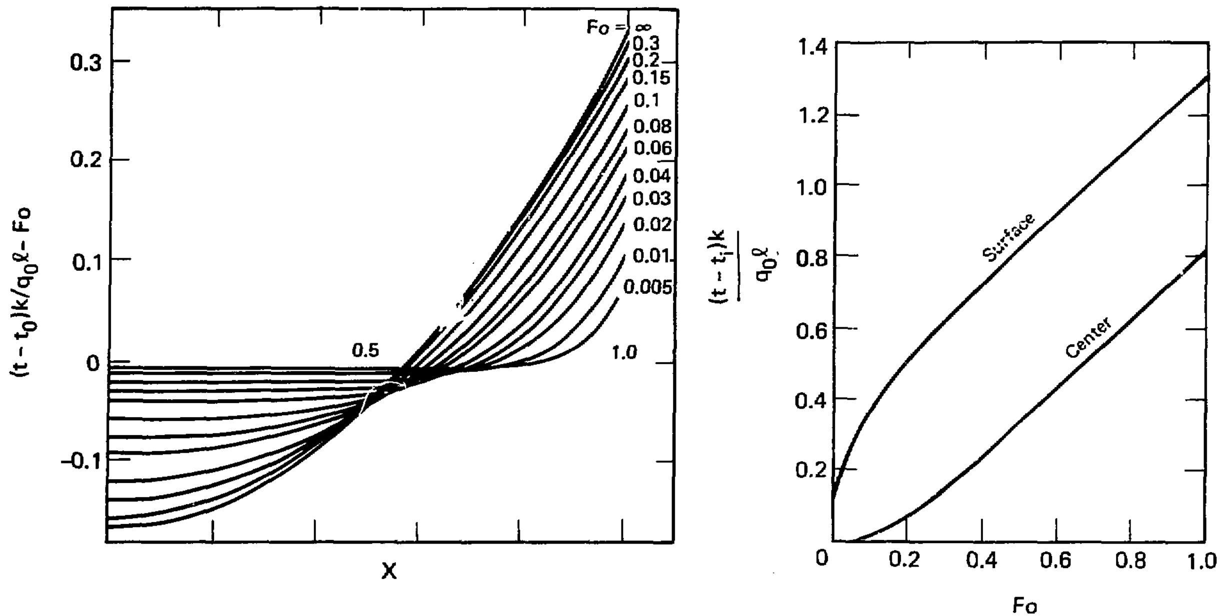


FIG. 8.7. Temperature distribution in an infinite plate with no heat flow at $x = 0$ and constant heat flux q_0 at $x = \ell$ (case 8.1.18, source: Ref. 9, p. 113, Fig. 15).

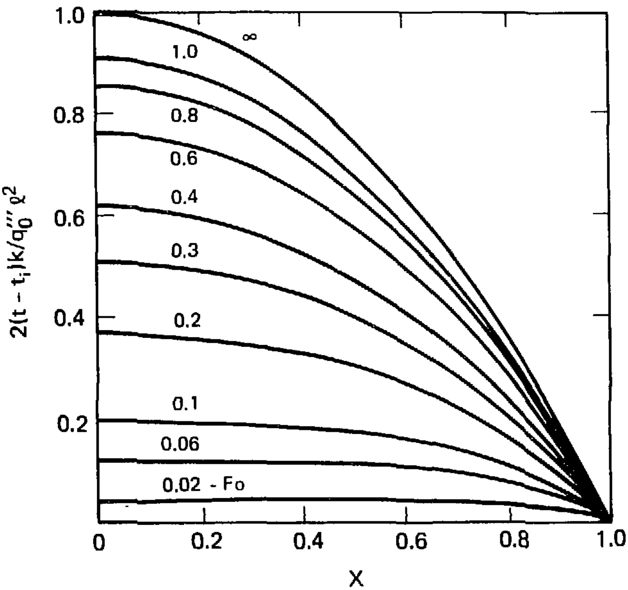


FIG. 8.8. Temperatures in an infinite plate with constant internal heating q''' and surface temperature t_i (case 8.2.1, source: Ref. 9, p. 131, Fig. 20).

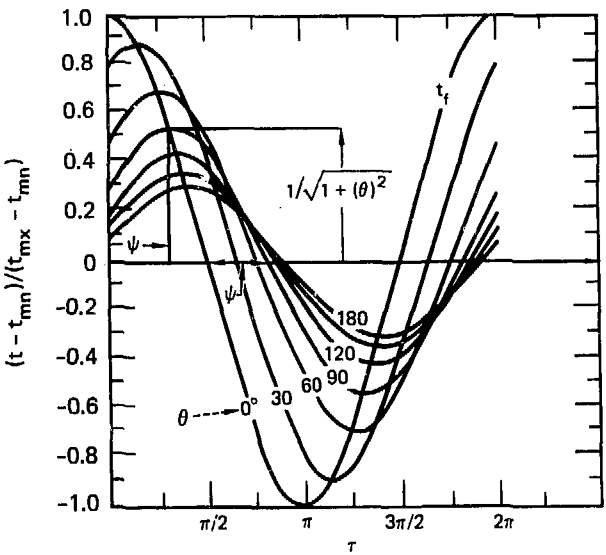


FIG. 8.9. Quasi-steady state temperature of an infinite conductivity plate with a harmonic fluid temperature (case 8.1.49, source: Ref. 19, Fig. 17b, p. 3-31).

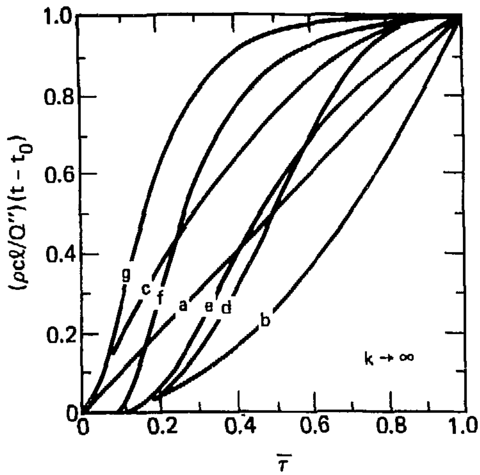


FIG. 8.10. Plate temperature response for prescribed heat inputs (case 8.2.15, source: Ref. 19, p. 3-35, Fig. 18).

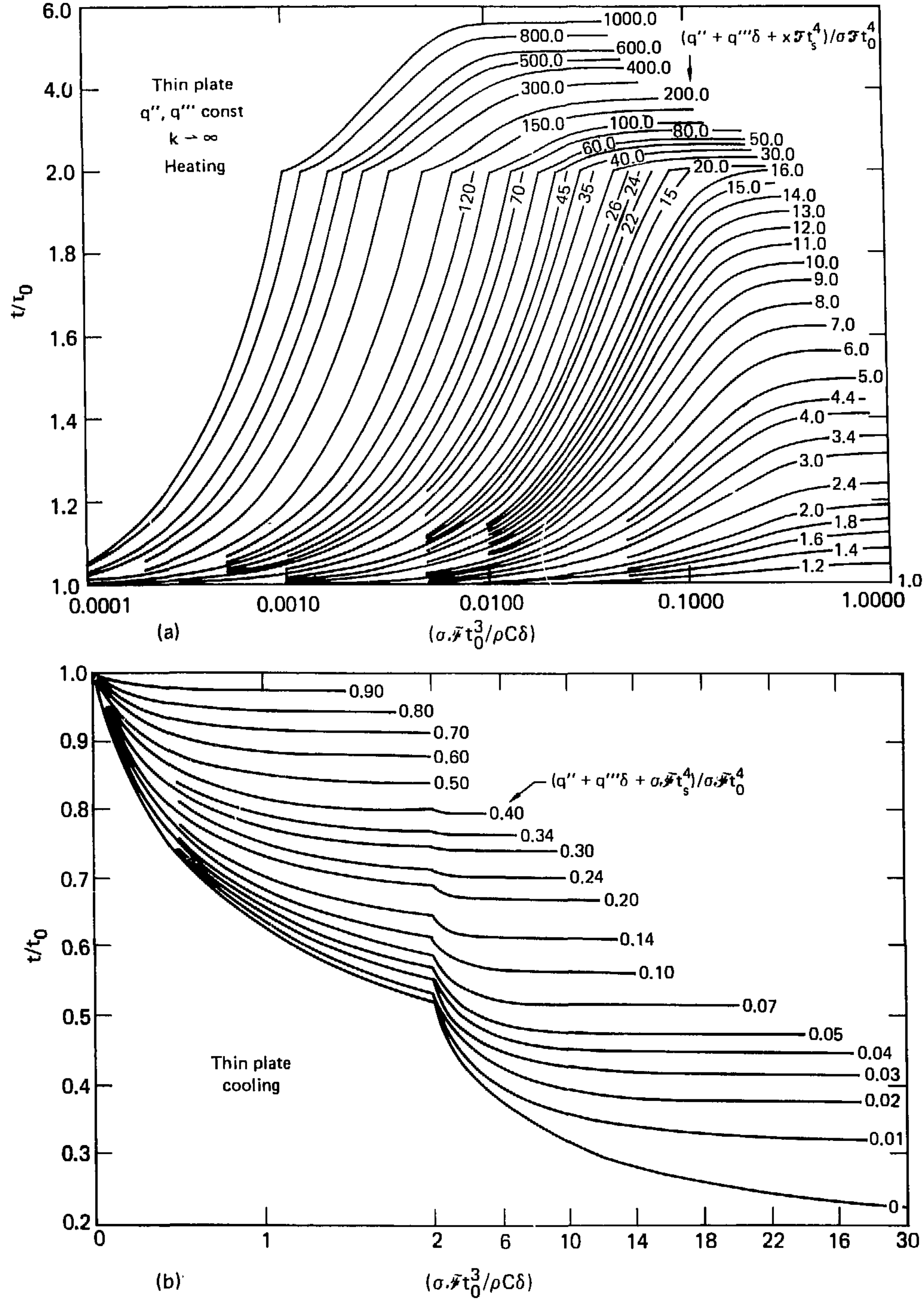
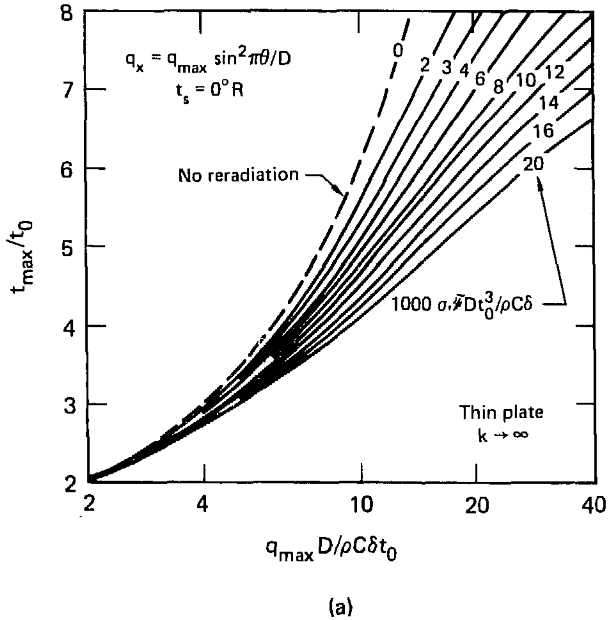


FIG. 8.11. Temperature response of thin generating plate suddenly exposed to constant heat input with surface reradiation: (a) heating, (b) cooling (case 8.2.16, source: Ref. 19, p. 3-36, Fig. 19).



(a)

FIG. 8.12. Maximum temperature of thin insulated plate suddenly exposed to circular heat pulse with surface reradiation: (case 8.1.50, source: Ref. 19, p. 3-38, Fig. 20).

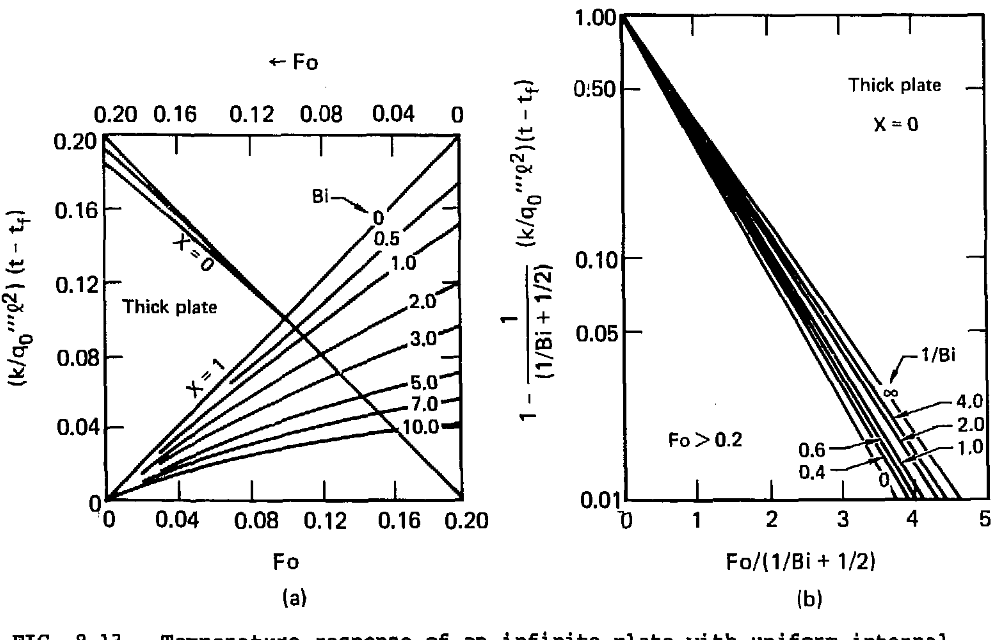


FIG. 8.13. Temperature response of an infinite plate with uniform internal heating, ℓ = half thickness (case 8.2.6, source: Ref. 19, p. 3-43, Fig. 24).

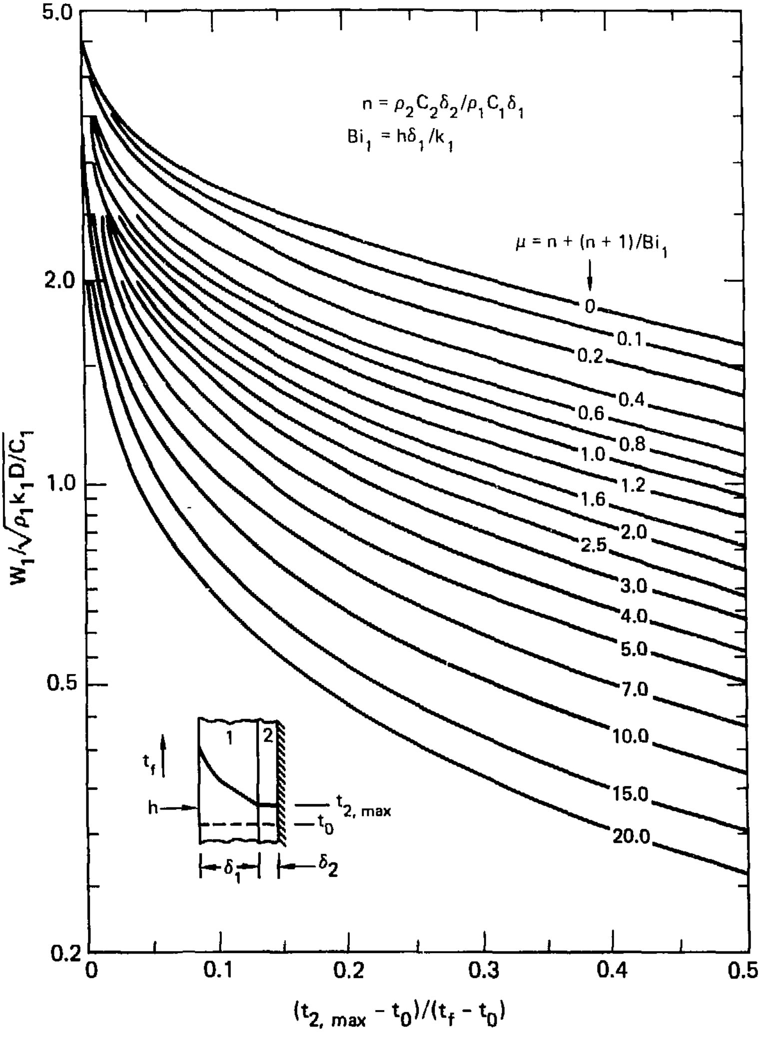


FIG. 8.14a. Insulation weight for substructure protection to $t_{2,max}$ in heating duration D, $W_1 = \rho_1\delta_1$ (case 8.1.51, source: Ref. 19 p. 3-48, Fig. 27).

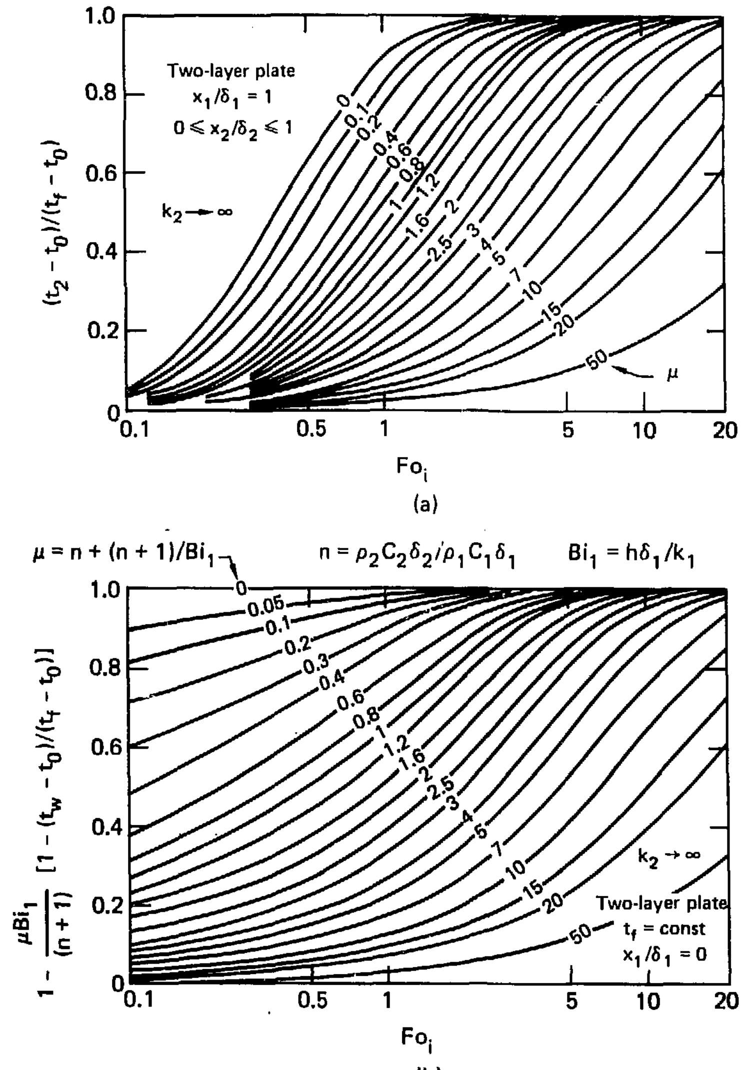


FIG. 8.14b. Temperature response of thick plate ($0 \leq x_1 \leq \delta_1$) convectively heated at $x_1 = 0$ and in perfect contact at its rear face $x_1 = \delta_1$ with a thin plate ($0 \leq x_2 \leq \delta_2$) insulated at $x_2 = \delta_2$: (a) $x_1/\delta_1 = 0$. (b) $x_1/\delta_1 = 1$ (case 8.1.51, source: Ref. 19, p. 3-46, Fig. 26).

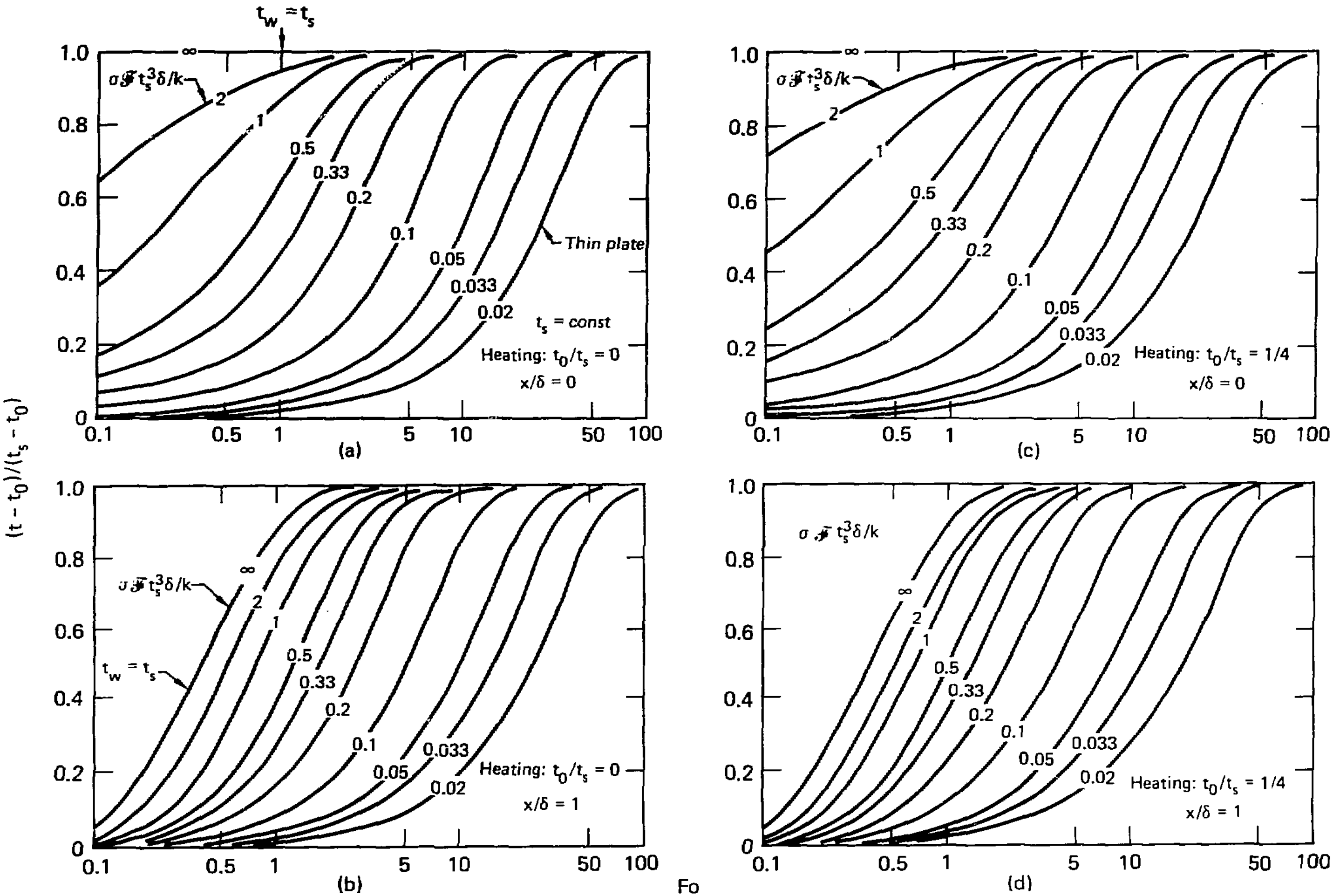


FIG. 8.15. Temperature response of thick plate ($0 \leq x \leq \delta$) with insulated rear face $x = \delta$ after sudden exposure to uniform radiative environment t_s at $x = 0$: (a) heating, $t_0/t_s = 0$, $x/\delta = 0$, (b) heating, $t_0/t_s = 0$, $x/\delta = 1$, (c) heating, $t_0/t_s = 1/4$, $x/\delta = 0$, (d) heating, $t_0/t_s = 1/4$, $x/\delta = 1$ (case 8.1.52, source: Ref. 19, pp. 3-50, Fig. 29).

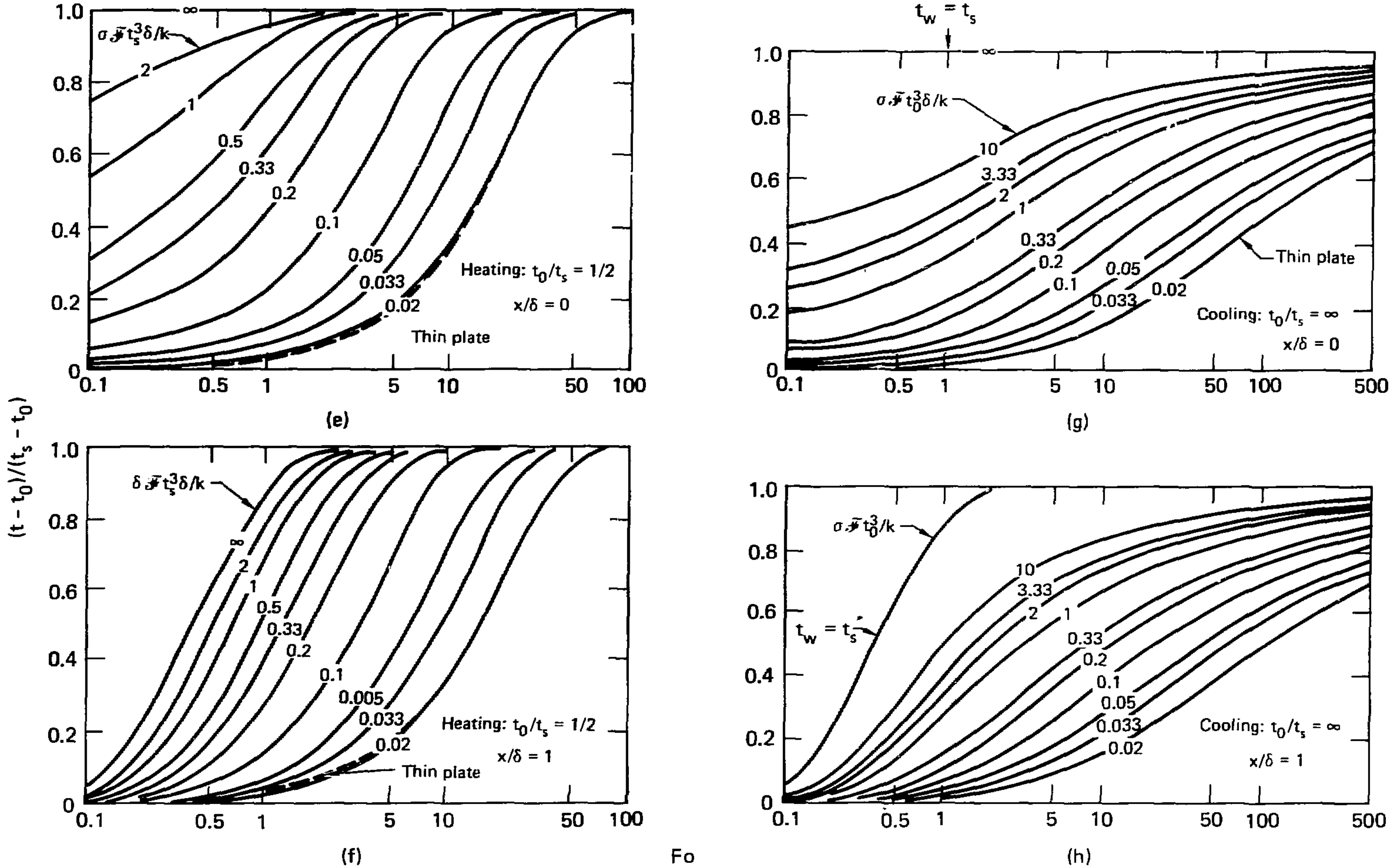
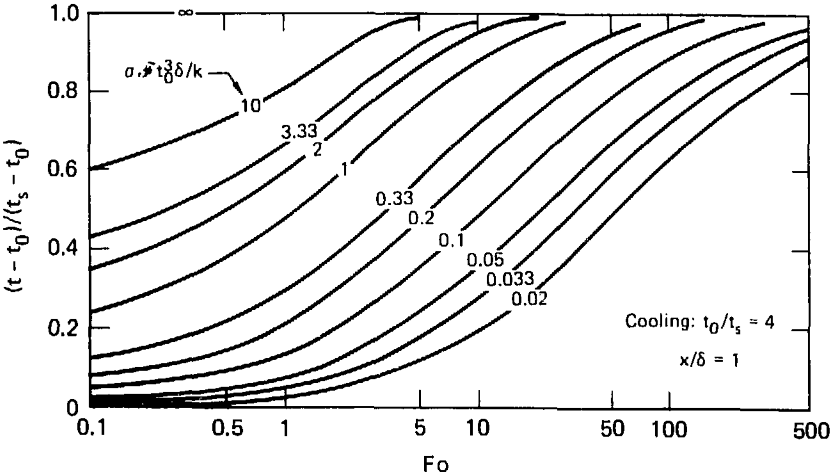
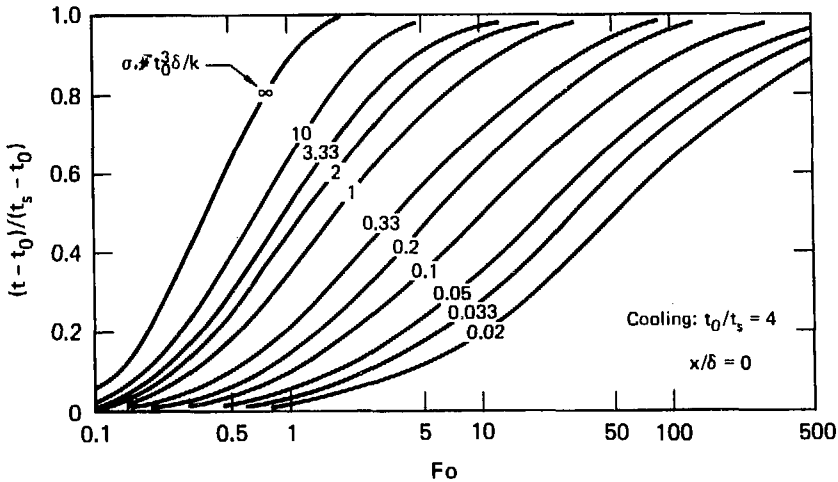


FIG. 8.15 (Cont.). Temperature response of thick plate ($0 \leq x \leq \delta$) with insulated rear face $x = \delta$ after sudden exposure to uniform radiative environment t_s at $x = 0$: (e) heating, $t_0/t_s = 1/2$, $x/\delta = 0$, (f) heating, $t_0/t_s = 1/2$, $x/\delta = 1$, (g) cooling, $t_0/t_s = \delta$, $x/\delta = 0$, (h) cooling, $t_0/t_s = \delta$, $x/ \ = 1$.

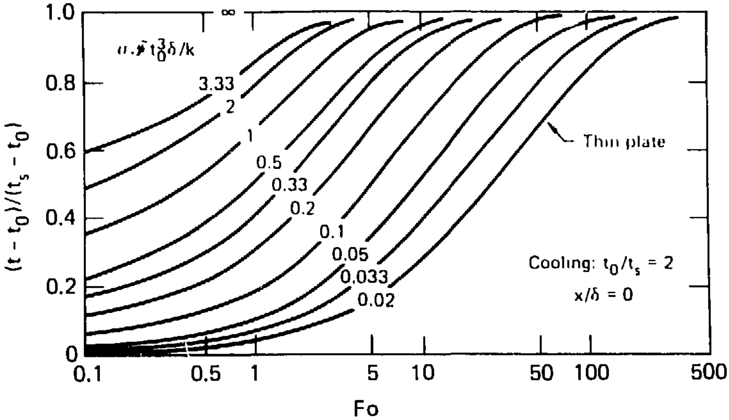


(i)

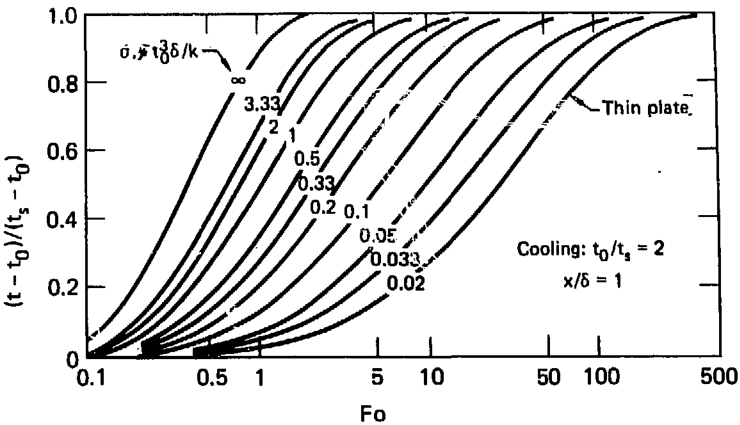


(j)

FIG. 8.15 (Cont.). Temperature response of thick plate ($0 \leq x \leq \delta$) with insulated rear face $x = \delta$ after sudden exposure to uniform radiative environment t_s at $x = 0$: (k) cooling, $t_0/t_s = 2$, $x/\delta = 0$, (l) cooling, $t_0/t_s = 2$, $x/\delta = 1$.



(k)



(l)

FIG. 8.15 (Cont.). Temperature response of thick plate ($0 \leq x \leq \delta$) with insulated rear face $x = \delta$ after sudden exposure to uniform radiative environment t_s at $x = 0$: (i) cooling, $t_0/t_s = 4$, $x/\delta = 0$, (j) cooling, $t_0/t_s = 4$, $x/\delta = 1$.

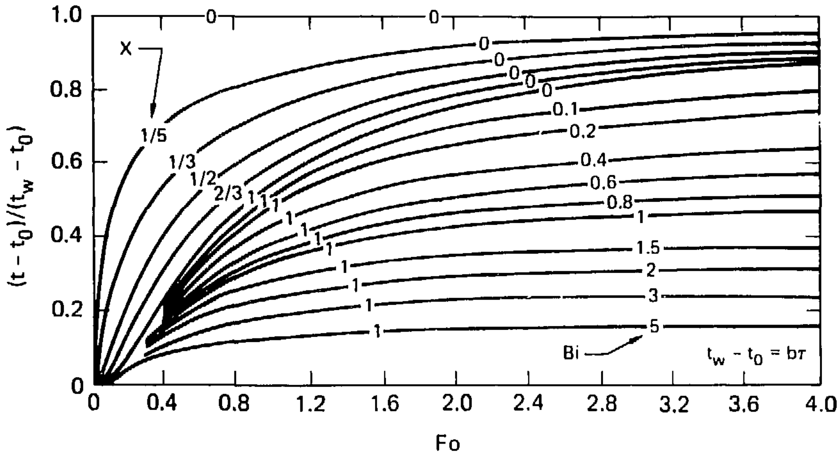


FIG. 8.16. Temperature response of thick plate ($0 \leq x \leq \delta$) with surface temperature t_w at $x = 0$, increasing linearly with time and rear face $x = \delta$ insulated or exposed to uniform convective environment t_0 (case 8.1.8, source: Ref. 19, pp. 3-60, Fig. 33).

