Fiber Generator Controlled Parameters

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Definitions

Fiber Count

The is is the integer number m of fibers in the image.

Mean Fiber Length

The path of a fiber is defined by a tuple $P = (\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_n)$. The fiber length l is given by

$$l = \sum_{i=1}^{n-1} |\vec{p}_{i+1} - \vec{p}_i|.$$

If we assume that all segments have equal length l_s, then the fiber length is simply

$$l = (n-1) \cdot l_s.$$

Mean Fiber Width

The varying width is given by a tuple W with $n-1$ elements. w_j gives the width of the fiber between points $\vec{p}_j$ and $\vec{p}_{j+1}$. Because all segments have the same length, the average for a fiber is

$$\bar{w} = \frac{1}{n-1} \sum_{j=1}^{n-1} w_j$$

The image-wide average is the average over fibers weighted by the length of each fiber. Let there be m fibers, each with width tuple W_i and number of segments n_i.

$$\bar{w}_{\text{tot}} = \frac{1}{\sum_{i=1}^{m} n_i} \sum_{i=1}^{m} \sum_{j=1}^{n_i-1} w_{i,j}$$

This is equivalent to taking the fiber-independent mean of segment widths.

Mean Fiber Angle

The angle of the i^{th} fiber is simply the angle between its endpoints, $\vec{p}_{i,1}$ and $\vec{p}_{i,n}$.

$$\theta_i = \cos^{-1}\left(\frac{(\vec{p}_{i,n} - \vec{p}_{i,1})_x}{|\vec{p}_{i,n} - \vec{p}_{i,1}|}\right)$$

A normalized "mean vector" is found by multiplying all fiber angles by 2 and taking the normalized, length-weighted mean of the resultant direction vectors.

$$\bar{\theta} = \frac{1}{2} \cdot \arg\left(\frac{1}{\sum_{i=1}^{m} n_i} \sum_{i=1}^{m} n_i \cdot e^{2i \cdot \theta_i}\right)$$

Overall Fiber Alignment

The alignment A is the magnitude of the "mean vector" from the previous part.

$$A = \text{mag}\left(\frac{1}{\sum_{i=1}^{m} n_i} \sum_{i=1}^{m} n_i \cdot e^{2i \cdot \theta_i}\right)$$

Mean Fiber Straightness

The straightness of an individual fiber is the ratio of its length to the distance between its endpoints. Let l_i be the length of the i^{th} fiber and assume it contains n points. Then the straightness s_i is

$$s_i = \frac{|\vec{p}_n - \vec{p}_1|}{l_i}.$$

The image-wide mean is weighted by fiber length.

$$\bar{s} = \frac{1}{\sum_{i=1}^{m} n_i} \sum_{i=1}^{m} n_i \cdot s_i$$

Parameter Generation

Output

For each generated fiber, the following details will be recorded as output:

- The tuple $P = (\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_n)$ of points which define the path of the fiber.
- The tuple $W = (\vec{w}_i, \vec{w}_2, \ldots, \vec{w}_{n-1})$ of widths.

Other details (local density, individual fiber angle, etc.) from these tuples.