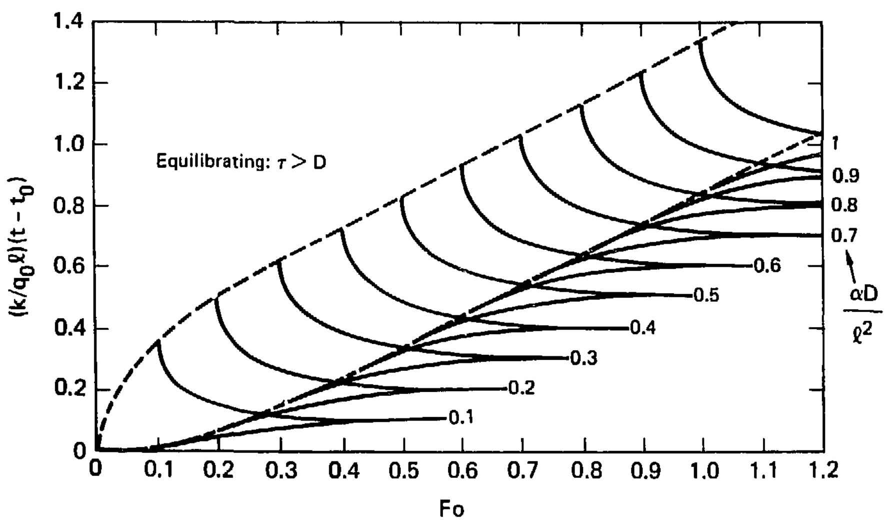


FIG. 8.17. Surface temperature response of a plate of thickness 2ℓ exposed to a steady heat flux q_0 on both sides for a time duration D (case 8.1.18, source: Ref. 19, pp. 3-67, Fig. 40).

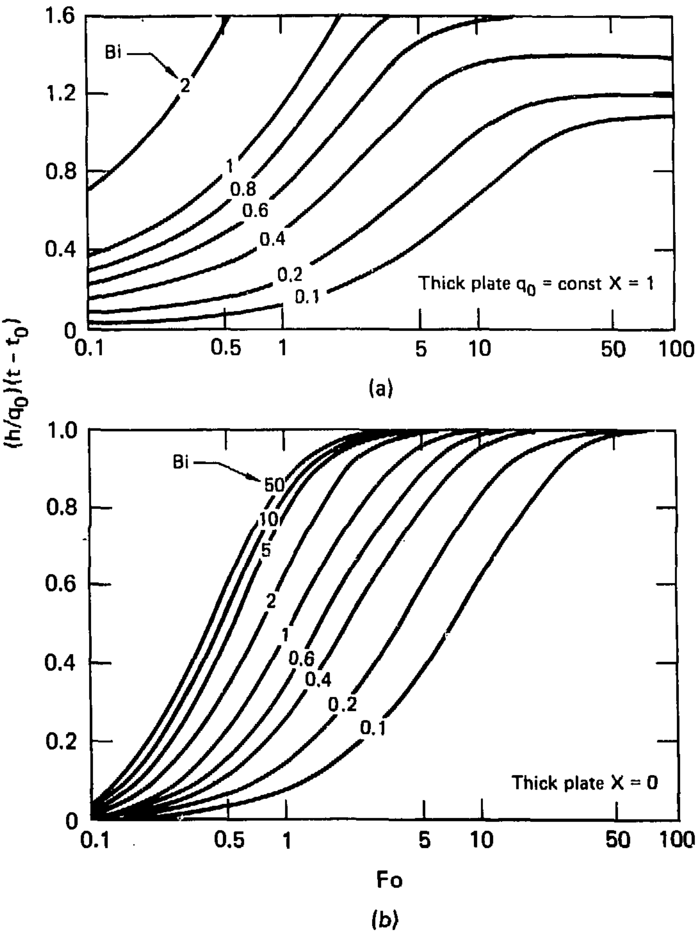


FIG. 8.18. Temperature response of a plate with steady heating q_0 at $x = \ell$ and convection boundary at $x = 0$ to t_0 (a) $x/\ell = 1$, (b) $x/\ell = 0$ (case 8.1.17, source: Ref. 19, pp. 3-69, Fig. 41).

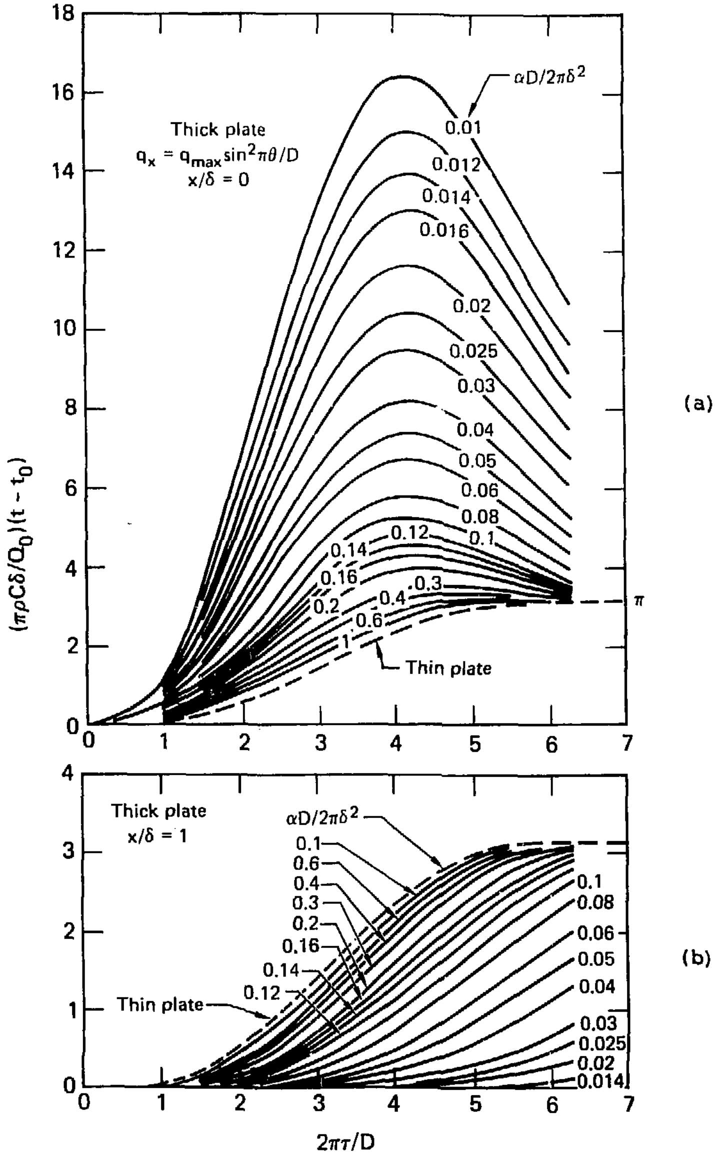


FIG. 8.19. Temperature response of an infinite plate exposed to a circular heat pulse q_{max} $\sin^2$ $(\pi\tau/D)$ at x = 0 and insulated at x = δ (a) x/δ = 0, (b) x/δ = 1 (case 8.1.54, source: Ref. 19, pp. 3-70, Fig. 42).

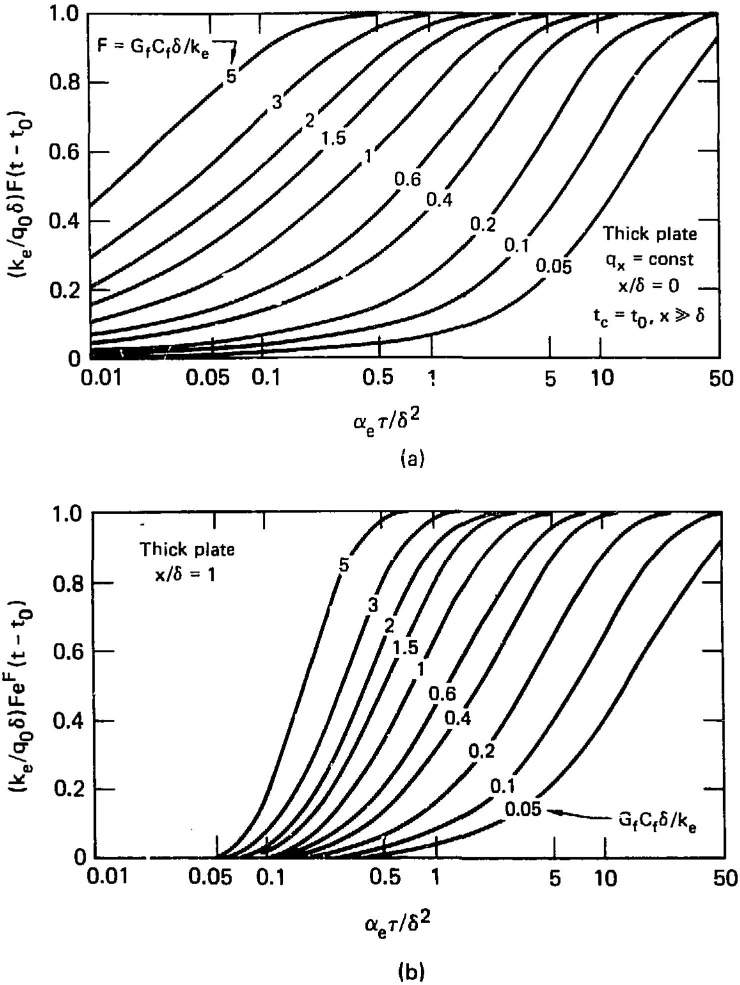


FIG. 8.20. Temperature response of an infinite porous plate after sudden exposure to a constant heat flux at x = 0 and cooled by steady flow through plate from x = δ of a fluid initially at t_0 (a) $x/\delta = 0$, (b) $x/\delta = 1$ (case 8.1.55, source: Ref. 19, pp. 3-74, Fig. 44).

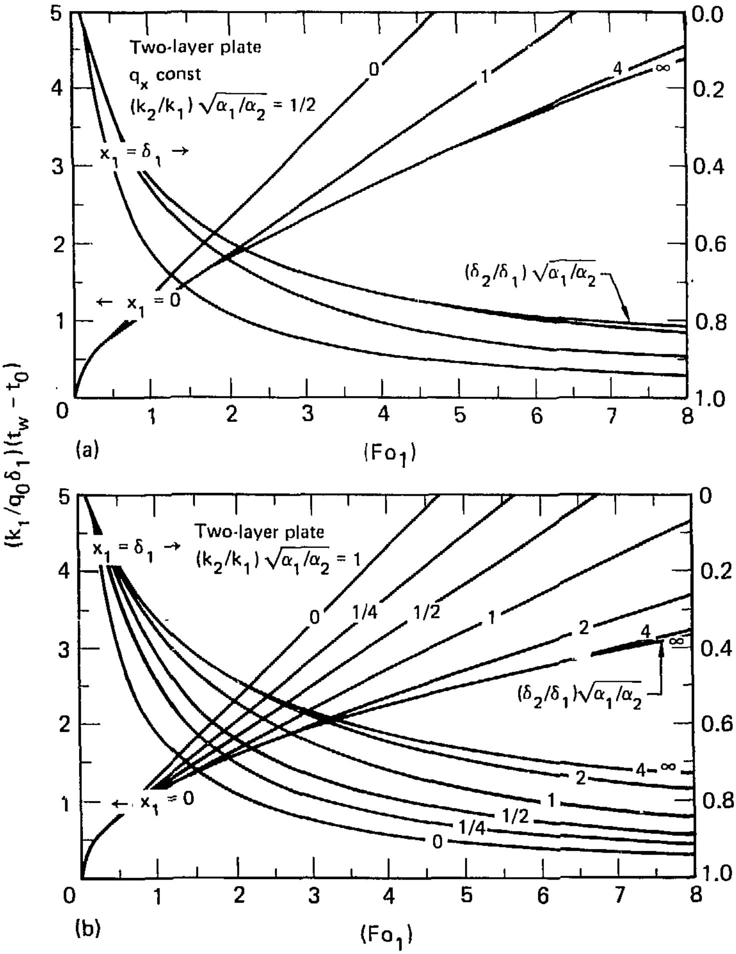


FIG. 8.21. Temperature response of an infinite plate with a constant heat flux q_0 at $x_1 = 0$, in contact with a plate at $x_1 = \delta_1$ and insulated on exposed surface of second plate $x_2 = \delta_2$ (case 8.1.56, source: Ref. 19, pp. 3-76, Fig. 45).

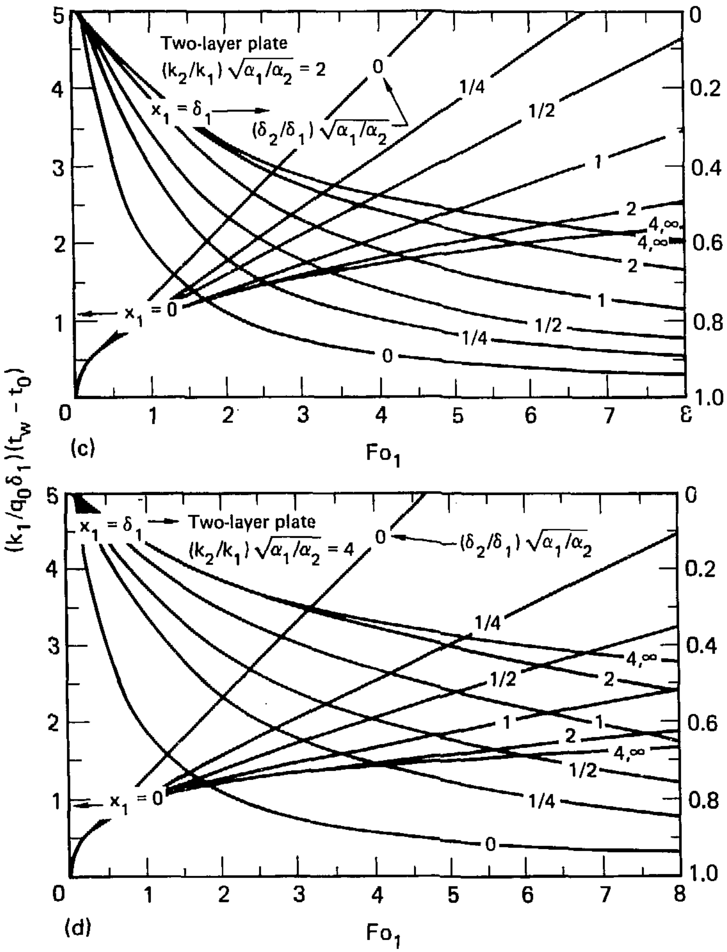


FIG. 8.21. (continued).

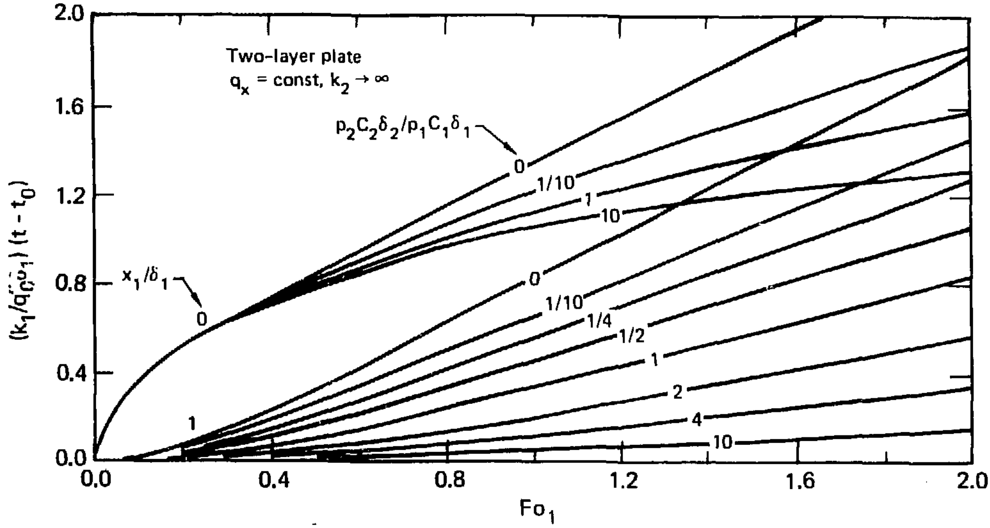
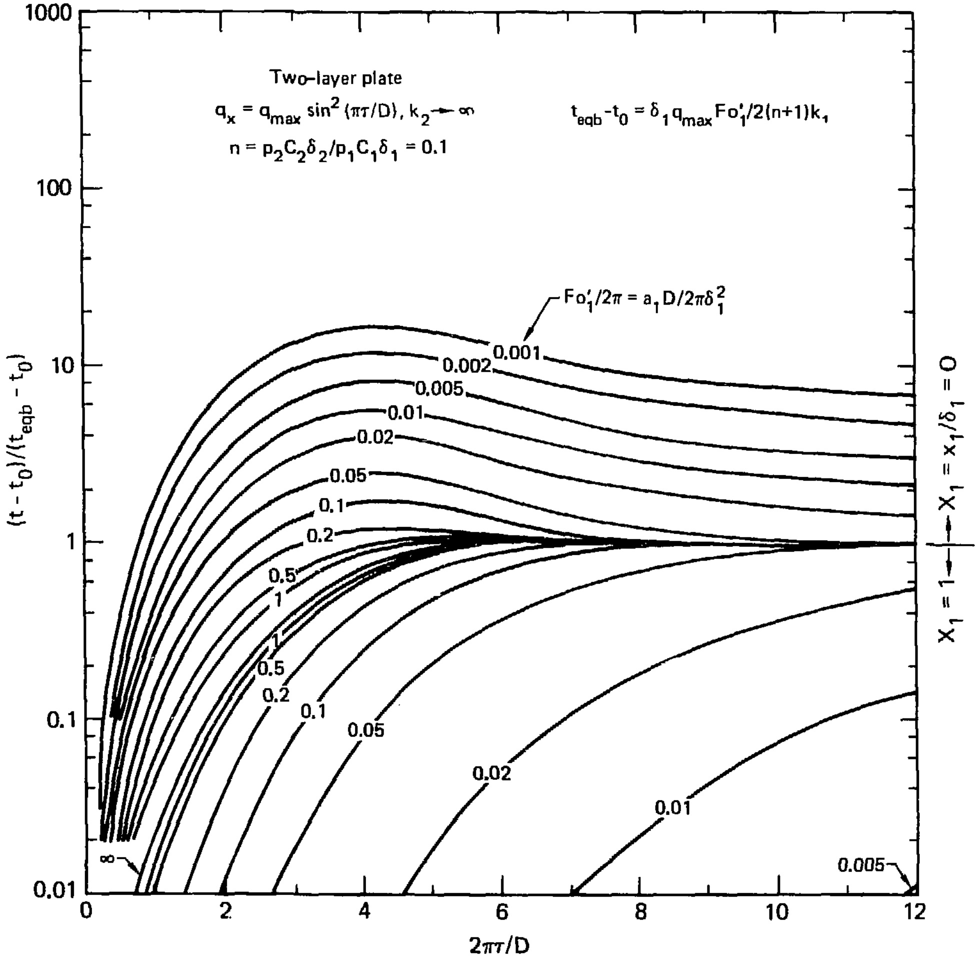


FIG. 8.22. Temperature response of an infinite plate exposed to a constant heat flux q'' at $x_1 = 0$ and in perfect contact at $x_1 = \delta_1$ with a plate of thickness δ_2 insulated at $x_2 = \delta_2$ (case 8.1.57, source: Ref. 19, pp. 3-78, Fig. 46).



(a)

FIG. 8.23. Temperature response of an infinite plate exposed to a circular heat pulse at $x^1 = 0$ and in perfect contact with an infinite conductivity plate of thickness δ_2 (case 8.1.58, source: Ref. 19, pp. 3-78, Fig. 47).

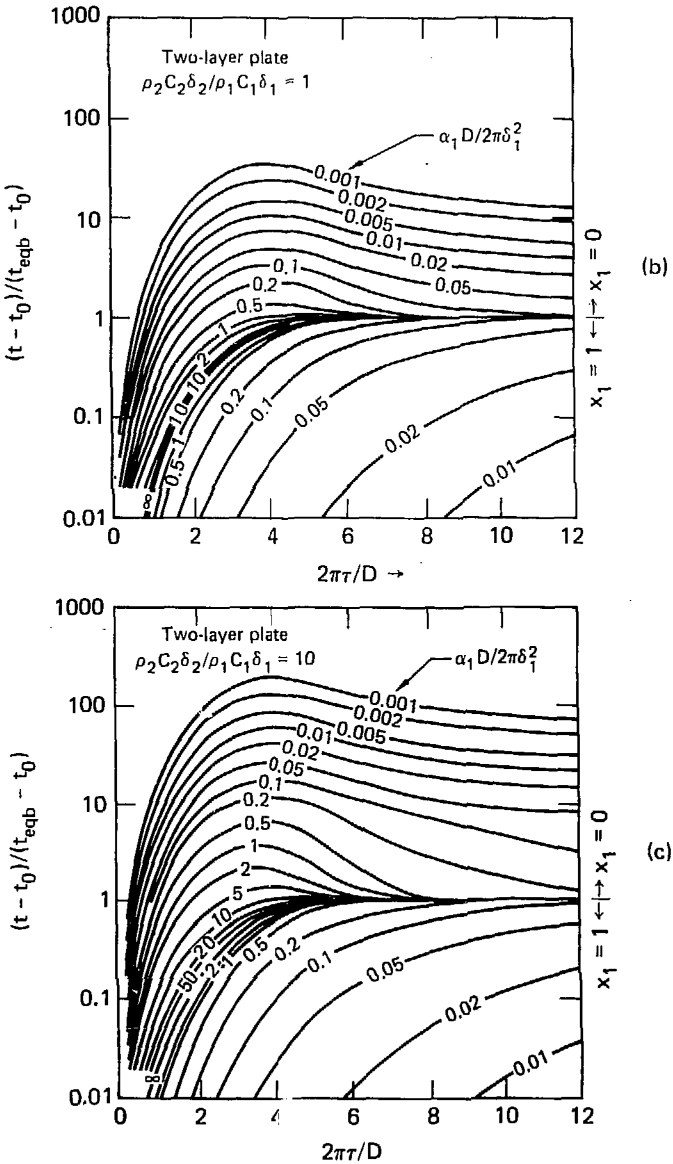


FIG. 8.23. (continued).

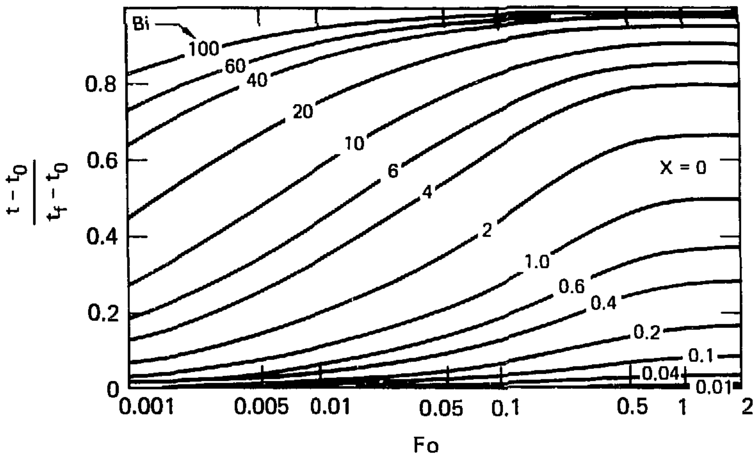


FIG. 8.24. Temperature response of an infinite plate surface having steady temperature and convection boundaries (case 8.1.25.2, source: Ref. 74, p. 240, Fig. 6.12).

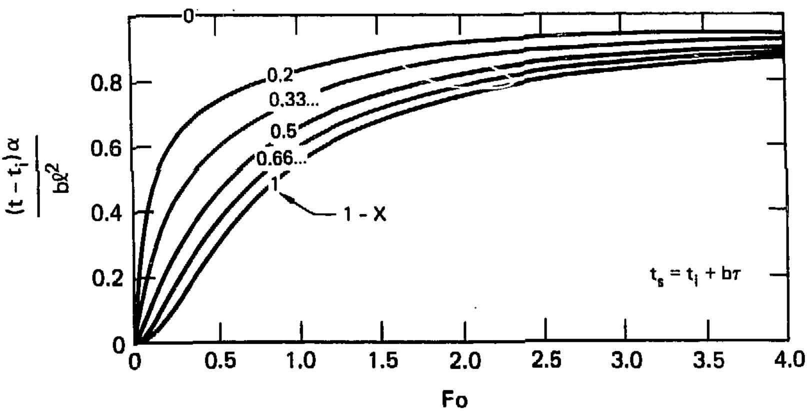


FIG. 8.25. Temperature of a flat plate having a ramp surface temperature equal to $t_s = t_i + b\tau$ (case 8.1.33, source: Ref. 74, p. 305, Fig. 7.1).

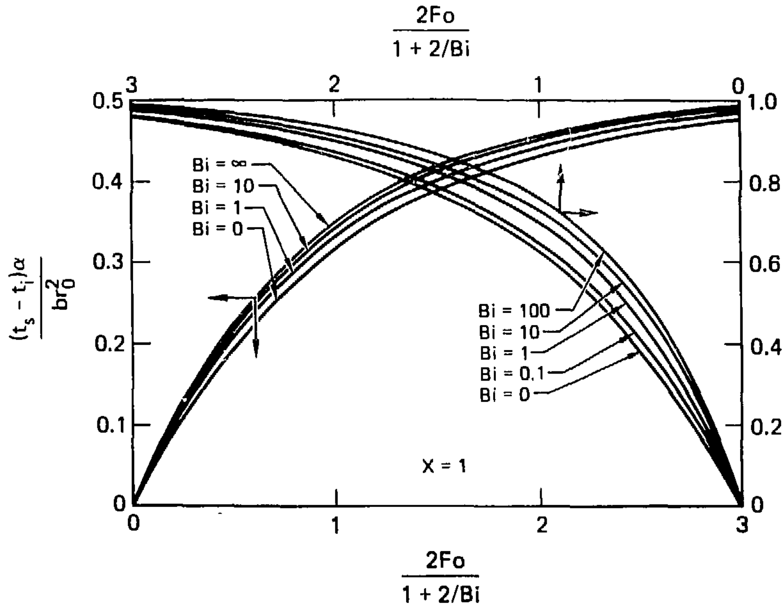


FIG. 8.26. Transient surface temperature of a slab convectively coupled to a linearly changing environment temperature equal to $t_f = t_i + b\tau$ (case 8.1.33, source: Ref. 89).

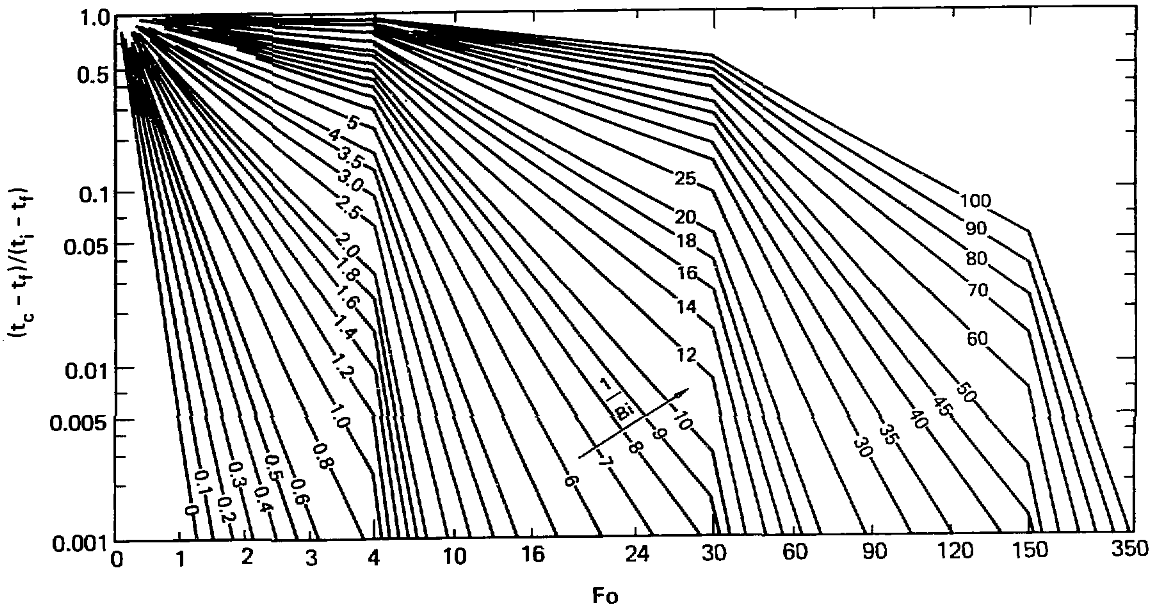


FIG. 9.1a. Axis temperature for an infinite cylinder of radius r_0 (case 9.1.3, source: Ref. 5, p. 84, Fig. 4-8 and Ref. 12).

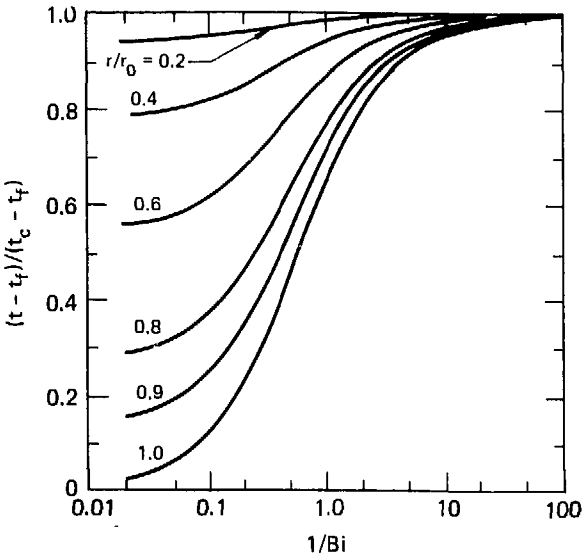


FIG. 9.1b. Temperature as a function of axis temperature in an infinite cylinder of radius r_0 (case 9.1.3, source: Ref. 5, p. 87 and Ref. 12).

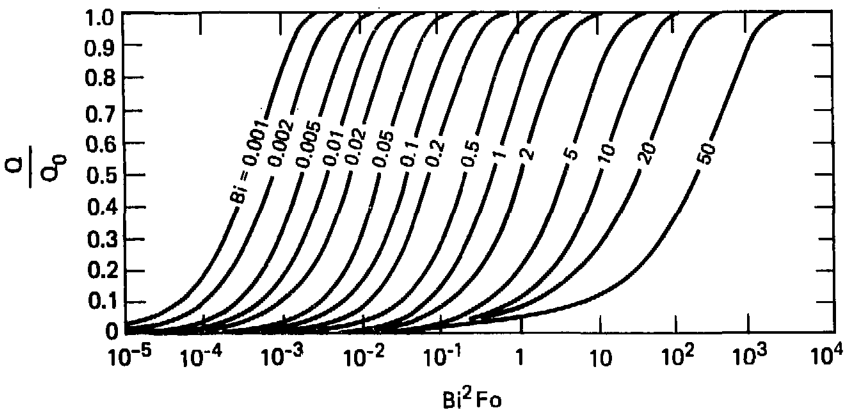


FIG. 9.1c. Dimensionless heat loss Q/Q_0 of an infinite cylinder of radius r_0 with time (case 9.1.3, source: Ref. 5, p. 90, Fig. 4-15 and Ref. 13).

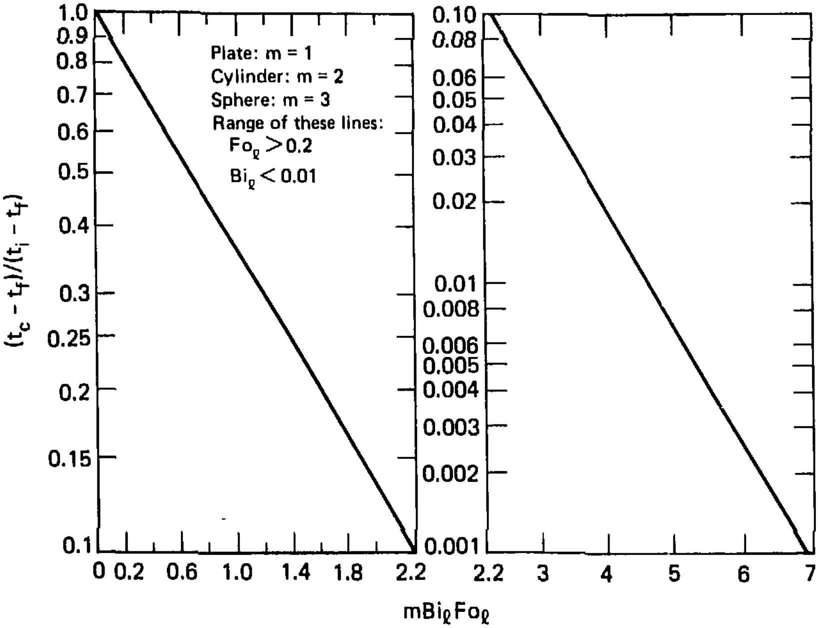


FIG. 9.1d. Center temperatures for plates, cylinders, and spheres for small values of h (case 9.1.3, 8.1.7 and 10.1., source: Ref. 5, p. 89, Fig. 4-13 and Ref. 12), ℓ is half thickness of radius.

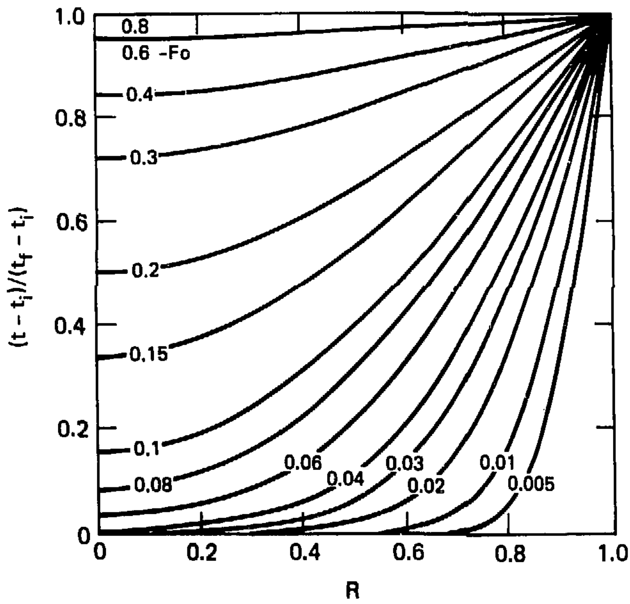


FIG. 9.2. Temperature distribution in an infinite cylinder with initial temperature t_i and steady surface temperature t_0 (case 9.1.1, source: Ref. 9, p. 200, Fig. 24).

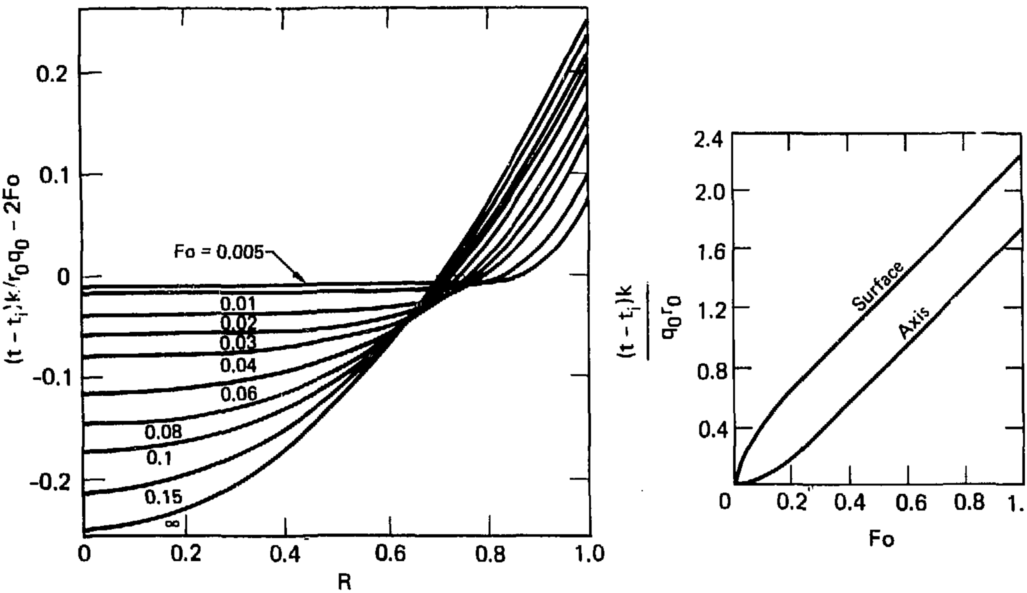
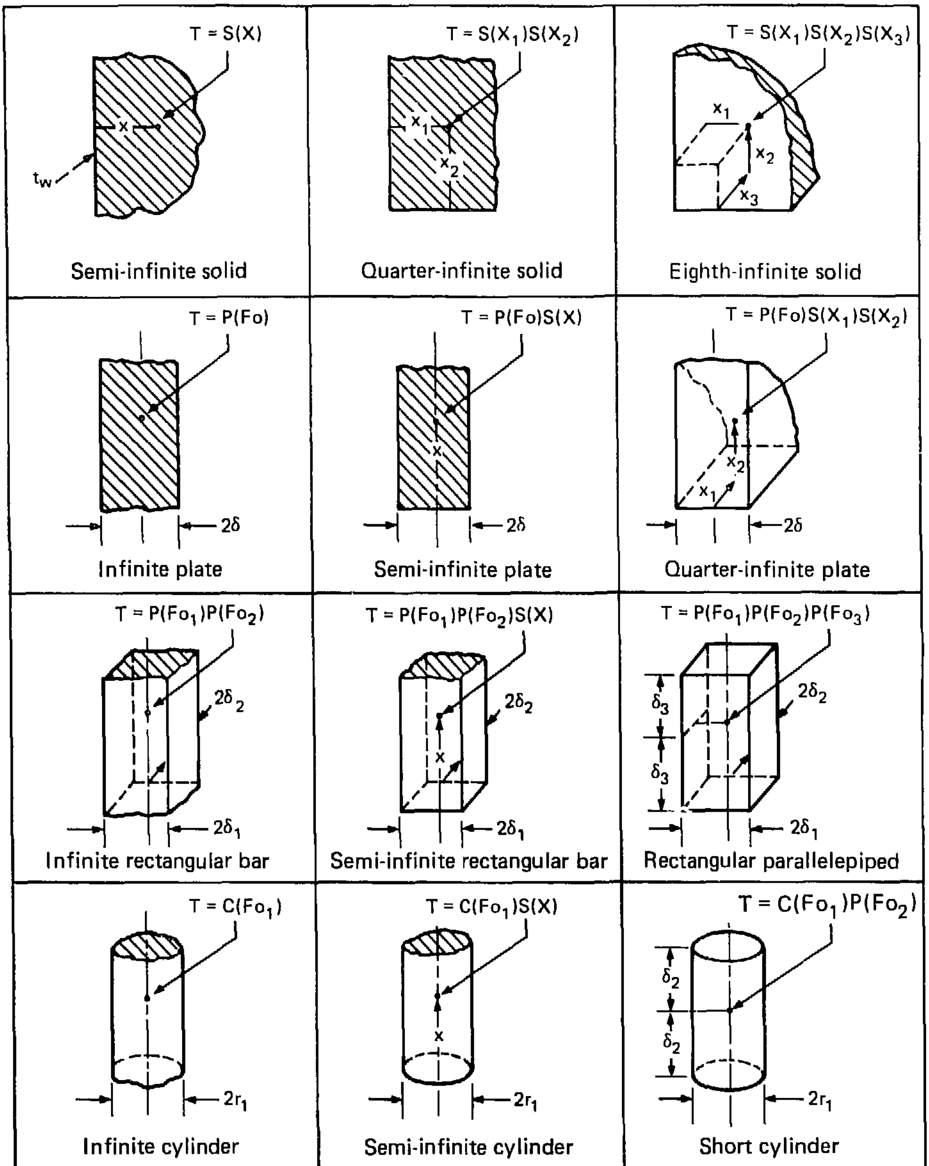


FIG. 9.3. Temperature in an infinite cylinder with constant heat flux at the surface (case 9.1.8, source: Ref. 9, p. 203, Fig. 25).



S = semi-infinite solid
P = plate
C = cylinder

$Fo_1 = a\tau/\delta_1^2 = a\tau/r_1^2$

$X_1 = x_1/2\sqrt{a\tau} = Fo_x^*$

$T = (t - t_w)/(t_0 - t_w)$

FIG. 9.4a. Product solutions for internal and central temperatures in solids with step change in surface temperature. S(X) given in Fig. 7.2, P(Fo) given in Fig. 8.4, C(Fo) given in Fig. 9.1 (case 7.1.19, 7.1.20, 8.1.22, 9.1.16, and 9.1.17, source: Ref. 74, pp. 3-65, Fig. 38).

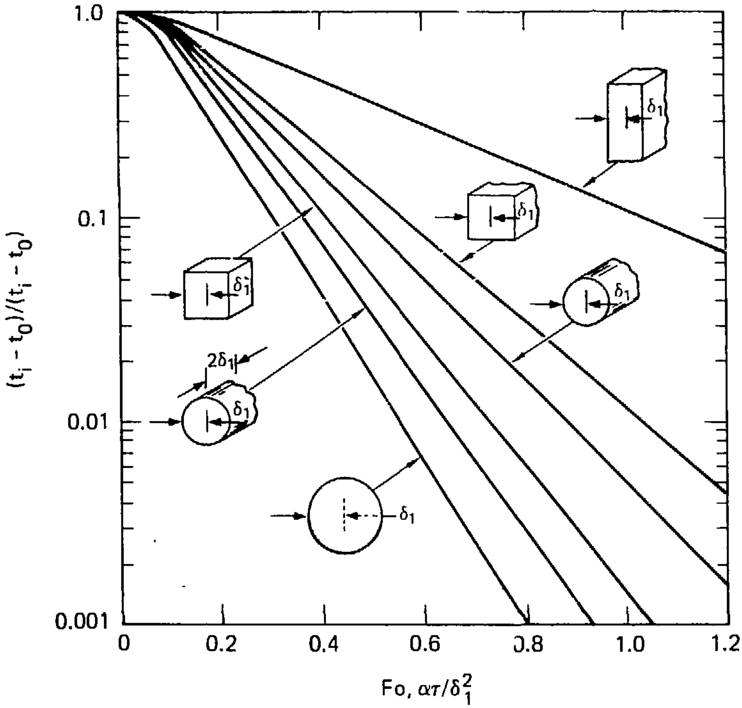


FIG. 9.4.b. Central temperatures in an infinite plate, an infinite rod, an infinite cylinder, a cube, a sphere, and a finite cylinder of length equal to its diameter, with all surfaces at temperature t_0 (case 7.1.29, 7.1.20, 9.1.16, 9.1.17, and 8.1.22, source: Ref. 2, p. 248, Fig. 10-8).

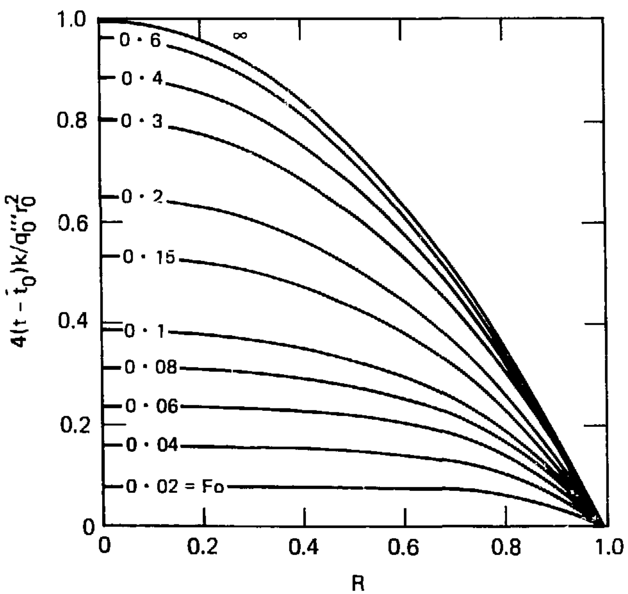


FIG. 9.5. Temperature distribution in an infinite cylinder with steady internal heating and a surface temperature t_0 (case 9.2.1, source: Ref. 9, p. 205, Fig. 26).

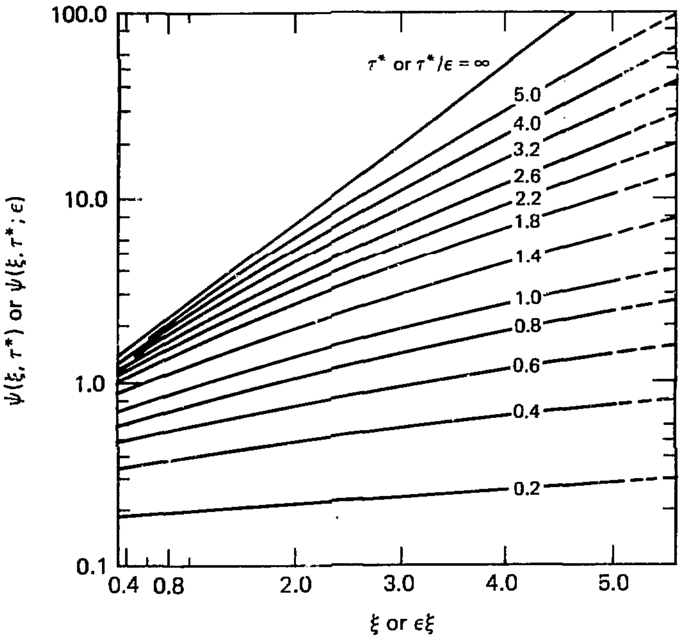


FIG. 9.6a. (case 9.2.4, source: Ref. 3, p. 356, Fig. 7-5).

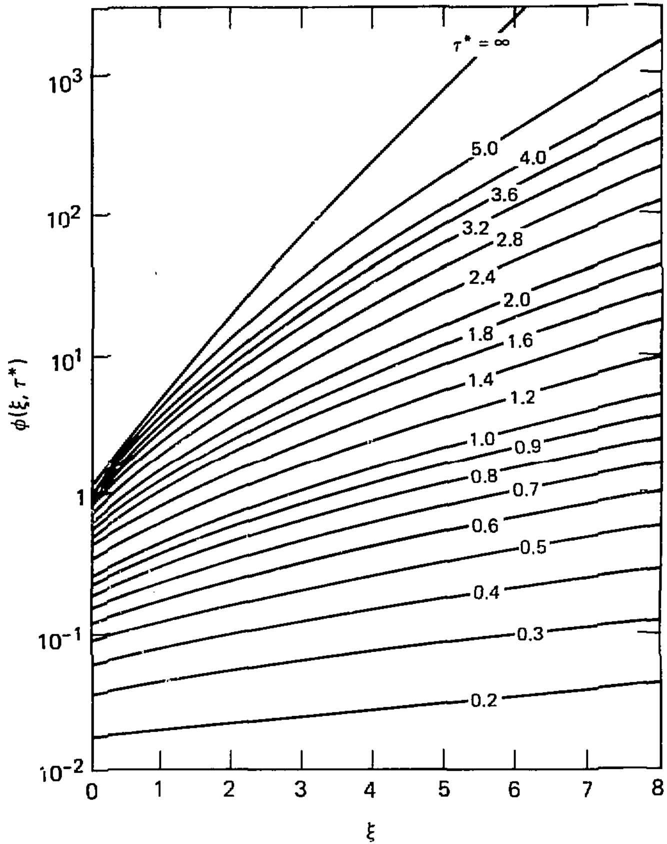


FIG. 9.6b. (case 9.2.4, source: Ref. 3, p. 364, Fig. 7-8).

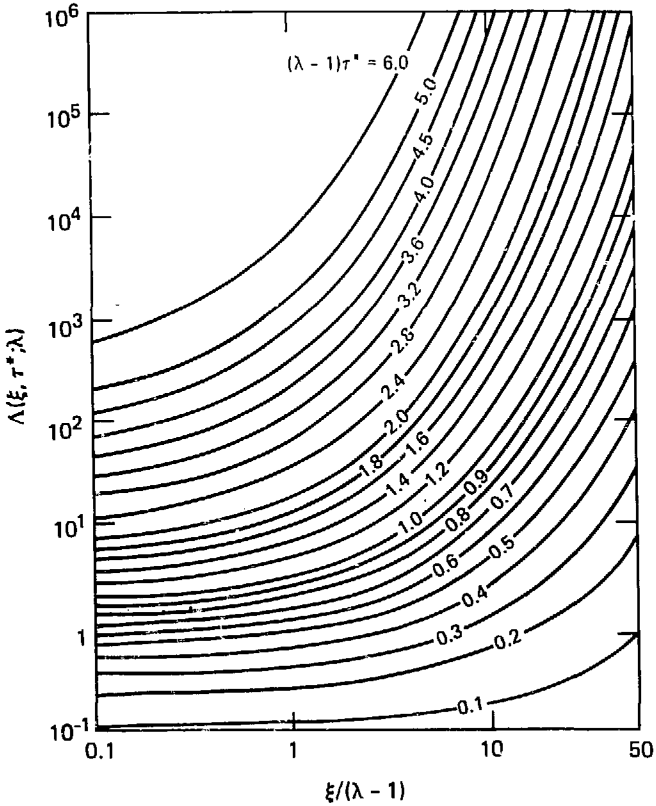


FIG. 9.6c. (case 9.2.4, source: Ref. 3, p. 365, Fig. 7-9).

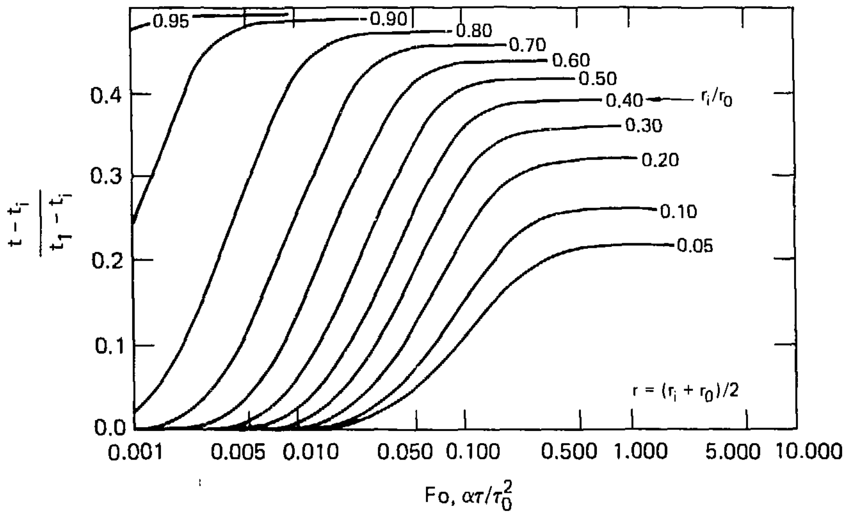


FIG. 9.7a. Temperature response of a hollow cylinder with a steady inside surface temperature equal to the initial temperature, t_i (case 9.1.10, source: Ref. 74, p. 156, Fig. 4.24).

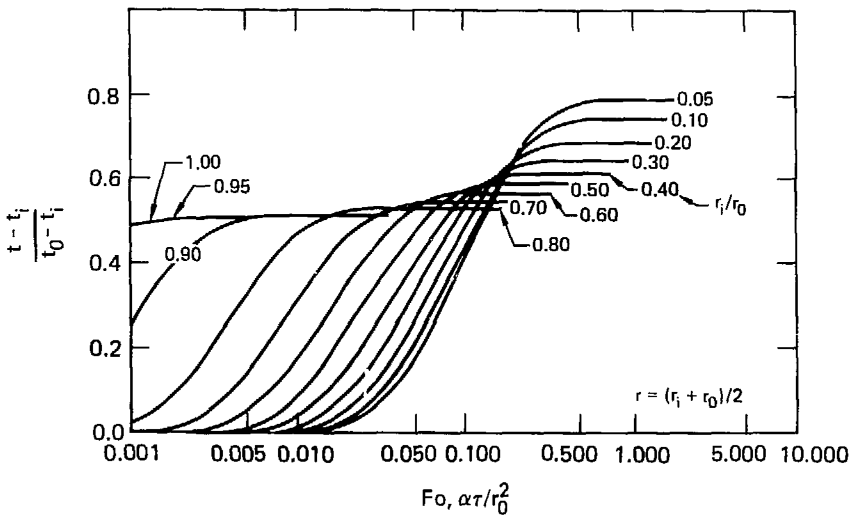


FIG. 9.7b. Temperature response of a hollow cylinder with a steady outside surface equal to the initial temperature, t_i (case 9.1.10, source: Ref. 74, p. 157, Fig. 4.25).

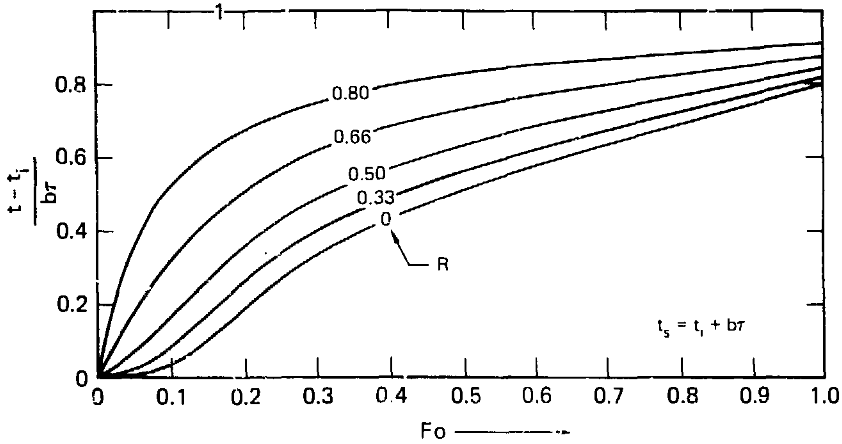


FIG. 9.8. Temperature of an infinite cylinder having a ramp surface temperature equal to $t_s = t_i + b\tau$ (case 9.1.2, source: Ref. 74, p. 312, Fig. 7.3).

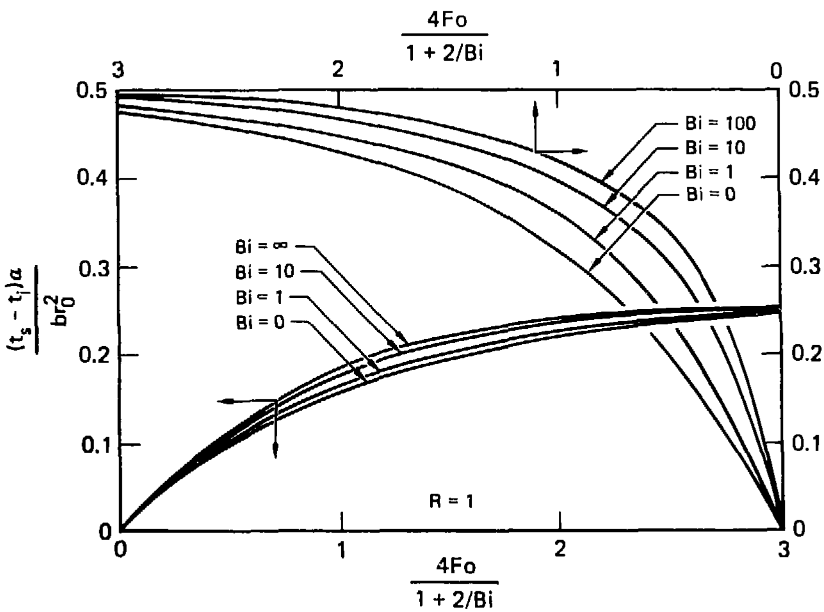


FIG. 9.9. Transient surface temperature of a cylinder convectively coupled to a linearly changing environment temperature equal to $t_f = t_i + b\tau$ (case 9.1.6, source: Ref. 89).

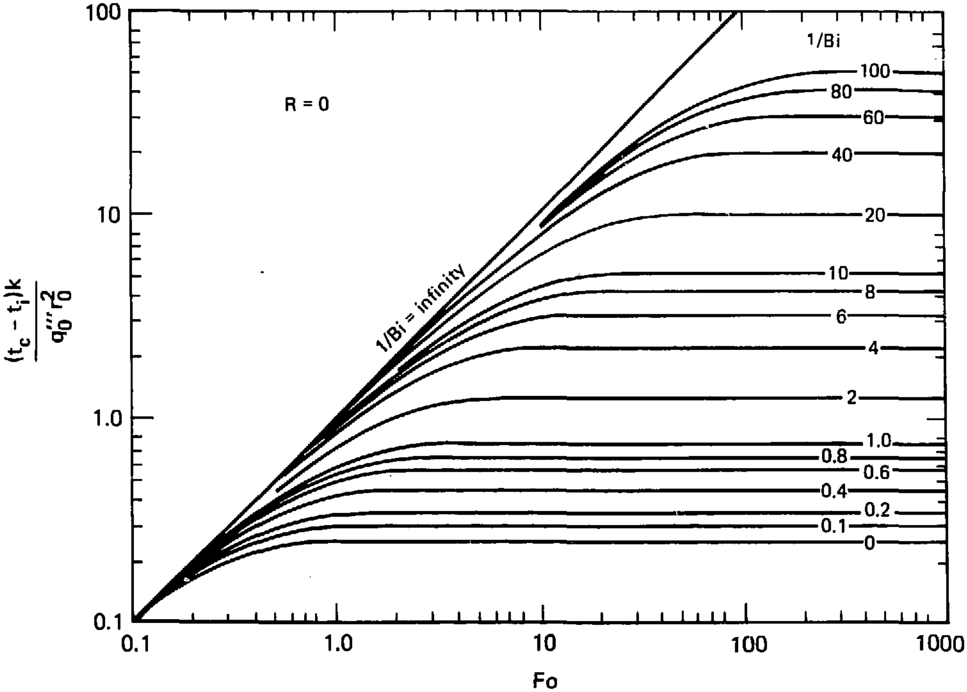


FIG. 9.10. Central temperature of an infinite cylinder with uniform internal heating and convection boundary (case 9.2.2, source: Ref. 90).

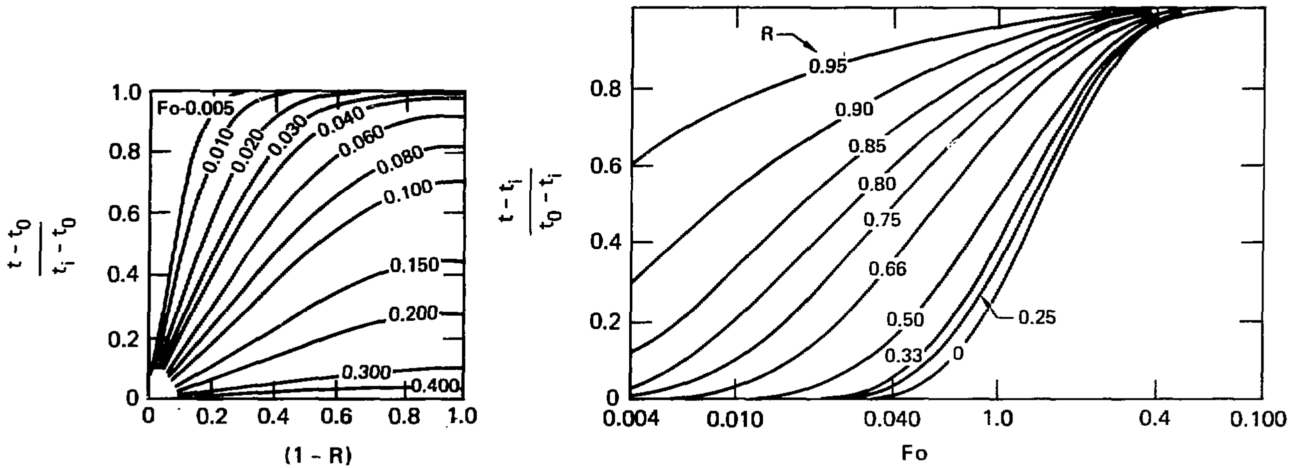


FIG. 10.1. Temperature distribution in a sphere of radius r_o with initial temperature t_i and surface temperature t_0 (case 10.1.1, source: Ref. 74, p. 127, Figs. 4.15 and 4.16).

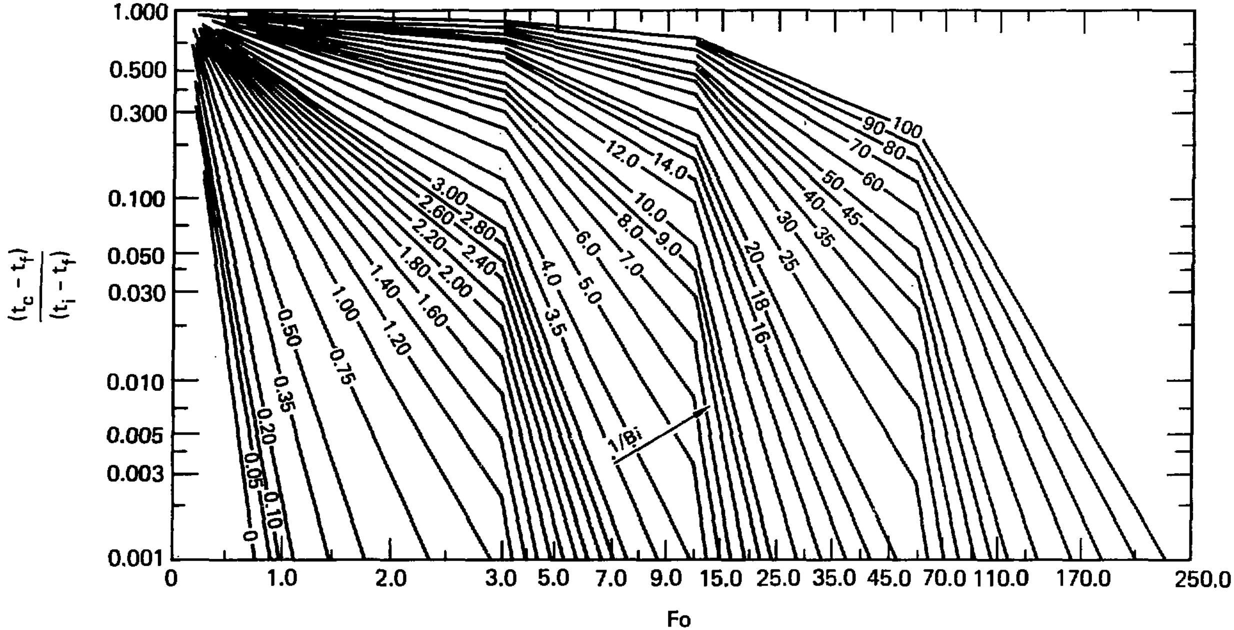


FIG. 10.2a. Center temperature of a convectively cooled sphere (case 10.1.2, source: Ref. 5, p. 85, Fig. 4-9 and source: Ref. 12).

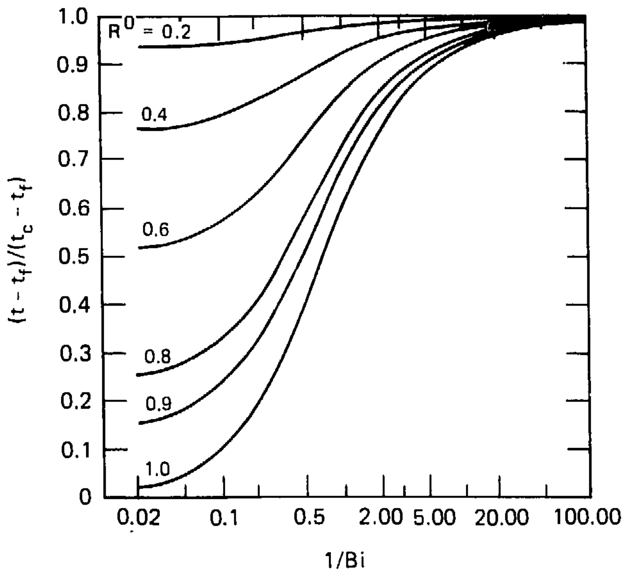


FIG. 10.2b. Temperature as a function of center temperature of a convectively cooled sphere (case 10.1.2, source: Ref. 5, p. 88, Fig. 4-12, and Ref. 12).

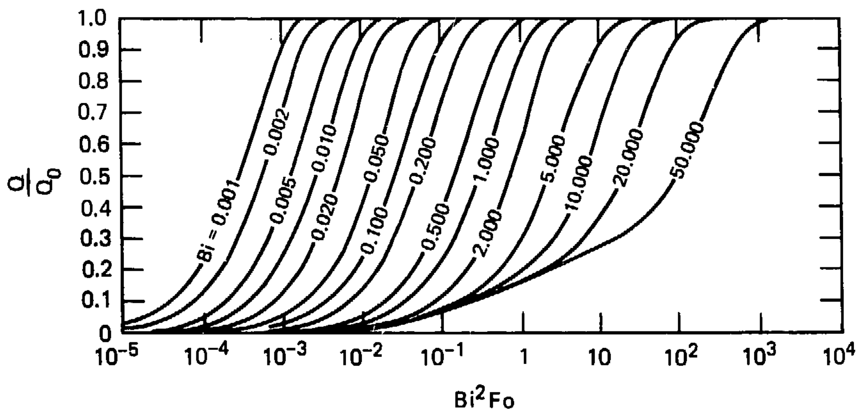


FIG. 10.2c. Dimensionless heat loss Q/Q_0 of a convectively cooled sphere (case 10.1.2, source: Ref. 5, p. 91, Fig. 4-16, and Ref. 13).

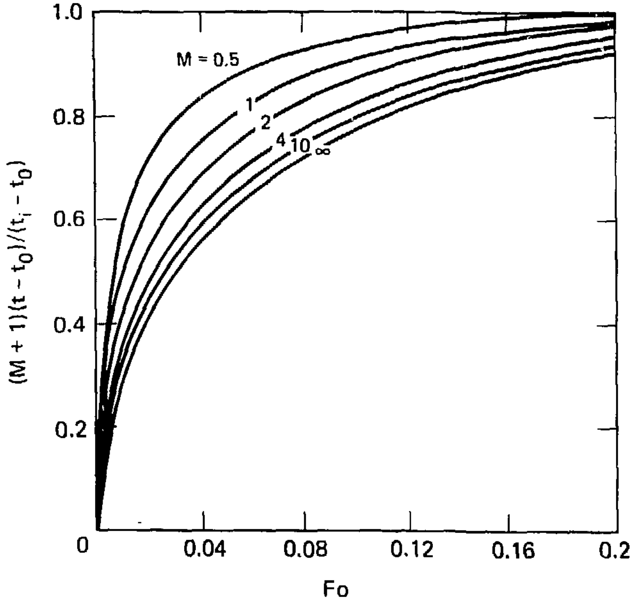


FIG. 10.3. Temperature of an infinite conductivity medium surrounding a spherical solid (case 10.1.13, source: Ref. 9, p. 241, Fig. 30).

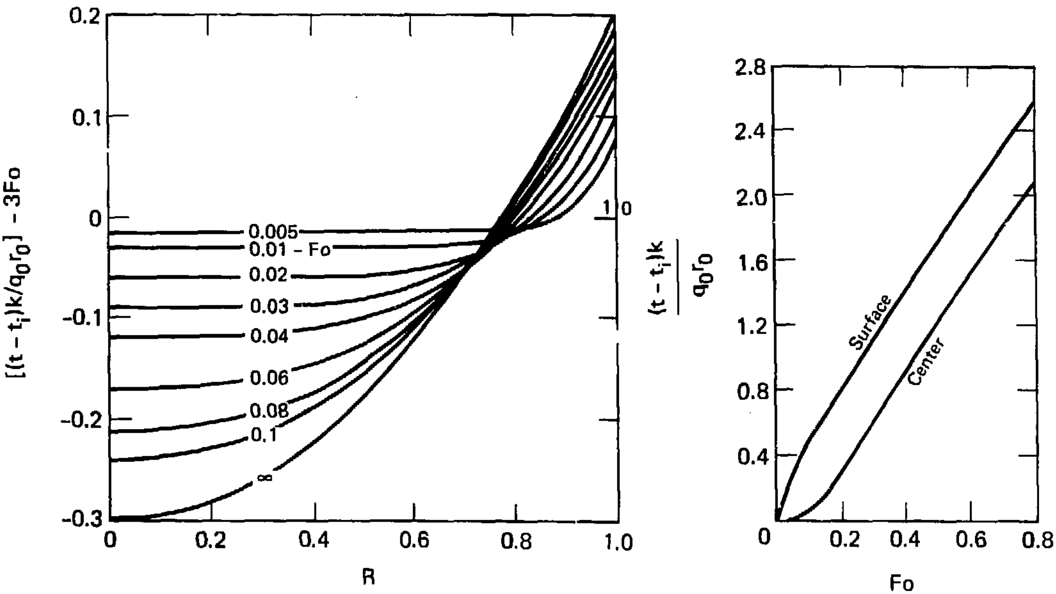


FIG. 10.4. Temperature in a sphere caused by a steady surface heat flux (case 10.1.15, source: Ref. 9, p. 242, Fig. 31).

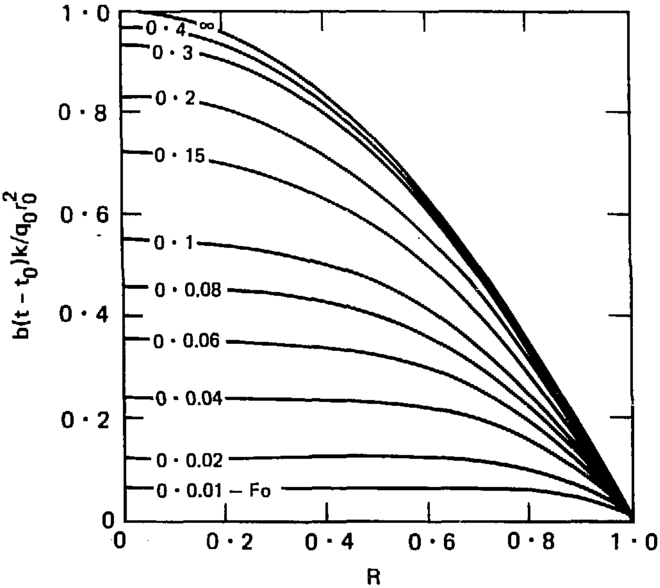


FIG. 10.5. Temperature distribution in a sphere with steady internal heating (case 10.2.1, source: Ref. 9, p. 244, Fig. 32).

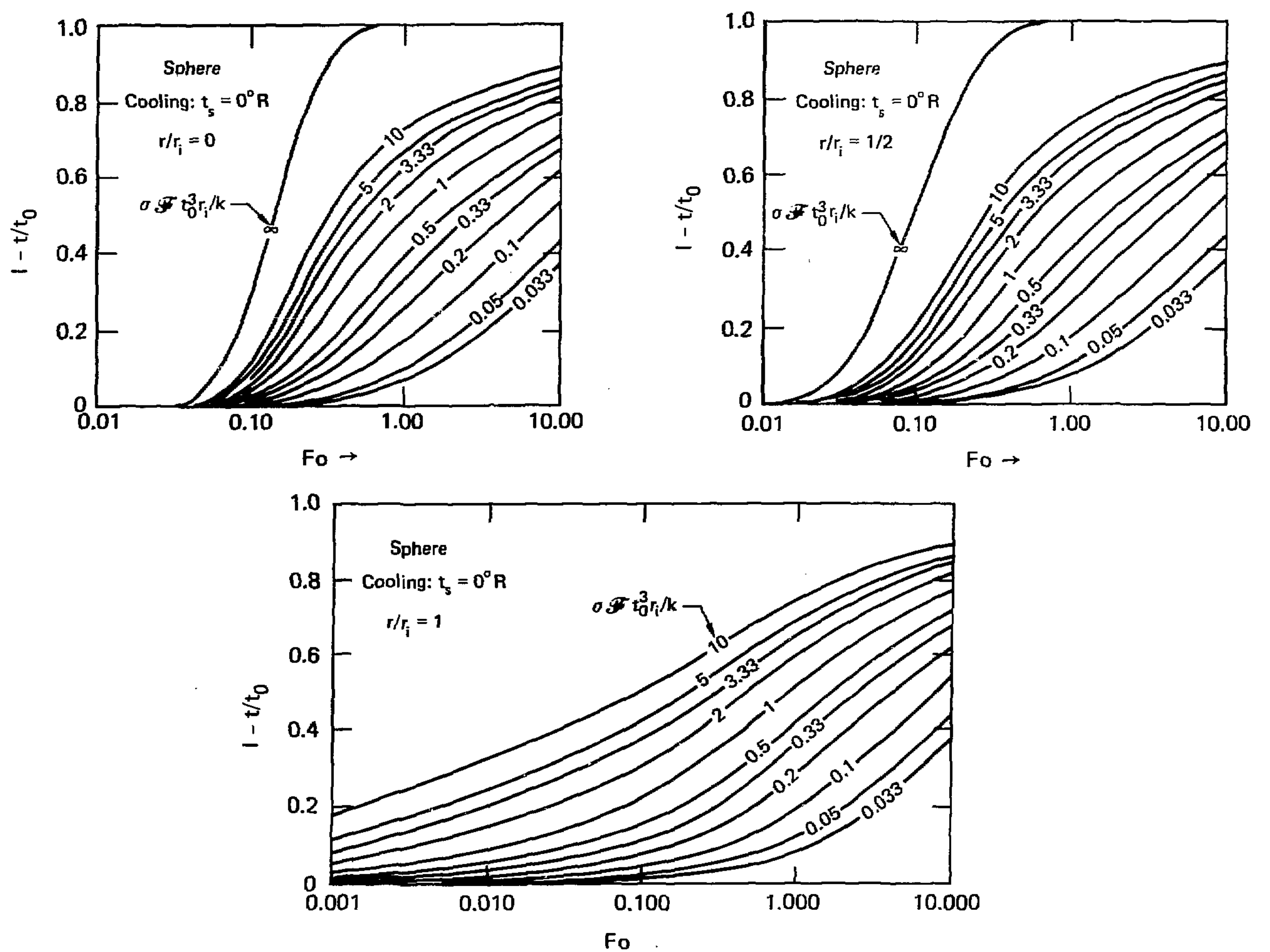


FIG. 10.6. Temperature response of solid sphere ($0 \leq r \leq r_1$) cooling by radiation to a sink temperature of $0^{o}R$: (a) $r/r_1 = 0$, (b) $r/r_1 = 1/2$, (c) $r/r_1 = 1$ (case 10.1.20, source: Ref. 19, pp. 3-57, Fig. 30).

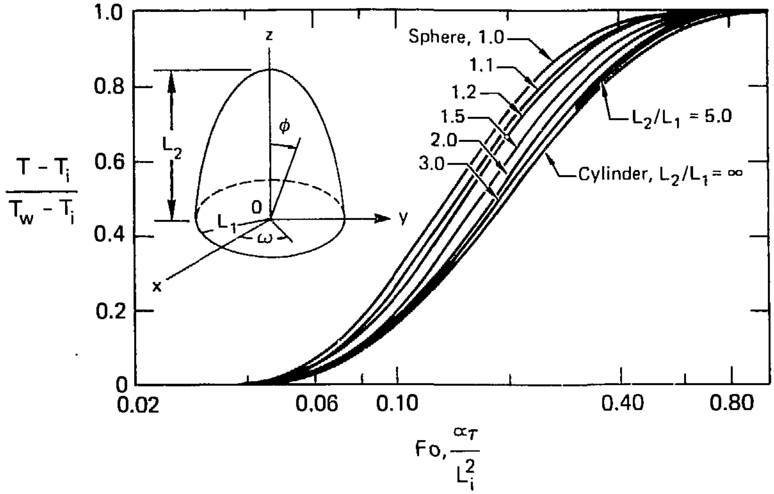


FIG. 10.7a. Transient temperature distribution at the center point (x = y = z = 0) for various prolate spheroids (case 10.1.21, source: Ref. 72, Fig. 1).

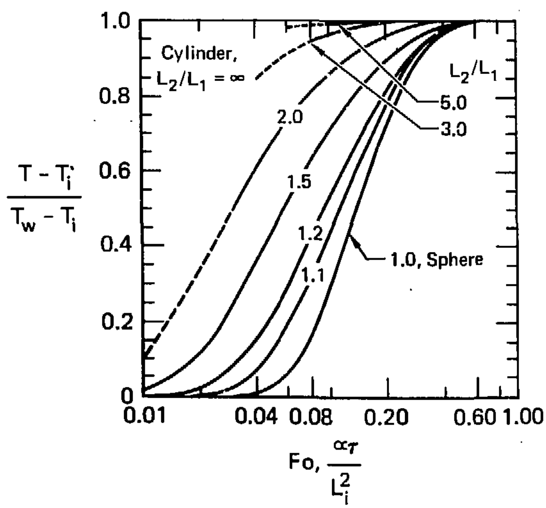


FIG. 10.7b. Transient temperature distribution at the focal point (x = y = 0, z = L_2) for various prolate solids (case 10.1.21, source: Ref. 72, Fig. 2).

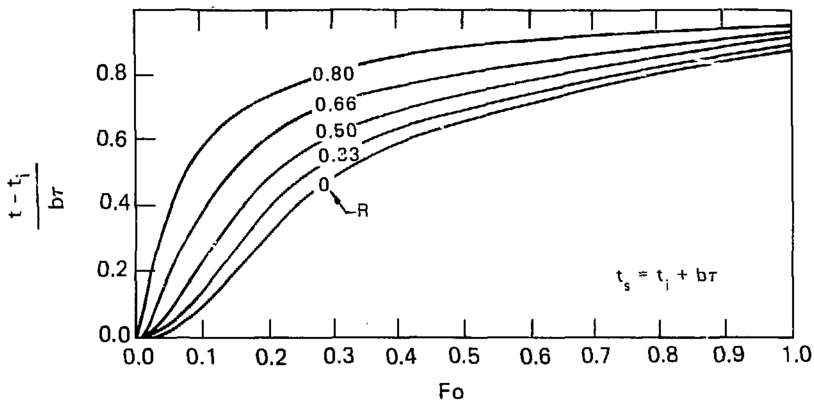


FIG. 10.8. Temperature of a sphere having a ramp surface te.:perature equal to $t_s = t_i + b\tau$ (case 10.1.4, source: Ref. 74, p. 310, Fig. 7.?\.

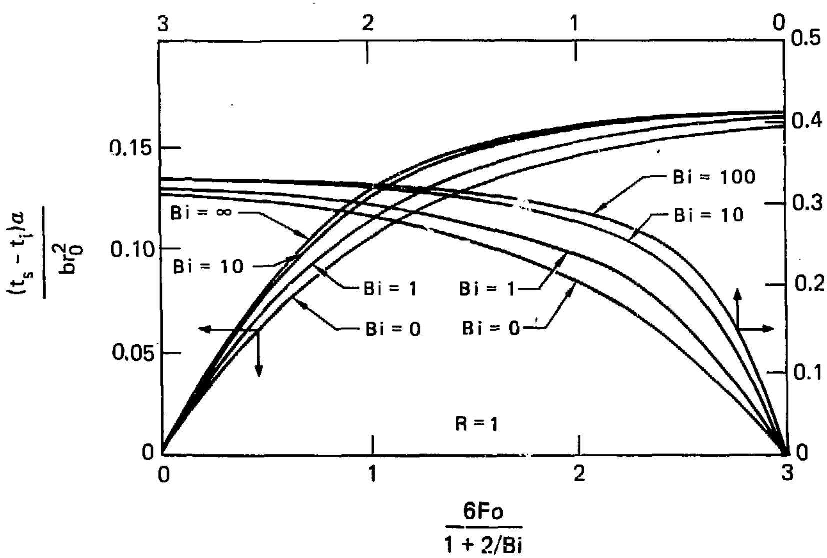


FIG. 10.9. Transient surface temperature of a sphere convectively coupled to a linearly changing environment temperature equal to $t_f = t_1 + b\tau$ (case 10.1.11, source: Ref. 89).

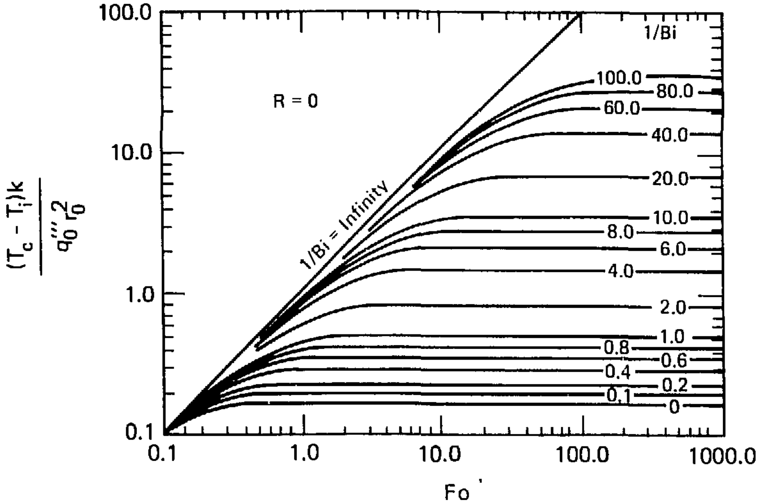


FIG. 10.10. Central temperature of a sphere with uniform internal heating and convection boundary (case 10.2.9, source: Ref. 90).

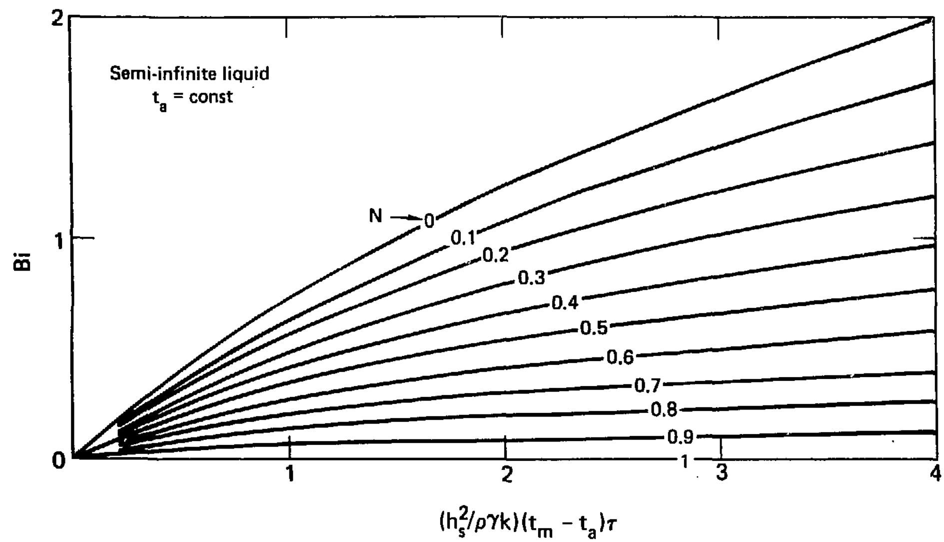


FIG. 11.1. Solidification depth in semi-infinite liquid for which solidified phase has negligible thermal capacity and is exposed to convective environments t_a at free surface x = 0 and t_o at interface x = w (case 11.1.10, source: Ref. 19, pp. 3-88, Fig. 54).

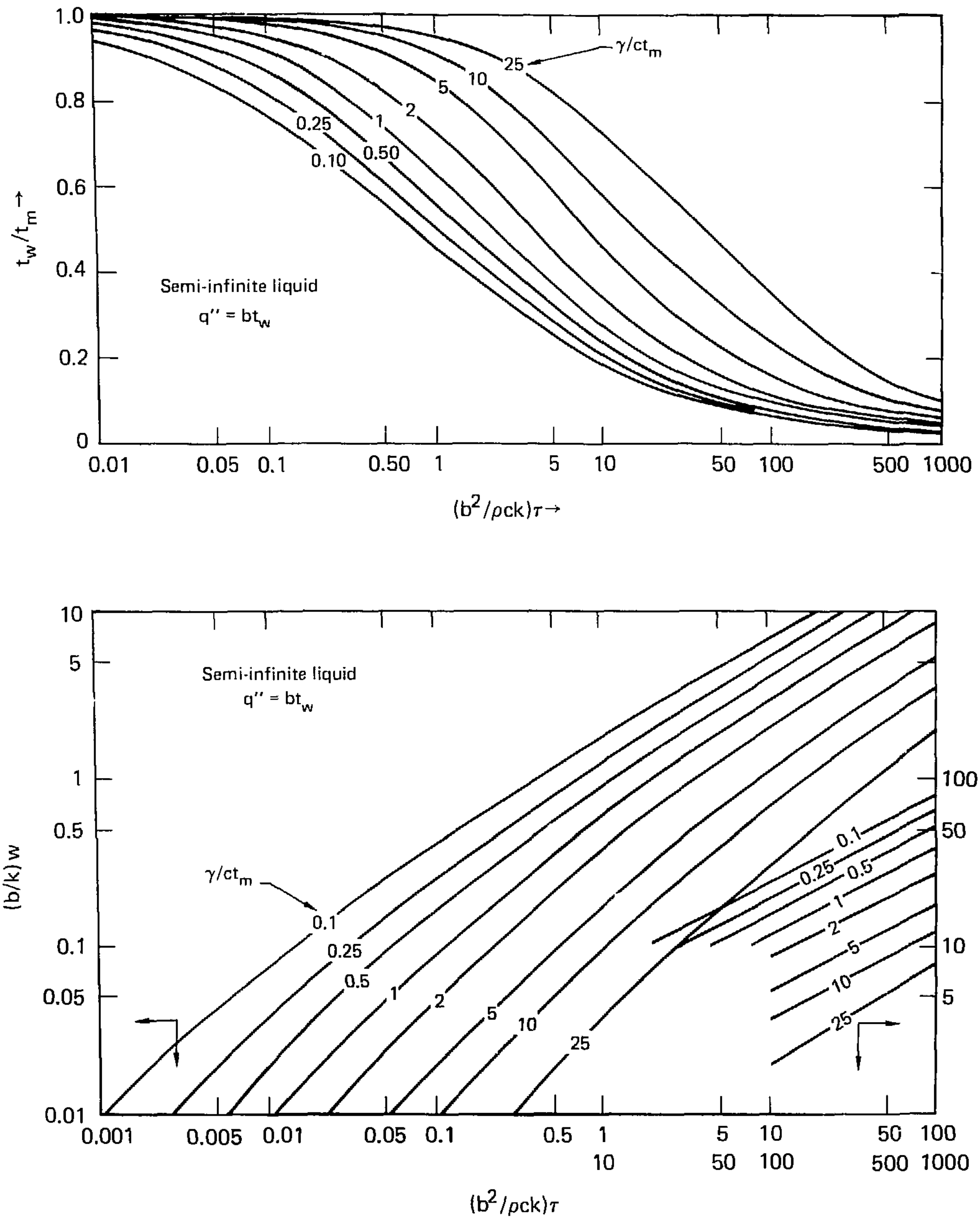


FIG. 11.2. Temperature response and solidification depth in semi-infinite liquid initially at t_m (a) surface temperature for convective cooling $q = bt_w$ at $x = 0$, (b) solidification depth (case 11.1.11, source: Ref. 19, pp. 3-89, Fig. 55).

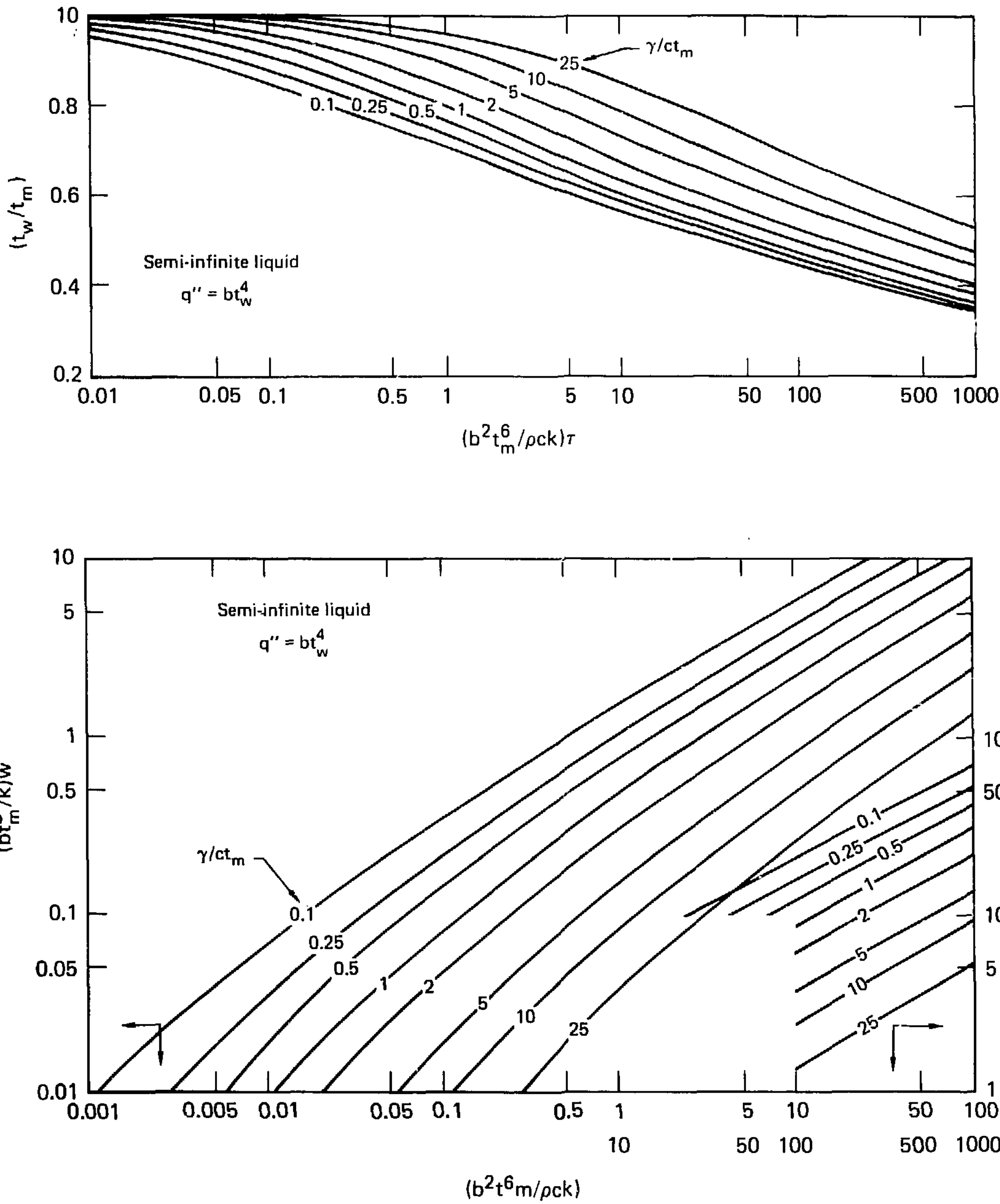
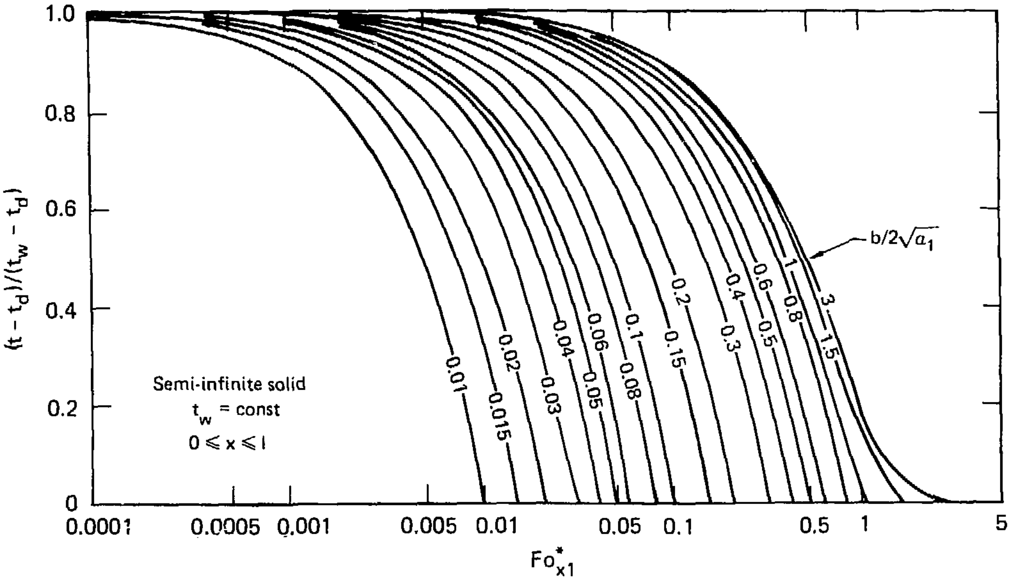


FIG. 11.3. Temperature response and solidification depth in semi-infinite liquid initially at t_m with radiative cooling $q = bt_w^4$ at $x = 0$ (a) surface temperature at $x = 0$, (b) solidification depth (case 11.1.12, source: Ref. 19, pp. 3-90, Fig. 55).



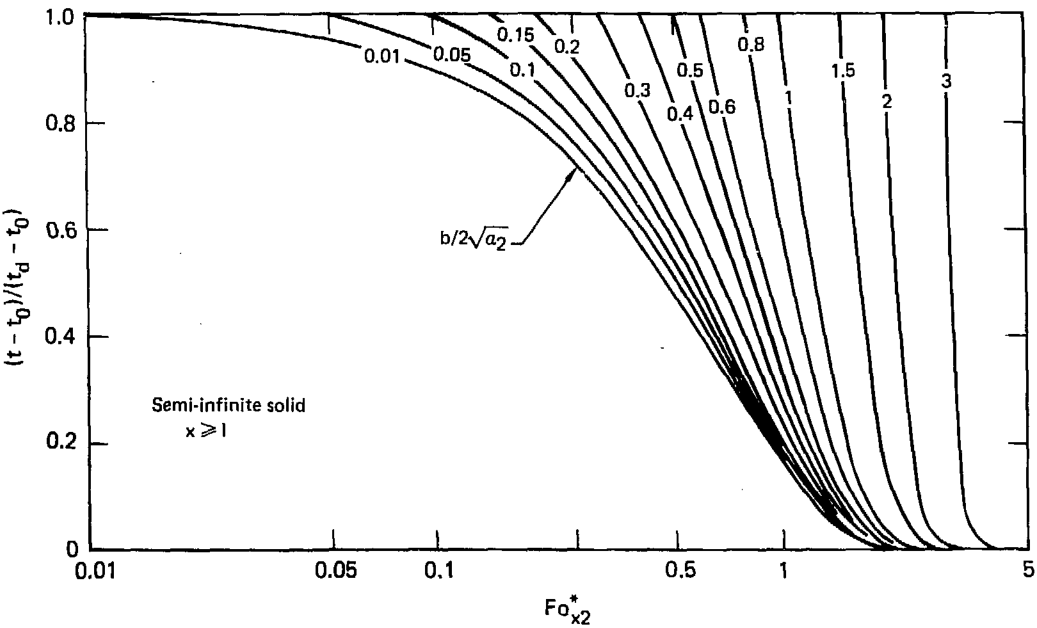


FIG. 11.4. Temperature response of decomposing semi-infinite solid after sudden change in surface temperature from t_0 to t_w: (a) $0 \leq x \leq \ell$, (b) $x \leq \ell$ (case 11.1.13, source: Ref. 19, pp. 3-91, Fig. 56).

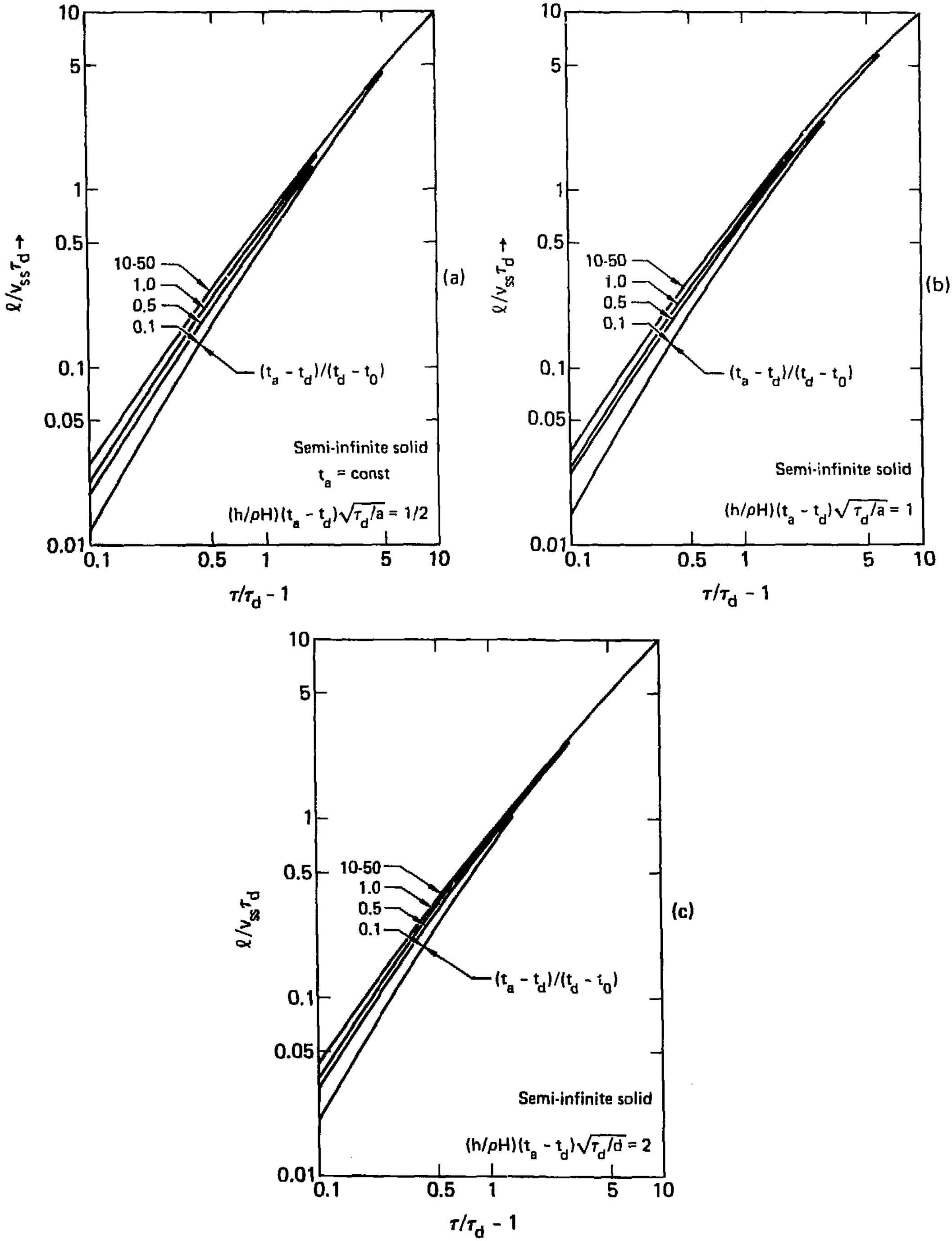


FIG. 11.5. Ablation depth of semi-infinite solid ($x \geq 0$) after sudden exposure to convective environment $t_a (> t_d)$: (a) $(h/\rho H)(t_a - t_d)\sqrt{\tau_d/\alpha} = 1/2$, (b) $(h/\rho H)(t_a - t_d)\sqrt{\tau_d/\alpha} = 1$, (c) $(h/\rho H)(t_a - t_d)\sqrt{\tau_d/\alpha} = 2$ (case 11.1.14, source: Ref. 19, pp. 3-94, Fig. 58).

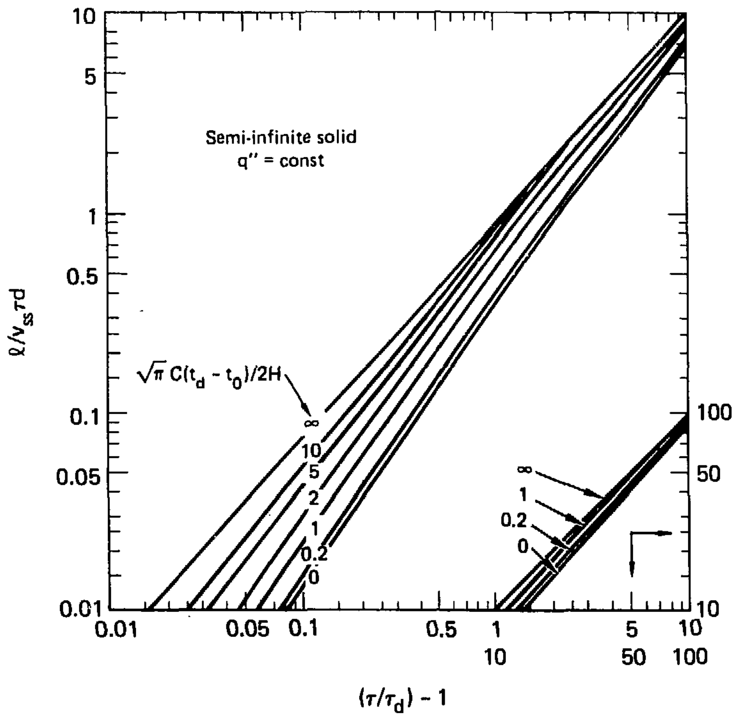


FIG. 11.6. Ablation depth of semi-infinite solid ($x \geq 0$) after sudden exposure to constant surface heat input q" (case 11.1.15, source: Ref. 19, pp. 3-96, Fig. 59).

TABLE 1.1. Mean and maximum temperature excesses and their ratio for electrical coils of rectangular and circular cross sections, $\theta_m = t_{mean} - t_0$, $\theta_0 = t_{max} - t_0$ (case 1.2.8, source: Ref. 3, p. 220, Table 10-3).

Cross Section,	Rectangular						Circular,
b/a	∞	10	5	2.5	1.5	1	$r = 2a/\sqrt{\pi}$
$(2k/q'''a^2)\theta_m$	0.66	0.625	0.58			0.28	0.32
$(2k/q'''a^2)\theta_0$	1.00	1.00	1.00			0.59	0.64
$\phi = \theta_m/\theta_0$	0.66	0.625	0.58	0.52	0.485	0.475	0.50

TABLE 1.2a. Values of $(t - t_1)/(t_f - t_1)$ on the surface y = 0 of a semi-infinite strip (case 1.1.29, source: Ref. 15, Table 2).

x/w	hw/k = 100	hw/k = 10	hw/k = 1	hw/k = 0.10	hw/k = 0.01
0.05	0.889	0.448	0.094	0.011	0.001
0.10	0.931	0.594	0.146	0.018	0.002
0.20	0.964	0.725	0.212	0.027	0.003
0.30	0.973	0.781	0.250	0.032	0.003
0.40	0.977	0.807	0.270	0.035	0.004
0.50	0.978	0.814	0.277	0.036	0.004

TABLE 1.2b. Heat transfer rates through the surface y = 0 of a semi-infinite strip (case 1.1.29, source: Ref. 15, Table 1) $\theta = (t_1 - t_f)$.

	hw/k								
	1000	200	100	20	10	1.0	0.1	0.01	0.001
$Q/k\theta_g$	8.603	6.827	5.984	3.971	3.107	0.792	0.097	0.01	0.001
% Error	1.8	0.5	0.3	0.08	0.05	0.02	0.02	0.02	0.02

TABLE 1.3. Conductance data for heat flow normal to wall cuts in an infinite plate (case 1.1.37, source: Ref. 19, p. 3-124). These groups of data are for the four kinds of wall plates shown in case 1.1.37.

(a)	K/K_{uncut}		
c/a	b/a = 1/2	1/4	1/8
4	0.902	0.760	0.646
2	0.818	0.618	0.480
1	0.704	0.465	0.339
1/2	0.610	0.363	0.235
1/4	0.564	0.303	0.182

(b)	K/K_{uncut}		
c/a	b/a = 1/2	1/4	1/8
3/2	0.747	0.520	0.381
3/4	0.575	0.315	0.217
1/2	0.430	0.209	0.146
1/3	0.296	0.113	0.080

(c)		K/K_{uncut}		
c/a	d/a	b/a = 1/2	1/4	1/8
3	1/8	0.822	0.634	0.478
	3/8	0.767	0.546	0.374
	5/8	0.721	0.480	0.311
	12/8	0.599	0.339	0.190
3/2	1/16	0.740	0.486	0.342
	3/16	0.698	0.430	0.284
	5/16	0.661	0.390	0.242
	12/16	0.570	0.299	0.164
1/2	0	0.627	0.403	0.270
	1/16	0.561	0.312	0.186
	3/16	0.527	0.276	0.152

(d)		K/K_{uncut}		
c/a	d/a	b/a = 1/4	1/8	1/16
3/2	0.402	0.507		
	0.469		0.362	
	0.502			0.310
	0.866	0.479		
	1.010		0.338	
	1.083			0.254
1/2	1/2	0.329	0.213	0.141

TABLE 2.1. Values of Δ_j for a triangular cooling array of cylinders (case 2.2.18, source: Ref. 64, Table 2).

s/r_0	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7
4.0	$-5.05072(10^{-2})$	$-8.0891(10^{-4})$	$-1.262(10^{-5})$	$-1.97(10^{-7})$	$-3.08(10^{-9})$	$-4.8(10^{-11})$	$-7(10^{-13})$
2.0	$-5.04988(10^{-2})$	$-8.0542(10^{-4})$	$-1.217(10^{-5})$	$-1.64(10^{-7})$	$-1.33(10^{-9})$	$2.8(10^{-11})$	$2(10^{-12})$
1.5	$-5.02447(10^{-2})$	$-6.9920(10^{-4})$	$-1.374(10^{-6})$	$8.34(10^{-7})$	$5.23(10^{-7})$	$2.4(10^{-9})$	$8(10^{-11})$
1.3	$-4.90792(10^{-2})$	$-2.1203(10^{-4})$	$6.354(10^{-5})$	$5.42(10^{-6})$	$2.99(10^{-7})$	$1.3(10^{-8})$	$4(10^{-10})$
1.2	$-4.69398(10^{-2})$	$6.7812(10^{-4})$	$1.749(10^{-4})$	$1.31(10^{-5})$	$6.27(10^{-7})$	$1.7(10^{-8})$	$-1(10^{-10})$
1.175	$-4.60073(10^{-2})$	$1.0619(10^{-3})$	$2.205(10^{-4})$	$+1.56(10^{-5})$	$6.32(10^{-7})$	$9.8(10^{-10})$	$-2(10^{-9})$
1.15	$-4.48345(10^{-2})$	$1.5384(10^{-3})$	$2.736(10^{-4})$	$1.76(10^{-5})$	$4.21(10^{-7})$	$-5\ (10^{-8})$	$-7(10^{-9})$
1.10	$-4.15694(10^{-2})$	$2.8029(10^{-3})$	$3.782(10^{-4})$	$1.08(10^{-5})$	$-2.40(10^{-6})$	$-5\ (10^{-7})$	$-5(10^{-9})$
1.05	$-3.68059(10^{-2})$	$4.3358(10^{-3})$	$3.169(10^{-4})$	$5.67(10^{-5})$	$-1.51(10^{-5})$	$-1.7(10^{-6})$	$-9(10^{-8})$

TABLE 2.2. Values of δ_j for a square cooling array of cylinders (case 2.2.19, source: Ref. 64, Table 1).

s/r_0	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
4.0	$-1.25382(10^{-1})$	$-1.0583(10^{-2})$	$-6.120(10^{-4})$	$-3.898(10^{-5})$	$-2.42(10^{-6})$	$-1.5(10^{-7})$	$-8\ (10^{-9})$
2.0	$-1.25098(10^{-1})$	$-1.0428(10^{-2})$	$-5.713(10^{-4})$	$-3.177(10^{-5})$	$-1.39(10^{-6})$	$-2\ (10^{-8})$	$3\ (10^{-9})$
1.5	$-1.22597(10^{-1})$	$-9.060\ (10^{-3})$	$-2.08\ (10^{-4})$	$3.33\ (10^{-5})$	$8.1\ (10^{-6})$	$1\ (10^{-6})$	$1\ (10^{-7})$
1.3	$-1.17022(10^{-1})$	$-5.995\ (10^{-3})$	$6.13\ (10^{-4})$	$1.83\ (10^{-4})$	$3.1\ (10^{-5})$	$4\ (10^{-6})$	$4\ (10^{-7})$
1.2	$-1.10421(10^{-1})$	$-2.387\ (10^{-3})$	$1.566(10^{-3})$	$3.50\ (10^{-4})$	$5.3\ (10^{-5})$	$6\ (10^{-6})$	$5\ (10^{-7})$
1.15	$-1.05352(10^{-1})$	$3.172\ (10^{-4})$	$2.233(10^{-3})$	$4.44\ (10^{-4})$	$5.8\ (10^{-5})$	$4.3(10^{-6})$	$1\ (10^{-8})$
1.1	$-9.8721\ (10^{-2})$	$3.6596(10^{-3})$	$2.911(10^{-3})$	$4.655(10^{-4})$	$2.44(10^{-5})$	$-1.1(10^{-5})$	$-5.3(10^{-6})$
1.05	$-9.0358\ (10^{-2})$	$7.310\ (10^{-3})$	$3.210(10^{-3})$	$2.06\ (10^{-4})$	$-1.20(10^{-4})$	$-5.8(10^{-5})$	$-1.6(10^{-5})$

TABLE 8.1. Values of $(t - t_i)/(t_s - t_i)$ for an infinitely wide plate whose surface temperature increases proportionally to time, $t_s = t_i + b\tau$ (case 8.1.8, source: Ref. 1, p. 268, Table 13.5).

	Fo								
X	0.08	0.16	0.20	0.32	0.40	0.80	1.60	2.00	4.00
0	0.004	0.045	0.074	0.170	0.231	0.464	0.694	0.752	0.875
0.33	0.030	0.104	0.142	0.245	0.305	0.522	0.728	0.779	0.889
0.5	0.082	0.194	0.238	0.346	0.402	0.594	0.770	0.814	0.906
0.66	0.214	0.353	0.398	0.498	0.545	0.698	0.830	0.862	0.930
0.8	0.418	0.548	0.586	0.664	0.699	0.802	0.889	0.911	0.955
1	1	1	1	1	1	1	1	1	1

TABLE 8.2. Values of the function ζ (case 8.1.12, source: Ref. 1, p. 299).

	X								
σ	0	1/8	2/8	3/8	4/8	5/8	6/8	7/8	1
0	1	1	1	1	1	1	1	1	1
0.5	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.99	1
1.0	0.77	0.77	0.77	0.78	0.79	0.81	0.85	0.91	1
1.5	0.47	0.47	0.47	0.48	0.52	0.58	0.68	0.83	1
2.0	0.27	0.27	0.28	0.30	0.36	0.45	0.58	0.77	1
4.0	0.04	0.04	0.05	0.08	0.13	0.22	0.37	0.64	1
8.0	0.00	0.00	0.01	0.01	0.02	0.05	0.14	0.36	1
∞	0	0	0	0	0	0	0	0	

TABLE 9.1. Values of $(t - t_i)/(t_s - t_i)$ for an infinitely long cylinder whose surface increases proportionally to time $(t_s - t_i = c\tau)$ (case 9.1.2, source: Ref. 1, p. 269).

R	Fo			
	0.08	0.16	0.32	0.80
0	0.016	0.123	0.354	0.691
0.33	0.054	0.191	0.420	0.725
0.5	0.122	0.287	0.505	0.768
0.66	0.268	0.443	0.628	0.828
0.8	0.470	0.621	0.755	0.888
1	1	1	1	1

TABLE 10.1. Temperatures for a sphere whose surface temperature increases proportionally to time $(t_0 = t_i + b\tau)$ (case 10.1.4, source: Ref. 1, p. 269, Table 13-7).

R	Fo				
	0.016	0.08	0.16	0.32	0.80
0	0.00	0.054	0.219	0.506	0.792
0.33	0.00	0.090	0.290	0.560	0.725
0.5	0.00	0.162	0.385	0.626	0.844
0.66	0.02	0.312	0.529	0.722	0.884
0.8	0.14	0.516	0.686	0.819	0.925
1	1	1	1	1	1

Misc. Data

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13.1. The Error Function and Related Functions.

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-z^2)\, dz$$

$$= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)\, n!} \text{ , for small values of } x$$

$$= 1 - \frac{\exp(-x^2)}{\sqrt{\pi}} \left(\frac{1}{x} - \frac{1}{2x^3} + \frac{1}{2^2} \cdot \frac{3}{x^5} - \frac{1}{2^3} \cdot \frac{3}{x^7} \cdot 5 + \ldots \right) \text{ . for large values of } x$$

$\mathrm{erf}(0) = 0$, $\mathrm{erf}(\infty) = 1$

$\mathrm{erf}(-x) = -\,\mathrm{erf}(x)$

$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$

$$i^n\mathrm{erfc}(x) = \int_x^{\infty} i^{n-1}\mathrm{erfc}(z)\, dz \quad , n = 1, 2, \ldots,$$

$i^0\mathrm{erfc}(x) = \mathrm{erfc}(x)$

$$\mathrm{ierfc}(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2) - x\, \mathrm{erfc}(x)$$

$$i^2\mathrm{erfc}(x) = \frac{1}{4} \left[\mathrm{erfc}(x) - 2x\, \mathrm{ierfc}(x)\right]$$

$$2n\, i^n\mathrm{erfc}(x) = i^{n-2}\mathrm{erfc}(x) - 2x\, i^{n-1}\mathrm{erfc}(x)$$

$$\frac{d}{dx} \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \exp(-x^2)$$

$$\frac{d^2}{dx^2} \mathrm{erf}(x) = -\frac{4x}{\sqrt{\pi}} \exp(-x^2)$$

TABLE 13.1. The error function, its derivatives and integrals (source: Ref. 9, p. 485).

x	e^{x^2} erfc(x)	$4\pi^{-1/2}xe^{-x^2}$	$2\pi^{-1/2}e^{-x^2}$	erf(x)	erfc(x)	2 ierfc(x)	4 i^2erfc(x)	6 i^2erfc(x)	8 i^4erfc(x)	10 i^5erfc(x)	12 i^4erfc(x)
0	1.0	0	1.1284	0	1.0	1.284	1.0	0.5642	0.25	0.0940	0.0313
0.05	0.9460	0.1126	1.1256	0.056372	0.943628	1.0312	0.8921	0.4933	0.2148	0.0795	0.0261
0.1	0.8965	0.2234	1.1172	0.112463	0.887537	0.9396	0.7936	0.4301	0.1841	0.0671	0.0217
0.15	0.8509	0.3310	1.1033	0.167996	0.832004	0.8537	0.7040	0.3740	0.1573	0.0564	0.0180
0.2	0.8090	0.4336	1.0841	0.222703	0.777297	0.7732	0.6227	0.3243	0.1341	0.0474	0.0149
0.25	0.7703	0.5300	1.0600	0.276326	0.723674	0.6982	0.5491	0.2805	0.1139	0.0396	0.0123
0.3	0.7346	0.6188	1.0313	0.328627	0.671373	0.6284	0.4828	0.2418	0.0965	0.0331	0.0101
0.35	0.7015	0.6988	0.9983	0.379382	0.620618	0.5639	0.4233	0.2079	0.0816	0.0275	0.0083
0.4	0.6708	0.7692	0.9615	0.428392	0.571608	0.5043	0.3699	0.1782	0.0687	0.0228	0.0068
0.45	0.6423	0.8294	0.9215	0.475482	0.524518	0.4495	0.3223	0.1522	0.0577	0.0189	0.0055
0.5	0.6157	0.8788	0.8788	0.520500	0.479500	0.3993	0.2799	0.1297	0.0484	0.0156	0.0045
0.55	0.5909	0.9172	0.8338	0.563323	0.436677	0.3535	0.2423	0.1101	0.0404	0.0128	0.0036
0.6	0.5678	0.9447	0.7872	0.603856	0.396144	0.3119	0.2090	0.0932	0.0336	0.0105	0.0029
0.65	0.5462	0.9614	0.7395	0.642029	0.357971	0.2742	0.1798	0.0787	0.0279	0.0086	0.0024
0.7	0.5259	0.9678	0.6913	0.677801	0.322199	0.2402	0.1541	0.0662	0.0231	0.0070	0.0019
0.75	0.5069	0.9644	0.6429	0.711156	0.288844	0.2097	0.1316	0.0555	0.0190	0.0057	0.0015
0.8	0.4891	0.9520	0.5950	0.742101	0.257899	0.1823	0.1120	0.0464	0.0156	0.0046	0.0012
0.85	0.4723	0.9314	0.5479	0.770668	0.229332	0.1580	0.0950	0.0386	0.0128	0.0037	0.0010
0.9	0.4565	0.9035	0.5020	0.796908	0.203092	0.1364	0.0803	0.0321	0.0104	0.0030	0.0008
0.95	0.4416	0.8695	0.4576	0.820891	0.179109	0.1173	0.0677	0.0265	0.0085	0.0024	0.0006
1.0	0.4276	0.8302	0.4151	0.842701	0.157299	0.1005	0.0568	0.0218	0.0069	0.0019	0.0005
1.1	0.4017	0.7403	0.3365	0.880205	0.119795	0.0729	0.0396	0.0147	0.0045	0.0012	0.0003
1.2	0.3785	0.6416	0.2673	0.910314	0.089686	0.0521	0.0272	0.0097	0.0029	0.0007	0.0002
1.3	0.3576	0.5413	0.2082	0.934008	0.065992	0.0366	0.0184	0.0063	0.0019	0.0004	0.0001
1.4	0.3387	0.4450	0.1589	0.952285	0.047715	0.0253	0.0122	0.0041	0.0011	0.0003	0.0001
1.5	0.3216	0.3568	0.1189	0.966105	0.033895	0.0172	0.0080	0.0026	0.0007	0.0002	
1.6	0.3060	0.2791	0.0872	0.976348	0.023652	0.0115	0.0052	0.0016	0.0004	0.0001	
1.7	0.2917	0.2132	0.0627	0.903790	0.016210	0.0076	0.0033	0.0010	0.0003		
1.8	0.2786	0.1591	0.0442	0.989091	0.010909	0.0049	0.0021	0.0006	0.0002		
1.9	0.2665	0.1160	0.0305	0.992790	0.007210	0.0031	0.0013	0.0003	0.0001		
2.0	0.2554	0.0827	0.0207	0.995322	0.004678	0.0020	0.0008	0.0002	0.0001		
2.1	.02451	0.0576	0.0137	0.997021	0.002979	0.0012	0.0005	0.0001			
2.2	0.2356	0.0393	0.0089	0.998137	0.001863	0.0007	0.0003				
2.3	0.2267	0.0262	0.0057	0.998857	0.001143	0.0004	0.0002				
2.4	0.2185	0.0171	0.0036	0.999311	0.000689	0.0002	0.0001				
2.5	0.2108	0.0109	0.0022	0.999593	0.000407	0.0001					
2.6	0.2036	0.0068	0.0013	0.000236	0.000236	0.0001					
2.7	0.1969	0.0042	0.0008	0.999866	0.000134						
2.8	0.1905	0.0025	0.0004	0.999925	0.000075						
2.9	0.1846	0.0015	0.0003	0.999959	0.000041						
3.0	0.1790	0.0008	0.0001	0.999978	0.000022						

13.2. Exponential and Hyperbolic Functions.

$e = 2.71828 \ldots$

$\exp(u) = e^{u}$

$$\sinh(u) = \frac{e^{u} - e^{-u}}{2}, \quad \sinh(0) = 0, \quad \sinh(\infty) = \infty$$

$$\cosh(u) = \frac{e^{u} + e^{-u}}{2}, \quad \cosh(0) = 1, \quad \cosh(\infty) = \infty$$

$$\tanh(u) = \frac{e^{2u} - 1}{e^{2u} + 1}, \quad \tanh(0) = 0, \quad \tanh(\infty) = 1$$

TABLE 13.2. Exponential functions (source: Ref. 2, pp. 367-9).

u	e^u	e^{-u}	u	e^u	e^{-u}	u	e^u	e^{-u}
0.00	1.000	1.000	0.60	1.822	0.549	1.20	3.320	0.301
0.02	1.020	0.980	0.62	1.859	0.538	1.22	3.387	0.295
0.04	1.041	0.961	0.64	1.896	0.527	1.24	3.456	0.289
0.06	1.062	0.942	0.66	1.935	0.517	1.26	3.525	0.284
0.08	1.083	0.923	0.68	1.974	0.507	1.28	3.597	0.278
0.10	1.105	0.905	0.70	2.014	0.497	1.30	3.669	0.272
0.12	1.128	0.887	0.72	2.054	0.487	1.32	3.743	0.267
0.14	1.150	0.869	0.74	2.096	0.477	1.34	3.819	0.262
0.16	1.174	0.852	0.76	2.138	0.468	1.36	3.896	0.257
0.18	1.197	0.835	0.78	2.182	0.458	1.38	3.975	0.252
0.20	1.221	0.819	0.80	2.226	0.449	1.40	4.055	0.247
0.22	1.246	0.802	0.82	2.270	0.440	1.42	4.137	0.242
0.24	1.271	0.787	0.84	2.316	0.432	1.44	4.221	0.237
0.26	1.297	0.771	0.86	2.363	0.423	1.46	4.306	2.232
0.28	1.323	0.756	0.88	2.411	0.415	1.48	4.393	0.228
0.30	1.350	0.741	0.90	2.460	0.407	1.50	4.482	0.223
0.32	1.377	0.726	0.92	2.509	0.398	1.52	4.572	0.219
0.34	1.405	0.712	0.94	2.560	0.391	1.54	4.665	0.214
0.36	1.433	0.698	0.96	2.612	0.383	1.56	4.759	0.210
0.38	1.462	0.684	0.98	2.664	0.375	1.58	4.855	0.206
0.40	1.492	0.670	1.00	2.718	0.368	1.60	4.953	0.202
0.42	1.522	0.657	1.02	2.773	0.361	1.62	5.053	0.198
0.44	1.553	0.644	1.04	2.829	0.353	1.64	5.155	0.194
0.46	1.584	0.631	1.06	2.886	0.346	1.66	5.259	0.190
0.48	1.616	0.619	1.08	2.945	0.340	1.68	5.366	0.186
0.50	1.649	0.606	1.10	3.004	0.333	1.70	5.474	0.183
0.52	1.682	0.594	1.12	3.065	0.326	1.72	5.584	0.179
0.54	1.716	0.583	1.14	3.127	0.320	1.74	5.697	0.176
0.56	1.751	0.571	1.16	3.190	0.313	1.76	5.812	0.172
0.58	1.786	0.560	1.18	3.254	0.307	1.78	5.930	0.169

TABLE 13.2. (Continued).

u	e^u	e^{-u}	u	e^u	e^{-u}	u	e^u	e^{-u}
1.80	6.050	0.165	2.40	11.023	0.091	3.00	20.08	0.050
1.82	6.172	0.162	2.42	11.246	0.089	3.10	22.20	0.045
1.84	6.296	0.159	2.44	11.473	0.087	3.20	24.53	0.041
1.86	6.424	0.156	2.46	11.705	0.085	3.30	27.11	0.037
1.88	6.554	0.153	2.48	11.941	0.084	3.40	29.96	0.033
1.90	6.686	0.150	2.50	12.18	0.082	3.50	33.11	0.030
1.92	6.821	0.147	2.52	12.43	0.080	3.60	36.60	0.027
1.94	6.959	0.144	2.54	12.68	0.079	3.70	40.45	0.025
1.96	7.099	0.141	2.56	12.94	0.077	3.80	44.70	0.022
1.98	7.243	0.138	2.58	13.20	0.076	3.90	49.40	0.020
2.00	7.389	0.135	2.60	13.46	0.074	4.00	54.60	0.018
2.02	7.538	0.133	2.62	13.74	0.073	4.20	66.69	0.015
2.04	7.691	0.130	2.64	14.01	0.071	4.40	81.45	0.012
2.06	7.846	0.127	2.66	14.30	0.070	4.60	99.48	0.010
2.08	8.004	0.125	2.68	14.58	0.069	4.80	121.51	0.008
2.10	8.166	0.122	2.70	14.88	0.067	5.00	148.4	0.007
2.12	8.331	0.120	2.72	15.18	0.066	5.20	181.3	0.006
2.14	8.499	0.118	2.74	15.49	0.065	5.40	221.4	0.004
2.16	8.671	0.115	2.76	15.80	0.063	5.60	270.4	0.004
2.18	8.846	0.113	2.78	16.12	0.062	5.80	330.3	0.003
2.20	9.025	0.111	2.80	16.44	0.061	6.00	403.4	0.002
2.22	9.207	0.109	2.82	16.78	0.060	6.50	665.1	0.002
2.24	9.393	0.106	2.84	17.12	0.058	7.00	1096.6	0.001
2.26	9.583	0.104	2.86	17.46	0.057	7.50	1808.0	0.001
2.28	9.777	0.102	2.88	17.81	0.056	8.00	2981.0	0.000
2.30	9.974	0.100	2.90	18.17	0.055	8.50	4914.8	0.000
2.32	10.176	0.098	2.92	18.54	0.054	9.00	8103.1	0.000
2.34	10.381	0.096	2.94	18.92	0.053	9.50	13360	0.000
2.36	10.591	0.094	2.96	19.30	0.052	10.00	22026	0.000
2.38	10.805	0.093	2.98	19.69	0.051			

13.3. The Gamma Function.

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u}\, du, \quad x > 0$$

$$\Gamma(x) \simeq x^x e^{-x} \sqrt{\frac{2\pi}{x}} \left[1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{5184x^3} - \frac{571}{2488320x^4} + \cdots\right],$$

for large positive values of x

$$\Gamma(x + 1) = x\Gamma(x)$$

TABLE 13.3. The gamma function (source: Ref. 17, p. 136).

x	Γ(x)	x	Γ(x)	x	Γ(x)	x	Γ(x)	x	Γ(x)
1.01	0.99 433	1.21	0.91 558	1.41	0.88 676	1.61	0.89 468	1.81	0.93 408
1.02	0.98 884	1.22	0.91 311	1.42	0.88 636	1.62	0.89 592	1.82	0.93 685
1.03	0.98 355	1.23	0.91 075	1.43	0.88 604	1.63	0.89 724	1.83	0.93 969
1.04	0.97 844	1.24	0.90 852	1.44	0.88 581	1.64	0.89 864	1.84	0.94 261
1.05	0.97 350	1.25	0.90 640	1.45	0.88 566	1.65	0.90 012	1.85	0.94 561
1.06	0.97 874	1.26	0.90 440	1.46	0.88 560	1.66	0.90 167	1.86	0.94 869
1.07	0.96 415	1.27	0.90 250	1.47	0.88 563	1.67	0.90 330	1.87	0.95 184
1.08	0.95 973	1.28	0.90 072	1.48	0.88 575	1.68	0.90 500	1.88	0.95 507
1.09	0.95 546	1.29	0.89 904	1.49	0.88 595	1.69	0.90 678	1.89	0.95 838
1.10	0.95 135	1.30	0.89 747	1.50	0.88 623	1.70	0.90 864	1.90	0.96 177
1.11	0.94 740	1.31	0.89 600	1.51	0.88 659	1.71	0.91 057	1.91	0.96 523
1.12	0.94 359	1.32	0.89 464	1.52	0.88 704	1.72	0.91 258	1.92	0.96 877
1.13	0.93 993	1.33	0.89 338	1.53	0.88 757	1.73	0.91 467	1.93	0.97 240
1.14	0.93 642	1.34	0.89 222	1.54	0.88 818	1.74	0.91 683	1.94	0.97 610
1.15	0.93 304	1.35	0.89 115	1.55	0.88 887	1.75	0.91 906	1.95	0.97 988
1.16	0.92 980	1.36	0.89 018	1.56	0.88 964	1.76	0.92 137	1.96	0.98 374
1.17	0.92 670	1.37	0.88 931	1.57	0.89 049	1.77	0.92 376	1.97	0.98 768
1.18	0.92 373	1.38	0.88 854	1.58	0.89 142	1.78	0.92 623	1.98	0.99 171
1.19	0.92 089	1.39	0.88 785	1.59	0.89 243	1.79	0.92 877	1.99	0.99 581
1.20	0.91 817	1.40	0.88 726	1.60	0.89 352	1.80	0.93 138	2.00	1.00 000

the positive sign used if $-\pi/2 < \arg u < 3\pi/2$ and the negative sign used if $-3\pi/2 < \arg u < \pi/2$.

$$K_n(u) = \frac{\pi}{2} \frac{I_{-n}(u) - I_n(u)}{\sin (n\pi)} \quad .$$

For n any positive integer:

$$K_n(u) = (-1)^{n+1} \left\{ \ln (u/2) + \gamma \right\} I_n(u) + \frac{(-1)^n}{2} \sum_{j=0}^{\infty} \frac{(u/2)^{n+2j}}{j!\,(n+j)!}$$

$$\times \left[\sum_{m=1}^{n+j} m^{-1} + \sum_{m=1}^{j} m^{-1} \right] + \frac{1}{2} \sum_{j=0}^{\infty} (-1)^j (u/2)^{-n+2j} \frac{(n-j-1)!}{j!} \, ,$$

where for $j=0$, $\sum_{m=1}^{j} m^{-1} = 0$.

For large values of u:

$$K_n(u) = \sqrt{\frac{\pi}{2u}}\, e^{-u} \left\{ 1 + \frac{4n^2 - 1^2}{1!\,8u} + \frac{(4n^2 - 1^2)(4n^2 - 3^2)}{2!\,(8u)^2} + O(u^{-3}) \right\}$$

$$\frac{d}{du} J_n(u) = J_{n-1}(u) - \frac{n}{u} J_n(u) = \frac{n}{u} J_n(u) - J_{n+1}(u)$$

$$\frac{d}{du} Y_n(u) = Y_{n-1}(u) - \frac{n}{u} Y_n(u) = \frac{n}{u} Y_n(u) - Y_{n+1}(u)$$

$$\frac{d}{du} I_n(u) = I_{n-1}(u) - \frac{n}{u} I_n(u) = I_{n+1}(u) + \frac{n}{u} I_n(u)$$

$$\frac{d}{du} K_n(u) = -K_{n-1}(u) - \frac{n}{u} K_n(u) = \frac{n}{u} K_n(u) - K_{n+1}(u)$$

13.4. Bessel Functions.

$$J_n(u) = \sum_{j=0}^{\infty} \frac{(-1)^j (u/2)^{n+2j}}{j!\Gamma(n+j+1)} \text{ , n is real and u may be complex}$$

$$J_n(u) = (-1)^n J_{-n}(u) \text{ , if n is an integer}$$

$$Y_n(u) = \frac{J_n(u)\cos(n\pi) - J_{-n}(u)}{\sin(n\pi)}$$

$$\pi Y_n(u) = 2\,\{\ell n\,(u/2) + \gamma\}\,J_n(u) - \sum_{j=0}^{\infty} \frac{(-1)^j (u/2)^{n+2j}}{j!(n+j)!}$$

$$\times \left[\sum_{m=1}^{j+n} m^{-1} + \sum_{m=1}^{j} m^{-1}\right] - \sum_{j=0}^{n-1} (u/2)^{-n+2j} \frac{(n-j-1)!}{j!}$$

where $\gamma = 0.5772...$ is Euler's constant, and n is any positive integer

$$\frac{\pi}{2} Y_0(u) = \{\ell n\,(u/2) + \gamma\}J_0(u) + (u/2)^2 - \left(1 + \frac{1}{2}\right)\frac{(u/2)^4}{(2!)^2}$$

$$+ \left(1 + \frac{1}{2} + \frac{1}{3}\right)\frac{(u/2)^6}{(3!)^2} - \ldots$$

$$I_n(u) = \sum_{j=0}^{\infty} \frac{(u/2)^{n+2j}}{j!\Gamma(n+j+1)}$$

For large values of u:

$$I_n(u) = \frac{1}{\sqrt{2\pi u}} \exp(u) \left\{1 - \frac{4n^2 - 1^2}{1!8u} + \frac{\left(4n^2 - 1^2\right)\left(4n^2 - 3^2\right)}{2!(8u)^2} + O\left(u^{-3}\right)\right\}$$

$$+ \frac{1}{\sqrt{2\pi u}} \exp\left[-u \pm \left(n + \frac{1}{2}\right)\pi i\right]\left\{1 + O\left(u^{-1}\right)\right\},$$

TABLE 13.4.1. Zero and first-order Bessel functions of the first kind (source: Ref 2, pp. 369-71).

u	$J_0(u)$	$J_1(u)$	u	$J_0(u)$	$J_1(u)$
0.0	1.0000	0.0000	3.0	-0.2600	0.3391
0.1	0.9975	0.0499	3.1	-0.2921	0.3009
0.2	0.9900	0.0995	3.2	-0.3202	0.2613
0.3	0.9776	0.1483	3.3	-0.3443	0.2207
0.4	0.9604	0.1960	3.4	-0.3643	0.1792
0.5	0.9385	0.2423	3.5	-0.3801	0.1374
0.6	0.9120	0.2867	3.6	-0.3918	0.0955
0.7	0.8812	0.3290	3.7	-0.3992	0.0538
0.8	0.8463	0.3688	3.8	-0.4026	0.0128
0.9	0.8075	0.4059	3.9	-0.4018	-0.0272
1.0	0.7652	0.4400	4.0	-0.3971	-0.0660
1.1	0.7196	0.4709	4.1	-0.3887	-0.1033
1.2	0.6711	0.4983	4.2	-0.3766	-0.1386
1.3	0.6201	0.5220	4.3	-0.3610	-0.1719
1.4	0.5669	0.5419	4.4	-0.3423	-0.2028
1.5	0.5118	0.5579	4.5	-0.3205	-0.2311
1.6	0.4554	0.5699	4.6	-0.2961	-0.2566
1.7	0.3980	0.5778	4.7	-0.2693	-0.2791
1.8	0.3400	0.5815	4.8	-0.2404	-0.2985
1.9	0.2818	0.5812	4.9	-0.2097	-0.3147
2.0	0.2239	0.5767	5.0	-0.1776	-0.3276
2.1	0.1666	0.5683	5.1	-0.1443	-0.3371
2.2	0.1104	0.5560	5.2	-0.1103	-0.3432
2.3	0.0555	0.5399	5.3	-0.0758	-0.3460
2.4	0.0025	0.5202	5.4	-0.0412	-0.3453
2.5	-0.0484	0.4971	5.5	-0.0068	-0.3414
2.6	-0.0968	0.4708	5.6	0.0270	-0.3343
2.7	-0.1424	0.4416	5.7	0.0599	-0.3241
2.8	-0.1850	0.4097	5.8	0.0917	-0.3110
2.9	-0.2243	0.3754	5.9	0.1220	-0.2951

TABLE 13.4.1. (Continued.)

u	$J_0(u)$	$J_1(u)$	u	$J_0(u)$	$J_1(u)$
6.0	0.1506	-0.2767	9.0	-0.0903	0.2453
6.1	0.1773	-0.2559	9.1	-0.1142	0.2324
6.2	0.2017	-0.2329	9.2	-0.1368	0.2174
6.3	0.2238	-0.2081	9.3	-0.1577	0.2004
6.4	0.2433	-0.1816	9.4	-0.1768	0.1816
6.5	0.2601	-0.1538	9.5	-0.1939	0.1613
6.6	0.2740	-0.1250	9.6	-0.2090	0.1395
6.7	0.2851	-0.0953	9.7	-0.2218	0.1166
6.8	0.2931	-0.0652	9.8	-0.2323	0.0928
6.9	0.2981	-0.0349	9.9	-0.2403	0.0684
7.0	0.3001	-0.0047	10.0	-0.2459	0.0435
7.1	0.2991	0.0252	10.1	-0.2490	0.0184
7.2	0.2951	0.0543	10.2	-0.2496	-0.0066
7.3	0.2882	0.0826	10.3	-0.2477	-0.0313
7.4	0.2786	0.1096	10.4	-0.2434	-0.0555
7.5	0.2663	0.1352	10.5	-0.2366	-0.0788
7.6	0.2516	0.1592	10.6	-0.2276	-0.1012
7.7	0.2346	0.1813	10.7	-0.2164	-0.1224
7.8	0.2154	0.2014	10.8	-0.2032	-0.1422
7.9	0.1944	0.2192	10.9	-0.1881	-0.1604
8.0	0.1716	0.2346	11.0	-0.1712	-0.1768
8.1	0.1475	0.2476	11.1	-0.1528	-0.1913
8.2	0.1222	0.2580	11.2	-0.1330	-0.2028
8.3	0.0960	0.2657	11.3	-0.1121	-0.2143
8.4	0.0692	0.2708	11.4	-0.0902	-0.2224
8.5	0.0419	0.2731	11.5	-0.0677	-0.2284
8.6	0.0146	0.2728	11.6	-0.0446	-0.2320
8.7	-0.0125	0.2697	11.7	-0.0213	-0.2333
8.8	-0.0392	0.2641	11.8	0.0020	-0.2323
8.9	-0.0652	0.2559	11.9	0.0250	-0.2290

TABLE 13.4.1. (Continued.)

u	$J_0(u)$	$J_1(u)$	u	$J_0(u)$	$J_1(u)$
12.0	0.0477	-0.2234	13.6	0.2101	0.0590
12.1	0.0697	-0.2158	13.7	0.2032	0.0791
12.2	0.0908	-0.2060	13.8	0.1943	0.0984
12.3	0.1108	-0.1943	13.9	0.1836	0.1165
12.4	0.1296	-0.1807	14.0	0.1711	0.1334
12.5	0.1469	-0.1655	14.1	0.1570	0.1488
12.6	0.1626	-0.1487	14.2	0.1414	0.1626
12.7	0.1766	-0.1307	14.3	0.1245	0.1747
12.8	0.1887	-0.1114	14.4	0.1065	0.1850
12.9	0.1988	-0.0912	14.5	0.0875	0.1934
13.0	0.2069	-0.0703	14.6	0.0679	0.1998
13.1	0.2129	-0.0488	14.7	0.0476	0.2043
13.2	0.2167	-0.0271	14.8	0.0271	0.2066
13.3	0.2183	-0.0052	14.9	0.0064	0.2069
13.4	0.2177	0.0166	15.0	0.0142	0.2051
13.5	0.2150	0.0380			

TABLE 13.4.2. Zero and first-order Bessel functions of the second kind (source: Ref. 2 p. 371-3).

u	$Y_0(u)$	$Y_1(u)$	u	$Y_0(u)$	$Y_1(u)$
0.0	$-\infty$	$-\infty$	3.0	0.3768	0.3247
0.1	-1.5342	-6.4590	3.1	0.3431	0.3496
0.2	-1.0811	-3.3238	3.2	0.3070	0.3707
0.3	0.8073	-2.2931	3.3	0.2691	0.3878
0.4	-0.6060	-1.7809	3.4	0.2296	0.4010
0.5	-0.4445	-1.4715	3.5	0.1890	0.4102
0.6	-0.3085	-1.2604	3.6	0.1477	0.4154
0.7	-0.1907	-0.1032	3.7	0.1061	0.4167
0.8	-0.0868	-0.9781	3.8	0.0645	0.4141
0.9	0.0056	-0.8731	3.9	0.0234	0.4078
1.0	0.0883	-0.7812	4.0	-0.0169	0.3979
1.1	0.1622	-0.6981	4.1	-0.0561	0.3846
1.2	0.2281	-0.6211	4.2	-0.0938	0.3680
1.3	0.2865	-0.5485	4.3	-0.1296	0.3484
1.4	0.3379	-0.4791	4.4	-0.1633	0.3260
1.5	0.3824	-0.4123	4.5	-0.1947	0.3010
1.6	0.4204	-0.3476	4.6	-0.2235	0.2737
1.7	0.4520	-0.2847	4.7	-0.2494	0.2445
1.8	0.4774	-0.2237	4.8	-0.2723	0.2136
1.9	0.4968	-0.1644	4.9	-0.2921	0.1812
2.0	0.5104	-0.1070	5.0	-0.3085	0.1479
2.1	0.5183	-0.0517	5.1	-0.3216	0.1137
2.2	0.5208	0.0015	5.2	-0.3312	0.0792
2.3	0.5181	0.0523	5.3	-0.3374	0.0445
2.4	0.5104	0.1005	5.4	-0.3402	0.0101
2.5	0.4981	0.1459	5.5	-0.3395	-0.0238
2.6	0.4813	0.1884	5.6	-0.3354	-0.0568
2.7	0.4605	0.2276	5.7	-0.3282	-0.0887
2.8	0.4359	0.2635	5.8	-0.3177	-0.1192
2.9	0.4079	0.2959	5.9	-0.3044	-0.1481

TABLE 13.4.2. (Continued.)

u	$Y_0(u)$	$Y_1(u)$	u	$Y_0(u)$	$Y_1(u)$
6.0	-0.2882	-0.1750	9.0	0.2499	0.1043
6.1	-0.2694	-0.1998	9.1	0.2383	0.1275
6.2	-0.2483	-0.2223	9.2	0.2245	0.1491
6.3	-0.2251	-0.2422	9.3	0.2086	0.1691
6.4	-0.2000	-0.2596	9.4	0.1907	0.1871
6.5	-0.1732	-0.2741	9.5	0.1712	0.2032
6.6	-0.1452	-0.2857	9.6	0.1502	0.2171
6.7	-0.1162	-0.2945	9.7	0.1279	0.2287
6.8	-0.0864	-0.3002	9.8	0.1045	0.2379
6.9	-0.0562	-0.3029	9.9	0.0804	0.2447
7.0	-0.0260	-0.3027	10.0	0.0557	0.2490
7.1	0.0042	-0.2995	10.1	0.0307	0.2508
7.2	0.0338	-0.2934	10.2	0.0056	0.2502
7.3	0.0628	-0.2846	10.3	-0.0193	0.2471
7.4	0.0907	-0.2731	10.4	-0.0437	0.2416
7.5	0.1173	-0.2591	10.5	-0.0675	0.2337
7.6	0.1424	-0.2428	10.6	-0.0904	0.2236
7.7	0.1658	-0.2243	10.7	-0.1122	0.2114
7.8	0.1872	-0.2039	10.8	-0.1326	0.1973
7.9	0.2065	-0.1817	10.9	-0.1516	0.1813
8.0	0.2235	-0.1581	11.0	-0.1688	0.1637
8.1	0.2381	-0.1332	11.1	-0.1843	0.1446
8.2	0.2501	-0.1072	11.2	-0.1977	0.1243
8.3	0.2595	-0.0806	11.3	-0.2091	0.1029
8.4	0.2662	-0.0535	11.4	-0.2183	0.0807
8.5	0.2702	-0.0262	11.5	-0.2252	0.0579
8.6	0.2715	0.0011	11.6	-0.2299	0.0348
8.7	0.2700	0.0280	11.7	-0.2322	0.0114
8.8	0.2659	0.0544	11.8	-0.2322	-0.0118
8.9	0.2592	0.0799	11.9	-0.2298	-0.0347

TABLE 13.4.2. (Continued.)

u	$Y_0(u)$	$Y_1(u)$	u	$Y_0(u)$	$Y_1(u)$
12.0	-0.2252	-0.0571	13.5	0.0301	-0.2140
12.1	-0.2184	-0.0787	13.6	0.0512	-0.2084
12.2	-0.2095	-0.0994	13.7	0.0717	-0.2007
12.3	-0.1986	-0.1190	13.8	0.0913	-0.1912
12.4	-0.1858	-0.1371	13.9	0.1099	-0.1798
12.5	-0.1712	-0.1538	14.0	0.1272	-0.1666
12.6	-0.1551	-0.1689	14.1	0.1431	-0.1520
12.7	-0.1375	-0.1821	14.2	0.1575	-0.1359
12.8	-0.1187	-0.1935	14.3	0.1703	-0.1186
12.9	-0.0989	-0.2028	14.4	0.1812	-0.1003
13.0	-0.0782	-0.2101	14.5	0.1903	-0.0810
13.1	-0.0569	-0.2152	14.6	0.1974	-0.0612
13.2	-0.0352	-0.2182	14.7	0.2025	-0.0408
13.3	-0.0134	-0.2190	14.8	0.2056	-0.0202
13.4	0.0085	-0.2176	14.9	0.2066	0.0005

TABLE 13.4.3. Zero and first order modified Bessel functions of the first kind (source: Ref. 2, p. 373-4).

u	$I_0(u)$	$I_1(u)$	u	$I_0(u)$	$I_1(u)$
0.0	1.0000	0.0000	3.0	4.881	3.953
0.1	1.0025	0.0501	3.1	5.294	4.326
0.2	1.0100	0.1005	3.2	5.747	4.734
0.3	1.0226	0.1517	3.3	6.243	5.181
0.4	1.0404	0.2040	3.4	6.785	5.670
0.5	1.0635	0.2579	3.5	7.378	6.206
0.6	1.0920	0.3137	3.6	8.028	6.793
0.7	1.1263	0.3719	3.7	8.739	7.436
0.8	1.1665	0.4329	3.8	9.517	8.140
0.9	1.2130	0.4971	3.9	10.369	8.913
1.0	1.2661	0.5652	4.0	11.30	9.76
1.1	1.3262	0.6375	4.1	12.32	10.69
1.2	1.3937	0.7147	4.2	13.44	11.71
1.3	1.4693	0.7973	4.3	14.67	12.82
1.4	1.5534	0.8861	4.4	16.01	14.05
1.5	1.6467	0.9817	4.5	17.48	15.39
1.6	1.7500	1.0848	4.6	19.09	16.86
1.7	0.8640	1.1963	4.7	20.86	18.48
1.8	1.9806	1.3172	4.8	22.79	20.25
1.9	2.1277	1.4482	4.9	24.91	22.20
2.0	2.280	1.591	5.0	27.24	24.34
2.1	2.446	1.746	5.1	29.79	26.68
2.2	2.629	1.914	5.2	32.58	29.25
2.3	2.830	2.098	5.3	35.65	32.08
2.4	3.049	2.298	5.4	39.01	35.18
2.5	3.290	2.517	5.5	42.70	38.59
2.6	3.553	2.755	5.6	46.74	42.33
2.7	3.842	3.016	5.7	51.17	46.44
2.8	4.157	3.301	5.8	56.04	50.95
2.9	4.503	3.613	5.9	61.38	55.90

TABLE 13.4.4. Zero and first order modified Bessel functions of the second kind (source: Ref. 2, p. 374-5).

u	$\frac{2}{\pi}K_0(u)$	$\frac{2}{\pi}K_1(u)$	u	$\frac{2}{\pi}K_0(u)$	$\frac{2}{\pi}K_1(u)$
0.0	∞	∞	2.0	0.072	0.089
0.1	1.545	6.270	2.1	0.064	0.078
0.2	1.116	3.040	2.2	0.057	0.069
0.3	0.874	1.946	2.3	0.050	0.060
0.4	0.710	1.391	2.4	0.045	0.053
0.5	0.588	1.054	2.5	0.040	0.047
0.6	0.495	0.829	2.6	0.035	0.042
0.7	0.420	0.669	2.7	0.031	0.037
0.8	0.360	0.549	2.8	0.028	0.032
0.9	0.310	0.456	2.9	0.025	0.029
1.0	0.268	0.383	3.0	0.022	0.026
1.1	0.233	0.324	3.1	0.020	0.023
1.2	0.203	0.277	3.2	0.018	0.020
1.3	0.177	0.237	3.3	0.016	0.018
1.4	0.155	0.204	3.4	0.014	0.016
1.5	0.136	0.177	3.5	0.012	0.014
1.6	0.120	0.153	3.6	0.011	0.013
1.7	0.105	0.133	3.7	0.010	0.011
1.8	0.093	0.116	3.8	0.009	0.010
1.9	0.082	0.102	3.9	0.008	0.009

13.5. Legendre Polynomials

The Legendre polynomial of degree n, of the first kind:

$$P_n(u) = (-1)^{n/2} \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \Big[1 - \frac{n(n+1)}{2!} u^2$$

$$+ \frac{n(n-2)(n+1)(n+3)}{4!} u^4 + \cdots\Big], \quad n = 2, 4, 6, \ldots$$

$$P_n(u) = (-1)^{(n-1)/2} \frac{1 \cdot 3 \cdot 5 \cdots n}{2 \cdot 4 \cdot 6 \cdots (n-1)} \Big[u - \frac{(n-1)(n+2)}{3!} u^3$$

$$+ \frac{(n-1)(n-3)(n+2)(n+4)}{5!} u^5 + \cdots\Big], \quad n = 3, 5, 7, \ldots, \ |u|<1$$

$$P_0(u) = 1, \ P_1(u) = u, \ P_2(u) = (3u^2 - 1)/2 ,$$

$$P_3(u) = (5u^3 - 3u)/2, \ P_4(u) = (35u^4 - 30u^2 + 3)/8 ,$$

$$P_5(u) = (63u^5 - 70u^3 + 15u)/8, \ |u|<1$$

The Legendre polynomial of degree n, of the second kind:

$$Q_n(u) = (-1)^{(n+1)/2} \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdots n} \Big[1 - \frac{n(n+1)}{2!} u^2$$

$$+ \frac{n(n-2)(n+1)(n+3)}{4!} u^4 + \cdots\Big], \quad n = 3, 5, 7, \ldots,$$

$$Q_n(u) = (-1)^{n/2} \frac{2 \cdot 4 \cdot 6 \cdots n}{1 \cdot 3 \cdot 5 \cdots (n-1)} \Big[u - \frac{(n-1)(n+2)}{3!} u^3$$

$$+ \frac{(n-1)(n-3)(n+2)(n+4)}{5!} u^5 + \cdots\Big], \quad n = 2, 4, 6, \ldots,$$

$$Q_0(u) = \begin{cases} \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right), & |u|<1 \\ \frac{1}{2} \ln\left(\frac{u+1}{u-1}\right), & |u|<1 \end{cases}$$

$$Q_1(u) = Q_0(u)\ P_1(u) - 1,$$

$$Q_2(u) = Q_0(u)\ P_2(u) - 3u/2$$

$$Q_3(u) = Q_0(u)\ P_3(u) - 5u^2/2 + \frac{2}{3},$$

$$Q_4(u) = Q_0(u)\ P_4(u) - 35u^3/8 + 55u^2/24\ ,$$

$$Q_n(u) = Q_0(u)\ P_n(u) - \frac{(2n-1)}{1-n} P_{n-1}(u) - \frac{(2n-5)}{3(n-1)} P_{n-3}(u) - \dots ,$$

TABLE 13.5. The first five Legendre polynomials of the first kind (source: Ref. 2, p. 375-7).

u	$P_1(u)$	$P_2(u)$	$P_3(u)$	$P_4(u)$	$P_5(u)$
0.00	0.0000	-0.5000	0.0000	0.3750	0.0000
0.01	0.0100	-0.4998	-0.0150	0.3746	0.0187
0.02	0.0200	-0.4994	-0.0300	0.3735	0.0374
0.03	0.0300	-0.4986	-0.0449	0.3716	0.0560
0.04	0.0400	-0.4976	-0.0598	0.3690	0.0744
0.05	0.0500	-0.4962	-0.0747	0.3657	0.0927
0.06	0.0600	-0.4946	-0.0895	0.3616	0.1106
0.07	0.0700	-0.4926	-0.1041	0.3567	0.1283
0.08	0.0800	-0.4904	-0.1187	0.3512	0.1455
0.09	0.0900	-0.4878	-0.1332	0.3449	0.1624
0.10	0.1000	-0.4850	-0.1475	0.3379	0.1788
0.11	0.1100	-0.4818	-0.1617	0.3303	0.1947
0.12	0.1200	-0.4784	-0.1757	0.3219	0.2101
0.13	0.1300	-0.4746	-0.1895	0.3129	0.2248
0.14	0.1400	-0.4906	-0.2031	0.3032	0.2389
0.15	0.1500	-0.4662	-0.2166	0.2928	0.2523
0.16	0.1600	-0.4616	-0.2298	0.2819	0.2650
0.17	0.1700	-0.4566	-0.2427	0.2703	0.2769
0.18	0.1800	-0.4514	-0.2554	0.2581	0.2880
0.19	0.1900	-0.4458	-0.2679	0.2453	0.2982
0.20	0.2000	-0.4400	-0.2800	0.2320	0.3075
0.21	0.2100	-0.4338	-0.2918	0.2181	0.3159
0.22	0.2300	-0.4274	-0.3034	0.2037	0.3234
0.23	0.2300	-0.4206	-0.3146	0.1889	0.3299
0.24	0.2400	-0.4136	-0.3254	0.1735	0.3353
0.25	0.2500	-0.4062	-0.3359	0.1577	0.3397
0.26	0.2600	-0.3986	-0.3461	0.1415	0.3431
0.27	0.2700	-0.3906	-0.3558	0.1249	0.3453
0.28	0.2800	-0.3824	-0.3651	0.1079	0.3465
0.29	0.2900	-0.3738	-0.3740	0.0906	0.3465

TABLE 13.5. (Continued.)

u	$P_1(u)$	$P_2(u)$	$P_3(u)$	$P_4(u)$	$P_5(u)$
0.30	0.3000	-0.3650	-0.3825	0.0729	0.3454
0.31	0.3100	-0.3558	-0.3905	0.0550	0.3431
0.32	0.3200	-0.3464	-0.3981	0.0369	0.3397
0.33	0.3300	-0.3366	-0.4052	0.0185	0.3351
0.34	0.3400	-0.3266	-0.4117	0.0000	0.3294
0.35	0.3500	-0.3162	-0.4178	-0.0187	0.3225
0.36	0.3600	-0.3056	-0.4234	-0.0375	0.3144
0.37	0.3700	-0.2946	-0.4284	-0.0564	0.3051
0.38	0.3800	-0.2834	-0.4328	-0.0753	0.2948
0.39	0.3900	-0.2718	-0.4367	-0.0942	0.2833
0.40	0.4000	-0.2600	-0.4400	-0.1130	0.2706
0.41	0.4100	-0.2478	-0.4427	-0.1317	0.2569
0.42	0.4200	-0.2354	-0.4448	-0.1504	0.2421
0.43	0.4300	-0.2226	-0.4462	-0.1688	0.2263
0.44	0.4400	-0.2096	-0.4470	-0.1870	0.2095
0.45	0.4500	-0.1962	-0.4472	-0.2050	0.1917
0.46	0.4600	-0.1826	-0.4467	-0.2226	0.1730
0.47	0.4700	-0.1686	-0.4454	-0.2399	0.1534
0.48	0.4800	-0.1544	-0.4435	-0.2568	0.1330
0.49	0.4900	-0.1398	-0.4409	-0.2732	0.1118
0.50	0.5000	-0.1250	-0.4375	-0.2891	0.0898
0.51	0.5100	-0.1098	-0.4334	-0.3044	0.0673
0.52	0.5200	-0.0944	-0.4285	-0.3191	0.0441
0.53	0.5300	-0.0786	-0.4228	-0.3332	0.0204
0.54	0.5400	-0.0626	-0.4163	-0.3465	-0.0037
0.55	0.5500	-0.0462	-0.4091	-0.3590	-0.0282
0.56	0.5600	-0.0296	-0.4010	-0.3707	-0.0529
0.57	0.5700	-0.0126	-0.3920	-0.3815	-0.0779
0.58	0.5800	0.0046	-0.3822	-0.3914	-0.1028
0.59	0.5900	0.0222	-0.3716	-0.4002	-0.1278

TABLE 13.5. (Continued.)

u	$P_1(u)$	$P_2(u)$	$P_3(u)$	$P_4(u)$	$P_5(u)$
0.60	0.6000	0.0400	-0.3600	-0.4080	-0.1526
0.61	0.6100	0.0582	-0.3475	-0.4146	-0.1772
0.62	0.6200	0.0766	-0.3342	-0.4200	-0.2014
0.63	0.6300	0.0954	-0.3199	-0.4242	-0.2251
0.64	0.6400	0.1144	-0.3046	-0.4270	-0.2482
0.65	0.6500	0.1338	-0.2884	-0.4284	-0.2705
0.66	0.6600	0.1534	-0.2713	-0.4284	-0.2919
0.67	0.6700	0.1734	-0.2531	-0.4268	-0.3122
0.68	0.6800	0.1936	-0.2339	-0.4236	-0.3313
0.69	0.6900	0.2142	-0.2137	-0.4187	-0.3490
0.70	0.7000	0.2350	-0.1925	-0.4121	-0.3652
0.71	0.7100	0.2562	-0.1702	-0.4036	-0.3796
0.72	0.7200	0.2776	-0.1469	-0.3933	-0.3922
0.73	0.7300	0.2994	-0.1225	-0.3810	-0.4026
0.74	0.7400	0.3214	-0.0969	-0.3666	-0.4107
0.75	0.7500	0.3438	-0.0703	-0.3501	-0.4164
0.76	0.7600	0.3664	-0.0426	-0.3314	-0.4193
0.77	0.7700	0.3894	-0.0137	-0.3104	-0.4193
0.78	0.7800	0.4126	0.0164	-0.2871	-0.4162
0.79	0.7900	0.4362	0.0476	-0.2613	-0.4097
0.80	0.8000	0.4600	0.0800	-0.2330	-0.3995
0.81	0.8100	0.4842	0.1136	-0.2021	-0.3855
0.82	0.8200	0.5086	0.1484	-0.1685	-0.3674
0.83	0.8300	0.5334	0.1845	-0.1321	-0.3449
0.84	0.8400	0.5584	0.2218	-0.0928	-0.3177
0.85	0.8500	0.5838	0.2603	-0.0506	-0.2857
0.86	0.8600	0.6094	0.3001	-0.0053	-0.2484
0.87	0.8700	0.6354	0.3413	0.0431	-0.2056
0.88	0.8800	0.6616	0.3837	0.0947	-0.1570
0.89	0.8900	0.6882	0.4274	0.1496	-0.1023

TABLE 13.5. (Continued.)

u	$P_1(u)$	$P_2(u)$	$P_3(u)$	$P_4(u)$	$P_5(u)$
0.90	0.9000	0.7150	0.4725	0.2079	-0.0411
0.91	0.9100	0.7422	0.5189	0.2698	0.0268
0.92	0.9200	0.7696	0.5667	0.3352	0.1017
0.93	0.9300	0.7974	0.6159	0.4044	0.1842
0.94	0.9400	0.8254	0.6665	0.4773	0.2744
0.95	0.9500	0.8538	0.7184	0.5541	0.3727
0.96	0.9600	0.8824	0.7718	0.6349	0.4796
0.97	0.9700	0.9114	0.8267	0.7198	0.5954
0.98	0.9800	0.9406	0.8830	0.8089	0.7204
0.99	0.9900	0.9702	0.9407	0.9022	0.8552
1.00	1.0000	1.0000	1.0000	1.0000	1.0000

13.6. Sine, Cosine, and Exponential Integrals.

Sine integral: $\mathrm{Si}(x) = \int_0^x \frac{\sin(u)}{u}\,du,$

$\mathrm{Si}(\infty) = \pi/2$

Cosine integral: $\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos(u)}{u}\,du = \ell n(\gamma x) - \int_0^x \frac{1-\cos(u)}{u}\,du$

$$\ell n\ \gamma = 0.577215\ldots \text{ (Euler's constant)}$$

Exponential integral: $\mathrm{Ei}(x) = -\int_x^\infty \frac{e^{-u}}{u}\,du, \; <0, \infty > x > 0$

Logarithmic integral: $\ell i(x) = \int_0^x \frac{du}{\ell n(u)} = \overline{\mathrm{Ei}}(\ell n\ x)$

$$\ell i(e^x) = \overline{\mathrm{Ei}}(x)$$

For small values of x:

$\mathrm{Si}(x) \simeq x,$

$\ell i(x) \simeq -x/\ell n(1/x)$

$\mathrm{Ci}(x) \simeq \mathrm{Ei}(-x) \simeq \overline{\mathrm{Ei}}(x) \simeq \ell n(1/\gamma x) = \gamma + \ell n(x) - x + \left(x^2/4\right) + O(x^3)$

For large values of x:

$\mathrm{Si}(x) \simeq \pi/2 - \cos(x)/x$

$\mathrm{Ci}(x) = \sin(x)/x$

$\overline{\mathrm{Ei}}(x) = (e^x/x)(1 + 1!/x + 2!/x + 3!/x + \cdots), \; |x| >> 1$

TABLE 13.6. Values of the sine, cosine, logarithmic, and exponential integrals. (source: Ref. 18, p. 6-9.)

x	Si(x)	Ci(x)	$\overline{Ei}(x)$	Ei(-x)
0.00	+0.000000	$-\infty$	$-\infty$	$-\infty$
0.01	+0.010000	-4.0280	-4.0179	-4.0379
0.02	+0.019999	-3.3349	-3.3147	-3.3547
0.03	+0.029998	-2.9296	-2.8991	-2.9591
0.04	+0.039996	-2.6421	-2.6013	-2.6813
0.05	+0.04999	-2.4191	-2.3679	-2.4679
0.06	+0.05999	-2.2371	-2.1753	-2.2953
0.07	+0.06998	-2.0833	-2.0108	-2.1508
0.08	+0.07997	-1.9501	-1.8669	-2.0269
0.09	+0.08996	-1.8328	-1.7387	-1.9187
0.10	+0.09994	-1.7279	-1.6228	-1.8229
0.11	+0.10993	-1.6331	-1.5170	-1.7371
0.12	+0.11990	-1.5466	-1.4193	-1.6595
0.13	+0.12988	-1.4672	-1.3287	-1.5889
0.14	+0.13985	-1.3938	-1.2438	-1.5241
0.15	+0.14981	-1.3255	-1.1641	-1.4645
0.16	+0.15977	-1.2618	-1.0887	-1.4092
0.17	+0.16973	-1.2020	-1.0172	-1.3578
0.18	+0.1797	-1.1457	-0.9491	-1.3098
0.19	+0.1896	-1.0925	-0.8841	-1.2649
0.20	+0.1996	-1.0422	-0.8218	-1.2227
0.21	+0.2095	-0.9944	-0.7619	-1.1829
0.22	+0.2194	-0.9490	-0.7042	-1.1454
0.23	+0.2293	-0.9057	-0.6485	-1.1099
0.24	+0.2392	-0.8643	-0.5947	-1.0762
0.25	+0.2491	-0.8247	-0.5425	-1.0443
0.26	+0.2590	-0.7867	-0.4919	-1.0139
0.27	+0.2689	-0.7503	-0.4427	-0.9849
0.28	+0.2788	0.7153	-0.3949	-0.9573
0.29	+0.2886	-0.6816	-0.3482	-0.9309

TABLE 13.6. (Continued.)

x	Si(x)	Ci(x)	$\overline{Ei}(x)$	Ei(-x)
0.30	+0.2985	-0.6492	-0.3027	-0.9057
0.31	+0.3083	-0.6179	-0.2582	-0.8815
0.32	+0.3182	-0.5877	-0.2147	-0.8583
0.33	+0.3280	-0.5585	-0.17210	-0.8361
0.34	+0.3378	-0.5304	-0.13036	-0.8147
0.35	+0.3476	-0.5031	-0.08943	-0.7942
0.36	+0.3574	-0.4767	-0.04926	-0.7745
0.37	+0.3672	-0.4511	-0.00979	-0.7554
0.38	+0.3770	-0.4263	+0.02901	-0.7371
0.39	+0.3867	-0.4022	+0.06718	-0.7194
0.40	+0.3965	-0.3788	+0.10477	-0.7024
0.41	+0.4062	-0.3561	+0.14179	-0.6859
0.42	+0.4159	-0.3341	+0.17828	-0.6700
0.43	+0.4256	-0.3126	+0.2143	-0.6546
0.44	+0.4353	-0.2918	+0.2498	-0.6397
0.45	+0.4450	-0.2715	+0.2849	-0.6253
0.46	+0.4546	-0.2517	+0.3195	-0.6114
0.47	+0.4643	-0.2325	+0.3537	-0.5979
0.48	+0.4739	-0.2138	+0.3876	-0.5848
0.49	+0.4835	-0.1956	+0.4211	-0.5721
0.50	+0.4931	-0.17778	+0.4542	-0.5598
0.51	+0.5027	-0.16045	+0.4870	-0.5478
0.52	+0.5123	-0.14355	+0.5195	-0.5362
0.53	+0.5218	-0.12707	+0.5517	-0.5250
0.54	+0.5313	-0.11099	+0.5836	-0.5140
0.55	+0.5408	-0.09530	+0.6153	-0.5034
0.56	+0.5503	-0.07999	+0.6467	-0.4930
0.57	+0.5598	-0.06504	+0.6778	-0.4830
0.58	+0.5693	-0.05044	+0.7087	-0.4732
0.59	+0.5787	-0.03619	+0.7394	-0.4636

TABLE 13.6. (Continued.)

x	Si(x)	Ci(x)	$\overline{\mathrm{Ei}}(x)$	Ei(-x)
0.60	+0.5881	-0.02227	+0.7699	-0.4544
0.61	+0.5975	-0.008675	+0.8002	-0.4454
0.62	+0.6069	+0.004606	+0.8302	-0.4366
0.63	+0.6163	+0.01758	+0.8601	-0.4280
0.64	+0.6256	+0.03026	+0.8898	-0.4197
0.65	+0.6349	+0.04265	+0.9194	-0.4115
0.66	+0.6442	+0.05476	+0.9488	-0.4036
0.67	+0.6535	+0.06659	+0.9780	-0.3959
0.68	+0.6628	+0.07816	+1.0071	-0.3883
0.69	+0.6720	+0.08946	+1.0361	-0.3810
0.70	+0.6812	+0.10051	+1.0649	-0.3738
0.71	+0.6904	+0.11132	+1.0936	-0.3668
0.72	+0.6996	+0.12188	+1.1222	-0.3599
0.73	+0.7087	+0.13220	+1.1507	-0.3532
0.74	+0.7179	+0.14230	+1.1791	-0.3467
0.75	+0.7270	+0.15216	+1.2073	-0.3403
0.76	+0.7360	+0.16181	+1.2355	-0.3341
0.77	+0.7451	+0.17124	+1.2636	-0.3280
0.78	+0.7541	+0.1805	+1.2916	-0.3221
0.79	+0.7631	+0.1895	+1.3195	-0.3163
0.80	+0.7721	+0.1983	+1.3474	-0.3106
0.81	+0.7811	+0.2069	+1.3752	-0.3050
0.82	+0.7900	+0.2153	+1.4029	-0.2996
0.83	+0.7989	+0.2235	+1.4306	-0.2943
0.84	+0.8078	+0.2316	+1.4582	-0.2891
0.85	+0.8166	+0.2394	+1.4857	-0.2840
0.86	+0.8254	+0.2471	+1.5132	-0.2790
0.87	+0.8342	+0.2546	+1.5407	-0.2742
0.88	+0.8430	+0.2619	+1.5681	-0.2694
0.89	+0.8518	+0.2691	+1.5955	-0.2647

TABLE 13.6. (Continued.)

x	Si(x)	Ci(x)	$\overline{Ei}(x)$	Ei(-x)
0.90	+0.8605	+0.2761	+1.6228	-0.2602
0.91	+0.8692	+0.2829	+1.6501	-0.2557
0.92	+0.8778	+0.2896	+1.6774	-0.2513
0.93	+0.8865	+0.2961	+1.7047	-0.2470
0.94	+0.8951	+0.3024	+1.7319	-0.2429
0.95	+0.9036	+0.3086	+1.7591	-0.2387
0.96	+0.9122	+0.3147	+1.7864	-0.2347
0.97	+0.9207	+0.3206	+1.8136	-0.2308
0.98	+0.9292	+0.3263	+1.8407	-0.2269
0.99	+0.9377	+0.3319	+1.8679	-0.2231
1.00	+0.9461	+0.3374	+1.8951	-0.2194
1.0	+0.9461	+0.3374	+1.8951	-0.2194
1.1	+1.0287	+0.3849	+2.1674	-0.1860
1.2	+1.1080	+0.4205	+2.4421	-0.1584
1.3	+1.1840	+0.4457	+2.7214	-0.1355
1.4	+1.2562	+0.4620	+3.0072	-0.1162
1.5	+1.3247	+0.4704	+3.3013	-0.1000
1.6	+1.3892	+0.4717	+3.6053	-0.08631
1.7	+1.4496	+0.4670	+3.9210	-0.07465
1.8	+1.5058	+0.4568	+4.2499	-0.06471
1.9	+1.5578	+0.4419	+4.5937	-0.05620
2.0	+1.6054	+0.4230	+4.9542	-0.04890
2.1	+1.6487	+0.4005	+5.3332	-0.04261
2.2	+1.6876	+0.3751	+5.7326	-0.03719
2.3	+1.7222	+0.3472	+6.1544	-0.03250
2.4	+1.7525	+0.3173	+6.6007	-0.02844
2.5	+1.7785	+0.2859	+7.0738	-0.02491
2.6	+1.8004	+0.2533	+7.5761	-0.02185
2.7	+1.8182	+0.2201	+8.1103	-0.01918
2.8	+1.8321	+0.1865	+8.6793	-0.01686
2.9	+1.8422	+0.1529	+9.2860	-0.01482
3.0	+1.8487	+0.1196	+9.9338	-0.01304

TABLE 13.6. (Continued.)

x	Si(x)	Ci(x)	$\overline{\mathrm{Ei}}(x)$	Ei(−x)
3.1	+1.8517	+0.08699	+10.6263	−0.01149
3.2	+1.8514	+0.05526	+11.3673	−0.01013
3.3	+1.8481	+0.02468	+12.1610	$-0.0^{2}8939$
3.4	+1.8419	+0.004518	+13.0121	$-0.0^{2}7890$
3.5	+1.8331	−0.03213	+13.9254	$-0.0^{2}6970$
3.6	+1.8219	−0.05797	+14.9063	$-0.0^{2}6160$
3.7	+1.8086	−0.08190	+15.9606	$-0.0^{2}5448$
3.8	+1.7934	−0.1038	+17.0948	$-0.0^{2}4820$
3.9	+1.7765	−0.1235	+18.3157	$-0.0^{2}4267$
4.0	+1.7582	−0.1410	+19.6309	$-0.0^{2}3779$
4.1	+1.7387	−0.1562	+21.0485	$-0.0^{2}3349$
4.2	+1.7184	−0.1690	+22.5774	$-0.0^{2}2969$
4.3	+1.6973	−0.1795	+24.2274	$-0.0^{2}2633$
4.4	+1.6758	−0.1877	+26.0090	$-0.0^{2}2336$
4.5	+1.6541	−0.1935	+27.9337	$-0.0^{2}2073$
4.6	+1.6325	−0.1970	+30.0141	$-0.0^{2}1841$
4.7	+1.6110	−0.1984	+32.2639	$-0.0^{2}1635$
4.8	+1.5900	−0.1976	+34.6979	$-0.0^{2}1453$
4.9	+1.5696	−0.1948	+37.3325	$-0.0^{2}1291$
5.0	+1.5499	−0.1900	+40.1853	$-0.0^{2}1148$
6	+1.4247	−0.06806	+85.9898	$-0.0^{3}3601$
7	+1.4546	+0.07670	+191.505	$-0.0^{3}1155$
8	+1.5742	+0.1224	+440.380	$-0.0^{4}3767$
9	+1.6650	+0.05535	+1037.88	$-0.0^{4}1245$
10	+1.6583	−0.04546	+2492.23	$-0.0^{5}4157$
11	+1.5783	−0.08956	+6071.41	$-0.0^{5}1400$
12	+1.5050	−0.04978	+14959.5	$-0.0^{6}4751$
13	+1.4994	+0.02676	+37197.7	$-0.0^{6}1622$
14	+1.5562	+0.06940	+93192.5	$-0.0^{7}5566$
15	+1.6182	+0.04628	+234 956	$-0.0^{7}1918$

TABLE 13.6. (Continued.)

x	Si(x)	Ci(x)	x	Si(x)	Ci(x)
20	+1.5482	+0.04442	140	+1.5722	+0.007011
25	+1.5315	-0.00685	150	+1.5662	-0.004800
30	+1.5668	-0.03303	160	+1.5769	+0.001409
35	+1.5969	-0.01148	170	+1.5653	+0.002010
40	+1.5870	+0.01902	180	+1.5741	-0.004432
45	+1.5587	+0.01863	190	+1.5704	+0.005250
50	+1.5516	-0.00563	200	+1.5684	-0.004378
55	+1.5707	-0.01817	300	+1.5709	-0.003332
60	+1.5867	-0.00481	400	+1.5721	-0.002124
65	+1.5792	+0.01285	500	+1.5726	-0.0009320
70	+1.5616	+0.01092	600	+1.5725	+0.0000764
75	+1.5586	-0.00533	700	+1.5720	+0.0007788
80	+1.5723	-0.01240	800	+1.5714	+0.001118
85	+1.5824	-0.001935	900	+1.5707	+0.001109
90	+1.5757	+0.009986	10^2	+1.5702	+0.000826
95	+1.5630	+0.007110	10^4	+1.5709	-0.0000306
100	+1.5622	-0.005149	10^5	+1.5708	+0.0000004
110	+1.5799	-0.000320	10^6	+1.5708	-0.0000004
120	+1.5640	+0.004781	10^7	+1.5708	+0.0
130	+1.5737	-0.007132	∞	$1/2\pi$	0.0

SECTION 14. ROOTS OF SOME CHARACTERISTIC EQUATIONS

TABLE 14.1. First six roots of $\lambda_n \tan \lambda_n = C$. (Source: Ref. 74, p. 217.)

C	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0	0.0000	3.1416	6.2832	9.4248	12.5664	15.7080
0.001	0.0316	3.1419	6.2833	9.4249	12.5665	15.7080
0.002	0.0447	3.1422	6.2835	9.4250	12.5665	15.7081
0.004	0.0632	3.1429	6.2838	9.4252	12.5667	15.7082
0.006	0.0774	3.1435	6.2841	9.4254	12.5668	15.7083
0.008	0.0893	3.1441	6.2845	9.4256	12.5670	15.7085
0.01	0.0998	3.1448	6.2848	9.4258	12.5672	15.7086
0.02	0.1410	3.1479	6.2864	9.4269	12.5680	15.7092
0.04	0.1987	3.1543	6.2895	9.4290	12.5696	15.7105
0.06	0.2425	3.1606	6.2927	9.4311	12.5711	15.7118
0.08	0.2791	3.1668	6.2959	9.4333	12.5727	15.7131
0.1	0.0311	3.1731	6.2991	9.4354	12.5743	15.7143
0.2	0.4328	3.2039	6.3148	9.4459	12.5823	15.7207
0.3	0.5218	3.2341	6.3305	9.4565	12.5902	15.7270
0.4	0.5932	3.2636	6.3461	9.4670	12.5981	15.7334
0.5	0.6533	3.2923	6.3616	9.4775	12.6060	15.7397
0.6	0.7051	3.3204	6.3770	9.4879	12.6139	15.7460
0.7	0.7506	3.3477	6.3923	9.4983	12.6218	15.7524
0.8	0.7910	3.3744	6.4074	9.5087	12.6296	15.7587
0.9	0.8274	3.4003	6.4224	9.5190	12.6375	15.7650
1.0	0.8603	3.4256	6.4373	9.5293	12.6453	15.7713
1.5	0.9882	3.5422	6.5097	9.5801	12.6841	15.8026
2.0	1.0769	3.6436	6.5783	9.6296	12.7223	15.8336
3.0	1.1925	3.8088	6.7040	9.7240	12.7966	15.8945
4.0	1.2646	3.9352	6.8140	9.8119	12.8678	15.9536
5.0	1.3138	4.0336	6.9096	9.8928	12.9352	16.0107
6.0	1.3496	4.1116	6.9924	9.9667	12.9988	16.0654
7.0	1.3766	4.1746	7.0640	10.0339	13.0584	16.1177
8.0	1.3978	4.2264	7.1263	10.0949	13.1141	16.1675
9.0	1.4149	4.2694	7.1806	10.1502	13.1660	16.2147
10.0	1.4289	4.3058	7.2281	10.2003	13.2142	16.2594

TABLE 14.1. (Continued.)

C	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
15.0	1.4729	4.4255	7.3959	10.3898	13.4078	16.4474
20.0	1.4961	4.4915	7.4954	10.5117	13.5420	16.5864
30.0	1.5202	4.5615	7.6057	10.6543	13.7085	16.7691
40.0	1.5325	4.5979	7.6647	10.7334	13.8048	16.8794
50.0	1.5400	4.6202	7.7012	10.7832	13.8666	16.9519
60.0	1.5451	4.6353	7.7259	10.8172	13.9094	17.0026
80.0	1.5514	4.6543	7.7573	10.8606	13.9644	17.0686
100.0	1.5552	4.6658	7.7764	10.8871	13.9981	17.1093
∞	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788

TABLE 14.2. First five roots of $1 - \lambda_n \cot \lambda_n = C$. (source: Ref. 20 p. 442.)

C	λ_1	λ_2	λ_3	λ_4	λ_5
0.000	0.0000	4.4934	7.7253	10.9041	14.0662
0.005	0.1224	4.4945	7.7259	10.9046	14.0666
0.010	0.1730	4.4956	7.7265	10.9050	14.0669
0.020	0.2445	4.4979	7.7278	10.9060	14.0676
0.030	0.2991	4.5001	7.7291	10.9069	14.0683
0.040	0.3450	4.5023	7.7304	10.9078	14.0690
0.050	0.3854	4.5045	7.7317	10.9087	14.0697
0.060	0.4217	4.5068	7.7330	10.9096	14.0705
0.070	0.4551	4.5090	7.7343	10.9105	14.0712
0.080	0.4860	4.5112	7.7356	10.9115	14.0719
0.090	0.5150	4.5134	7.7369	10.9124	14.0726
0.100	0.5423	4.5157	7.7382	10.9133	14.0733
0.200	0.7593	4.5379	7.7511	10.9225	14.0804
0.300	0.9208	4.5601	7.7641	10.9316	14.0875
0.400	1.0528	4.5822	7.7770	10.9408	14.0946
0.500	1.1656	4.6042	7.7899	10.9499	14.1017
0.600	1.2644	4.6261	7.8028	10.9591	14.1088
0.700	1.3525	4.6479	7.8156	10.9682	14.1159
0.800	1.4320	4.6696	7.8284	10.9774	14.1230
0.900	1.5044	4.6911	7.8412	10.9865	14.1301
1.000	1.5708	4.7124	7.8540	10.9956	14.1372
1.500	1.8366	4.8158	7.9171	11.0409	14.1724
2.000	2.0288	4.9132	7.9787	11.0856	14.2075
3.000	2.2889	5.0870	8.0962	11.1727	14.2764
4.000	2.4557	5.2329	8.2045	11.2560	14.3434
5.000	2.5704	5.3540	8.3029	11.3349	14.4080
6.000	2.6537	5.4544	8.3914	11.4086	14.4699
7.000	2.7165	5.5378	8.4703	11.4773	14.5288
8.000	2.7654	5.6078	8.5406	11.5408	14.5847
9.000	2.8044	5.6669	8.6031	11.5994	14.6374
10.000	2.8363	5.7172	8.6587	11.6532	14.6870
11.000	2.8628	5.7606	8.7083	11.7027	14.7335
16.000	2.9476	5.9080	8.8898	11.8959	14.9251
21.000	2.9930	5.9921	9.0019	12.0250	15.0625

TABLE 14.2. (Continued.)

C	λ_1	λ_2	λ_3	λ_4	λ_5
31.000	3.0406	6.0831	9.1294	12.1807	15.2380
41.000	3.0651	6.1311	9.1987	12.2688	15.3417
51.000	3.0801	6.1606	9.2420	12.3247	15.4090
101.000	3.1105	6.2211	9.3317	12.4426	15.5537

TABLE 14.3. First five roots of $J_m(\lambda_n) = 0$. (Source: Ref. 20, p. 443.)

m	λ_1	λ_2	λ_3	λ_4	λ_5
0	2.4048	5.5201	8.6537	11.7915	14.9309
1	3.8317	7.0156	10.1735	13.3237	16.4706
2	5.1356	8.4172	11.6198	14.7960	17.9598
3	6.3802	9.7610	13.0152	16.2235	19.4094
4	7.5883	11.0647	14.3725	17.6160	20.8269

TABLE 14.4. First six roots of $\lambda_n J_1(\lambda_n) - CJ_0(\lambda_n) = 0$. (Source: Ref. 74. p. 217)

C	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0	0	3.8317	7.0156	10.1735	13.3237	16.4706
0.01	0.1412	3.8343	7.0170	10.1745	13.3244	16.4712
0.02	0.1995	3.8369	7.0184	10.1754	13.3252	16.4718
0.04	0.2814	3.8421	7.0213	10.1774	13.3267	16.4731
0.06	0.3438	3.8473	7.0241	10.1794	13.3282	16.4743
0.08	0.3960	3.8525	7.0270	10.1813	13.3297	16.4755
0.1	0.4417	3.8577	7.0298	10.1833	13.3312	16.4767
0.15	0.5376	3.8706	7.0369	10.1882	13.3349	16.4797
0.2	0.6170	3.8835	7.0440	10.1931	13.3387	16.4828
0.3	0.7465	3.9091	7.0582	10.2029	13.3462	16.4888
0.4	0.8516	3.9344	7.0723	10.2127	13.3537	16.4949
0.5	0.9408	3.9594	7.0864	10.2225	13.3611	16.5010
0.6	1.0184	3.9841	7.1004	10.2322	13.3686	16.5070
0.7	1.0873	4.0085	7.1143	10.2419	13.3761	16.5131
0.8	1.1490	4.0325	7.1282	10.2516	13.3835	16.5191
0.9	1.2048	4.0562	7.1421	10.2613	13.3910	16.5251
1.0	1.2558	4.0795	7.1558	10.2710	13.3984	16.5312
1.5	1.4569	4.1902	7.2233	10.3188	13.4353	16.5612
2.0	1.5994	4.2910	7.2884	10.3658	13.4719	16.5910
3.0	1.7887	4.4634	7.4103	10.4566	13.5434	16.6499
4.0	1.9081	4.6018	7.5201	10.5423	13.6125	16.7073
5.0	1.9898	4.7131	7.6177	10.6223	13.6786	16.7630
6.0	2.0490	4.8033	7.7039	10.6964	13.7414	16.8168
7.0	2.0937	4.8772	7.7797	10.7646	13.8008	16.8684
8.0	2.1286	4.9384	7.8464	10.8271	13.8566	16.9179
9.0	2.1566	4.9897	7.9051	10.8842	13.9090	16.9650
10.0	2.1795	5.0332	7.9569	10.9363	13.9580	17.0099
15.0	2.2509	5.1773	8.1422	11.1367	14.1576	17.2008
20.0	2.2880	5.2568	8.2534	11.2677	14.2983	17.3442
30.0	2.3261	5.3410	8.3771	11.4221	14.4748	17.5348
40.0	2.3455	5.3846	8.4432	11.5081	14.5774	17.6508
50.0	2.3572	5.4112	8.4840	11.5621	14.6433	17.7272
60.0	2.3651	5.4291	8.5116	11.5990	14.6889	17.7807
80.0	2.3750	5.4516	8.5466	11.6461	14.7475	17.8502
100.0	2.3809	5.4652	8.5678	11.6747	14.7834	17.8931
∞	2.4048	5.5201	8.6537	11.7915	14.9309	18.0711

TABLE 14.5. First five roots of $J_0(\lambda_n)Y_0(C\lambda_n) = Y_0(\lambda_n)J_0(C\lambda_n)$. (Source: Ref. 9, p. 493.)

C	λ_1	λ_2	λ_3	λ_4	λ_5
1.2	15.7014	31.4126	47.1217	62.8302	78.5385
1.5	6.2702	12.5598	18.8451	25.1294	31.4133
2.0	3.1230	6.2734	9.4182	12.5614	15.7040
2.5	2.0732	4.1773	6.2754	8.3717	10.4672
3.0	1.5485	3.1291	4.7038	6.2767	7.8487
3.5	1.2339	2.5002	3.7608	5.0196	6.2776
4.0	1.0244	2.0809	3.1322	4.1816	5.2301

TABLE 14.6. First six roots of $\tan(\lambda_n) = -\lambda_n/C$. (Source: Ref. 74, p. 322.)

C	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788
0.1	1.6320	4.7335	7.8667	11.0047	14.1443	17.2845
0.2	1.6887	4.7544	7.8794	11.0137	14.1513	17.2903
0.3	1.7414	4.7751	7.8920	11.0228	14.1584	17.2961
0.4	1.7906	4.7956	7.9046	11.0318	14.1654	17.3019
0.5	1.8366	4.8158	7.9171	11.0409	14.1724	17.3076
0.6	1.8798	4.8358	7.9295	11.0498	14.1795	17.3134
0.7	1.9203	4.8556	7.9419	11.0588	14.1865	17.3192
0.8	1.9586	4.8751	7.9542	11.0677	14.1935	17.3249
0.9	1.9947	4.8943	7.9665	11.0767	14.2005	17.3306
1.0	2.0288	4.9132	7.9787	11.0856	14.2075	17.3364
1.5	2.1746	5.0037	8.0385	11.1296	14.2421	17.3649
2.0	2.2889	5.0870	8.0962	11.1727	14.2764	17.3932
3.0	2.4557	5.2329	8.2045	11.2560	14.3434	17.4490
4.0	2.5704	5.3540	8.3029	11.3349	14.4080	17.5034
5.0	2.6537	5.4544	8.3914	11.4086	14.4699	17.5562
6.0	2.7165	5.5373	8.4703	11.4773	14.5288	17.6072
7.0	2.7654	5.6078	8.5406	11.5408	14.5847	17.6562
8.0	2.8044	5.6669	8.6031	11.5994	14.6374	17.7032
9.0	2.8363	5.7172	8.6587	11.6532	14.6870	17.7481
10.0	2.8628	5.7606	8.7083	11.7027	14.7335	17.7908
15.0	2.9476	5.9080	8.8898	11.8959	14.9251	17.9742
20.0	2.9930	5.9921	9.0019	12.0250	15.0625	18.1136
30.0	3.0406	6.0831	9.1294	12.1807	15.2380	18.3018
40.0	3.0651	6.1311	9.1986	12.2688	15.3417	18.4180
50.0	3.0801	6.1606	9.2420	12.3247	15.4090	18.4953
60.0	3.0901	6.1805	9.2715	12.3632	15.4559	18.5497
80.0	3.1028	6.2058	9.3089	12.4124	15.5164	18.6209
100.0	3.1105	6.2211	9.3317	12.4426	15.5537	18.6650
∞	3.1416	6.2832	9.4248	12.5664	15.7080	18.8496

SECTION 15. CONSTANTS AND CONVERSION FACTORS.

15.1. Mathematical Constants.

$e = 2.7182818...$

$\ln 10 = 2.3025851...$

$\pi = 3.1415926...$

$\gamma = 0.5772156...$ (Euler's constant)

15.2. Physical Constants.

Standard acceleration of gravity

$g_0 = 9.80665\ m/s^2$

$= 32.1742\ ft/s^2$

Joule's constant

$J_c = 1.0\ N{\cdot}m/J$

$= 778.16\ ft{\cdot}lbf/Btu$

Stefan-Boltzmann constant

$\sigma = 1.355 \times 10^{-12}\ cal/s{\cdot}cm^2\text{-}K^4$

$= 1712 \times 10^{-12}\ Btu/hr{\cdot}ft^2\text{-}R^4$

$= 5.673 \times 10^{-8}\ W/m^2{\cdot}K^4$

Universal gas constant

$R = 8.3143\ J/mol{\cdot}K$

$= 0.08205\ \ell{\cdot}atm/mol{\cdot}K$

$= 1.9859\ Btu/lbm{\cdot}mol{\cdot}R$

$= 1545.33\ ft{\cdot}lbf/lbm{\cdot}mol{\cdot}R$

$= 6.41 \times 10^{-4}\ J/kg{\cdot}mol{\cdot}K$

15.3. Conversion Factors.

TABLE 15.1. Conversion factors for length.

	m	cm	μm	Å	in.	ft	yd
1 m	= 1	100	10^6	10^{10}	39.37	3.280	1.0936
1 cm	= 0.01	1	10^4	10^8	0.3937	0.0328	0.0109
1 μm	= 10^{-6}	10^{-4}	1	10^4	0.3937×10^{-4}	0.0328×10^{-4}	0.0109×10^{-4}
1 Å	= 10^{-10}	10^{-8}	10^{-4}	1	0.3937×10^{-8}	0.0328×10^{-4}	0.0109×10^{-8}
1 in.	= 0.0254	2.540	25.4×10^4	2.540×10^8	1	0.0833	0.0277
1 ft	= 0.3048	30.48	30.48×10^4	30.43×10^8	12	1	0.3333
1 yd	= 0.9144	91.440	91.440×10^4	91.440×10^8	36	3	1

TABLE 15.2. Conversion factors for area.

		cm^2	m^2	$in.^2$	ft^2	yd^2
1 cm^2	=	1	10^{-4}	0.1550	1.07639	1.1960×10^{-4}
1 m^2	=	10^4	1	1550	10.7639	1.1960
1 $in.^2$	=	6.4516	6.4516×10^{-4}	1	6.9444	7.7160×10^{-4}
1 ft^2	=	929.034	929.034×10^{-4}	144	1	0.11111
1 yd^2	=	8361.307	8361.307×10^{-4}	1296	9	1

TABLE 15.3. Conversion factors for volume.

	cm^3	$in.^3$	ft^3	ml	liter	gal
1 cm^3	= 1	610.23×10^{-4}	35.3145×10^{-4}	999.972×10^{-3}	999.972×10^{-6}	264.170×10^{-6}
1 $in.^3$	= 16.3872	1	5.7870×10^{-4}	16.3867	16.3867×10^{-3}	432.900×10^{-5}
1 ft^3	= 283.170×10^{2}	1728	1	28.3162×10^{3}	28.3162	7.4805
1 ml	= 1.000028	610.251×10^{-4}	353.154×10^{-7}	1	0.001	264.178×10^{-6}
1 liter	= 1000.028	61.0251	353.154×10^{-4}	1000	1	264.178×10^{-3}
1 gal	= 3785.434	231	133.680×10^{-3}	3785.329	3.785329	1

TABLE 15.4. Conversion factors for mass.

	lb	slugs	g	kg	ton
1 lb	= 1	0.03108	453.59	0.45359	0.0005
1 slug	= 32.174	1	1.4594×10^{4}	14.594	0.016087
1 g	= 2.2046×10^{-3}	6.8521×10^{-5}	1	10^{-3}	1.1023×10^{-6}
1 kg	= 2.2046	6.8521×10^{-3}	10^{3}	1	1.1023×10^{-3}
1 ton	= 2000	62.162	9.0718×10^{5}	907.18	1

TABLE 15.5. Conversion factors for density.

	lbm/ft^3	$slug/in.^3$	$lbm/in.^3$	lbm/gal	g/cm^3
1 lbm/ft^3 =	1	0.03108	5.787×10^{-4}	0.13386	0.01602
1 $slug/ft^3$ =	32.174	1	0.01862	4.3010	0.51543
1 $lbm/in.^3$ =	1728	53.706	1	231	27.680
1 lbm/gal =	7.4805	0.2325	4.329×10^{-3}	1	0.11983
1 g/cm^3 =	62.428	1.9403	0.03613	8.345	1

TABLE 15.6. Conversion factors for pressure.

	$lbf/in.^2$	$dyne/cm^2$	kgf/cm^2	in. Hg	mm Hg	in. H_2O	atm	bar
1 $lbf/in.^2$ =	1	689.473×10^2	0.07031	2.0360	51.715	27.71	0.06805	0.06895
1 $dyne/cm^2$ =	145.0383×10^{-7}	1	101.972×10^{-8}	295.299×10^{-7}	750.062×10^{-6}	4.0188×10^{-4}	986.923×10^{-9}	10^{-6}
1 kgf/cm^2 =	14.2234	980.665×10^3	1	28.959	735.559	394.0918	967.841×10^3	980.665×10^{-3}
1 in. Hg =	0.4912	338.64×10^2	0.03453	1	25.40	13.608	0.03342	0.03386
1 mm Hg =	0.01934	1333.223	1.3595×10^{-3}	0.03937	1	0.5358	1.315×10^{-3}	1.333×10^{-3}
1 in. H_2O =	0.03609	24.883×10^2	2.537×10^{-3}	0.0735	1.8665	1	2.458×10^{-3}	2.488×10^{-3}
1 atm =	14.6960	101.325×10^4	1.03323	29.9212	760	460.80	1	1.01325
1 bar =	14.5038	10^6	1.01972	29.5299	750.0617	401.969	986.923×10^{-3}	1

TABLE 15.7. Conversion factors for energy.

	ft•lbf	abs joule	int joule	cal	I.T. cal
1 ft•lbf =	1	1.35582	1.355597	0.32405	0.32384
1 abs joule =	0.73756	1	0.999835	0.23885	0.238849
1 int joule =	0.737682	1.000165	1	0.239045	0.238889
1 cal =	3.08596	4.18401	4.1833	1	0.99934
1 I.T. cal =	3.08799	4.18676	4.18605	1.000657	1
1 Btu =	778.16	1055.045	1054.866	252.161	251.996
1 int kW•hr =	265.567×10^{4}	360.0612×10^{4}	360.000×10^{4}	860.565×10^{3}	860.000×10^{3}
1 hp•hr =	198.0000×10^{4}	268.4525×10^{4}	268.082×10^{4}	641.615×10^{3}	641.194×10^{3}
1 liter•atm =	74.7354	101.3278	101.3111	24.2179	24.2020

	Btu	int kW•hr	hp•hr	liter•atm
1 ft•lbf =	128.5083×10^{-5}	376.553×10^{-9}	505.051×10^{-9}	133.8054×10^{-4}
1 abs joule =	947.827×10^{-6}	277.731×10^{-9}	372.505×10^{-9}	986.896×10^{-5}
1 int joule =	0.947988×10^{-3}	2.777778×10^{-7}	3.7256×10^{-7}	9.87058×10^{-3}
1 cal =	396.572×10^{-5}	116.2028×10^{-8}	155.8566×10^{-8}	412.918×10^{-4}
1 I.T. cal =	396.832×10^{-5}	116.2791×10^{-8}	155.9590×10^{-8}	413.189×10^{-4}
1 Btu =	1	293.018×10^{-6}	293.010×10^{-6}	10.4122
1 int kW•hr =	3412.76	1	1.3412	255.343×10^{2}
1 hp•hr =	2544.46	0.74558	1	264.935×10^{2}
1 liter•atm =	0.09604	281.718×10^{-7}	377.452×10^{-7}	1

Table 15.8. Conversion factors for specific energy.

		abs joule/h	cal/g	I.T. cal/gm	Btu/lb	ft•lbf/lbm	int. kW hr/g	hp hr/lb	ft^2/s^2
abs joule/g	=	1	0.2390	0.2388	0.4299	334.53	2.777×10^{-7}	1.690×10^{-4}	10763
cal/g	=	4.184	1	0.9993	1.7988	1399.75	1.162×10^{-6}	7.069×10^{-4}	4.504×10^{4}
I.T. cal/g	=	4.186	1.0007	1	1.8	1400.69	1.163×10^{-6}	7.074×10^{-4}	4.506×10^{4}
Btu/lb	=	2.326	0.5559	0.5556	1	778.16	6.460×10^{-7}	3.930×10^{-4}	25,037
ft•lbf/lbm	=	2.989×10^{-3}	7.144×10^{-4}	7.139×10^{-4}	1.285×10^{-3}	1	8.302×10^{-10}	5.051×10^{-7}	32.174
int. kW•hr/g	=	3.610×10^{6}	860,565	86[illegible],000	[illegible].548 $\times 10^{6}$	1.2046×10^{9}	1	608.4	3.876×10^{10}
hp•hr/lb	=	5919	1414.5	1413.6	2545	1.980×10^{6}	0.001644	1	6.370×10^{7}
ft^2/s^2	=	9.291×10^{-5}	2.220×10^{-5}	2.219×10^{-5}	3.994×10^{-5}	0.03108	2.580×10^{-11}	1.567×10^{-8}	1

Table 15.9. Conversion factors for specific energy per degree.

		abs joule/g•K	Cal/g•K	I.T. cal/g•K	Btu/lb•R	W•s/kg•K
abs joule/g•K	=	1	0.2390	0.2388	0.2388	10^{3}
cal/g•K	=	4.184	1	0.9993	0.9993	4184
I.T. cal/g•K	=	4.186	1.0007	1	1	4186
btu/lb•R	=	4.186	1.0007	1	1	4186
W•s/kg•K	=	10^{-3}	2.390×10^{-4}	2.388×10^{-4}	2.388×10^{-4}	1

Table 15.10. Conversion factors for thermal conductivity.

		cal/s•cm-°C	Btu/hr•ft•°F	Btu/hr•ft^2•°F/in.	W/ m•°C
1 cal/s•cm• C	=	1	241.9	2903	418.6
1 Btu/hr•ft- F	=	4.13×10^{-3}	1	12	1.73
1 Btu/hr•ft^2- F/in.	=	3.45×10^{-4}	0.0833	1	1.44×10^{-1}
1 W/m C	=	23.89	5780	69350	1

Table 15.11. Conversion fa[illegible] for thermal and momentum diffusivities

		ft^2/hr	stokes	m^2/hr	m^2/s
ft^2/hr	=	1	0.25806	0.092903	2.58×10^{-5}
stokes	=	3.885	1	0.36	10^{-4}
m^2/hr	=	10.764	2.778	1	1.778×10^{-4}
m^2/s	=	38,750	10^4	3600	1

TABLE 15.12. Conversion factors for heat flux.

		$\frac{Btu}{ft^2 \cdot hr}$	$\frac{W}{m^2}$	$\frac{kcal}{hr \cdot m^2}$	$\frac{cal}{s \cdot cm^2}$
$Btu/ft^2 \cdot hr$	=	1	3.154	2.713	7.536×10^{-5}
W/m^2	=	3.170×10^{7}	1	8.600×10^{7}	2389
$kcal/hr\text{-}m^2$	=	0.3687	1.163	1	36000
$cal/s \cdot cm^2$	=	13277	41868.	2.778×10^{-5}	1

TABLE 15.13. Conversion factors for heat transfer coefficient.

		$\frac{Btu}{hr \cdot ft^2 \cdot °F}$	$\frac{W}{m^2 \cdot °C}$	$\frac{cal}{s \cdot cm^2 \cdot °C}$	$\frac{kcal}{hr \cdot m^2 \cdot °C}$
$Btu/hr \cdot ft^2 \cdot °F$	=	1	5.678	1.356×10^{-4}	4.883
$W/m^2 \cdot °C$	=	1.761×10^{7}	1	2391	8.600×10^{7}
$cal/sec\text{-}cm^2\text{-}°C$	=	7376	4.186×10^4	1	36000
$kcal/hr\text{-}m^2\text{-}°C$	=	0.2049	1.163	2.778×10^{-5}	1

SECTION 16. CONVECTION COEFFICIENTS

16.1. Forced Flow in Smooth Tubes.

16.1.1 Fully Developed Laminar Flow (Source: Ref. 21):

$$\frac{hD}{k} = 4.364 \text{ (constant heat rate)}$$

$$\frac{hD}{k} = 3.658 \text{ (constant surface temp)}$$

16.1.2 Fully Developed Turbulent Flow (Source: Ref. 21):

$$\frac{hD}{k} = 6.3 + 0.003 \text{ (Re Pr)}, \text{ Pr} < 0.1 \text{ (constant heat rate)}$$

$$\frac{hD}{k} = 4.8 + 0.003 \text{ (Re Pr)}, \text{ Pr} < 0.1 \text{ (constant surface temp)}$$

$$\frac{hD}{k} = 0.022 \text{ Pr}^{0.6}\text{Re}^{0.8}, \ 0.5 < \text{Pr} < 1.0 \text{ (constant heat rate)}$$

$$\frac{hD}{k} = 0.021 \text{ Pr}^{0.6}\text{Re}^{0.8}, \ 0.5 < \text{Pr} < 1.0 \text{ (constant surface temp)}$$

$$\frac{hD}{k} = 0.0155 \text{ Pr}^{0.5}\text{Re}^{0.9}, \ 1.0 < \text{Pr} < 20$$

$$\frac{hD}{k} = 0.011 \text{ Pr}^{0.3}\text{Re}^{0.9}, \ \text{Pr} > 20$$

Pr = Prandtl number, Re = Reynolds number

16.2. Forced Flow Between Smooth Infinite Parallel Plates.

16.2.1. Fully developed laminar flow (Source: Ref. 21):

$$\frac{hs}{k} = 4.118 \text{ (constant heat rate on both sides)}$$

$$\frac{hs}{k} = 2.693 \text{ (constant heat rate on one side. other side insulated)}$$

$$\frac{hs}{k} = 3.77 \text{ (constant surface temperature on both sides)}$$

$$\frac{hs}{k} = 2.43 \text{ (constant surface temperature on one side, other side insulated)}$$

s = spacing of plates

16.3. Forced Flow Parallel to Smooth Semi-Infinite Flat Plates Laminar Flow.

16.3.1. Laminar flow:

$$\frac{hx}{k} = 0.332\ Pr^{1/3} Re_x^{1/2} \left[1 - (X_0/X)^{3/4}\right]^{-1/3}$$, (constant surface temperature) (Source: Ref. 7)

X_0 = unheated starting length
X = distance from leading edge

$$\frac{hx}{k} = 0.453\ Pr^{1/3} Re_x^{1/2} \left[1 - (X_0/X)^{3/4}\right]^{-1/3}$$, (constant heat rate) (Source: Ref. 21)

16.3.2. Turbulent flow (Source: Ref. 21):

$$\frac{hx}{k} = 0.0295\ Pr^{0.6} Re_x^{0.8} \left[1 - (X_0/X)^{9/10}\right]^{-1/9}$$, (constant surface temperature)

$$\frac{hx}{k} = 0.0307\ Pr^{0.6} Re_x^{0.8} \left[1 - (X_0/X)^{9/10}\right]^{-1/9}$$, (constant heat rate)

16.4. Fully Developed Flow in Smooth Tube Annuli.

16.4.1. Laminar flow (Source: Ref. 21):

$$\frac{h_i(d_o - d_i)}{k} = \frac{Nu_{ii}}{1 - (q_o/q_i)\theta_i^+}$$

$$\frac{h_o(d_o - d_i)}{k} = \frac{Nu_{oo}}{1 - (q_i/q_o)\theta_o^*}$$

(Subscripts i and o refer to inner and outer surfaces, q is surface heat flux, Nu_{ii} and Nu_{oo} are inner and outer surface Nusselt numbers when only one surface is heated and Θ is an influence coefficient given in Table 16.1.)

TABLE 16.1. Tube-annulus solutions for constant heat rate in fully developed laminar flow and temperature profiles.

$r_i r_o$	Nu_{ii}	Nu_{oo}	Θ_i^*	Θ_o^*
0	∞	4.364	∞	0
0.2	8.499	4.883	0.905	0.1041
0.4	6.583	4.979	0.603	0.1823
0.6	5.912	5.099	0.473	0.2455
0.8	5.580	5.240	0.401	0.299
1.0	5.385	5.385	0.346	0.346

16.4.2. Turbulent flow (Source: Ref. 22):

$$\frac{h}{k}(d_o - d_i) - 0.023\ Re_{\Delta d}^{0.8}\ Pr^{0.4}\ (d_o/d_i)^{0.45},\ Re_{\Delta d}\ 10^4,\ \Delta d = d_o - d_i$$

16.5. Forced Flow Normal to Circular Cylinders.

16.5.1. Local Coefficients (Source: Ref. 22):

$$\frac{hd}{k} = 1.14\ Pr^{0.4}\ Re_d^{0.5}\ \left[1 - (\Theta/90)^3\right],\ 0 < \Theta < 80°$$

Θ = cylinder angle from stagnation point

16.5.2. Average Coefficients (Source: Ref. 7):

$$\frac{hd}{k} = 0.43 + C\ Re_d^m\ Pr^{0.31}$$

TABLE 16.2.

Re_d	C	m
1-4,000	0.533	0.500
4,000-40,000	0.193	0.618
40,000-400,000	0.0265	0.805

16.6. Forced Flow Normal to Spheres.

16.6.1. Average Coefficients (Source: Ref. 7):

$$\frac{hd}{k} = 0.37\ Re_d^{0.6}\ Pr^{0.33},\ 20 < Re_d < 150{,}000$$

$$\frac{hd}{k} = 2 + 0.37\ Re_d^{0.6}\ Pr^{0.33},\ Re_d < 20$$

16.7. Free Convection on Vertical Plates and Cylinders.

16.7.1. Local Coefficients (Source: Ref. 7):

$$\frac{hx}{k} = 0.508(Pr)^{1/2}(0.952 + Pr)^{-1/4}(Gr_x)^{1/4},\ GrPr < 10^4$$

$$\frac{hx}{k} = 0.0295(Pr)^{7/5}\left[1 + 0.494(Pr)^{2/3}\right]^{-2/5}(Gr)^{2/5},\ GrPr > 10^4$$

16.8. Free Convection on Horizontal Cylinders.

16.8.1. Average Coefficients (Source: Ref. 23):

$$\frac{hd}{k} = C\ (Gr_d\ Pr)^m$$

TABLE 16.3.

Gr_dPr	C	m
$0-10^{-5}$	0.40	0
$10^{-5}-10^{-1}$	0.97	1/16
$10^{-1}-10^{-4}$	1.14	1/7
$10^{4}-10^{9}$	0.53	1/4
$10^{9}-10^{12}$	0.13	1/3

16.9. Free Convection From Horizontal Square Plates.

16.9.1. Average Coefficients (Source: Ref. 23):

$$\frac{hL}{k} = C\ (Gr_L\ Pr)^m,\ L = \text{plate dimension}$$

TABLE 16.4.

Condition	Gr_L Pr	C	m
Upper surface heated or lower surface cooled	$10^5 - 2 \times 10^7$	0.54	1/4
Lower surface heated or upper surface cooled	$3 \times 10^5 - 3 \times 10^{10}$	0.27	1/4
Upper surface heated or lower surface cooled	$2 \times 10^7 - 3 \times 10^{10}$	0.14	1.3

16.10. Free Convection from Spheres.

16.10.1. Average Coefficients (Source: (Ref. 5):

$$\frac{hd}{k} = 2 + 0.43\ (Gr_d\ Pr)^{1/4},\ 10^\circ < Gr_d\ Pr < 10^5$$

16.11. Free Convection in Enclosed Spaces.

16.11.1. Enclosed Vertical Air Spaces (Source: Ref. 1):

$$\frac{k_e}{k} = \begin{cases} 1, & Gr_s < 2{,}000 \\ 0.18\ (Gr_s)^{1/4}(s/L)^{1/9}, & 2{,}000 < Gr_s < 20{,}000 \\ 0.065(Gr_s)^{1/3}(s/L)^{1/9}, & 20{,}000 < Gr_s < 10^7 \end{cases}$$

k_e = effective thermal conductivity

s = width of air space

L = length of air space

16.11.2. Enclosed Horizontal Air Spaces (Source: Ref. 1):

$$\frac{k_e}{k} = \begin{cases} 0.195\ (Gr_s)^{1/4}, & 10^4 < Gr_s < 4 \times 10^5 \\ 0.068\ (Gr_s)^{1/3}, & 4 \times 10^5 < Gr_s \end{cases}$$

16.12. Film Condensation.

16.12.1. Vertical Plates--Average Coefficients (Source: Ref. 5):

$$\frac{h}{k_\ell}\left[\frac{\mu_\ell}{\rho_\ell(\rho_\ell - \rho_v)g}\right]^{1/3} = 1.47\ Re_\ell^{-1/3},\ Re_\ell < 1800$$

$$\frac{h}{k_\ell}\left[\frac{\mu_\ell}{\rho_\ell(\rho_\ell - \rho_v)g}\right]^{1/3} = 0.007\ Re_\ell^{0.4},\ Re_\ell > 1800$$

$Re_\ell = (4h\Delta t_{vw})/i_{\ell v}\mu_\ell$

Δt_{vw} = temp difference between saturated vapor and wall

$i_{\ell v}$ = heat of evaporation

$\mu_\ell, k_\ell, \rho_\ell$ = viscosity, thermal conductivity and density of liquid at saturation vapor temp

16.12.2. Horizontal Tubes--Average Coefficients (Source: Ref. 5):

$$\frac{hd}{k_\ell} = 0.725\left[\frac{\rho_\ell\left(\rho_\ell - \rho_1^v\right)g\ i_{\ell v}d^3}{k_\ell\mu_\ell\Delta t_{vw}}\right]^{1/4}$$

16.13. Pool Boiling,

$$\frac{q}{\mu_\ell i_{\ell v}}\left[\frac{\sigma}{g(\rho_\ell - \rho_v)}\right]^{1/2} = \left[\frac{c_\ell \Delta t_{wv}}{i_{\ell v} C_{sf}\ Pr_\ell\ n}\right]^{1/3} \qquad \text{(Source: Ref. 19)}$$

Δt_{wv} = temp difference between wall and saturated vapor

$c_\ell, \mu_\ell, \rho_\ell, Pr_\ell$ = specific heat, viscosity, density and Prandtl number at saturated liquid temperature

σ = surface tension at liquid vapor interface

$i_{\ell v}$ = heat of evaporation

C_{sf} = surface coefficient (see Table 16.5)

TABLE 16.5. Values of C_{sf} and n (source: Ref. 19).

Surface-fluid combination	C_{sf}	n
Water-nickel	0.006	1.0
Water-platinum	0.013	1.0
Water-copper	0.013	1.0
Water-brass	0.006	1.0
CCl_4-copper	0.013	1.7
Benzene-chromium	0.101	1.7
n-Pentane-chromium	0.015	1.7
Ethyl alcohol-chromium	0.0027	1.7
Isopropyl alcohol-copper	0.0025	1.7
35% K_2CO_3-copper	0.0054	1.7
50% K_2CO_3-copper	0.0027	1.7
n-Butyl alcohol-copper	0.0030	1.7

SECTION 17. CONTACT COEFFICIENTS

An empirical correlation developed by Shevts and Dyban (Ref. 24) gives estimated thermal contact coefficients for many common ferrous and non-ferrous metals in contact, including dissimilar metals.

$$hr/k = (\pi/4)\left[1 + 85(P/S)^{0.8}\right]$$

k = thermal conductivity of the gas phase

P = contact pressure

r = height of a micro-element of roughness plus the height of the wave for one surface

S = permissible rupture stress

A theoretical approximation of contact coefficients developed by French and Rohsenow (Ref. 25) can be found by using Fig. 17.1 and the following properties:

C = constriction number, P/M

P = contact pressure

M = Meyer hardness of the softer contact material

B = gap number = $0.335\ C^m$

$m = 0.315(\sqrt{A/l})^{0.137}$

A = interface area (one side)

ℓ = effective gap thickness, $3.56(\ell_1 + \ell_2)$ if $(\ell_1 + \ell_2) < 280$ µin. (smooth contacts), or $0.46\ (\ell_1 + \ell_2) > 280$ µin. (rough contacts)

ℓ_1, ℓ_2 = mean (or rms) depths of surface roughness

k_f = equivalent conductivity of interstitial fluid, for liquids use $k_f = k_o$ evaluated at $t = (t_1 + t_2)/2$, for gases:

$$k_f = \frac{k_o}{1 + 8\ \gamma\ (v/\bar{v})(a_1 + a_2 - a_1 - a_2)/Pr(\gamma + 1)\ell a_1 + a_2} + \frac{4\sigma\ell\varepsilon_1\varepsilon_2 t^3}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2} .$$

k_0 = fluid conductivity at zero contact pressure

Pr = Prandtl number

$\bar{v}$ = mean molecular velocity

γ = ratio of specific heats

ν = kinematic viscosity evaluated at t

a = accommodation coefficient

ε = surface emissivity

σ = Stefan-Boltzmann constant

K = conductivity number, $k_f(k_1 + k_2)/2k_1k_2$

k_1 = conductivity of first solid evaluated at
$t = \left[t_1 + (k_1t_1 + k_2t_2)/(k_1 + k_2)\right]/2$

k_2 = conductivity of second solid evaluated at
$t = \left[t_2 + (k_1t_1 + k_2t_2)/(k_1 + k_2)\right]/2$

Some thermal contact data assembled by P. J. Schneider from various sources (Ref. 19) is given in Fig. 17.2. Also, some additional data from Ref. 26 showing the effects of machining processes and surface matching is given in Fig. 17.3.

TABLE 17.1. Interface conditions for contact data given in Fig. 17.2.

Curve	Material pair	RMS surface finish (μin.)	Gap material	Mean contact temp (°F)
1	aluminum (2024-T3)	48-65	vacuum (10^{-4} mm Hg)	110
2	aluminum (2024-T3)	8-18	vacuum (10^{-4} mm Hg)	110
3	aluminum (2024-T3	6-0 (not flat)	vacuum (10^{-4} mm Hg)	110
4	aluminum (75S-T6)	120	air	200
5	aluminum (75S-T6)	65	air	200
6	aluminum (75S-T6)	10	air	200
7	aluminum (2024-T3)	6-8 (not flat)	lead foil (0.008 in.)	110
8	aluminum (75S-T6)	120	brass foil (0.001 in.)	200
9	stainless (304)	42-60	vacuum (10^{-4} mm Hg)	85
10	stainless (304)	10-15	vacuum (10^{-4} mm Hg)	85
11	stainless (416)	100	air	200
12	stainless (416)	100	brass foil (0.001 in.)	200
13	magnesium (AZ-31B)	50-60 (oxidized)	vacuum (10^{-4} mm Hg)	85
14	magnesium (AZ-31B)	8-16 (oxidized)	vacuum (10^{-4} mm Hg)	85
15	copper (OFHC)	7-9	vacuum (10^{-4} mm Hg)	115
16	stainless/aluminum	30-65	air	200
17	iron/aluminum	—	air	80
18	tungsten/graphite	—	air	270

TABLE 17.2. Interface conditions for contact data given in Fig. 17.3.

Curve	Material	Finish	Roughness RMS (μin.) Block 1 2	Fluid in Gap	Temp (°F)	Condition
a	Cold rolled steel	Shaped	1000-1000	Air	200	Parallel cuts, rusted
b	Cold rolled steel	Shaped	1000-1000	Air	200	Parallel cuts, clean
c	Cold rolled steel	Shaped	1000-1000	Air	200	Perpendicular cuts, clean
d	Cold rolled steel	Milled	125-125	Air	200	Parallel cuts, rusted
e	Cold rolled steel	Milled	125-125	Air	200	Parallel cuts, clean
f	Cold rolled steel	Shaped	63-63	Air	200	Perpendicular cuts, clean
g	Cold rolled steel	Shaped	63-63	Air	200	Parallel cuts, clean
h	Cold rolled steel	Lapped	4-4	Air	200	Clean
i	416 Stainless	Ground	100-100	Air	200	
j	416 Stainless	Ground	100-100	Air	400	
k	416 Stainless	Ground	30-30	Air	200	
l	416 Stainless	Ground	30-30	Air	400	
m	Stainless[6]	Milled	195-195	Air		Clean

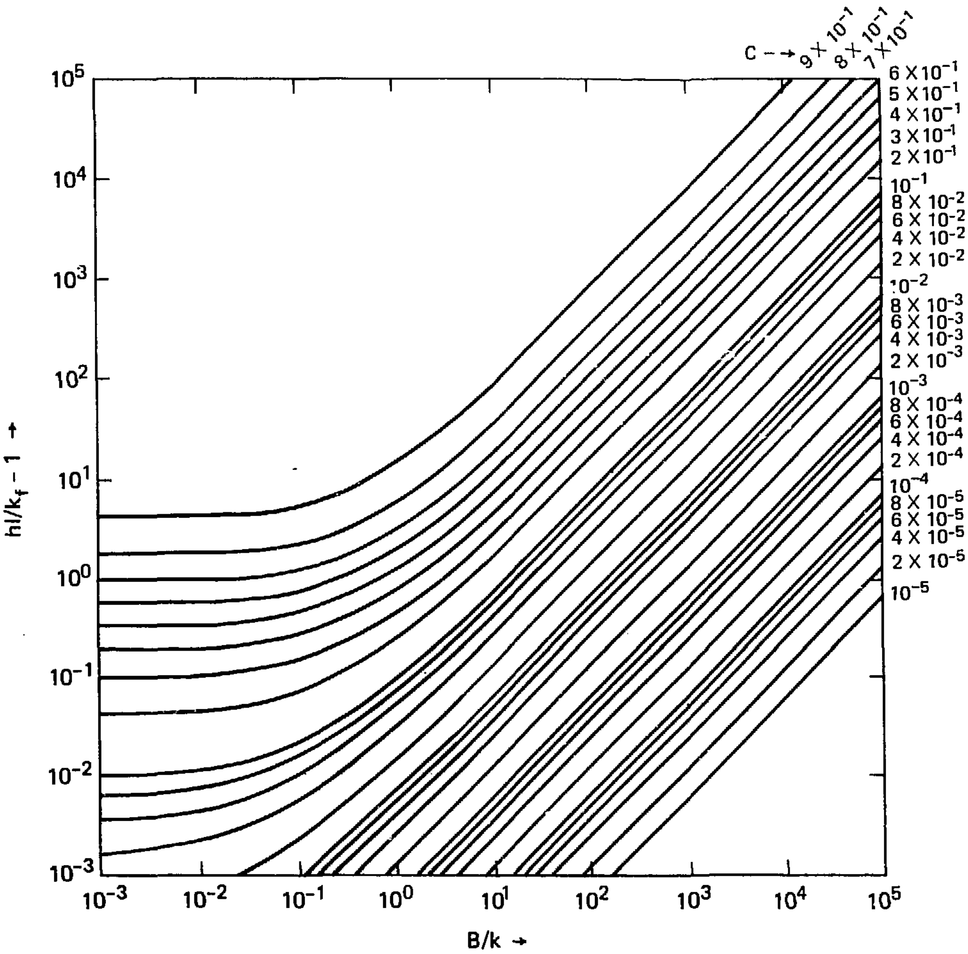

FIG. 17.1. Thermal contact coefficient from theory (source: Ref. 25).

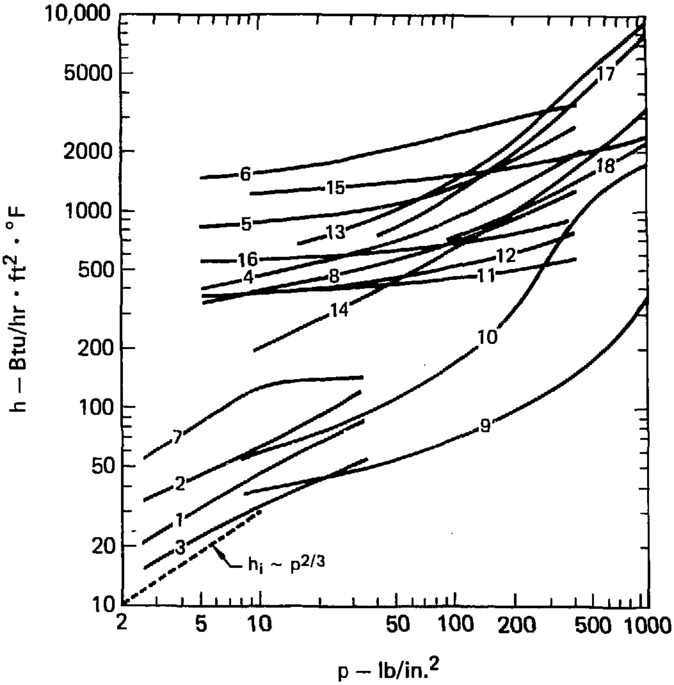

FIG. 17.2. Thermal contact coefficient data (source: Ref. 19).

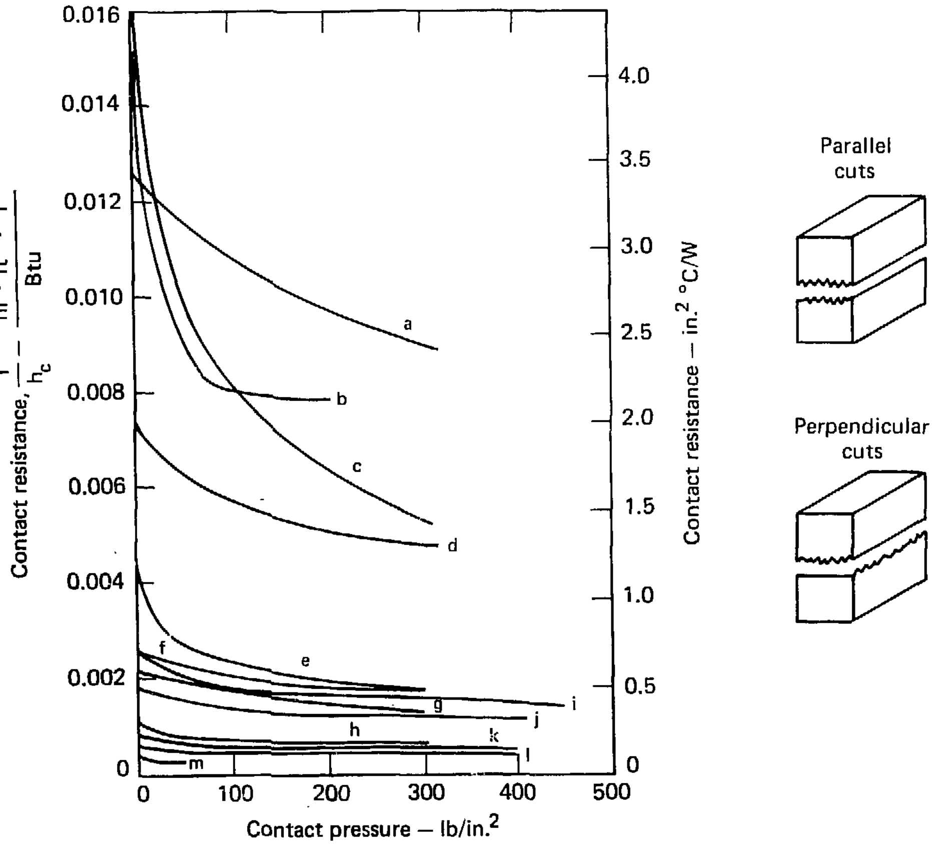

FIG. 17.3. Thermal contact coefficient for bare steel surfaces, (source: Ref. 26, section G502.5, p. 5).

Table 18.1 Thermal properties of selected metals (source: Refs. 35 to 39).

Metal	Properties at 300 K				Thermal conductivity (k), (W/m·K) x 10^{-2}									
	ρ, (kg/m^3) x 10^{-3}	c, (J/kg·K) x 10^{-3}	k, (W/m·K) x 10^{-2}	α, (m^2/s) x 10^4	10 K	20	50	100	200	400	600	800	1000	1200
Aluminum: Pure	2.70	0.88	2.4	1.01	235.	117.	12.3	3.0	2.4	2.4	2.3	2.2		
2024 (4.5% Cu, 1.5% Mg)	2.77	0.84	1.3	0.56	0.08	0.17	0.40	0.65	1.0	1.4				
5086 (4.1% Mg, 0.5% Mn)	2.65	0.90	1.3	0.55	0.08	0.17	0.40	0.63	0.85					
6063 (0.7% Mg, 0.4% Si)	2.70	0.92	2.1	0.85	0.86	1.7	2.8	2.1	2.0					
Beryllium: Pure	1.85	1.84	2.0	0.59	18.0	34.8	40.0	9.9	3.01	1.61	1.26	1.07	0.89	0.73
Chromium: Pure	7.19	4.56	0.90	0.03	3.9	6.0	3.2	1.6	1.1	0.87	0.81	0.71	0.65	0.62
Copper: Pure	8.96	0.38	4.0	1.17	196.	105.	12.2	4.8	4.1	3.9	3.8	3.7	3.6	3.4
Commercial bronze (10% Zn)	8.80	0.41	1.8	0.50	0.20	0.42	0.81	1.2						
Yellow brass (35% Zn)	8.47	0.41	1.2	0.35				0.59	0.94	1.3	1.5			
Naval brass (39% Zn, 0.8% Sn)	8.41	0.38	0.75	0.23						0.85	1.01			
German silver (22% Zn, 15% Ni)	8.55	0.40	0.25	0.07	0.028	0.07	0.15	0.17	0.20					
Cupronickel (20% Ni, 2% Zn)	8.94	0.38	1.5	0.44		0.11	0.20	0.38	0.92					
Constantan (40% Ni)	8.90	0.42	0.23	0.06		0.08	0.15	0.17	0.19					
Manganin (13% Mn, 4% Ni)	8.19	0.40	0.22	0.07	0.015	0.032	0.082	0.13	0.17	0.28				
Gold: Pure	19.32	0.13	3.2	1.27	28.2	15.0	4.2	3.5	3.3	3.1	3.0	2.9	2.8	2.6
Iron: Pure	7.87	0.45	0.80	0.23	7.1	10.0	3.7	1.3	0.94	0.69	0.55	0.43	0.33	0.28
Wrought iron (<0.5% C)	7.87	0.46	0.60	0.17						0.58	0.49	0.39		
Gray cast iron (3.0% C, 0.6% Si)	7.15	0.42	0.29	0.10						0.31	0.34	0.29	0.22	0.20
SAE 1095 steel (0.9% C, 0.3% Mn)	7.83	0.47	0.44	0.12		0.08	0.22	0.34	0.43					
AISI 4340 steel (1.9% Ni, 0.7% Cr)	7.84	0.46	0.36	0.10				0.26	0.31	0.38	0.38	0.34		
Nickel steel (9% Ni, 0.8% Mn)	7.96	0.46	0.29	0.08				0.18	0.24	0.33	0.34	0.33		
Invar (31% Ni, 5% Co)	8.00	0.46	0.14					0.08	0.12	0.16	0.19	0.22		
SAE 4130 steel (1% Cr, 0.5% Mn)	7.86	0.46	0.41	0.11		0.06	0.16	0.17	0.37					
AISI 304 stainless (19% Cr, 10% Ni)	7.90	0.38	0.15	0.05	0.008	0.021	0.06	0.09	0.13	0.17	0.20	0.22	0.25	0.28
AISI 316 stainless (17% Cr, 12% Ni)	8.00	0.46	0.14	0.40	0.008	0.021	0.06	0.09	0.12	0.15	0.18	0.21	0.23	0.25
Lead: Pure	11.34	0.13	0.35	0.24	1.78	0.59	0.44	0.40	0.37	0.34	0.31			
Solder (80% Pb, 20% Sn)	10.20		0.37											
Solder (50% Pb, 50% Sn)	8.89	0.18	0.46	0.29										
Lithium: Pure	0.53	3.97	0.77	0.37	6.1	7.2	2.3	1.1	0.88	0.72				
Magnesium: Pure	1.74	1.00	1.56	0.90	11.7	13.9	3.8	1.7	1.6	1.5	1.5	1.5		
(6% Al, 2% Si)	1.84	1.00	0.70	0.38				0.50	0.62	9.80				
(12% Al, 2% Si)	1.81	0.99	0.56	0.31				0.30	0.44	0.68				
Molybednum: Pure	10.22	0.25	1.4	0.55	1.45	2.77	3.0	1.8	1.4	[illegible]	1.3	1.2	1.1	1.0

Table 18.1 (Continued).

Metal	Properties at 300 K ρ, $(kg/m^3) \times 10^{-3}$	c, $(J/kg \cdot K) \times 10^{-3}$	k, $(W/m \cdot K) \times 10^{-2}$	α, $(m^2/s) \times 10^4$	Thermal conductivity (k), $(W/m \cdot K) \times 10^{-2}$ 10 K	20	50	100	200	400	600	800	1000	1200
Nickel: Pure	8.90	0.45	0.91	0.23	6.0	8.6	3.4	1.6	1.1	0.80	0.66	0.67	0.72	0.76
Duranickel (4.5% Al)	8.26	0.54	0.19	0.04										
Monel (30% Cu, 1.4% Fe)	8.84	0.43	0.23	0.05	0.08	0.14	0.17	0.20						
Inconel X-750 (16% Cr, 8% Fe)	8.51	0.46	0.12	0.03	0.010	0.025	0.065	0.085	0.10	0.13	0.17	0.21	0.25	0.29
Nichrome (20% Cr)	8.40	0.45	0.13	0.03						0.14	0.17	0.21	0.25	0.28
Nichrome V (24% Fe, 16% Cr)	8.25	0.44	0.10	0.03						0.12	0.15	0.19	0.22	0.26
Niobium: Pure	8.57	0.27	0.54	0.23	2.2	2.3	0.76	0.55	0.53	0.55	0.58	0.61	0.64	0.68
Platinum: Pure	21.45	0.13	0.71	0.25	12.3	4.9	1.1	0.78	0.72	0.72	0.73	0.76	0.79	0.83
Plutonium: Pure	19.84	0.15	0.67	0.23				0.03	0.48	0.90				
Rhenium: Pure	21.04	0.14	0.48	0.16	14.0	8.4	0.96	0.60	0.51	0.46	0.44	0.44	0.45	0.46
Silver: Pure	10.49	0.24	4.3	1.71	168.	51.	7.0	4.5	4.3	4.2	4.1	3.9	3.7	3.6
Sterling (7.5% Cu)	10.38		3.5											
Eutectic (28% Cu)	10.06		3.3											
Tin: Pure	7.30	0.22	0.67	0.42	19.3	3.2	1.2	0.85	0.73	0.62				
Solder (40% Pb)	8.44	0.19	0.51	0.32	0.43	0.56	0.52	0.54	0.52					
Tantalum: Pure	16.6	0.14	0.58	0.25	1.08	1.5	0.72	0.59	0.58	0.58	0.59	0.59	0.60	0.61
Tungsten: Pure	19.3	0.13	1.8	0.72	82.	32.	4.7	2.4	2.0	1.6	1.4	1.3	1.2	1.1
Titanium: Pure	4.51	0.50	0.22	0.10	0.14	0.28	0.40	0.31	0.25	0.20	0.19	0.20	0.21	0.22
A-110 AT (5% Al, 2.5% Sn)	4.46	0.53	0.075	0.03						0.088	0.11	0.14		
Uranium: Pure	19.07	0.12	0.28	0.12	0.098	0.16	0.19	0.22	0.25	0.30	0.34	0.39	0.44	0.49
Zinc: Pure	7.13	0.39	1.2	0.43	43.	10.7	2.1	1.2	1.2	1.2	1.1			

TABLE 18.2. Thermal properties of miscellaneous solids (source: Refs. 35 to 38).

Material	ρ, $(kg/m^3) \times 10^{-3}$	c, $(J/kg{\cdot}K) \times 10^{-3}$	k, $W/m{\cdot}K$	α, $(m^2/s) \times 10^4$	10 K	20	50	100	200	400	600	800	1000	1200
	Properties at 300 K				Thermal conductivity (k), $(W/m{\cdot}K) \times 10$									
Ceramics: (Polycrystalline, 99.5% purity, 98% solid)														
Al_2O_3	3.84	0.79	36.0	0.12				1330.	550.	260.	160.	100.	80.	
BeO	2.97	1.00	272.0	0.92					4240.	1960.	1110.	700.	470.	
MgO	3.21	0.92	48.0	0.16					750.	350.	220.	140.	90.	
SiO_2 (high purity fused)	2.21	0.75	1.4	0.008	1.3	1.5	3.4	6.9	11.	15.	18.	22.	29.	
ThO_2	9.58	0.23	13.0	0.06					180.	100.	70.	50.	40.	
TiO_2	3.91	0.71	8.0	0.03					100.	70.	50.	40.	30.	
ZrO_2	5.28	0.46	1.6	0.007						17.	18.	19.	19.	
Glasses:														
Borosilicate (Pyrex)	2.21	0.71	1.1	0.007				6.	9.	12.	15.	19.		
Soda lime (75% SiO_2)	2.52	0.66	0.9	0.005					8.	11.				
Vitreous silica (100% SiO_2)	2.21	1.00	1.4	0.006				8.	11.	15.				
Zinc crown (65% SiO_2)	2.60	0.67	1.1	0.006					9.	13.				
Insulations: (For high temps.)														
Alumina, fused (90% Al_2O_3)	2.70	0.83	5.2	0.02						56.	60.	66.	73.	
Asbestos paper (laminated)	0.35	0.84	0.048	0.002						0.73				
Diatomaceous earth silica (powder)	0.30		0.064							0.79	1.0	1.3		
Firebrick (58% SiO_2, 37% Al_2O_3)	0.81	0.92	0.34	0.005							5.2	6.6	8.4	10.
Magnesite (85% MgO)	0.19	1.13	0.050	0.002						0.61				
Micro quartz fiber (blanket)	0.05	0.84	0.036	0.009						0.48	0.89	1.3	1.7	
Rockwool (loose)	0.16		0.046							0.66	1.0	1.3		
Zirconia (grain)	1.81	0.50	0.18	0.002						1.9	2.1	2.2	2.8	3.5
Insulations: (For low temps.)														
Expanded glass (Foamglas)	0.17		0.062					0.31	0.51					
Fiberglass (board)	0.18		0.040					0.12	0.24					
Fiberglass (blanket)	0.05		0.038											
Silica aerogel (powder)	0.08		0.022						0.17					
Polystyrene foam (1atmabs)	0.08		0.038			0.05	0.07	0.12	0.24					
Polystyrene foam (10^{-3} atmabs)	0.08		0.017						0.12					

TABLE 18.2. (Continued.)

Material	Properties at 300 K: ρ, (kg/m^3) × 10^{-3}	c, (J/kg·K) × 10^{-3}	k, W/m·K	α, (m^2/s) × 10^4	Thermal conductivity (k), (W/m·K) × 10^{-2}: 10 K	20	50	100	200	400	600	800	1000	1200
Plastics:														
Acrylic, PMMA (Plexiglas)	1.18	1.46	0.16	0.001	0.6	0.7			1.2					
Nylon 6	1.16	1.59	0.25	0.001	0.3	0.9								
Polyvinyl chloride (rigid)	1.40	1.00	0.15	0.001				1.5	1.6	1.5				
Teflon, PTFE	2.18	1.05	0.40	0.002	1.0	1.5	2.0	2.5		4.5	5.5			
Polyethylene, high density	0.95	2.30	0.50	0.002										
Rocks:														
Granite	2.60	0.79	3.4	0.017						30.	24.			
Marble	2.50	0.88	1.8	0.008						17.	11.			
Sandstone	2.20	0.92	5.3	0.026						44.	30.			
Shale	2.60	0.71	1.8	0.010						15.	14.			
Woods: (across grain, oven dry)														
Balsa	0.16		0.059											
Douglas fir	0.46	2.72	0.11	0.001										
Oak, red	0.67	2.38	0.17	0.001										
Pine, white	0.40	2.80	0.10	0.001										
Redwood	0.42	2.90	0.11	0.001										
Miscellaneous:														
Carbon black (powder)	0.19	0.84	0.021	0.001										
Carbon (petroleum coke)	2.10	0.84	1.9	0.011						21.	25.	28.	30.	
Graphite (ATJ)														
(∥ to grains)	1.73	0.84	129.0	0.89	5.	25.	170.	580.	1200.	1180.	950.	770.	640.	
(⊥ to grains)	1.73	0.84	98.0	0.67	4.	19.	130.	420.	860.	900.	730.	590.	490.	
Graphite (pyrolytic)														
(∥ to grains)	2.20	0.84	2000.0	10.8	810.	4200.	23000.	50000.	32000.	14600.	9300.	6800.	5300.	
(⊥ to grains)	2.20	0.84	9.0	0.05	270.	1100.	1000.	390.	150.	70.	40.	30.	20.	
Concrete	2.28	0.67	1.8	0.012										
Mica	1.96	0.88	0.43	0.002										
Rubber (hard)	1.19	1.88	0.16	0.001										
Gypsum board	0.82		0.11											
Sand (dry)	1.52	0.80	0.33	0.003										

TABLE 18.3. Thermal properties of some metals at 10 K above their melting point (source: Refs. 35 and 38).

Metal	mp, K	k, W/m·k	ρ, kg/m^3	c, kJ/kg·k	γ kJ/kg
Aluminum	933.2	91	2390	1.09	395.4
Copper	1356.	167	7940	0.49	205.
Iron	1810.	41	7020	0.82	281.6
Lead	600.6	16	10700	0.21	24.7
Lithium	453.7	43	520	4.25	663.2
Mercury	234.3	7	13650	1.34	11.3
Potassium	336.8	54	820	0.84	61.5
Sodium	371.0	87	930	1.34	114.6
NaK (eutectic)	262.	13	850	1.00	--
Tin	505.1	31	6980	0.23	60.2
Zinc	692.7	50	6640	0.50	102.1

TABLE 18.4. Room temperature total emissivities (source: Ref. 37).

Silver (highly polished)	0.02	Brass (polished)	0.60
Platinum (highly polished)	0.05	Oxidized copper	0.60
Zinc (highly polished)	0.05	Oxidized steel	0.70
Aluminum (highly polished)	0.08	Bronze paint	0.80
Monel metal (polished)	0.09	Black gloss paint	0.90
Nickel (polished)	0.12	White lacquer	0.95
Copper (polished)	0.15	White vitreous enamel	0.95
Stellite (polished)	0.18	Asbestos paper	0.95
Cast iron (polished)	0.25	Green paint	0.95
Monel metal (oxidized)	0.43	Gray paint	0.95
Aluminum paint	0.55	Lamp black	0.95

TABLE 18.5. Total emissivities of miscellaneous materials (source: Ref. 37).

Material	Temp, °C	Emis-sivity	Material	Temp, °C	Emis-sivity
Alloys			Iron, rusted	25	0.65
20Ni-25Cr-55Fe, oxidized	200	0.90	wrought, dull	25	0.94
	500	0.97	oxidized	350	0.94
60Ni-12Cr-28Fe, oxidized	270	0.89	Lead, unoxidized	100	0.05
	560	0.82	oxidized	200	0.63
80Ni-20Cr, oxidized	100	0.87	Mercury, unoxidized	25	0.10
	600	0.87		100	0.12
	1300	0.89	Molybdenum, unoxidized	1000	0.13
Aluminum, unoxidized	25	0.022		1500	0.19
	100	0.028		2000	0.24
	500	0.60	Monel metal, oxidized	200	0.43
oxidized	200	0.11		600	0.43
	600	0.19	Nickel, unoxidized	25	0.045
Bismuth, unoxidized	25	0.048		100	0.06
	100	0.061		500	0.12
Brass, oxidized	200	0.61		1000	0.19
	600	0.59	oxidized	200	0.37
unoxidized	25	0.035		1200	0.85
	100	0.035	Platinum, unoxidized	25	0.037
Carbon, unoxidized	25	0.81		100	0.047
	100	0.81		500	0.096
	500	0.81		1000	0.152
Chromium, unoxidized	100	0.08		1500	0.191
Cobalt, unoxidized	500	0.13	Silica brick	1000	0.80
	1000	0.23		1100	0.85
Columbium, unoxidized	1500	0.19	Silver, unoxidized	100	0.02
	2000	0.24		500	0.035
Copper, unoxidized	100	0.02	Steel, unoxidized	100	0.08
	liquid	0.15		liquid	0.28
oxidized	200	0.6	oxidized	25	0.80
	1000	0.6		200	0.79
calorized	100	0.26		600	0.79
	500	0.26	Steel plate, rough	40	0.94
calorized, oxidized	200	0.18		400	0.97
	600	0.19	calorized, oxidized	200	0.52
Fire brick	1000	0.75		600	0.57
Gold, unoxidized	100	0.02	Tantalum, unoxidized	1500	0.21
	500	0.03		2000	0.26
Gold enamel	100	0.37	Tin, unoxidized	25	0.043
Iron, unoxidized	100	0.05		100	0.05
oxidized	100	0.74	Tungsten, unoxidized	25	0.024
	500	0.84		100	0.032
	1200	0.89		500	0.071
cast, unoxidized	100	0.21		1000	0.15
	liquid	0.29		1500	0.23
cast, oxidized	200	0.64		2000	0.28
	600	0.78	Zinc, unoxidized	300	0.05
cast, strongly oxidized	40	0.95			
	250	0.95			

TABLE 18.6. Electrical resistivity of some common metals (source: Ref. 37).

Metal	Resistivity, μΩ•cm 20°C	Temp. coefficient 20°C	Specific gravity, g/cm³	Melting point, °C
Advance. See constantan				
Aluminum	2.824	0.0039	2.70	659
Antimony	41.7	.0036	6.6	630
Arsenic	33.3	.0042	5.73	--
Bismuth	120	.004	9.8	271
Brass	7	.002	8.6	900
Cadmium	7.6	.0038	8.6	321
Calido. See nichrome				
Climax	87	.0007	8.1	1250
Cobalt	9.8	.0033	8.71	1480
Constantan	49	.00001	8.9	1190
Copper: annealed	1.7241	.00393	8.89	1083
hard-drawn	1.771	.00382	8.89	--
Eureka. See constantan				
Excello	92	.00016	8.9	1500
Gas carbon	5000	-.0005	--	3500
German silver, 18% Ni	33	.0004	8.4	1100
Gold	2.44	.0034	19.3	1063
Ideal. See constantan				
Iron, 99.98% pure	10	.005	7.8	1530
Lead	22	.0039	11.4	327
Magnesium	4.6	.004	1.74	651
Manganin	44	.00001	8.4	910
Mercury	95.783	.00089	13.546	-38.9
Molybdenum, drawn	5.7	.004	9.0	2500
Monel metal	42	.0020	8.9	1300
Nichrome[a]	100	.0004	8.2	1500
Nickel	7.8	.006	8.9	1452
Palladium	11	.0033	12.2	1550
Phosphor bronze	7.8	.0018	8.9	750
Platinum	10	.003	21.4	1755
Silver	1.59	.0038	10.5	960
Steel, E. B. B.	10.4	.005	7.7	1510
Steel, B. B.	11.9	.004	7.7	1510
Steel, Siemens-Martin	18	.003	7.7	1510
Steel, manganese	70	.001	7.5	1260
Tantalum	15.5	.0031	16.6	2850
Therlo[a]	47	.00001	8.2	--
Tin	11.5	.0042	7.3	232
Tungsten, drawn	5.6	.0045	19	3400
Zinc	5.8	.0037	7.1	419

[a]Trade mark.

TABLE 18.7. Thermal conductivity integrals of miscellaneous materials (source: Ref. 76).

$$Q = \frac{A}{L}(k_I)\ ,\ k_I = \int_{4K}^{t} k\,dt\ ,\ W/m \times 10^{-2}$$

Temp, K	Coppers			Aluminum		Steel			Glass and plastics			Misc. Alloys		
	O.F.H.C.	Hi-purity annealed	Brass (Pb)	2024-T4	6063-T5	SAE 1020	Stainless	Inconel (annealed)	glass	nylon	perspex	Constantan	Ag solder	Soft solder
6	6.1	166	0.053	0.080	0.850	0.088	0.0063	0.0133	2.11×10^{-3}	0.32×10^{-3}	1.18×10^{-3}	0.024	0.059	0.425
8	14.5	382	0.129	0.197	2.05	0.231	0.0159	0.0348	4.43	0.80	2.38	0.066	0.148	1.05
10	25.2	636	0.229	0.347	3.60	0.431	0.0293	0.0653	6.81	1.48	3.59	0.128	0.268	1.83
15	61.4	1270	0.594	0.872	9.00	1.17	0.0816	0.182	13.1	4.10	6.69	0.375	0.688	4.18
20	110	1790	1.12	1.60	16.5	2.22	0.163	0.356	20.0	8.23	10.1	0.753	1.25	6.86
25	168	2160	1.81	2.51	25.8	3.52	0.277	0.592	27.9	13.9	14.4	1.24	1.92	9.66
30	228	2410	2.65	3.61	36.5	5.02	0.424	0.882	36.8	20.8	19.6	1.81	2.67	12.5
35	285	2580	3.63	4.91	48.8	6.74	0.607	1.22	47.1	29.0	25.9	2.44	3.52	15.3
40	338	2700	4.76	6.41	62.0	8.67	0.824	1.60	58.6	38.5	33.0	3.12	4.47	18.1
50	426	2860	7.36	9.91	89.5	13.1	1.35	2.47	84.6	60.4	49.5	4.57	6.62	23.4
60	496	2960	10.4	14.1	117	18.1	1.98	3.45	115	85.9	68.3	6.12	9.12	28.5
70	554	3030	13.9	18.9	143	23.6	2.70	4.52	151	113	88.5	7.75	12.0	33.6
76	586	3070	16.2	22.0	158	27.1	3.17	5.19	175	131	101.	8.75	13.9	36.7
80	606	3090	17.7	24.2	167	29.5	3.49	5.66	194	142	110.	9.43	15.2	38.8
90	654	3140	22.0	30.1	190	35.5	4.36	6.85	240	173	132.	11.1	18.7	44.1
100	700	3180	26.5	36.3	211	41.7	5.28	8.06	292	204	155.	12.8	22.6	49.4
120	788	3270	36.5	50.1	253	54.5	7.26	10.6	408	269	200.	16.2	31.1	60.3
140	874	3360	47.8	65.4	293	67.5	9.39	13.1	542	336	247.	19.7	40.6	71.4
160	956	3440	60.3	82.1	333	80.5	11.7	15.7	694	405	294.	23.2	51.0	82.6
180	1040	3520	73.8	100	373	93.5	14.1	18.3	858	475	342.	26.9	62.2	93.8
200	1120	3600	88.3	119	413	107	16.6	21.0	1030	545	390.	30.6	74.0	105
250	1320	3800	128	171	513	139	23.4	28.0	1500	720	510.	40.6	105	133
300	1520	4000	172	229	613	172	30.6	35.4	1990	895	630.	51.6	138	162

Table 18.8 Specific heat of miscellaneous solids (source: Refs. 35 to 38).

Material	Specific heat (c), J/kg·K									
	10	20	50	100	200	400	600	800	1000	1200
Aluminum: Pure	1.4	8.9	142	481	790	161	1026	1092	1054	1096
: 2024			418	515	737	920	1029	1318		
Beryllium	0.389	1.61	19.2	199	1113	2134	1046	2804	3022	3223
Copper: Pure	0.86	7.7	99	254	360	423	427	434	439	443
: Brass (30% Zn)	0.65	10.5	120	255	360	401	428	448	469	
Gold	2.2	15.9	72	108	122	134	135	138	140	142
Iron: Pure	1.24	4.5	55	216	389	485	582	677	774	665
: AISI 316 stainless	0.80	13.1	110	265	420	510	560	580	590	600
: SAE 1010 steel	1.88	6.69	75	251	418	628	669	690	711	724
Lead	13.7	53	103	118	121	141	121	108	103	96
Nickel	1.62	5.8	68	232	385	502	589	539	540	540
Platinum	1.12	7.4	55	100	121	133	142	146	150	155
Silver	1.8	15.5	108	187	226	243	251	259	267	280
Tin	4.2	40	130	189	200	242				
Tungsten	0.234	1.89	33	88	122	133	138	142	146	150
Zinc	2.5	26	171	293	352	401	472	489	460	434
Carbon: Graphite	1.38	71	41.8	138	418	1004	1339	1590	1716	1799
Ceramic: Al_2O_3	0.08	0.84	14.2	125	502	920	1088	1172	1213	1255
Glasses: Borosilicate										
(pyrex)	4.2	25.9	125	272	544	837	1088			
Quartz	16.7	16.7	96	255	502	879	1046	1172	1213	1255
Plastics: Acrylic, PMMA										
(plexiglas)	22	79	250	540	360	1640				
NEMA G10	18	89	240	360	630	1110	1600			
Teflon, PTFE	20	75	200	380	700	1300				

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